CS180 HW1

Jingyi Zuo 804997146
Dis 1A

October 9, 2019

1. Exercise 3, Page 22

If Network A has a set of TV shows with rating 1 and 3, while Network B has a set of TV shows in rating 2 and 4, there will be no stable pair of schedules.

Case 1: S=(1,3) T=(2,4)

Then we can find another schedule for Network A, such that S'=(3,1). Therefore, Network A wins more slots with the pair (S', T) than it did with the pair (S, T). It's not a stable pair.

Case 2: S=(3,1) T=(2,4)

There is a schedule T'=(4,2). Therefore, Network B wins more slots with the pair (S, T') than it did with the pair (S, T)

Case 3: S=(1,3) T=(4,2)

Similarly, we can find T'=(2,4) just as Case 2 to help Network B win more slots.

Case 4: S=(3,1) T=(4,2)

Similarly, we can find S'=(1,3) just as Case 1 to help Network A win more slots.

Since we've listed all four possible pairs of schedules and there is no stable pairs, we can conclude that for every set of TV shows and ratings, there is not always a stable pair of schedules

2. Exercise 4, Page 22

Initially all the hospitals and students are free.
While there is a hospital with available spots,
    Choose such a hospital h
    Let s be the highest-ranked student in h's preference list to which h has not yet asked
    If s is free then
        add (s,h) as a match
        the number of available position of h decrease by 1
    Else s is currently assigned to hospital h'
        If s prefered h' to h then
            keep (s, h')
            the number of available position of h remains same
        Else s prefered h to h'
            add (s, h) as a match
            remove (s, h')
            the number of available position of h' increase by 1
            the number of available position of h decrease by 1
        Endif
    Endif
Endwhile

Prove there is always a stable match:
(1) Assume if we end up with a instable match with students s and s', and a hospital h, so that

- s is assigned to h
- s' is assigned to no hospital
- h prefers s' to s

Then we can consider that if h asked s before s'.
If yes, it contradict with the algorithm that the hospital should ask the one ranked higher in the list.
If no, then s' will accept the offer, which contradict with the assumption that s' is assigned to no hospital.
Another instable match case is
(2)Assume if we end up with a instable match with students s and s', and a hospital h, so that

- s is assigned to h
- s' is assigned to h'
- h prefers s' to s
- s' prefers h to h'

Then we can consider that if h asked s before s'.
If yes, it contradict with the algorithm that the hospital should ask the one ranked higher in the list.
If no, then we can conclude that s' rejected h, which follows that there is a hospital h" ranked higher than h and asked for s' too. Similarly, since s' is eventually assigned to h', h' ranks higher than h" in the list. Reach a contradiction.

3.

Let each boat has a list of ports in the order of visit.
Let each port has a list of ships in the reversed order of visit. That is, the ship comes latter has higher priority in the list.
Initially all the ports and ships are free.

```
While there is a ship available,
    Choose such a ship s
    Let p be the highest-ranked port in s's preference list that has not yet occupied
    If p is free then
        s will be schedule to stop at p
    Else p is currently scheduled to boat s'
        If p prefered s' to s (s' comes latter)then
            keep (p, s')
            ship s keep available
        Else p prefered s to s'(s comes latter)
            s will be schedule to stop at p, add (p, s) as a match
            ship s' becomes available
        Endif
    Endif
Endwhile
```

Proof:

Let us assume that there are 2 ships s and s' in the same port p on the same day in the final result.
We may assume s' comes after s arriving.
By the algorithm, the port p will choose the one of s and s' higher in preference list, but cannot accept both of them.
There is a contradiction.

4.

$g_1 < g_5 < g_3 < g_4 < g_2 < g_7 < g_6$

$g_1 < g_5$

comparing $2^{\sqrt{logn}}$ with n^{logn} is the same comparing with $2^{logn*logn}$

Since $\sqrt{logn} < logn * logn$ for large enough n, we know $g_1 < g_5$

$g_5 < g_3$

$g_3 < g_4$

since $logn < n^{1/9}$, we have $n(logn)^3 < n^{\frac{4}{3}}$

$g_4 < g_2$

comparing $n^{\frac{4}{3}}$ with 2^n is the same with comparison between $2^{\frac{4}{3}*logn}$ and 2^n

Since $\frac{4}{3} * logn < n$ for large enough n, we know $g_4 < g_2$

$g_2 < g_7$

Since $n < n^2$, we have $2^n < 2^{n^2}$

$g_7 < g_6$

Since $n^2 < 2^n$, we have $2^{n^2} < 2^{2^n}$

5. (a)
Base step:
For n=1, the statement holds.

$$1 = \frac{1 * (1 + 1)}{2} \tag{1}$$

Inductive step:
We can assume that for the k-th term, the statement holds, such that:

$$1 + 2 + \cdots + k = \frac{k * (1 + k)}{2} \tag{2}$$

Then for k+1-th term, we have

$$1 + 2 + \cdots + k + (k + 1) = \frac{k * (1 + k)}{2} + (k + 1) \tag{3}$$

$$= (\frac{k + 2}{2}) * (1 + k) \tag{4}$$

$$= \frac{(k + 1) * (2 + k)}{2} \tag{5}$$

that satisfies the statement.
Therefore, we can conclude that this statement is true by mathematical induction.
5. (b)

$$1^3 + 2^3 + \cdots + n^3 = \frac{(n)^2 * (n + 1)^2}{4} \tag{6}$$

Base step: For n=1, the statement holds.

$$1^3 = \frac{(1)^2 * (1 + 1)^2}{4} = 1 \tag{7}$$

Inductive step:
We can assume that for the k-th term, the statement holds, such that:

$$1^3 + 2^3 + \cdots + k^3 = \frac{(k)^2 * (k + 1)^2}{4} \tag{8}$$

Then for k+1-th term, we have

$$1^3 + 2^3 + \cdots + k^3 + (k + 1)^3 = \frac{(k)^2 * (k + 1)^2}{4} + (k + 1)^3 \tag{9}$$

$$= (k + 1)^2 * (\frac{k^2 + 4k + 1}{4}) \tag{10}$$

$$= (k + 1)^2 * \frac{(k + 2)^2}{4} \tag{11}$$

$$= \frac{(k + 1)^2 * (k + 2)^2}{4} \tag{12}$$

that satisfies the statement.
Therefore, we can conclude that this statement is true by mathematical induction.

6.
For 200 steps, let us drop the egg from x-th floor.

$$x + (x - 1) + (x - 2) + \cdots + 1 \geq 200 \tag{13}$$

by solving this inequility, we have x=20
For N steps,

$$x + (x - 1) + (x - 2) + \cdots + 1 \geq N \tag{14}$$

by solving this inequility, we have

$$x = \lceil \frac{-1 \pm \sqrt{1 + 8N}}{2} \rceil \tag{15}$$