Section 1.1.

7. (a) $\frac{8i-1}{i} = 8+i$

(b) $\frac{-1+5i}{2+3i} = \frac{(-1+5i)(2-3i)}{(2+3i)(2-3i)} = \frac{13+13i}{13} = 1+i$

(c) $\frac{3}{i} + \frac{i}{3} = -3i + \frac{i}{3} = -\frac{8}{3}i$

13. $((3-i)^2-3)i = (3^2-6i+i^2-3)i = 5i+6.$

14. $Re(iz) = -Im z$

proof: $\forall z \in \mathbb{C}$ $\exists a,b \in \mathbb{R}$. s.t. $z=a+bi$

Then $Re(iz) = Re(ai-b) = -b$

$-Im z = -b$

Thus $Re(iz) = -Im z$.

15. (a) $i^{4k} = (i^4)^k = 1^k = 1$

(b) $i^{4k+1} = i^{4k}\cdot i = 1\cdot i = i$

(c) $i^{4k+2} = i^{4k}\cdot i^2 = 1\cdot i^2 = -1$

(d) $i^{4k+3} = i^{4k}\cdot i^3 = 1\cdot i^3 = -i$

20. (a) $iz = 4 - zi$

$2iz = 4$

$z = \frac{4}{2i} = -2i$

(c) $(2-i)z + 8z^2 = 0$

$z(2 - i + 8z) = 0$

$z = 0$ or $z = -\frac{1}{4} + \frac{1}{8}i$

21. $(1-i)z_1 + 3z_2 = 2-3i$ ①

$iz_1 + (1+2i)z_2 = 1.$ ②

Then. ①×i : $i(1-i)z_1 + 3iz_2 = (2-3i)i$

$(1+i)z_1 + 3iz_2 = 3+2i$ ③

②×(1-i) $(1-i)iz_1 + (1-i)(1+2i)z_2 = 1-i$

$(1+i)z_1 + (3+i)z_2 = 1-i$ ④

④-③:

$(3-2i)z_2 = -2-3i$

$z_2 = \frac{-2-3i}{3-2i} = -i$

$z_1 = 1+i$

Section 1.2.

10. let $z = a+bi$ $\quad a, b \in \mathbb{R}$.

Then $|\text{Re } z|^2 = |a|^2 \le a^2+b^2 = |z|^2$

Then $|\text{Re } z| \le |z|$.

$|\text{Im } z|^2 = |b|^2 \le a^2+b^2 = |z|^2$

Then $|\text{Im } z| \le |z|$

13. proof: let $z = a+bi$ $\quad a.b \in \mathbb{R}$.
$\bar{z} = a-bi$

$(\bar{z})^2 = (a-bi)(a-bi)$
$= a^2 - 2abi - b^2$

$z^2 = (a+bi)(a+bi) = a^2+2abi-b^2$

if $(\bar{z})^2 = z^2$, we have $4abi = 0$

This implies that either $a=0$ or $b=0 \Rightarrow$ z is pure real or pure imaginary.

Section 1.3.

5. ~~(a)~~ (b) $|\overline{(1+i)}(2-3i)(4i-3)|$

$= |1+i| \cdot |2-3i| |4i-3|$

$= \sqrt{2} \cdot \sqrt{13} \cdot \sqrt{25} = 5\sqrt{26}$.

Scanned with CamScanner

(c) $\left|\dfrac{i(2+i)^3}{(1-i)^2}\right| = \dfrac{|i|\,|2+i|^3}{|1-i|^2}$

$= \dfrac{\sqrt{1}\ \sqrt{5}^3}{\sqrt{2}^2} = \dfrac{5\sqrt{5}}{2}$

7. (b) $\cos\theta + i\sin\theta = \dfrac{-3+3i}{|-3+3i|} = -\dfrac{3}{\sqrt{18}} + \dfrac{3}{\sqrt{18}}i$

$\cos\theta = -\frac{1}{\sqrt{2}}$
$\sin\theta = \frac{1}{\sqrt{2}}$ $\Rightarrow \theta = \frac{3}{4}\pi.$ $\arg(-3+3i)$
$= \frac{3}{4}\pi + 2k\pi$, $k = 0, \pm1, \pm2 \cdots$

$-3+3i = 3\sqrt{2}\,\mathrm{cis}\,\frac{3\pi}{4}.$

(c) $\cos\theta + i\sin\theta = \dfrac{-\pi i}{|-\pi i|} = -i$

$\cos\theta = 0$
$\sin\theta = -1$ $\Rightarrow \theta = -\frac{\pi}{2}$ $\arg(-\pi i) = -\frac{\pi}{2} + 2k\pi$ $k = 0, \pm1, \pm2 \cdots$

$-\pi i = \pi\,\mathrm{cis}(-\frac{\pi}{2})$

(g) $-1+\sqrt{3}i = 2\,\mathrm{cis}\,(\frac{2}{3}\pi)$

$2+2i = 2\sqrt{2}\,\mathrm{cis}\,(\frac{\pi}{4})$ $\arg\left(\dfrac{-1+\sqrt{3}i}{2+2i}\right) = \frac{5}{12}\pi + 2k\pi$ $k = 0, \pm1, \pm2 \cdots$

Then $\dfrac{-1+\sqrt{3}i}{2+2i} = \frac{1}{\sqrt{2}}\mathrm{cis}\,\frac{5\pi}{12}$

19. "$\Rightarrow$" ~~$\exists r_1 r_2 \in \mathbb{R}$. $z_1 = r_1 \operatorname{cis}\theta$~~

let $\arg z_1 = \arg z_2 = \theta$

$\exists r_1, r_2 \in \mathbb{R}$ s.t. $z_1 = r_1 \operatorname{cis}\theta$

$r_1, r_2 > 0$ $z_2 = r_2 \operatorname{cis}\theta$.

Then $\exists c > 0$ $r_1 = c r_2$.

Then $z_1 = r_1 \operatorname{cis}\theta = c r_2 \operatorname{cis}\theta = c z_2$

"$\Leftarrow$" $z_1 = c z_2$ then $|z_1| = c|z_2|$

Then $z_1 = c z_2 = c|z_2| \operatorname{cis}(\arg z_2)$

$= |z_1| \operatorname{cis}(\arg z_2)$

$= |z_1| \operatorname{cis}(\arg z_1)$

Then $\arg z_1 = \arg z_2$.

Section 1.4.

4. (a) $\left(\cos\frac{2\pi}{9} + i\sin\frac{2\pi}{9}\right)^3 = e^{i\frac{2\pi}{9}\cdot 3} = e^{i\frac{2}{3}\pi}$

(b) $\frac{2+2i}{-\sqrt{3}+i} = \sqrt{2}\operatorname{cis}\left(-\frac{7}{12}\pi\right) = \sqrt{2}\, e^{i\left(-\frac{7}{12}\pi\right)}$

(c) $\frac{2i}{3e^{4+i}} = \frac{2i}{3e^4 e^i} = \frac{2e^{i\frac{\pi}{2}}}{3e^4 e^i} = \frac{2}{3e^4} e^{i\left(\frac{\pi}{2}-1\right)}$

8. (a) $e^{z+\pi i} = e^{-z}$

proof: let $z = x+yi$

$e^{x+yi+\pi i} = e^x e^{(y+\pi)i}$

$= e^x\left(\cos(y+\pi) + i\sin(y+\pi)\right) = -e^x(\cos y + i\sin y)$

$= -e^{x+yi} = -e^z$

let $z = x + yi$

b. $\overline{e^z} = \overline{e^x(\cos y + i\sin y)}$

$= e^x(\cos y - i\sin y)$

$= e^x(\cos(-y) + i\sin(-y))$

$= e^x e^{-iy} = e^{x-iy} = e^{\bar{z}}$

11. (a) T. let $z = a + bi$

$e^z = e^{a+bi} = e^a \cdot (\cos b + i\sin b) \neq 0$

since $e^a \neq 0$ and $\cos b + i\sin b \neq 0$.

(b). F $e^{z+2\pi i} = e^z$

(c) T: $\forall z \in \mathbb{C}$ $z = a + bi$ $a, b \in \mathbb{R}$

Then $e^z = e^{a+bi} = e^a(\cos b + i\sin b)$

(d) T:

$e^{-z} = e^{-a-bi} = e^{-a}(\cos(-b) + i\sin(-b))$

$= \frac{1}{e^a}(\cos b - i\sin b)$

$= \frac{1}{e^a} \cdot \frac{1}{\cos b + i\sin b}$

$= \frac{1}{e^a(\cos b + i\sin b)} = \frac{1}{e^{a+bi}}$

$= \frac{1}{e^z}$

Scanned with CamScanner

23. (a)

$$\int_0^{2\pi} \cos^8\theta d\theta$$

$$= \int_0^{2\pi} \left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)^8 d\theta$$

$$= \frac{1}{2^8}\cdot\int_0^{2\pi} (e^{i\theta}+e^{-i\theta})d\theta = \frac{1}{256}\int_0^{2\pi}\left[C(8,0)e^{8i\theta}+C(8,1)e^{6i\theta}+\cdots\right.$$

$$\left.+C(8,1)e^{-6i\theta}+C(8,0)e^{-8i\theta}\right]d\theta$$

$$= \frac{1}{256}\cdot(140\pi) = \frac{35}{64}\pi.$$

(b) $\int_0^{2\pi} \sin^6(2\theta)\,d\theta$

$$= \int_0^{2\pi}\left(\frac{e^{i2\theta}-e^{-i2\theta}}{2i}\right)^6 d\theta$$

$$= \left(\frac{1}{2i}\right)^6\int_0^{2\pi}(e^{i2\theta}-e^{-i2\theta})\,d\theta$$

$$= -\frac{1}{64}\cdot(-40\pi) = \frac{5}{8}\pi.$$

Section 1.5.

5. (d) $(1-\sqrt{3}i)^{\frac{1}{3}} = \left[2e^{i(-\frac{\pi}{3}+2k\pi)}\right]^{\frac{1}{3}}$

$$= 2^{\frac{1}{3}}e^{\frac{1}{3}i(-\frac{\pi}{3}+2k\pi)} \qquad k=0,\pm1,\pm2\cdots$$

(f) $\left(\frac{2i}{1+i}\right)^{\frac{1}{6}} = (1+i)^{\frac{1}{6}}$

$$= \left[\sqrt{2}\,e^{i(\frac{\pi}{4}+2k\pi)}\right]^{\frac{1}{6}}$$

$$= 2^{\frac{1}{12}}e^{\frac{1}{6}i(\frac{\pi}{4}+2k\pi)} \qquad k=0,\pm1,\pm2\cdots$$

14.

$$(z^m)^{\frac{1}{n}} = (|z|^m e^{im\theta})^{\frac{1}{n}}$$

$$= |z|^{\frac{m}{n}} e^{i\frac{(m\theta+2km\pi)}{n}}$$

$$= |z|^{\frac{m}{n}} e^{i\frac{m}{n}(\theta+2k\pi)}$$

$$= \left(|z|^{\frac{1}{n}} e^{i\frac{\theta+2k\pi}{n}}\right)^m = (z^{\frac{1}{n}})^m.$$

Section 1.7.

1. a. $(0,1,0)$

   b. $\left(\frac{12}{101}, -\frac{16}{101}, \frac{99}{101}\right)$

   c. $\left(-\frac{12}{25}, \frac{16}{25}, -\frac{3}{5}\right)$

5. a. $x_1 > 0$

   b. $x_3 < -\frac{3}{5}$

   c. $0 < x < \frac{3}{5}$

   d. $x_3 > \frac{4}{5}$

   e. $x_1 = x_2$ and $-1 < x_3 < 1$