MAT 128A: Homework 5
Hangshi Jin
913142686
Rohit Thomas

Section 4.3

2.d.

$$\frac{\frac{1}{(e+1)\ln(e+1)}+\frac{1}{e}}{2} = 0.286334172478$$

4.d.

$$f''(x) = \frac{2\ln^2(x) + 3\ln x + 2}{x^3 \ln^3 x}$$

$$\text{error bound} = \frac{1}{12}\frac{2\ln^2 e + 3\ln e + 2}{e^3\ln^3 e} = 0.0290424565479$$

$$\text{actual error} = \int_e^{e+1} \frac{1}{x\ln x}dx - \frac{\frac{1}{(e+1)\ln(e+1)}+\frac{1}{e}}{2} = \ln(\ln(e+1)) - \frac{\frac{1}{(e+1)\ln(e+1)}+\frac{1}{e}}{2} = 0.0138202919758$$

6.d.

$$\int_e^{e+1} \frac{1}{x\ln x}dx \approx \frac{1}{6}[f(e) + 4f(e+\frac{1}{2}) + f(e+1)] = 0.2726705$$

8.d.

$$f^{(4)}(x) = \frac{24\ln^4 x + 50\ln^3 x + 70\ln^2(x) + 60\ln(x) + 24}{x^5\ln^5 x}$$

$$\text{error bound} = \frac{24\ln^4 e + 50\ln^3 e + 70\ln^2(e) + 60\ln(e) + 24}{e^5\ln^5 e}\cdot\frac{1}{90\cdot 2^5} = 0.000533420804094$$

$$\text{actual error} = \left|\int_e^{e+1}\frac{1}{x\ln x}dx - \frac{1}{6}[f(e)+4f(e+\frac{1}{2})+f(e+1)]\right| = 0.00015657194238$$

10.d.

$$\int_e^{e+1}\frac{1}{x\ln x}dx \approx f(e+\frac{1}{2}) = 0.265838592$$

12.d.

$$\text{error bound} = \frac{1}{2^3\cdot 3}\frac{2\ln^2 e + 3\ln e + 2}{e^3\ln^3 e} = 0.014521228274$$

$$\text{actual error} = \left|\int_e^{e+1}\frac{1}{x\ln x}dx - f(e+\frac{1}{2})\right| = 0.00667528807258$$

14.

$$5 = f(0) + f(2), 4 = 2f(1) \Rightarrow f(1) = 2$$

Simpson rule gives

$$\frac{1}{3}[f(0) + 4f(1) + f(2)] = \frac{13}{3}$$

18.

$$f(x) = x^0, \int_0^2 1dx = 2 = c_0 + c_1 + c_2$$

$$f(x) = x, \int_0^2 xdx = 2 = c_1 + 2c_2$$

$$f(x) = x^2, \int_0^2 x^2dx = \frac{8}{3} = c_1 + 4c_2$$

$$\Rightarrow c_0 = \frac{1}{3}, c_1 = \frac{4}{3}, c_2 = \frac{1}{3}$$

Section 4.4

```
function px=composite(a,b,n,f)
h=(b-a)/n;              %Calculate step size
sum=f(a)+f(b);          %Initiate sum with f(a)+f(b)
for i=2:n
    sum=sum+2*f(a+(i-1)*h);
                        %Apply composite trapzoid rule
end
px=sum*h/2;             %Return the result
end
```

2.d.

Use composite.m, we get

$$\int_e^{e+2} \frac{1}{x \ln x} dx \approx 0.44034501347$$

4.d.

$$\frac{1}{12}\left(\frac{1}{e\ln(e)} + \frac{4}{\left(\frac{1}{4}+e\right)\ln\left(\frac{1}{4}+e\right)} + \frac{2}{\left(\frac{2}{4}+e\right)\ln\left(\frac{2}{4}+e\right)} + \frac{4}{\left(\frac{3}{4}+e\right)\ln\left(\frac{3}{4}+e\right)} + \frac{2}{\left(\frac{4}{4}+e\right)\ln\left(\frac{4}{4}+e\right)}\right.$$

$$\left. + \frac{4}{\left(\frac{5}{4}+e\right)\ln\left(\frac{5}{4}+e\right)} + \frac{2}{\left(\frac{6}{4}+e\right)\ln\left(\frac{6}{4}+e\right)} + \frac{4}{\left(\frac{7}{4}+e\right)\ln\left(\frac{7}{4}+e\right)} + \frac{1}{(2+e)\ln(2+e)}\right) = 0.439199259506$$

6.d.

$$\frac{2}{5}\left(\frac{1}{\left(e+\frac{1}{5}\right)\ln\left(e+\frac{1}{5}\right)} + \frac{1}{\left(e+\frac{3}{5}\right)\ln\left(e+\frac{3}{5}\right)} + \frac{1}{(e+1)\ln(e+1)} + \frac{1}{\left(e+\frac{7}{5}\right)\ln\left(e+\frac{7}{5}\right)} + \frac{1}{\left(e+\frac{9}{5}\right)\ln\left(e+\frac{9}{5}\right)}\right) = 0.43771812299$$

10.

$$12 = 2f(0) \Rightarrow f(0) = 6$$

$$f(-0.5) + f(0.5) = 5 \Rightarrow f(0.5) = 3, f(-0.5) = 2$$

$$\frac{1}{6}[f(1) + f(-1) + 4f(-0.5) + 2f(0) + 4f(0.5)] = 6 \Rightarrow f(1) = f(-1) = 2$$

12.a.

$$\frac{\pi}{12}h^2 f''(\pi) \leq 10^{-4} \Rightarrow n \geq \sqrt{\frac{((2-\pi^2)\cos\pi - 4\pi\sin\pi)\cdot\frac{\pi^3}{12}}{10^{-4}}} = 451, h < \frac{\pi}{451} = 0.00696689453213$$

b.

$$\frac{\pi}{180}h^4 f^{(4)}(2.181) \leq 10^{-4} \Rightarrow n \geq \left(\frac{\left(8\cdot 2.181\sin 2.181 + \left(2.181^2 - 12\right)\cos 2.181\right)\cdot\frac{\pi^5}{180}}{10^{-4}}\right)^{-4} = 24, h < \frac{\pi}{24} = 0.132749440081$$

c.

$$\frac{\pi}{6}h^2 f''(\pi) \leq 10^{-4} \Rightarrow n \geq \sqrt{\frac{((2-\pi^2)\cos\pi - 4\pi\sin\pi)\cdot\frac{\pi^3}{6}}{10^{-4}}} = 638, h < \frac{\pi}{638} = 0.00492633836748$$

Section 4.5

```
function intapprox =Jin_Hangshi_romberg(a,b,n,f)
%
% This function calculates the integral of f over [a,b] using Romberg
% Integration with degree n.
%
% Inputs:
%
% a, a real number
% b, a real number
% n, a positive integer
% f, a user-specified external function
%
% Outputs:
%
% intapprox, a real number
%
m=zeros(n,1);
for i=1:n
    m(i)=2.^(i-1);
end
h=(b-a)/m(1);
R=zeros(n,n);
R(1,1)=h*(f(a)+f(b))/2;
for i=2:n
    h=(b-a)/m(i);
    for j=2:m(i)
        R(i,1)=R(i,1)+h*(f(a+(j-1)*h));
    end
    R(i,1)=R(i,1)+h*(f(a)+f(b))/2;
end
for i=2:n
    for j=i:n
        R(j,i)=(4.^(i-1)*R(j,i-1)-R(j-1,i-1))/(4.^(i-1)-1);
    end
end
intapprox = vpa(R(n,n));
end
```

2.d.

Use romberg.m, we get

$$R_{3,3} = 0.526815545$$

4.d.

Use romberg.m, we get

$$R_{4,4} = 0.52659385$$

6.d.

Use romberg.m, we found that

$$|R_{5,5} - R_{6,6}| = |0.52658907812 - 0.526589034294| = 4.3826000051 \times 10^{-8} < 10^{-6}$$

$$actualerror = |0.526589034294 - 0.526589034139| = 1.549999018 \times 10^{-10}$$