**6.** Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

**a.** $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

**b.** $f(0.25)$ if $f(-1) = 0.86199480$, $f(-0.5) = 0.95802009$, $f(0) = 1.0986123$, $f(0.5) = 1.2943767$

**7.** **a.** Use Algorithm 3.2 to construct the interpolating polynomial of degree three for the unequally spaced points given in the following table:

| $x$ | $f(x)$ |
|---|---|
| $-0.1$ | 5.30000 |
| 0.0 | 2.00000 |
| 0.2 | 3.19000 |
| 0.3 | 1.00000 |

**b.** Add $f(0.35) = 0.97260$ to the table, and construct the interpolating polynomial of degree four.

**8.** **a.** Use Algorithm 3.2 to construct the interpolating polynomial of degree four for the unequally spaced points given in the following table:

| $x$ | $f(x)$ |
|---|---|
| 0.0 | $-6.00000$ |
| 0.1 | $-5.89483$ |
| 0.3 | $-5.65014$ |
| 0.6 | $-5.17788$ |
| 1.0 | $-4.28172$ |

**b.** Add $f(1.1) = -3.99583$ to the table, and construct the interpolating polynomial of degree five.

**9.** **a.** Approximate $f(0.05)$ using the following data and the Newton forward-difference formula:

| $x$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---|---|---|---|---|---|
| $f(x)$ | 1.00000 | 1.22140 | 1.49182 | 1.82212 | 2.22554 |

**b.** Use the Newton backward-difference formula to approximate $f(0.65)$.

**c.** Use Stirling's formula to approximate $f(0.43)$.

**10.** Show that the polynomial interpolating the following data has degree 3.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 4 | 11 | 16 | 13 | $-4$ |

**11.** **a.** Show that the cubic polynomials

$$P(x) = 3 - 2(x+1) + 0(x+1)(x) + (x+1)(x)(x-1)$$

and

$$Q(x) = -1 + 4(x+2) - 3(x+2)(x+1) + (x+2)(x+1)(x)$$

both interpolate the data

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | $-1$ | 3 | 1 | $-1$ | 3 |

**b.** Why does part (a) not violate the uniqueness property of interpolating polynomials?

**12.** A fourth-degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$.