

# Currency Risk and Capital Accumulation

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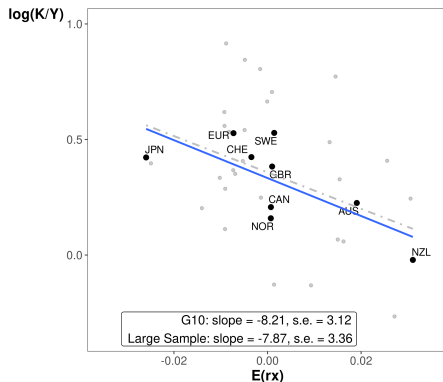
November 11, 2021

# The Lucas Paradox and Currency Risk

- **Heterogeneity in capital-output ratios** across countries are large and persistent.
  - Suggest large and persistent cross-country **heterogeneity in returns to capital**.
  - Also evident in the developed world: Capital-output ratio in Japan is 44% higher than in New Zealand.
- Idea in finance: currency risk induce cross-country **variations in currency returns**
  - Risk-free bonds in safe currencies offer lower returns.
  - Currency return in JPN is 5.70% percent lower than NZL
- This paper: **link currency risk to capital-output ratios**.
  - Currency risk should also matter for returns to capital and thus capital accumulation!

Details

# Negative Correlation between Log K/Y and $\mathbb{E}(rx)$ : G10



Currency risk premium:  $\mathbb{E}_t(rx_{t+1}^i) = r_{f,t}^i - \mathbb{E}_t[\Delta ex_{t+1}] - r_{f,t}^{US}$

- Large variations in  $\log(K/Y)$  and  $\mathbb{E}(rx)$ .
- Safe currency feature lower  $\mathbb{E}(rx)$  and higher  $\log(K/Y)$

# This Paper...

- Endogenize capital accumulation within a **quantitative** international asset pricing framework. The model features:
  - **heterogeneous loadings** on a global shock;
  - **external habit**.
- Estimate the model using **GDP data** of countries(regions) issuing the G10 currencies.

## Main Findings

- Loadings that are estimated **from correlations of GDP with the world alone** are highly correlated with capital-output ratios.
- Model generated capital-output ratio differences accounts for roughly **55%** of that in the data for the G10 countries.
- Model generated currency risk premia comes predominately from interest rate differences, consistent with the data.

# Literature Review

- Papers that explain interest rate differentials with riskiness of exchange rates.
  - Reduced form or qualitative: Lustig and Verdelhan (2007), Lustig, Roussanov and Verdelhan (2011, 2014), Hassan (2013), Richmond (2019), Ready, Roussanov and Ward (2017), among others
  - Quantitative: Colacito, Croce, Gavazzoni and Ready (2018), Gourio, Siemer and Verdelhan (2013), Bansal and Shaliastovich (2013)  
This paper: **focus on K/Y, quantitative, better match  $r_f^* - r_f$**
- Lucas Paradox. [Details](#)
  - Karabarbounis and Neiman (2014), Hsieh and Klenow (2009), Caselli and Feyrer (2007), Monge-Naranjo, Sanchez and Santaaulalia-Llopis (2019). Hall and Jones (1997), Jorgenson (1996), Alfaro, Kalemli-Ozcan, and Volosovych (2008), David, Henriksen and Simonovska (2016)
  - Hassan, Mertens and Zhang (2016), Richers (2021)  
This paper: **focus on  $\mathbb{E}(r)$  and G10, link to GDP and heter loadings**
- External Habit
  - Verdelhan (2010), Heyerdahl-Larsen (2014), Stathopoulos (2017)
  - Campbell and Cochrane (1999), Chen (2017)  
This paper: **endogenize K in a heterogeneous-country framework.**

# Outline

- Set-up
- Intuition and Mechanism
- Estimation & Results

## Model Setup: Households

**Population:** There are  $N$  countries, indexed by  $i \in \{1, 2, \dots, N\}$ , each populated with a unit measure of households.

**Preference:**

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \eta^t \frac{(C_t^i - H_t^i)^{1-\gamma} - 1}{1-\gamma}$$

where  $H_t^i$  denote the habit level.

**Surplus Consumption Ratio:** Define the surplus consumption ratio as  $S_t^i = \frac{C_t^i - H_t^i}{C_t^i}$ ,  $s_t^i = \log(S_t^i)$  follows (Chen (2017))

$$s_t^i = (1 - \rho_s)\bar{s} + \rho_s s_{t-1}^i + \lambda_s (\Delta c_t^i - \mu)$$

**Labor Supply:** Households supply 1 unit of labor inelastically.

## Model Setup: Firms

**Production Function:** In each country, a representative firm produces a country specific good under the production function:

$$Y_t^i = e^{z_t^i} (K_t^i)^\alpha (e^{\mu t} N_t^i)^{1-\alpha}$$

**Productivity process:** Different loadings on a global shock.

$$z_t^i = \rho z_{t-1}^i + \beta_z^i \sigma_g \varepsilon_{z,t}^g + \sigma_z^i \varepsilon_{z,t}^i$$

**Capital accumulation:**

$$K_{t+1}^i = \Phi(I_t^i / K_t^i) K_t^i + (1 - \delta) K_t^i$$

where the capital adjustment cost follows Jermann(1998)

$$\Phi\left(\frac{I}{K}\right) = a_1 + \frac{a_2}{1 - \frac{1}{\xi}} \left(\frac{I}{K}\right)^{1 - \frac{1}{\xi}}$$



# Model Setup: Final Good and Resource Constraints

**Final Good:** with home bias parameter  $\nu > 0$

$$F_t^i = (X_{i,t}^i)^\nu \prod_{j=1}^N (X_{j,t}^i)^{\frac{1-\nu}{N}}$$

**Resource Constraints**

$$F_t^i = C_t^i + I_t^i$$

$$Y_t^i = \sum_{j=1}^N X_{i,t}^j \quad \forall i, t$$

Markets are complete. Solve the model by solving a social planner's problem with all the resource constraints.

# Examining the Mechanism: A Simplified Version

Suppose

- $N = 2$ .
- The economy is at its deterministic steady state at period 0 and the world ends at period 1.
- No capital adjustment cost:  $\Phi(\frac{I}{K}) = \frac{I}{K}$ .
- Capital fully depreciates:  $\delta = 1$ .
- Country specific shocks feature the same volatility:  
 $\sigma_z^* = \sigma_z = \sigma$

## Result #1: Change in Exchange Rate and Currency Risk

Change in exchange rate (foreign/home) is given by:

$$\Delta ex = m - m^*$$

$$\approx \frac{\nu\gamma(1+\lambda_s)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu) + \nu^2} [(\beta_z^* - \beta_z)\sigma_g\varepsilon_g + \sigma(\varepsilon^* - \varepsilon)]$$

### Proposition 1

If  $\beta_z > \beta_z^*$

- if  $\varepsilon_g < 0$ ,  $\Delta ex > 0$ : the real exchange rate increases (appreciation of the high loading home currency) when a negative global shock hits.
- expected change in exchange rate is 0, and currency risk premium are driven by interest rate differences.  $E(rx) = r_f^* - r_f$

Intuition: when a negative global shock hits:

- The country-specific good of the high loading country is especially scarce, and more expensive;
- Because of home bias, the price of its consumption bundle increase: its currency appreciates.

## Result #2: Currency Risk, $r_f$ and Capital Accumulation

### Proposition 2

*Under the simplified specification, the higher loading country*

- ...features lower currency risk premium and risk-free rates .

$$\begin{aligned}\mathbb{E}(r_x) &= r_f^* - r_f \\ &\approx -\frac{1}{2} \frac{\nu \gamma^2 (1 + \lambda_s)^2}{\gamma (1 + \lambda_s) (1 + \nu) (1 - \nu) + \nu^2} [(\beta_z^*)^2 - (\beta_z)^2] \sigma_g^2\end{aligned}$$

- ...features lower required return to capital

$$\begin{aligned}\mathbb{E}(r^* - r) &\approx -\frac{1}{2} [\gamma (1 + \lambda_s) (1 + \nu (1 - \alpha)) (1 - \nu) + \nu^2 (1 - \alpha)] \\ &\quad \times \frac{\nu^2 (1 - \gamma (1 + \lambda_s))^2}{(\gamma (1 + \lambda_s) (1 + \nu) (1 - \nu) + \nu^2)^2} [(\beta_z^*)^2 - (\beta_z)^2] \sigma_g^2\end{aligned}$$

- ...accumulates more capital.

$$k^* - k \approx \frac{1}{2} \frac{\nu^2 (1 - \gamma (1 + \lambda_s))^2}{\gamma (1 + \lambda_s) (1 + \nu) (1 - \nu) + \nu^2} [(\beta_z^*)^2 - (\beta_z)^2] \sigma_g^2$$

# Linking K/Y to Currency Risk Premium

High currency risk premium country accumulates less capital and has higher return to capital.

$$\mathbb{E}(r^* - r) \approx \nu \left(1 - \frac{1}{(1 + \lambda_s)\gamma}\right)^2 B \mathbb{E}(rx)$$
$$k^* - k \approx -\nu \left(1 - \frac{1}{(1 + \lambda_s)\gamma}\right)^2 \mathbb{E}(rx)$$

- Currency risk premia passes through to required return to capital and thus capital accumulation.
- Currency risk jointly determines currency risk premia and capital-output ratios.

# Quantitative Challenge

Recall that  $\mathbb{E}(rx_{NZL-JPN}) = 5.70\%$ , and

$$\mathbb{E}_t(rx_{t+1}) = -\frac{1}{2}(\text{var}_t(m_{t+1}^*) - \text{var}_t(m_{t+1}))$$

But under standard CRRA preferences,  $\text{var}_t(m_{t+1}) = \gamma^2 \text{var}_t(\Delta c_{t+1})$

- “Currency Premium Puzzle”: *Difference* in variances of aggregate consumption growth is too small

## Result #3: Quantitative Performance: Role of Habit

With habit, the variance of the log SDF is given by

$$\text{var}(m) = \text{var}(-\gamma s - \gamma c) = \gamma^2(1 + \lambda_s)^2 \text{var}(\Delta c)$$

- Agents fear the state when consumption is close to the habit level.
- They have high "effective risk aversion" w.r.t consumption risk.

### Proposition 3

*If countries share the same constant sensitivity parameter  $\lambda_s$ , currency risk premium is given by*

$$\mathbb{E}(rx) = -\frac{1}{2}\gamma^2(1 + \lambda_s)^2(\text{var}(\Delta c^*) - \text{var}(\Delta c))$$

Remark: with capital accumulation, risk-free rate is smooth even with large, constant  $\lambda_s$  (Chen (2017)).

risk-free rate volatility    difference from Verdelhan (2010)

# Summary of the Theoretical Results

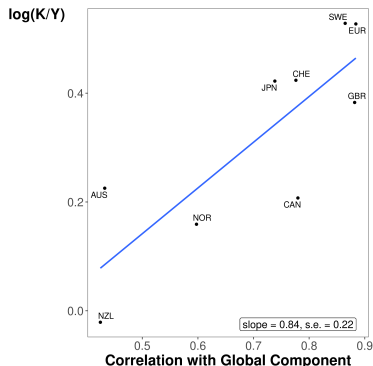
In the simplified model

- ① Currencies of high loading countries appreciate in global bad times and are thus safe.
- ② High loading country features lower risk-free rate, lower required return to capital and accumulates more capital.
- ③ Capital accumulation is negatively correlated with currency risk premium and expected return to capital.
- ④ Habit generates large currency risk premium as in the data;
- ⑤ Expected change in exchange rate is 0 and currency risk premia are driven by interest rate differences;

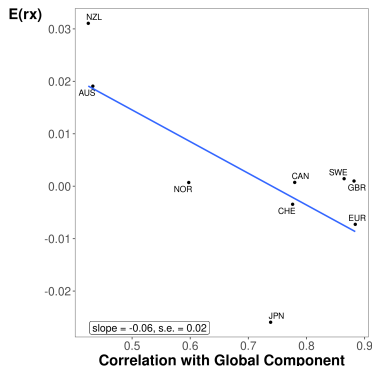


# Evidence on Heter Loadings on a Global Shock

Figure: Correlation of GDP with Global Component



(a) Capital Output Ratio



(b) Currency Risk Premium

Countries that covary more with the world have low currency risk premium and accumulates more capital.

# Estimation by SMM: Data and Targets

- Quarterly GDP data (from OECD National Accounts Dataset) for countries issuing G10 currencies.
- Parameters to be estimated: loadings on the global shock  $\beta_z^i$  and volatility of country specific shocks  $\sigma_z^i$
- Target moments: standard deviation of HP-filtered GDP for each country, as well as the correlations of HP-filtered GDP with its average across countries.

Objective Function:

$$\hat{\Theta} = \arg \min_{\Theta} \left( \frac{H(\Theta) - H_D}{H_D} \right)' \left( \frac{H(\Theta) - H_D}{H_D} \right)$$

Target Matching

# Calibrated Parameters

Description	Value	Source
<b>Preference and Production:</b>		
Relative risk aversion $[\gamma]$	4	
Capital Share $[\alpha]$	0.35	
Subjective discount factor $[\eta]$	0.995	Chen(2017)
Degree of home bias $[\nu]$	0.98	Colacito et al. (2018)
Depreciation Rate $[\delta]$	0.016	Chen (2017)
Elasticity of I/K wrt Tobin's Q $[\xi]$	0.7	Kaltenbrunner and Lochstoer (2010)
<b>TFP:</b>		
Mean of TFP growth(%) $[\mu]$	0.45	Chen (2017)
Persistence of TFP growth $\rho$	0.98	Chen (2017)
<b>Habit:</b>		
Mean surplus consumption ratio(%) $[\bar{S}]$	7	Verdelhan (2010)
Persistence $[\rho_s]$	0.995	Verdelhan (2010)

# Estimated Loadings

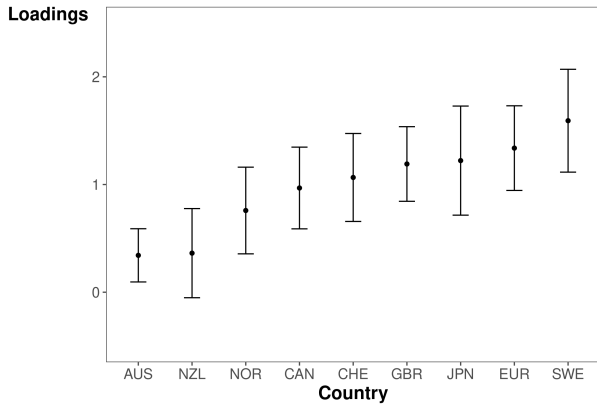
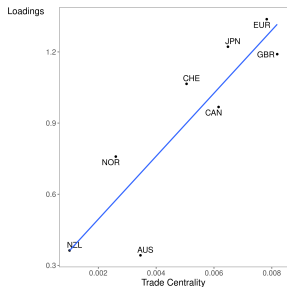
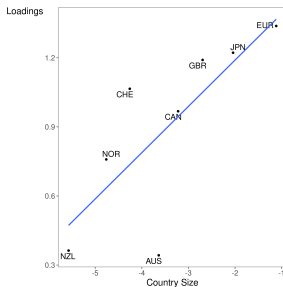
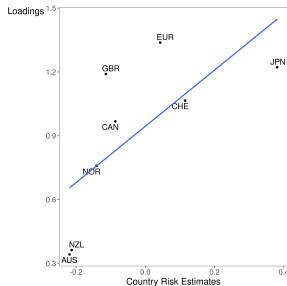
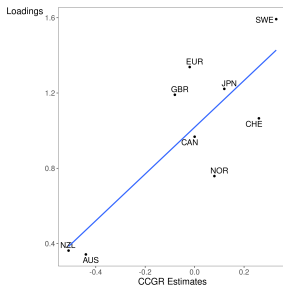


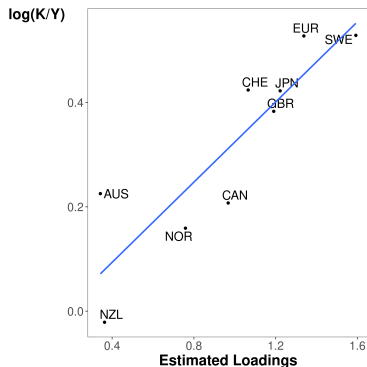
Table of  $\beta_z^i$

# Correlations with other Estimates/Potential Drivers

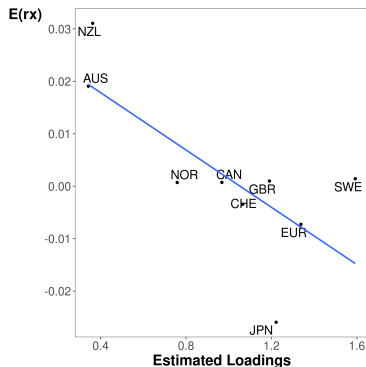


# Estimated Loadings, $K/Y$ and Currency Risk Premia

Figure: Estimated Loadings



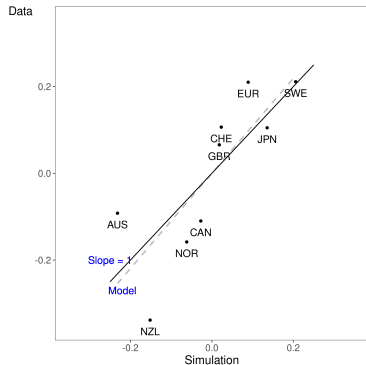
(a) Capital Output Ratio



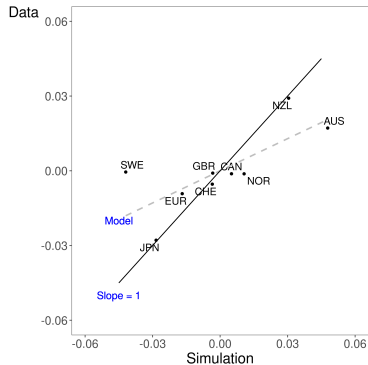
(b) Currency Risk Premium

- The  $R^2$  for currency risk premia is 0.49 and for  $K/Y$  is 0.70.
- Loadings estimated **from only GDP data** are highly correlated with currency risk premia and capital-output Ratios.

# Simulated Data vs Empirical Data: K/Y



(a) Capital Output Ratio



(b) Currency Risk Premium

Interest Rate Differences

## JPL-NZL Example

Use Japan as the base country:

	Diff in $\log(K/Y)$	$\mathbb{E}(r_x)$	$r_f^{NZL} - r_f^{JPN}$	$\mathbb{E}(\Delta ex)$
Data	-0.44	5.70%	5.08%	-0.62%
Model	-0.29	5.89%	6.06%	0.17%

- The model explains a large portion of the difference in capital-output ratios.
- The model matches currency risk premium very well.
- The model generates large difference in risk-free rates, with minimal unconditional movements in exchange rates.



# Variance Decomposition: Average Performance

Write capital-output ratio in the data as

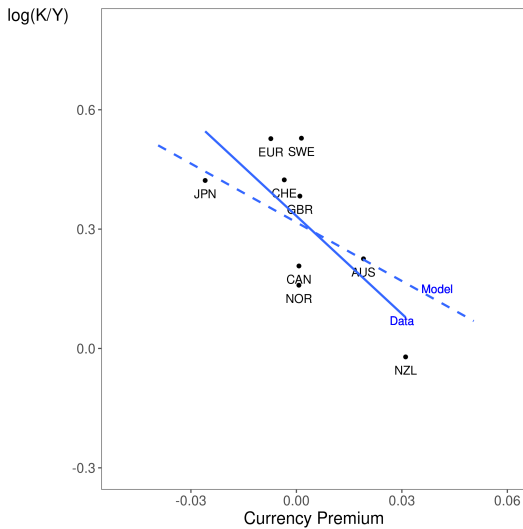
$$\kappa_D^i = \kappa_M^i + e^i$$

Taking variance on both side:

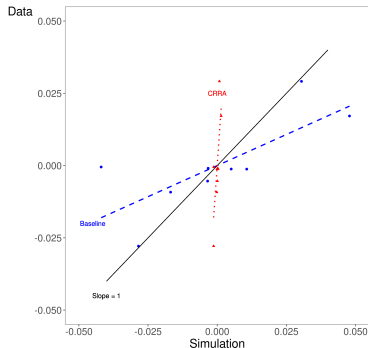
$$\underbrace{\text{var}(\kappa_D^i)}_{0.0345} = \underbrace{\text{var}(\kappa_M^i)}_{0.0189} + \underbrace{\text{var}(e^i)}_{0.0119} + \underbrace{2 \text{cov}(\kappa_M^i, e^i)}_{0.0036}$$

$\frac{\text{var}(\kappa_M^i)}{\text{var}(\kappa_D^i)} = 54.76\%$ : the model can account for 54.76% of the cross-country variations in capital-output ratios among countries issuing the G10 currencies!

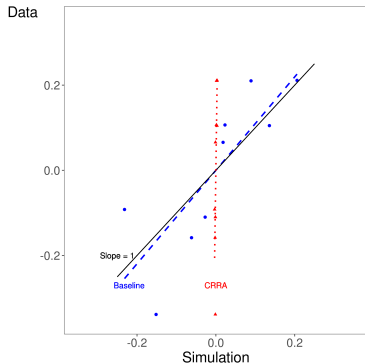
# Correlation of $\log(K/Y)$ and $\mathbb{E}(r_x)$ : Model vs Data



# Habit vs CRRA: Significant Quantitative Improvement



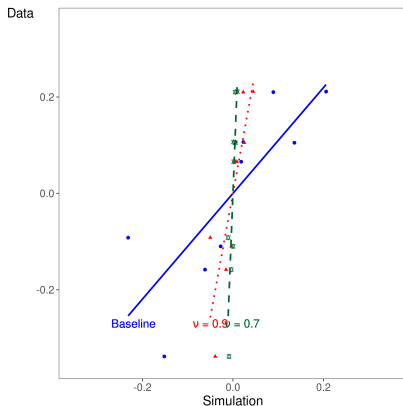
(a) Currency Risk Premium



(b) Capital-output Ratio

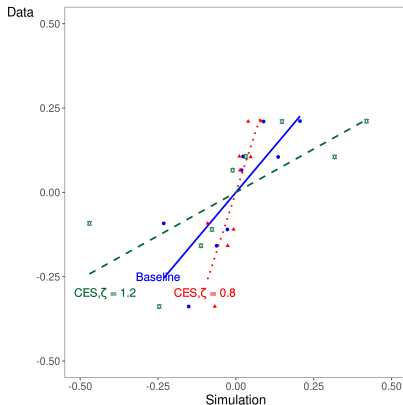
- Although CRRA fails quantitatively, the simulated moments are still highly correlated with the data.  $R^2$  : 0.58, 0.59

# Robustness: Home Bias



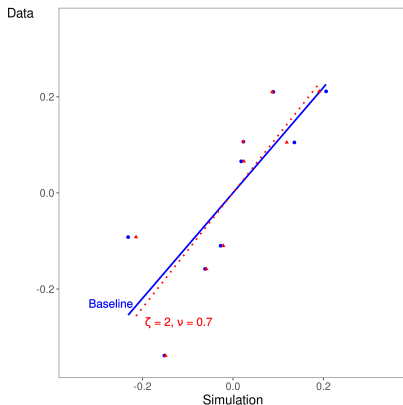
The model-generated differences in  $\log(K/Y)$  is smaller when lower home bias  $\nu$ , but still highly correlated.

# Robustness: CES Aggregator



The model-generated differences in  $\log(K/Y)$  is increasing in elasticity of substitution  $\zeta$ .

## Robustness: High Elasticity with Low Home Bias



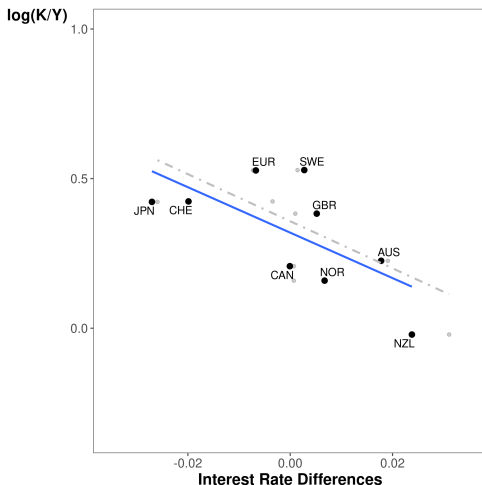
When a high elasticity is allowed ( $\zeta = 2$ ), a lower home bias ( $\nu = 0.7$ ) can be allowed for similar performance as the baseline.

# Conclusion

- Currency with higher loading on global shock is safe, and thus feature lower currency risk premia, lower required return on capital, and accumulate more capital.
  - Heterogenous loadings estimated from GDP data is informative of currency risk premia and capital-output ratios.
  - Currency risk can explain 55% of the cross-country variations in capital-output ratios among countries issuing the G10 currencies.
- External habit can generate large currency risk premia through large differences in risk-free rates.

Thank you very much!

# A1: Negative Correlation between Log K/Y and InRate Diff



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## A2: Deviation from Verdelhan (2010)

Under Verdelhan (2010):

- Countries are symmetric so no unconditional variance in variance of consumption growth,  $E(rx) = 0$ ;
- Even if there are difference in  $\text{var}(\Delta c)$ , it would cancel out because the specific functional form of the sensitivity function.

$$(1 + \lambda(s))^2 = \frac{1}{\text{var}(\Delta c)} \frac{1 - \rho_s}{\gamma} (1 - 2(s - \bar{s}))$$

Currency Risk Premium

$$\begin{aligned}\mathbb{E}(rx) &= -\frac{1}{2} \mathbb{E}(\text{var}(m^*) - \text{var}(m)) \\ &= -\frac{1}{2} \gamma^2 [(1 + \lambda(s))^2 \text{var}(\Delta c^*) - (1 + \lambda(s^*))^2 \text{var}(\Delta c)] \\ &= \mathbb{E}(s - \bar{s} - (s^* - \bar{s}^*)) = 0\end{aligned}$$

## A3: Change in Exchange Rate Under EZ

Under Epstein and Zin (1989) preference Colacito, Croce, Gavazzoni and Ready (2018), there is a hard-wired relationship between first and second moment of the log SDF.

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1 - \gamma) \left( \frac{1}{\psi} - \gamma \right) \mathbb{E}(\text{var}_t(u_{t+1}))$$

$$\frac{1}{2} \mathbb{E}(\text{var}_t(m_{t+1})) = \frac{1}{2} \left( \frac{1}{\psi} - \gamma \right)^2 \mathbb{E}(\text{var}_t(u_{t+1}))$$

- high interest rate currency appreciate a lot.
- $\mathbb{E}(\Delta \text{ex}_{t+1}) = -\frac{\gamma-1}{\gamma-\frac{1}{\psi}} \mathbb{E}(r x_{t+1})$ , most of the currency risk premium is accounted for by expected change in exchange rates, and risk-free rate difference is tiny.

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## A4: Simulated Data vs Empirical: Target Moments

Country	s.d. of GDP (%)		correlation	
	Data	Model	Data	Model
AUS	0.58	0.58	0.43	0.43
CAN	1.06	1.06	0.78	0.78
CHE	1.12	1.12	0.78	0.78
EUR	1.12	1.12	0.87	0.87
GBR	1.05	1.05	0.88	0.88
JPN	1.41	1.41	0.74	0.74
NOR	1.11	1.11	0.60	0.60
NZL	0.99	0.99	0.42	0.42
SWE	1.48	1.48	0.87	0.87

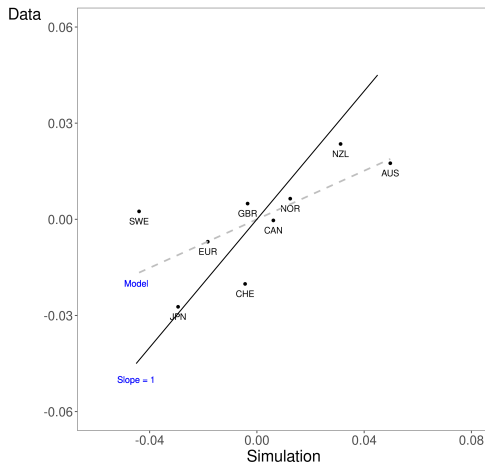
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## A5: Estimated Parameter Values

Country	$\beta_z^i$	$\sigma_z^i(\%)$
AUS	0.34 (0.12)	0.44 (0.05)
CAN	0.97 (0.19)	0.57 (0.08)
CHE	1.07 (0.20)	0.61 (0.08)
EUR	1.34 (0.20)	0.46 (0.07)
GBR	1.19 (0.17)	0.39 (0.06)
JPN	1.22 (0.25)	0.83 (0.10)
NOR	0.76 (0.20)	0.70 (0.09)
NZL	0.36 (0.21)	0.77 (0.09)
SWE	1.59	0.58
Global	1	0.64

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## A6: Simulated Data vs Empirical Data: Interest Rate Differences



## A7: Capital Accumulation and Currency Risk Premium

Cobb-Douglas production function implies:

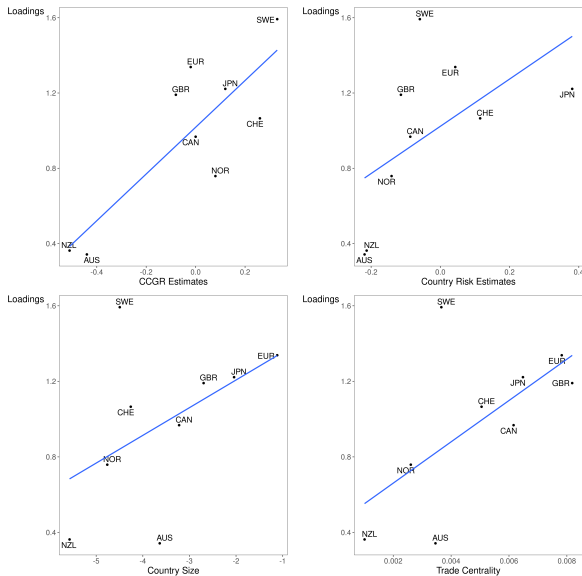
$$\mathbb{E}_t(Y_{t+1}/K_{t+1}) = \frac{[\mathbb{E}_t[r_{t+1}] + \delta] \tau}{\alpha}$$

- Previous literature focuses on  $\alpha$ ,  $\tau$  and  $\delta$ .
- Relatively little attention on  $\mathbb{E}_t[r_{t+1}] = r_{f,t} + \text{risk premium}$

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[back to literature](#)

## A8: Correlations with other Estimates/Potential Drivers



## A9: Volatility of Interest Rate Differences: Model vs Data

Country	Data(%)	Model(%)
AUS	0.50	0.46
CAN	0.29	0.41
CHE	0.46	0.42
EUR	0.42	0.37
GBR	0.36	0.33
JPN	0.63	0.52
NOR	0.56	0.48
NZL	0.48	0.57
SWE	0.72	0.44

Model generated interest rates are stable: capital offers extra channel of intertemporal substitution and consumption smoothing.

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# A10: Controlling for Institutions

	<i>Dependent variable:</i>							
	Capital-output Ratios Relative to the US							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
E( $\alpha$ )	-7.945* (3.405)	-8.742** (3.319)	-7.010 (4.340)	-7.722* (3.609)	-7.549* (3.300)	-10.662 (5.617)	-7.223 (4.071)	-8.193 (4.399)
FDI	0.143 (0.397)							
FOI		-0.140 (0.195)						
CC			-0.077 (0.182)					
GE				-0.085 (0.245)				
PS					-0.150 (0.187)			
RQ						0.194 (0.359)		
RL							-0.107 (0.257)	
VA								-0.001 (0.350)
Constant	0.226 (0.300)	0.635 (0.426)	0.476 (0.339)	0.477 (0.420)	0.492* (0.204)	0.031 (0.561)	0.515 (0.441)	0.334 (0.491)
Observations	9	9	9	9	9	9	9	9
R <sup>2</sup>	0.508	0.537	0.512	0.507	0.546	0.521	0.511	0.497
Adjusted R <sup>2</sup>	0.344	0.382	0.349	0.343	0.395	0.361	0.348	0.330
Residual Std. Error (df = 6)	0.150	0.146	0.150	0.150	0.144	0.148	0.150	0.152
F Statistic (df = 2; 6)	3.098	3.476*	3.148	3.088	3.610*	3.259	3.140	2.968

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

# A11: Controlling for Institutions, Larger Sample [back](#)

	<i>Dependent variable:</i>	
	Capital-output Ratios Relative to the US	
	(1)	(2)
$E(rx)$	-3.276 (3.768)	-9.283** (3.435)
FDI	0.139 (0.393)	0.050 (0.436)
FOI	0.184*** (0.066)	
CC	-0.816*** (0.220)	-0.696*** (0.240)
GE	0.734** (0.276)	0.452 (0.286)
PS	0.051 (0.132)	0.139 (0.142)
RQ	0.059 (0.257)	0.346 (0.262)
RL	0.033 (0.268)	0.012 (0.299)
VA	0.071 (0.071)	0.063 (0.079)
Constant	-0.177 (0.191)	0.043 (0.194)
Observations	37	37
R <sup>2</sup>	0.601	0.487
Adjusted R <sup>2</sup>	0.468	0.341
Residual Std. Error	0.219 (df = 27)	0.243 (df = 28)
F Statistic	4.520*** (df = 9; 27)	3.323*** (df = 8; 28)

Note: \* p<0.1; \*\* p<0.05; \*\*\* p<0.01