MAT 128A: Homework 2
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## Section 3.2

6.

$$P_{1,2} = \frac{x-0.7}{0.4-0.7}\cdot 2.8 + \frac{x-0.4}{0.7-0.4}\cdot P_2 = \frac{28}{15} + \frac{P_2}{3}$$
$$P_{0,1,2} = \frac{x-0.7}{0-0.7}\cdot 3.5 + \frac{x-0}{0.7-0}\cdot P_{1,2} = 1 + \frac{28}{21} + \frac{5P_2}{21} = \frac{27}{7}$$
$$\Rightarrow P_2 = 6.4$$

## Section 3.3

```
function px = newton(x,y)
n=size(x,1);                        %Get the number of rows in x
sum1=y(1);                          %Initiate sum with F0,0
b=sym('x');                         %Claim variable x
F=zeros(n,n);                       %Initiate matrix to store F
F(:,1)=y;                           %Let the first column be y
for i=2:n
    for j=2:i
        F(i,j)=(F(i,j-1)-F(i-1,j-1))/(x(i)-x(i-j+1));%Apply Algorithm 3.2
    end
    a=F(i,j);                       %Store the coef
    for k=2:i
    a=a*(b-x(k-1));                 %Multiply coef with each term
    end
    sum1=sum1+a;                    %Add up to P(x)
end
px= vpa(sum1,7);                    %Round coef to 7 digits
end
```

8.

By using newton.m, we get

a.

$P_4(x) = 1.0517*x+0.5725*x*(x-0.1)+0.215*x*(x-0.3)*(x-0.1)+0.06301587*x*(x-0.3)*(x-0.6)*(x-0.1)-6.0$

b.

$$P_5(x) = 1.0517*x+0.5725*x*(x-0.1)+0.215*x*(x-0.3)*(x-0.1)+0.06301587*x*(x-0.3)*(x-0.6)*(x-0.1)$$
$$+0.01415945*x*(x-1.0)*(x-0.3)*(x-0.6)*(x-0.1)-6.0$$

10.

By using newton.m, we get

$$P(x) = 3.0*x+(x+1.0)*(2.0*x+4.0)-1.0*x*(x+1.0)*(x+2.0)+7.0$$

which is of degree 3.

11.

a.

By using newton.m, we get

$$4.0 * x - 1.0 * (x + 1.0) * (3.0 * x + 6.0) + x * (x + 1.0) * (x + 2.0) + 7.0 = Q(x)$$

⇒ Q(x) interpolates the data.

Expand Q(x) and P(x), we get

$$P(x) = x^3 - 3.0 * x + 1.0 = Q(x)$$

⇒ P(x) also interpolates the data.

We can also plug in x into both P(x) and Q(x) to see if they equal to y, which also tells they interpolate the data.

b.

Because P(x) and Q(x) are resulted from two different methods with different formats, where P(x) has a catastrophic cancellation for degree 2 term.

## Section 3.4

```
function hx = Hermite(x,y,y1)
b=sym('x');                              %Claim variable x
n=size(x,1);                             %Get the number of rows in x
X=[x(1) x(1)];                           %Initiate the first entries of vectors
Y=[y(1) y(1)];                           %X and Y with the first entries of x and y
for i=2:n
    X=[X x(i) x(i)];                     %Complete X and Y by copying each entry
    Y=[Y y(i) y(i)];                     %and inserting it right below the one copied
end
sum1=y(1)+y1(1)*(b-x(1));                %Initiate sum at F(0,0)
F=zeros(2*n,2*n);                        %Initiate matrix to store F
F(:,1)=Y';                               %Store the first column of F with Y
Y1=[0 y1(1)];                            %Initiate Y1 with first entry of y1
for i=2:n
    Y1=[Y1 (y(i)-y(i-1))/(x(i)-x(i-1)) y1(i)];%Construct Y1 with each of y1 on
                                          %odd position and the
                                          %computation on even.
end
F(:,2)=Y1';                              %Store the second column of F with Y1
for i=3:2*n
    for j=3:i
        F(i,j)=(F(i,j-1)-F(i-1,j-1))/(X(i)-X(i-j+1));%Apply Algorithm 3.3
    end
    a=F(i,j);                            %Store the coef
    for k=2:i
    a=a*(b-X(k-1));                      %Multiply coef with each term
    end
    sum1=sum1+a;                         %Add up to H(x)
end
hx= vpa(sum1,7);                         %Round coef to 7 digits
```

**end**

---

2.c.

By using Hermite.m, we get

$H_5(x) = (0.945237*x-0.0945237)*(x-0.1)-2.801997*x-1.0*(0.47935*x-0.047935)*(x-0.1)*(x-0.2)^2$

$-1.0*(x-0.1)*(x-0.2)*(0.297*x-0.0297)-1.0*(x-0.3)*(x-0.1)*(x-0.2)^2*(1299.972*x-129.9972)-0.00985021$

6.

$$f'(x) = -2e^{2x} + 3xe^x + 3e^x, f(x_0) = 3e^1 - e^2, f(x_1) = 3.15e^{1.05} - e^{2.1}$$

a.

By using Hermite.m, we get

$H_3(x) = 1.531579*x-1.0*(x-1.0)*(174.234*x-174.234)+(6853.612*x-6853.612)*(x-1.0)*(x-1.05)-0.7657894$

$$H_3(1.03) = 0.8093248562, R_3(x) = \frac{-e^x \cdot (16e^x - 3x - 12)}{4 \cdot 3 \cdot 2} (x-1)^2 (x-1.05)^2$$

$$\text{Error bound} \leq \left| \frac{-e^{1.025} \cdot (16e^{1.025} - 3 \cdot 1.025 - 12)}{4 \cdot 3 \cdot 2} (1.025-1)^2 (1.025-1.05)^2 \right| = 0.00000133904546929$$

$$\text{actual error} = |f(1.03) - H_3(1.03)| = 0.00000123731717216 < 0.00000133904546929$$

10.a.

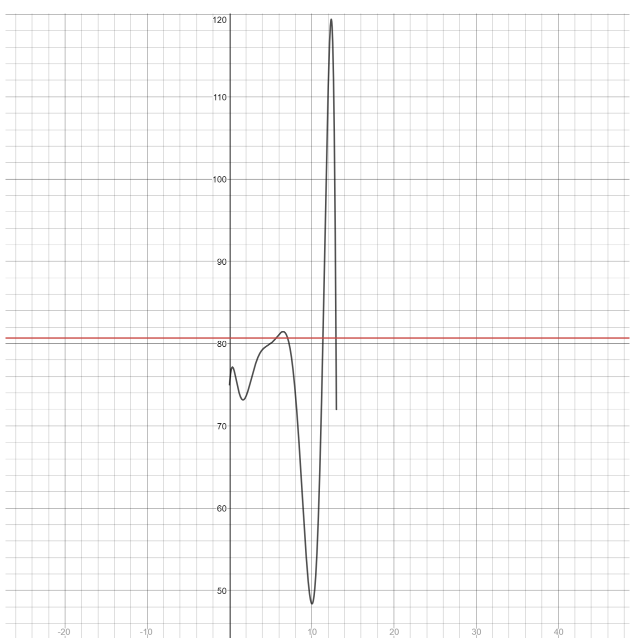
By using Hermite.m, we get

$H_9(x) = 75.0*x+0.2222222*x^2*(x-3.0)-0.03111111*x^2*(x-3.0)^2+0.002263889*x^2*(x-5.0)^2*(x-3.0)^2$

$$-0.006444444*x^2*(x-5.0)*(x-3.0)^2 - 0.0009131944*x^2*(x-8.0)*(x-5.0)^2*(x-3.0)^2$$

$+0.0001305268*x^2*(x-8.0)^2*(x-5.0)^2*(x-3.0)^2-0.00002022363*x^2*(x-8.0)^2*(x-5.0)^2*(x-3.0)^2*(x-13.0)$

$$H_9(10) = 742.50283909877$$



b.

$$55mi/hr = 80.67ft/sec$$

Through the graph, we see the car exceeds 55mi/hr first at x=5.651sec.

c.

Through the graph, we see the approximate maximum speed for the car is 119.417ft/sec=81.421mi/hr at t=12.372s.

## Section 3.5

```
function cpsplinecalc(x,y,y1)
q=sym('x');                          %Claim variable x
n=size(x,1);                         %Get the number of rows in x
h=zeros(n-1,1);a=y;                  %Initiate vectors needed and store y into a
al=zeros(n+1,1);
l=zeros(n,1);u=zeros(n-1,1);
z=zeros(n,1);b=zeros(n-1,1);
c=zeros(n,1);d=zeros(n-1,1);
l(1)=1;FPO=y1(1);FPN=y1(2);          %Set the first entry of l to 1,
                                     %and set FPO to f'(x0), FPN to f'(xn)
for i=1:n-1
    h(i)=x(i+1)-x(i);                %Apply Algorithm 3.5
end
al(1)=3*(y(2)-y(1))/h(1)-3*FPO;      %Initiate the first and last entry
al(n)=3*FPN-3*(a(n)-a(n-1))/h(n-1);  %of al
for i=2:n-1
    al(i)=3*(a(i+1)-a(i))/h(i)-3*(a(i)-a(i-1))/h(i-1);
end
l(1)=2*h(1);                         %Initiate the first entry of l,u, and z
u(1)=0.5;
z(1)=al(1)/l(1);
for i=2:n-1
    l(i)=2*(x(i+1)-x(i-1))-h(i-1)*u(i-1);
    u(i)=h(i)/l(i);
    z(i)=(al(i)-h(i-1)*z(i-1))/l(i);
end
l(n)=h(n-1)*(2-u(n-1));              %Set the last entry of l,z
z(n)=(al(n)-h(n-1)*z(n-1))/l(n);
c(n)=z(n);                           %store the last entry of z to the last of c
for i=n-1:-1:1
    c(i)=z(i)-u(i)*c(i+1);
    b(i)=(a(i+1)-a(i))/h(i)- h(i)*(c(i+1)+2*c(i))/3;
    d(i)=(c(i+1)-c(i))/(3*h(i));
end
a(n)=[];c(n)=[];                     %Remove the last entry of a and c
for i=1:n-1
    s{i}=vpa((a(i)+b(i)*(q-x(i))+c(i)*(q-x(i)).^2+d(i)*(q-x(i)).^3),7);
                                     %Store each natural cubic spline into s
end
celldisp(s)                          %Display each natural cubic spline
end
```

2.

By using cpsplinecalc.m, we get

$$s_0(x) = s_1(x) = x$$

```
function psplinecalc(x,y)
q=sym('x');                        %Claim variable x
n=size(x,1);                       %Get the number of rows in x
h=zeros(n-1,1);a=y;                %Initiate vectors needed and store y into a
al=zeros(n-2,1);
l=zeros(n,1);u=zeros(n-1,1);
z=zeros(n,1);b=zeros(n-1,1);
c=zeros(n,1);d=zeros(n-1,1);
l(1)=1;                            %Set the first entry of l to 1
for i=1:n-1
    h(i)=x(i+1)-x(i);              %Apply Algorithm 3.4
end
for i=1:n-2
    al(i)=3*(y(i+2)-y(i+1))/h(i+1)-3*(y(i+1)-y(i))/h(i);
end
for i=2:n-1
    l(i)=2*(x(i+1)-x(i-1))-h(i-1)*u(i-1);
    u(i)=h(i)/l(i);
    z(i)=(al(i-1)-h(i-1)*z(i-1))/l(i);
end
l(n)=1;z(n)=0;                     %Set the last entry of l and
                                   %z to 1 and 0 respectively
for i=n-1:-1:1
    c(i)=z(i)-u(i)*c(i+1);
    b(i)=(a(i+1)-a(i))/h(i)- h(i)*(c(i+1)+2*c(i))/3;
    d(i)=(c(i+1)-c(i))/(3*h(i));
end
a(n)=[];c(n)=[];                   %Remove the last entry of a and c
for i=1:n-1
    s{i}=vpa((a(i)+b(i)*(q-x(i))+c(i)*(q-x(i)).^2+d(i)*(q-x(i)).^3),7);
                                   %Store each natural cubic spline into s
end
celldisp(s)                        %Display each natural cubic spline
end
```

4.c.

By using psplinecalc.m, we get

$$s_0(x) = 4.38125 * (x - 0.1)^3 - 2.751286 * x - 0.01492133$$

$$s_1(x) = 1.314375 * (x - 0.2)^2 - 2.619849 * x - 4.38125 * (x - 0.2)^3 - 0.03682758$$

8.c.

By using cpsplinecalc.m, we get

$$s_0(x) = 12.81506 * (x - 0.1)^3 - 0.3512485 * (x - 0.1)^2 - 2.8005 * x - 0.01$$

$$s_1(x) = 3.493271 * (x - 0.2)^2 - 2.486297 * x - 39.52535 * (x - 0.2)^3 - 0.06353787$$

14.

$$s_0(1) = 1 + B = 1 = s_1(1), \Rightarrow B = 0, s_0'(1) = B - 2 = b = s_1'(1), \Rightarrow b = -2$$

$$\Rightarrow f'(0) = s_0'(0) = B = 0, f'(2) = s_1'(2) = b + 13 = 11$$

16.

$$y = [1e^{-0.25}e^{-0.75}e^{-1}]$$

By using psplinecalc.m, we get

$$s_0(x) = 0.6208652 * x^3 - 0.9236009 * x + 1.0$$

$$s_1(x) = 0.4656489 * (x - 0.25)^2 - 0.8071887 * x - 0.1540168 * (x - 0.25)^3 + 0.980598$$

$$s_2(x) = 0.2346237 * (x - 0.75)^2 - 0.4570524 * x - 0.3128316 * (x - 0.75)^3 + 0.8151559$$

$$\text{approx} = \int_0^{0.25} s_0(x)dx + \int_{0.25}^{0.75} s_1(x)dx + \int_{0.75}^{1} s_2(x)dx = 0.631966361168031$$

$$s'(0.5) = s_1'(0.5) = -0.6032424115, s''(0.5) = s_1''(0.5) = 0.7002726321541$$

$$\text{actual} = \int_0^1 e^{-x}dx = 0.63212055882, f'(0.5) = -e^{-0.5} = -0.60653065971263 < s'(0.5), f''(0.5) = e^{-0.5} = 0.60653065971263$$

18.

Use cpsplinecalc.m, we get

$$s_0(x) = -0.154515 * x^3 + 0.4994413 * x^2 - 1.0 * x + 1.0$$

$$s_1(x) = 0.383555 * (x - 0.25)^2 - 0.7792509 * x - 0.1015802 * (x - 0.25)^3 + 0.9736135$$

$$s_2(x) = 0.2311847 * (x - 0.75)^2 - 0.471881 * x - 0.06181745 * (x - 0.75)^3 + 0.8262773$$

$$\text{approx} = \int_0^{0.25} s_0(x)dx + \int_{0.25}^{0.75} s_1(x)dx + \int_{0.75}^{1} s_2(x)dx = 0.63207773208960$$

$$s'(0.5) = s_1'(0.5) = -0.60651969936339, s''(0.5) = s_1''(0.5) = 0.61473976438377$$

$$\text{actual} = \int_0^1 e^{-x}dx = 0.63212055882, f'(0.5) = -e^{-0.5} = -0.60653065971263 < s'(0.5), f''(0.5) = e^{-0.5} = 0.60653065971263$$

19.Let

$$f(x) = a + bx + cx^2 + dx^3,$$

by the definition of a cubic spline interpolant S, f interpolates itself for $x_0 ... x_n$.

Since the property of clamped spline holds, f is its own clamped cubic spline.

$f''(x) = 2c + 6dx \Rightarrow f''(x) = 0$ only at $x = \frac{-c}{3d} \neq x_0 \neq x_n$.

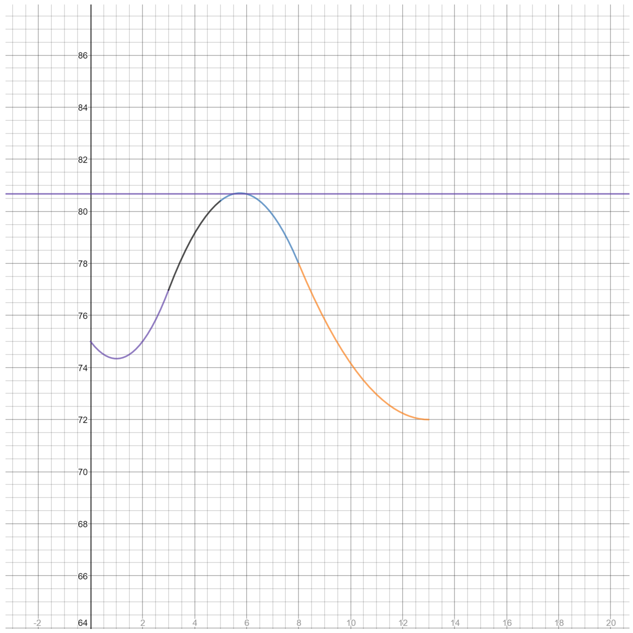
$\Rightarrow$ f cannot be a natural cubic spline.

29.a.

Use cpsplinecalc.m, we get

$$s_0(x) = 0.219764 * x^3 - 0.659292 * x^2 + 75.0 * x$$

$$s_1(x) = 76.97788 * x + 1.318584 * (x - 3.0)^2 - 0.1537611 * (x - 3.0)^3 - 5.933628$$
$$s_2(x) = 80.40708 * x + 0.3960177 * (x - 5.0)^2 - 0.177237 * (x - 5.0)^3 - 19.0354$$
$$s_3(x) = 77.99779 * x - 1.199115 * (x - 8.0)^2 + 0.0799115 * (x - 8.0)^3 - 0.9823009$$
$$S(10) = s_3(10) = 774.838407079646$$




b.

$$55mi/hr = 80.67ft/sec$$

Through the graph, we see the car exceeds 55mi/hr first at x=5.499sec.

c.

Through the graph, we see the approximate maximum speed for the car is 80.702ft/sec=55.024mi/hr at t=5.745s.

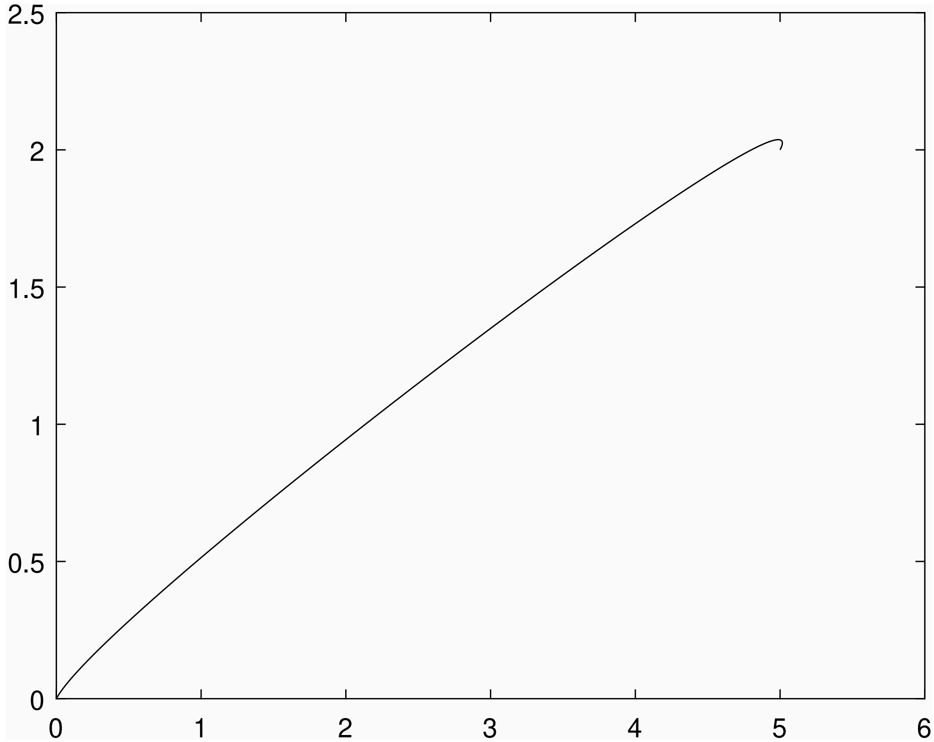
## Section 3.5

```
function parahermite(x,y,a,b)
q=sym('t');                            %Claim variable x
x1=zeros(2,1);y1=zeros(2,1);           %Initiate vectors x1 and y1
x1(1)=a(1)-x(1);x1(2)=x(2)-a(2);       %Calculate and store alpha to x1 and beta to y1
y1(1)=b(1)-y(1);y1(2)=y(2)-b(2);
                                       %Compute x(t) and y(t) explicitly
xt=(2*(x(1)-x(2))+(x1(1)+x1(2)))*q.^3+(3*(x(2)-x(1))-(x1(2)+2*x1(1)))*q.^2+x1(1)*q+x(1)
yt=(2*(y(1)-y(2))+(y1(1)+y1(2)))*q.^3+(3*(y(2)-y(1))-(y1(2)+2*y1(1)))*q.^2+y1(1)*q+y(1)
end
```

1.c.

By using parahermite.m, we get

$$x(t) = -10 * t^3 + 14 * t^2 + t, y(t) = -4 * t^3 + 5 * t^2 + t, \text{and clearly } 0 \le t \le 1$$


2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6