Currency Risk and Capital Accumulation*

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Abstract

The "Lucas Paradox" states that there are large and persistent differences in capital-output ratios across countries, suggesting capital is not flowing to countries where it is relatively scarce. In the data, capital-output ratios vary a lot even within developed countries and they are negatively correlated with currency premia and risk-free rates. I build a quantitative multi-country model of capital accumulation with external habit and heterogenous exposures to a global productivity shock. I show that currency risk in this model generates cross-country variations in capital-output ratios that is quantitatively consistent with the data. I estimate the model using GDP data from countries issuing the G10 currencies and find two main results: (1) the heterogenous loadings that I extract from GDP data alone are highly correlated with capital-output ratios; (2) when I feed the estimated loadings to the model, model generated capital-output ratios account for roughly 55% of the cross-country variations in the data.

Keywords: currency risk, currency return, capital-output ratio, external habit, safe currency

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1. Introduction

Differences in capital-output ratios are large and persistent across countries, a phenomenon known as the "Lucas Paradox" (Lucas (1990)). A lesser known fact is that such phenomenon is also evident within developed countries. For example, the capital-output ratio in Japan is 44% higher than in New Zealand. Everything else being equal, this is suggesting that there is significant and long-lasting cross-country differences in returns to capital, and capital is not flowing across borders to eliminate such differences. Existing explanations include various frictions that prevent capital from moving freely, heterogeneity in the production functions, and misallocation of resources. In this paper, I argue that currency risk induces persistent differences in required rates of return to capital across countries and is a quantitatively important driver of cross-country variations in capital-output ratios.

A growing literature has established that some currencies are "safer" in the sense that they appreciate in global bad times. This property is widely used to understand cross-country variations in currency returns and risk-free rates[1]. Again, using Japan and New Zealand as an example, as a well-known safe currency, the Japanese yen offers 5.70% lower annual return on average than New Zealand, the currency of which is considered risky. Intuitively, from an international investor's perspective, risk-free bonds denominated in safe haven currencies are attractive because they offer higher returns in terms of appreciation in global downturns. In equilibrium, these bonds should feature lower risk-free rates, leading to lower currency returns. But, by the same logic, investing in capital in countries with safe haven currencies should also be attractive. If currency risk induces systematic variations in risk-free rates and currency returns across countries[2], it should pass through to required rates of return to capital[3]. Countries with safe haven currencies should thus feature lower cost of capital and higher capital-output ratios[4].

In this paper, I study the link between currency risk and capial-output ratios by endogenizing capital accumulation within a quantitative international asset-pricing model with external habit. To induce different responses to global downturns and thus currency riskiness,

[1]Examples are Lustig and Verdelhan (2007), Lustig et al. (2011),Lustig et al. (2014), Menkhoff et al. (2012), Hassan (2013), Lettau et al. (2014), Farhi and Gabaix (2016), Maggiori (2017), Ready et al. (2017), Mueller et al. (2017), Colacito et al. (2018a), Richmond (2019), Wiriadinata (2021), Jiang (2021), among others.

[2]Empirically, risk-free rate differences across countries are large and persistent, and they are linked to safeness of the corresponding currency. See Lustig et al. (2011), Hassan and Mano (2018), for example.

[3]Richers (2021) finds direct empirical evidence for this. Specifically, he finds that violation of the uncovered interest rate parity strongly passes through to firm borrowing and cost of capital.

[4]For example, consider a Cobb-Douglas production function, one can easily derive $\mathbb{E}(Y/K) = \frac{\mathbb{E}[r]+\delta}{\alpha}$, where Y is output, K is capital, $\mathbb{E}[r]$ is expected return to capital net of depreciation, δ is depreciation and α is capital share. Obviously capital-output ratio is negatively linked to expected return to capital.

I follow Lustig et al. (2011) and Colacito et al. (2018a) and allow countries' productivities to differ in their exposures to a global shock[5]. In addition, agents are sensitive to currency risk because of external habit, which enables the model to generate large cross-country variations in currency premia and capital-output ratios as in the data. I theoretically show that in this model, countries with higher exposures to a global shock feature lower currency premia, lower required rates of return to capital, and accumulate more capital.

I then take the model to the data and estimate the exposures to the global shock using GDP data from countries (regions) issuing the G10 currencies. Two main findings emerge: First, the estimated exposures that I extract just from correlations of GDP across countries are highly correlated with capital-output ratios, suggesting that heterogenous exposures on a global shock, which is often seen as an asset-pricing tool to rationalize currency premia, are tightly linked to economic fundamentals and have real implications. Second, when I feed the estimated exposures to my model, the model generated cross-country variations in capital-output ratios can account for roughly 55% of that in the data.

To better motivate the model, I first explore the cross-country variations in capital-output ratios and currency premia, and the correlations between them in the data. I define currency premium of country i relative to the U.S., $\mathbb{E}_t(rx^i_{t+1})$, as the expected return a U.S. investor would get if she borrows at the risk-free rate in the U.S. and invests it in a foreign risk-free bond:

$$\mathbb{E}_t(rx^i_{t+1}) = r^i_{f,t} - \mathbb{E}_t[\Delta ex_{t+1}] - r^{U.S.}_{f,t}$$

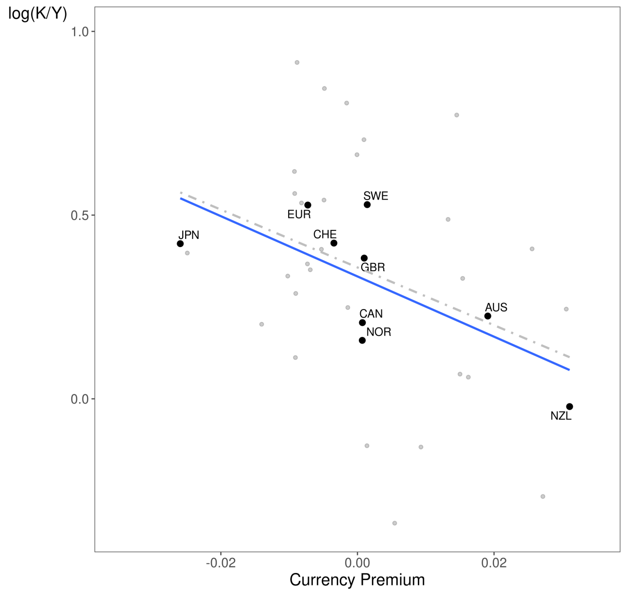
where $r^i_{f,t}$ and $r^{U.S.}_{f,t}$ denote the risk-free rates of country i and the U.S. respectively, and Δex_{t+1} is the change in exchange rates quoted in units of country i currencies per U.S. dollar[6]. If currency safeness jointly determines currency premia and capital-output ratios, we would expect countries with safe currencies to be associated with lower currency premia and higher capital-output ratios. In our Japan-New Zealand example, it is indeed true that Japan features a higher capital-output ratio and a low currency premium.

Figure 1 confirms this conjecture for a broader set of countries. In Figure 1, I plot log capital-output ratios relative to the U.S. against U.S.-based currency premia for countries

[5]These exposures can be seen as a reduced form representation of a number of mechanism that can induce currency risk, for example, country size (Hassan (2013)), trade centrality (Richmond (2019)), and financial development (Maggiori (2017)). See Hassan and Zhang (2021) for a survey.

[6]In the data, expected change in exchange rate is close to zero unconditionally (See Hassan and Mano (2018)), so cross-country variations in risk-free rates and currency premia are quantitatively very similar. In this paper, I focus on currency premia because global investors should take change in exchange rates into account, but all results are similar if I use risk-free rate differences instead. All results using risk-free rates are available upon request.

Figure 1: Negative Correlation between Log Capital Output Ratios and Currency Premia



This figure plots unconditional log capital-output ratios relative to the U.S. against unconditional currency premia relative to the U.S. for countries issuing the G10 currencies (black dots) and a broader sample of 37 currencies (grey dots). The line of best fit has a slope of -8.21 (s.e. 3.12) for G10 (solid blue line) and -7.87 (s.e. 3.36) for the broader sample (dashed grey line). All moments are annual.
Data source: Capital stock and output are from PWT 10.0. Currency premia are from Adrien Verdelhan's website. Data ranges from 1994 to 2019. Details on the data construction can be found in section 4.1 and Appendix B.

(regions) issuing the G10 currencies[7] (Australia, Canada, the Euro area, Great Britain, Japan, Norway, New Zealand, Sweden, Switzerland and United States. Throughout the paper, I treat the Euro area as one country). We can clear see that there is indeed large variation in capital-output ratios and currency premia across countries. In addition, there is a strong negative relationship between capital-output ratios and currency premia (solid blue line): countries with lower currency premia and thus lower risk-free rates accumulate more capital. To the extend that currency premia are tightly connected to currency risk, Figure 1 empirically links currency risk to capital-output ratios.

Although the Lucas Paradox is often referred to as a phenomenon between developed (rich) and developing (poor) countries, Figure 1 shows that it is also evident within the developed world, which is perhaps even more puzzling. I consciously restrict my sample

[7] Throughout this paper, I always use G10 to refer to countries issuing the G10 currencies (the most traded ten currencies), not the usual The Group of Ten countries.

to countries issuing the G10 currencies[8] for two reasons: (1) all the countries (regions) in this group are western developed countries and are relatively homogeneous in terms of institutions, tax systems and market structures, which mitigates the concern that these factors might confound my analysis[9]; and (2) this set of countries closely resembles complete markets, which I am going to assume in my model. Nevertheless, if I extend my sample to a set of 37 countries (grey dots in Figure 1), the same pattern persists: there is large cross-country variation in capital-output ratios and currency premia across countries, and there is a clear negative relationship between the two. In fact, the line of best fit (dashed grey line) has almost the same slope as the G10 currencies (solid blue line), and they are both significant at the 5% level. In general, capital-output ratios and currency premia vary significantly across countries and they are negatively correlated with each other in the data.

Motivated by the empirical fact and recognizing that currency premia are connected to currency risk, I extend a standard international asset pricing model of currency risk to incorporate capital accumulation and study the link between currency risk and capital-output ratios. There are two key ingredients in the model. First, I allow countries to have different loadings (exposures) on a global productivity shock; second, agents in my model extracts utility from an exogenously given habit level.

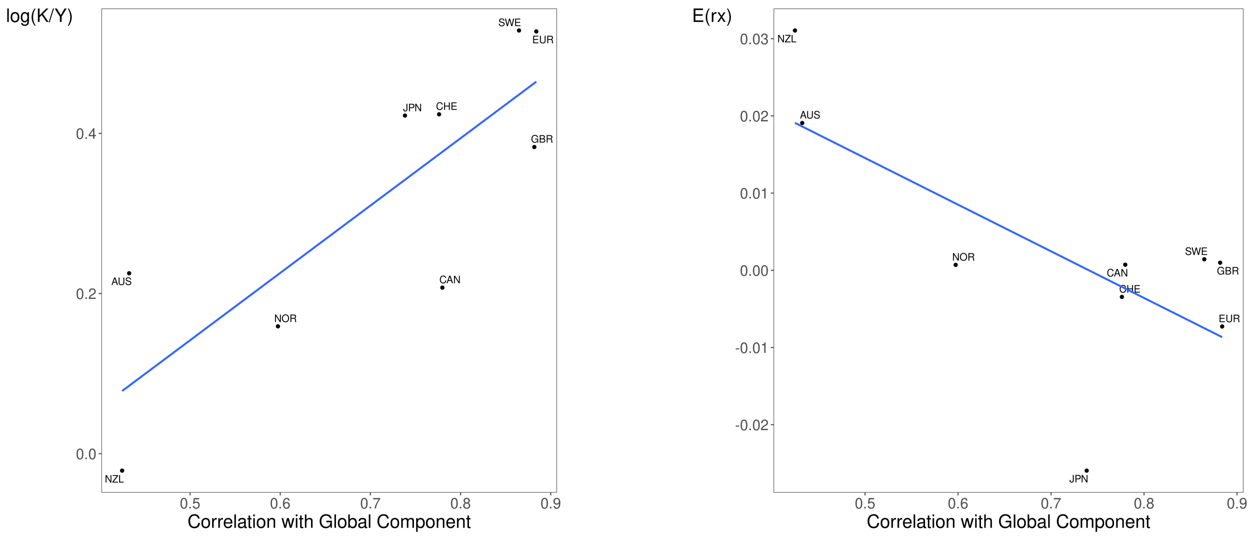
Heterogeneous loadings on a global shock is widely used in the international asset pricing literature to link currency risk to cross-country variations in currency premia[10]. Extending this framework to a general equilibrium model of capital accumulation, my model generates cross-country variation in capital-output ratios in addition to currency premia in a way that is consistent with the data. Intuitively, the country-specific good of the high-loading country becomes relatively more expensive when a global negative shock hits. With home bias, the price of the consumption bundle of said country increases, which translates to appreciation of the real exchange rate. In this sense, the high loading currency appreciates in global bad times and is thus safe. In equilibrium, global investors require a lower return on assets of high-loading countries, including risk-free assets and investments in capital, and high-loading countries feature lower risk-free rates, lower required rates of return to capital, and install

[8]Note that as the base country, U.S. does not show up in Figure 1 so my sample has effectively 9 countries. I make this choice because (1) currency premia need a base country to calculate, and the U.S. dollar is the standard choice; and (2) it is well known that as the center of the global trade, financial, and political network, the U.S. is special in terms of capital accumulation, so I leave the proper treatment of the U.S. for future research.

[9]In Appendix A, I show that the negative correlations between capital-output ratios and currency premia is robust to controlling for financial development, governance efficiency and financial openness, suggesting that these factors are not main drivers of cross-country variation in capital-output ratios within the countries issuing G10 currencies.

[10]Examples are Lustig et al. (2011, 2014), who assume SDFs load differently on a global shock, and Colacito et al. (2018a), who assume long-run endowment processes load differently on a global shock.

Figure 2: Correlations with Global Component



(a) Capital-output Ratio

(b) Currency Premium

This figure plots log capital-output ratios (Panel(a)) and currency premia (Panel(b)) relative to the U.S. against correlations of each country's HP-filtered log GDP with the cross-country average (the global component). The line of best fit has a slope of 0.84 (s.e. 0.22) for capital-output ratios and a slope of -0.06 (s.e. 0.02) for currency premia.
Source: Capital stock and output data is from PWT 10.0. Currency premia data is from Adrien Verdelhan's website. GDP data is from OECD National Accounts Statistics. Data ranges from 1994 to 2019. Details on the data construction can be found in section 4.1.

more capital. Heterogeneous loadings on a global shock thus induce cross-country variation in currency safeness, currency premia, expected returns to capital, and capital-output ratios.

Once capital accumulation and production is introduced, heterogenous loadings on a global shock also have implications on correlations of output across countries. In particular, output of countries with higher exposure to the global shock should covary more with world output. I show direct empirical evidence for this in Figure 2. Panel (a) plots capital-output ratios against the correlation of each country's HP-filtered log GDP with a global component (the cross-country average of HP-filtered log GDP); Panel (b) plots the same figure for currency premia. As we can see clearly, if the GDP of a country correlates more with the world, it features a lower currency premium and a higher capital-output ratio.

Motivated by this empirical fact, in my estimation exercise, I extract the loadings on the global shock solely from correlations of GDP with the average across countries, and show that the estimated loadings are highly correlated with currency premia and capital-output ratios. This fact in itself is interesting: the correlation of GDP with the world average is informative on how much capital a country is going to install. Heterogenous loadings on a global shock, which is typically used to understand currency returns and often linked to asset prices[11], can actually be traced back to a macro variable, namely GDP, and have implications for capital accumulation.

While heterogenous exposures on a global shock can induce cross-country variations in currency premia and capital-output ratios that are qualitatively consistent with the data, external habit is essential for the quantitative success of the model. There is a quantitative difficulty in generating currency premia as large as in the data using standard CRRA preferences, similar to the famous equity premium puzzle[12]. I thus use external habit, which is proven to be successful in resolving the equity premium puzzle, to boost currency premia and cross-country variations in capital-output ratios to their empirical levels.

In my model, agents extract utility from their consumption over an externally given habit level (Campbell and Cochrane (1999)), which is approximately a geometric average of the history of consumption. Thus, their marginal utility of consumption are jointly determined by consumption and the habit level. Because the habit level is also sensitive to consumption, overall, an agent's marginal utility of consumption is much more responsive to changes in consumption than without habit. This feature effectively makes the agents more risk-averse to consumption risk and helps the model to generate large risk premium. Following Chen (2017), I assume constant sensitivity of habit with respect to changes in consumption, as opposed to time varying sensitivity in standard habit models[13], and I set the sensitivity parameter to be the same across countries. Under this setup, my model generates unconditional cross-country variations in currency premia and capital-output ratios comparable to the data.

In addition to generating large currency premia, using external habit in a heterogenous country set-up also resolves a more subtle quantitative puzzle in international asset pricing: in the data, currency premia are mostly accounted for by interest rate differences, not expected change in exchange rates. Take Japan and New Zealand as an example, the currency premium between the two countries is 5.70%, with 5.08% coming from interest rate

[11] For example, Colacito et al. (2018a) estimates these loadings using dividend data.

[12] See Hassan and Zhang (2021) for a broad discussion of how the cross-country "currency premium puzzle" and the equity premium puzzle are related.

[13] Chen (2017) made this choice for convenience because the sensitivity function in standard Campbell and Cochrane (1999) external habit models are reverse engineered, which is not possible when capital accumulation is considered. In Section 3.3, I show that in my setup, this choice is actually economically important in terms of generating large currency premia.

differences[14]. Existing quantitative models tend to struggle with this fact[15]. My model with external habit and heterogenous loadings on a global shock generates a currency premium of 6.06% between Japan and New Zealand, all which it comes from interest rate differences: expected change in exchange rate is exactly zero in my model. To my knowledge, my model is the first risk-based general equilibrium heterogenous country framework that generates unconditional interest rate differences as large as in the data.

To evaluate the quantitative performance of the model, I estimate the loadings on the global shock using GDP data of countries issuing the G10 currencies, then simulate the model taking the estimated loadings as given and contrast the simulated capital-output ratios with the data. In a variance decomposition exercise, I find that the estimated model accounts for roughly 55% of the cross-country variations in capital-output ratios in the data. I conduct robustness check by exploring how habit, the degree of home-bias and the elasticity of substitution between different goods affects my results.

My paper makes contributions to three strands of literature. First, it contributes to the large body of work rationalizing the significant, persistent difference in capital-output ratios across countries. Existing explanations include heterogeneity in the protection of property rights (Hall and Jones (1997)), in the capital share of output (Karabarbounis and Neiman (2014)), in the misallocation of resources (Hsieh and Klenow (2009)), in natural resource endowments (Caselli and Feyrer (2007) and Monge-Naranjo et al. (2019)), and in institutional quality (Alfaro et al. (2008)). However, all of these explanations don't consider currency risk. I show that currency risk alone can induce significant cross-country variations in capital-output ratios. David et al. (2014) links capital-output ratios to correlations of returns to capital with the U.S. stock market. They find that the emerging markets covary more with the U.S. stock market and is thus risky, and returns to capital are high as a result. I focus on currency risk instead and show that currency risk is a key driver of capital-output ratio among the developed G10 countries.

Second, this paper complements the literature on currency risk with a quantitative approach. While various risk-based explanations exist on why some countries have lower interest rates than others, almost all of them are either reduced form (Lustig et al. (2011, 2014),

[14] In fact, Hassan and Mano (2018) shows that one can not reject the null that expected change in exchange rate is 0 for a representative country.

[15] For example, Gourio et al. (2013) study a model in which countries differ in severity of disasters and agents feature Epstein and Zin (1989) preferences. They generate a currency premium of 2.36%, with only 0.55% accounted for by interest rate differences; Colacito et al. (2018a) also study a model with Epstein and Zin (1989) preference, and they focus on heterogenous exposure to a long-run global shock. Their model generates a carry trade return of around 3%, with about 1% coming from interest rate differences. In ongoing work Hassan et al. (2021a), we show that this problem is embedded into the Epstein and Zin (1989) preferences.

Verdelhan (2018)) or qualitative (e.g., Hassan (2013), Richmond (2019), Ready et al. (2017), Maggiori (2017)). I adapt external habit preferences to a heterogenous country set-up and show that such preferences boost the difference in the volatility of stochastic discount factors across countries, and thus generate currency premia and risk-free rate differences comparable to the data. In addition, currency premia are fully accounted for by interest rate differences in my model, which is consistent with the data and differentiates my approach from Gourio et al. (2013) and Colacito et al. (2018a).

Also, the majority of the currency risk literature only focus on asset prices and do not consider capital accumulation, and this paper is one of the few that studies the real impications of the advancements in international asset pricing. Colacito et al. (2018b) studies capital flow patterns in an international set-up with long-run and short-run risk and Epstein and Zin (1989) preferences, but they only study symmetric countries and focus on capital flow instead of capital stock. Hassan et al. (2016) studies a model where countries feature different sizes and real exchange rate volatilities, and find that countries are either larger or more volatile tend to accumulate more capital. But their model is qualitative and they directly rely on real exchange rate dynamics to discipline their model. My paper uses GDP data instead, and can generate economically significant variation in capital-output ratios comparable to the data. Hassan et al. (2021b) studies the effect of exchange rate stabilization on capital accumulation, and finds that stabilizing relative to a safe currency promotes capital accumulation.

Third, this paper is also related to several others applying external habit models to international setups. Verdelhan (2010) uses external habit with time-varying risk-free rates to explain the UIP puzzle. Stathopoulos (2017) further allows for risk-sharing and home bias across countries, and resolves the Backus and Smith (1993) puzzle. Heyerdahl-Larsen (2014) uses deep habit to rationalize a series of puzzles in the international finance literature. However, all of these papers feature symmetric countries and thus yield no unconditional differences in risk-free rates. They also feature endowment economies and thus do not endogenize capital accumulation. Chen (2017) studies an external habit model with capital accumulation, but he only focuses on closed economy. I extend his framework to a heterogeneous-country setup, and study its implications for cross-country variations in capital accumulations.

The rest of my paper is organized as follows.In Section 2, I set up the model and derive the optimality conditions. In Section 3 , I solve a simplified two-period, two-country version of the model to study the mechanisms. In Section 4, I solve and estimate the full model and conduct robustness checks. I conclude in Section 5.

2. The Model

In this section, I extend a standard multi-country international asset pricing model to incorporate capital accumulation and heterogeneous countries, as well as external habit. The basic structure of the model largely follows Colacito et al. (2018a), except that instead of using Epstein and Zin (1989) preference, I use external habit, and I endogenize capital accumulation. The economy within each country closely resembles the closed-economy model of Chen (2017).

2.1. Setup

2.1.1. Households

There are N countries indexed by $i = \{1, 2, \ldots, N\}$, each populated by a unit measure of households. Households in country i extract utility from consumption over an externally given habit level H_t^i, and maximize

$$\mathbb{E}_0 \sum_{t=0}^{T} \eta^t \frac{(C_t^i - H_t^i)^{1-\gamma} - 1}{1-\gamma}$$

where η is the time discount factor, C_t^i denotes consumption, and γ governs relative risk aversion. T is a terminal period (which I set to 1 in Section 3 and ∞ in all other sections). Following Campbell and Cochrane (1999), instead of directly specifying an exogenous process for H_t^i, I assume that the surplus consumption ratio, $S_t^i = \frac{C_t^i - H_t^i}{C_t^i}$, follows

$$s_{t+1}^i = (1-\rho_s)\bar{s} + \rho_s s_t^i + \lambda_s(\Delta c_{t+1}^i - \mu) \tag{1}$$

where $s_t^i = \log(S_t^i)$. Throughout the paper, lower case letters denotes logs so that $x = \log(X)$. $\bar{s}$ is the steady state level of s_t^i, ρ_s governs the persistence, and μ is the steady state growth rate of technology. $\bar{s}$, ρ_s and μ are assumed to be the same across countries.

$\lambda_s \geq 0$ governs the sensitivity of the log surplus consumption ratio to consumption growth. Following Chen (2017), I deviate from the standard external habit model by setting λ_s to be a constant instead of a function. In particular,

$$\lambda_s = \frac{1}{\bar{S}} - 1$$

is set to be the steady-state level of Campbell and Cochrane (1999). There are two reasons for this deviation. First, the sensitivity function $\lambda(\cdot)$ in Campbell and Cochrane (1999) is reverse-engineered to ensure a constant risk-free rate. Once capital accumulation is introduced, reverse-engineering is no longer feasible[16]. Second, as we will see in Section 3.3, setting λ_s to be a constant and to be the same across countries helps with generating large currency premium.

Households supply 1 unit of labor inelastically.

2.1.2. Firms

Each household owns a firm that produces a country specific good Y_t^i. The production process is identical across all firms within a country and can be summarized by the production function

$$Y_t^i = e^{z_t^i}(K_t^i)^\alpha(e^{\mu t}N_t^i)^{1-\alpha} \tag{2}$$

where K_t^i denotes capital and N_t^i denotes labor. α is the capital's share of output. The productivity process z_t^i follows

$$z_{t+1}^i = \rho z_t^i + \beta_z^i \sigma_g \varepsilon_{z,t+1}^g + \sigma^i \varepsilon_{z,t+1}^i \tag{3}$$

where ρ governs the persistence of the productivity process and is assumed to be the same across countries. z_t^i is subject to two shocks: a country specific shock $\varepsilon_{z,t+1}^i \overset{i.i.d.}{\sim} N(0,1)$ and a global shock $\varepsilon_{z,t+1}^g \overset{i.i.d.}{\sim} N(0,1)$. σ^i and σ_g are the corresponding volatilities. I assume that all shocks are orthogonal to each other.

Following the international asset pricing literature, I assume that each country's productivity has a different loading β_z^i on the global shock. Under reduced form frameworks (e.g. Lustig et al. (2011)) and endowment economies (e.g. Colacito et al. (2018a)), β_z^i is simply interpreted as a closed form way of capturing any country heterogeneities that may induce one currency to be safer than others, and asset prices are typically used to estimate them. In my framework with capital accumulation and production, these loadings can be directly linked to correlations of outputs, allowing for estimation using GDP data.

Capital accumulation follows

$$K_{t+1}^i = \Phi(I_t^i/K_t^i)K_t + (1-\delta)K_t^i \tag{4}$$

where δ is the depreciation rate and I_t^i is investment. Following Jermann (1998), I assume

[16] See Chen (2017) for a complete analysis on the advantage and disadvantages of setting λ_s to be a constant. In particular, Chen (2017) shows that the habit level H_t^i is approximately a geometric average of the history of consumption, and risk-free rates remain stable because capital offers an alternative way of transferring resources across time. I confirm that my model generated smooth risk-free rates in Appendix D.

that firms face a convex capital adjustment cost:

$$\Phi(\frac{I}{K}) = a_1 + \frac{a_2}{1-\frac{1}{\xi}} \left(\frac{I}{K}\right)^{1-\frac{1}{\xi}}$$

where ξ governs the elasticity of investment-capital ratio with respect to Tobin's Q. $a_1 = \frac{\exp(\mu)-1+\delta}{1-\xi}$ and $a_2 = (\exp(\mu) - 1 + \delta)^{\frac{1}{\xi}}$ are chosen so that at the steady state, $\Phi(I/K) = \exp(\mu) - 1 + \delta$ and $\Phi'(I/K) = 1$.

2.1.3. Final Good and Resource Constraints

Households produce a final good F_t^i according to a Cobb-Douglas aggregator[17]:

$$F_t^i = (X_{i,t}^i)^{\nu} \prod_{j=1}^{N} \left(X_{j,t}^i\right)^{\frac{1-\nu}{N}} \tag{5}$$

where $X_{j,t}^i$ denotes the amount of country-j good used by a typical household in country i. Following a large literature surveyed by Lewis (2011), I assume households feature home bias and have extra preference for their own good, which is governed by $\nu > 0$. Home bias is important for the model to generate real exchange rate dynamics and differences in returns, which will become clear in Section 3.

The final good can be used for consumption or investment, so the resource constraint for it is given by

$$F_t^i = C_t^i + I_t^i \qquad \forall i, t. \tag{6}$$

The goods market clears for each country specific good

$$Y_t^i = \sum_{j=1}^{N} X_{i,t}^j \qquad \forall i, t \tag{7}$$

I assume complete market and I solve the model by solving a social planner's problem.

2.2. Solving the Model

For simplicity, I assume unit Pareto weights for all households and abstract away from the effect that heterogeneous loadings may have on the initial distribution of wealth across

[17] I use a Cobb-Douglas aggregator for simplicity and for comparison with Colacito et al. (2018a). In Section 4.4.2, I conduct robustness check using a constant elasticity of substitution (CES) aggregator.

countries. The social planner solves

$$\max \sum_{i=1}^{N} \left[\left(\mathbb{E}_0 \sum_{t=0}^{T} \eta^t \frac{(C_t^i - H_t^i)^{1-\gamma} - 1}{1-\gamma} \right) \right]$$

subject to the resource constraints (6) and (7). The first-order condition with respect to consumption is given by:

$$(S_t^i)^{-\gamma}(C_t^i)^{-\gamma} = \Lambda_{C,t}^i \qquad \forall i, t \tag{8}$$

where $\Lambda_{C,t}^i$ is the shadow price (marginal utility) of consumption. Note that S_t^i, the surplus consumption ratio, directly affects $\Lambda_{C,t}^i$. Because surplus consumption ratio is sensitive to consumption growth (see (1)), the marginal utility of consumption is more sensitive to consumption risk than standard CRRA preferences (where $S_t^i = 1$).

The first-order conditions with respect to each country-specific good are given by:

$$\Lambda_{C,t}^i (\nu + \frac{1}{N}(1-\nu)) \frac{F_t^i}{X_{i,t}^i} = \Lambda_{X,t}^i \qquad \forall i, t \tag{9}$$

$$\Lambda_{C,t}^j \frac{1}{N}(1-\nu) \frac{F_t^j}{X_{i,t}^j} = \Lambda_{X,t}^i \qquad \forall i, j \neq i, t \tag{10}$$

where $\Lambda_{X,t}^i$ is shadow prices of the country specific good of country i. First-order conditions (9) and (10) highlights the role of home bias in generating real exchange rate dynamics. Under complete market, the log real exchange rate between two countries is given by $Ex_t^{i,j} = \frac{\Lambda_{C,t}^j}{\Lambda_{C,t}^i}$. If $\nu = 0$ and there is no home bias, it can be shown that $\Lambda_{C,t}^i = \Lambda_{C,t}^i$ so that real exchange rate is one[18].

And we can derive the Euler equation as

$$1 = \mathbb{E}_t(M_{t+1}^i R_{t+1}^i) \tag{11}$$

where the the stochastic discount factor (SDF) is the growth rate of marginal utility of consumption:

$$M_{t+1}^i = \eta \frac{\Lambda_{C,t+1}^i}{\Lambda_{C,t}^i} \tag{12}$$

[18]This is well known in the international asset pricing literature. Intuitively, without home bias, all countries essentially consume the same final good and there is only one price for it. This will become clear in Section 3.1.

The required return to capital is given by

$$(13) \quad R^i_{t+1} = \frac{1}{Q^i_t}\left[(\nu + \frac{1}{N}(1-\nu))\frac{F^i_{t+1}}{X^i_{i,t+1}}\alpha\frac{Y^i_{t+1}}{K^i_{t+1}} - I^i_{t+1}/K^i_t + Q^i_{t+1}(\Phi(I^i_{t+1}/K^i_{t+1}) + 1 - \delta)\right]$$

where $Q^i_t = \frac{1}{\Phi'(I^i_t/K^i_t)}$ is Tobin's Q.

Now we have a recursive system of $(11+N)N$ equations (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), which determines policy functions for $(11+N)N$ endogenous variables $\{s^i_{t+1}, Y^i_t, z^i_{t+1}, K^i_{t+1}, I^i_t, F^i_t, C^i_t, \Lambda^i_{C,t}, \Lambda^i_{X,t}, M^i_t, R^i_t, \{X^i_{j,t}\}_{j=1,\ldots,N}\}_{i=1,\ldots,N}$.

3. Examining the Mechanism

In this section, I examine a simplified version of the model to study the mechanism. I first establish how different loadings on a global shock can induce different levels of currency riskiness, then show that it can jointly determine currency premia, required rates of return and capital-output ratio in a way that is consistent with the data. I also examine the role of habit in the quantitative success of the model. To that end, I consider a two country, two period, simplified version of the model and utilize log-linearization (with risk adjustments) to get closed form solutions.

In particular, in addition to setting $N = 2$ and $T = 1$, I shut down capital adjustment cost so that $\Phi(\frac{I}{K}) = \frac{I}{K}$, I set $\delta = 1$ so that capital fully depreciates between periods, and I set $\sigma^\star = \sigma$ so that the only source of heterogeneity between the home and foreign country is the loadings β_z and $\beta^\star_z$. To simplify notation, in this section, I omit the country indicator i and simply denote the foreign country with a star $\star$. I also omit the time indicator for period 1. I assume that the economy is at its deterministic steady state at period 0 and study agents' incentive to accumulate capital for production in period 1.

To get closed form solution and make the link between capital accumulation and currency risk explicit, I log-linearize the system around its deterministic steady state[19] where all shocks are 0 and all households hold a capital stock that is fixed at its steady state level. That is, I study households' incentive to accumulate different levels of capital while holding the initial capital stock fixed[20].

[19]The log-linearized system can be found in Appendix C.

[20]Fixing the initial capital stock avoids solving for a quadratic system of equations and makes closed-form solutions feasible.

3.1. Real Exchange Rate and Currency Risk

I first examine real exchange rates and show how different exposures on the global shock induce currencies to behave differently in global bad times. Under complete market, change in log real exchange rate (quoted in units foreign currencies per home currency) is given by the difference between log SDFs (Backus et al. (2001)):

$$\Delta ex = m - m^\star$$

where m and $m^\star$ are the log SDFs. Under my log-linear solution, real exchange rate is given by

$$\Delta ex \approx \underbrace{\frac{\nu\gamma(1+\lambda_s)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}}_{>0}[(\beta_z^\star - \beta_z)\sigma_g\varepsilon_g + \sigma(\varepsilon^\star - \varepsilon)] \tag{14}$$

where I use $\approx$ to highlight the fact that the result relies on log-linear approximation[21]. As discussed in Section 2.2, home bias ($\nu > 0$) is essential for there to be any exchange rate dynamics. If $\nu = 0$, both countries would consume the same consumption bundle, which can only have one price, and real exchange rate would be constant.

Expression (14) shows how different loadings β and $\beta^\star$ are connected to currency risk and induce one currency to be safer than the other. Without loss of generality, assume $\beta > \beta^\star$ so that the home country has a higher loading on the global shock. When a negative global shock hits ($\varepsilon_g < 0$), it is easy to see that $\Delta ex > 0$ so that the home currency appreciates. The intuition is as follows: when a negative global shock hits, the higher loading home country experiences a deeper drop in its productivity and produces less of its country-specific good, which becomes relatively expensive. Because of home bias ($\nu > 0$), its consumption bundle also becomes relatively expensive, resulting in appreciation of the home currency as real exchange rate is defined to be the relative price of the final consumption good. The higher loading home currency thus appreciates in global bad times and is perceived to be safer by global investors.

Another immediate implication of (14) is that expected change in exchange rate is 0. Recall that currency premia has two parts: expected change in exchange rate and risk-free rate differences. This property thus ensures that any currency premia generated by this model are fully accounted for by interest rate differences[22], which is consistent with the

[21] Alternatively one can denote $\hat{x} = x - \bar{x}$ for all endogenous variables and replace all $\approx$ with $=$.

[22] Such property is not satisfied under Epstein and Zin (1989) preferences. See Appendix E.2 for a brief discussion. We discuss this issue and its implication for international asset pricing in ongoing work Hassan et al. (2021a).

data.

I summarize the properties of the real exchange rate in the following proposition.

Proposition 1. *Under the simplified set-up, real exchange rate is given by (14). Suppose $\beta_z > \beta_z^\star$, we have*

- *the real exchange rate increases (appreciation of the high loading home currency) when a negative global shock hits.*
- *expected change in exchange rate is 0.*

Proof. See Appendix C. □

3.2. Currency Premium, Expected Return on Capital, and Capital Accumulation

In Proposition 1, we have established that the currency of the country with a higher loading on the global shock tends to appreciate when a negative global shock hits and is thus safe. In this section, I further link currency risk to currency premia, required returns on capital, and capital accumulations. I summarize the model predictions in the following proposition:

Proposition 2. *Under the simplified specification, with home bias ($\nu > 0$), the higher loading country*

- *...features lower currency premium and risk-free rates .*

$$\begin{aligned}\mathbb{E}(rx) &= r_f^\star - r_f - \underbrace{\mathbb{E}(\Delta ex)}_{\approx 0} \\ &\approx -\frac{1}{2}\nu A\left[(\beta_z^\star)^2 - (\beta_z)^2\right]\sigma_g^2\end{aligned}$$

- *...features lower required rate of return to capital*

$$\mathbb{E}(r^\star - r) \approx -\frac{1}{2}\nu^2\left(1 - \frac{1}{(1+\lambda_s)\gamma}\right)^2 AB\left[(\beta_z^\star)^2 - (\beta_z)^2\right]\sigma_g^2$$

- *...accumulates more capital*

$$k^\star - k \approx \frac{1}{2}\nu^2\left(1 - \frac{1}{(1+\lambda_s)\gamma}\right)^2 A\left[(\beta_z^\star)^2 - (\beta_z)^2\right]\sigma_g^2$$

where $A = \frac{\gamma^2(1+\lambda_s)^2}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2} > 0$ *and* $B = \frac{\gamma(1+\lambda_s)(1+\nu(1-\alpha))(1-\nu)+\nu^2(1-\alpha)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2} > 0$

Proof. See Appendix C. □

As in the data, the model predicts that the country with a safe currency (the high loading country) features lower currency premium and accumulate more capital. From an asset pricing perspective, this is intuitive: the higher loading country's currency appreciate in global bad times (Proposition 1), it is thus a good hedge against global risk. Investors require a lower rate of return on assets from this country, thus it features lower currency premium, lower required rate of return to capital, and accumulates more capital.

It is also intuitive from a social planner's perspective. Installing more capital in the high loading country is desirable for two reasons. First, because the high loading country features a more volatile productivity profile by construction and they prefer their own good, the social planner finds it optimal to install more capital there out of precautionary saving motive. Second, installing more capital in the high loading country also benefits the low loading country because when facing a negative shock, less resources needs to be transferred to the high loading country. To understand this, notice that because of heterogenous loadings, there exists risk-sharing and transfer of resources even when the shock is global. If more capital is installed in the high loading country, their marginal utility of consumption does not drop as much when a negative global shock hits. They need less help from abroad, which makes the low loading country better off as well. In this sense, installing more capital in the high loading country insures the world from downturns as a whole, so the social planner finds it optimal to do so.

3.3. Quantitative Performance: the Role of External Habit

Propositions 1 and 2 shows that when I extend a model of currency risk and currency premia to incorporate capital accumulation, the model predictions are qualitatively consistent with the data. I then use external habit to boost the heterogenous-loading induced currency premia and capital-output ratio differences to their empirical levels and ensure the quantitative success of the model.

I highlight the role of external habit in the following proposition:

Proposition 3. *If countries share the same constant sensitivity parameter λ_s, currency premium is given by*

$$\mathbb{E}(rx) = -\frac{1}{2}\gamma^2(1+\lambda_s)^2(var(\Delta c^\star) - var(\Delta c))$$

Proof. Under the simplified set-up, $s_0 = \bar{s}$ and c_0 is a constant. From (1), (8) and (12), we have

$$\text{var}(m) = \text{var}(-\gamma s - \gamma c) = \gamma^2(1+\lambda_s)^2 \,\text{var}(\Delta c) \tag{15}$$

Under complete market, currency premium is given by (Backus et al. (2001))

$$\begin{aligned} \mathbb{E}(rx) &= -\frac{1}{2}(\text{var}(m) - \text{var}(m^\star)) \\ &= -\frac{1}{2}\gamma^2(1+\lambda_s)^2(\text{var}(\Delta c^\star) - \text{var}(\Delta c)) \end{aligned}$$

□

If $\lambda_s = 0$, we are back to the standard CRRA preference and currency premium is given by difference in volatilities of consumption growth, which is known to be tiny[23]. With habit ($\lambda_s > 0$), difference in consumption variances are boosted by a factor of $(1+\lambda_s)^2$. Under standard calibration used in the external habit literature, λ_s is a large number[24], which is apparently helpful in generating large currency premium and thus differences in capital-output ratios in the data.

The intuition is as follows. Households care about the difference between their consumption level and the externally given habit and hate states when consumption is close to the habit level. Because consumption affects habit level through the sensitivity parameter (see equation (1)), habit provides an additional channel through which consumption could affect marginal utility (this is evident in equation (8)). As a result, marginal utility is much more sensitive to consumption growth than without habit, and the effective risk-aversion is much higher. Habit thus generates large currency premium even with mild differences in consumption growth volatilities.

Setting λ_s to be constant[25] and to be the same across countries is important for this result. In Appendix E.1, I show that in an endowment economy as in Verdelhan (2010) where λ_s takes the functional form used in standard habit models and is not constant across

[23]This quantitative challenge that CRRA preferences face in international asset pricing is similar to the famous equity premium puzzle. The equity premium puzzle states aggregate consumption is too smooth to account for the observed Sharpe ratio of the stock market (Hansen and Jagannathan (1991)), while here the *difference* between aggregate consumption growth volatilities is too small to account for the observed currency premium. For example, Hassan (2013) and Richmond (2019) both use CRRA preferences and can not generate quantitatively viable currency premia. See Hassan and Zhang (2021) for a brief discussion.

[24]For example, $\lambda_s = 16.54$ in Campbell and Cochrane (1999) and $\lambda_s = 13.29$ in Verdelhan (2010).

[25]In an endowment economy, setting λ_s to be a constant typically induces excess risk-free rate volatilities (see Campbell and Cochrane (1999)), but once capital accumulation is introduced, risk-free rates remain stable even with constant λ_s because agents have another channel (capital) to smooth their consumption profile. See Chen (2017) for a detailed analysis. I confirm that risk-free rates are stable in my model in Appendix D.

countries, currency premium is always 0 even if countries feature heterogeneous variance of consumption growth. In that sense, while in closed-economy asset pricing (Chen (2017)) setting λ_s to constant is a choice of convenience, the same trick is economically important in an international setup because it boosts the difference between variances of SDFs across countries and helps with generating large currency premium.

Together with Proposition 1, the model is capable of generating large currency premia while keeping expected change in exchange rate at 0. In other words, the model can generate large differences in risk-free rates and currency premia, which is consistent with the data. To my knowledge, this is the first quantitative risk-based framework with heterogenous countries which induces large cross-country variations in risk-free rates.

I summarize the main findings in the simplified model as follows. First, heterogenous loadings on a global shock induces currency risk. The currency of a high loading country appreciates when a negative global shock hits and is thus safe. Second, the high loading country features lower currency premium, lower required rate of return to capital and accumulate more capital. Third, external habit can generate large cross-country variations in currency premia while keeping expected change in exchange rate at 0.

4. Estimation of the Full Model

We have established a list of theoretical predictions from the simplified model in the last section. In this section, I take the full model to the data and estimate the loadings on the global shock for countries issuing the G10 currencies and investigate the quantitative implications of the model.

4.1. Data

I consider countries(regions) issuing the G10 currencies: Australia, Canada, the Euro area, Japan, New Zealand, Norway, United Kingdom, Sweden, Switzerland, and United States. I use quarterly GDP (in 2015 U.S. dollars) data from the OECD National Account Statistics starting from 1994Q1 to 2019Q4 (before COVID-19). I generate capital-output ratios from Pen World Table 10.0 (Feenstra et al. (2015)) by dividing capital stock by GDP and then averaging across the sample periods[26]. I get currency premium and exchange rate data from Adrien Verdelhan's website[27] by annualizing monthly returns and then taking average across the sample periods.

[26]For the Euro area, I divide the total capital of all countries in the Euro area by their total GDP.
[27]Verdelhan (2018): `http://web.mit.edu/adrienv/www/Data_for_AugmentedUIP_allcountries.xls`

Table 1: Externally Calibrated Parameters (Quarterly)

Description	Value	Source
Preference and Production:		
Relative risk aversion [γ]	4	
Capital Share [α]	0.35	
Time discount factor [η]	0.995	Chen (2017)
Degree of home bias [ν]	0.98	Colacito et al. (2018a)
Depreciation Rate [δ]	0.016	Chen (2017)
Elasticity of I/K wrt Tobin's Q [ξ]	0.7	Kaltenbrunner and Lochstoer (2010)
TFP:		
Mean of TFP growth(%) [μ]	0.45	Chen (2017)
Persistence of TFP [ρ]	0.98	Chen (2017)
Habit:		
Mean surplus consumption ratio(%) [$\bar{S}$]	7	Verdelhan (2010)
Persistence [ρ_s]	0.995	Verdelhan (2010)

Notes: This table summarizes calibrated parameters. Relative risk aversion γ and capital share α are calibrated to standard values. The rest of the parameters are taken from various papers in the literature. I list the source for each parameter under Source.

Country size are constructed following Hassan (2013) as the long-run GDP share[28]. Trade centrality are obtained from Robert J. Richmond's website[29] and are constructed by taking average across time.

4.2. Estimation

I begin by externally calibrating a set of parameters to standard values used in the literature, summarized in Table 1. These parameters are set to be the same across countries. The model is quarterly so one period represents a quarter.

The value of relative risk aversion γ is larger than what is typically used in standard habit models, but is the same as Heyerdahl-Larsen (2014) and lower than Van Binsbergen (2016). It is also well within the standard values used in macro models. I follow Colacito et al. (2018a) and set home bias ν to 0.98 for easy comparison to the literature. I discuss the effect of changing this parameter in Section 4.4.2 and show that under a CES aggregator, this value can be lowered. All other variables are standard.

I estimate the remaining 20 parameters $\Theta = \{\sigma_g^i, \sigma^i\}_{i=1,\ldots,N}$ using simulated method

[28] Take GDP share of each country relative to the sum across countries for each period, and then average across periods. See Hassan (2013) for details.

[29] Richmond (2019): `https://robertjrichmond.com/data/Richmond_Centrality.xlsx`

of moments (SMM). Here I estimate $\sigma_g^i = \beta_z^i \sigma_g$ because β_z^i and σ_g can not be estimated separately. I assume the average country has a loading of 1 on the global shock and set $\sigma_g = \frac{1}{N}\sum \sigma_g^i$, then calculate β_z^i for each country accordingly. All the standard errors are scaled accordingly.

Heterogenous loadings on a global shock are widely used as an asset pricing modeling tool and are typically estimated using asset prices. In my model with capital and production, it has direct implications for covariances of outputs across countries: high loading country should covary more with the world. Motivated by this theoretical link as well as the empirical patterns in Figure 2, I choose the following 20 data moments as targets: the standard deviations of HP-filtered GDP and the *correlation* of each country's HP-filtered GDP with the sample average across countries. I estimate the parameter vector $\hat{\Theta}$ by minimizing the distance between data moments and model simulated moments:

$$\hat{\Theta} = \arg\,\min_{\Theta} \left(\frac{H(\Theta) - H_D}{H_D}\right)' \left(\frac{H(\Theta) - H_D}{H_D}\right)$$

where H_D is a vector of the target moments in the data and $H(\Theta)$ is a vector of simulated moments for a given parameter vector Θ. I get $H(\Theta)$ by solving the model using second-order perturbation and simulate the model for 500 samples, each with 104 periods[30], then extract the target moments for each sample and take average across samples.

Because I have the same number of target data moments as the number of parameters to be estimated, the parameters are exactly identified[31]. The estimated parameter values are summarized in Figure 3, with two standard deviation bands[32].

The estimated loadings β_z^i are largely inline with common examples of safe and risky currencies. For example, Japan has a loading of 1.22 and New Zealand has a loading of 0.36 while Japanese yen is widely conceived as a safe haven currency and the New Zealand dollar is considered risky. To further validate my estimates, I contrast my estimation with other estimates in the literature as well as potential drivers of heterogenous loadings in Figure 4.

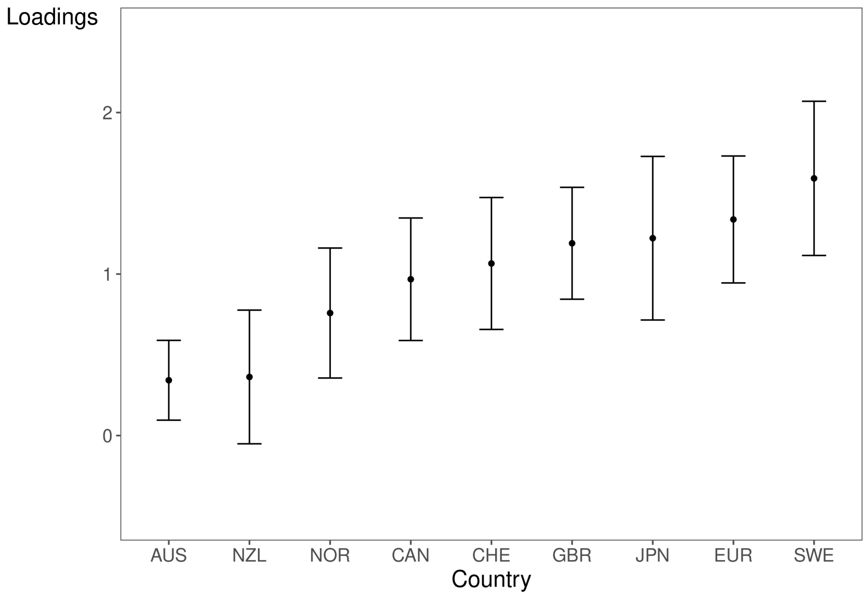
Figure 4, Panel (a) compares my estimates with Colacito et al. (2018a), who use dividend data to estimate heterogenous loadings on a global shock in an endowment economy. My estimates using only GDP data is highly in line with theirs, suggesting that heterogenous loadings on a global shock can be extracted not only from asset prices, but also from fun-

[30]I use Dynare 4.6.4 to achieve this. I use second order approximation for efficiency and because for the purpose of this paper, I only focus on unconditional moments and does not consider time-varying risk premia. I use pruning (Andreasen et al. (2018)) to reduce the exploding behavior induced by high order terms, but the result is very similar if I use standard perturbation. For higher order solutions, Mertens and Judd (2018) provides an efficient method.

[31]See Appendix F for the list of target moments and the moment matching results.

[32]See Appendix F for the table of estimates and standard errors, as well as details on SMM.

Figure 3: Estimated Loadings



This figure plots the estimated loadings on the global shock and two standard deviation bands β_z^i for countries issuing the G10 currencies (without United States). Details on estimation can be found in Appendix F.

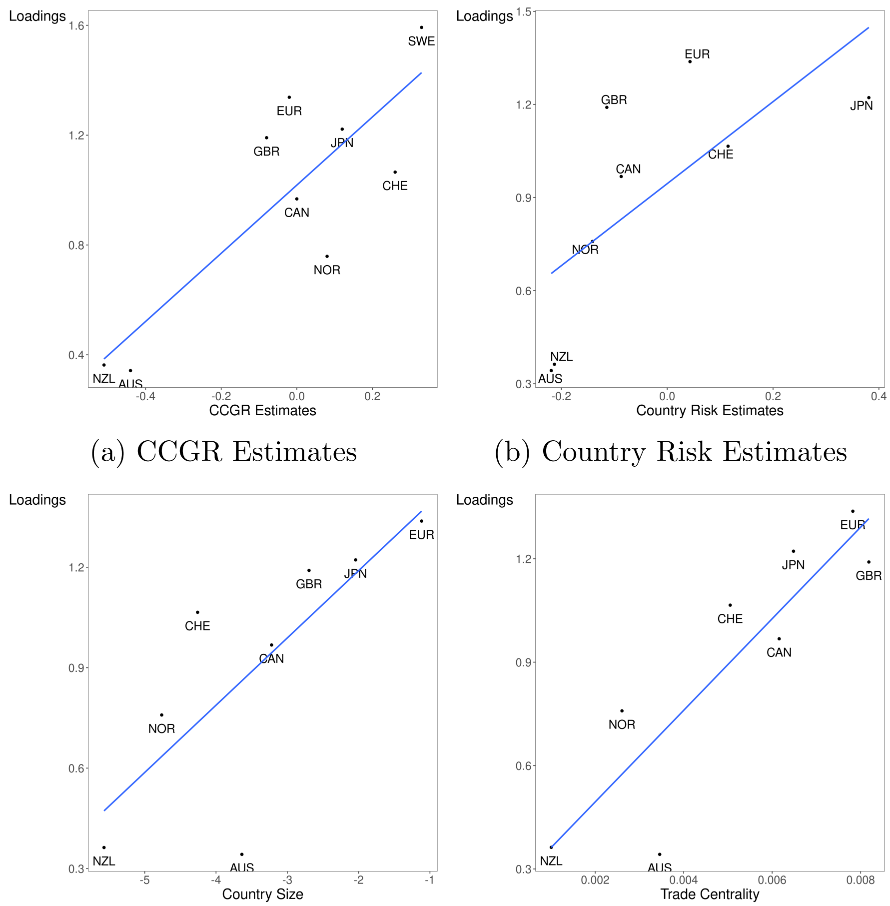
damentals like GDP. Interestingly, both of our estimates show that Sweden, which is not typically considered a country with a safe currency, has a very high loading on the global shock, suggesting Sweden is somehow special. In Panel (b), (c) and (d) I omit Sweden for it being an obvious outlier[33]. Panel (b) compares my estimates with Hassan et al. (2021c)[34], who uses text analysis on conference calls of listed firms to directly estimate which country is perceived as "safe". My estimations using only GDP data is inline with their estimates. Panel (c) and (d) plot my estimates against two potential drivers of these heterogenous loadings proposed in the literature, country size (Hassan (2013)) and trade centrality (Richmond (2019)). Both are highly correlated with my estimated loadings, suggesting that the loadings can potentially be driven by these factors. The line of best fit in all four panels are statistically significant with high R^2s.

My approach in silent on the economic origins of the heterogeneity in loadings on a

[33]See Appendix Figure A1 for plots with Sweden included.

[34]I thank the authors for sending me their estimates.

Figure 4: Estimated Loadings: Validation



(a) CCGR Estimates

(b) Country Risk Estimates

(c) Country Size

(d) Trade Centrality

This figure plots my estimates against: the estimates obtained by Colacito et al. (2018a) in Panel (a) (slope: 1.11 (s.e. 0.37, $R^2 = 0.60$)); the estimates obtained by Hassan et al. (2021c) in Panel (b) (slope: 1.32 (s.e. 0.55, $R^2 = 0.48$)); country size (Hassan (2013)) in Panel (c) (slope: 0.20 (s.e. 0.07, $R^2 = 0.59$)); and trade centrality (Richmond (2019)) in Panel (d) (slope: 132.89 (s.e. 28.67, $R^2 = 0.78$)). Panel (b), (c) and (d) excludes Sweden.

global shock. The fact that estimated loadings are highly correlated with potential drivers like country size and trade centrality suggests that a micro-founded model is possible, which I leave for future research. In particular, the quantitative framework developed in this paper could potentially be used to evaluate relative importance of various drivers proposed in the literature. One caveat is that some potential drivers of these loadings might be endogenous to capital accumulation. I argue that because of the relative homogeneity of the countries in my sample, the scope of the endogeneity problem is limited[35]. Even if there is a path through which capital-output ratios could affect the loadings, the channel developed in this

[35] In Appendix A, I show that the negative relationship between capital-output ratios and currency premia in Figure 1 is robust to including a wide number of controls.

paper is still valuable: it becomes an amplification mechanism. I leave the identification of potential drivers of the heterogenous loadings as well as their interaction with the mechanism developed in the paper for future research.

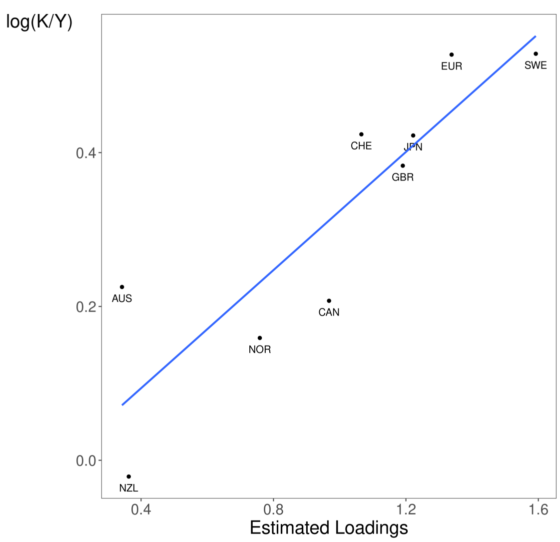
To summarize, the loadings on the global shock that I estimated from only GDP data are consistent with conventional views of currency risk, and are also inline with other estimates using alternative datasets. In addition, they are also highly correlated with potential drivers of these heterogenous loadings proposed in the literature.

4.3. Properties of the Estimated Model

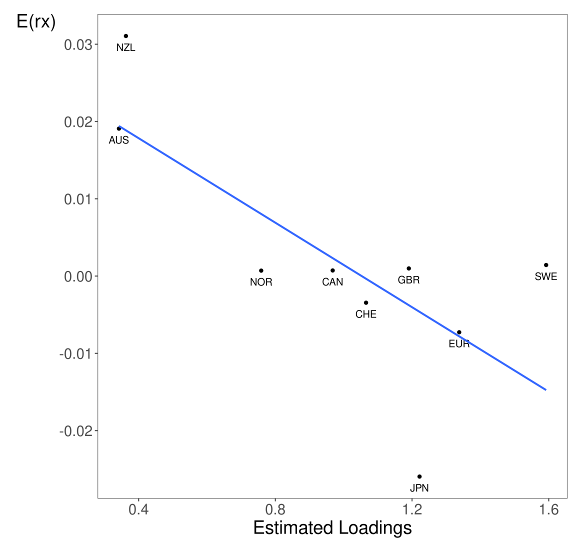
In this subsection I take the estimated loadings as given and simulate the model to evaluate its performance. I make two main findings: first, the loadings on a global shock that I estimated from only GDP data is highly correlated with currency premia and capital-output ratios. Second, when I feed the estimated loadings to the model for simulation, the simulated currency premia and capital-output ratios are highly inline with their data counterparts.

4.3.1. Result 1: Correlation between Loadings and Key Variables

Figure 5: Estimated Loadings



(a) Capital-output Ratio



(b) Currency Premium

This figure plots log capital-output ratios (Panel(a)) and currency premia (Panel(b)) relative to the U.S. against estimated loadings on the global shock. The line of best fit has a slope of 0.38 (s.e. 0.08) for log capital-output ratio and a slope of -0.03 (s.e. 0.01) for currency premium. All moments are annualized.
Data Source: PWT 10.0 and Adrien Verdelhan's webset. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

I first explore the correlation of the estimated loadings with currency premia and capital-output ratios in the data. To that end, I plot capital-output ratios (Panel (a)) and currency premia (Panel (b)) against the estimated loadings in Figure 5. Consistent with the predictions of Proposition 2, countries with higher loadings on the global shock features lower currency premia and higher capital-output ratios. The R^2 is 0.70 for capital-output ratios and 0.49 for currency premia[36]. The estimated loadings are indeed highly correlated with currency premia and capital-output ratios across countries.

These high correlations themselves are interesting even without the model considering the fact that the loadings on the global shock are estimated solely from GDP data. The way that a country covaries with the world is informative on its currency premium (and thus risk-free rate) and capital-output ratio. This is suggesting that different loadings on a global shock, which is long thought as an asset pricing tool to rationalize currency premia, have support in fundamental variables like GDP. Countries that covary more with the world and thus have higher estimated loadings on the global shock features higher capital-output ratios and lower currency premia. In this sense, my paper can be seen as a framework that jointly matches these facts in the data.

4.3.2. Result 2: Data Moments v.s. Model Simulations

I investigate the quantitative performance of the model by plotting moments in the data against model generated moments for capital-output ratios (Panel (a)) and currency premia (Panel (b)) in Figure 6.

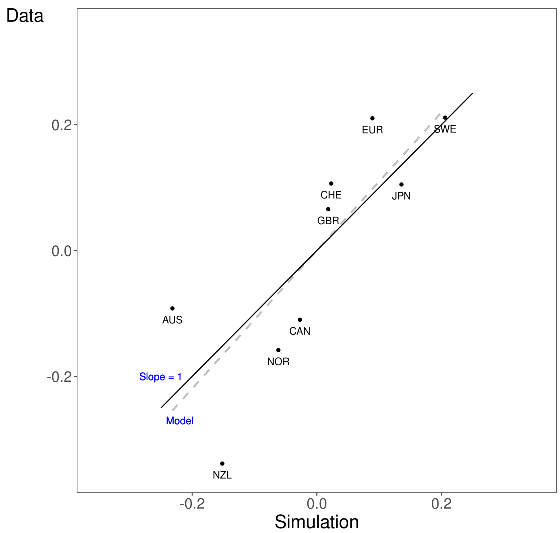
If the model perfectly matches the data, we would expect the model simulated moments and the data moments to be identical across countries and all points should lie on the 45° line. In Figure 6, Panel (a), we can see that for capital-output ratios, the points all very close to the 45° line. In fact, the line of best fit (dashed line) a slope of 1.09 (s.e. 0.30), which is very close to 1, suggesting on average the model does a remarkably good job in matching the cross-country variations in capital-output ratios. High loading countries feature higher capital-output ratios in a way that is quantitatively consistent with the data.

Figure 6, Panel (b) plots currency premia in the data against model simulations. Again the model does a fairly good job at matching the data. All countries lie almost perfectly on the 45° line (solid line) except for Sweden and Australia. Recognizing that as asset prices, currency premia are highly volatile and hard to predict, especially with fundamentals like GDP, the model is doing reasonably well in matching the currency premia in the data.

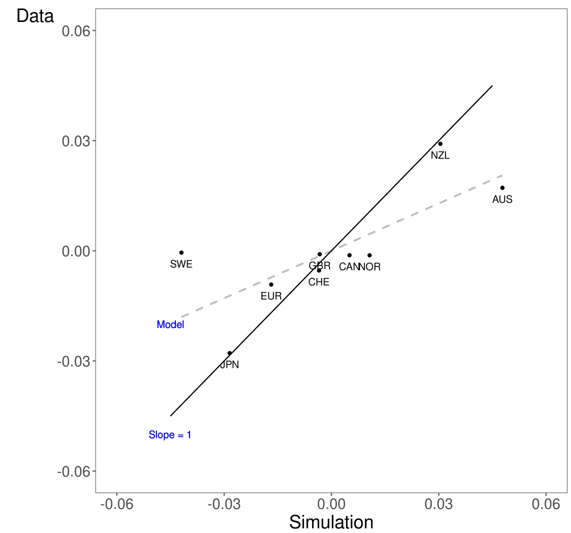
Overall, the model simulations of capital-output ratios and currency premia match the data very well. To further evaluate the quantitative performance of the model, I again use Japan and New Zealand as an example. Table 2 compares the data and the model generated moments for difference in log capital-output ratios between the two countries, as

[36]It is not surprising that R^2 is lower for currency premia because as an asset price, currency premia are much more volatile and harder to estimate and predict compared to capital-output ratios.

Figure 6: Data v.s. Simulations



(a) Capital-output Ratio



(b) Currency Premium

This figure plots log capital-output ratios(Panel(a)) and currency premium (Panel(b)) relative to the cross-country average against their simulated counterparts. The line of best fit has a slope of 1.09 (s.e. 0.30) for log capital-output ratio and a slope of 0.43 (s.e. 0.14) for currency premium. Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking average. All moments are annualized.
Data Source: PWT 10.0 and Adrien Verdelhan's webset. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

well as currency premium, interest rate difference and expected change in exchange rates. The model can generate roughly 66% of the observed difference in log capital output ratios between the two countries. In terms of currency premium, the model (5.89%) can almost perfectly match the data (5.70%). In the data, most of the currency premium comes from interest rate differences, with expected change in exchange rate close to 0. In the model, we have the same pattern. The model generated interest rate difference is 6.06%, compared to 5.08% in the data. New Zealand dollar is appreciating a little bit (0.62%) in the sample, but depreciating a little bit in the model[37].

For the Japan and New Zealand example, the model does remarkably well in matching currency premium, interest rate differences, and expected change in exchange rates, and can also explain a significant portion of the difference in capital-output ratios. To quantify the model's average performance across countries in terms of matching the capital-output ratios, I perform the following variance decomposition. Let $\kappa_D^i = k_D^i - y_D^i$ denote the log

[37] The interest rate differences in the data become larger and expected change in exchange rates become positive when we extend our sample to the 1980s. In general, unconditionally, expected change in exchange rate is close to 0. If anything, high interest rate currency tend to depreciate a little bit. See Hassan and Mano (2018),

Table 2: Japan and New Zealand: An Example

	Diff in $\log(K/Y)$	$\mathbb{E}(rx)$	$r_f^{NZL} - r_f^{JPN}$	$\mathbb{E}(\Delta ex)$
Data	-0.44	5.70%	5.08%	-0.62%
Model	-0.29	5.89%	6.06%	0.17%

Notes: This table compares key moments of Japan and New Zealand between the model simulations and the data. Model moments are obtained by simulating 500 samples of 104 periods and taking average. All moments are annualized.

capital-output ratio in the data for country i and κ_M^i denote the same variable predicted by the model, and write

$$\kappa_D^i = \kappa_M^i + e^i$$

where e^i is the prediction error of the model. If the model perfectly matches the data, $e^i = 0$ for all i. Taking variances across countries on both side[38], we have

$$\underbrace{\mathrm{var}(\kappa_D^i)}_{0.0344} = \underbrace{\mathrm{var}(\kappa_M^i)}_{0.0189} + \underbrace{\mathrm{var}(e^i)}_{0.0119} + \underbrace{2\,\mathrm{cov}(\kappa_M^i, e^i)}_{0.0037}$$

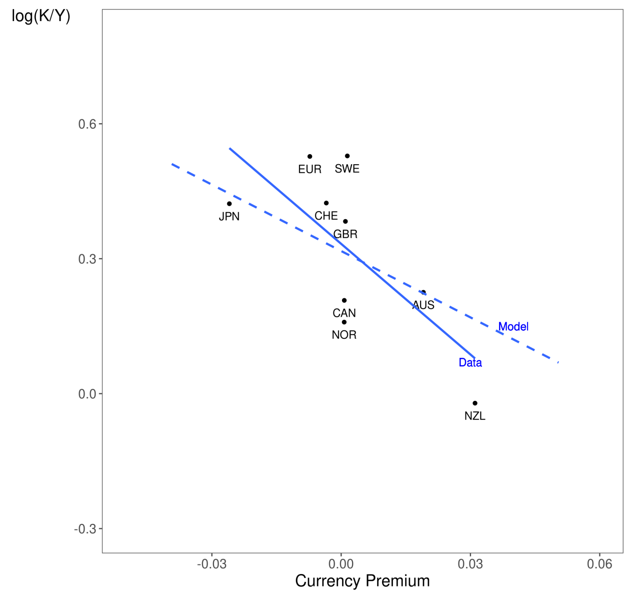
Under this decomposition, $\frac{\mathrm{var}(\kappa_M^i)}{\mathrm{var}(\kappa_D^i)} = 54.76\%$, so the model roughly accounts for 55% of the cross-country variations in capital-output ratios. Note that $2\,\mathrm{cov}(\kappa_M^i, e^i) = 0.0037$ is close to 0, suggesting the error terms are not correlated with κ_M^i, and the model's predictions is on average very close to the data.

Overall, the model does very well in matching the cross-country variations in capital-output ratios, and can explain roughly 55% of the variations in the data. The model also have predictions for the relationship between capital-output ratios and currency premia. Figure 7 compares the regression line of capital-output ratios on currency premia in the data (solid line) with the regression line implied by model simulations (dashed line). The model predicts a slope of -4.91, compared to -8.21 in the data. The model under-predicts the slope, but still explain a significant portion. This result also confirms the findings of Proposition 2, which states that there is a tight link between currency premia and capital-output ratios in the model.

To summarize, the loadings that I estimated from only GDP data are highly correlated with capital-output ratios and currency premia across countries. Taking the estimated loadings as given, the model does remarkably well in predicting cross-country variations in capital-output ratios, explaining around 55% of it. The model also does a fairly good job at

[38] This is similar to calculation of R^2, but with the coefficient restricted to 1.

Figure 7: Regression Line of K/Y on Currency Premium, Data v.s. Model



This figure plots the regression line of capital-output ratios on currency premia for the G10 currencies in the data and the same regression line implied by the model simulations. The points represents the data. The regression line in the data has slope of -8.21 (s.e. 3.12). The regression line implied by the model simulations has a slope of -4.91 (s.e. 0.07). Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking average. All moments are annualized.
Data Source: PWT 10.0 and Adrien Verdelhan's webset. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

matching the currency premia and the correlations between capital-output ratios and currency premia in the data. Model generated currency premia are large, and all of them are accounted for by interest rate differences as in the data.

4.4. Role of Parameters and Robustness

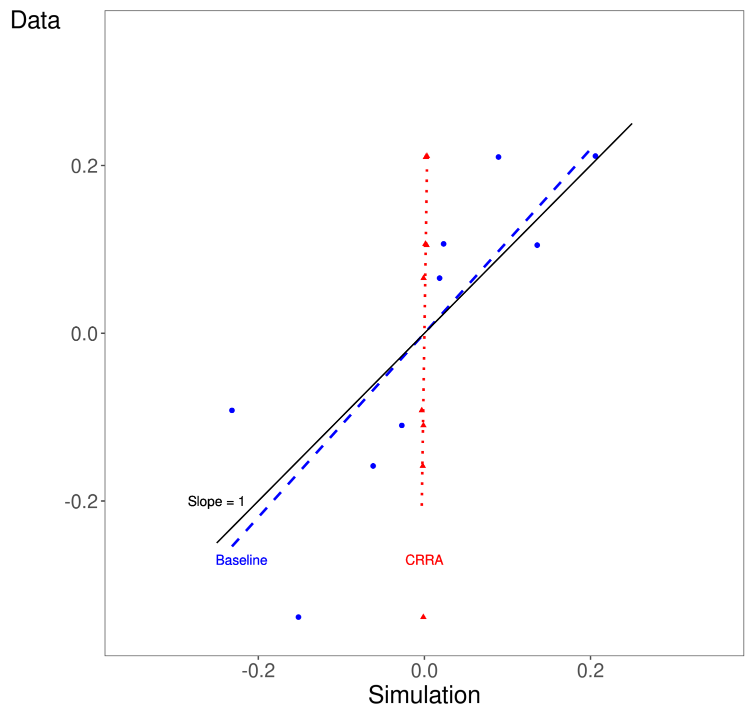
In this section, I check the sensitivity of my findings in Section 4.3.2 to changes of parameters and set-ups. In particular, I take the estimated loadings as given, use the calibration in Table 1 as baseline, and test how changing some of the parameters would affect the quantitative performance of the model. In particular, I test how using (1) CRRA preference; (2) CES aggregator; (3) a different home bias parameter would affect the model's ability to match the data. I focus on capital-output ratios in this section.

4.4.1. The role of habit

To highlight the role of habit in the quantitative success of the model, I compare the simulated results generated from our baseline habit model with standard CRRA preferences.

Figure 8 is generated by adding simulated moments from the CRRA model (red triangles) to Figure 6, Panel (a). The CRRA model does poorly on the quantitative front. The lines of

Figure 8: Habit v.s. CRRA



This figure plots log capital-output ratio relative to the cross-country average against their simulated counterparts. Blue dots represent the baseline model with habit, and red triangles represent the CRRA model. The dashed blue line represents the line of best fit for the baseline model with habit, and features a slope of 1.09 (s.e. 0.30, $R^2 = 0.66$). The dotted red line represents the line of best fit for the CRRA model, and features a slope of 64.75 (s.e. 20.28, $R^2 = 0.59$) for log capital-output ratio. All moments are annualized.
Model simulated moments are obtained by simulating the model for 10000 periods and then taking average.
Data Source: PWT 10.0. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

best fit (red dotted line) is close to being vertical, and is far away from the 45° line, suggesting the model is generating differences in capital-output ratios that are too small compared to the data. In comparison, the baseline model with habit features a line of best fit with a slope very close to 1 (the blue dashed line). This confirms our findings in Proposition 3, which states habit is crucial for the quantitative success of the model.

One interesting fact is that although the model with CRRA preference fails on the quantitative front, its predictions are still qualitatively consistent with the data. Under CRRA (the red dotted line), the R^2 is 0.59, suggesting that the model generated moments are still highly correlated with the data. This is not surprising because the estimated loadings are highly correlated with currency premia and capital-output ratios (Figure 5), which would induce different responses to the global shock as long as agents are risk averse and feature home bias. The level of sensitivity to risk then governs the quantitative importance of this mechanism. With external habit, the model generates large differences in currency premia and capital-output ratios, which is consistent with the data.

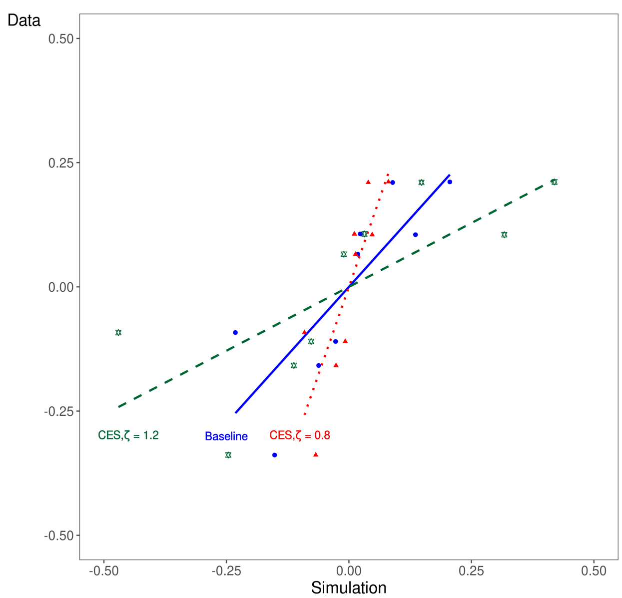
4.4.2. CES Aggregator

In this section, I test how the quantitative predictions of the model reacts to using a CES aggregator for the final good and changing the elasticity of substitution between goods. I start by changing the production function of the final good (5) to the following CES aggregator:

$$F_t^i = \left(\nu (X_{i,t}^i)^{\frac{\zeta-1}{\zeta}} + \sum_{j=1}^{N} \frac{1}{N}(1-\nu) \left(X_{j,t}^i\right)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}}$$

where ζ governs the elasticity of substitution between the country-specific goods. In my baseline specification, I use Cobb-Douglas aggregator so that $\zeta = 1$. To see how the model predictions change with respect to ζ, in Figure 9 I re-produce Figure 6, Panel (a) for $\zeta = 0.8$ and $\zeta = 1.2$ and put them in the same figure as my baseline specification with $\zeta = 1$.

Figure 9: Changing Elasticity of Substitutions



This figure plots log capital-output ratios relative to the U.S. against their simulated counterparts for the baseline model and the model with CES aggregators ($\zeta = 0.8$ and $\zeta = 1.2$). Blue dots represent the baseline model, red triangle represents $\zeta = 0.8$ and green stars represent $\zeta = 1.6$. The line of best fit has a slope of 1.09 (s.e. 0.30, $R^2 = 0.66$) for the baseline model (solid blue line), a slope of 2.83 (s.e. 0.69, $R^2 = 0.70$) for $\zeta = 0.8$ (dotted red line) and a slope of 0.51 (s.e. 0.17, $R^2 = 0.58$) for $\zeta = 1.2$ (dashed green line). Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking average. All moments are annualized.

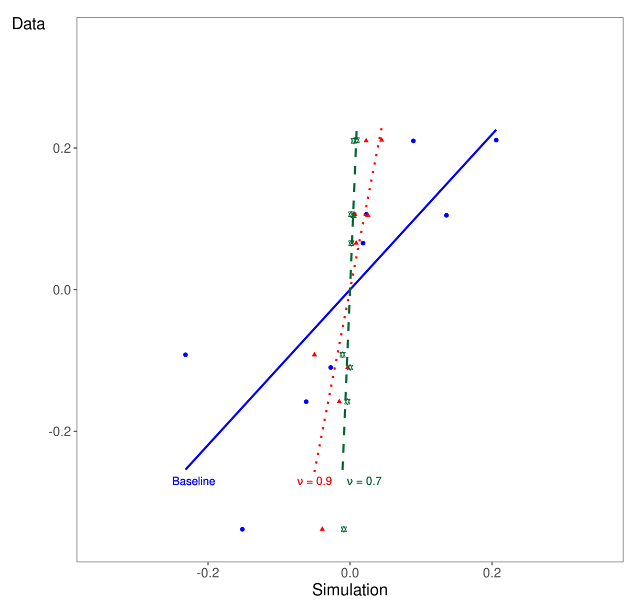
Data Source: PWT 10.0. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

I use the slope of the lines of best fit as a rough measure on how alternative specifications compare to the baseline. A steeper slope indicates the alternative model on average over-predicts the cross-country variations in capital-output ratios across countries relative to the baseline model and vice versa. As shown in Figure 9, as the elasticity gets larger ($\zeta = 1.2$, green dashed line), the model generated cross-country variations are larger, and the opposite is also true ($\zeta = 0.8$, red dotted line).

This result is intuitive. As elasticity of substitution becomes larger, goods become more substitutable and agents optimally choose to consume more of their home good because it has a higher weight. Higher ζ thus enhances the effect of home bias. When a negative global shock hits and ζ is high, the increase in the price of high-loading country's good passes through more to the price of its final good, leading to more pronounced appreciation. As a result, more capital is installed in the high loading country.

Again, changing the elasticity of substitution between goods only affects the quantitative performance of the model, but not the qualitative performance. The R^2 for all three specifications are all quite high (with a minimum of 0.58).

Figure 10: Changing Degree of Home Bias



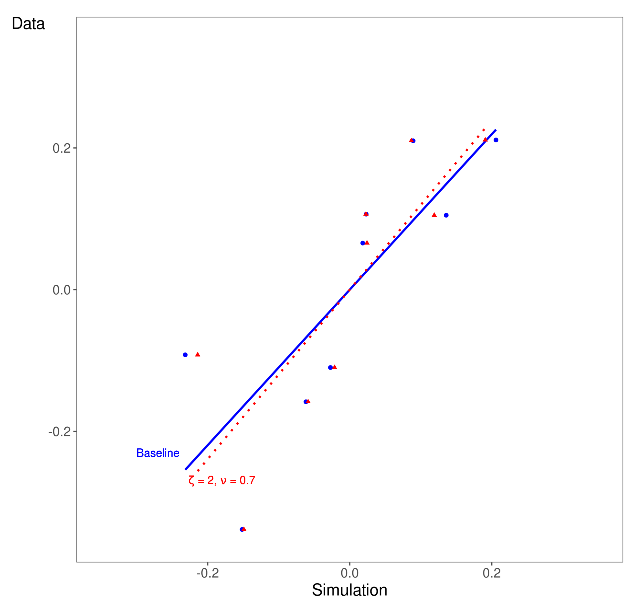
This figure plots log capital-output ratios relative to the U.S. against their simulated counterparts for the baseline model and the model with $\nu = 0.9$ and $\nu = 0.7$). Blue dots represent the baseline model, red triangle represents $\nu = 0.9$ and green stars represent $\nu = 0.7$. The line of best fit has a slope of 1.09 (s.e. 0.30, $R^2 = 0.66$) for the baseline model (solid blue line), a slope of 5.14 (s.e. 1.21, $R^2 = 0.72$) for $\nu = 0.9$ (dotted red line) and a slope of 24.21 (s.e. 5.50, $R^2 = 0.73$) for $\nu = 0.7$ (dashed green line). Model simulated moments are obtained by simulating the model for 500 samples of 104 periods and then taking average. All moments are annualized.
Data Source: PWT 10.0. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

4.4.3. *Changing home bias*

I next turn to the home bias parameter, ν. In international asset pricing models with Cobb-Douglas aggregators, a high degree of home bias is typically assumed[39]. In my baseline calibration, I follow Colacito et al. (2018b) and set $\nu = 0.98$. I illustrate how changing ν affects the results in Figure 10.

When using Cobb-Douglas aggregator, it is important to use large home bias. As we lower home bias, the model generated cross-country variations becomes much smaller (dotted red line and dashed green line) compared to the baseline (solid blue line). Because Cobb-Douglas aggregator restricts elasticity of substitution to 1, a high level of home bias is needed so that price changes in country specific goods are sufficiently passed through to prices of final goods, which represents real exchange rates. Interestingly, Figure 9 and 10 suggests that we can set ν to lower, more reasonable levels if we use CES aggregators and set $\zeta > 1$. Figure 11 confirms this conjure by showing that a model with $\zeta = 2$ and $\alpha = 0.7$ generates results very similar to the baseline. Again, all specifications feature high R^2 (with a minimum of 0.66) regardless of their quantitative performance.

Figure 11: High Elasticity with Low Home Bias



This figure plots log capital-output ratios relative to the U.S. against their simulated counterparts for the baseline model and the model with $\zeta = 2$, $\nu = 0.7$). Blue dots represent the baseline model, red triangle represents $\nu = 0.7$, $\zeta = 2$. The line of best fit has a slope of 1.09 (s.e. 0.30, $R^2 = 0.66$) for the baseline model (solid blue line) and a slope of 1.19 (s.e. 0.31, $R^2 = 0.66$) for $\nu = 0.7$, $\zeta = 2$. Model simulated moments are obtained by simulating the model for 200 samples of 104 periods and then taking average. All moments are annualized.
Data Source: PWT 10.0. Data Range: 1994-2019. Details on the data construction can be found in section 4.1.

[39] For example, Stathopoulos (2017) uses $\nu = 0.952$, Colacito and Croce (2013) uses $\nu = 0.97$.

To summarize, standard CRRA preferences only generate tiny cross-country variations in capital-output ratios. The model predicted variations in capital-output ratios are increasing in elasticity of substitutions between goods, ζ, and are decreasing in degree of home bias ν. All specifications in this section feature high R^2s, confirming that the estimated loadings and thus currency risk is tightly linked to capital-output ratios. While the exact variation induced by the model depends on parameterization, one can achieve similar results to the baseline model with a high ζ and a low ν. In other words, the qualitative prediction of the model is very robust to changes in parameter values, and there is a set of parameters that generates similar quantitative results as the baseline model.

5. Conclusion

Heterogenous loadings on a global shock has been a standard modeling device in international asset pricing literature to capture the idea that some currencies are safer than others. It is widely used to understand violations of UIP, exchange rate dynamics and currency premia. Intuitively, currency of high loading countries appreciate in global bad times, making them a good hedge against global risk and investors require a lower return for assets in such countries. In this paper, I have argued that heterogenous loadings and currency risk should also have quantitatively important real implications: high loading countries should feature lower expected returns to capital and accumulate more capital. I confirm that the GDP correlations between countries issuing the G10 currencies are consistent with heterogenous loadings on a global shock, and the loadings estimated using only GDP data is highly correlated with currency premia and capital-output ratios. When feeding the estimated loadings into my model with external habit, the model can explain around 55% of the cross-country variations in capital-output ratios among countries issuing the G10 currencies. External habit is essential for the quantitative success of the model in terms of matching currency premia and capital-output ratios. Compared to existing international asset pricing frameworks using Epstein and Zin (1989) preferences , it has the advantage of generating large currency premia with 0 expected change in exchange rates thus better match the data.

My paper is silent on the economic sources of the heterogeneity in loadings on the global shock. One interesting extension is thus to apply external habit to existing frameworks that are trying to micro-found the loadings and evaluate the relative importance of potential channels. I have also consciously abstracted away from other sources of heterogeneities that could have important implications for capital output ratios, for example capital share, but the modeling tools developed in this paper can easily be extended to a more complicated set-up and potentially be used to evaluate the relative importance of different drivers of capital-output ratios. In addition, it is an empirical fact that risk-free rates differ persistently across countries, which should have implications beyond capital-output ratios. Because my model can induce long-lasting differences in risk-free rates across countries as in the data, it can potentially be used to study cross-country patterns of other important economic variables that are tightly linked to risk-free rates.

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A. Controlling for Country Characteristics

Table A1: Regression of Capital-output ratios on Currency Premia with Controls, G10

	Dependent variable:							
	Capital-output Ratios Relative to the US							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\mathbb{E}(rx)$	-7.945^{*}	-8.742^{**}	-7.010	-7.722^{*}	-7.549^{*}	-10.662	-7.223	-8.193
	(3.405)	(3.319)	(4.340)	(3.609)	(3.300)	(5.617)	(4.071)	(4.399)
FDI	0.143							
	(0.397)							
FOI		-0.140						
		(0.195)						
CC			-0.077					
			(0.182)					
GE				-0.085				
				(0.245)				
PS					-0.150			
					(0.187)			
RQ						0.194		
						(0.359)		
RL							-0.107	
							(0.257)	
VA								-0.001
								(0.350)
Constant	0.226	0.635	0.476	0.477	0.492^{*}	0.031	0.515	0.334
	(0.300)	(0.426)	(0.339)	(0.420)	(0.204)	(0.561)	(0.441)	(0.491)
Observations	9	9	9	9	9	9	9	9
R^2	0.508	0.537	0.512	0.507	0.546	0.521	0.511	0.497
Adjusted R^2	0.344	0.382	0.349	0.343	0.395	0.361	0.348	0.330
Residual Std. Error (df = 6)	0.150	0.146	0.150	0.150	0.144	0.148	0.150	0.152
F Statistic (df = 2; 6)	3.098	3.476^{*}	3.148	3.088	3.610^{*}	3.259	3.140	2.968

Note: *p<0.1; **p<0.05; ***p<0.01

This table summarizes regressions of log capital-output ratio relative to the U.S. on currency premia and different controls for countries issuing the G10 currencies:

$$ky^i = \alpha\,\mathbb{E}(rx^i) + \beta \text{control}^i + \epsilon^i$$

FDI stands for Financial Development Index, FOI stands for Chinn-Ito financial openness index (Chinn and Ito (2006)). CC stands for control of corruption, GE stands for government effectiveness, PS stands for political stability and absence of violence/terrorism, RQ stands for regulatory quality, RL stands for rule of law, and VA stands for voice and accountability.
Data Source: Capital-output ratios are from PWT 10.0. Currency premia are from Verdelhan's website. FDI is from the IMF Financial Development Indicator dataset. FOI is from the Chinn-Ito Index website. All the other variables are from the World Governance Indicators by the World Bank. Data range from 1994 to 2019.

As is shown in Table A1 , the strong negative relationship between capital-output ratios and currency premia is robust to including a series of controls representing financial development, governance and financial openness within the G10 countries. Because of limitation on the sample size, I can only control for each variable separately.

Table A2: Regression of Capital-output ratios on Currency Premia with Controls, All Countries

	Dependent variable:	
	Capital-output Ratios Relative to the US	
	(1)	(2)
$\mathbb{E}(rx)$	-3.276	-9.283^{**}
	(3.768)	(3.435)
FDI	0.139	0.050
	(0.393)	(0.436)
FOI	0.184^{***}	
	(0.066)	
CC	-0.816^{***}	-0.696^{***}
	(0.220)	(0.240)
GE	0.734^{**}	0.452
	(0.276)	(0.286)
PS	0.051	0.139
	(0.132)	(0.142)
RQ	0.059	0.346
	(0.257)	(0.262)
RL	0.033	0.012
	(0.268)	(0.299)
VA	0.071	0.063
	(0.071)	(0.079)
Constant	-0.177	0.043
	(0.191)	(0.194)
Observations	37	37
R^2	0.601	0.487
Adjusted R^2	0.468	0.341
Residual Std. Error	0.219 (df = 27)	0.243 (df = 28)
F Statistic	4.520^{***} (df = 9; 27)	3.323^{***} (df = 8; 28)
Note:		*p<0.1; **p<0.05; ***p<0.01

This table summarizes regressions of log capital-output ratio relative to the U.S. on currency premia and different controls for a sample of 37 countries (See Appendix B for the list of countries).

$$ky^i = \alpha\,\mathbb{E}(rx^i) + \beta \text{control}^i + \epsilon^i$$

FDI stands for Financial Development Index, FOI stands for Chinn-Ito financial openness index (Chinn and Ito (2006)). CC stands for control of corruption, GE stands for government effectiveness, PS stands for political stability and absence of violence/terrorism, RQ stands for regulatory quality, RL stands for rule of law, and VA stands for voice and accountability. Data Source: Capital-output ratios are from PWT 10.0. Currency premia are from Verdelhan's website. FDI is from the IMF Financial Development Indicator dataset. FOI is from the Chinn-Ito Index website. All the other variables are from the World Governance Indicators by the World Bank. Data range from 1994 to 2019.

As is shown in Table A2 , for a broader range of 37 countries, the strong negative relationship between capital-output ratios and currency premia is robust to including a series of controls representing financial development and governance (Panel (2)) but is weakened when financial openness is included (Panel (1)). This is to be expected because in the broader sample with developing countries, financial openness is crucial how each economy covaries with the world, and thus the loadings on the global shock and currency premia. But within the countries issuing the G10 currencies, all countries have very high openness and thus financial openness is less relevant.

B. The List of Countries in the Broader Sample

The broader sample in Figure 1 include: United Arab Emirats, Australia, Austria, Belgium, Canada, Switzerland, Czech Republic, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Hong Kong SAR China, Hungary, Indonesia, India, Ireland, Italy, Japan, South Korea, Kuwait, Mexico, Malaysia, Netherlands, Norway, New Zealand, Philippines, Poland, Portugal, Saudi Arabia, Singapore, Sweden, Thailand, United States, South Africa.

Currency premia for countries in the Euro Zone are replaced by the Euro currency premia after Euro is introduced.

C. Solving the Simplified Model

Under the simplified set-up, the return to capital (13) simplifies to

$$R = \frac{\Lambda_X}{\Lambda_C} \alpha \frac{Y}{K}$$

Substituting back to the Euler equation (11), moving capital to the left hand side and taking logs, we have

$$k = \log(\alpha\eta) + \log\left[\mathbb{E}\left(e^{\lambda_X + y - \lambda_{C,0}}\right)\right]$$

Assuming log-normality, we can already see that the risk plays a role in choosing the optimal level of capital:

$$k = \log(\alpha\eta) + \mathbb{E}\left(\lambda_X + y - \lambda_{C,0}\right) + \underbrace{\frac{1}{2}\operatorname{var}\left(\lambda_X + y - \lambda_{C,0}\right)}_{\text{risk adjustment}}$$

The last term is a risk adjustment term and represents the effect of risk on agent's incentive to accumulate capital. Higher variance induces higher precautionary saving motive, and thus leads to more capital.

I solve the model using log-linearization.

Under the simplified model, the system of equations in period 1 can be log-linearized as:

$$
\begin{aligned}
\hat{c}^i &= \frac{1}{2}(1+\nu)\hat{x}_i^i + \frac{1}{2}(1-\nu)\hat{x}_j^i \\
\hat{\lambda}_C^i &= -\gamma\hat{s}^i - \gamma\hat{c}^i \\
\hat{\lambda}_X^i &= \hat{\lambda}_C^i + \hat{c}^i - \hat{x}_i^i \\
\hat{\lambda}_X^i &= \hat{\lambda}_C^j + \hat{c}^j - \hat{x}_i^j \\
\hat{y}^i &= \hat{z}^i + \alpha\hat{k}_0^i \\
Y^{i\star}\hat{y}^i &= X_i^{i\star}\hat{x}_i^i + X_i^{j\star}\hat{x}_i^j \\
\hat{s}^i &= \rho_s\hat{s}_0^i + \lambda_s\Delta c^i
\end{aligned}
$$

The Mathematica file that derives this system is available upon request.

We want to solve for

$$
\hat{k}_0^i = \log(\nu\eta) + \log\left[\mathbb{E}_0\left(e^{\hat{\lambda}_X^i + \hat{y}^i - \hat{\lambda}_{C0}^i}\right)\right] \tag{A1}
$$

Now, note that we have $\frac{X_i^{i\star}}{Y^{i\star}} = \frac{1}{2}(1+\alpha)$, $\frac{X_i^{j\star}}{Y^{i\star}} = \frac{1}{2}(1-\alpha)$, we solve for $\hat{\lambda}_X^i + \hat{y}^i$ as a function of the exogenous shocks.

We start from the resource constraints,

$$
\hat{y}^i = \frac{1}{2}(1+\nu)\hat{x}_i^i + \frac{1}{2}(1-\nu)\hat{x}_i^j
$$

which gives us

$$
\hat{x}_i^j = \frac{1}{\frac{1}{2}(1-\nu)}\hat{y}^i - \frac{1+\nu}{1-\nu}\hat{x}_i^i \tag{A2}
$$

Now we have

$$
\hat{c}^i = \frac{1}{2}(1+\nu)\hat{x}_i^i + \hat{y}^j - \frac{1}{2}(1+\nu)\hat{x}_j^j \tag{A3}
$$

Then marginal utility of consumption is given by

$$
\begin{aligned}
\hat{\lambda}_C^i &= -\gamma \hat{s}^i - \gamma \hat{c}^i \\
&= -\gamma \lambda_s (\hat{c}^i - \hat{c}_0^i) - \gamma \hat{c}^i \\
&= -\gamma (1+\lambda_s) \hat{c}^i - \gamma \lambda_s \hat{c}_0^i \\
&= -\gamma (1+\lambda_s) \left(\frac{1}{2}(1+\nu)\hat{x}_i^i + \hat{y}^j - \frac{1}{2}(1+\nu)\hat{x}_j^j \right) + \gamma \lambda_s \hat{c}_0^i
\end{aligned} \tag{A4}
$$

Now note that

$$
\begin{aligned}
\hat{\lambda}_C^i + \hat{c}^i - \hat{x}_i^i &= \hat{\lambda}_C^j + \hat{c}^j - \hat{x}_i^j \\
\hat{\lambda}_C^i + \hat{c}^i - \hat{x}_j^i &= \hat{\lambda}_C^j + \hat{c}^j - \hat{x}_j^j
\end{aligned}
$$

which implies

$$
-\hat{x}_i^i + \hat{x}_j^i = -\hat{x}_i^j + \hat{x}_j^j
$$

And we have

$$
\hat{x}_i^i + \hat{x}_j^j = \hat{y}^i + \hat{y}^j \tag{A5}
$$

To get another equation, we examine

$$
\lambda_C^i + \hat{c}^i - \hat{x}_i^i = \lambda_C^j + \hat{c}^j - \hat{x}_i^j
$$

Plugging in (A2), (A3) and (A4), we have

$$
\begin{aligned}
&- \gamma(1+\lambda_s) \left(\frac{1}{2}(1+\nu)\hat{x}_i^i + \hat{y}^j - \frac{1}{2}(1+\nu)\hat{x}_j^j \right) + \gamma \lambda_s \hat{c}_0^i - \frac{1}{2}(1-\nu)\hat{x}_i^i + \hat{y}^j - \frac{1}{2}(1+\nu)\hat{x}_j^j \\
&= -\gamma(1+\lambda_s) \left(\frac{1}{2}(1+\nu)\hat{x}_j^j + \hat{y}^i - \frac{1}{2}(1+\nu)\hat{x}_i^i \right) + \gamma \lambda_s \hat{c}_0^j + \frac{1}{2}(1+\nu)x_j^j - \frac{1+\nu}{1-\nu}\hat{y}^i + \frac{1}{2}\frac{(1+\nu)^2}{1-\nu}\hat{x}_i^i
\end{aligned}
$$

Combined with (A5), we can then easily solve for $\hat{x}_i^i$

$$
\hat{x}_i^i = \frac{1}{2} \frac{(\gamma(1+\lambda_s)-1)(1-\nu)\nu}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}(\hat{y}^i + \hat{y}^j) + \frac{\gamma(1+\lambda_s)(1-\nu)+\nu}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i
$$

Substituting back to (A4), we can easily obtain

$$\begin{aligned}
\hat{\lambda}_C^i &= -\frac{1}{2}\gamma(1+\lambda_s)\frac{(1+\nu)\gamma(1+\lambda_s)(1-\nu)+\nu(1+\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i \\
&\quad -\frac{1}{2}\gamma(1+\lambda_s)\frac{(1-\nu)\gamma(1+\lambda_s)(1+\nu)-\nu(1-\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^j \\
&= -\frac{1}{2}\gamma(1+\lambda_s)\frac{(1-\nu)\gamma(1+\lambda_s)(1+\nu)-\nu(1-\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}(\hat{y}^i+\hat{y}^j) \\
&\quad -\gamma(1+\lambda_s)\frac{\nu}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i
\end{aligned} \tag{A6}$$

and we have

$$\begin{aligned}
\hat{\lambda}_X^i &= \hat{\lambda}_C^i + \hat{c}^i - \hat{x}_i^i \\
&= \frac{1}{2}(1-\gamma(1+\lambda_s))\frac{(1-\nu)\gamma(1+\lambda_s)(1+\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}(\hat{y}^i+\hat{y}^j) - \frac{\gamma(1+\lambda_s)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i
\end{aligned} \tag{A7}$$

Then we have

$$\hat{\lambda}_X^i + \hat{y}^i = \frac{1}{2}(1-\gamma(1+\lambda_s))\frac{(1-\nu)\gamma(1+\lambda_s)(1+\nu)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}(\hat{y}^i+\hat{y}^j) + \frac{\nu^2(1-\gamma(1+\lambda_s))}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\hat{y}^i$$

Substituting back to (A1), we can then solve for capital stock as

$$\hat{k}_0^i = \frac{1}{2}\nu\left(1-\frac{1}{(1+\lambda_s)\gamma}\right)^2 \mathrm{var}(\hat{\lambda}_C^i) + const \tag{A8}$$

where *const* is a constant that is commom across countries. The Mathematica file that solves the full expression for it is available upon request.

Naturally, we would expect expected return to capital to be closely related to consumption risk as well. Indeed,

$$\mathbb{E}(\hat{r}^i) = -\frac{1}{2}\nu\left(1-\frac{1}{(1+\lambda_s)\gamma}\right)^2 B\,\mathrm{var}(\hat{\lambda}_C^i) + const \tag{A9}$$

where $0 < B = \frac{\gamma(1+\lambda_s)(1+\nu(1-\alpha))(1-\nu)+\nu^2(1-\alpha)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}$. The Mathematica file that solves the full expression for it is available upon request.

As shown in the seminal paper by Backus et al. (2001), under complete market and log-normality of the SDF, currency premium can be written as differences between variance of the log SDF:

$$\mathbb{E}(rx) = r_f^\star - \mathbb{E}(\Delta ex) - r_f$$
$$= -\frac{1}{2}(\text{var}(m^\star) - \text{var}(m)) \tag{A10}$$

Since variance of log SDF shows up in (A8) and (A9), it is obvious that there is a tight link between capital accumulation, expected return to capital, and currency premium.

C.1. Proof of Proposition 1

Proof. Recall that the two countries are at the deterministic steady state at period one so $\hat{\lambda}^i_{C,0} = \hat{\lambda}^j_{C,0}$. Change in exchange rate is given by

$$\begin{aligned}
\Delta ex &= \hat{m}^i - \hat{m}^j \\
&= \hat{\lambda}^i_C - \hat{\lambda}^j_C \\
&= \frac{\nu\gamma(1+\lambda_s)}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}[(\beta_z^\star - \beta_z)\sigma_g\varepsilon_g + \sigma(\varepsilon^\star - \varepsilon)]
\end{aligned}$$

The last equality is obvious from (A6).

□

C.2. Proof of Proposition 2

Proof. Taking cross-country differences using A8 and substitute in A6, we have

$$\begin{aligned}
\hat{k}^j_0 - \hat{k}^i_0 &= \log\left[\mathbb{E}_0\left(e^{\hat{\lambda}^j_X + \hat{y}^j - \hat{\lambda}^j_{C0}}\right)\right] - \log\left[\mathbb{E}_0\left(e^{\hat{\lambda}^i_X + \hat{y}^i - \hat{\lambda}^i_{C0}}\right)\right] \\
&= \frac{1}{2}\frac{\nu^2(1-\gamma(1+\lambda_s))^2}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\left[(\beta_z^j)^2 - (\beta_z^i)^2\right]\sigma_g^2
\end{aligned}$$

Recall that risk-free rate is given by

$$R^f_{i,0} = \frac{1}{\mathbb{E}_0\left(\beta\frac{\Lambda^i_{C,1}}{\Lambda^i_{C,0}}\right)}$$

And the expected return to capital is given by

$$\mathbb{E}(R^I_{i,1}) = \mathbb{E}\left(\frac{\Lambda^i_{X,1}}{\Lambda^i_{C,1}}\alpha\frac{Y^i_1}{K^i_0}\right)$$

Write them in log-deviation terms, we have

$$\hat{r}^f_{i,0} = -\log(\eta) - \log\left(\mathbb{E}_0(e^{\hat{\lambda}^i_C - \hat{\lambda}^i_{C0}})\right)$$
$$\hat{r}^I_{i,1} = \log(\nu) + \hat{\lambda}^i_X - \hat{\lambda}^i_C + \hat{y}^i - \hat{k}^i_0$$

Substitute in our solution (A6), (A7) and (A8),, we have

$$\hat{r}^f_{j,0} - \hat{r}^f_{i,0} = -\frac{1}{2}\frac{\nu\gamma^2(1+\lambda_s)^2}{\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2}\left[(\beta^j_z)^2 - (\beta^i_z)^2\right]\sigma^2_g$$
$$\hat{r}^I_{j,1} - \hat{r}^I_{i,1} = -\frac{1}{2}\frac{\nu^2(1-\gamma(1+\lambda_s))^2[\gamma(1+\lambda_s)(1+\nu(1-\alpha))(1-\nu)+\nu^2(1-\alpha)]}{(\gamma(1+\lambda_s)(1+\nu)(1-\nu)+\nu^2)^2}\left[(\beta^j_z)^2 - (\beta^i_z)^2\right]\sigma^2_g$$

□

D. Risk-free Rate Volatility

Table A3: Volatility of Risk-free Rates, Model v.s. Data

Country	Data(%)	Model(%)
AUS	2.04	1.19
CAN	2.05	1.27
CHE	0.59	0.93
EUR	2.76	0.89
GBR	1.28	0.89
JPN	3.46	1.18
NOR	2.23	1.00
NZL	1.88	1.28
SWE	2.01	0.78

This table summarizes volatility of real interst rates in the data and in model simulations. Model moments are obtained by simulating the model for 200 samples of 104 periods.
Data Source: Real interest rates are from the World Development Indicators of the World Bank.

This table confirms the findings in Chen (2017) that risk-free rates are stable in the model when the sensitivity is set to a constant in (1). Note that the real interest rate volatilities in this table are rough estimates and are presented here purely to give a broad idea of the risk-free rate volatilities in the data. The main takeaway is that the model generated risk-free rates are stable.

E. Other Quantitative Approaches

E.1. Currency Premium Under Verdelhan (2010)

Verdelhan (2010) extends the standard habit model of Campbell and Cochrane (1999) to an international setup. Under his specification, consumption growth is exogenously given and $\lambda(\cdot)$ is a function. In particular,

$$(1+\lambda(s_0))^2 = \frac{1}{\operatorname{var}(\Delta c)}\frac{1-\rho_s}{\gamma}(1-2(s_0-\bar{s}))$$

Note that consumption growth, which is constant under the setup in Verdelhan (2010), is built into the sensitivity function $\lambda(\cdot)$. Verdelhan (2010) also assumes symmetric countries so $\operatorname{var}(\Delta c) = \operatorname{var}(\Delta c^\star)$, but even if we allow the two countries to have different variance of consumption growth so that $\operatorname{var}(\Delta c) \neq \operatorname{var}(\Delta c^\star)$, we still end up with

$$\begin{aligned}
\mathbb{E}(rx) &= \mathbb{E}\left(-\frac{1}{2}(\operatorname{var}(m^\star) - \operatorname{var}(m))\right) \\
&= -\frac{1}{2}\,\mathbb{E}[\gamma^2(1+\lambda^\star(s^\star))^2\operatorname{var}(\Delta c^\star) - \gamma^2(1+\lambda(s))^2\operatorname{var}(\Delta c)] \\
&= \frac{1-\rho_s}{\gamma}\,\mathbb{E}(s_0 - \bar{s} - (s_0^\star - \bar{s}^\star)) \\
&= 0
\end{aligned}$$

So setting the sensitivity function to a constant and to be the same across countries, while being a simple tweak, is essential for external habit to generate large currency premia.

E.2. Expected Change in Exchange Rates under Epstein and Zin (1989) Preference

Under Epstein and Zin (1989) preference, there is a hard-wired relationship between first and second moment of the log SDF.

$$\begin{aligned}
\mathbb{E}(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\operatorname{var}_t(u_{t+1})) \\
\frac{1}{2}\,\mathbb{E}(\operatorname{var}_t(m_{t+1})) &= \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2\mathbb{E}(\operatorname{var}_t(u_{t+1}))
\end{aligned}$$

The first moment of the log SDF is tightly linked to the second moments, and is not independent of heterogenous loadings. So if there is large heterogenous-loading induced unconditional currency premia (large differences in $\mathbb{E}(\operatorname{var}_t(m_{t+1}))$), there is large unconditional movements in change in exchange rates $\mathbb{E}(\Delta ex) = \mathbb{E}(m - m^\star)$. In fact, $\mathbb{E}(\Delta ex_{t+1}) =$

$-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}\mathbb{E}(rx_{t+1})$, under standard calibrations, expected change in exchange rates account for a large portion of the currency premia.

F. Details on Estimation

I estimate the model using simulated method of moments following Jalali et al. (2015). The model is exactly identified, as shown in Table A4.

Table A4: Moment Matching

Country	s.d. of GDP (%)		correlation	
	Data	Model	Data	Model
AUS	0.58	0.58	0.43	0.43
CAN	1.06	1.06	0.78	0.78
CHE	1.12	1.12	0.78	0.78
EUR	1.12	1.12	0.87	0.87
GBR	1.05	1.05	0.88	0.88
JPN	1.41	1.41	0.74	0.74
NOR	1.11	1.11	0.60	0.60
NZL	0.99	0.99	0.42	0.42
SWE	1.48	1.48	0.87	0.87

This table shows target moments used in Section 4. "s.d. of GDP" stands for standard deviation of HP-filtered GDP, and "correlation" stands for the correlation between each country's HP-filtered GDP with the average across countries in the sample. All moments are quarterly.
Data source: OECD National Account Statistics.

To calculate the standard errors of the estimates, I use parametric boostrapping. In particular, I take the estimated parameters as given, and simulate the model for 2000 times, each using a different set of random shocks. I then estimate the moment variance-covariance matrix as

$$\hat{S} = \frac{1}{L1}\sum_{l1=1}^{L1}\left[\tilde{M}_S^{l1}(\hat{\theta}) - \frac{1}{L2}\sum_{l2=1}^{L2}\tilde{M}_S^{l2}(\hat{\theta})\right]\cdot\left[\tilde{M}_S^{l1}(\hat{\theta}) - \frac{1}{L2}\sum_{l2=1}^{L2}\tilde{M}_S^{l2}(\hat{\theta})\right]'$$

where $L1 = L2 = 1000$, $\tilde{M}_S^{l1}(\hat{\theta})$ denotes the moments obtained in sample $l1$ taken $\hat{\theta}$ as given. With this moment variance-covariance matrix, I then calculate the standard errors by taking the diagonal elements of

$$Q = \left(1 + \frac{1}{K}\right) \left[\left(\frac{\partial m}{\partial \theta}\right)' \cdot \hat{S}^{-1} \cdot \frac{\partial m}{\partial \theta} \right]^{-1}$$

where K is the number of samples that I used in estimating $\hat{\theta}$, and $\frac{\partial m}{\partial \theta}$ is calculated numerically using using forward differentiation.

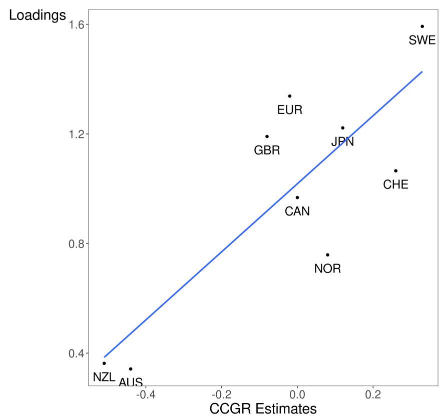
And the estimated parameters are given by (standard errors in brackets)

Table A5: Estimated Parameters

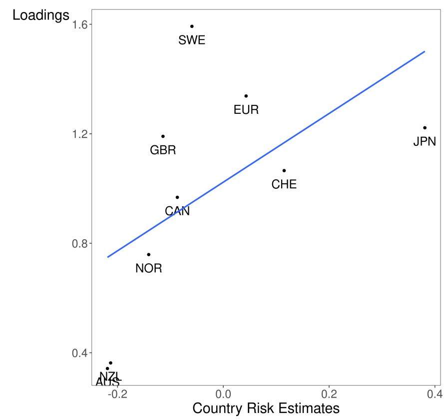
Country	β_z^i	$\sigma_z^i(\%)$
AUS	0.34	0.44
	(0.12)	(0.05)
CAN	0.97	0.57
	(0.19)	(0.08)
CHE	1.07	0.61
	(0.20)	(0.08)
EUR	1.34	0.46
	(0.20)	(0.07)
GBR	1.19	0.39
	(0.17)	(0.06)
JPN	1.22	0.83
	(0.25)	(0.10)
NOR	0.76	0.70
	(0.20)	(0.09)
NZL	0.36	0.77
	(0.21)	(0.09)
SWE	1.59	0.58
Global	1	0.64

G. Robustness Figures

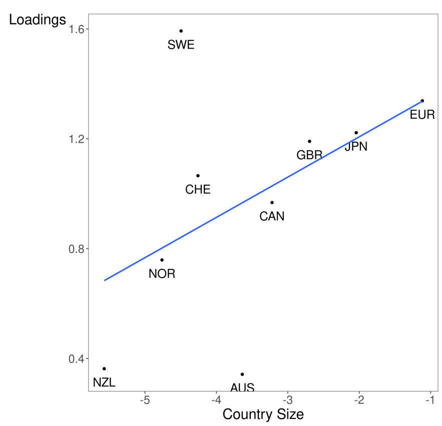
Figure A1: Estimated Loadings: Validation, with Sweden



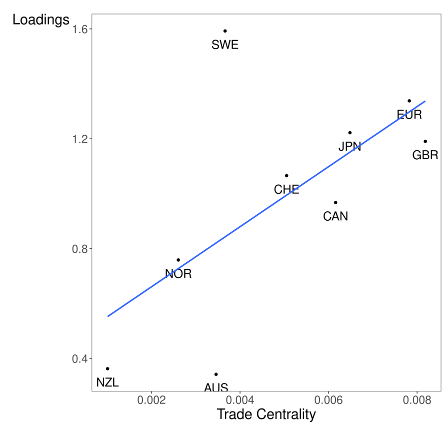
(a) CCGR Estimates



(b) Country Risk Estimates



(c) Country Size



(d) Trade Centrality

This figure plots my estimates against: the estimates obtained by Colacito et al. (2018a) in Panel (a); the estimates obtained by Hassan et al. (2021c) in Panel (b); country size (Hassan (2013)) in Panel (c); and trade centrality (Richmond (2019)) in Panel (d).

Figure A1 is the same as Figure 4, but includes Sweden for Panel (b), (c) and (d). We can clearly see that Sweden is an obvious outlier, suggesting it is special in terms of its correlation with the world average. I leave the source of its specialty for future research.