A Currency Premium Puzzle

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Motivation

- Surge in interest in the role of **risk premia** in international finance/macro

  e.g. exchange rates, interest rates, capital flows, and financial stability.
  (Mendoza (2010), Forbes (2013), Miranda-Agrippino & Rey (2020), ...)

- Key to understanding UIP violations, contagion, global financial cycle, capital retrenchments, and sudden stops.

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- Key to understanding UIP violations, contagion, global financial cycle, capital retrenchments, and sudden stops.
- Nevertheless lack **quantitative model** that can reconcile the observed FX with **large and persistent differences in interest rates** across countries.
  - e.g. NZ and AUS persistently have a 3-4 pp. higher risk-free rate than JP and US.
- Key roadblock to understanding effect of risk premia on allocation of capital across countries.

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- Key roadblock to understanding effect of risk premia on allocation of capital across countries.

This paper

- Highlight fundamental tension between canonical asset pricing models and empirically observed behavior of interest rates and exchange rates.

A currency premium puzzle

This Paper

- Classical asset pricing puzzles:
  - High equity premium (Mehra and Prescott (1985))
  - Low and stable risk-free rates (Weil (1989)):
- Canonical long-run risk and habit models
  - Increase variance of log SDF to generate high equity premium.
  - A **negative functional relationship** between the variance and the mean of the log SDF to keep risk-free rates low and stable.
- This "trick" has proven highly successful in accounting for closed-economy asset prices and quantities.

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- This "trick" has proven highly successful in accounting for closed-economy asset prices and quantities.

- This same trick is also the *fundamental reason* why these models struggle to account for long-lasting diffs in risk-free rates and currency returns.
- Large currency premia pose a fundamentally different challenge to these models than the classical asset pricing puzzles.

Main Findings

In the data, FX are **largely unpredictable** (Meese & Rogoff, 1983) and differences in interest rates across countries are **large and persistent** (Hassan & Mano, 2019).

1. No model with complete markets and canonical long-run-risk or habit preferences can jointly match these facts.
2. Canonical models require vast majority (94%) of any differences in currency returns must result from **predictable appreciations**, with **tiny** interest rate differentials.
3. Counterfactual prediction is hard-wired in the utility function, independent of any features of the economic environment (differences in country size, volatility, financial development, trade centrality...)
4. Affects virtually all leading international macro models with asset prices and macro quantities.
5. Market incompleteness (limited spanning) is no easy fix.

Related Literature

- **Macro / financial effects of international risk premia**
  Forbes (2013), Miranda-Agrippino and Rey (2020), Forbes and Warnock (2021), Mendoza (2010), Colacito and Croce (2011), Bansal and Shaliastovich (2013), Colacito and Croce (2013), Colacito et al. (2018b), Colacito et al. (2018a), Verdelhan (2010), Stathopoulos (2017), Heyerdahl-Larsen (2014),Gourio, Siemer, and Verdelhan (2013)

$\rightarrow$ Highlight a major challenge to the development of this literature.

- **"Classic" approaches to equity and risk-free rate puzzles**
  Campbell and Cochrane (1999); Bansal and Yaron (2004)

$\rightarrow$ International data place new restrictions on these approaches.

- **Models with asymmetries in economic environment across countries**
  Martin (2011), Hassan (2013), Richmond (2019), Ready, Roussanov, and Ward (2017), Maggiori (2017), Wiriadinata (2021), Gourinchas, Govillot, and Rey (2017)

$\rightarrow$ Manifest as predictable appreciations with LRR/habit preferences.

- **Applicability of (in)complete markets in international asset pricing**
  Sandulescu, Trojani, and Vedolin (2021), Jiang et al. (2022), Jiang (2023), Jiang, Krishnamurthy, and Lustig (2023), Chernov, Haddad, and Itskhoki (2023)

$\rightarrow$ No easy fix.

Outline

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A Highly Successful Trick

Fundamental equation of asset pricing:

$$1 = \mathbb{E}_t(M_{t+1} R_{t+1})$$

Risk-free rate (lognormality)

$$r_{f,t} = -\,\mathbb{E}_t(m_{t+1}) - \frac{1}{2}\,\mathsf{var}_t(m_{t+1})$$

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- **Equity premium puzzle**: the equity premium is high
  - Need high $\mathsf{var}_t(m_{t+1})$ to justify high equity premium. (HJ Bound)
- **Risk-free rate puzzle**: the risk-free rate is low and stable.
  - Whatever increases $\mathsf{var}_t(m_{t+1})$ also has to decrease $\mathbb{E}_t(m_{t+1})$ to match low and stable r_f.

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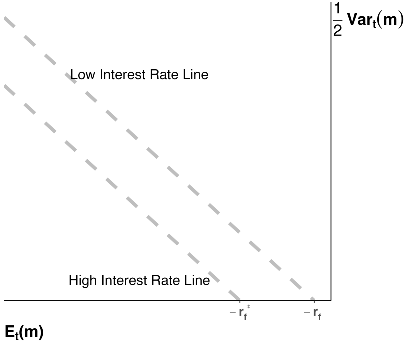
  - Whatever increases $\mathsf{var}_t(m_{t+1})$ also has to decrease $\mathbb{E}_t(m_{t+1})$ to match low and stable r_f.
- Canonical long-run risk and habit models achieve this by creating a **functional form** between the two.

The Iso-rf Line in the SDF Space



For a given r_f,

$$\frac{1}{2}\,\mathsf{var}_t(m_{t+1}) = -\,\mathbb{E}_t(m_{t+1}) - r_{f,t}$$

represents a negative 45° line in the "SDF space".

Exchange Rates, Currency Premium and the SDF

Intl. asset prices provide additional information on the two moments!
If markets are complete (Backus, Foresi, and Telmer, 2001),

- Data on **exchange rates** (F per H): how much $\mathbb{E}(m_{t+1})$ differs across countries.

$$\mathbb{E}(\Delta s_{t+1}) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m^*_{t+1})$$

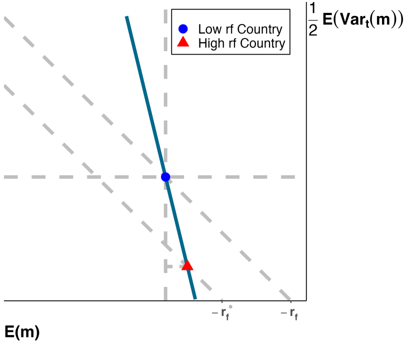
- Data on **currency premium**: how much $\frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}))$ differs across countries.

$$\begin{aligned}\mathbb{E}(rx_{t+1}) &= \mathbb{E}(r^*_t - r_t) - \mathbb{E}(\Delta s_{t+1}) \\ &= \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}) - \text{var}_t(m^*_{t+1}))\end{aligned}$$

  - Define FX-share= $\frac{-\mathbb{E}(\Delta s)}{\mathbb{E}(rx)}$.
- Note that each country has a mean-variance pair $(\mathbb{E}(m_{t+1}), \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1})))$, which is a point in the SDF space.
  - Data on exchange rate and currency premium determines the **relative positions** of countries in the SDF Space!

Interest Rate Differences

High Interest Rate Currency Appreciates

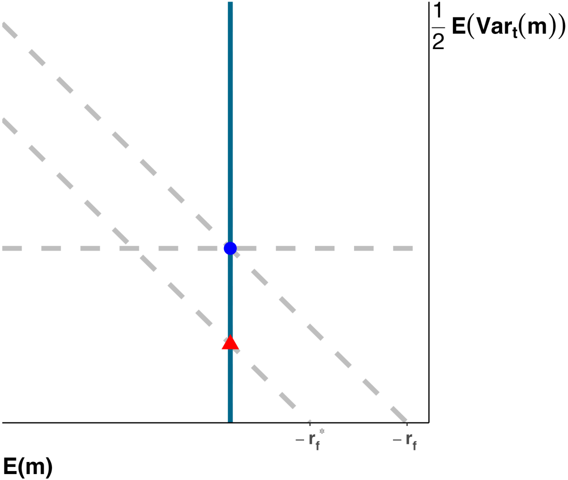


Remark: The *FX-Slope* that connects two points is a visualization of the composition of currency premia.

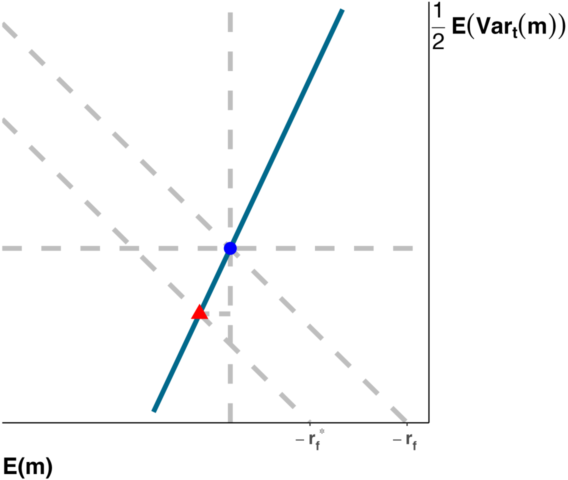
$$\text{FX-slope} = \frac{\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1}) - \text{var}_t(m^*_{t+1}))}{\mathbb{E}(m_{t+1} - m^*_{t+1})}$$

$$= \frac{\mathbb{E}(rx)}{\mathbb{E}(\Delta s)} = -\frac{1}{\frac{-\mathbb{E}(\Delta s)}{\mathbb{E}(rx)}} = -\frac{1}{\text{FX-share}}$$

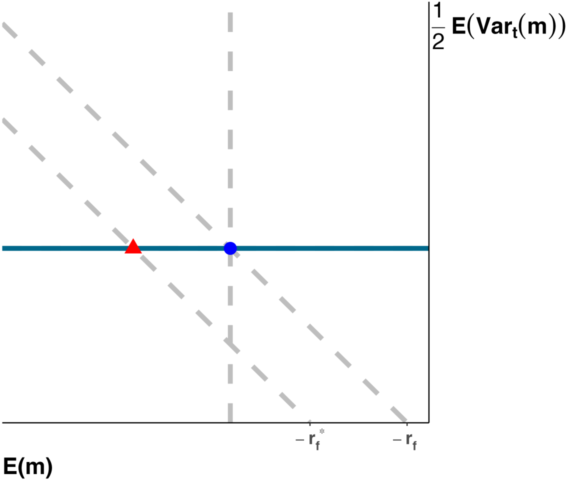
Unpredictable Exchange Rates



High Interest Rate Currency Depreciates



UIP Holds



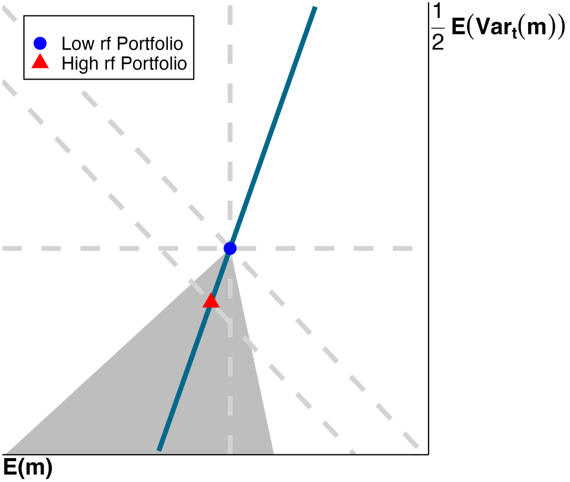
Data

- Dataset used in Hassan and Mano (2019)
  - Time span: Oct1983 - May2010. 15 countries.
  - Static trade: long(short) a fixed, weighted portfolio of currencies that, on average, have high(low) interest rates.
    - Low-interest rate currencies: JPN, CHE, SGP, DNK, SWE, CAN, HKG, SAU
    - High-interest rate currencies: MYS, NOR, KWT, GBR, AUS, NZL, ZAF

	Return (%)	Change in FX (%)	FX-share	FX-slope
	$\mathbb{E}(rx)$	$-\mathbb{E}(\Delta s)$	$\frac{-\mathbb{E}(\Delta s)}{\mathbb{E}(rx)}$	$-\frac{1}{\text{FX-share}}$
Static Trade	3.46	-1.30	-0.37	2.67
	[1.18,5.54]	[-3.82,0.60]	[-1.16,0.23]	$[0.86, \infty) \cup (-\infty, -4.42]$

Portfolio Construction | With Interest Rate Differences

Data: Currency Returns in the SDF Space, Static Trade



HM Estimates Individual Countries

Bounds on log SDF

- To match the data on currency returns, a model needs to generate the following patterns.

Property (SDF bound)

- *Property 1: Large difference in the variances of log SDFs*

$$\frac{1}{2}\,\mathbb{E}(\mathit{var}_t(m) - \mathit{var}_t(m^*)) \geq 0.0346$$

- *Property 2: FX-slope is weakly positive*

$$\frac{\frac{1}{2}\,\mathbb{E}(\mathit{var}_t(m) - \mathit{var}_t(m^*))}{\mathbb{E}(m_{t+1} - m^*_{t+1})} \geq 0$$

Remark:

- Compare to HJ bound $\frac{1}{2}\,\mathrm{var}_t(m) > 0.125$.

Outline

Long-Run Risk: Model Setup

- Epstein-Zin preferences (analogous setup for other country (*))

$$U_t = \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}.$$

- Consumption growth governed by

$$\begin{aligned} \Delta c_{t+1} &= \mu + z_t \\ z_t &= \rho z_{t-1} + \sigma \varepsilon_t \end{aligned}$$

(no short-run shocks)

- Log SDF is given by

$$\begin{aligned} m_{t+1} = \log(\delta) &- \frac{1}{\psi} \Delta c_{t+1} \\ &+ \left(\frac{1}{\psi} - \gamma \right) \left(u_{t+1} - \frac{1}{1-\gamma} \log \left(\mathbb{E}_t[\exp((1-\gamma) u_{t+1}]) \right) \right) \end{aligned}$$

Long-Run Risk: Moments of SDFs

- Assuming u_{t+1} is normal, SDF unconditionally

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\mathsf{var}_t(u_{t+1}))$$

$$\frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\mathsf{var}_t(u_{t+1}))$$

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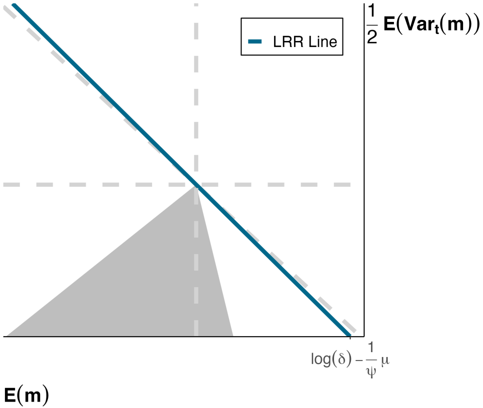
- Substituting out $\mathbb{E}(\mathsf{var}_t(u_{t+1}))$,

$$\frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m_{t+1})) = -\frac{\frac{1}{\psi} - \gamma}{1-\gamma}\,\mathbb{E}(m_{t+1}) + \frac{\frac{1}{\psi} - \gamma}{1-\gamma}\left(\log(\delta) - \frac{1}{\psi}\mu\right)$$

- LRR models imply a **functional relationship** between mean and variance of the log SDFs!
- Note that this is a line in the SDF space.

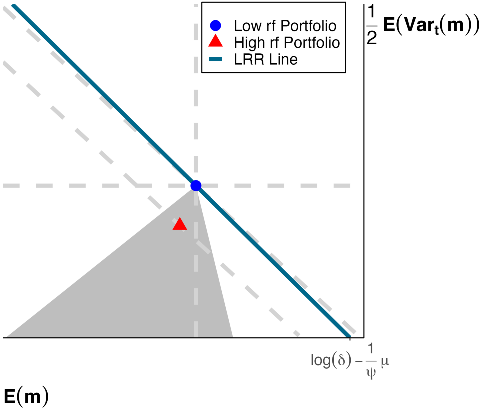
General Case

Long-run Risk: the LRR Line ($\gamma = 6.5, \psi = 1.6$)



- The LRR line is very close to the iso-rf line;
- Helpful to resolve the equity premium puzzle and the risk-free rate puzzle: one can increase volatility of the SDF without changing r_f much.
- Intuition: under EZ preferences, agents have a preference for the timing of resolution of uncertainty, generating a link between first and second moments of marginal utility growth (SDF).

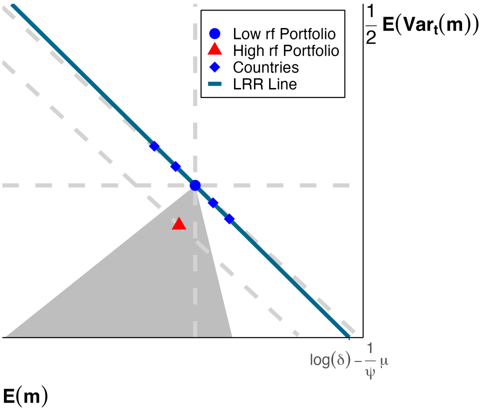
Long-Run Risk: The LRR Line when $\gamma = 6.5, \psi = 1.6$



- Regardless of the economic environment (country size, trade centrality, resource endowments, loadings on shocks...), all countries with the same preferences and growth rates are on the **same** LRR line.
- No combination of LRR preferences with any of the existing risk-based theories of currency returns can fit the data.

CCGR Setup

Long-Run Risk: The LRR Line when $\gamma = 6.5, \psi = 1.6$



- For example, Colacito et al (2018) (CCGR) use heterogeneous loadings on a global long-run shock.
- Currency premia (vertical difference) are mostly accounted for by expected change in exchange rates (horizontal difference).
- Universal problem with LRR models.

CCGR Setup

Long-Run Risk: International Asset Pricing

Proposition

If γ, ψ, μ and δ are symmetric across countries, then FX-slope is given by

$$\frac{\mathbb{E}(rx_{t+1})}{\mathbb{E}(\Delta s_{t+1})} = -\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}$$

Furthermore, If agents prefer early resolution of uncertainty so that $\gamma > 1/\psi$, and assume $\gamma > 1$, then the model can not match Properties 1 and 2 at the same time: as long as $\mathbb{E}(rx) > 0$, FX-slope is negative.

- In particular, if $\gamma > 2 - \frac{1}{\psi}$, we have $\frac{-\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{1-\gamma}{\frac{1}{\psi}-\gamma} > \frac{1}{2}$, $-\mathbb{E}(\Delta s_{t+1})$ accounts for more than 50% of the currency premium.

additional results

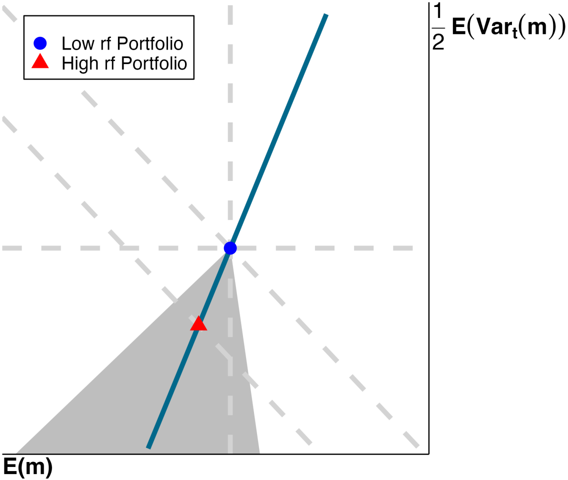
Long-run Risk Models: Simulation

Table: Static Trade Returns

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $-\mathbb{E}(\Delta s^{st})$	FX-share $\frac{-\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	-0.37 [-1.16,0.23]	2.67 [0.86, ∞)$\cup$ ($-\infty$, -4.42]	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	7.10	5.98	0.93	-1.07	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	0.00	0.00	0.98	-1.02	No	No
Bansal and Shaliastovich (2013) RFS	0.00	0.00	0.96	-1.04	No	No
Colacito and Croce (2013) JF	0.00	0.00	0.95	-1.05	No	No
Bansal and Yaron (2004) JF	-	-	0.94	-1.06	No	No

With Interest Rates

Data: Currency Returns in the SDF Space, Carry Trade



- Low (high) rf portfolio: a weighted portfolio of currencies with low (high) risk-free rates each period.

Long-run Risk Models: Simulation

Table: Carry Trade Returns

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	-0.43 [-1.10,0.15]	2.30 [0.90, ∞)$\cup$ ($-\infty$, -6.56]	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	4.47	2.76	0.62	-1.62	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	-0.09	-0.52	6.11	-0.16	No	No
Bansal and Shaliastovich (2013) RFS	-0.03	-0.26	9.54	-0.10	No	No
Colacito and Croce (2013) JF	0.05	-0.35	-7.02	0.14	No	No

With Interest Rates

Long-Run Risk Models: Extension

What if add time-varying volatility?

$$\begin{aligned}\Delta c_{t+1} &= \mu + z_t \\ z_t &= \rho z_{t-1} + w_{t-1}\varepsilon_{LR,t} \\ w_t^2 &= (1-\phi)w_0^2 + \phi w_{t-1}^2 + \sigma_w \varepsilon_{w,t}\end{aligned}$$

Results:

$$\begin{aligned}\mathbb{E}(\hat{m}_{t+1}) &= -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\color{red}{(A_{vw}^2\sigma_w^2 + A_{vz}^2 w_0^2)} \\ \frac{1}{2}\,\mathsf{var}_t(\hat{m}_{t+1}) &= \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \color{red}{(A_{vw}^2\sigma_w^2 + A_{vz}^2 w_0^2)}\end{aligned}$$

We again get the same negative functional relationship:

$$\frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(\hat{m}_{t+1})) = -\frac{\frac{1}{\psi} - \gamma}{1-\gamma}\,\mathbb{E}(\hat{m}_{t+1})$$

Long-Run Risk Models: Summary

- Long run risk models with EZ preferences impose a strict functional relationship between the first and second moments of log SDFs
- Stationary models with complete markets, EZ, and a preference for early resolution of uncertainty cannot match the data on currency returns, regardless of the economic environment.
- In particular, adding differences in country size, trade centrality, resource endowments, loadings on shocks, any of the sources of heterogeneity suggested in the literature, will not help match the data.
- If markets are complete, LRR preferences themselves are at odds with the exchange rate data.

Outline

Habit: Model Setup (1/2)

- Habit utility (analogous equations for country $(^*)$)

$$\mathbb{E}\sum_{t=0}^{\infty}\delta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma}$$

- Following Campbell and Cochrane (1999), we define the surplus consumption ratio as

$$X_t \equiv \frac{C_t - H_t}{C_t}$$

- The pricing kernel is given by

$$M_{t+1} = \delta\left(\frac{X_{t+1}}{X_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$

- Consumption growth follows

$$\Delta c_{t+1} = \mu + \sigma\varepsilon_{t+1}$$

- Shocks can be correlated across countries.

Habit: Model Setup (2/2)

- Assume a log surplus consumption ratio of

$$x_{t+1} = (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)(\Delta c_{t+1} - \mu)$$

- with a sensitivity function $\lambda(x_t)$

$$\lambda(x_t) = \begin{cases} \frac{1}{\bar{X}}\sqrt{1-2(x_t-\bar{x})} - 1 & \text{when } x < x_{max} \\ 0 & \text{elsewhere} \end{cases}$$

- where surplus consumption ratio has steady-state $\bar{X}$

$$\bar{X} = \sigma\sqrt{\frac{\gamma}{1-\phi-B/\gamma}}$$

- and its log an upper bound of x_{max}

$$x_{max} = \bar{x} + \frac{1-\left(\bar{X}\right)^2}{2}$$

- Note that $\gamma(1-\phi) - B > 0$ for existence of steady state.
- Parameter B nests different SDFs from the literature.

Habit: Moments of SDF

- Under this setup, log SDF is given by

$$m_{t+1} = \log(\delta) - \gamma(\Delta c_{t+1} + \Delta x_{t+1})$$

- The conditional moments are given by

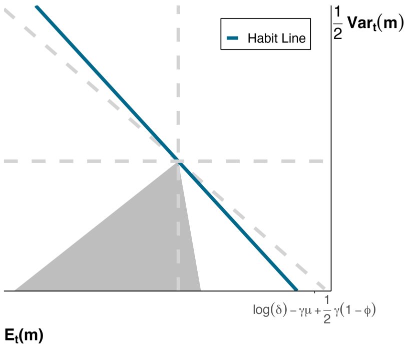
$$\begin{aligned}
\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \gamma\mu + \gamma(1-\phi)(x_t - \bar{x}) \\
\frac{1}{2}\mathsf{var}_t(m_{t+1}) &= \frac{1}{2}\gamma^2(1+\lambda(x_t))^2\sigma^2 \\
&= \frac{1}{2}(\gamma(1-\phi) - B) - (\gamma(1-\phi) - B)(x_t - \bar{x})
\end{aligned}$$

- Substituting out $x_t - \bar{x}$,

$$\begin{aligned}
\frac{1}{2}\mathsf{var}_t(m_{t+1}) = & -\frac{\gamma(1-\phi) - B}{\gamma(1-\phi)}\,\mathbb{E}_t(m_{t+1}) \\
& + \frac{\gamma(1-\phi) - B}{\gamma(1-\phi)}(\log(\delta) - \gamma\mu) + \frac{1}{2}(\gamma(1-\phi) - B)
\end{aligned}$$

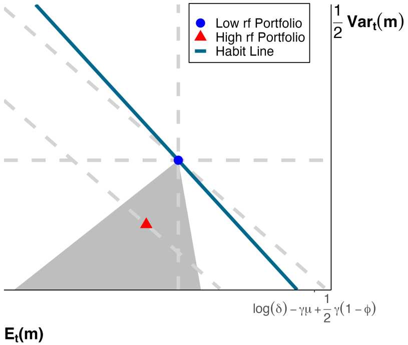
  - Habit models also imply a **functional relationship** between the conditional mean and variance of the log SDFs!

Habit Line in the SDF Space, $\gamma = 2, \phi = 0.98, B = -0.01$



- The Habit line is close to the iso-rf line.
- Helpful to resolve the equity premium puzzle and the risk-free rate puzzle.
- Intuition: $\lambda()$ is specifically designed to balance intertemporal substitution and precautionary saving so that risk-free rate is stable. In fact, when $B = 0$, risk-free rate is constant.

Habit Line in the SDF Space, $\gamma = 2, \phi = 0.98, B = -0.01$



- Countries with the same preferences lie on the same line.
- Can not match the P1 and P2 unless preferences or μ differ across countries.

Habit: International Asset Pricing

Proposition

If preferences are symmetric across countries, then

$$\frac{\mathbb{E}_t(rx_{t+1})}{\mathbb{E}_t(\Delta s_{t+1})} = -\frac{\gamma(1-\phi)-B}{\gamma(1-\phi)}.$$

Because $\gamma(1-\phi)-B>0$ *is required by stationarity, the model cannot satisfy Properties 1 and 2 at the same time.*

Furthermore, if $\gamma(1-\phi)>-B$, *we have* $\frac{-\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{\gamma(1-\phi)}{\gamma(1-\phi)-B} > \frac{1}{2}$*: Appreciation of the high interest currency accounts for more than 50% of the currency premium.*

Habit Models: Simulation

Table: Carry Trade

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	-0.43 [-1.10,0.15]	2.30 [0.90, ∞)$\cup$ ($-\infty$, -6.56]	-	-
Verdelhan (2010) JF	4.54	2.19	0.48	-2.07	Yes	No
Stathopoulos (2017) RFS	-1.23	-2.40	1.95	-0.51	No	No
Heyerdahl-Larsen (2014) RFS	3.48	3.05	0.88	-1.14	Yes	No
Campbell and Cochrane (1999) JPE	-	-	1.00	-1.00	No	No

With Interet Rates

Habit Models: Summary

- Habit models mechanically link the first and second moments of the log SDF to ensure a stable risk-free rate.
- Under complete market and standard calibration, a significant portion of the carry trade return is accounted for by expected change in exchange rates, contradicting the data.

Outline

Going Beyond Log-normality

In general, risk-free rate is given by

$$\begin{aligned} r_{f,t} &= -\log(\mathbb{E}_t\, M_{t+1}) \\ &= -\,\mathbb{E}_t(m_{t+1}) - \underbrace{[\log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})]}_{\text{Entropy, } \Xi_t(m_{t+1})} \end{aligned}$$

The entropy equals $\frac{1}{2}\,\mathsf{var}_t(m_{t+1})$ when log normal.

$\Rightarrow$ We just need to re-label.

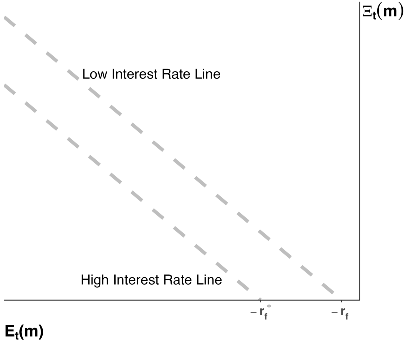
$$\begin{aligned} \mathbb{E}_t(\Delta s_{t+1}) &= \mathbb{E}_t(m_{t+1}) - \mathbb{E}_t(m^*_{t+1}) \\ \mathbb{E}_t(rx_{t+1}) &= \Xi_t(m_{t+1}) - \Xi_t(m^*_{t+1}) \end{aligned}$$

Going Beyond Log-normality

Now for a given r_f,

$$\Xi_t(m_{t+1}) = -\mathbb{E}_t(m_{t+1}) - r_{f,t}$$

represents a negative 45° line in the new "SDF space".



In general, a similar tension between high equity premium, low and stable risk-free rates, and currency premium is present with or without log-normality.

Going Beyond Log-normality: GSV

Gourio, Siemer, and Verdelhan (2013)

- A disaster model with EZ preferences.

We can show that (assuming $\Xi_t(\Delta c_{t+1}) = 0$):

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\,\mathbb{E}(\Delta c_{t+1}) \tag{1}$$

$$+\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\,\mathbb{E}\left[((1-\gamma)u_{t+1}) - \log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right)\right] \tag{2}$$

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\,\mathbb{E}\left[\left(\frac{1}{\psi}-\gamma\right)u_{t+1} - \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right)\right]. \tag{3}$$

We again see a tight relationship between the entropy and the first moment of the SDF.

Going Beyond Log-normality: GSV

- If we set $\psi = 1$,

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\mathbb{E}(m_{t+1}) + \log(\delta) - \mathbb{E}(\Delta c_{t+1}).$$

- All countries lie on the same iso-rf line and share exactly the same risk-free rate.

Going Beyond Log-normality: Skewness

Using cumulant generating function (BFT2001) We show that:

$$
\begin{aligned}
\mathbb{E}_t(m_{t+1}) - \left(\log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1})\right)
&= -\frac{1}{2}\,(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\kappa_{2,t}(u_{t+1}) \\
&\quad - \frac{1}{6}\,(1-\gamma)^2\left(\frac{1}{\psi} - \gamma\right)\kappa_{3,t}(u_{t+1}) + \dots \\
\Xi_t(m_{t+1}) &= \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1}) \\
&= \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \kappa_{2,t}(u_{t+1}) \\
&\quad + \frac{1}{6}\left(\frac{1}{\psi} - \gamma\right)^3 \kappa_{3,t}(u_{t+1}) + \dots
\end{aligned}
$$

Where $\kappa_{i,t}(u_{t+1})$ is the ith cumulant of u_{t+1}.

Going Beyond Log-normality: Skewness

- If we allow only the skewness to differ across countries, we have

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\left(\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\right)^2 \mathbb{E}(m_{t+1}) + \text{constant}$$

- Again, we see a tight functional relationship between $\mathbb{E}(m_{t+1})$ and $\mathbb{E}(\Xi_t(m_{t+1}))$, just like the log-normal case.
- Under standard calibrations, this implies the vast majority of currency premium is accounted for by appreciations.
- The currency premium puzzle generalizes to non-normal cases.

Going Beyond Log-normality : Disaster Models

Table: Carry Trade Returns

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	-0.43 [-1.10,0.15]	2.30 [0.90, ∞)$\cup$ ($-\infty$, -6.56]	-	-
Gourio, Siemer and Verdelhan (2013) JIE	2.36	1.81	0.77	-1.31	Yes	No
Gourio (2012) AER	-	-	1.00	-1.00	No	No
Farhi and Gabaix (2016) QJE (UN)	4.9	3.39	0.69	-1.44	Yes	No
Farhi and Gabaix (2016) QJE (ND)	-	-	0.75	-1.33	Yes	No

The return, FX-share and FX-slope for Gourio, Siemer and Verdelhan (2013) are calculated from their tables 2 and 4; return for Farhi and Gabaix (2016) is from their table III, FX-shares and FX-slopes are calculated using their calibrations in Tables I and II, and their equations (24) and (25). UN stands for unconditional, ND stands for conditional on no disaster in the sample.

Outline

Incomplete Spanning

- Agents have access to their domestic risk-free asset, but not necessarily any foreign assets.
- In this case, a wedge η appears (Lustig and Verdelhan, 2019)

$$\mathbb{E}(\Delta s_{t+1}) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m^*_{t+1}) - \mathbb{E}(\eta_{t+1})$$

- Currency returns are then

$$\begin{aligned}\mathbb{E}(rx_{t+1}) &= \mathbb{E}(r^*_f - r_f) - \mathbb{E}(\Delta s) \\ &= \mathbb{E}(\eta_{t+1}) + \frac{1}{2}\mathbb{E}(\mathrm{var}_t(m_{t+1}) - \mathrm{var}_t(m^*_{t+1}))\end{aligned}$$

Incomplete Spanning

- Agents have access to their domestic risk-free asset, but not necessarily any foreign assets.
- In this case, a wedge η appears (Lustig and Verdelhan, 2019)

$$\mathbb{E}(\Delta s_{t+1}) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m^*_{t+1}) - \mathbb{E}(\eta_{t+1})$$
$$= \mathbb{E}(m_{t+1}) - \mathbb{E}(m^{im,*}_{t+1})$$

- Currency returns are then

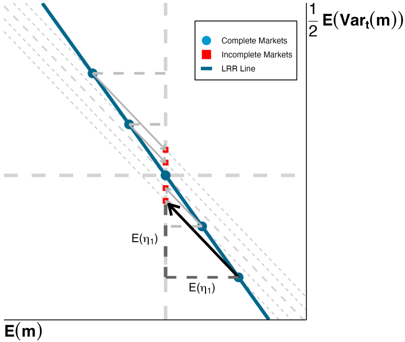
$$\mathbb{E}(rx_{t+1}) = \mathbb{E}(r^*_f - r_f) - \mathbb{E}(\Delta s)$$
$$= \mathbb{E}(\eta_{t+1}) + \frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m_{t+1}) - \mathsf{var}_t(m^*_{t+1}))$$
$$= \frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m_{t+1}) - \mathsf{var}_t(m^{im,*}_{t+1}))$$

where

$$\mathbb{E}(m^{im,*}_{t+1}) \equiv \mathbb{E}(m^*_{t+1}) + \mathbb{E}(\eta_{t+1})$$
$$\frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m^{im,*}_{t+1})) \equiv \frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m^*_{t+1})) - \mathbb{E}(\eta_{t+1})$$

are the incomplete-market-wedge-adjusted moments of the foreign SDF.

Incomplete Spanning: LRR Example



Remarks

- The wedge can only move the foreign country along its iso-rf line!
- Because the LRR line is close to the iso-rf line, it can only generate small risk-free rate differences even with incomplete spanning.
- What model could rationalize each country having just the right $(E[\eta^i])$ to remove FX predictability?

Incomplete Spanning: Properties

- The right wedges ($\mathbb{E}[\eta^i]$) could remove FX predictability.
- However, they would do so by shrinking the currency premia towards zero, thus fixing P2 but exaggerating P1.
- In particular, the wedge does not affect the risk-free rates at all!

$$\begin{aligned}
\mathbb{E}(r^i_{f,t}) &= -\,\mathbb{E}(m^i_{t+1}) - \frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m^i_{t+1})) \\
&= -\left[\mathbb{E}(m^i_{t+1}) + \mathbb{E}(\eta^i_{t+1})\right] - \left[\frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m^i_{t+1})) - \mathbb{E}(\eta^i_{t+1})\right] \\
&= -\,\mathbb{E}(m^{im,i}_{t+1}) - \frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m^{im,i}_{t+1}))
\end{aligned}$$

Incomplete Spanning: Numerical Example

Table: Implied Wedges For Countries on the LRR Line

Country	Return (%)		Change in FX (%)		Interest Rate Diff (%)	Implied Wedge
	Complete	Incomplete	Complete	Incomplete		
1	2.91	0.16	-2.75	0.00	0.16	-2.75
2	1.52	0.08	-1.44	0.00	0.08	-1.44
3	0.00	0.00	0.00		0.00	0.00
4	-1.64	-0.07	1.57	0.00	-0.07	1.57
5	-3.41	-0.12	3.28	0.00	-0.12	3.28

Remark: while the wedge can remove exchange rate predictability, it also takes that part entirely out of the currency premium!

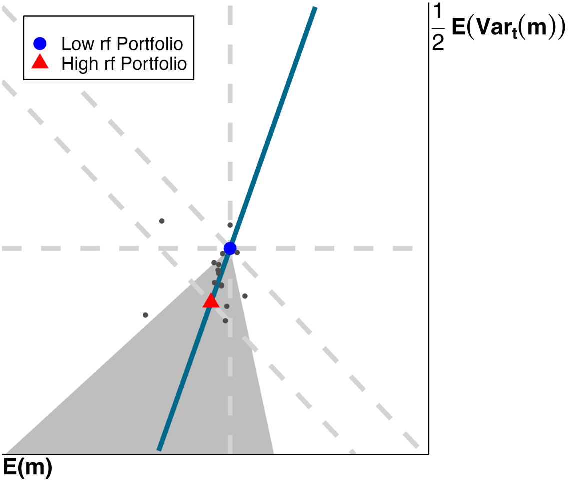
Incomplete Spanning: Summary

- When agents feature the same preference and have access to domestic risk-free assets, incomplete spanning can only remove predicted appreciation of the foreign currency (P2) by shrinking the currency premium by the same amount (P1).
- Incomplete market wedge has no effect on the risk-free rate differences, which are tiny under LRR and habit models by construction.
- The currency premium puzzle can also be thought of as a risk-free rate difference puzzle.

Conclusion

- Canonical models with long-run risk and external habits models link the first and second moments of the SDF.
- This feature helps to resolve closed-economy equity premium and risk-free rate puzzles.
- Internationally, these models either generate negligible currency premia or large predictable FX appreciations, with tiny cross-country interest rate differences: a currency premium puzzle.
- When countries share the same preferences, LRR or habit models cannot match the currency data, regardless of the economic environment.
- Non-normality/Incomplete spanning are not easy fixes.

Appendix: Currency Returns in the SDF Space, Static Trade



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Appendix: Interest Rate Differences

Risk-free rate difference

$$\begin{aligned}\mathbb{E}(r_t^* - r_t) &= \mathbb{E}(m_{t+1}) - \mathbb{E}(m_{t+1}^*) \\ &\quad - \frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m_{t+1}^*) - \mathsf{var}_t(m_{t+1}))\end{aligned}$$

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Appendix: Portfolio Construction

Take the high-interest rate portfolio as an example. The portfolio is given by:

$$\Sigma_{i \in \{\forall i \text{ s.t. } \overline{fp}_i - \overline{fp} > 0\}, t}[rx_{i,t+1}(\overline{fp}_i - \overline{fp})]$$

Where $fp_i = r_f^i - r_f^{US}$ (assuming CIP holds) is the forward premium relative to the US.

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Appendix: Table with Interest Rate Differences

	Return (%)	Change in FX (%)	Interest Rate Diff (%)	FX-share	FX-slope
	$\mathbb{E}(rx)$	$-\mathbb{E}(\Delta s)$	$\mathbb{E}(r^{\star} - r)$	$\frac{-\mathbb{E}(\Delta s)}{\mathbb{E}(rx)}$	$-\frac{1}{\text{FX-share}}$
Static Trade	3.46	-1.30	4.76	-0.37	2.67
	[1.18,5.54]	[-3.82,0.60]	[1.30,8.46]	[-1.16,0.23]	[0.86, ∞)$\cup$ ($-\infty$, -4.42]

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Appendix: Long-Run Risk: General Case

Let $V_t = \frac{U_t}{C_t}$, we have

- Assuming u_{t+1} is normal,

$$\begin{aligned}
\mathbb{E}(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mu \\
&\quad - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}\left(\mathsf{var}_t(v_{t+1} + \Delta c_{t+1})\right) \\
\frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m_{t+1})) &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_{SR}^2 - \frac{1}{\psi}\left(\frac{1}{\psi} - \gamma\right)\sigma_{SR}^2 \\
&\quad + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\mathsf{var}_t(v_{t+1} + \Delta c_{t+1}))
\end{aligned}$$

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Appendix: CCGR Setup

- Model taken from Colacito, Croce, Gavazzoni, and Ready (2018).
- Endowments in country i:

$$y_{i,t} = \mu + y_{i,t-1} + z_{i,t-1} - \tau(y_{i,t-1} - \frac{1}{N}\sum_j y_{j,t-1}) + \varepsilon_{i,t}^{SR}$$

- Long-run risk

$$z_{i,t} = \rho z_{i,t-1} + (1 + \beta_i^z)\varepsilon_{global,t}^{LR} + \varepsilon_{i,t}^{LR}$$

- Consumption bundle

$$C_t^i = (I_{i,t}^i)^\alpha \left(I_{j,t}^i\right)^{1-\alpha}$$

- CCGR considers five countries with β_i^z ranging from -0.65 to 0.65.
- Get closed-form solutions by using risk-adjusted affine approximation

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Appendix: Long-Run Risk, Additional Results

$$\left(1-\frac{1}{\psi}\right)\mathbb{E}(rx_{t+1})=\left(\gamma-\frac{1}{\psi}\right)\mathbb{E}(r^*_{f,t}-r_{f,t})$$
$$-\left(1-\frac{1}{\psi}\right)\mathbb{E}(\Delta s_{t+1})=(\gamma-1)\,\mathbb{E}(r^*_{f,t}-r_{f,t})$$

Additional Results

- If $\psi<1$, $\mathbb{E}(rx_{t+1})$ has opposite sign of $\mathbb{E}(r^*_{f,t}-r_{f,t})$: high interest rate currency yields negative currency premium.
- If $\psi=1$, $\mathbb{E}(r^*_{f,t}-r_{f,t})=0$, interest rates are identical across countries;
- If $\psi>1$, $-\mathbb{E}(\Delta s_{t+1})=\frac{\gamma-1}{1-\frac{1}{\psi}}\,\mathbb{E}(r^*_{f,t}-r_{f,t})$, high interest rate currency appreciates unconditionally.

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Long-run Risk Models: Simulation with rf

Table: Carry Trade Returns

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 $[0.90, \infty) \cup (-\infty, -6.56]$	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	4.47	2.76	1.71	0.62	-1.62	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	-0.09	-0.52	0.44	6.11	-0.16	No	No
Bansal and Shaliastovich (2013) RFS	-0.03	-0.26	0.23	9.54	-0.10	No	No
Colacito and Croce (2013) JF	0.05	-0.35	0.41	-7.02	0.14	No	No

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Long-run Risk Models: Simulation with rf

Table: Static Trade Returns

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $-\mathbb{E}(\Delta s^{st})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,st} - r^{st})$	FX-share $\frac{-\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]	2.67 [0.86, ∞)$\cup$ ($-\infty$, -4.42]	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	7.10	5.98	1.12	0.93	-1.07	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	0.00	0.00	0.00	0.98	-1.02	No	No
Bansal and Shaliastovich (2013) RFS	0.00	0.00	0.00	0.96	-1.04	No	No
Colacito and Croce (2013) JF	0.00	0.00	0.00	0.95	-1.05	No	No
Bansal and Yaron (2004) JF	-	-	-	0.94	-1.06	No	No

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Habit Models: Simulation with rf

Table: Carry Trade

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 [0.90, ∞)$\cup$ ($-\infty$, -6.56]	-	-
Verdelhan (2010) JF	4.54	2.19	2.35	0.48	-2.07	Yes	No
Stathopoulos (2017) RFS	-1.23	-2.40	1.17	1.95	-0.51	No	No
Heyerdahl-Larsen (2014) RFS	3.48	3.05	0.43	0.88	-1.14	Yes	No
Campbell and Cochrane (1999) JPE	-	-	-	1.00	-1.00	No	No

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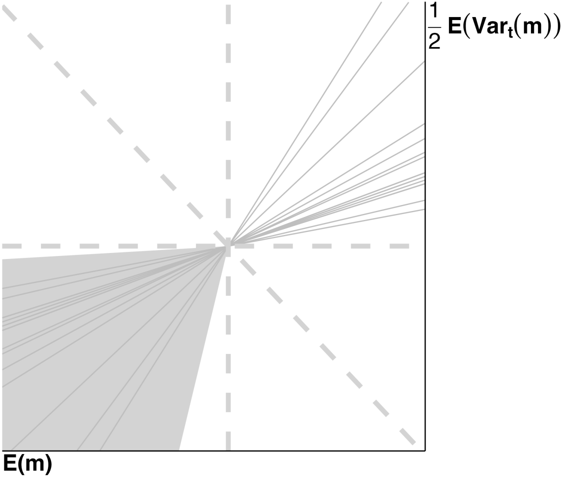
Appendix: Estimation of FX-slope in HM2019

Table: Estimation of FX-slope

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Horizons (months)	1	1	6	12	1	1	6	12
Sample	1 Reblance				3 Reblance			
Static T: β^{stat}	0.47	0.37	0.56	0.60	0.26	0.18	0.26	0.25
	[0.31, 0.63]	[0.19, 0.55]	[0.36, 0.76]	[0.40, 0.80]	[0.16, 0.36]	[0.08, 0.28]	[0.18, 0.34]	[0.13, 0.37]
Static T: FX-slope	0.89	0.59	1.27	1.50	0.35	0.22	0.35	0.33
	[0.46, 1.68]	[0.24, 1.20]	[0.57, 3.10]	[0.68, 3.90]	[0.19, 0.56]	[0.09, 0.39]	[0.22, 0.51]	[0.15, 0.58]
Carry T: β^{ct}	0.68	0.55	0.62	0.71	0.57	0.45	0.42	0.43
	[0.15, 1.21]	[0.04, 1.06]	[0.05, 1.19]	[0.20, 1.22]	[0.20, 0.94]	[0.10, 0.80]	[0.01, 0.83]	[0.06, 0.80]
Carry T: FX-slope	2.13	1.22	1.63	2.45	1.33	0.82	0.72	0.75
	$[0.18,+\infty)\cup(-\infty,-5.78]$	$[0.04,+\infty)\cup(-\infty,-17.78]$	$[0.05,+\infty)\cup(-\infty,-6.31]$	$[0.25,+\infty)\cup(-\infty,-5.55]$	[0.25, 16.36]	[0.11, 4.07]	[0.01, 4.94]	[0.06, 4.06]
Sample	6 Reblance				12 Reblance			
Static T: β^{stat}	0.23	0.15	0.25	0.24	0.34	0.23	0.31	0.30
	[0.13, 0.33]	[0.05, 0.25]	[0.17, 0.33]	[0.14, 0.34]	[0.18, 0.50]	[0.05, 0.41]	[0.15, 0.47]	[0.14, 0.46]
Static T: FX-slope	0.30	0.18	0.33	0.32	0.52	0.30	0.45	0.43
	[0.15, 0.49]	[0.05, 0.33]	[0.21, 0.49]	[0.17, 0.51]	[0.22, 0.99]	[0.06, 0.68]	[0.18, 0.88]	[0.17, 0.84]
Carry T: β^{ct}	0.56	0.45	0.45	0.11	0.67	0.52	0.57	0.22
	[0.21, 0.91]	[0.12, 0.78]	[0.08, 0.82]	[-0.16, 0.38]	[0.36, 0.98]	[0.21, 0.83]	[0.26, 0.88]	[-0.11, 0.55]
Carry T: FX-slope	1.27	0.82	0.82	0.12	2.03	1.08	1.33	0.28
	[0.26,10.47]	[0.13,3.61]	[0.08,4.63]	[-0.14,0.62]	[0.55, 59.98]	[0.26, 5.01]	[0.34, 7.59]	[-0.10, 1.24]

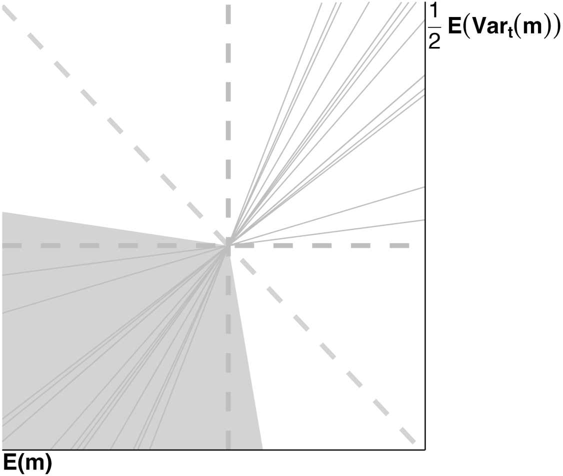
Appendix: Estimation of FX-slope in HM2019

Figure: Estimation of FX-slope: Static Trade



Appendix: Estimation of FX-slope in HM2019

Figure: Estimation of FX-slope: Carry Trade



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