MAT 128A: Homework 3
Hangshi Jin
913142686
Rohit Thomas

Section 8.5

2.

$$a_k = 0, a_0 = 0$$

$$\Rightarrow S_n(x) = \sum_{k=1}^{n-1} \frac{2\sin kx(\sin \pi k - \pi k \cos \pi k)}{\pi k^2}$$

6.

$$\text{For } -\pi < x \leq 0, a_k = \frac{\int_{-\pi}^{0} -\cos kx dx}{\pi} = -\frac{\int_0^{\pi} \cos kx dx}{\pi}$$

$$a_0 = -1, a_n = -\frac{\int_0^{\pi} \cos kx dx}{\pi}$$

$$b_k = \frac{\int_{-\pi}^{0} -\sin kx dx}{\pi} = \frac{1 - cos(k\pi)}{k\pi}$$

$$\text{For } 0 < x \leq \pi, a_k = \frac{\int_0^{\pi} \cos kx dx}{\pi}$$

$$a_0 = 1, a_n = \frac{\int_0^{\pi} \cos kx dx}{\pi}$$

$$b_k = \frac{\int_0^{\pi} \sin kx dx}{\pi} = \frac{1 - cos(k\pi)}{k\pi}$$

It is clear that a_0, a_n, a_k cancel out.

$$\Rightarrow S_n(x) = \sum_{k=1}^{n-1} \frac{2(1-(-1)^k)}{k\pi} \sin kx$$

```
function sx=dtripoly(fx,l,u,n,m)
a=zeros(n+1,1);b=zeros(n,1);          %Initiate a_k and b_k
q=sym('x');                           %Claim variable x
x=zeros(2*m,1);y=zeros(2*m,1);        %Initiate x_j and y_j
for i=1:2*m
    x(i)=l+((i-1)/m)*u;               %Calculate x_j
end
for i=1:2*m
    y(i)=subs(fx,q,x(i));             %Calculate y_j
end
for i=1:n+1
    for j=1:2*m
        a(i)=a(i)+y(j)*cos((i-1)*x(j));
    end
    a(i)=a(i)/m;                      %Calculate a_k
end
for i=2:n
    for j=1:2*m
    b(i)=b(i)+y(j)*sin((i-1)*x(j));
```

```
    end
    b(i)=b(i)/m;                    %Calculate b_k
end
sum=0;                              %Initiate sum
for i=2:n
    sum= sum+a(i)*cos((i-1)*q)+b(i)*sin((i-1)*q);
                                    %Calculate the sum for k=1,...,n-1
end
sx=vpa(a(1)/2+a(n+1)*cos(n*q)+ sum,7);
                                    %Calculate S_n(x)
end
```

7.

Use dtripoly.m, we get

a.

$$S_2(x) = cos(2.0 * x) - 1.530808e^{-17} * cos(x) - 2.775558e^{-17} * sin(x)$$

b.

$$S_2(x) = 2.775558e^{-17} * sin(x) - 5.551115e^{-17} * cos(x) - 9.880895e^{-17} * cos(2.0 * x)$$

```
function E=edtripoly(fx,l,u,n,m)
a=zeros(n+1,1);b=zeros(n,1);        %Initiate a_k and b_k
q=sym('x');                         %Claim variable x
x=zeros(2*m,1);y=zeros(2*m,1);      %Initiate x_j and y_j
for i=1:2*m
    x(i)=l+((i-1)/m)*u;             %Calculate x_j
end
for i=1:2*m
    y(i)=subs(fx,q,x(i));           %Calculate y_j
end
for i=1:n+1
    for j=1:2*m
        a(i)=a(i)+y(j)*cos((i-1)*x(j));
    end
    a(i)=a(i)/m;                    %Calculate a_k
end
for i=2:n
    for j=1:2*m
    b(i)=b(i)+y(j)*sin((i-1)*x(j));
    end
    b(i)=b(i)/m;                    %Calculate b_k
end
sum=0;                              %Initiate sum
for i=2:n
    sum= sum+a(i)*cos((i-1)*q)+b(i)*sin((i-1)*q);
                                    %Calculate the sum for k=1,...,n-1
end
sx=a(1)/2+a(n+1)*cos(n*q)+ sum;
                                    %Calculate S_n(x)
E=0;                                %Initiate error
```

```
for i=1:2*m
    E=E+(y(i)-subs(sx,q,x(i))).^2; %Calculate error
end
E=vpa(E,7);
end
```

8.

Use edtripoly.m, we get

a.

$$E(S_2) = 4.018838e^{-33}$$

b.

$$E(S_2) = 4.0$$

Section 4.1

2.b.

$$f'(1) = \frac{1 - 1.2625}{-0.2} = 1.3125, f'(1.2) = \frac{1.2625 - 1}{0.2} = 1.3125, \frac{1.6595 - 1.2625}{0.2} = 1.985$$

4.b.

$$f'(x) = 2x \ln x + x, f''(x) = 2 \ln x + 3$$

$$\text{actual error: } f'(1) : |1 - 1.3125| = 0.3125$$

$$f'(1.2) : |2 \cdot 1.2 \cdot \ln(1.2) + 1.2 - 1.3125| = 0.325071736305$$

$$f'(1.4) : |2 \cdot 1.4 \cdot \ln(1.4) + 1.4 - 1.985| = 0.357122262539$$

$$\text{error bound:} f'(1) : |(2 \ln 1.2 + 3) \cdot \frac{0.2}{2}| = 0.336464311359$$

$$f'(1.2) : |(2 \ln 1.2 + 3) \cdot \frac{-0.2}{2}| = 0.336464311359$$

$$f'(1.4) : |(2 \ln 1.4 + 3) \cdot \frac{0.2}{2}| = 0.367294447324$$

6.b.

$$f'(7.4) = \frac{1}{2 \cdot 0.2}(-3 \cdot -68.3193 + 4 \cdot -71.6982 + 75.1576) = -16.69325$$

$$f'(7.6) = \frac{1}{2 \cdot 0.2}(-3 \cdot -71.6982 + 4 \cdot -75.1576 + 78.6974) = -17.096$$

$$f'(7.8) = \frac{1}{2 \cdot -0.2}(-3 \cdot -75.1576 + 4 \cdot -71.6982 + 68.3193) = -17.49825$$

$$f'(8.0) = \frac{1}{2 \cdot -0.2}(-3 \cdot -78.6974 + 4 \cdot -75.1576 + 71.6982) = -17.9$$

8.b.

$$f'(x) = \frac{1}{x+2} - 2(x+1), f''(x) = \frac{-1}{(x+2)^2} - 2, f'''(x) = \frac{2}{(x+2)^3}$$

$$\text{actual error: } f'(7.4) : |\frac{1}{7.4+2} - 2(7.4+1) + 16.69325| = 0.000367021276599$$

$$f'(7.6) : |\frac{1}{7.6+2} - 2(7.6+1) + 17.096| = 0.000166666666669$$

$$f'(7.8) : |\frac{1}{7.8+2} - 2(7.8+1) + 17.49825| = 0.00029081632653$$

$$f'(8.0) : |\frac{1}{8+2} - 2\,(8+1) + 17.9| = 0$$

$$\text{error bound:} f'(7.4) : \frac{0.2^2}{3}\frac{2}{(7.4+2)^3} = 0.000032105923864$$

$$f'(7.6) : \frac{0.2^2}{6}\frac{2}{(7.4+2)^3} = 0.000016052961932$$

$$f'(7.8) : \frac{0.2^2}{6}\frac{2}{(7.6+2)^3} = 0.0000150704089506$$

$$f'(8.0) : \frac{0.2^2}{3}\frac{2}{(7.6+2)^3} = 0.0000301408179012$$

14.

$$f'(3) = \frac{3.0976 - 2.6734}{2} = 0.2121$$

$$\text{Error bound} : \frac{4\cdot 1^2}{6} = \frac{4}{6} = 0.666666666667$$

Section 4.2

```
function N=Richex1(fx,d,h,x0)
N=zeros(d,d);                           %Initiate N as a dXd matrix
q=sym('x');                             %Claim variable x
for i=1:d
    N(i,1)=(subs(fx,q,x0+(h/(2.^(i-1))))-subs(fx,q,x0))/(h/(2.^(i-1)));
                                        %Calculate O(h)
end
for j=2:d
    for i=j:d
        N(i,j)=(1/(2.^(j-1)-1))*(2.^(j-1)*N(i,j-1)-N(i-1,j-1));
                                        %Calculate O(h^j)
    end
end
end
```

2.b.

Use Richex1.m, we get

$$N_4(0.4) = 1.99999622$$

```
function N=Richex(fx,d,h,x0)
N=zeros(d,d);                           %Initiate N as a dXd matrix
q=sym('x');                             %Claim variable x
for i=1:d
    N(i,1)=(subs(fx,q,x0+(h/(2.^(i-1))))-subs(fx,q,x0))/(h/(2.^(i-1)));
                                        %Calculate O(h^2)
end
for j=2:d
    for i=j:d
        N(i,j)=((N(i,j-1)-N(i-1,j-1))/(4.^(j-1)-1))+N(i,j-1);
                                        %Calculate O(h^(2j))
    end
end
end
```

6.

Type in $N_1(h), N_1(\frac{h}{2}), N_1(\frac{h}{4}), N_1(\frac{h}{8})$ to Richex.m, we get

$$N_4(h) = -1.00013515$$

8.

$$N_1(h) = \frac{1}{h}[f(x_0+h) - f(x_0)]$$

$$O(h) = N_1(\frac{h}{2}) - \frac{h}{4}f''(x_0) - \frac{h^2}{24}f'''(x_0)$$

$$O(h^2) = 2N_1(\frac{h}{2}) - N_1(h) + \frac{h^2}{12}f'''(x_0)$$

$$N_2(h) = 2N_1(\frac{h}{2}) - N_1(h)$$

$$O(h^3) = \frac{4}{3}N_2(\frac{h}{2}) - \frac{1}{3}N_2(h) = \frac{4}{3}(2N_1(\frac{h}{4}) - N_1(\frac{h}{2})) - \frac{1}{3}(2N_1(\frac{h}{2}) - N_1(h))$$

$$= \frac{4}{3}(2(\frac{4}{h}(f(x_0+\frac{h}{4}) - f(x_0))) - \frac{2}{h}(f(x_0+\frac{h}{2} - f(x_0)))) - \frac{1}{3}(2(\frac{2}{h}(f(x_0+\frac{h}{2}) - f(x_0))) - \frac{1}{h}(f(x_0+h) - f(x_0)))$$

$$= \frac{32}{3h}(f(x_0+\frac{h}{4}) - f(x_0)) - \frac{8}{3h}(f(x_0+\frac{h}{2}) - f(x_0)) - \frac{4}{3h}(f(x_0+\frac{h}{2}) - f(x_0)) + \frac{1}{3h}(f(x_0+h) - f(x_0))$$

Replace h with 4h, we get

$$\frac{8}{3h}(f(x_0+h) - f(x_0)) - \frac{1}{h}(f(x_0+2h) - f(x_0)) + \frac{1}{12h}(f(x_0+4h) - f(x_0))$$