DS-5620 HW3

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Contents

Overview

Assumptions: 1. Each game is independent.
2. The per-game win probability for the Braves is constant (P_B) across games.

Key fact: In a best-of-n (odd) series, a team needs $w = (n+1)/2$ wins. The probability of winning the series is:

$$\mathbb{P}(\text{win}) \;=\; \sum_{m=0}^{w-1} \binom{(w-1)+m}{m} p^w (1-p)^m$$

For coding, it's easier to compute with the binomial:

$$\mathbb{P}(\text{win series}) = 1 - \text{pbinom}(w-1, n, p).$$

```r
prob_win_best_of_n <- function(p, n){
  stopifnot(n %% 2 == 1)
  w <- floor(n/2) + 1
  1 - pbinom(w - 1, n, p)
}

prob_win_best_of_7 <- function(p){
  prob_win_best_of_n(p, n = 7)
}

min_odd_n_for_target <- function(p, target = 0.8, n_max = 4001){
  for(n in seq(1, n_max, by = 2)){
```

```r
    if(prob_win_best_of_n(p, n) >= target) return(as.integer(n))
  }
  return(NA_integer_)
}
```

Q1

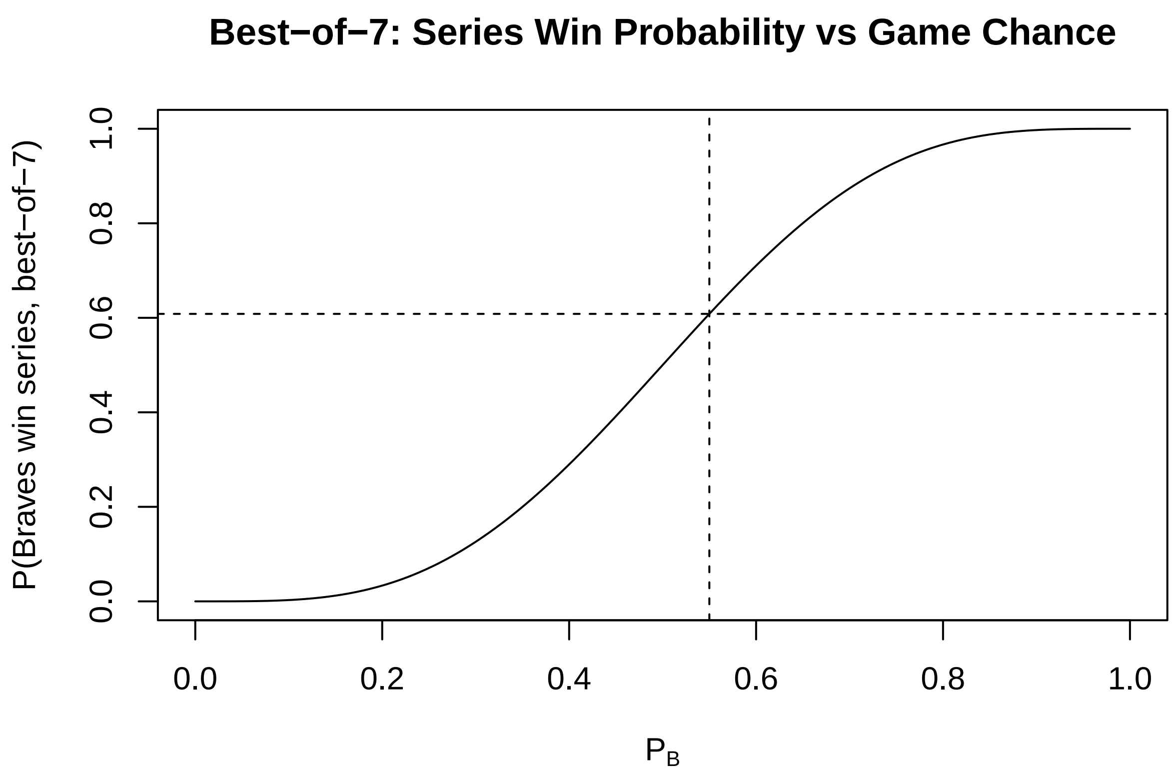
Result: 60.8%

```r
prob_q1 <- prob_win_best_of_7(0.55)
prob_q1
```

```
## [1] 0.6082878
```

Q2

```r
x <- seq(0, 1, by = 0.001)
y <- prob_win_best_of_7(x)

plot(x, y, type = "l",
     xlab = expression(P[B]),
     ylab = "P(Braves win series, best-of-7)",
     main = "Best-of-7: Series Win Probability vs Game Chance")
abline(v = 0.55, lty = 2)
abline(h = prob_win_best_of_7(0.55), lty = 2)
```



Q3

Answer: best-of-71.

```
min_n_q3 <- min_odd_n_for_target(0.55, 0.8, n_max = 4001)
min_n_q3
```

```
## [1] 71
```

```
prob_win_best_of_n(0.55, min_n_q3)
```

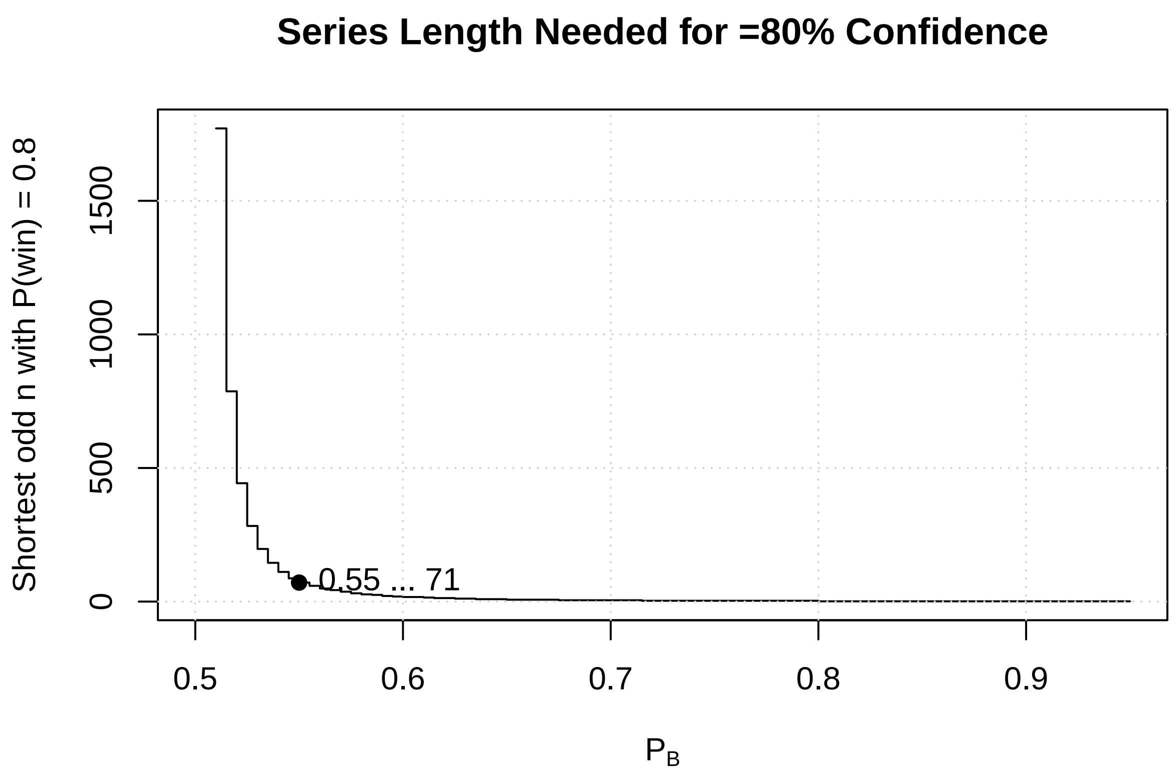
```
## [1] 0.8017017
```

Q4

```
p_grid <- seq(0.5, 0.95, by = 0.005)
n_needed <- vapply(p_grid, function(p) min_odd_n_for_target(p, 0.8, 4001),
                   FUN.VALUE = integer(1))

plot(p_grid, n_needed, type = "s",
```

```r
     xlab = expression(P[B]),
     ylab = "Shortest odd n with P(win)   0.8",
     main = "Series Length Needed for  80% Confidence")
grid()
points(0.55, min_n_q3, pch = 19)
text(0.55, min_n_q3, labels = "0.55 → 71", pos = 4)
```

```
## Warning in text.default(0.55, min_n_q3, labels = "0.55 → 71", pos = 4):
## 'mbcsToSbcs'  '0.55  <92> 71'  <e2>  dot

## Warning in text.default(0.55, min_n_q3, labels = "0.55 → 71", pos = 4):
## 'mbcsToSbcs'  '0.55  <92> 71'  <86>  dot

## Warning in text.default(0.55, min_n_q3, labels = "0.55 → 71", pos = 4):
## 'mbcsToSbcs'  '0.55  <92> 71'  <92>  dot
```



Explaination:
At $P_B \geq 0.8$: a single game is enough.
Around 0.70: need best-of-5. Around 0.60: need best-of-17.
At 0.55: need best-of-71.
As $P_B \rightarrow 0.5$: required length explodes.

Q5

Likelihood under p: $\binom{6}{3}p^4(1-p)^3$.
Posterior:

$$P(p = 0.55 \mid \text{win in 7}) = \frac{0.55}{0.55 + 0.45} = 0.55.$$

```r
like <- function(p){ choose(6,3) * p^4 * (1-p)^3 }
post_055 <- like(0.55) / (like(0.55) + like(0.45))
post_055
```

```
## [1] 0.55
```

Explaination: Braves win in 7 = first 6 games are 3–3 and Braves win game 7. Likelihood for per-game win probability p:

$$L(p) = \binom{6}{3} p^4 (1-p)^3.$$

With equal priors on $p = 0.55$ and $p = 0.45$, Bayes' rule gives

$$P(p = 0.55 \mid \text{data}) = \frac{L(0.55)}{L(0.55) + L(0.45)} = \frac{0.55^4 0.45^3}{0.55^4 0.45^3 + 0.45^4 0.55^3} = \frac{0.55}{0.55 + 0.45} = 0.55.$$