Currency Risk and Capital Accumulation*

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December 2021

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Abstract

The "Lucas Paradox" states that there are large and persistent differences in capital-output ratios across countries, suggesting capital is not flowing to countries where it is relatively scarce. In the data, capital-output ratios vary a lot cross-sectionally even within developed countries, and they are negatively correlated with currency risk premia and risk-free rates. To rationalize these patterns, I build a quantitative multi-country model of capital accumulation with external habit and heterogeneous exposures to a global productivity shock. I show that currency risk in this model generates cross-country variations in risk-free rates and capital-output ratios that are consistent with the data. I estimate the model using GDP data from countries issuing the G10 currencies and find two main results: (1) The heterogenous loadings that I extract from GDP data alone are highly correlated with capital-output ratios; and (2) when I feed the estimated loadings into the model, model-generated capital-output ratios account for roughly 55% of the cross-country variation in the data. I conclude that variation in currency risk and therefore currency risk premia have significant effects on the real economy.

Keywords: currency risk, currency return, capital-output ratio, external habit, safe currency
JEL Codes: F31, F41, G12

*I am grateful for advice, support, and guidance from Tarek A. Hassan, Thomas M. Mertens, Stephen J. Terry, and Andrea Vedolin during this project. For helpful discussions and comments, I thank Marianne Baxter, Joaquin Blaum, Anusha Chari, Javier Cravino, Adam Guren, Leyla Han, Anna Pavlova, Pascual Restrepo, Robert Richmond, Maxime Sauzet, Rosen Valchev, Adrien Verdelhan, Tony Zhang, and seminar participants at the Boston University Macro Dissertation Workshop, Questrom School of Business Brownbag Seminar, and Shanghai University of Finance and Economics (School of Finance). All errors are my own.

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1. Introduction

Differences in capital-output ratios are large and persistent across countries, a phenomenon sometimes referred to as the "Lucas Paradox" (Lucas (1990)). A lesser-known fact is that this paradox is also present within developed countries. For example, the capital-output ratio in Japan is 44% higher than in New Zealand. Everything else being equal, this suggests that there are significant and long-lasting cross-country differences in returns to capital, and that capital is not flowing across borders to eliminate such differences. In this paper, I argue that currency risk induces persistent differences in required rates of return to capital across countries and is a quantitatively important driver of cross-country variations in capital-output ratios.

A growing literature has established that some currencies are "safer" in the sense that they appreciate in global bad times. This property is widely used to understand cross-country variations in currency risk premia and risk-free rates.[1] Again, using Japan and New Zealand as examples, as a well-known safe currency, the Japanese yen offers a 5.70% lower annual return on average than the New Zealand dollar, which is generally considered risky. Intuitively, from an international investor's perspective, risk-free bonds denominated in safe-haven currencies are attractive because they offer higher returns in global downturns and are thus good hedges against global bad times. In equilibrium, these bonds should feature lower risk-free rates, leading to lower currency risk premia (see, for example, Lustig et al. (2011), Hassan (2013), Richmond (2019), Ready et al. (2017), and Colacito et al. (2018a)). However, by the same logic, investing in capital in countries with safe-haven currencies should also be attractive. If currency risk induces systematic variations in risk-free rates and currency risk premia across countries,[2] it should pass through to the required returns to capital.[3] Countries with safe-haven currencies should thus feature a lower cost of capital and have higher capital-output ratios.

In this paper, I study the link between currency risk and capital-output ratios by endogenizing capital accumulation within a quantitative international asset-pricing model. To induce different responses to global downturns and thus currency riskiness, I follow Lustig

[1] Examples are Lustig and Verdelhan (2007), Lustig et al. (2011),Lustig et al. (2014), Menkhoff et al. (2012), Hassan (2013), Lettau et al. (2014), Farhi and Gabaix (2016), Corte et al. (2016), Maggiori (2017), Ready et al. (2017), Mueller et al. (2017), Colacito et al. (2018a), Richmond (2019), Wiriadinata (2021), Jiang (2021), among others.

[2] Empirically, risk-free rate differences across countries are large and persistent, and they are linked to the safeness of the corresponding currency. See Lustig et al. (2011) and Hassan and Mano (2018), for example.

[3] Richers (2021) finds direct empirical evidence for the passthrough of currency risk premia to required rates of return to capital. Specifically, he finds that violation of the uncovered interest rate parity strongly passes through to firm borrowing and the cost of capital.

et al. (2011) and allow countries' productivities to differ in their exposures to a global shock.[4] I then generate large currency premia and risk-free rate differences as in the data, which is proven to be a challenge in the literature,[5] using external habit. Finally, I endogenize capital accumulation within this framework and show that the model can generate cross-country variations in currency risk premia, risk-free rates, and capital-output ratios in a way that is quantitatively consistent with the data.

I estimate the exposures to the global shock using GDP data from countries issuing the G10 currencies.[6] Two main findings emerge: First, the estimated exposures I extract from correlations of GDP across countries alone are highly correlated with capital-output ratios, suggesting heterogeneous exposures to a global shock, which is often seen as an asset-pricing tool to rationalize currency risk premia, are tightly linked to economic fundamentals and have implications on the real economy. Second, when I feed the estimated exposures into my model, the model-generated cross-country variations in capital-output ratios can account for roughly 55% of that in the data. In this sense, currency risk can explain half of the cross-country variation in capital-output ratios.

To better motivate my approach, I first explore the cross-country variations in capital-output ratios and currency risk premia, and the correlations between them in the data. I define the currency risk premium of country i relative to the U.S. dollar, $\mathbb{E}_t(rx^i_{t+1})$, as the expected return a U.S. investor would get if she borrows at the risk-free rate in the U.S. and invests it in a foreign risk-free bond:

$$\mathbb{E}_t(rx^i_{t+1}) = r^i_{f,t} - \mathbb{E}_t[\Delta ex_{t+1}] - r^{U.S.}_{f,t}, \tag{1}$$

where $r^i_{f,t}$ and $r^{U.S.}_{f,t}$ denote the risk-free rates of country i and the U.S. respectively, and Δex_{t+1} is the change in exchange rates quoted in units of country i currencies per U.S. dollar.[7] If currency safety jointly determines currency risk premia and capital-output ratios,

[4]These exposures can be seen as a reduced-form representation of a number of mechanisms that can induce currency risk, for example, country size (Hassan (2013)), trade centrality (Richmond (2019)), and financial development (Maggiori (2017)). See Hassan and Zhang (2021) for a survey.

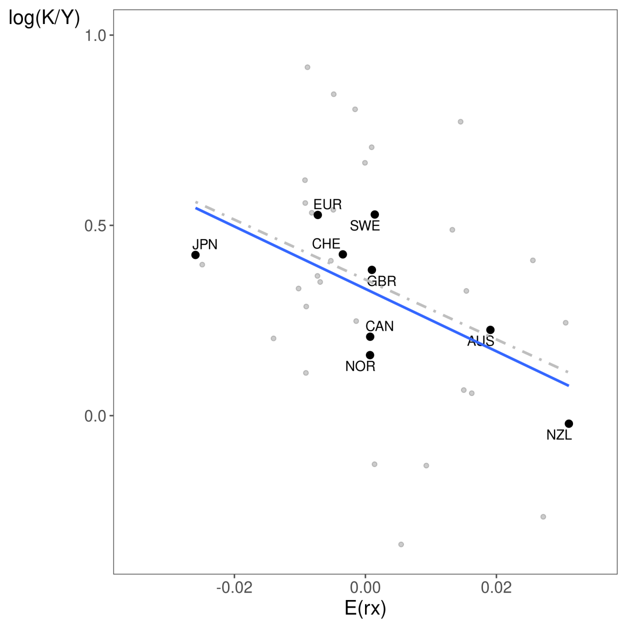
[5]Models with conventional CRRA preferences tend to generate small currency premia, see, for example, Hassan (2013), Richmond (2019), and Ready et al. (2017). Models with Epstein and Zin (1989) preferences tend to generate small risk-free rate differences, see Colacito et al. (2018a) and Gourio et al. (2013).

[6]These countries are: Australia, Canada, the euro area, Great Britain, Japan, Norway, New Zealand, Sweden, Switzerland, and the U.S. Throughout the paper, I treat the euro area as one country. In addition, I always use G10 to refer to countries issuing the G10 currencies (the most traded ten currencies), not the usual the Group of Ten countries.

[7]In the data, the expected change in exchange rates is close to zero unconditionally (see Hassan and Mano (2018)), so cross-country variations in risk-free rates and currency risk premia are quantitatively very similar. In this paper, I focus on currency risk premia because global investors should take into account changes in exchange rates when making investment decisions, but all results are similar if I use risk-free rate differences instead. All results using risk-free rates are available upon request.

we would expect countries with safe currencies to be associated with lower currency risk premia and higher capital-output ratios. In our Japan-New Zealand example, it is indeed true that Japan features a high capital-output ratio and a low currency risk premium.

Figure 1: Log Capital Output Ratios and Currency risk premia



This figure plots unconditional log capital-output ratios relative to the U.S. against unconditional currency risk premia relative to the U.S. for countries issuing the G10 currencies (black dots) and a broader sample of 37 currencies (grey dots). The line of best fit has a slope of -8.21 (s.e. 3.12) for G10 (solid blue line) and -7.87 (s.e. 3.36) for the broader sample (dashed grey line). All moments are annual.
Data source: Capital stock and output are from PWT 10.0. Currency risk premia are from Adrien Verdelhan's website. Data ranges from 1994 to 2019. Details on data construction can be found in section 4.1 and Appendix B.

Figure 1 confirms this conjecture for a broader set of countries. In Figure 1, I plot log capital-output ratios relative to the U.S. against U.S.-based currency risk premia for countries issuing the G10 currencies. We can clearly see that there are indeed large variations in capital-output ratios and currency risk premia across countries. In addition, there is a strong negative relationship between capital-output ratios and currency risk premia (solid blue line): countries with lower currency risk premia and thus lower risk-free rates accumulate more capital. To the extent that currency risk premia are tightly connected to currency risk, Figure 1 empirically links currency risk to capital-output ratios.

Although the Lucas Paradox is often referred to as a phenomenon between developed (rich) and developing (poor) countries, Figure 1 shows that it also extends to the developed world, which is perhaps even more puzzling. For two reasons, I consciously restrict my sample to countries issuing the G10 currencies:[8] (1) All the countries in this group are Western developed countries and are relatively homogeneous in terms of institutions, tax systems, and market structures, which mitigates the concern that these factors might confound my analysis;[9] and (2) this set of countries closely resembles complete markets, an assumption underpinning my model. Nevertheless, if I extend my sample to a set of 37 countries (grey dots in Figure 1), the same pattern persists: Large cross-country variations exist in capital-output ratios and currency risk premia across countries, and a clear negative relationship exists between the two. In fact, the line of best fit (dashed grey line) has almost the same slope in the larger sample as for the G10 currencies (solid blue line), and both slopes are significant at the 5% level. Therefore, I conclude that capital-output ratios and currency risk premia vary significantly across countries, and they are negatively correlated with each other in the data.

Motivated by this empirical fact and recognizing that currency risk premia are tightly connected to currency risk, I extend a standard international asset-pricing model of currency risk to incorporate capital accumulation and study the link between currency risk and capital-output ratios. My model is based on two key ingredients. First, I allow countries to have different loadings (exposures) on a global productivity shock; second, agents in my model extract utility from an exogenously given habit level.

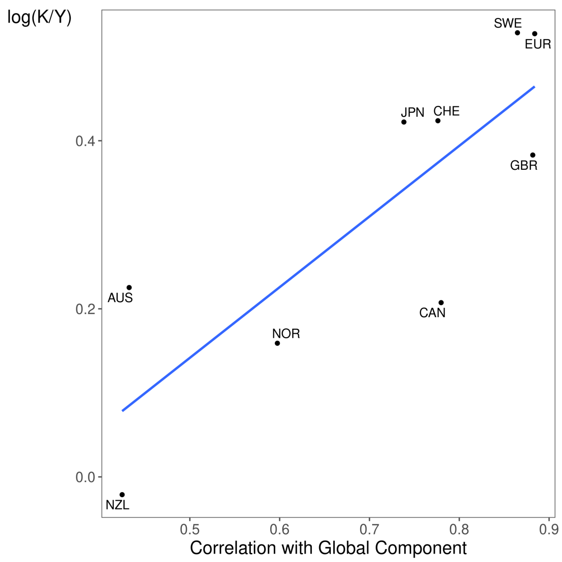
Heterogeneous loadings on a global shock is widely used in the international asset-pricing literature to link currency risk to cross-country variations in currency risk premia.[10] Extending this framework to a general equilibrium model with capital accumulation, my model generates cross-country variation in capital-output ratios in addition to currency risk premia in a way that is consistent with the data. Intuitively, when a global negative shock hits, the country-specific good of the high-loading country becomes relatively more expensive. With home bias, the price of the consumption bundle of said country increases, which translates to an appreciation of the real exchange rate. In this sense, the high-loading currency appreciates in global bad times so that assets denominated in this currency are safer. In equilibrium, global investors require a lower return on assets of high-loading countries, including risk-free

[8]Note that as the base country, U.S. does not show up in Figure 1, so my sample has effectively nine countries. I make this choice because (1) currency risk premia need a base country to calculate, and the U.S. dollar is the standard choice; and (2) as the center of the global trade, financial, and political network, the U.S. is known to be special in terms of capital accumulation, so I leave the proper treatment of the U.S. for future research.

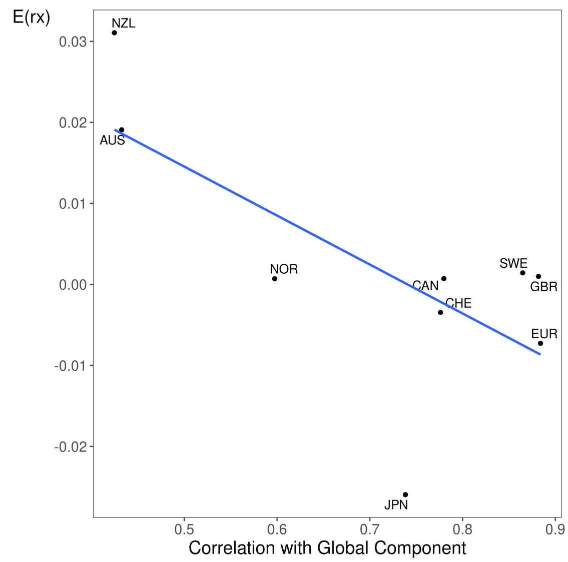
[9]In Appendix A, I show that the negative correlations between capital-output ratios and currency risk premia are robust to controlling for financial development, governance efficiency, and financial openness, suggesting that these factors are not the main drivers of cross-country variation in capital-output ratios within the countries issuing G10 currencies.

[10]Examples are Lustig et al. (2011, 2014), who assume SDFs load differently on a global shock, and Colacito et al. (2018a), who assume long-run endowment processes load differently on a global shock.

Figure 2: Correlations with Global Component



(a) Capital-Output Ratio



(b) Currency Risk Premium

This figure plots log capital-output ratios (panel(a)) and currency risk premia (panel(b)) relative to the U.S. against correlations of each country's HP-filtered log GDP with the cross-country average (the global component). The line of best fit has a slope of 0.84 (s.e. 0.22) for capital-output ratios and a slope of -0.06 (s.e. 0.02) for currency risk premia.
Source: Capital stock and output data are from PWT 10.0. Currency risk premia data are from Adrien Verdelhan's website. GDP data are from OECD National Accounts Statistics. Data ranges from 1994 to 2019. Details on data construction can be found in section 4.1.

assets and investments in capital, and high-loading countries feature lower risk-free rates, lower required rates of return to capital, and install more capital. Heterogeneous loadings on a global shock thus induces cross-country variations in currency risk premia, the required returns to capital, and capital-output ratios.

Heterogenous loadings on a global shock also have implications on correlations of output across countries. In particular, output of countries with higher exposure to the global shock should covary more with world output. I show direct empirical evidence for this in Figure 2. Panel (a) plots capital-output ratios against the correlation of each country's HP-filtered log GDP with a global component (the cross-country average of HP-filtered log GDP); panel (b) plots the same figure for currency risk premia. As we can clearly see, if a country's GDP correlates more with the world, it features a lower currency risk premium and a higher capital-output ratio.

Motivated by this empirical fact, in my estimation exercise, I extract the loadings on the global shock solely from correlations of GDP with the average GDP across countries, and show that the estimated loadings are highly correlated with currency risk premia and capital-output ratios. This fact in itself is interesting: the correlation of GDP with the world average is informative of how much capital a country installs. In this sense, I trace heterogenous loadings on a global shock, which is typically used to understand currency returns and is often linked to asset prices,[11] to a fundamental macro variable, namely GDP, and show that it has implications for capital accumulation.

While heterogeneous exposures to a global shock can induce cross-country variations in currency risk premia and capital-output ratios that are qualitatively consistent with the data, external habit is essential for the quantitative success of the model. There is a quantitative challenge in generating currency risk premia as large as in the data using standard CRRA preferences, similar to the famous equity premium puzzle:[12] consumption growths are too smooth to generate large risk premia unless an implausibly high risk aversion is assumed. I thus use external habit, a modeling device that is proven to be successful in resolving the equity premium puzzle, to boost currency risk premia and cross-country variations in capital-output ratios to their empirical levels.

In my model, agents extract utility from their consumption over an externally given habit level (Campbell and Cochrane (1999)), which is approximately a geometric average of the history of consumption. Thus, their marginal utility of consumption is jointly determined by consumption and the habit level. Because the habit level is also sensitive to consumption, overall, an agent's marginal utility of consumption is much more responsive to changes in consumption than without habit. This feature effectively makes the agents more risk-averse to consumption risk and helps the model generate large risk premia. Following Chen (2017), I assume constant sensitivity of habit with respect to changes in consumption, as opposed to time-varying sensitivity in standard habit models,[13] and I set the sensitivity parameter to be the same across countries. Under this setup, my model generates unconditional cross-country variations in currency risk premia and capital-output ratios comparable to the data.

In addition to generating large currency risk premia, using external habits in a heterogenous country set-up also resolves a more subtle quantitative puzzle in international asset pricing. In the data, currency risk premia are mostly accounted for by interest-rate differences, not the expected changes in exchange rates. Taking Japan and New Zealand as an example, the currency risk premium between the two countries is 5.70%, with 5.08% com-

[11] For example, Colacito et al. (2018a) estimates these loadings using dividend data.

[12] See Hassan and Zhang (2021) for a broad discussion of how the cross-country "currency risk premium puzzle" and the equity premium puzzle are related.

[13] Chen (2017) shows that once capital accumulation is endogenized, production and precautionary motives endogenously generate countercyclical volatility of marginal utility, and thus time-varying sensitivity function is unnecessary. In Section 3.3 I show that in my setup, constant sensitivity is also important in generating large currency risk premia.

ing from interest-rate differences.[14] Existing quantitative models tend to struggle with this fact.[15] My model with external habit and heterogenous loadings on a global shock generates a currency risk premium of 6.06% between Japan and New Zealand, all of which comes from interest-rate differences: the expected change in exchange rates is exactly zero in my model. To my knowledge, my model is the first risk-based general-equilibrium heterogenous-country framework that generates unconditional interest-rate differences as large as in the data.

To evaluate the quantitative performance of the model, I estimate the loadings on the global shock using GDP data of countries issuing the G10 currencies, then simulate the model taking the estimated loadings as given and contrast the simulated capital-output ratios with the data. In a variance decomposition exercise, I find that the estimated model accounts for roughly 55% of the cross-country variation in capital-output ratios in the data. Finally, I conduct robustness checks by exploring how habit, the degree of home bias, and the elasticity of substitution between different goods affect my results.

My paper makes contributions to three strands of literature. First, it contributes to the large body of work rationalizing the significant, persistent difference in capital-output ratios across countries. Existing explanations include heterogeneity in the protection of property rights (Hall and Jones (1997)), in the capital share of output (Karabarbounis and Neiman (2014)), in the misallocation of resources (Hsieh and Klenow (2009)), in natural resource endowments (Caselli and Feyrer (2007) and Monge-Naranjo et al. (2019)), and in institutional quality (Alfaro et al. (2008)). I show that currency risk alone can induce significant cross-country variations in capital-output ratios. Closely related to my work, David et al. (2014) links capital-output ratios to correlations of returns to capital with the U.S. stock market. Chari and Rhee (2020) argue that financial returns across countries are similar, which provide little incentive for capital to flow across boarders even when marginal products of capital differ. I focus on currency risk instead and show that currency risk is a key driver of capital-output ratios among the developed G10 countries.

Second, this paper complements the literature on currency risk with a quantitative approach. Although various risk-based explanations exist on why some countries have lower interest rates than others, almost all of them are either reduced form (Lustig et al. (2011,

[14] In fact, Hassan and Mano (2018) shows that one can not reject the null that expected change in exchange rates is zero for a representative country.

[15] For example, Gourio et al. (2013) study a model in which countries differ in the severity of disasters and agents feature Epstein and Zin (1989) preferences. They generate a currency risk premium of 2.36%, with only 0.55% accounted for by interest-rate differences; Colacito et al. (2018a) also study a model with Epstein and Zin (1989) preferences, and they focus on heterogeneous exposures to a long-run global shock. Their model generates a carry trade return of around 3%, with about 1% coming from interest-rate differences. In ongoing work, Hassan et al. (2021a), we show that this problem is embedded in the Epstein and Zin (1989) preferences.

2014), Verdelhan (2018)) or qualitative (e.g., Hassan (2013), Richmond (2019), Ready et al. (2017), Maggiori (2017)). I adapt external habit preferences to a heterogenous-country setup and show that such preferences boost the difference in the volatility of stochastic discount factors across countries and thus generate currency risk premia and risk-free rate differences comparable to the data. In this sense, my work relates closely to Colacito et al. (2018a), who quantitatively match the size of currency risk premia within a long-run risk framework, but without endogenizing capital accumulation.[16]

Also, the majority of the currency risk literature only focuses on asset prices and does not consider capital accumulation; this paper is one of the few that studies the real implications of the advancements in international asset pricing. Colacito et al. (2018b) study capital flow patterns in an international setup with long-run and short-run risk and symmetric countries. Hassan et al. (2016) links stochastic properties of real exchange rates to capital accumulation in a qualitative framework. My paper features heterogeneous countries and can generate economically significant variation in capital-output ratios comparable to the data. Hassan et al. (2021b) study the effect of exchange rate stabilization on capital accumulation and find that stabilizing relative to a safe currency promotes capital accumulation.

Third, this paper builds on several others applying external habit models to international setups. Verdelhan (2010) uses external habit with time-varying risk-free rates to explain the UIP puzzle. Stathopoulos (2017) further allows for risk-sharing and home bias across countries and resolves the Backus and Smith (1993) puzzle. Heyerdahl-Larsen (2014) uses deep habit to rationalize a series of puzzles in the international finance literature. However, all of these papers feature symmetric countries and thus yield no unconditional differences in risk-free rates. They also feature endowment economies and thus do not endogenize capital accumulation. Chen (2017) studies an external habit model with capital accumulation, but he focuses only on the closed economy. I extend his framework to a heterogeneous-country setup, and study its implications for cross-country variations in capital accumulations.

The rest of my paper is organized as follows. In Section 2, I set up the model and derive the optimality conditions. In Section 3 , I solve a simplified two-period, two-country version of the model to study the mechanisms. In Section 4, I solve and estimate the full model and conduct robustness checks. I conclude in Section 5.

[16] An added difficulty for the literature using Epstein and Zin (1989) preferences to match currency risk premia is that strong forces in these models push for return differentials to be generated by predicted appreciations rather than interest differentials (another example is Gourio et al. (2013)), an issue I avoid in my setup by using habit preferences and elaborate on further below.

2. The Model

In this section, I extend a standard multi-country international asset-pricing model to incorporate capital accumulation and heterogeneous countries, as well as external habits. The basic structure of the model largely builds on Colacito et al. (2018a), but deviates from it in two critical ways: First, I use external habits to generate large currency risk premia and risk-free rate spreads across countries; second, I endogenize capital accumulation and link currency risk to capital-output ratios. The economy within each country closely resembles the closed-economy model of Chen (2017).

2.1. Setup

2.1.1. Households

There are N countries indexed by $i = \{1, 2, \dots, N\}$, each populated by a unit measure of households. Households in country i extract utility from consumption over an externally given habit level H_t^i, and maximize

$$\mathbb{E}_0 \sum_{t=0}^{T} \eta^t \frac{(C_t^i - H_t^i)^{1-\gamma} - 1}{1-\gamma},$$

where η is the time discount factor, C_t^i denotes consumption, and γ governs relative risk aversion. T is a terminal period (which I set to 1 in Section 3 and ∞ in all other sections). Following Campbell and Cochrane (1999), instead of directly specifying an exogenous process for H_t^i, I assume that the surplus consumption ratio, $S_t^i = \frac{C_t^i - H_t^i}{C_t^i}$, follows

$$s_{t+1}^i = (1-\rho_s)\bar{s} + \rho_s s_t^i + \lambda_s(\Delta c_{t+1}^i - \mu), \tag{2}$$

where $s_t^i = \log(S_t^i)$. Throughout the paper, lowercase letters denote logs so that $x = \log(X)$. $\bar{s}$ is the steady-state level of s_t^i, ρ_s governs the persistence of surplus consumption ratio, and μ is the steady-state growth rate of technology. All preference parameters, as well as $\bar{s}$, ρ_s and μ are assumed to be the same across countries.

$\lambda_s \geq 0$ governs the sensitivity of the log surplus consumption ratio to consumption growth. Following Chen (2017), I deviate from the standard external habit model by setting λ_s to be a constant instead of a function. In particular,

$$\lambda_s = \frac{1}{\bar{S}} - 1$$

is set to be the Campbell and Cochrane (1999) steady-state value. There are two reasons for this deviation. First, the sensitivity function $\lambda(\cdot)$ in Campbell and Cochrane (1999) is formulated to ensure a constant risk-free rate. Once capital accumulation is introduced, capital offers an alternative way of transferring resources across time and the risk-free rate is stable even with a constant sensitivity parameter λ_s.[17] Second, as I show in Section 3.3, setting λ_s to be a constant and to be the same across countries helps with generating large currency risk premia that behave as in the data.

Finally, households supply one unit of labor inelastically.

2.1.2. Firms

Each household owns a firm that produces a country-specific good Y_t^i. The production process is identical across all firms within a country and can be summarized by the production function

$$Y_t^i = e^{z_t^i}(K_t^i)^\alpha (e^{\mu t} N_t^i)^{1-\alpha}, \tag{3}$$

where K_t^i denotes capital and N_t^i denotes labor. α is the capital's share of output. The productivity process z_t^i follows

$$z_{t+1}^i = \rho z_t^i + \beta_z^i \sigma_g \varepsilon_{z,t+1}^g + \sigma^i \varepsilon_{z,t+1}^i, \tag{4}$$

where ρ governs the persistence of the productivity process and is assumed to be the same across countries. z_t^i is subject to two shocks: a country-specific shock $\varepsilon_{z,t+1}^i \overset{i.i.d.}{\sim} N(0,1)$ and a global shock $\varepsilon_{z,t+1}^g \overset{i.i.d.}{\sim} N(0,1)$. σ^i and σ_g are the corresponding volatilities. I assume that all shocks are orthogonal to each other.

Following the international asset-pricing literature, I assume that each country's productivity has a different loading β_z^i on the global shock. β_z^i can be interpreted as a closed-form way to capture any country heterogeneities that may induce one currency to be safer than others. While asset prices are typically used to estimate them in the literature, in my framework with capital accumulation and production, these loadings can be directly linked to correlations of outputs, allowing for estimation using GDP data.

Capital accumulation follows

$$K_{t+1}^i = \Phi(I_t^i/K_t^i)K_t + (1-\delta)K_t^i, \tag{5}$$

[17] See Chen (2017) for a complete analysis on the advantage and disadvantages of setting λ_s to be a constant. In particular, Chen (2017) shows that the habit level H_t^i is approximately a geometric average of the history of consumption, and risk-free rates remain stable. I confirm that my model generated smooth risk-free rates in Appendix D.

where δ is the depreciation rate and I_t^i is investment. Following Jermann (1998), I assume that firms face a convex capital adjustment cost:

$$\Phi(\frac{I}{K}) = a_1 + \frac{a_2}{1-\frac{1}{\xi}}\left(\frac{I}{K}\right)^{1-\frac{1}{\xi}},$$

where ξ governs the elasticity of the investment-capital ratio with respect to Tobin's Q. $a_1 = \frac{\exp(\mu)-1+\delta}{1-\xi}$ and $a_2 = (\exp(\mu)-1+\delta)^{\frac{1}{\xi}}$ are chosen so that at the steady state, $\Phi(I/K) = \exp(\mu)-1+\delta$ and $\Phi'(I/K) = 1$.

2.1.3. Final Good and Resource Constraints

Households produce a final good F_t^i according to a Cobb-Douglas aggregator[18]:

$$F_t^i = (X_{i,t}^i)^{\nu} \prod_{j=1}^{N} \left(X_{j,t}^i\right)^{\frac{1-\nu}{N}}, \tag{6}$$

where $X_{j,t}^i$ denotes the amount of country-j good used by a typical household in country i. Following a large literature surveyed by Lewis (2011), I assume households feature home bias and have extra preference for their own good, which is governed by $\nu > 0$. Home bias is important for the model to generate real exchange rate dynamics and differences in returns, which will become clear in Section 3.

The final good can be used for consumption or investment, so the resource constraint for it is given by

$$F_t^i = C_t^i + I_t^i \qquad \forall i, t. \tag{7}$$

The goods market clears for each country specific good

$$Y_t^i = \sum_{j=1}^{N} X_{i,t}^j \qquad \forall i, t. \tag{8}$$

I assume complete market and solve the model by solving a social planner's problem.

2.2. Solving the Model

For simplicity, I assume unit Pareto weights for all households and abstract away from the effect that heterogeneous loadings may have on the initial distribution of wealth across

[18] I use a Cobb-Douglas aggregator for simplicity and for comparison with Colacito et al. (2018a). In Section 4.4.2, I conduct robustness check using a constant elasticity of substitution (CES) aggregator.

countries. The social planner solves

$$\max \sum_{i=1}^{N} \left[\left(\mathbb{E}_0 \sum_{t=0}^{T} \eta^t \frac{(C_t^i - H_t^i)^{1-\gamma} - 1}{1-\gamma} \right) \right]$$

subject to the resource constraints (7) and (8). The first-order condition with respect to consumption is given by:

$$(S_t^i)^{-\gamma}(C_t^i)^{-\gamma} = \Lambda_{C,t}^i \qquad \forall i, t, \tag{9}$$

where $\Lambda_{C,t}^i$ is the shadow price (marginal utility) of consumption. Note that S_t^i, the surplus consumption ratio, directly affects $\Lambda_{C,t}^i$. Because surplus consumption ratio is sensitive to the growth rate rather than the level of consumption (see (2)), the marginal utility of consumption is more sensitive to consumption risk than standard CRRA preferences (where $S_t^i = 1$).

The first-order conditions with respect to each country-specific good are given by:

$$\Lambda_{C,t}^i(\nu + \frac{1}{N}(1-\nu))\frac{F_t^i}{X_{i,t}^i} = \Lambda_{X,t}^i \qquad \forall i, t, \tag{10}$$

$$\Lambda_{C,t}^j \frac{1}{N}(1-\nu)\frac{F_t^j}{X_{i,t}^j} = \Lambda_{X,t}^i \qquad \forall i, j \neq i, t, \tag{11}$$

where $\Lambda_{X,t}^i$ is the shadow prices of the country-specific good of country i. First-order conditions (10) and (11) highlight the role of home bias in generating real exchange rate dynamics. Under complete markets, the log real exchange rate between two countries is given by $Ex_t^{i,j} = \frac{\Lambda_{C,t}^j}{\Lambda_{C,t}^i}$. If $\nu = 0$ and there is no home bias, it can be shown that $\Lambda_{C,t}^i = \Lambda_{C,t}^i$ so that real exchange rate is one.[19]

The Euler equation is given by

$$1 = \mathbb{E}_t(M_{t+1}^i R_{t+1}^i), \tag{12}$$

where the stochastic discount factor (SDF) for households in country i is the growth rate of the marginal utility of consumption:

$$M_{t+1}^i = \eta \frac{\Lambda_{C,t+1}^i}{\Lambda_{C,t}^i}. \tag{13}$$

[19]This is well known in the international asset-pricing literature. Intuitively, without home bias, all countries essentially consume the same final good and it has only one price. This point will become clear in Section 3.1.

The required return to capital is given by

$$(14)\quad R^i_{t+1} = \frac{1}{Q^i_t}\left[(\nu + \frac{1}{N}(1-\nu))\frac{F^i_{t+1}}{X^i_{i,t+1}}\alpha\frac{Y^i_{t+1}}{K^i_{t+1}} - I^i_{t+1}/K^i_t + Q^i_{t+1}(\Phi(I^i_{t+1}/K^i_{t+1}) + 1 - \delta)\right],$$

where $Q^i_t = \frac{1}{\Phi'(I^i_t/K^i_t)}$ is Tobin's Q.

Now we have a recursive system of $(11+N)N$ equations (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), and (14), which determines policy functions for $(11+N)N$ endogenous variables $\{s^i_{t+1}, Y^i_t, z^i_{t+1}, K^i_{t+1}, I^i_t, F^i_t, C^i_t, \Lambda^i_{C,t}, \Lambda^i_{X,t}, M^i_t, R^i_t, \{X^i_{j,t}\}_{j=1,...,N}\}_{i=1,...,N}$.

3. Examining the Mechanism

I begin by examining a simplified version of the model to show clearly the economic forces behind the key results. I first establish how different loadings on a global shock can induce different levels of currency risk, then show how these heterogeneous levels of currency risk determine differences in required rates of return to capital and capital-output ratio in a way that is consistent with the data. I also examine the role of habit in the quantitative success of the model. Specifically, in this section, I consider a two-country, two-period, simplified version of the model and utilize log-linearization (with risk adjustments) to get closed-form solutions.

In particular, in addition to setting $N = 2$ and $T = 1$, I shut down the capital-adjustment cost so that $\Phi(\frac{I}{K}) = \frac{I}{K}$. I set $\delta = 1$ so that capital fully depreciates between periods, and I set $\sigma^\star = \sigma$ so that the only source of heterogeneity between the home and foreign country is the loadings β_z and $\beta^\star_z$. To simplify notation, in this section, I omit the country indicator i and simply denote the foreign country with a star $\star$. I also omit the time indicator for period 1. I assume that the economy is at its deterministic steady state at period 0 and study agents' incentive to accumulate capital for production in period 1.

To get closed-form solutions and make the link between capital accumulation and currency risk explicit, I log-linearize the system around its deterministic steady state[20] where all shocks are 0 and all households hold a capital stock that is fixed at its steady-state level. That is, I study households' incentive to accumulate different levels of capital while holding the initial capital stock fixed.[21]

[20] The log-linearized system can be found in Appendix C.

[21] Fixing the initial capital stock avoids solving for a quadratic system of equations and makes closed-form solutions feasible.

3.1. *Real Exchange Rates and Currency Risk*

I first examine real exchange rates and show how different exposures to the global shock induce currencies to behave differently in global bad times. Under complete markets, changes in log real exchange rates (quoted in units foreign currencies per home currency) are given by the difference between log SDFs (Backus et al. (2001)):

$$\Delta ex = m - m^\star,$$

where m and $m^\star$ are the log SDFs as in equation (13). Under my log-linear solution, the real exchange rate is then given by

$$\Delta ex \approx \underbrace{\frac{\nu\gamma(1+\lambda_s)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}}_{>0}[(\beta_z^\star - \beta_z)\sigma_g\varepsilon_g + \sigma(\varepsilon^\star - \varepsilon)], \tag{15}$$

where I use $\approx$ to highlight the fact that the result relies on a log-linear approximation.[22] As discussed in Section 2.2, home bias ($\nu > 0$) is essential for there to be any variation in exchange rates. If $\nu = 0$, both countries would consume the same consumption bundle, which can only have one price, and the real exchange rate would be constant ($\Delta ex = 0$ in (15)).

Expression (15) shows how different loadings β and $\beta^\star$ are connected to currency risk and induce one currency to be safer than the other. Without loss of generality, assume $\beta > \beta^\star$ so that the home country has a higher loading on the global shock. When a negative global shock hits ($\varepsilon_g < 0$), it is easy to see that $\Delta ex > 0$ so that the home currency appreciates. The intuition is as follows: when a negative global shock hits, the higher-loading, more affected home country experiences a deeper drop in its productivity and produces less of its country-specific good, which becomes relatively expensive. Because of home bias ($\nu > 0$), its consumption bundle also becomes relatively expensive, resulting in appreciation of the home currency because the real exchange rate is defined as the relative price of the two countries' final consumption goods. In this sense, the higher loading home currency appreciates in global bad times. Any assets whose payments are fixed in units of its currency (such as its risk-free bond) are thus perceived to be safer by global investors.

Another immediate implication of (15) is that the expected change in exchange rates is zero. Recall from (1) that currency risk premia consist of two parts: the expected change in exchange rates and risk-free rate differences. This property thus ensures that interest-rate

[22] Alternatively, one can denote $\hat{x} = x - \bar{x}$ for all endogenous variables and replace all $\approx$ with $=$.

differences fully account for any currency risk premia generated by this model,[23] which is consistent with the data (Meese and Rogoff (1983)).

I summarize the properties of the real exchange rate in the following proposition.

Proposition 1. *The change in real exchange rates is given by* (15)*. If $\beta_z > \beta_z^\star$, we have the following:*

- *If $\varepsilon_g < 0$, $\Delta ex > 0$. The real exchange rate increases (appreciation of the high loading home currency) when a negative global shock hits.*
- *The expected change in exchange rates is zero.*

Proof. See Appendix C. ☐

3.2. *Currency risk premium, Required Return on Capital, and Capital Accumulation*

In Proposition 1, I have established that the currency of the country with a higher loading on the global shock tends to appreciate when a negative global shock hits. Assets denominated in this currency then provide insurance against these shocks and are thus safe. In this section, I further link currency risk to currency risk premia, required returns to capital, and capital accumulations. I summarize the model predictions in the following proposition:

Proposition 2. *With home bias ($\nu > 0$), the higher loading country*

- *features a lower currency risk premium and risk-free rate:*

$$\begin{aligned}\mathbb{E}(rx) &= r_f^\star - r_f - \underbrace{\mathbb{E}(\Delta ex)}_{\approx 0} \\ &\approx -\frac{1}{2}\nu A\left[(\beta_z^\star)^2 - (\beta_z)^2\right]\sigma_g^2\end{aligned}$$

- *features a lower required rate of return to capital*

$$\mathbb{E}(r^\star - r) \approx -\frac{1}{2}\nu^2\left(1 - \frac{1}{(1+\lambda_s)\gamma}\right)^2 AB\left[(\beta_z^\star)^2 - (\beta_z)^2\right]\sigma_g^2$$

- *accumulates more capital:*

$$k^\star - k \approx \frac{1}{2}\nu^2\left(1 - \frac{1}{(1+\lambda_s)\gamma}\right)^2 A\left[(\beta_z^\star)^2 - (\beta_z)^2\right]\sigma_g^2,$$

[23] Such a property is not satisfied under Epstein and Zin (1989) preferences. See Appendix E.2 for a brief discussion. We discuss this issue and its implication for international asset pricing in ongoing work Hassan et al. (2021a).

where $A = \frac{\gamma^2(1+\lambda_s)^2}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2} > 0$ *and* $B = \frac{\gamma(1+\lambda_s)(1+\nu(1-\alpha))(1-\nu)+\nu^2(1-\alpha)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2} > 0.$

Proof. See Appendix C. □

As in the data, the model predicts that the country with a safe currency (the high-loading country) features a lower currency risk premium and accumulates more capital. From an asset-pricing perspective, this result is intuitive: the higher-loading country's currency appreciates in global bad times (Proposition 1) and is thus a good hedge against global risk. Investors require a lower rate of return on assets from this country; thus, it features a lower currency risk premium, a lower required rate of return to capital, and accumulates more capital.

It is also intuitive from a social planner's perspective. Installing more capital in the high-loading country is desirable for two reasons. First, because the high-loading country features a more volatile productivity profile by construction and prefers its own good, the social planner finds it optimal to install more capital for precautionary reasons. Second, installing more capital in the high-loading country also benefits the low-loading country. Following a negative shock, fewer resources need to be transferred to the high-loading country. To understand this intuition, notice that because of heterogenous loadings, risk-sharing and the transfer of resources occur even when the shock is global. If more capital is installed in the high-loading country, the marginal utility of consumption does not drop as much when a negative global shock hits. As a result, the country needs less help from abroad, which makes the low-loading country better off as well. In this sense, installing more capital in the high-loading country ensures the world from downturns as a whole, so the social planner finds it optimal to do so.

3.3. Quantitative Performance: the Role of External Habit

Propositions 1 and 2 show that when I extend a model of currency risk and currency risk premia to incorporate capital accumulation, the model predictions are qualitatively consistent with the data. However, under standard CRRA preferences, model generated currency risk premia and therefore difference in required returns to capital and capital-output ratios are small relative to the data. I illustrate this quantitative challenge using currency risk premia as an example.[24] Under complete markets and log-normal SDFs, the currency premium

[24] I make this choice for its simplicity and for the fact that such quantitative challenge is present in a number of papers in the literature. Examples are Hassan (2013), Richmond (2019), and Ready et al. (2017).

between the two countries is given by (Backus et al. (2001))

$$\mathbb{E}(rx) = r_f^{\star} - \mathbb{E}(\Delta ex) - r_f$$
$$(16) \qquad = -\frac{1}{2}(\mathrm{var}(m) - \mathrm{var}(m^{\star})).$$

Under CRRA preference, $\mathrm{var}(m) = \gamma^2 \,\mathrm{var}(\Delta c)$, so that currency risk premia is given by

$$\mathbb{E}(rx) = -\gamma^2(\mathrm{var}(\Delta c) - \mathrm{var}(\Delta c^{\star})).$$

Consumption growths are smooth in the data. The variance of consumption growth is 0.035% in New Zealand and 0.047% in Japan. Recall that the currency risk premium between these two countries is 5.70% in the data, which implies a risk aversion parameter that is too large ($\gamma \approx 30$). International asset pricing then posits a quantitative challenge similar to the well-known equity premium puzzle. The equity premium puzzle states that aggregate consumption growth is too smooth to account for the observed Sharpe ratio of the stock market (Hansen and Jagannathan (1991)), while here the *difference* between aggregate consumption-growth volatilities is too small to account for the observed currency risk premia.[25]

I use external habit, which is proven to be successful in resolving the equity premium puzzle, to boost the heterogenous-loading induced currency risk premia and capital-output ratio differences to their empirical levels and ensure the quantitative success of the model.

I highlight the role of external habit in the following proposition:

Proposition 3. *If countries share the same constant sensitivity parameter λ_s, the currency risk premium is given by*

$$\mathbb{E}(rx) = -\frac{1}{2}\gamma^2(1+\lambda_s)^2(var(\Delta c^{\star}) - var(\Delta c)).$$

Proof. Under the simplified set-up, $s_0 = \bar{s}$ and c_0 is a constant. From (2), (9), and (13), we have

$$(17) \qquad \mathrm{var}(m) = \mathrm{var}(-\gamma s - \gamma c) = \gamma^2(1+\lambda_s)^2\,\mathrm{var}(\Delta c).$$

[25] See Hassan and Zhang (2021) for a brief discussion.

The currency risk premium is then given by

$$\begin{aligned}\mathbb{E}(rx) &= -\frac{1}{2}(\mathrm{var}(m) - \mathrm{var}(m^\star)) \\ &= -\frac{1}{2}\gamma^2(1+\lambda_s)^2(\mathrm{var}(\Delta c^\star) - \mathrm{var}(\Delta c)).\end{aligned}$$

□

If $\lambda_s = 0$, we are back to the standard CRRA preferences. With habit ($\lambda_s > 0$), differences in consumption variances are boosted by a factor of $(1+\lambda_s)^2$. Under standard calibration used in the external-habit literature, λ_s is a large number (for example, $\lambda_s = 16.54$ in Campbell and Cochrane (1999) and $\lambda_s = 13.29$ in Verdelhan (2010)), which is helpful in generating large currency risk premia and thus differences in capital-output ratios in the data.

The intuition is as follows. Households care about the difference between their consumption level and the externally given habit and dislike states when consumption is close to the habit level. Because consumption affects habit level through the sensitivity parameter (see equation (2)), habit provides an additional channel through which consumption could affect marginal utility (evident in equation (9)). As a result, marginal utility is much more sensitive to consumption growth risk than without habit, and the effective risk-aversion is higher. Habit thus generates large currency risk premia even with mild differences in consumption-growth volatilities.

Setting λ_s to be constant[26] and the same across countries is important for this result. In Appendix E.1, I show that in an endowment economy as in Verdelhan (2010), where λ_s takes the functional form used in standard habit models and is not constant across countries, the currency risk premium is always zero even if countries feature heterogeneous variance of consumption growth. In that sense, setting λ_s to a constant is economically important in an international setup because it boosts the difference between variances of SDFs across countries and helps with generating large currency risk premia while keeping the risk-free rates stable as in the data.

Together with Proposition 1, the model is capable of generating significant currency risk premia while keeping the expected change in exchange rates at zero. In other words, the model can generate large differences in risk-free rates and currency risk premia, which is

[26] In an endowment economy, setting λ_s to be a constant typically induces excess risk-free rate volatilities (see Campbell and Cochrane (1999)), but once capital accumulation is introduced, risk-free rates remain stable even with constant λ_s because agents have another channel (capital) to smooth their consumption profile. See Chen (2017) for a detailed analysis. I confirm that risk-free rates are stable in my model in Appendix D.

consistent with the data. To my knowledge, this model is the first quantitative risk-based framework with heterogenous countries that induces large cross-country variations in risk-free rates.[27]

I summarize the main findings in the simplified model as follows. First, heterogenous loadings on a global shock induce currency risk. The currency of a high-loading country appreciates when a negative global shock hits and is thus safe. Second, the high-loading country features a lower currency risk premium, a lower required rate of return to capital, and therefore accumulates more capital. Third, external habits can generate large cross-country variation in currency risk premia while keeping the expected change in exchange rates at zero, consistent with the data.

4. Estimation of the Full Model

I have established a list of theoretical predictions from the simplified model in the last section. In this section, I take the full model to the data and estimate the loadings on the global shock for countries issuing the G10 currencies and investigate the quantitative implications of the model.

4.1. Data

I consider countries (regions) issuing the G10 currencies: Australia, Canada, the euro area, Japan, New Zealand, Norway, United Kingdom, Sweden, Switzerland, and the U.S. I use quarterly GDP (in 2015 U.S. dollars) data from the OECD National Account Statistics starting from 1994Q1 to 2019Q4, and I generate capital-output ratios from the Penn World Tables 10.0 (Feenstra et al. (2015)) by dividing capital stock by GDP and then averaging across the sample periods.[28] I get currency risk premium and exchange rate data from Adrien Verdelhan's website[29] by annualizing monthly returns and then taking the average across the sample periods.

Country sizes are constructed following Hassan (2013) as the long-run GDP share,[30] Trade centrality is obtained from Robert J. Richmond's website[31] and is constructed by taking the average across time. I use these measures to validate my estimation.

[27] Colacito et al. (2018a) and Gourio et al. (2013) also generate large currency risk premia using Epstein and Zin (1989) preferences, but only with mild cross-country spread in risk-free rates.

[28] For the Euro area, I divide the total capital of all countries in the Euro area by their total GDP.

[29] Verdelhan (2018): `http://web.mit.edu/adrienv/www/Data_for_AugmentedUIP_allcountries.xls`

[30] I take the GDP share of each country relative to the sum across countries for each period, and then average across periods. See Hassan (2013) for details.

[31] Richmond (2019): `https://robertjrichmond.com/data/Richmond_Centrality.xlsx`

Table 1: Externally Calibrated Parameters (Quarterly)

Description	Value	Source
Preference and Production:		
Relative risk aversion [γ]	4	
Capital Share [α]	0.35	
Time discount factor [η]	0.995	Chen (2017)
Degree of home bias [ν]	0.98	Colacito et al. (2018a)
Depreciation Rate [δ]	0.016	Chen (2017)
Elasticity of I/K wrt Tobin's Q [ξ]	0.7	Kaltenbrunner and Lochstoer (2010)
TFP:		
Mean of TFP growth(%) [μ]	0.45	Chen (2017)
Persistence of TFP [ρ]	0.98	Chen (2017)
Habit:		
Mean surplus consumption ratio(%) [$\bar{S}$]	7	Verdelhan (2010)
Persistence [ρ_s]	0.995	Verdelhan (2010)

Notes: This table summarizes calibrated parameters. Relative risk aversion γ and capital share α are calibrated to standard values. The rest of the parameters are taken from various papers in the literature. The last column lists the source.

4.2. Estimation

I begin by externally calibrating a set of parameters to standard values used in the literature, summarized in Table 1. These parameters are set to be the same across countries. The model is quarterly, so one period represents a quarter.

The value of relative risk aversion γ is larger than what is typically used in standard habit models but is the same as Heyerdahl-Larsen (2014) and lower than Van Binsbergen (2016). It is also well within the standard values used in macro models. I follow Colacito et al. (2018a) and set home bias ν to 0.98 for easy comparison to the literature. I discuss the effect of changing this parameter in Section 4.4.2 and show that under a CES aggregator, this value can be lowered. All other variables are standard.

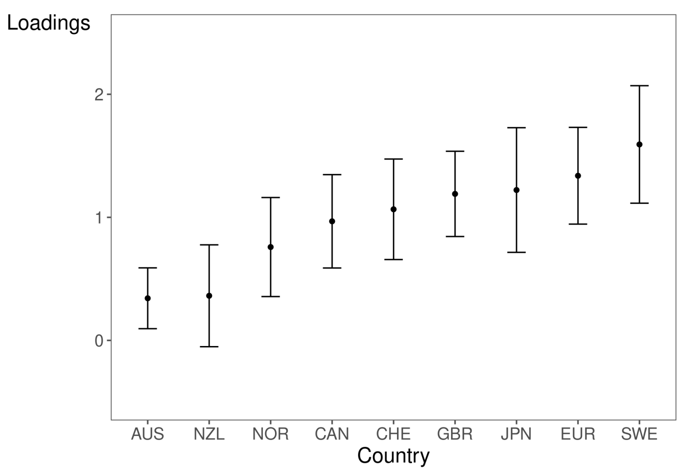
I estimate the remaining 20 parameters $\Theta = \{\sigma_g^i, \sigma^i\}_{i=1,\dots,N}$ using the simulated method of moments (SMM). Here I estimate $\sigma_g^i = \beta_z^i \sigma_g$ because β_z^i and σ_g cannot be estimated separately. I assume the average country has a loading of 1 on the global shock and set $\sigma_g = \frac{1}{N} \sum \sigma_g^i$, then calculate β_z^i for each country accordingly. All standard errors are scaled accordingly.

Heterogenous loadings on a global shock are widely used as an asset-pricing modeling tool and are typically estimated using asset prices. My model with capital and production has direct implications for covariances of outputs across countries: the high-loading countries should covary more with the world. Motivated by this theoretical link as well as the empirical patterns in Figure 2, I choose the following 20 data moments as targets: the standard deviations of HP-filtered GDP and the *correlation* of each country's HP-filtered GDP with the sample average across countries. I estimate the parameter vector $\hat{\Theta}$ by minimizing the distance between data moments and model-simulated moments:

$$\hat{\Theta} = \arg\min_{\Theta} \left(\frac{H(\Theta) - H_D}{H_D} \right)' \left(\frac{H(\Theta) - H_D}{H_D} \right),$$

where H_D is a vector of the target moments in the data and $H(\Theta)$ is a vector of simulated moments for a given parameter vector Θ. I get $H(\Theta)$ by solving the model using second-order perturbation and simulating the model for 500 samples, each with 104 periods,[32] then extract the target moments for each sample and take the average across samples.

Figure 3: Estimated Loadings



This figure plots the estimated loadings on the global shock β_z^i and two standard deviation bands for countries issuing the G10 currencies (without United States). Details on estimation can be found in Appendix F.

Because I have the same number of target data moments as the number of parameters to be estimated, the parameters are exactly identified.[33] The estimated loadings β_z^i are summarized in Figure 3, with two standard deviation bands.[34]

The estimated loadings β_z^i are largely in line with common examples of safe and risky currencies. For example, Japan has a loading of 1.22 and New Zealand has a loading of 0.36,

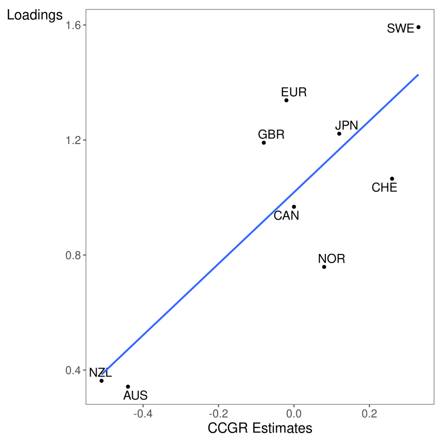
[32] I use Dynare 4.6.4 to achieve the perturbation solutions. I use second-order approximation for efficiency and because for the purpose of this paper, I only focus on unconditional moments and do not consider time-varying risk premia. I use pruning (Andreasen et al. (2018)) to reduce the exploding behavior induced by high order terms, but the result is very similar if I use standard perturbation. For higher-order solutions, Mertens and Judd (2018) provides an efficient method.

[33] See Appendix F for the list of target moments and the moment matching results.

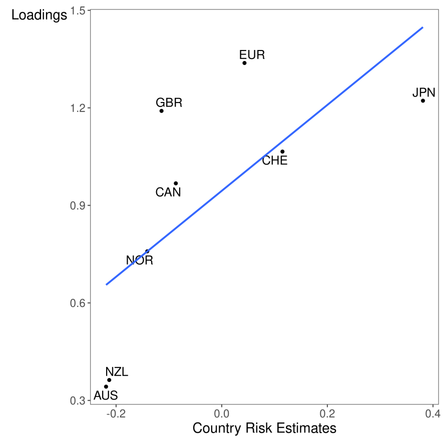
[34] See Appendix F for the table of estimates and standard errors, as well as details on SMM.

while the Japanese yen is widely viewed as a safe-haven currency and the New Zealand dollar is considered risky. To further validate my estimates, I contrast my estimation with other estimates in the literature as well as potential drivers of heterogenous loadings in Figure 4.

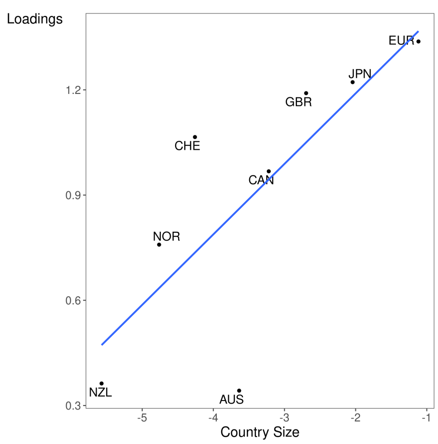
Figure 4: Estimated Loadings: Validation



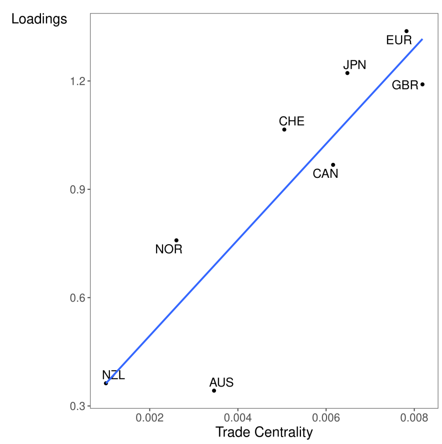
(a) CCGR Estimates



(b) Country Risk Estimates



(c) Country Size



(d) Trade Centrality

This figure plots my estimates against: the estimates obtained by Colacito et al. (2018a) in panel (a) (slope: 1.11 (s.e. 0.37, $R^2 = 0.60$)); the estimates obtained by Hassan et al. (2021c) in panel (b) (slope: 1.32 (s.e. 0.55, $R^2 = 0.48$)); country size (Hassan (2013)) in panel (c) (slope: 0.20 (s.e. 0.07, $R^2 = 0.59$)); and trade centrality (Richmond (2019)) in panel (d) (slope: 132.89 (s.e. 28.67, $R^2 = 0.78$)). Panels (b), (c), and (d) excludes Sweden.

Figure 4, panel (a) compares my estimates with Colacito et al. (2018a) (CCGR), who use dividend data to estimate heterogenous loadings on a long-run global shock in an endowment economy. My estimates using only GDP data are very similar to theirs, suggesting that heterogenous loadings on a global shock can be extracted not only from asset prices, but also directly from GDP data. Interestingly, both of our estimates show that Sweden, which is not typically considered a country with a safe currency, has a very high loading on the global shock, suggesting Sweden is somehow special.[35] In panels (b), (c), and (d), I omit Sweden because it is an obvious outlier.[36] Panel (b) compares my estimates with Hassan et al. (2021c),[37] who uses text analysis on conference calls of listed firms to directly estimate which country is perceived as "safe." Again, my estimations using only GDP data are in line with their estimates. Panels (c) and (d) plot my estimates against two potential drivers of these heterogenous loadings proposed in the literature, country size (Hassan (2013)) and trade centrality (Richmond (2019)). Both are highly correlated with my estimated loadings, suggesting that these factors can potentially drive the loadings. The lines of best fit in all four panels are positive and statistically significant at the 5% level with high R^2s.

My approach is silent on the economic origins of the heterogeneity in loadings on a global shock. The fact that estimated loadings are highly correlated with potential drivers like country size and trade centrality is comforting. However, one potential caveat is that some other potential drivers of these loadings might be endogenous to capital accumulation. I argue that because of the relative homogeneity of the countries in my sample, the scope of the endogeneity problem is limited.[38] Even if there is a path through which capital-output ratios could affect the loadings, the channel developed in this paper is still valuable: it becomes an amplification mechanism. I leave the identification of potential drivers of the heterogenous loadings as well as their interaction with the mechanism developed in the paper for future research.

To summarize, the loadings on the global shock that I estimate from GDP data alone are consistent with conventional views of currency risk and are in line with other estimates using alternative datasets. In addition, they are also highly correlated with potential drivers of these heterogenous loadings proposed in the literature.

4.3. Properties of the Estimated Model

Next, I take the estimated loadings as given and simulate the model to evaluate its performance. I document two main findings: First, the loadings on a global shock that I estimated from GDP data alone are highly correlated with currency risk premia and capital-output ratios. Second, when I feed the estimated loadings into the model for simulation, the simulated currency risk premia and capital-output ratios are highly in line with their empirical counterparts.

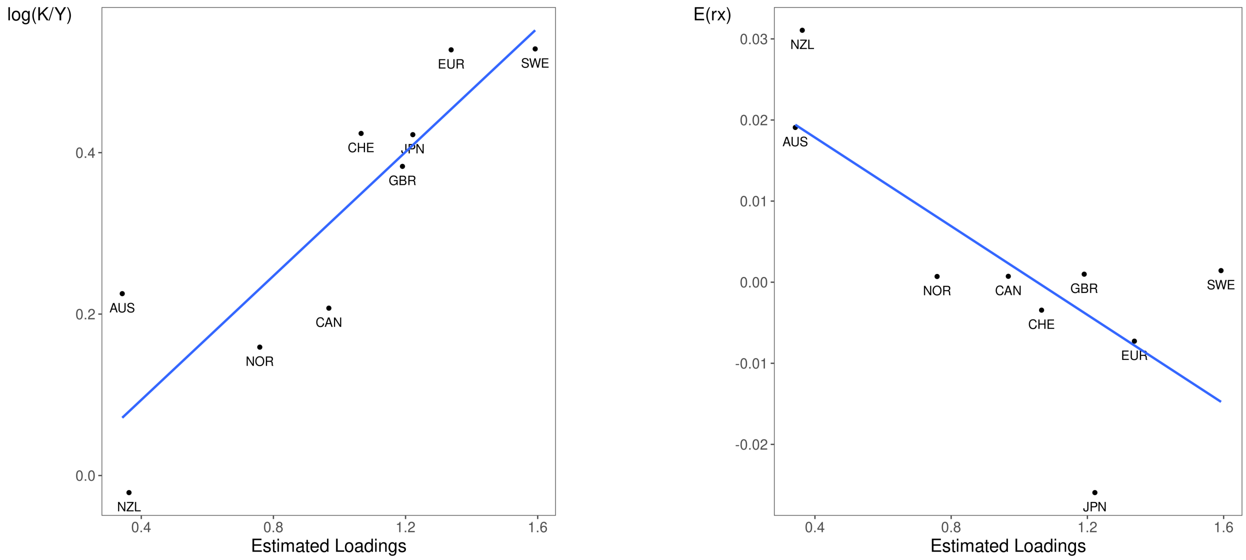
[35] One possible reason is that the Swedish krona is stabilized to the euro.

[36] See Appendix Figure A1 for plots with Sweden included.

[37] I thank the authors for sending me their estimates.

[38] In Appendix A, I show that the negative relationship between capital-output ratios and currency risk premia in Figure 1 is robust to including a wide number of controls.

Figure 5: Estimated Loadings



(a) Capital-Output Ratio (b) Currency Risk Premium

This figure plots log capital-output ratios (Panel(a)) and currency risk premia (Panel(b)) relative to the U.S. against estimated loadings on the global shock. The line of best fit has a slope of 0.38 (s.e. 0.08) for log capital-output ratio and a slope of -0.03 (s.e. 0.01) for currency risk premium. All moments are annualized.
Data Source: PWT 10.0 and Adrien Verdelhan's webset. Data Range: 1994-2019. Details on data construction can be found in section 4.1.

4.3.1. Result 1: Correlation between Loadings and Key Variables

I first explore the correlation of the estimated loadings with currency risk premia and capital-output ratios in the data. To that end, I plot capital-output ratios (panel (a)) and currency risk premia (panel (b)) against the estimated loadings in Figure 5. Consistent with the predictions of Proposition 2, countries with higher loadings on the global shock features lower currency risk premia and higher capital-output ratios. The R^2 is 0.70 for capital-output ratios and 0.49 for currency risk premia.[39] The estimated loadings are therefore indeed highly correlated with currency risk premia and capital-output ratios across countries.

These high correlations themselves are interesting even without the model, considering the fact that the loadings on the global shock are estimated solely from GDP data. The way that a country covaries with the world is informative of its currency risk premium (and thus risk-free rate) and capital-output ratio. This suggests that different loadings on a global shock, which has long been considered as an asset pricing tool to rationalize currency risk premia (e.g., Colacito et al. (2018a), Lustig et al. (2011)), have support in fundamental variables such as GDP. Countries that covary more with the world and thus

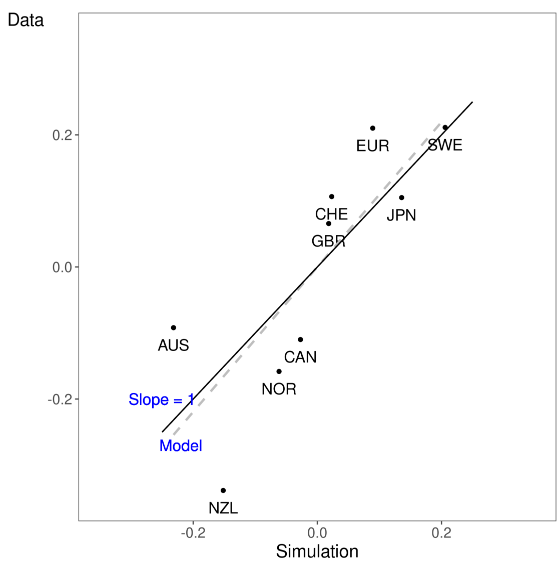
[39] Unsurprisingly, R^2 is lower for currency risk premia because, as an asset price, currency risk premia are much more volatile and harder to estimate and predict than capital-output ratios.

have higher estimated loadings on the global shock feature higher capital-output ratios and lower currency risk premia. In this sense, my paper can be seen as a framework that jointly matches these facts in the data.

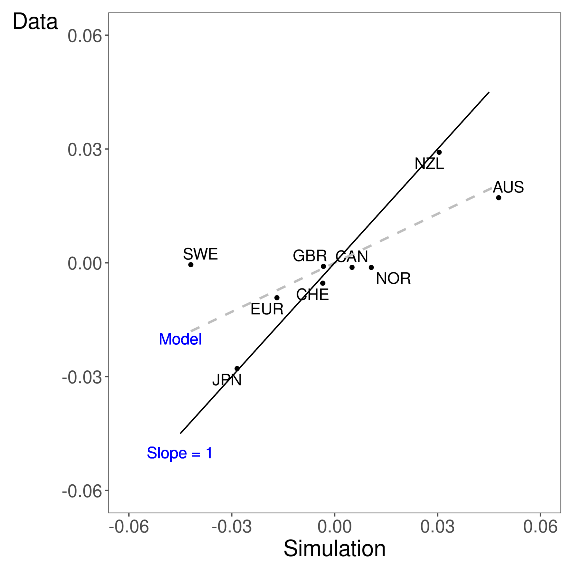
4.3.2. Result 2: Data Moments vs. Model Simulations

I investigate the quantitative performance of the model by plotting moments in the data against model-generated moments for capital-output ratios (panel (a)) and currency risk premia (panel (b)) in Figure 6.

Figure 6: Data vs. Simulations



(a) Capital-output Ratio



(b) Currency risk premium

This figure plots log capital-output ratios (panel(a)) and currency risk premia (panel(b)) relative to the cross-country average against their simulated counterparts. The line of best fit has a slope of 1.09 (s.e. 0.30) for log capital-output ratios and a slope of 0.43 (s.e. 0.14) for currency risk premia. Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking the average. All moments are annualized.
Data Source: PWT 10.0 and Adrien Verdelhan's webset. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

If the model perfectly matches the data, we would expect the model simulated moments and the data moments to be identical across countries, and all points should lie on the 45° line. In Figure 6, panel (a), we can see that for capital-output ratios, the points are all very close to the 45° line. In fact, the line of best fit (dashed line) has a slope of 1.09 (s.e. 0.30), which is very close to 1, suggesting, on average, the model does a remarkably good job in matching the cross-country variations in capital-output ratios. High-loading countries feature higher capital-output ratios in a way that is quantitatively consistent with the data.

Figure 6, panel (b), plots currency risk premia in the data against model simulations. Again, the model does a fairly good job of matching the data. All countries lie almost perfectly on the 45° line (solid line) except for Sweden and Australia. Recognizing that as asset prices, currency risk premia are highly volatile and hard to predict, especially with fundamentals like GDP, the model matches the currency risk premia in the data reasonably well.

Overall, the model simulations of capital-output ratios and currency risk premia match the data very well. To further evaluate the quantitative performance of the model, I again use Japan and New Zealand as an example. Table 2 compares the data and the model-generated moments for the difference in log capital-output ratios between the two countries, as well as the currency risk premium, the interest rate difference, and the expected change in exchange rates. The model can generate roughly 66% of the observed difference in log capital-output ratios between the two countries. Regarding currency risk premium, the model (5.89%) almost perfectly matches the data (5.70%). In the data, most of the currency risk premium comes from interest-rate differences, with the expected change in exchange rates close to 0. In the model, we have the same pattern. The model-generated interest rate difference is 6.06%, compared to 5.08% in the data.[40] The New Zealand dollar appreciates somewhat (0.62%) in the sample, but depreciates somewhat in the model.[41]

Table 2: Japan and New Zealand: An Example

	Diff in $\log(K/Y)$	$\mathbb{E}(rx)$	$r_f^{NZL} - r_f^{JPN}$	$\mathbb{E}(\Delta ex)$
Data	-0.44	5.70%	5.08%	-0.62%
Model	-0.29	5.89%	6.06%	0.17%

Notes: This table compares key moments of Japan and New Zealand between the model simulations and the data. Model moments are obtained by simulating 500 samples of 104 periods and taking the average. All moments are annualized.

For the Japan and New Zealand example, the model does remarkably well in matching the currency risk premium, interest rate difference, and the expected change in exchange rates, and can also explain a significant portion of the difference in capital-output ratios. To quantify the model's average performance across countries in terms of matching the capital-output ratios, I perform the following variance decomposition. Let $\kappa_D^i = k_D^i - y_D^i$ denote the log capital-output ratio in the data for country i, and let κ_M^i denote the same variable

[40] In comparison, Colacito et al. (2018a) generate a interest rate difference of about 1%, and Gourio et al. (2013) 0.55%.

[41] The interest rate differences in the data become larger and expected change in exchange rates become positive when we extend our sample to the 1980s. In general, unconditionally, the expected change in exchange rates is close to 0. If anything, the high-interest-rate currency tends to depreciate a little bit. See Hassan and Mano (2018).

predicted by the model, and write

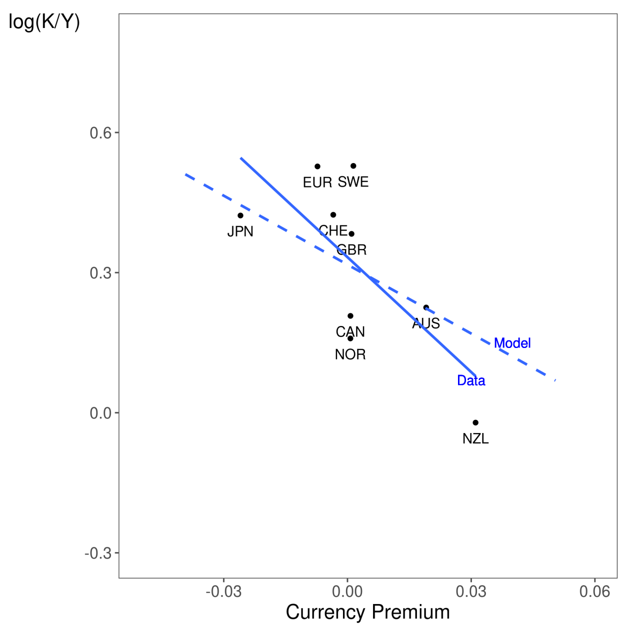
$$\kappa_D^i = \kappa_M^i + e^i,$$

where e^i is the prediction error of the model. If the model perfectly matches the data, $e^i = 0$ for all i. Taking variances across countries on both side,[42] we have

$$\underbrace{\mathrm{var}(\kappa_D^i)}_{0.0344} = \underbrace{\mathrm{var}(\kappa_M^i)}_{0.0189} + \underbrace{\mathrm{var}(e^i)}_{0.0119} + \underbrace{2\,\mathrm{cov}(\kappa_M^i, e^i)}_{0.0037}.$$

Under this decomposition, $\frac{\mathrm{var}(\kappa_M^i)}{\mathrm{var}(\kappa_D^i)} = 54.76\%$, so the model roughly accounts for 55% of the cross-country variation in capital-output ratios. Note that $2\,\mathrm{cov}(\kappa_M^i, e^i) = 0.0037$ is close to 0, suggesting the error terms are not correlated with κ_M^i, and the model's prediction is, on average, very close to the data.

Figure 7: Regression Line of K/Y on Currency risk premium, Data v.s. Model



This figure plots the regression line of capital-output ratios on currency risk premia for the G10 currencies in the data and the same regression line implied by the model simulations. The points represents the data. The regression line in the data has a slope of -8.21 (s.e. 3.12). The regression line implied by the model simulations has a slope of -4.91 (s.e. 0.07). Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking the average. All moments are annualized.
Data Source: PWT 10.0 and Adrien Verdelhan's webset. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

[42] This is similar to calculating an R^2, but with the coefficient restricted to 1.

Overall, the model does very well in matching the cross-country variations in capital-output ratios and can explain roughly 55% of the variations in the data. The model also provides predictions for the relationship between capital-output ratios and currency risk premia. Figure 7 compares the regression line of capital-output ratios on currency risk premia in the data (solid line) with the regression line implied by model simulations (dashed line). The model predicts a slope of -4.91, compared to -8.21 in the data. The model under-predicts the slope but still explains a significant portion. This result also confirms the findings of Proposition 2, which states a tight link exists between currency risk premia and capital-output ratios in the model.

To summarize, the loadings that I estimated from GDP data alone are highly correlated with capital-output ratios and currency risk premia across countries. Taking the estimated loadings as given, the model predicts cross-country variations in capital-output ratios remarkably well, explaining around 55% of it. The model matches the currency risk premia and the correlations between capital-output ratios and currency risk premia in the data fairly well. Model-generated currency risk premia are large, and most of them are accounted for by interest-rate differences as in the data.

4.4. Role of Parameters and Robustness

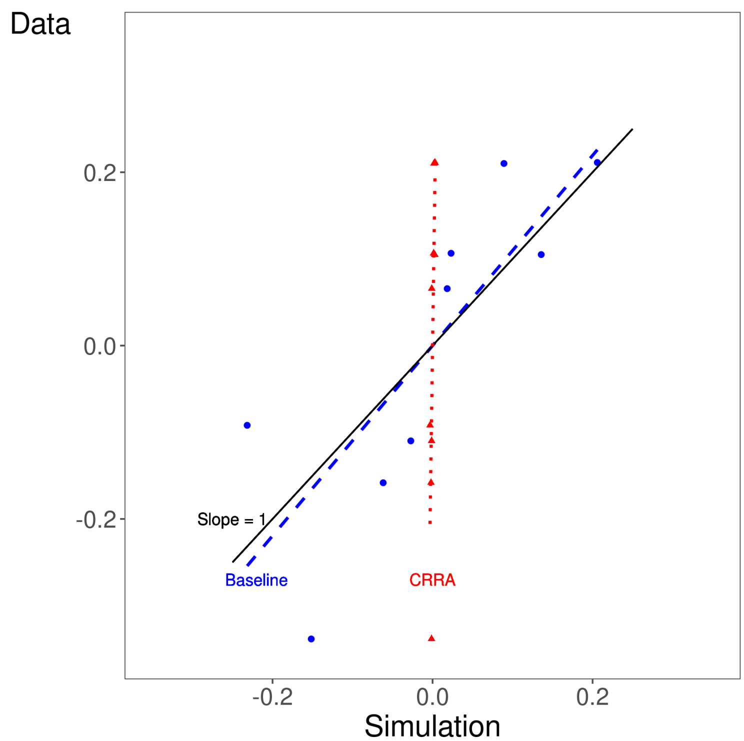
In this section, I check the sensitivity of my findings to changes of parameters and setups. In particular, I take the estimated loadings as given, use the calibration in Table 1 as the baseline, and test how changing some of the parameters would affect the quantitative performance of the model. In particular, I test how using (1) CRRA preferences; (2)a CES aggregator; (3) a different home-bias parameter would affect the model's ability to match the data. I focus on capital-output ratios in this section.

4.4.1. The role of habit

To highlight the role of habit in the quantitative success of the model, I compare the simulated results generated from our baseline habit model with standard CRRA preferences.

Figure 8 is generated by adding simulated moments from the CRRA model (red triangles) to Figure 6, panel (a). The CRRA model does poorly on the quantitative front. The line of best fit (red dotted line) is close to being vertical, and is far away from the 45° line, suggesting the model is generating differences in capital-output ratios that are too small relative to the data. In comparison, the baseline model with habit features a line of best fit with a slope very close to 1 (the blue dashed line). To give an example, under CRRA, the model generates a 0.004 difference in log capital-output ratios between Japan and New Zealand, compared to 0.29 in the baseline specification and 0.44 in the data. This confirms

Figure 8: Habit v.s. CRRA



This figure plots the log capital-output ratio relative to the cross-country average against their simulated counterparts. Blue dots represent the baseline model with habit, and red triangles represent the CRRA model. The dashed blue line represents the line of best fit for the baseline model with habit, and features a slope of 1.09 (s.e. 0.30, $R^2 = 0.66$). The dotted red line represents the line of best fit for the CRRA model, and features a slope of 64.75 (s.e. 20.28, $R^2 = 0.59$) for log capital-output ratio. Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking the average. All moments are annualized. Data Source: PWT 10.0. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

our findings in Proposition 3, which states habits are crucial for the quantitative success of the model.

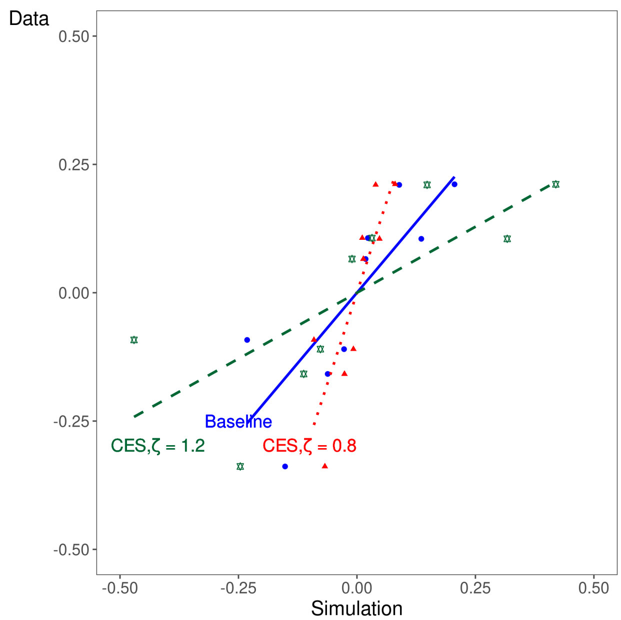
4.4.2. CES Aggregator

Next, I test how the quantitative predictions of the model differ when I use a CES aggregator for the final good and change the elasticity of substitution between goods. I start by changing the production function of the final good (6) to the following CES aggregator:

$$F_t^i = \left(\nu (X_{i,t}^i)^{\frac{\zeta-1}{\zeta}} + \sum_{j=1}^{N} \frac{1}{N} (1-\nu) \left(X_{j,t}^i \right)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}},$$

where ζ governs the elasticity of substitution between the country-specific goods. In my baseline specification, I use the Cobb-Douglas aggregator so that $\zeta = 1$. To see how the

Figure 9: Changing Elasticity of Substitutions



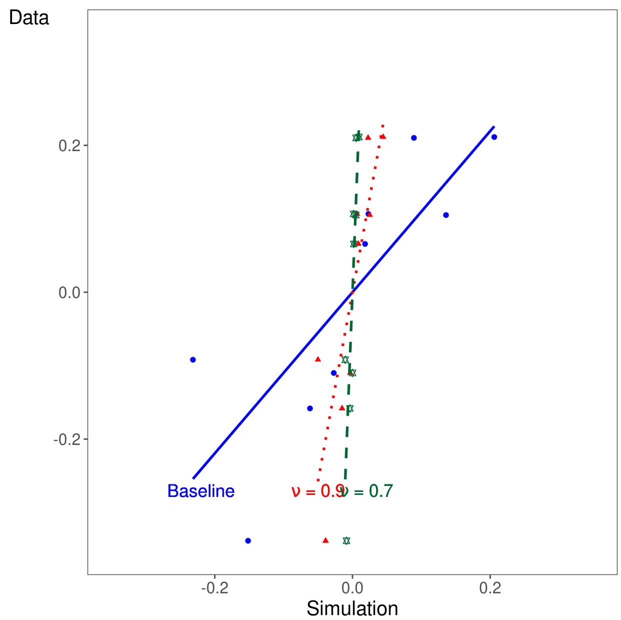
This figure plots log capital-output ratios relative to the U.S. against their simulated counterparts for the baseline model and the model with CES aggregators ($\zeta = 0.8$ and $\zeta = 1.2$). Blue dots represent the baseline model, red triangles represent $\zeta = 0.8$, and green stars represent $\zeta = 1.6$. The line of best fit has a slope of 1.09 (s.e. 0.30, $R^2 = 0.66$) for the baseline model (solid blue line), a slope of 2.83 (s.e. 0.69, $R^2 = 0.70$) for $\zeta = 0.8$ (dotted red line), and a slope of 0.51 (s.e. 0.17, $R^2 = 0.58$) for $\zeta = 1.2$ (dashed green line). Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking the average. All moments are annualized.
Data Source: PWT 10.0. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

model predictions change with respect to ζ, in Figure 9, I re-produce Figure 6, panel (a), for $\zeta = 0.8$ and $\zeta = 1.2$ and put them in the same figure as my baseline specification with $\zeta = 1$.

I use the slope of the line of best fit as a rough measure of how alternative specifications compare to the baseline. A steeper slope indicates the alternative model on average under-predicts the cross-country variation in capital-output ratios across countries relative to the baseline model and vice versa. As shown in Figure 9, as the elasticity gets larger ($\zeta = 1.2$, green dashed line), the model-generated cross-country variations are larger, and the opposite is also true ($\zeta = 0.8$, red dotted line).

This result is intuitive. As the elasticity of substitution becomes larger, goods become more substitutable, and agents optimally choose to consume more of their home good. Higher ζ thus enhances the effect of home bias. When a negative global shock hits and ζ is high, the increase in the price of a high-loading country's good passes through more to the price of its final good, leading to more pronounced appreciation. As a result, more capital is installed in the high-loading country.

Figure 10: Changing Degree of Home Bias



This figure plots log capital-output ratios relative to the U.S. against their simulated counterparts for the baseline model and the model with $\nu = 0.9$ and $\nu = 0.7$). Blue dots represent the baseline model, red triangles represent $\nu = 0.9$, and green stars represent $\nu = 0.7$. The line of best fit has a slope of 1.09 (s.e. 0.30, $R^2 = 0.66$) for the baseline model (solid blue line), a slope of 5.14 (s.e. 1.21, $R^2 = 0.72$) for $\nu = 0.9$ (dotted red line), and a slope of 24.21 (s.e. 5.50, $R^2 = 0.73$) for $\nu = 0.7$ (dashed green line). Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking the average. All moments are annualized.
Data Source: PWT 10.0. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

4.4.3. Changing home bias

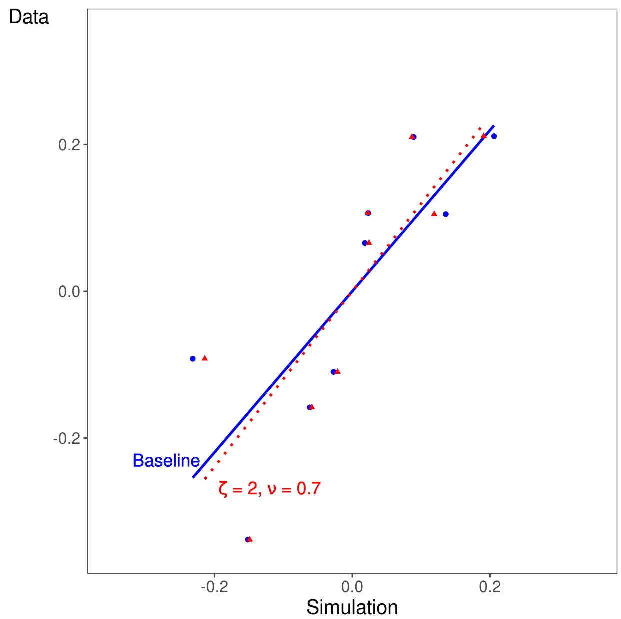
I next turn to the home-bias parameter, ν. In international asset-pricing models with Cobb-Douglas aggregators, a high degree of home bias is typically assumed.[43] In my baseline calibration, I follow Colacito et al. (2018b) and set $\nu = 0.98$. I illustrate how changing ν affects the results in Figure 10.

When using Cobb-Douglas aggregator, using a large home bias parameter is important. As we lower home bias, the model-generated cross-country variation becomes much smaller (dotted red line and dashed green line) compared with the baseline (solid blue line). Because the Cobb-Douglas aggregator restricts the elasticity of substitution to 1, a high level of home bias is needed so that price changes in country-specific goods are sufficiently passed through to prices of final goods, which represents real exchange rates. Interestingly, Figures 9 and

[43] For example, Stathopoulos (2017) uses $\nu = 0.952$, Colacito and Croce (2013) uses $\nu = 0.97$.

10 suggest that we can set ν to lower, more reasonable levels if we use CES aggregators and set $\zeta > 1$. Figure 11 confirms this conjure by showing that a model with $\zeta = 2$ and $\alpha = 0.7$ generates results similar to the baseline.

Figure 11: High Elasticity with Low Home Bias



This figure plots log capital-output ratios relative to the U.S. against their simulated counterparts for the baseline model and the model with $\zeta = 2$, $\nu = 0.7$). Blue dots represent the baseline model and red triangles represent $\nu = 0.7$, $\zeta = 2$. The line of best fit has a slope of 1.09 (s.e. 0.30, $R^2 = 0.66$) for the baseline model (solid blue line) and a slope of 1.19 (s.e. 0.31, $R^2 = 0.66$) for $\nu = 0.7$, $\zeta = 2$. Model simulated moments are obtained by simulating the model for 200 samples of 104 periods and then taking the average. All moments are annualized.
Data Source: PWT 10.0. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

To summarize, standard CRRA preferences only generate small cross-country variation in capital-output ratios. The model-predicted variations in capital-output ratios are increasing in elasticity of substitutions between goods, ζ, and are decreasing in the degree of home bias ν. Although the exact variation induced by the model depends on parameterization, one can achieve similar results to the baseline model with a high ζ and a low ν. In other words, the qualitative prediction of the model is robust to changes in parameter values, and a set of parameters exists that generates similar quantitative results to the baseline model.

5. Conclusion

Heterogenous loadings on a global shock have been a standard modeling device in the international asset-pricing literature to capture the idea that some currencies are safer than others. It is typically used to qualitatively understand currency risk premia. Intuitively, currencies of high-loading countries appreciate in global bad times, making them a good hedge against global risk, and investors require a lower return for assets in such countries. In this paper, I make two important extensions to this framework. First, I endogenize capital accumulation and show that these heterogenous loadings and currency risk have important implications on the real economy: high loading countries should feature lower required returns to capital and accumulate more capital. Second, I use external habit to quantitatively match the large cross-country variations in currency risk premia, risk-free rates, and capital-output ratios in the data. I confirm that the GDP correlations between countries issuing the G10 currencies are consistent with heterogenous loadings on a global shock, and the loadings estimated using GDP data alone are highly correlated with currency risk premia and capital-output ratios. When feeding the estimated loadings into my model with external habits, the model can explain around 55% of the cross-country variation in capital-output ratios among countries issuing the G10 currencies. External habit is essential for the quantitative success of the model in terms of matching currency risk premia and capital-output ratios. Compared to existing international asset pricing frameworks using Epstein and Zin (1989) preferences, it has the advantage of generating large currency risk premia with zero expected change in exchange rates, thus better matching the data.

My paper is silent on the economic sources of the heterogeneity in loadings on the global shock. One interesting extension is thus to apply external habit to existing frameworks that provide micro-foundations for the loadings and evaluate the relative importance of potential channels. I have also consciously abstracted away from other sources of heterogeneity that could have important implications for capital-output ratios, for example, capital share. However, the modeling tools developed in this paper can easily be extended to a more complicated set-up and potentially be used to evaluate the relative importance of different drivers of capital-output ratios. In addition, it is an empirical fact that risk-free rates differ persistently across countries, which should have implications beyond capital-output ratios. Because my model can induce long-lasting differences in risk-free rates across countries as in the data, it can potentially be used to study cross-country patterns of other important economic variables that are tightly linked to risk-free rates.

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A. Controlling for Country Characteristics

Table A1: Regression of Capital-Output Ratios on Currency Risk Premia with Controls, G10

	Dependent variable:							
	Capital-output Ratios Relative to the US							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\mathbb{E}(rx)$	-7.945^{*}	-8.742^{**}	-7.010	-7.722^{*}	-7.549^{*}	-10.662	-7.223	-8.193
	(3.405)	(3.319)	(4.340)	(3.609)	(3.300)	(5.617)	(4.071)	(4.399)
FDI	0.143							
	(0.397)							
FOI		-0.140						
		(0.195)						
CC			-0.077					
			(0.182)					
GE				-0.085				
				(0.245)				
PS					-0.150			
					(0.187)			
RQ						0.194		
						(0.359)		
RL							-0.107	
							(0.257)	
VA								-0.001
								(0.350)
Constant	0.226	0.635	0.476	0.477	0.492^{*}	0.031	0.515	0.334
	(0.300)	(0.426)	(0.339)	(0.420)	(0.204)	(0.561)	(0.441)	(0.491)
Observations	9	9	9	9	9	9	9	9
R^2	0.508	0.537	0.512	0.507	0.546	0.521	0.511	0.497
Adjusted R^2	0.344	0.382	0.349	0.343	0.395	0.361	0.348	0.330
Residual Std. Error (df = 6)	0.150	0.146	0.150	0.150	0.144	0.148	0.150	0.152
F Statistic (df = 2; 6)	3.098	3.476^{*}	3.148	3.088	3.610^{*}	3.259	3.140	2.968
Note:						*p<0.1; **p<0.05; ***p<0.01		

This table summarizes regressions of log capital-output ratio relative to the U.S. on currency risk premia and different controls for countries issuing the G10 currencies:

$$ky^i = \alpha\, \mathbb{E}(rx^i) + \beta \text{control}^i + \epsilon^i$$

FDI stands for Financial Development Index, FOI stands for Chinn-Ito financial openness index (Chinn and Ito (2006)). CC stands for control of corruption, GE stands for government effectiveness, PS stands for political stability and absence of violence/terrorism, RQ stands for regulatory quality, RL stands for rule of law, and VA stands for voice and accountability.
Data Source: Capital-output ratios are from PWT 10.0. Currency risk premia are from Verdelhan's website. FDI is from the IMF Financial Development Indicator dataset. FOI is from the Chinn-Ito Index website. All the other variables are from the World Governance Indicators by the World Bank. Data range from 1994 to 2019.

As is shown in Table A1, the strong negative relationship between capital-output ratios and currency risk premia is robust to including a series of controls representing financial development, governance, and financial openness within the G10 countries. Because of the limited sample size, I can only control for each variable separately.

Table A2: Regression of Capital-Output Ratios on Currency Risk Premia with Controls, All Countries

	Dependent variable:	
	Capital-output Ratios Relative to the US	
	(1)	(2)
$\mathbb{E}(rx)$	-3.276	-9.283^{**}
	(3.768)	(3.435)
FDI	0.139	0.050
	(0.393)	(0.436)
FOI	0.184^{***}	
	(0.066)	
CC	-0.816^{***}	-0.696^{***}
	(0.220)	(0.240)
GE	0.734^{**}	0.452
	(0.276)	(0.286)
PS	0.051	0.139
	(0.132)	(0.142)
RQ	0.059	0.346
	(0.257)	(0.262)
RL	0.033	0.012
	(0.268)	(0.299)
VA	0.071	0.063
	(0.071)	(0.079)
Constant	-0.177	0.043
	(0.191)	(0.194)
Observations	37	37
R^2	0.601	0.487
Adjusted R^2	0.468	0.341
Residual Std. Error	0.219 (df = 27)	0.243 (df = 28)
F Statistic	4.520^{***} (df = 9; 27)	3.323^{***} (df = 8; 28)
Note:		*p<0.1; **p<0.05; ***p<0.01

This table summarizes regressions of log capital-output ratio relative to the U.S. on currency risk premia and different controls for a sample of 37 countries (See Appendix B for the list of countries).

$$ky^i = \alpha\,\mathbb{E}(rx^i) + \beta \text{control}^i + \epsilon^i$$

FDI stands for Financial Development Index, FOI stands for Chinn-Ito financial openness index (Chinn and Ito (2006)). CC stands for control of corruption, GE stands for government effectiveness, PS stands for political stability and absence of violence/terrorism, RQ stands for regulatory quality, RL stands for rule of law, and VA stands for voice and accountability. Data Source: Capital-output ratios are from PWT 10.0. Currency risk premia are from Verdelhan's website. FDI is from the IMF Financial Development Indicator dataset. FOI is from the Chinn-Ito Index website. All the other variables are from the World Governance Indicators by the World Bank. Data range from 1994 to 2019.

As is shown in Table A2, for a broader range of 37 countries, the strong negative relationship between capital-output ratios and currency risk premia is robust to including a series of controls representing financial development and governance (Panel (2)) but is weakened when financial openness is included (Panel (1)). This finding is to be expected because in the broader sample with developing countries, financial openness is crucial in how each economy covaries with the world, and thus the loadings on the global shock and currency risk premia. But within the countries issuing the G10 currencies, all countries have high openness and thus financial openness is less relevant.

B. The List of Countries in the Broader Sample

The broader sample in Figure 1 include: United Arab Emirates, Australia, Austria, Belgium, Canada, Switzerland, Czech Republic, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Hong Kong SAR China, Hungary, Indonesia, India, Ireland, Italy, Japan, South Korea, Kuwait, Mexico, Malaysia, Netherlands, Norway, New Zealand, Philippines, Poland, Portugal, Saudi Arabia, Singapore, Sweden, Thailand, the U.S., South Africa.

Currency risk premia for countries in the euro zone are replaced by the euro currency risk premia after the euro is introduced.

C. Solving the Simplified Model

Under the simplified set-up, the return to capital (14) simplifies to

$$R = \frac{\Lambda_X}{\Lambda_C} \alpha \frac{Y}{K}.$$

Substituting back to the Euler equation (12), moving capital to the left-hand side and taking logs, we have

$$k = \log(\alpha\eta) + \log\left[\mathbb{E}\left(e^{\lambda_X + y - \lambda_{C,0}}\right)\right].$$

Assuming log-normality, we can already see that the risk plays a role in choosing the optimal level of capital:

$$k = \log(\alpha\eta) + \mathbb{E}\left(\lambda_X + y - \lambda_{C,0}\right) + \underbrace{\frac{1}{2}\operatorname{var}\left(\lambda_X + y - \lambda_{C,0}\right)}_{\text{risk adjustment}}.$$

The last term is a risk-adjustment term and represents the effect of risk on the agent's incentive to accumulate capital. Higher variance induces a higher precautionary saving motive, and thus leads to more capital.

I solve the model using log-linearization.

Under the simplified model, the system of equations in period 1 can be log-linearized as:

$$\begin{aligned}
\hat{c}^i &= \frac{1}{2}(1+\nu)\hat{x}_i^i + \frac{1}{2}(1-\nu)\hat{x}_j^i \\
\hat{\lambda}_C^i &= -\gamma\hat{s}^i - \gamma\hat{c}^i \\
\hat{\lambda}_X^i &= \hat{\lambda}_C^i + \hat{c}^i - \hat{x}_i^i \\
\hat{\lambda}_X^i &= \hat{\lambda}_C^j + \hat{c}^j - \hat{x}_i^j \\
\hat{y}^i &= \hat{z}^i + \alpha\hat{k}_0^i \\
Y^{i\star}\hat{y}^i &= X_i^{i\star}\hat{x}_i^i + X_i^{j\star}\hat{x}_i^j \\
\hat{s}^i &= \rho_s\hat{s}_0^i + \lambda_s\Delta c^i.
\end{aligned}$$

The Mathematica file that derives this system is available upon request.

We want to solve for

$$\hat{k}_0^i = \log(\nu\eta) + \log\left[\mathbb{E}_0\left(e^{\hat{\lambda}_X^i + \hat{y}^i - \hat{\lambda}_{C0}^i}\right)\right]. \tag{A1}$$

Now, note that we have $\frac{X_i^{i\star}}{Y^{i\star}} = \frac{1}{2}(1+\alpha)$, $\frac{X_i^{j\star}}{Y^{i\star}} = \frac{1}{2}(1-\alpha)$. We solve for $\hat{\lambda}_X^i + \hat{y}^i$ as a function of the exogenous shocks.

We start from the resource constraints,

$$\hat{y}^i = \frac{1}{2}(1+\nu)\hat{x}_i^i + \frac{1}{2}(1-\nu)\hat{x}_i^j,$$

which gives us

$$\hat{x}_i^j = \frac{1}{\frac{1}{2}(1-\nu)}\hat{y}^i - \frac{1+\nu}{1-\nu}\hat{x}_i^i. \tag{A2}$$

Now we have

$$\hat{c}^i = \frac{1}{2}(1+\nu)\hat{x}_i^i + \hat{y}^j - \frac{1}{2}(1+\nu)\hat{x}_j^j. \tag{A3}$$

Then marginal utility of consumption is given by

$$\begin{aligned}\hat{\lambda}_C^i &= -\gamma \hat{s}^i - \gamma \hat{c}^i \\ &= -\gamma\lambda_s(\hat{c}^i - \hat{c}_0^i) - \gamma \hat{c}^i \\ &= -\gamma(1+\lambda_s)\hat{c}^i - \gamma\lambda_s \hat{c}_0^i \\ &= -\gamma(1+\lambda_s)\left(\frac{1}{2}(1+\nu)\hat{x}_i^i + \hat{y}^j - \frac{1}{2}(1+\nu)\hat{x}_j^j\right) + \gamma\lambda_s\hat{c}_0^i. \end{aligned} \tag{A4}$$

Now, note

$$\begin{aligned}\hat{\lambda}_C^i + \hat{c}^i - \hat{x}_i^i &= \hat{\lambda}_C^j + \hat{c}^j - \hat{x}_i^j \\ \hat{\lambda}_C^i + \hat{c}^i - \hat{x}_j^i &= \hat{\lambda}_C^j + \hat{c}^j - \hat{x}_j^j,\end{aligned}$$

which implies

$$-\hat{x}_i^i + \hat{x}_j^i = -\hat{x}_i^j + \hat{x}_j^j,$$

and we have

$$\hat{x}_i^i + \hat{x}_j^j = \hat{y}^i + \hat{y}^j. \tag{A5}$$

To get another equation, we examine

$$\lambda_C^i + \hat{c}^i - \hat{x}_i^i = \lambda_C^j + \hat{c}^j - \hat{x}_i^j.$$

Plugging in (A2), (A3), and (A4), we have

$$\begin{aligned}&-\gamma(1+\lambda_s)\left(\frac{1}{2}(1+\nu)\hat{x}_i^i + \hat{y}^j - \frac{1}{2}(1+\nu)\hat{x}_j^j\right) + \gamma\lambda_s\hat{c}_0^i - \frac{1}{2}(1-\nu)\hat{x}_i^i + \hat{y}^j - \frac{1}{2}(1+\nu)\hat{x}_j^j \\ &= -\gamma(1+\lambda_s)\left(\frac{1}{2}(1+\nu)\hat{x}_j^j + \hat{y}^i - \frac{1}{2}(1+\nu)\hat{x}_i^i\right) + \gamma\lambda_s\hat{c}_0^j + \frac{1}{2}(1+\nu)x_j^j - \frac{1+\nu}{1-\nu}\hat{y}^i + \frac{1}{2}\frac{(1+\nu)^2}{1-\nu}\hat{x}_i^i.\end{aligned}$$

Combined with (A5), we can then easily solve for $\hat{x}_i^i$:

$$\hat{x}_i^i = \frac{1}{2}\frac{(\gamma(1+\lambda_s)-1)(1-\nu)\nu}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}(\hat{y}^i + \hat{y}^j) + \frac{\gamma(1+\lambda_s)(1-\nu)+\nu}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i.$$

Substituting back to (A4), we can easily obtain

$$\begin{aligned}\hat{\lambda}_C^i &= -\frac{1}{2}\gamma(1+\lambda_s)\frac{(1+\nu)\gamma(1+\lambda_s)(1-\nu)+\nu(1+\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i \\ &\quad -\frac{1}{2}\gamma(1+\lambda_s)\frac{(1-\nu)\gamma(1+\lambda_s)(1+\nu)-\nu(1-\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^j \\ &= -\frac{1}{2}\gamma(1+\lambda_s)\frac{(1-\nu)\gamma(1+\lambda_s)(1+\nu)-\nu(1-\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}(\hat{y}^i+\hat{y}^j) \\ &\quad -\gamma(1+\lambda_s)\frac{\nu}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i, \end{aligned} \tag{A6}$$

and we have

$$\begin{aligned}\hat{\lambda}_X^i &= \hat{\lambda}_C^i + \hat{c}^i - \hat{x}_i^i \\ &= \frac{1}{2}(1-\gamma(1+\lambda_s))\frac{(1-\nu)\gamma(1+\lambda_s)(1+\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}(\hat{y}^i+\hat{y}^j) - \frac{\gamma(1+\lambda_s)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i. \end{aligned} \tag{A7}$$

Then, we have

$$\hat{\lambda}_X^i + \hat{y}^i = \frac{1}{2}(1-\gamma(1+\lambda_s))\frac{(1-\nu)\gamma(1+\lambda_s)(1+\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}(\hat{y}^i+\hat{y}^j) + \frac{\nu^2(1-\gamma(1+\lambda_s))}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i.$$

Substituting back to (A1), we can then solve for capital stock as

$$\hat{k}_0^i = \frac{1}{2}\nu\left(1-\frac{1}{(1+\lambda_s)\gamma}\right)^2 \operatorname{var}(\hat{\lambda}_C^i) + const, \tag{A8}$$

where *const* is a constant that is common across countries. The Mathematica file that solves the full expression for it is available upon request.

Naturally, we would expect the required return to capital to be closely related to consumption risk as well. Indeed,

$$\mathbb{E}(\hat{r}^i) = -\frac{1}{2}\nu\left(1-\frac{1}{(1+\lambda_s)\gamma}\right)^2 B\operatorname{var}(\hat{\lambda}_C^i) + const, \tag{A9}$$

where $0 < B = \frac{\gamma(1+\lambda_s)(1+\nu(1-\alpha))(1-\nu)+\nu^2(1-\alpha)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}$. The Mathematica file that solves the full expression for it is available upon request.

As shown in the seminal paper by Backus et al. (2001), under complete market and log-normality of the SDF, currency risk premia can be written as differences between variance of the log SDF:

$$\mathbb{E}(rx) = r_f^\star - \mathbb{E}(\Delta ex) - r_f$$
$$= -\frac{1}{2}(\text{var}(m^\star) - \text{var}(m)). \tag{A10}$$

The variance of log SDF shows up in (A8) and (A9), it is obvious that a tight link exists between capital accumulation, the required return to capital, and currency risk premium.

C.1. Proof of Proposition 1

Proof. Recall that the two countries are at the deterministic steady state at period one so $\hat{\lambda}^i_{C,0} = \hat{\lambda}^j_{C,0}$. The change in exchange rates is given by

$$\begin{aligned}\Delta ex &= \hat{m}^i - \hat{m}^j \\ &= \hat{\lambda}^i_C - \hat{\lambda}^j_C \\ &= \frac{\nu\gamma(1+\lambda_s)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}[(\beta_z^\star - \beta_z)\sigma_g\varepsilon_g + \sigma(\varepsilon^\star - \varepsilon)].\end{aligned}$$

The last equality is obvious from (A6).

□

C.2. Proof of Proposition 2

Proof. Taking cross-country differences using A8 and substituting in A6, we have

$$\begin{aligned}\hat{k}^j_0 - \hat{k}^i_0 &= \log\left[\mathbb{E}_0\left(e^{\hat{\lambda}^j_X + \hat{y}^j - \hat{\lambda}^j_{C0}}\right)\right] - \log\left[\mathbb{E}_0\left(e^{\hat{\lambda}^i_X + \hat{y}^i - \hat{\lambda}^i_{C0}}\right)\right] \\ &= \frac{1}{2}\frac{\nu^2(1-\gamma(1+\lambda_s))^2}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\left[(\beta^j_z)^2 - (\beta^i_z)^2\right]\sigma_g^2.\end{aligned}$$

Recall that the risk-free rate is given by

$$R^f_{i,0} = \frac{1}{\mathbb{E}_0\left(\beta\frac{\Lambda^i_{C,1}}{\Lambda^i_{C,0}}\right)}.$$

And the required return to capital is given by

$$\mathbb{E}(R^I_{i,1}) = \mathbb{E}(\frac{\Lambda^i_{X,1}}{\Lambda^i_{C,1}}\alpha\frac{Y^i_1}{K^i_0}).$$

Writing them in log-deviation terms, we have

$$\hat{r}^f_{i,0} = -\log(\eta) - \log\left(\mathbb{E}_0(e^{\hat{\lambda}^i_C - \hat{\lambda}^i_{C0}})\right)$$
$$\hat{r}^I_{i,1} = \log(\nu) + \hat{\lambda}^i_X - \hat{\lambda}^i_C + \hat{y}^i - \hat{k}^i_0.$$

Substituting in our solution (A6), (A7), and (A8),, we have

$$\hat{r}^f_{j,0} - \hat{r}^f_{i,0} = -\frac{1}{2}\frac{\nu\gamma^2(1+\lambda_s)^2}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\left[(\beta^j_z)^2 - (\beta^i_z)^2\right]\sigma^2_g$$
$$\hat{r}^I_{j,1} - \hat{r}^I_{i,1} = -\frac{1}{2}\frac{\nu^2(1-\gamma(1+\lambda_s))^2[\gamma(1+\lambda_s)(1+\nu(1-\alpha))(1-\nu)+\nu^2(1-\alpha)]}{(\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2)^2}\left[(\beta^j_z)^2 - (\beta^i_z)^2\right]\sigma^2_g.$$

□

D. Risk-Free Rate Volatility

Table A3: Volatility of Risk-Free Rates, Model vs. Data

Country	Data(%)	Model(%)
AUS	2.04	1.19
CAN	2.05	1.27
CHE	0.59	0.93
EUR	2.76	0.89
GBR	1.28	0.89
JPN	3.46	1.18
NOR	2.23	1.00
NZL	1.88	1.28
SWE	2.01	0.78

This table summarizes volatility of real interest rates in the data and in model simulations. Model moments are obtained by simulating the model for 200 samples of 104 periods.
Data Source: Real interest rates are from the World Development Indicators of the World Bank.

This table confirms the findings in Chen (2017) that risk-free rates are stable in the model when the sensitivity is set to a constant in (2). Note that the real interest rate volatilities in this table are rough estimates and are presented here purely to give a broad idea of the risk-free rate volatilities in the data. The main takeaway is that the model generated risk-free rates are stable.

E. Other Quantitative Approaches

E.1. Currency risk premium under Verdelhan (2010)

Verdelhan (2010) extends the standard habit model of Campbell and Cochrane (1999) to an international setup. Under his specification, consumption growth is exogenously given and $\lambda(\cdot)$ is a function. In particular,

$$(1+\lambda(s_0))^2 = \frac{1}{\text{var}(\Delta c)}\frac{1-\rho_s}{\gamma}(1-2(s_0-\bar{s})).$$

Note that consumption growth, which is constant under the setup in Verdelhan (2010), is built into the sensitivity function $\lambda(\cdot)$. Verdelhan (2010) also assumes symmetric countries, so $\text{var}(\Delta c) = \text{var}(\Delta c^\star)$, but even if we allow the two countries to have different variance of consumption growth so that $\text{var}(\Delta c) \neq \text{var}(\Delta c^\star)$, we still end up with

$$\begin{aligned}
\mathbb{E}(rx) &= \mathbb{E}\left(-\frac{1}{2}(\text{var}(m^\star) - \text{var}(m))\right) \\
&= -\frac{1}{2}\,\mathbb{E}[\gamma^2(1+\lambda^\star(s^\star))^2\,\text{var}(\Delta c^\star) - \gamma^2(1+\lambda(s))^2\,\text{var}(\Delta c)] \\
&= \frac{1-\rho_s}{\gamma}\,\mathbb{E}(s_0 - \bar{s} - (s_0^\star - \bar{s}^\star)) \\
&= 0.
\end{aligned}$$

Thus, setting the sensitivity function to a constant and to be the same across countries, although a simple tweak, is essential for external habit to generate large currency risk premia.

E.2. Expected Change in Exchange Rates under Epstein and Zin (1989) Preferences

Under Epstein and Zin (1989) preferences, a hard-wired relationship exists between the first and second moment of the log SDF:

$$\begin{aligned}
\mathbb{E}(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\text{var}_t(u_{t+1})) \\
\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})) &= \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2\mathbb{E}(\text{var}_t(u_{t+1})).
\end{aligned}$$

The first moment of the log SDF is tightly linked to the second moment and is not independent of heterogenous loadings. Thus, if there are large heterogenous-loading induced unconditional currency risk premia (large differences in $\mathbb{E}(\text{var}_t(m_{t+1}))$), there are large unconditional movements in the change in exchange rates, $\mathbb{E}(\Delta ex) = \mathbb{E}(m - m^\star)$. In fact,

$\mathbb{E}(\Delta ex_{t+1}) = -\frac{\gamma-1}{\gamma-\frac{1}{\psi}}\,\mathbb{E}(rx_{t+1})$. Under standard calibrations, the expected change in exchange rates account for a large portion of the currency risk premia.

F. Details on Estimation

I estimate the model using the simulated method of moments following Jalali et al. (2015). The model is exactly identified, as shown in Table A4.

Table A4: Moment Matching

Country	s.d. of GDP (%)		correlation	
	Data	Model	Data	Model
AUS	0.58	0.58	0.43	0.43
CAN	1.06	1.06	0.78	0.78
CHE	1.12	1.12	0.78	0.78
EUR	1.12	1.12	0.87	0.87
GBR	1.05	1.05	0.88	0.88
JPN	1.41	1.41	0.74	0.74
NOR	1.11	1.11	0.60	0.60
NZL	0.99	0.99	0.42	0.42
SWE	1.48	1.48	0.87	0.87

This table shows target moments used in Section 4. "s.d. of GDP" stands for standard deviation of HP-filtered GDP, and "correlation" stands for the correlation between each country's HP-filtered GDP with the average across countries in the sample. All moments are quarterly.
Data source: OECD National Account Statistics.

To calculate the standard errors of the estimates, I use parametric boostrapping. In particular, I take the estimated parameters as given, and simulate the model for 2000 times, each using a different set of random shocks. I then estimate the moment variance-covariance matrix as

$$\hat{S} = \frac{1}{L1}\sum_{l1=1}^{L1}\left[\tilde{M}_S^{l1}(\hat{\theta}) - \frac{1}{L2}\sum_{l2=1}^{L2}\tilde{M}_S^{l2}(\hat{\theta})\right]\cdot\left[\tilde{M}_S^{l1}(\hat{\theta}) - \frac{1}{L2}\sum_{l2=1}^{L2}\tilde{M}_S^{l2}(\hat{\theta})\right]',$$

where $L1 = L2 = 1000$, $\tilde{M}_S^{l1}(\hat{\theta})$ denotes the moments obtained in sample $l1$ taken $\hat{\theta}$ as given. With this moment variance-covariance matrix, I then calculate the standard errors by taking the diagonal elements of

$$Q = \left(1 + \frac{1}{K}\right) \left[\left(\frac{\partial m}{\partial \theta}\right)' \cdot \hat{S}^{-1} \cdot \frac{\partial m}{\partial \theta} \right]^{-1},$$

where K is the number of samples that I used in estimating $\hat{\theta}$, and $\frac{\partial m}{\partial \theta}$ is calculated numerically using using forward differentiation.

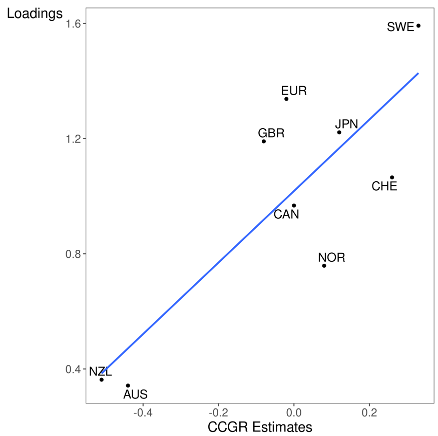
The estimated parameters are shown in Table A5 (standard errors in parentheses).

Table A5: Estimated Parameters

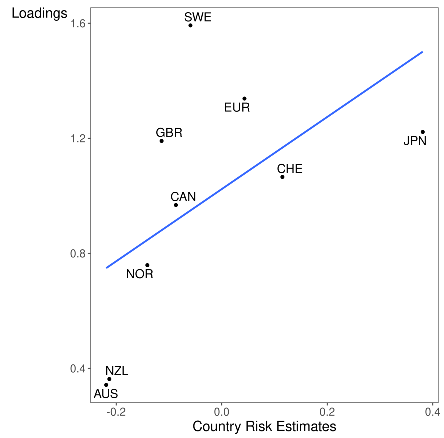
Country	β_z^i	$\sigma_z^i(\%)$
AUS	0.34	0.44
	(0.12)	(0.05)
CAN	0.97	0.57
	(0.19)	(0.08)
CHE	1.07	0.61
	(0.20)	(0.08)
EUR	1.34	0.46
	(0.20)	(0.07)
GBR	1.19	0.39
	(0.17)	(0.06)
JPN	1.22	0.83
	(0.25)	(0.10)
NOR	0.76	0.70
	(0.20)	(0.09)
NZL	0.36	0.77
	(0.21)	(0.09)
SWE	1.59	0.58
Global	1	0.64

G. Robustness Figures

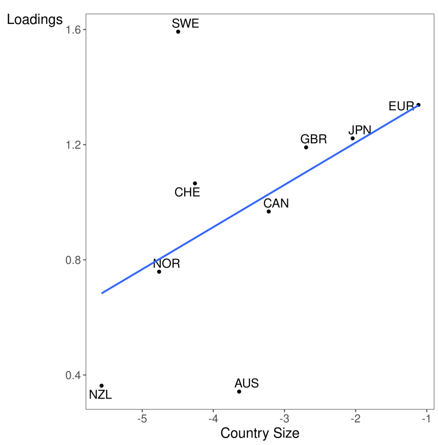
Figure A1: Estimated Loadings: Validation, with Sweden



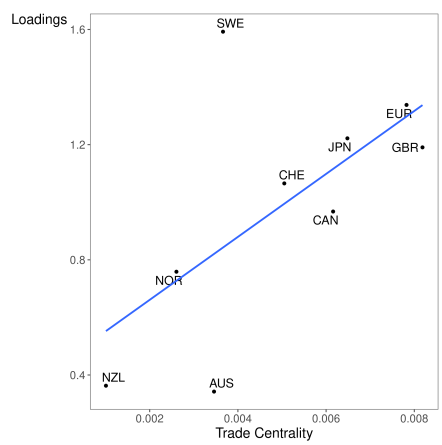
(a) CCGR Estimates



(b) Country Risk Estimates



(c) Country Size



(d) Trade Centrality

This figure plots my estimates against the estimates obtained by Colacito et al. (2018a) in Panel (a), the estimates obtained by Hassan et al. (2021c) in Panel (b), country size (Hassan (2013)) in Panel (c), and trade centrality (Richmond (2019)) in Panel (d).

Figure A1 is the same as Figure 4 but includes Sweden for Panel (b), (c) and (d). We can clearly see that Sweden is an obvious outlier, suggesting it is special in terms of its correlation with the world average. I leave the source of its specialty for future research.