Drunkard Random Walk Simulation

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Introduction

In this blog post, we simulate a two-dimensional random walk of a drunkard trying to get home.
The drunkard flips three coins each step:

- Two coins (coin 1 and 2) decide east/west/stay.
- One coin (coin 3) decides north/south.

Home is at (11, -6), meaning 11 steps east and 6 steps south from the origin (0,0).
If the drunkard cannot reach home in 5,000 flips, the attempt is a failure.
We will simulate this process 5,000 times and compute the average flips conditional on success.

R Setup

```r
set.seed(1)
m <- 5000
max_flips <- 5000
target_x <- 11
target_y <- -6
```

```r
# Flip a single coin: 1 for Head, 0 for Tail
flip_coin <- function(p_head = 0.5) rbinom(1, 1, p_head)

# We use two-coin rule for horizontal move
move_horizontal <- function(p1 = 0.5, p2 = 0.5) {
  c1 <- flip_coin(p1); c2 <- flip_coin(p2)
  if (c1 == 1 && c2 == 1) return(+1)
  if (c1 == 0 && c2 == 0) return(-1)
  return(0)
}

# We use one-coin rule for vertical move
move_vertical <- function(p3 = 0.5) if (flip_coin(p3) == 1) +1 else -1

# Simulate one walk
simulate_walk <- function(max_flips, target_x, target_y, p1=0.5, p2=0.5, p3=0.5) {
  x <- 0; y <- 0
  xs <- numeric(max_flips + 1); ys <- numeric(max_flips + 1)
```

```r
  xs[1] <- 0; ys[1] <- 0
  flips <- 0; success <- FALSE
  while (flips < max_flips) {
    flips <- flips + 1
    x <- x + move_horizontal(p1, p2)
    y <- y + move_vertical(p3)
    xs[flips+1] <- x; ys[flips+1] <- y
    if (x == target_x && y == target_y) { success <- TRUE; break }
  }
  path <- cbind(step = 0:flips, x = xs[1:(flips+1)], y = ys[1:(flips+1)])
  list(success = success, flips = flips, path = path)
}

# Repeat m simulations
run_sims <- function(m, max_flips, target_x, target_y, p1=0.5, p2=0.5, p3=0.5) {
  flips_vec <- integer(m); success_vec <- logical(m)
  paths <- list(); keep <- 0
  for (i in 1:m) {
    res <- simulate_walk(max_flips, target_x, target_y, p1, p2, p3)
    flips_vec[i] <- res$flips; success_vec[i] <- res$success
    if (res$success && keep < 3) { keep <- keep+1; paths[[keep]] <- res$path }
  }
  avg <- if (any(success_vec)) mean(flips_vec[success_vec]) else NA
  list(avg = avg, success_rate = mean(success_vec), paths = paths)
}
```

Scenario A: All Fair Coins

```r
resA <- run_sims(m, max_flips, target_x, target_y, 0.5, 0.5, 0.5)
resA$avg
```

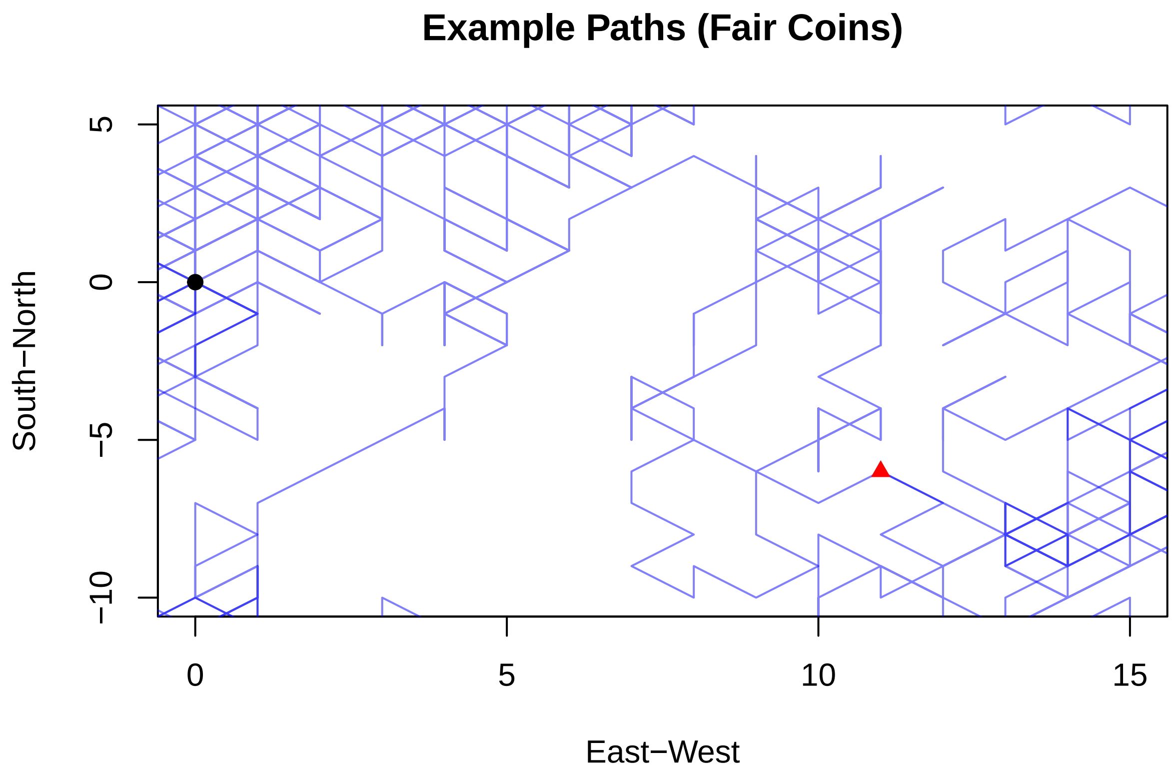
```
## [1] 1232.682
```

```r
resA$success_rate
```

```
## [1] 0.2492
```

Example

```r
plot(0,0,type="n",xlim=c(0,15),ylim=c(-10,5),
     xlab="East-West",ylab="South-North",
     main="Example Paths (Fair Coins)")
for (p in resA$paths) lines(p[,"x"], p[,"y"], col=rgb(0,0,1,0.5))
points(0,0,pch=19); points(11,-6,pch=17,col="red")
```



Scenario B: Coin 1 Biased (p=0.7)

```
resB <- run_sims(m, max_flips, target_x, target_y, 0.7, 0.5, 0.5)
resB$avg
```

```
## [1] 58.84589
```

```
resB$success_rate
```

```
## [1] 0.1168
```

Scenario C: Coin 1 & 2 Biased (p=0.7 each)

```
resC <- run_sims(m, max_flips, target_x, target_y, 0.7, 0.7, 0.5)
resC$avg
```

```
## [1] 28.80612
```

```r
resC$success_rate
```

```
## [1] 0.0784
```

Conclusion

- **Fair Coins (Scenario A):**
  When all 3 coins are fair, the drunkard doesn¡¯t have a natural push in any direction. He is just as likely to move west as to move east. Since his home requires 11 steps east, getting there doesn¡¯t happen very often. Even when he makes it, it usually takes many flips. This means the average number of flips (when he does succeed) is high, and the overall chance of success is low.

- **Coin 1 Biased Toward Heads (Scenario B):**
  When coin 1 is biased toward heads (`Pr(head) = 0.7`), the chance of flipping two heads goes up and the chance of two tails goes down. This makes the drunkard more likely to move east, which is the direction he needs. Because of this, the chance of reaching home goes up, and the average number of flips (when he succeeds) goes down. The north¨Csouth moves don¡¯t change on average, so he still has to be lucky to land exactly 6 steps south, but overall he gets home faster.

- **Coins 1 and 2 Both Biased Toward Heads (Scenario C):**
  When both coin 1 and coin 2 are biased toward heads (`Pr(head) = 0.7`), the drunkard moves east even more often. This makes reaching +11 steps east much easier, so the success rate goes up again and the average flips (when he succeeds) go down. The downside is that he almost always goes east too quickly, so the harder part becomes timing the ¨C6 steps south at the right moment.