Evaluating these sums produces the approximation

$$S_3(z) = 0.76201 + 0.77177\cos z + 0.017423\cos 2z + 0.0065673\cos 3z - 0.38676\sin z + 0.047806\sin 2z,$$

and converting back to the variable x gives

$$S_3(x) = 0.76201 + 0.77177\cos \pi(x-1) + 0.017423\cos 2\pi(x-1) + 0.0065673\cos 3\pi(x-1) - 0.38676\sin \pi(x-1) + 0.047806\sin 2\pi(x-1).$$

Table 8.12 lists values of $f(x)$ and $S_3(x)$. ■

Table 8.12

x	$f(x)$	$S_3(x)$	$\lvert f(x) - S_3(x)\rvert$
0.125	0.26440	0.24060	2.38×10^{-2}
0.375	0.84081	0.85154	1.07×10^{-2}
0.625	1.36150	1.36248	9.74×10^{-4}
0.875	1.61282	1.60406	8.75×10^{-3}
1.125	1.36672	1.37566	8.94×10^{-3}
1.375	0.71697	0.71545	1.52×10^{-3}
1.625	0.07909	0.06929	9.80×10^{-3}
1.875	−0.14576	−0.12302	2.27×10^{-2}

EXERCISE SET 8.5

1. Find the continuous least squares trigonometric polynomial $S_2(x)$ for $f(x) = x^2$ on $[-\pi, \pi]$.

2. Find the continuous least squares trigonometric polynomial $S_n(x)$ for $f(x) = x$ on $[-\pi, \pi]$.

3. Find the continuous least squares trigonometric polynomial $S_3(x)$ for $f(x) = e^x$ on $[-\pi, \pi]$.

4. Find the general continuous least squares trigonometric polynomial $S_n(x)$ for $f(x) = e^x$ on $[-\pi, \pi]$.

5. Find the general continuous least squares trigonometric polynomial $S_n(x)$ for

$$f(x) = \begin{cases} 0, & \text{if } -\pi < x \le 0, \\ 1, & \text{if } 0 < x < \pi. \end{cases}$$

6. Find the general continuous least squares trigonometric polynomial $S_n(x)$ in for

$$f(x) = \begin{cases} -1, & \text{if } -\pi < x < 0. \\ 1, & \text{if } 0 \le x \le \pi. \end{cases}$$

7. Determine the discrete least squares trigonometric polynomial $S_n(x)$ on the interval $[-\pi, \pi]$ for the following functions, using the given values of m and n:

a. $f(x) = \cos 2x,\ m = 4, n = 2$ **b.** $f(x) = \cos 3x,\ m = 4, n = 2$

c. $f(x) = \sin \frac{x}{2} + 2\cos \frac{x}{3},\ m = 6, n = 3$ **d.** $f(x) = x^2 \cos x,\ m = 6, n = 3$

8. Compute the error $E(S_n)$ for each of the functions in Exercise 7.

9. Determine the discrete least squares trigonometric polynomial $S_3(x)$, using $m = 4$ for $f(x) = e^x \cos 2x$ on the interval $[-\pi, \pi]$. Compute the error $E(S_3)$.

10. Repeat Exercise 9 using $m = 8$. Compare the values of the approximating polynomials with the values of f at the points $\xi_j = -\pi + 0.2j\pi$, for $0 \le j \le 10$. Which approximation is better?