c. $2x\cos(2x) - (x-2)^2 = 0$, $[2,3]$ and $[3,4]$

d. $x - (\ln x)^x = 0$, $[4,5]$

2. Find intervals containing solutions to the following equations.

a. $x - 3^{-x} = 0$

b. $4x^2 - e^x = 0$

c. $x^3 - 2x^2 - 4x + 2 = 0$

d. $x^3 + 4.001x^2 + 4.002x + 1.101 = 0$

3. Show that $f'(x)$ is 0 at least once in the given intervals.

a. $f(x) = 1 - e^x + (e-1)\sin((\pi/2)x)$, $[0,1]$

b. $f(x) = (x-1)\tan x + x\sin \pi x$, $[0,1]$

c. $f(x) = x\sin \pi x - (x-2)\ln x$, $[1,2]$

d. $f(x) = (x-2)\sin x \ln(x+2)$, $[-1,3]$

4. Find $\max_{a\le x\le b} |f(x)|$ for the following functions and intervals.

a. $f(x) = (2 - e^x + 2x)/3$, $[0,1]$

b. $f(x) = (4x-3)/(x^2-2x)$, $[0.5,1]$

c. $f(x) = 2x\cos(2x) - (x-2)^2$, $[2,4]$

d. $f(x) = 1 + e^{-\cos(x-1)}$, $[1,2]$

5. Use the Intermediate Value Theorem 1.11 and Rolle's Theorem 1.7 to show that the graph of $f(x) = x^3 + 2x + k$ crosses the x-axis exactly once, regardless of the value of the constant k.

6. Suppose $f \in C[a,b]$ and $f'(x)$ exists on (a,b). Show that if $f'(x) \ne 0$ for all x in (a,b), then there can exist at most one number p in $[a,b]$ with $f(p) = 0$.

7. Let $f(x) = x^3$.

a. Find the second Taylor polynomial $P_2(x)$ about $x_0 = 0$.

b. Find $R_2(0.5)$ and the actual error in using $P_2(0.5)$ to approximate $f(0.5)$.

c. Repeat part (a) using $x_0 = 1$.

d. Repeat part (b) using the polynomial from part (c).

8. Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = \sqrt{x+1}$ about $x_0 = 0$. Approximate $\sqrt{0.5}$, $\sqrt{0.75}$, $\sqrt{1.25}$, and $\sqrt{1.5}$ using $P_3(x)$, and find the actual errors.

9. Find the second Taylor polynomial $P_2(x)$ for the function $f(x) = e^x\cos x$ about $x_0 = 0$.

a. Use $P_2(0.5)$ to approximate $f(0.5)$. Find an upper bound for error $|f(0.5) - P_2(0.5)|$ using the error formula, and compare it to the actual error.

b. Find a bound for the error $|f(x) - P_2(x)|$ in using $P_2(x)$ to approximate $f(x)$ on the interval $[0,1]$.

c. Approximate $\int_0^1 f(x)\,dx$ using $\int_0^1 P_2(x)\,dx$.

d. Find an upper bound for the error in (c) using $\int_0^1 |R_2(x)\,dx|$, and compare the bound to the actual error.

10. Repeat Exercise 9 using $x_0 = \pi/6$.

11. Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = (x-1)\ln x$ about $x_0 = 1$.

a. Use $P_3(0.5)$ to approximate $f(0.5)$. Find an upper bound for error $|f(0.5) - P_3(0.5)|$ using the error formula, and compare it to the actual error.

b. Find a bound for the error $|f(x) - P_3(x)|$ in using $P_3(x)$ to approximate $f(x)$ on the interval $[0.5, 1.5]$.

c. Approximate $\int_{0.5}^{1.5} f(x)\,dx$ using $\int_{0.5}^{1.5} P_3(x)\,dx$.

d. Find an upper bound for the error in (c) using $\int_{0.5}^{1.5} |R_3(x)\,dx|$, and compare the bound to the actual error.

12. Let $f(x) = 2x\cos(2x) - (x-2)^2$ and $x_0 = 0$.

a. Find the third Taylor polynomial $P_3(x)$, and use it to approximate $f(0.4)$.

b. Use the error formula in Taylor's Theorem to find an upper bound for the error $|f(0.4) - P_3(0.4)|$. Compute the actual error.