Annals of Human Biology, March–April 2008; 35(2): 170–184

ORIGINAL ARTICLE

Estimating body surface area from mass and height: Theory and the formula of Du Bois and Du Bois

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(*Received 3 June 2007; revised 20 November 2007; accepted 9 January 2008*)

Abstract
Background: Body surface areas are usually estimated by means of a formula due in its general form to Du Bois and Du Bois (1916), i.e. area $= \mathrm{C} \times \mathrm{mass}^{a} \times \mathrm{height}^{b}$, where *C*, *a* and *b* are empirical constants. Its physical basis is unknown.
Aim: The present study aimed to explain this formula, correct some errors in the associated literature and provide a clear basis for future developments.
Subjects and methods: Use is made of published data, but arguments are largely based on mathematics and modelling.
Results: A more fundamental formula is as follows: area $= \alpha(\mathrm{mass} \times \mathrm{height})^{1/2} + \beta(\mathrm{mass/height})$, where α and β are constants. For realistic values of mass and height the two equations are numerically equivalent. For individuals, β cannot be negative and b cannot exceed a, but, as regression parameters, these conditions may not be satisfied. This could be due to systematic or statistical relationships between individual values of α or β and the ratio $\mathrm{height}^3/\mathrm{mass}$. Values of α, β, C, a and b are calculated for some published data.
Conclusions: The original type of formula suffices for practical purposes, but the new one is better in analytical contexts when other terms, e.g. for body shape, are to be incorporated.

Keywords: *Body surface area, Du Bois and Du Bois, anthropometry*

Introduction

Body surface area has been much used in physiology and clinical medicine as a measure of body size for indexing or normalizing variables such as glomerular filtration rate, cardiac output and parameters of pharmacokinetics (e.g. Gehan and George 1970; Jones et al. 1994; Shuter and Aslani 2000; Yu et al. 2003; Reading and Freeman 2005; Verbraecken et al 2006). The concern here is with the theoretical basis of the

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ISSN 0301–4460 print/ISSN 1464–5033 online © 2008 Informa UK Ltd.
DOI: 10.1080/03014460801908439

Table I. Parameters in Equation 1, $A = CW^aH^b$, where A is body surface area (m^2), W is body mass (kg) and H is height (cm).

Authors	n	C	a	b	$(3a + b)$
Du Bois and Du Bois (1916)	9	0.0072	0.425	0.725	2.000
Shuter and Aslani (2000)	42	0.0095	0.441	0.655	1.978
Boyd (1935)	197	0.0179	0.484	0.5	1.95
Brody (1945)	133	0.0240	0.53	0.4	1.99
Gehan and George (1970) (<5 years)	229	0.0267	0.539	0.382	2.000
Gehan and George (1970) (≥5 < 20 years)	42	0.0305	0.544	0.351	1.983
Gehan and George (1970) (≥20 years)	130	0.0155	0.463	0.545	1.935
Haycock et al. (1978)	81	0.0243	0.538	0.396	2.010
Livingston and Lee (2001)	460	0.0495	0.605	0.206	2.020
Tikuisis et al. (2001) men	394	0.0128	0.44	0.60	1.92
Tikuisis et al. (2001) women	246	0.0147	0.47	0.55	1.96

n is the number of individuals. The sum (3a + b) should be exactly 2 for dimensional correctness.

long-established formula of Du Bois and Du Bois (1916), and others like it, that relate surface area (A) to body mass (W) and height (H). In generalized form this is as follows:

$$A = CW^aH^b, \tag{1}$$

where C, a and b are empirical constants. Although Du Bois and Du Bois described how they arrived at their formula, its physical basis remains unexplained.

This paper explains the basis, corrects some errors in the literature, clarifies a few other points and so puts the subject on a firmer footing. By way of illustration, formulae are fitted to some published data and values for their respective constants are determined.

For Equation 1 to be dimensionally consistent, and on the assumption of near-constant body density so that mass is proportional to volume,

$$3a + b = 2 \tag{2}$$

Du Bois and Du Bois (1916) presented Equation 2 with '×' in place of '+' and Shuter and Aslani (2000), pointing this out, inadvertently replaced a and b with their reciprocals. However, both pairs of authors applied Equation 2 correctly. Indeed, they emphasized the importance of dimensional analysis in this context, as have others (Boyd 1935; Gehan and George 1970; Reading and Freeman 2005).

Values of C, a and b obtained by various authors are shown in Table I (with some parameters rounded to fewer significant figures). In most cases Equation 2 has not been enforced in the regression process, but departures of $(3a + b)$ from 2 are small. The parameter C is given in terms of m^2, kg and cm as are now commonly used in this context. (For cm^2, kg and cm, multiply by 10^4.) C has units of density^{-a}. These can be awkward and most authors do not give them. Numerical values of C depend both on the units and on the exponents (see below), as illustrated by Shuter and Aslani (2000).

Du Bois and Du Bois (1916) obtained their parameters using only nine of their subjects, choosing the exponents to conform to Equation 2. Their full data set for

42 individuals gives somewhat different values (Shuter and Aslani 2000). The variety of values for each parameter shown in Table I raises the question of whether they might all be approximations to a single, perhaps ideal, set that is adequate for practical purposes. For ease of calculation, Mosteller (1987) proposed the use of $a = b = 0.5$, so that:

$$A = CW^{0.5}H^{0.5}. \tag{3}$$

With the units again being m^2, kg and cm, Mosteller took C as $\surd(1/3600)$ $(=1/60 = 0.0167)$. He based the equation on results of Gehan and George (1970) for adolescent and adult subjects. With children, Lam and Leung (1988) found high correlations between estimates obtained with the equations of Mosteller (1987) and of Du Bois and Du Bois (1916), but did not comment on how well those estimates agreed. Verbraecken et al. (2006) have compared the results of applying these and other formulae to adults. Yu et al. (2003), also taking both a and b to be 0.5, found C to be 0.0159 for Chinese adults. Wang and Hihara (2004) derived Equation 3 theoretically, with $C = \surd[9\pi/10\,000] = 0.0168$ when expressed in terms of the same units. However, their derivation, based on a cylindrical approximation to the body with its surface area minimized at a fixed volume (for no stated reason) is unclear.

Reading and Freeman (2005) have given a derivation for Equation 3, with C again equal to 1/60 when the units are m^2, kg and cm. Because the human body is nowhere planar, they chose to model it as a prolate spheroid (ellipsoid of revolution) of constant density, but inadvertently used for its surface area a formula applying only to a degenerate ellipsoid with one axis reduced to zero (i.e. a flat elliptical disc). (The proper formula for the surface area of an ellipsoid is discussed below.) These assumptions, coupled with the standard formula for the volume, lead to the conclusion that the surface area is proportional to $W^{0.5}H^{0.5}$. The value of C obtained by Reading and Freeman is better justified, for it is chosen to give a good fit to data they obtained as follows. From masses and heights for 504 adults, they first calculated respective surface areas according to the formulae of Du Bois and Du Bois (1916), Boyd (1935), Gehan and George (1970) and Haycock et al. (1978) and then averaged them for each individual. Reading and Freeman tested their ellipsoid model against their formula on five subjects for which they had measured mass, height and volume. The surface areas based on the model, height and volume are very close to those calculated as $1/60(W^{0.5}H^{0.5})$, i.e. using m^2, kg and cm. In fact, as is mathematically inevitable, the discrepancies are almost exactly accounted for by variations in density, these being obtainable from the tabulated masses and volumes.

Other simple geometrical forms have been used in this context, as discussed by Boyd (1935). For each such form, say an upright cylinder or square prism, there is a fixed relationship of surface area to mass and height and this relationship has been assumed to apply to the human body when the whole of the relevant expression is multiplied by some empirical constant. If that were appropriate, one could also convert the formula for one simple shape to that for another in a similar manner, but that is not so (see Discussion). Such simple geometric shapes are discussed below, but only as a starting point for more realistic modelling. The purpose of the simple models is to explore the mathematical basis of Equation 1 and the meaning of the diverse exponents in Table I.

Mathematical analysis and modelling

For simplicity we initially consider volumes, V, rather than masses, using as units m, m^2 and m^3 (or cm, cm^2 and cm^3). Masses are introduced later. With K replacing C, Equation 1 becomes:

$$A = KV^aH^b. \tag{4}$$

However, according to geometrical principles applied to some simple shapes such as cylinders and square cuboids, the three variables are actually related thus:

$$A = \alpha(VH)^{0.5} + \frac{\beta V}{H}. \tag{5}$$

The terms $(VH)^{0.5}$ and V/H both have the dimensions of area, while α and β are dimensionless numbers. Equation 5 is not a standard formula, but is easily derived. To take the example of an upright cylinder of height H and radius R, its volume, V, is πR^2H and its surface area, A, is $(2\pi RH + 2\pi R^2)$. Therefore:

$$R = \left(\frac{V}{\pi H}\right)^{0.5}$$

and

$$A = (2\pi RH) + 2\pi R^2 = 2\pi\left(\frac{V}{\pi H}\right)^{0.5} H + 2\pi\left(\frac{V}{\pi H}\right) = 2\sqrt{\pi}(VH)^{0.5} + \frac{2V}{H}. \tag{6}$$

In this case $\alpha = 2\sqrt{\pi}$ and $\beta = 2$. For a rectangular cuboid of height H, with horizontal side lengths in ratio k, it can be shown similarly that $\alpha = 2(k^{0.5} + k^{-0.5})$ and $\beta = 2$. In the special case of an upright square cuboid, where $k = 1$, $\alpha = 4$ and $\beta = 2$.

We consider next whether Equation 5 may be applied, at least approximately, to the human body, with α and β having values unique to each individual. We then consider how Equation 5, is related to Equation 4.

The body may be modelled as a 'manikin' made up of any number of cylinders or square cuboids, a limb, for example, being made up of a succession of these of differing proportions. Then Equation 5 again applies. Let us briefly consider a simple manikin made up of six cylinders or square cuboids corresponding to trunk, limbs and head-with-neck. The vertical dimensions of the legs and the head-with-neck are specified as proportions of body height, and their widths as proportions of trunk width. The proportions of the arms, held horizontally, are specified as for the legs. The total area of the horizontal surfaces (with those of the arms excluded) equals the last term of Equation 5 namely $\beta V/H$. (Note that surfaces lost to the trunk by the attachment of appendages exist at their ends.) For realistic values of V and H, $\beta V/H$ is about an order of magnitude less than $\alpha(VH)^{0.5}$ – as is easily visualized for a real person.

Haycock et al. (1978) estimated surface areas by treating the body, manikin-like, as a set of 22 cylinders with the head represented by a sphere. Apart from the head, modelling and measurement are in accord here, for the number of cylinders is a matter of detail. Given the relevant measurements, one could calculate exactly the values of α and β that correspond to each total estimated area.

In principle, the manikin may be elaborated, with many more parts. However, there is a simpler approach. This is to assume realistic values for V, H and A and a realistic ratio of trunk height to total height. Again the trunk is taken as a cylinder or cuboid with parts of its surface displaced to the ends of the appendages and the top of the head. Other geometric details need not be specified and can therefore be indefinitely and realistically elaborated. Let us take the example of a man with $H = 1.784\,\text{m}$, $A = 2.11\,\text{m}^2$ and $V = 0.0924\,\text{m}^3$. These values are based on means given by Tikuisis et al. (2001) for 12 men (with V calculated from W taking body density as $1000\,\text{kg m}^{-3}$). With these values inserted into Equation 5, β equals $(41.0 - 7.86\alpha)$. If β is to be positive, α cannot then exceed 5.2. Based on data from the same 12 men, the trunk may be taken as having a volume of $0.050\,\text{m}^3$. If the trunk is seen as a cuboid, cylinder, or any simple shape with straight vertical sides, then, without legs and neck, the total area of its two horizontal surfaces is twice its volume divided by its height (as in Equation 6). The trunk height is not given, so let us assume that it represents 30–40% of the total height of 1.784 m. (Bardeen (1920), gives typical proportions in the middle of that range.) Then the area of the horizontal surfaces is 0.14–$0.19\,\text{m}^2$. As this equals $\beta V/H$ for the whole body, β is between 2.7 and 3.7. Because β also equals $(41.0 - 7.86\alpha)$, respective values of α are 4.9 and 4.7. It thus seems that typical values of α and β are likely to be near 5 and 3, respectively. It is important to realize that there can be no definitive values applicable to all bodies such as there are for the cylinder and square cuboid. Each body must have its own pair of values.

Haycock et al. (1978) modelled the head as a sphere. Realistic modelling calls for a variety of tapering and rounded elements, so let us consider two examples of these. For cylinders and cuboids, the term $\beta V/H$ is the combined area of the two horizontal surfaces. A shape without such a horizontal surface is the bicone, that is to say two equal cones joined base to base with vertical axes. (There is no suggestion that a whole bicone would be used in modelling, although parts of it might be.) Derived as for the cylinder, the formula for the surface area is as follows:

$$A = \left[3\pi VH + 36\left(\frac{V}{H}\right)^2\right]^{0.5}. \tag{7}$$

This is important in showing that Equation 5, is not correct for all shapes. However, with $\alpha = 3.05$ and $\beta = 1.00$ (values found by trial and error), Equation 5 gives values of A for the bicone within 1% of the true values provided that the ratio of height to width exceeds four.

Another shape without horizontal surfaces is the ellipsoid, including the prolate ellipsoid of Reading and Freeman (2005). Although its surface area cannot be expressed exactly by an elementary function (Maas 1994), it is given by the following formula within 1.061% (Thomsen 2004):

$$A = 4\pi\left[\frac{a^p b^p + a^p c^p + b^p c^p}{3}\right]^{t/p}, \tag{8}$$

where a, b and c are the semi-axes and $p = \ln(3)/\ln(2) = 1.6075$. (This reduces to the formula for the surface area of a sphere when $a = b = c$.) If both horizontal semi-axes are of length R and the length of the vertical semi-axis is $H/2$,

$$A = 4\pi\left[\left(R^{2p} + 2R^p\left(\frac{H}{2}\right)^p\right)/3\right]^{1/p}. \tag{9}$$

Table II. Values of α and β in Equation 5 that are appropriate to some simple upright geometric shapes. According to Equation 5, $A = \alpha(VH)^{0.5} + \beta V/H$, where A is the surface area, V is the volume and H is the height.

	α	β
Cylinder	$2\sqrt{\pi}$	2
Square cuboid	4	2
Bicone (with height/width >4)	3.05	1.00 (approximate)
Ellipsoid (with height/width = 0.7–4.0)	3.1	1.8 (approximate)

The volume, V, equals $^4/_3\pi abc$ and therefore $^2/_3\pi HR^2$, so that $R = (3/2\pi)^{0.5}(V/H)^{0.5}$. This allows R to be eliminated from Equation 9:

$$A = \left[\frac{2}{3}(6\pi)^{p/2}(VH)^{p/2} + 2^p 3^{(p-1)}\left(\frac{V}{H}\right)^p\right]^{1/p} = \left[7.06(VH)^{p/2} + 5.94\left(\frac{V}{H}\right)^p\right]^{1/p}. \quad (10)$$

Equation 10 cannot be written in the form of Equation 5, but for ellipsoids with height-to-width ratios of 0.7–4.0 Equation 5 gives similar areas, with less than 2% error, when $\alpha = 3.1$ and $\beta = 1.8$. Table II summarizes the values of α and β appropriate to this and other simple geometrical shapes.

It is evident that Equation 5 cannot be validated for the human body with mathematical rigour. Nevertheless it may reasonably be taken as satisfactory for descriptive purposes, with any algebraic discrepancy being of trivial significance in the context of measurement errors. Moreover, for the human body, even the precise definition of surface area may be debated. However, if the aim is to understand the general formula of Du Bois and Du Bois (1916), one may set aside one's reservations. This is because Equations 4 and 5 are very close numerical approximations one to another, with α and β being related to K, a and b. This is the next point to establish.

If the mean values of V and H for a group of individuals are denoted V_m and H_m, Equation 4 can be written as:

$$A = kx^g y^h, \quad (11)$$

where:

$$\begin{aligned} x &= [VH/(V_m H_m)]^{0.5}, \\ y &= (V/H)(H_m/V_m), \\ k &= K(V_m H_m)^{g/2}(V_m/H_m)^h, \\ g &= (a+b) \\ \text{and } h &= (a-b)/2. \end{aligned}$$

The new variables x and y each have means near 1. Equation 11 may be expanded about 1 by Taylor's series:

$$\frac{A}{k} = 1 + (x-1)g + (y-1)h + \frac{1}{2}\left\{(x-1)^2 g(g-1) + 2(x-1)(y-1)gh + (y-1)^2 h(h-1)\right\}\ldots \quad (12)$$

The second derivative term in the Taylor expansion (Equation 12) is numerically very small and contributes very little to A/k, as is confirmed in the Results section. This is not only because $(x-1)$ and $(y-1)$ are small, but because $g(g-1)$, gh and $h(h-1)$ are small too. (For the values of a and b in Table I, the latter three terms are all between -0.17 and $+0.17$ and all three never have the same sign.) If the final term is therefore dropped, Equation 12 rearranges to:

$$A = kgx + khy + k(1 - g - h). \tag{13}$$

When $(3a + b)$ equals 2, as is dimensionally appropriate, the term $k(1-g-h)$ is zero. Equation 13 is then equivalent to Equation 5 with

$$\alpha = kg = (a + b)KV_{\mathrm{m}}^{(a-1/2)}H_{\mathrm{m}}^{(b-1/2)} \tag{14}$$

$$\beta = kh = \frac{1}{2}(a - b)KV_{\mathrm{m}}{}^{(a-1)}H_{\mathrm{m}}{}^{(b+1)}. \tag{15}$$

The following approximate formulae relate a, b and K to α, β, V_{m}, H_{m} and the mean area, A_{m}.

$$a = \alpha\frac{(V_{\mathrm{m}}H_{\mathrm{m}})^{0.5}}{(2A_{\mathrm{m}})} + \frac{\beta V_{\mathrm{m}}}{(H_{\mathrm{m}}A_{\mathrm{m}})}, \tag{16}$$

$$b = \alpha\frac{(V_{\mathrm{m}}H_{\mathrm{m}})^{0.5}}{(2A_{\mathrm{m}})} - \frac{\beta V_{\mathrm{m}}}{(H_{\mathrm{m}}A_{\mathrm{m}})}, \tag{17}$$

$$A_{=}\frac{A_{\mathrm{m}}}{\left(V_{\mathrm{m}}^{a}H_{\mathrm{m}}^{b}\right)}. \tag{18}$$

This theoretical discussion is so far framed in terms of volumes rather than masses so that the formulae are not cluttered with density terms. In practice, however, it is mass that is measured, so that V may be replaced by W divided by density, ρ. Equation 5 then becomes:

$$A = \alpha\left(\frac{WH}{\rho}\right)^{0.5} + \beta\left[\frac{W}{(\rho H)}\right]. \tag{19}$$

The density of the body (with pulmonary air appropriately included here) is not usually determined in studies of surface area. One may therefore choose to treat it as constant, with a value close to 1000 kg m^{-3} or 0.001 kg cm^{-3}, and simplify Equation 19 as follows:

$$A = \alpha(WH)^{0.5} + \frac{\beta W}{H}. \tag{20}$$

Actual variations in density can then be seen as being amongst the determinants of α and β, and for practical purposes disregarded.

For the human body W/H is an order of magnitude less than $(WH)^{0.5}$ and the two are strongly correlated. This means that β, as determined by least-squares regression of A on $(WH)^{0.5}$ and W/H for group data, will have wider confidence intervals than those of α. In other words β should be more influenced by the vagaries of sampling. At the same time, uncertainties in β are correspondingly less important in estimating surface area.

It might be supposed that values of α and β typical of individuals might be obtained from group data by regression, but this cannot generally be true. Negative values of $\beta V/H$ and $\beta W/H$, being areas, lack physical meaning and therefore, according to the present analysis, these terms must be positive for individuals. Also, according to Equations 16 and 17, b cannot exceed a. Nevertheless, Table I shows that b, estimated from real group data by least-squares regression, can exceed a. Equation 15 implies that β is then negative. (This is confirmed below by direct estimates of β.) With small data sets such discrepancies could be due entirely to sampling effects, but it is also true that regression equations can depart systematically from true functional relationships (McArdle 1988).

Another possible explanation for negative values of β is that there exists some statistical trend in the way that body proportions vary within groups. Modelling of such a trend is best done in a manner that maintains dimensional correctness. Then, when Equation 1 is fitted by regression, $(3a+b)$ should remain close to 2, as is true of the examples in Table I. What is tested below is the possible variation in α and β with the dimensionless quantity H^3/V, or its equivalent H^3/W. This may be seen as a general measure of slenderness.

The Methods and Results sections are subdivided so as to preserve the distinction between model and real data. Modelling is mostly in terms of volumes, while real data are expressed as masses.

Methods

Calculations were carried out on spreadsheets (Microsoft Excel), except that the program Datafit (Oakdale Engineering, Oakdale, PA, USA) was used for non-linear regression analysis.

Model data

In order to explore aspects of the above theoretical treatment a set of realistic values of V and H was chosen. For easy specification here, these are the 42 values of W and H given by Du Bois and Du Bois (1916), but with masses in tonnes treated numerically as volumes (V) in cubic metres. Values of A were calculated for this data set in accordance with Equation 5 and for various values of α and β. The resulting artificial data sets were used both to check that the last term in Equation 12 is indeed insignificant and to check the numerical equivalence of Equations 4 and 5. Note that these modelled values of A are not those measured by Du Bois and Du Bois (1916).

'Experiments' were carried out with this same set of paired V and H values. The general method was to choose initial values of α and β for each pair (denoted α_1 and β_1), doing so in various ways that are described under 'Results'. Corresponding values of A were then calculated and these were collectively regressed against V and H to obtain new group parameters α_2 and β_2.

Real data

Equations 1 and 20 were fitted by multiple regression to all 42 of the real data of Du Bois and Du Bois (1916), not just the nine that they used, and also to those of Frontali (1927), Bradfield (1927), Banerjee and Sen (1955) and Banerjee and Bhattacharya (1961). With Equation 1, b was constrained to equal $(2-3a)$.

The possible influence of H^3/W on α was tested on the data of Du Bois and Du Bois (1916). To this end, A was regressed on W and H according to the following equation:

$$A = \gamma \left(\frac{H^3}{W}\right)^{\delta} (WH)^{0.5} + \frac{\beta W}{H}, \tag{21}$$

where γ and δ are regression parameters and β was taken arbitrarily as either 2 or 3. Note that $\gamma(H^3/W)^{\delta}$ corresponds to α. The mathematical form of the expression was chosen just for its convenience.

Results

Model data

The second-derivative term in the Taylor expansion (Equation 12) is confirmed as contributing very little to A/k. Thus, with the chosen model data set, and with α and β both taken as 5, this term, multiplied by k, contributes between –1.42% and 0.00% to A (mean –0.09%). With β lowered to 2 (with α again equal to 5) the corresponding range is narrower, namely –0.67% to 0.00% (mean –0.04%). The percentage is zero when β is zero. (Similarly low values were also obtained with real data.)

We look next at how closely Equations 4 and 5 correspond to each other numerically. Values of A were calculated for the chosen set of V and H taking both α and β as 5. The parameters of Equation 4 were then found by regression of A on V and H, the results being as follows: $K = 7.018$; $a = 0.5527$; $b = 0.3392$; $(3a + b) = 1.997$; $r = 0.99998$. When values of A are estimated from this regression equation for the same values of V and H, the discrepancies between these estimates and the original model values average 0.005%, with a range of –0.5% to +0.3%. The value of β, namely 5, was chosen as being perhaps unrealistically high and lower values improve the correspondence between Equations 4 and 5. Indeed the correspondence must become exact as β approaches zero. The parameters of Equation 4 were also estimated using Equations 16–18, producing similar, but not identical, results ($K = 6.96$; $a = 0.556$; $b = 0.353$).

Having confirmed the close equivalence of Equations 4 and 5 we now come to the first modelling experiment. To test a possible effect of varying body shape, β_1 was set arbitrarily at 2 and α_1 was made to increase with H^3/V, being taken for mathematical convenience as $0.70(H^3/V)^{0.08}$. Then, as obtained by regression, $\alpha_2 = 5.71$ and $\beta_2 = -4.57$ (i.e. negative). The mean of α_1 was 4.99 and its range was 4.58–5.25. The constants '0.70' and '0.08', and corresponding values of '0.00123' and '1.5' in the next paragraph, were chosen by trial error as giving suitable illustrative results. They have no meaning beyond that.

The effect of varying just β_1 was also tested. Thus, with α_1 constant at 5, β_1 was taken as $0.00123(H^3/V)^{1.5}$. Then $\alpha_2 = 5.83$ and $\beta_2 = -4.64$ (values similar to those obtained by varying α_1). The mean of β_1 was 3.06 and its range was 0.80–6.31. The high exponent (1.5) and the resulting wide range of β_1 make variations in β_1 alone an implausible explanation for large negative values of β_2.

To produce negative values of β_2, the dependence of α_1 or β_1 on H^3/V need not be as just described. More haphazard statistical correlations suffice, but experiments illustrating this are harder to summarize concisely.

In other experiments, either α_1 or β_1 were varied randomly around various mean values like those of the previous paragraphs. As expected, the regression parameters α_2 and β_2 resembled the means of α_1 and β_1, but they deviated (up or down) to an extent that depended on the chosen set of random numbers. The effect of varying α_1 or β_1 progressively with either V or H was also tested. The result was a breakdown of dimensional correctness, so that $(3a+b)$ departed much more markedly from 2 than is seen in Table I.

Real data

Table III summarizes results obtained with the published data. The parameter C is given in two versions: C_1 is calculated for kg, cm and m^2 for comparison with C in Table I; C_2 is for measurements in tonnes, m and m^2 ($C_2 = 1000^a 100^b C_1$). Equations, not shown, that combine the data of Banerjee and Sen (1955) for adults and of Banerjee and Bhattacharya (1961) for children overestimate A for the latter and underestimate it for the adults. Combining those data is therefore inappropriate. The tabulated values of α and β are negatively correlated ($r = -0.97$; $p = 0.005$).

Table III includes estimates of α and β calculated from C_2, a and b by means of Equations 14 and 15 (i.e. with W substituted for V). Estimates of α and β based instead on values of C_2, a and b that are calculated with no constraint on $(3a+b)$ are less satisfactory.

Table III. Results from five published data sets: mean body masses (W), heights (H) and surface areas (A); regression parameters for Equation 1 (C being given for two sets of units) and Equation 20; coefficient of multiple correlation (r).

	Du Bois and Du Bois (1916)	Bradfield (1927)	Frontali (1927)	Banerji and Sen (1955)	Banerjee and Bhattacharya (1961)
Number of individuals	42	47	33	15	13
Mean mass (kg)	52.7	56.9	9.1	49.6	19.3
Mean height (m)	1.62	1.63	0.74	1.63	1.15
Mean area (m^2)	1.51	1.58	0.43	1.58	0.77
Parameters of Equation 1					
C (m^2, kg, cm)	0.0087	0.0184	0.0059	0.0140	0.0096
C (m^2, tonne, m)	4.06	5.42	8.37	5.09	4.22
a	0.444	0.510	0.616	0.480	0.453
b	0.669	0.470	0.152	0.560	0.641
r (for both equations)	0.994	0.967	0.992	0.991	0.986
Parameters of Equation 20					
α	5.74	5.09	4.04	5.78	5.70
SE of α	0.15	0.21	0.33	0.24	0.71
β	−4.79*	0.86	9.02	−1.94	−4.65
SE of β	1.35	1.831	2.418	2.20	6.49
SE of estimate[a]	0.039	0.039	0.023	0.021	0.028
Estimates of α and β obtained from Equations 14 and 15					
α	5.79	5.09	4.15	5.79	5.67
β	−5.28	0.91	8.29	−2.07	−4.34

n is the number of data for each set.
*β is significantly below zero ($p < 0.001$).
[a]The standard errors of estimates for Equation 1 are within 0.001 of these values.

This is most notably true of the data of Frontali (1927) for which α and β were estimated in this way as 5.0 and –0.9, respectively.

There is evidence for the data of Du Bois and Du Bois (1916) that the negative value of β is associated with variations in H^3/W. The latter averages $87 \pm 21\,\mathrm{m}^3$ tonne^{-1} (range 36–143). Highly significant values of δ were obtained when Equation 21 was fitted to the data ($p = 0.00001$). With β_1 taken as 2, $\gamma = 3.46$ and $\delta = 0.083$. With β_1 taken as 3, $\gamma = 3.18$ and $\delta = 0.097$. For further evidence on this point we return to modelling, but using these same real data. Values of α and β (i.e. α_1 and β_1) were chosen and used to calculate new surface areas, denoted A′, such that A′/A averages 1 (A being the true value in each case). With $\beta_1 = 0$, α_1 needs to be 5.21 for A′/A to average 1 and this ratio then shows a negative correlation with H^3/W ($r = -0.45$, $p < 0.01$). With $\beta_1 = 3$, α_1 needs to be 4.89 and the correlation is stronger ($r = -0.62$, $p < 0.001$).

Discussion

The fundamental formula relating surface area to height and volume is regarded as being Equation 5 rather than Equation 4. It is dimensionally correct and remains so as a regression equation. When mass replaces volume, Equation 5 becomes Equation 20. Bouchard (1897) and Bardeen (1920) likened the body respectively to a cylinder and a square prism and for these they gave formulae essentially equivalent to Equations 5 and 20. However, the way they fitted these to human data was to multiply their whole formulae by single empirical constants so that, in present terms, α and β varied proportionately to each other. That this is inappropriate is illustrated by the impossibility of converting the expression for the surface area of a cylinder, $[2\sqrt{\pi}(WH)^{0.5} + 2W/H]$, to that of a square cuboid, $[4(WH)^{0.5} + 2W/H]$, just by multiplying the whole expression by one number. It is curious that the essential step of varying α and β independently has not previously been taken. Applied to the human body, Equations 5 and 20 may be approximations, just as they are for bicones and ellipsoids, but in practice they should suffice to well within the limits of measurement error.

Despite these reservations, it is important to realize that Equation 20 is the appropriate formula to use in exploring the properties of Equation 1. This is because the two are essentially equivalent, just as has been demonstrated for Equations 4 and 5. Furthermore, the manikin model almost exactly accords with the method of measuring surface areas used by Haycock et al. (1978).

The modelling experiments demonstrate that the values of α and β obtained as regression parameters (α_2 and β_2) can be very different from the means of the true individual values (α_1 and β_1). Typical true values of the latter are therefore not known for real people. One can merely note that crude modelling in terms of the manikin suggests respective values of about 5 and 3. Although this is unsatisfying, it is the regression parameters that matter for predictive purposes.

We come now to the results for real data (Table III). The data sets are all small and this could be why Banerjee and Sen (1955) and Banerjee and Bhattacharya (1961) simply accepted the values of a and b given by Du Bois and Du Bois (1916) and optimized fit by adjusting C. It was shown that their results, for Indian adults and children, respectively, could not appropriately be combined, and this suggests the need for caution in combining other data sets.

According to the basic model, individual values of β cannot be negative and yet, as regression parameters, they can be (Table III), and significantly so in the case of the data of Du Bois and Du Bois (1916). For their data, b far exceeds a (Tables I and III). A possible explanation for both points is that true individual values of α and β within a population vary systematically in some way. As demonstrated by modelling, one possibility is that they correlate with the dimensionless quantity H^3/V or its equivalents $\rho H^3/W$ or H^3/W. These, with their reciprocals or cube roots, are the only measures of body shape available from the data. In the model examples described, true values of α or β are made to vary progressively with H^3/V according to a precise rule, but mere statistical correlations would suffice to make β negative as a regression parameter. The results suggest that the main effect of body build (H^3/W) would be on α rather than on β, but both could be affected. Analysis of the data of Du Bois and Du Bois (1916) strongly supports the hypothesis that the negative value of β, and thus the high value of b, are associated with variations in H^3/W. One of the two lines of evidence presented is that β can be allotted a plausible positive value, with Equation 21 then fitted to the data by non-linear regression to obtain α as $\gamma(H^3/W)^\delta$. With β between 2 and 3, δ is 0.08–0.1. Incidentally, with $\delta = 0.08$, the term $(H^3/W)^\delta(WH)^{0.5}$ can be written as $W^{0.42}H^{0.74}$, which is suggestive both of Equation 1 and of the exponents given by Du Bois and Du Bois (1916) (Table I). Remember, however, that Equation 21 is just a mathematically convenient way of representing the dependence of α on H^3/W.

Although b exceeds a in several examples in Tables I and III, this is not general. Livingston and Lee (2001) pointed out that no previous study of surface area had included obese subjects. They added results for 47 subjects, some seriously obese, to 413 estimates previously published (Boyd 1935) and obtained the lowest value of b in Table I (i.e. 0.206). The obese subjects (with H^3/W as low as about 16 m^3 tonne^{-1}) were all at the upper ends of the height and surface area ranges and might therefore be expected to have had a large influence on the calculated values of a and b. If H^3/W is again relevant, then it is having the opposite influence on these exponents than the effect discussed in the previous paragraph, where a correlation between α and H^3/W results in a high value of b. Indeed, on the basis of a few representative data estimated from the Figures of Livingston and Lee (2001) the exponent δ in Equation 21 is negative. The index H^3/W can reflect more than one detail of body shape (adiposity, Cormic index, etc.) and only one of the subjects of Du Bois and Du Bois (1916) was obese. Estimates of surface area could clearly be improved by taking other measurements into account.

On the general question of the variability and reliability of regression parameters, it has already been noted that β, as determined by regression, will have wide confidence intervals and yet, at the same time, is of minor importance in the estimation of surface area. On the simplifying view that it might suffice to take β as zero in prediction equations, Equation 20 becomes:

$$A = \alpha W^{0.5} H^{0.5}. \tag{22}$$

This is equivalent to Equation 3 that is discussed in the Introduction.

Various authors have explored the extents to which prediction equations derived from one set of data fit other sets (Haycock et al. 1978; Lam and Leung 1988; Wang et al. 1992; Jones et al. 1994; Shuter and Aslani 2000; Livingston and Lee 2001; Tikuisis et al. 2001; Vu 2002; Yu et al. 2003; Reading and Freeman 2005; Verbraecken et al. 2006). The general method has been to compare surface areas calculated for particular sets of body mass and height according to two or more different formulae. The comparisons have been in terms of either correlations or mean values. Another approach, requiring no data, would be to compare surface areas calculated for particular, perhaps extreme, combinations of mass and height that are of interest. For example, an individual of 200 kg and height 160 cm (and body mass index 78 kg m^{-2}) has a surface area of 3.5 m^2 according to the values of C, a and b given by Livingston and Lee (2001), but of only 2.7 m^2 according to the equivalent parameters given by Shuter and Aslani (2000). One may also combine two such versions of Equation 1 to find the relationship between mass and height for which the formulae give identical surface areas, or areas differing by some chosen percentage. Thus those same two equations give identical surface areas when, according to the parameters in Table I, $0.0495W^{0.605}H^{0.206}$ equals $0.0095W^{0.441}H^{0.655}$. That condition is satisfied when W equals $4.25 \times 10^{-5} \times H^{2.74}$ (with W in kg and H in cm).

Although these comparisons may be useful, there cannot be a single set of parameters that is ideal for all groups or populations as is sometimes supposed (e.g. by Verbraecken et al. 2006). This is because people vary in build. Tikuisis et al. (2001) give different parameters for men and women and separate values were found necessary for the adults of Banerjee and Sen (1955) and the children studied by Banerjee and Bhattacharya (1961).

Shuter and Aslani (2000) graph five published pairs of the parameters C and b and obtained a line of best fit having the following equation:

$$C = 1086.2e^{-3.779b}. \tag{23}$$

The curve fits the data points well, but no theoretical basis is suggested. An alternative approach is to apply the approximate relationship of Equation 18 (with W substituted for V and C for K) together with Equation 2 and suitably chosen means for W, H and A. Almost exactly the same curve is generated if the latter are taken as 52 kg, 1.62 m and 1.51 m^2, respectively (resembling means for the data of Du Bois and Du Bois 1916). For values of b ranging from 0 to 0.7, the mean and maximum discrepancies between the two curves are only 0.1% and 0.4%. This theoretical approach is useful in emphasizing the determinants of C.

As well as fitting Equation 1 to their data and obtaining the parameters given in Table I, Livingston and Lee (2001) derived a simplified equation in which surface area is proportional to the two-thirds power of mass and which lacks a height term. This provides as good a fit to their data set as does their full Equation 1. However, their parameter a in Equation 1 (i.e. 0.605) is unusually close to 2/3, while the corresponding height exponent b (i.e. 0.206) is unusually low. The simplified equation is necessarily appropriate when all individuals have a similar shape, but, in other studies, b has been found to differ significantly from zero. Moreover, the height term is needed in Equation 1 to allow for variations in body build.

For the practical purpose of estimating surface area from mass and height, there is no strong reason to abandon the form of equation introduced by Du Bois and Du Bois (1916). However, the ideal parameters may represent compromises between convenience and accuracy, for no one formula can fit all shapes of people. Better formulae cannot be found just by combining data on mass, height and area for more individuals of deliberately diverse body build. New formulae should be obtained instead for particular groups of individuals that are categorized, for example, by sex, age, fatness (or body mass index) and relative limb length. Alternatively, some of these features, if quantified, might be included within the formulae as components of α or β. It is at this level of analysis that the new, theory-based formula (Equation 5 or 20) should prove valuable, even if the resulting parameters (α and β) are finally translated into those of Equation 1 (i.e. C, a and b) for general use. If the key aspects of body shape affecting α and β are identified, the number of subjects required for measurement could be quite small. A useful new way of comparing formulae is discussed above. As to other suggestions regarding future studies, parameters for both types of equation could usefully be given, together with the means of mass, height and surface area that have not generally been specified in the past. Dimensional consistency should be enforced in the estimation of the exponents a and b by regression.

Declaration of interest: The author reports no conflicts of interest. The author is alone responsible for the content and writing of the paper.

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