Then the command

*h*5 := *PolynomialInterpolation*(*xy*, *method* = *hermite*, *independentvar* = '*x*')

produces an array whose nonzero entries correspond to the values in Table 3.17. The Hermite interpolating polynomial is created with the command

Interpolant(*h*5))

This gives the polynomial in (almost) Newton forward-difference form

$$1.29871616 - 0.5220232x - 0.08974266667(x - 1.3)^2 + 0.06636555557(x - 1.3)^2(x - 1.6) + 0.002666666633(x - 1.3)^2(x - 1.6)^2 - 0.002774691277(x - 1.3)^2(x - 1.6)^2(x - 1.9)$$

If a standard representation of the polynomial is needed, it is found with

expand(*Interpolant*(*h*5))

giving the Maple response

$$1.001944063 - 0.0082292208x - 0.2352161732x^2 - 0.01455607812x^3 + 0.02403178946x^4 - 0.002774691277x^5$$

EXERCISE SET 3.4

1. Use Theorem 3.9 or Algorithm 3.3 to construct an approximating polynomial for the following data.

a.

x	$f(x)$	$f'(x)$
8.3	17.56492	3.116256
8.6	18.50515	3.151762

b.

x	$f(x)$	$f'(x)$
0.8	0.22363362	2.1691753
1.0	0.65809197	2.0466965

c.

x	$f(x)$	$f'(x)$
−0.5	−0.0247500	0.7510000
−0.25	0.3349375	2.1890000
0	1.1010000	4.0020000

d.

x	$f(x)$	$f'(x)$
0.1	−0.62049958	3.58502082
0.2	−0.28398668	3.14033271
0.3	0.00660095	2.66668043
0.4	0.24842440	2.16529366

2. Use Theorem 3.9 or Algorithm 3.3 to construct an approximating polynomial for the following data.

a.

x	$f(x)$	$f'(x)$
0	1.00000	2.00000
0.5	2.71828	5.43656

b.

x	$f(x)$	$f'(x)$
−0.25	1.33203	0.437500
0.25	0.800781	−0.625000

c.

x	$f(x)$	$f'(x)$
0.1	−0.29004996	−2.8019975
0.2	−0.56079734	−2.6159201
0.3	−0.81401972	−2.9734038

d.

x	$f(x)$	$f'(x)$
−1	0.86199480	0.15536240
−0.5	0.95802009	0.23269654
0	1.0986123	0.33333333
0.5	1.2943767	0.45186776

3. The data in Exercise 1 were generated using the following functions. Use the polynomials constructed in Exercise 1 for the given value of x to approximate $f(x)$, and calculate the absolute error.

a. $f(x) = x \ln x$; approximate $f(8.4)$.

b. $f(x) = \sin(e^x - 2)$; approximate $f(0.9)$.

c. $f(x) = x^3 + 4.001x^2 + 4.002x + 1.101$; approximate $f(-1/3)$.

d. $f(x) = x \cos x - 2x^2 + 3x - 1$; approximate $f(0.25)$.

4. The data in Exercise 2 were generated using the following functions. Use the polynomials constructed in Exercise 2 for the given value of x to approximate $f(x)$, and calculate the absolute error.

a. $f(x) = e^{2x}$; approximate $f(0.43)$.

b. $f(x) = x^4 - x^3 + x^2 - x + 1$; approximate $f(0)$.

c. $f(x) = x^2 \cos x - 3x$; approximate $f(0.18)$.

d. $f(x) = \ln(e^x + 2)$; approximate $f(0.25)$.

5. **a.** Use the following values and five-digit rounding arithmetic to construct the Hermite interpolating polynomial to approximate $\sin 0.34$.

x	$\sin x$	$D_x \sin x = \cos x$
0.30	0.29552	0.95534
0.32	0.31457	0.94924
0.35	0.34290	0.93937

b. Determine an error bound for the approximation in part (a), and compare it to the actual error.

c. Add $\sin 0.33 = 0.32404$ and $\cos 0.33 = 0.94604$ to the data, and redo the calculations.

6. Let $f(x) = 3xe^x - e^{2x}$.

a. Approximate $f(1.03)$ by the Hermite interpolating polynomial of degree at most three using $x_0 = 1$ and $x_1 = 1.05$. Compare the actual error to the error bound.

b. Repeat (a) with the Hermite interpolating polynomial of degree at most five, using $x_0 = 1$, $x_1 = 1.05$, and $x_2 = 1.07$.

7. Use the error formula and Maple to find a bound for the errors in the approximations of $f(x)$ in parts (a) and (c) of Exercise 3.

8. Use the error formula and Maple to find a bound for the errors in the approximations of $f(x)$ in parts (a) and (c) of Exercise 4.

9. The following table lists data for the function described by $f(x) = e^{0.1x^2}$. Approximate $f(1.25)$ by using $H_5(1.25)$ and $H_3(1.25)$, where H_5 uses the nodes $x_0 = 1$, $x_1 = 2$, and $x_2 = 3$; and H_3 uses the nodes $\bar{x}_0 = 1$ and $\bar{x}_1 = 1.5$. Find error bounds for these approximations.

x	$f(x) = e^{0.1x^2}$	$f'(x) = 0.2xe^{0.1x^2}$
$x_0 = \bar{x}_0 = 1$	1.105170918	0.2210341836
$\bar{x}_1 = 1.5$	1.252322716	0.3756968148
$x_1 = 2$	1.491824698	0.5967298792
$x_2 = 3$	2.459603111	1.475761867

10. A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

a. Use a Hermite polynomial to predict the position of the car and its speed when $t = 10$ s.

b. Use the derivative of the Hermite polynomial to determine whether the car ever exceeds a 55 mi/h speed limit on the road. If so, what is the first time the car exceeds this speed?

c. What is the predicted maximum speed for the car?

11. **a.** Show that $H_{2n+1}(x)$ is the unique polynomial of least degree agreeing with f and f' at $x_0, \ldots, x_n$. [*Hint:* Assume that $P(x)$ is another such polynomial and consider $D = H_{2n+1} - P$ and D' at $x_0, x_1, \ldots, x_n$.]