

Normalized Discounted Cumulative Gain – Multilabel Ranking Metrics | ML

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Discounted Cumulative Gain

Discounted Cumulative Gain (DCG) is the metric of measuring ranking quality. It is mostly used in information retrieval problems such as measuring the effectiveness of the search engine algorithm by ranking the articles it displays according to their relevance in terms of the search keyword.

Let's consider that a search engine that outputs 5 documents named (D_1, D_2, D_3, D_4, D_5) are output in that order. We need to define the relevance scale (0-3) where:

- **0** : not relevant
- **1-2** : somewhat relevant
- **3** : completely relevant


Suppose these documents have relevance scores:

- D_1 : 3
- D_2 : 2
- D_3 : 0
- D_4 : 0
- D_5 : 1

The **Cumulative Gain** is the sum of these relevance scores and can be calculated as:

$$CG = \sum_{i=1}^5 (rel)_i = 3 + 2 + 0 + 0 + 1 = 6$$

The discounted cumulative gain can be calculated by the formula:


$$CG = \sum_{i=1}^5 \frac{rel_i}{\log_2(i+1)}$$

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$$DCG_5 = \frac{3}{\log_2(2)} + \frac{2}{\log_2(3)} + \frac{0}{\log_2(4)} + \frac{0}{\log_2(5)} + \frac{1}{\log_2(6)}$$

$$DCG_5 = 3 + \frac{2}{1.585} + 0 + 0 + \frac{1}{2.585}$$

$$DCG_5 = 3 + 1.26 + 0.3868$$

$$DCG_5 \simeq 4.67$$

Now we need to arrange these articles in descending order by rankings and calculate DCG to get the Ideal Discounted Cumulative Gain (IDCG) ranking.

$$IDCG_5 = \frac{3}{\log_2(2)} + \frac{2}{\log_2(3)} + \frac{1}{\log_2(4)} + \frac{0}{\log_2(5)} + \frac{0}{\log_2(6)}$$

$$IDCG_5 = 3 + \frac{2}{1.585} + \frac{1}{2} + 0 + 0$$

$$IDCG_5 = 3 + 1.26 + 0.5$$

$$IDCG_5 = 4.76$$

Now, we calculate our **Normalized DCG** using the following formula :

$$nDCG = \frac{DCG_5}{IDCG_5}$$

$$nDCG = \frac{4.67}{4.76}$$

$$nDCG \simeq 0.98$$

Code : Python program for *Normalized Discounted Cumulative Gain*

Python3

```
# import required package
from sklearn.metrics import ndcg_score, dcg_score
import numpy as np

# Relevance scores in Ideal order
true_relevance = np.asarray([[3, 2, 1, 0, 0]])

# Relevance scores in output order
relevance_score = np.asarray([[3, 2, 0, 0, 1]])

# DCG score
dcg = dcg_score(true_relevance, relevance_score)
print("DCG score : ", dcg)

# IDCG score
idcg = dcg_score(true_relevance, true_relevance)
print("IDCG score : ", idcg)

# Normalized DCG score
ndcg = dcg / idcg
print("nDCG score : ", ndcg)

# or we can use the scikit-learn ndcg_score package
```

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Output:

DCG score : 4.670624189796882

IDCG score : 4.761859507142915

nDCG score : 0.980840401274087

nDCG score (from function) : 0.980840401274087

Limitations of Normalized Discounted Cumulative Gain (NDCG):

- NDCG metrics does not penalize the bad documents outputs. For Example: $[3]$ and $[3, 0, 0]$ has same NDCG but in second out there are two irrelevant documents.
- Because no specific standard defined for the number of output documents. DCG does not seem to deal with missing any relevant document in the output. For example, two outputs $[3, 3, 3]$ and $[3, 3, 3, 3]$ of similar input are considered equally good. For output-1 the DCG_3 is calculated but for output-2 the DCG_4 is calculated.

References:

- [DCG Wikipedia article](#)
- Jarvelin, K., & Kekalainen, J. (2002). Cumulated gain-based evaluation of IR techniques. ACM Transactions on Information Systems (TOIS), 20(4), 422-446.

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