MAT 128A: Homework 1
Hangshi Jin
913142686
Rohit Thomas

Section 1.1

10(a)

$$f'(x) = e^x \cos x - e^x \sin x, f''(x) = -2e^x \sin x, f'''(x) = -2e^x \sin x - 2e^x \cos x$$

$$P_2(x) = f(\frac{\pi}{6}) + (e^{\frac{\pi}{6}} \cos\frac{\pi}{6} - e^{\frac{\pi}{6}} \sin\frac{\pi}{6})(x - \frac{\pi}{6}) - e^{\frac{\pi}{6}} \sin\frac{\pi}{6}(x - \frac{\pi}{6})^2$$

$$P_2(0.5) = 1.44687901012, R_2(0.5) = \frac{f'''(X)}{6}(0.5 - \frac{\pi}{6})$$

$$\Rightarrow \text{upper bound for error} = \frac{f'''(\frac{\pi}{6})}{6}(0.5 - \frac{\pi}{6}) = 0.0000101018750376$$

$$\text{actual error} = |P_2(0.5) - f(0.5)| = 0.00001002646 < \text{upper bound for error}$$

(b)

$$R_2(x) = \frac{f'''(X)}{6}(x - \frac{\pi}{6})$$

$$\Rightarrow f'''(1) \leq f'''(X) \leq f'''(0) \Rightarrow |R_2(x)| \leq \frac{2e(\sin 1 + \cos 1)}{6}\left(1 - \frac{\pi}{6}\right)^3$$

$$\Rightarrow |P_2(x) - f(x)| \leq 0.135371932212$$

(c)

$$\int_0^1 P_2(x)dx = f(\frac{\pi}{6})+\frac{1}{2}(e^{\frac{\pi}{6}} \cos\frac{\pi}{6}-e^{\frac{\pi}{6}} \sin\frac{\pi}{6})-(e^{\frac{\pi}{6}} \cos\frac{\pi}{6}-e^{\frac{\pi}{6}} \sin\frac{\pi}{6})\frac{\pi}{6}-\frac{1}{3}e^{\frac{\pi}{6}} \sin\frac{\pi}{6}+\frac{\pi}{6}e^{\frac{\pi}{6}} \sin\frac{\pi}{6}-\frac{\pi^2}{36}e^{\frac{\pi}{6}} \sin\frac{\pi}{6} = 1.376541852$$

$$\int_0^1 f(x)dx = e^x \sin x|_0^1 - \int_0^1 e^x \sin x dx = \left.\frac{e^x \sin x + e^x \cos x}{2}\right|_0^1 = 1.37802461355$$

(d)

$$\int_0^1 |R_2(x)|dx \leq 1 \cdot 0.135371932212 = 0.135371932212$$

$$\text{actual error} = |1.376541852 - 1.37802461355| = 0.00148276155$$

Section 1.2

1(a)

$$\text{absolute error} = |\pi - \frac{22}{7}| = 0.00126448926735, \text{relative error} = \frac{|\pi - \frac{22}{7}|}{\pi} = 0.000402499434771$$

2(a)

$$3.14127849432 = 10^{-4} \cdot \pi - \pi \leq \text{approx} \leq 10^{-4} \cdot \pi + \pi = 3.14190681286$$

6(a)

$$\text{absolute error} = |133.921 - 133.9| = 0.021, \text{relative error} = \frac{|133.921 - 133.9|}{133.921} = 0.000156808864928$$

8(a)

$$\text{absolute error} = |133.921 - 133.9| = 0.021, \text{relative error} = \frac{|133.921 - 133.9|}{133.921} = 0.000156808864928$$

21(b)

$$f(x) = 1.01 \cdot (4.62)^4 - (4.62)^4 - 3.11 \cdot (4.62)^2 + 12.2 \cdot 4.62 - 1.99 \approx 460 - 455 - 66.2 + 56.4 - 1.99 = -6.79$$

(c)

$$f(x) = (((1.01e^x - 4.62)e^x - 3.11)e^x + 12.2)e^x - 1.99 = (((1.01 \cdot 4.62 - 4.62) \cdot 4.62 - 3.11) \cdot 4.62 + 12.2) \cdot 4.62 - 1.99 =$$

$$= -1.1 \cdot 4.62 - 1.99 = -7.07$$

(d)

$$\text{absolute error of (b)} = |-7.61 + 6.79| = 0.82 \geq \text{absolute error of (c)} = |-7.61 + 7.07| = 0.54$$

Section 3.1

2(b)

$$L_1(x) = \frac{x - 1.6}{-0.35} \text{at} (1.25, \sqrt[3]{0.25}), L_2(x) = \frac{x - 1.25}{0.35} \text{at} (1.6, \sqrt[3]{0.6})$$

$$\Rightarrow P_1(x) = \sqrt[3]{0.25} L_1(x) + \sqrt[3]{0.6} L_2(x) = 0.609920401012x - 0.132439976318 \Rightarrow P_1(1.4) = 0.721448585099$$

$$\text{absolute error} = |0.721448585099 - \sqrt[3]{0.4}| = 0.0153577146291$$

$$L_3(x) = \frac{(x - 1.25)(x - 1.6)}{(1 - 1.25)(1 - 1.6)} \text{at} (1, 0),$$

$$L_4(x) = \frac{(x - 1)(x - 1.6)}{(1.25 - 1)(1.25 - 1.6)} \text{at} (1.25, \sqrt[3]{0.25}),$$

$$L_5(x) = \frac{(x - 1)(x - 1.25)}{(1.6 - 1)(1.6 - 1.25)} \text{at} (1.6, \sqrt[3]{0.6})$$

$$\Rightarrow P_2(x) = \sqrt[3]{0.25} L_4(x) + \sqrt[3]{0.6} L_5(x) =$$

$$-3.1832028313x^2 + 9.68204847021x - 6.49884563891 \Rightarrow P_2(1.4) = 0.816944670036$$

$$\text{absolute error} = |0.816944670036 - \sqrt[3]{0.4}| = 0.0801383703079$$

4(b)

$$R_1(x) = \frac{-1}{9(x-1)^{\frac{5}{3}}}(x - 1.25)(x - 1.6) \leq f''(1.25)(1.425 - 1.25)(1.425 - 1.6) = 0.0342978508027$$

There is no $R_2(x)$ because $f'''(1.4)$ goes to ∞.

6(b)

$$L_1(x) = \frac{x - 0.25}{-0.5}, L_2(x) = \frac{x + 0.25}{0.5}$$

$$P_1(0) = \frac{-0.25 \cdot 1.33203}{-0.5} + \frac{0.25 \cdot 0.800781}{0.5} = 1.0664055$$

$$L_3(x) = \frac{(x + 0.25)(x - 0.25)}{(-0.5 + 0.25)(-0.5 - 0.25)}$$

$$L_4(x) = \frac{(x + 0.5)(x - 0.25)}{(-0.25 + 0.5)(-0.25 - 0.25)}$$

$$L_5(x) = \frac{(x + 0.5)(x + 0.25)}{(0.25 + 0.5)(0.25 + 0.25)}$$

$$P_2(0) = \frac{-0.0625 \cdot 1.93750}{0.1875} + \frac{-0.125 \cdot 1.33203}{-0.125} + \frac{0.125 \cdot 0.800781}{0.375} = 0.953123666667$$

$$P_3(0) = \frac{0.25 \cdot (-0.25)(-0.5) \cdot 1.93750}{(-0.5+0.25)(-0.5-0.25)(-0.5-0.5)} + \frac{0.5 \cdot (-0.25)(-0.5) \cdot 1.33203}{(-0.25+0.5)(-0.25-0.25)(-0.25-0.5)} +$$
$$\frac{0.25 \cdot 0.5 \cdot (-0.5) \cdot 0.800781}{(0.25+0.25)(0.25+0.5)(0.25-0.5)} + \frac{0.25 \cdot 0.5 \cdot (-0.25) \cdot 0.687500}{(0.5+0.25)(0.5+0.5)(0.5-0.25)} = 0.984374$$

8(b)

$|R_1(x)| = |(6x^2 - 3x + 1)(x - 0.25)(x + 0.25)| \leq f''(-0.25)(-0.0625) = 0.1328125 >$ actual error $= 0.0664055$

$|R_2(x)| = |(4x-1)(x+0.25)(x+0.5)(x-0.25)| \leq f'''(-0.5)(-0.033009559128) = 0.099028677384 >$ actual error $= 0.046876333$

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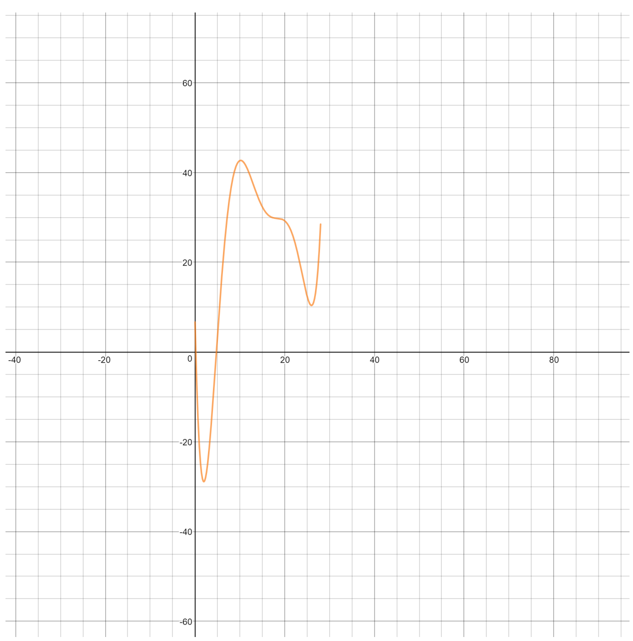
```
n=size(x,1);                                %Get the number of rows in x
sum1=0;                                      %Initiate sum with 0
b=sym('x');                                 %Claim variable x
for j=1:n
    a=x;
    a(j)=[];                                %For each iteration, exclude the jth entry
    L=expand(prod((b-a)./(x(j)-a)));%Get each L(x) and expand the polynomial
    sum1=sum1+L*y(j);                         %Add each L(x) to get P(x)
end
c = sym2poly(sum1);                          %Get the coefficient of each term
vpa(sum1)                                    %Show P(x) with floating-point coeff
```

By inputing the data manually, this code results

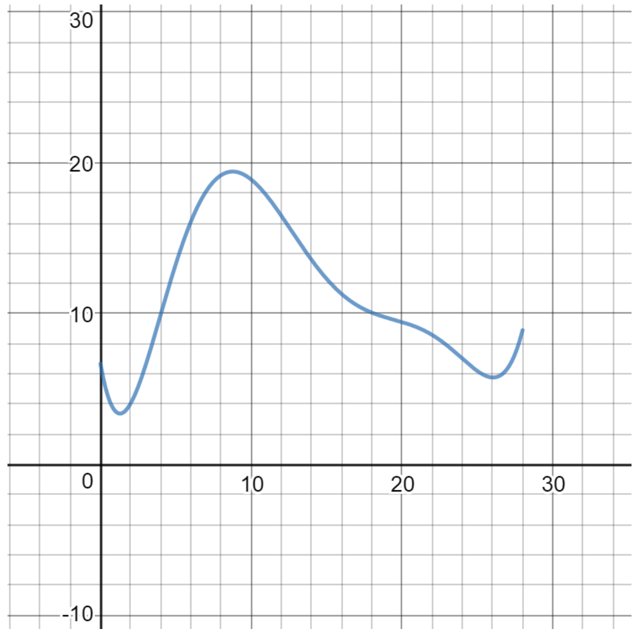
For sample 1: $P_6(x) = 0.000040945756790390098258700244949 29*x^6 - 0.0036716794001983077613329714 17005*x^5$

$$+0.12690236338593147225691152352726 * x^4 - 2.0946390777258137838733713447235 * x^3$$
$$+16.142724359576068896160569193419 * x^2 - 42.643480886166715043720391313974 * x + 6.67$$



Through the graph, we see the approximate maximum average weight for sample 2 is 42.701 mg.

For sample 2: $P_6(x) = 0.00000836159788843134908833915709393 17 * x^6 - 0.000752546222451684636558586138418 07 * x^5$
$+0.025841283442193010565814232735547 * x^4 - 0.41379865066497037467243731567643 * x^3$
$+2.9128091017755551750968099325624 * x^2 - 5.6782069577778133927866548187404 * x + 6.67$



Through the graph, we see the approximate maximum average weight for sample 2 is 19.416 mg.