EXERCISE SET 4.3

1. Approximate the following integrals using the Trapezoidal rule.

a. $\int_{0.5}^{1} x^4\,dx$ **b.** $\int_{0}^{0.5} \frac{2}{x-4}\,dx$

c. $\int_{1}^{1.5} x^2 \ln x\,dx$ **d.** $\int_{0}^{1} x^2 e^{-x}\,dx$

e. $\int_{1}^{1.6} \frac{2x}{x^2-4}\,dx$ **f.** $\int_{0}^{0.35} \frac{2}{x^2-4}\,dx$

g. $\int_{0}^{\pi/4} x \sin x\,dx$ **h.** $\int_{0}^{\pi/4} e^{3x}\sin 2x\,dx$

2. Approximate the following integrals using the Trapezoidal rule.

a. $\int_{-0.25}^{0.25} (\cos x)^2\,dx$ **b.** $\int_{-0.5}^{0} x\ln(x+1)\,dx$

c. $\int_{0.75}^{1.3} \left((\sin x)^2 - 2x\sin x + 1\right)\,dx$ **d.** $\int_{e}^{e+1} \frac{1}{x\ln x}\,dx$

3. Find a bound for the error in Exercise 1 using the error formula, and compare this to the actual error.

4. Find a bound for the error in Exercise 2 using the error formula, and compare this to the actual error.

5. Repeat Exercise 1 using Simpson's rule.

6. Repeat Exercise 2 using Simpson's rule.

7. Repeat Exercise 3 using Simpson's rule and the results of Exercise 5.

8. Repeat Exercise 4 using Simpson's rule and the results of Exercise 6.

9. Repeat Exercise 1 using the Midpoint rule.

10. Repeat Exercise 2 using the Midpoint rule.

11. Repeat Exercise 3 using the Midpoint rule and the results of Exercise 9.

12. Repeat Exercise 4 using the Midpoint rule and the results of Exercise 10.

13. The Trapezoidal rule applied to $\int_0^2 f(x)\,dx$ gives the value 4, and Simpson's rule gives the value 2. What is $f(1)$?

14. The Trapezoidal rule applied to $\int_0^2 f(x)\,dx$ gives the value 5, and the Midpoint rule gives the value 4. What value does Simpson's rule give?

15. Find the degree of precision of the quadrature formula

$$\int_{-1}^{1} f(x)\,dx = f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right).$$

16. Let $h = (b-a)/3$, $x_0 = a$, $x_1 = a+h$, and $x_2 = b$. Find the degree of precision of the quadrature formula

$$\int_{a}^{b} f(x)\,dx = \frac{9}{4}hf(x_1) + \frac{3}{4}hf(x_2).$$

17. The quadrature formula $\int_{-1}^{1} f(x)\,dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.

18. The quadrature formula $\int_{0}^{2} f(x)\,dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.

19. Find the constants c_0, c_1, and x_1 so that the quadrature formula

$$\int_{0}^{1} f(x)\,dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision.

20. Find the constants x_0, x_1, and c_1 so that the quadrature formula

$$\int_{0}^{1} f(x)\,dx = \frac{1}{2}f(x_0) + c_1 f(x_1)$$

has the highest possible degree of precision.