Three-dimensional curves are generated in a similar manner by additionally specifying third components z_0 and z_1 for the nodes and $z_0+\gamma_0$ and $z_1-\gamma_1$ for the guidepoints. The more difficult problem involving the representation of three-dimensional curves concerns the loss of the third dimension when the curve is projected onto a two-dimensional computer screen. Various projection techniques are used, but this topic lies within the realm of computer graphics. For an introduction to this topic and ways that the technique can be modified for surface representations, see one of the many books on computer graphics methods, such as [FVFH].

EXERCISE SET 3.6

1. Let $(x_0, y_0) = (0, 0)$ and $(x_1, y_1) = (5, 2)$ be the endpoints of a curve. Use the given guidepoints to construct parametric cubic Hermite approximations $(x(t), y(t))$ to the curve, and graph the approximations.
   - **a.** $(1, 1)$ and $(6, 1)$
   - **b.** $(0.5, 0.5)$ and $(5.5, 1.5)$
   - **c.** $(1, 1)$ and $(6, 3)$
   - **d.** $(2, 2)$ and $(7, 0)$
2. Repeat Exercise 1 using cubic Bézier polynomials.
3. Construct and graph the cubic Bézier polynomials given the following points and guidepoints.
   - **a.** Point $(1, 1)$ with guidepoint $(1.5, 1.25)$ to point $(6, 2)$ with guidepoint $(7, 3)$
   - **b.** Point $(1, 1)$ with guidepoint $(1.25, 1.5)$ to point $(6, 2)$ with guidepoint $(5, 3)$
   - **c.** Point $(0, 0)$ with guidepoint $(0.5, 0.5)$ to point $(4, 6)$ with entering guidepoint $(3.5, 7)$ and exiting guidepoint $(4.5, 5)$ to point $(6, 1)$ with guidepoint $(7, 2)$
   - **d.** Point $(0, 0)$ with guidepoint $(0.5, 0.5)$ to point $(2, 1)$ with entering guidepoint $(3, 1)$ and exiting guidepoint $(3, 1)$ to point $(4, 0)$ with entering guidepoint $(5, 1)$ and exiting guidepoint $(3, -1)$ to point $(6, -1)$ with guidepoint $(6.5, -0.25)$
4. Use the data in the following table and Algorithm 3.6 to approximate the shape of the letter $\mathcal{N}$.

i	x_i	y_i	α_i	β_i	α'_i	β'_i
0	3	6	3.3	6.5		
1	2	2	2.8	3.0	2.5	2.5
2	6	6	5.8	5.0	5.0	5.8
3	5	2	5.5	2.2	4.5	2.5
4	6.5	3			6.4	2.8

5. Suppose a cubic Bézier polynomial is placed through (u_0, v_0) and (u_3, v_3) with guidepoints (u_1, v_1) and (u_2, v_2), respectively.
   - **a.** Derive the parametric equations for $u(t)$ and $v(t)$ assuming that

$$u(0) = u_0, \quad u(1) = u_3, \quad u'(0) = u_1 - u_0, \quad u'(1) = u_3 - u_2$$

and

$$v(0) = v_0, \quad v(1) = v_3, \quad v'(0) = v_1 - v_0, \quad v'(1) = v_3 - v_2.$$

   - **b.** Let $f(i/3) = u_i$, for $i = 0, 1, 2, 3$ and $g(i/3) = v_i$, for $i = 0, 1, 2, 3$. Show that the Bernstein polynomial of degree 3 in t for f is $u(t)$ and the Bernstein polynomial of degree three in t for g is $v(t)$. (See Exercise 23 of Section 3.1.)