a bound independent of h (and n). This means that, even though we may need to divide an interval into more parts to ensure accuracy, the increased computation that is required does not increase the round-off error. This result implies that the procedure is stable as h approaches zero. Recall that this was not true of the numerical differentiation procedures considered at the beginning of this chapter.

EXERCISE SET 4.4

1. Use the Composite Trapezoidal rule with the indicated values of n to approximate the following integrals.

a. $\int_1^2 x \ln x \, dx, \quad n = 4$ **b.** $\int_{-2}^2 x^3 e^x \, dx, \quad n = 4$

c. $\int_0^2 \frac{2}{x^2+4} \, dx, \quad n = 6$ **d.** $\int_0^{\pi} x^2 \cos x \, dx, \quad n = 6$

e. $\int_0^2 e^{2x} \sin 3x \, dx, \quad n = 8$ **f.** $\int_1^3 \frac{x}{x^2+4} \, dx, \quad n = 8$

g. $\int_3^5 \frac{1}{\sqrt{x^2-4}} \, dx, \quad n = 8$ **h.** $\int_0^{3\pi/8} \tan x \, dx, \quad n = 8$

2. Use the Composite Trapezoidal rule with the indicated values of n to approximate the following integrals.

a. $\int_{-0.5}^{0.5} \cos^2 x \, dx, \quad n = 4$ **b.** $\int_{-0.5}^{0.5} x \ln(x+1) \, dx, \quad n = 6$

c. $\int_{.75}^{1.75} (\sin^2 x - 2x \sin x + 1) \, dx, \quad n = 8$ **d.** $\int_e^{e+2} \frac{1}{x \ln x} \, dx, \quad n = 8$

3. Use the Composite Simpson's rule to approximate the integrals in Exercise 1.

4. Use the Composite Simpson's rule to approximate the integrals in Exercise 2.

5. Use the Composite Midpoint rule with $n + 2$ subintervals to approximate the integrals in Exercise 1.

6. Use the Composite Midpoint rule with $n + 2$ subintervals to approximate the integrals in Exercise 2.

7. Approximate $\int_0^2 x^2 \ln(x^2 + 1) \, dx$ using $h = 0.25$. Use

a. Composite Trapezoidal rule.

b. Composite Simpson's rule.

c. Composite Midpoint rule.

8. Approximate $\int_0^2 x^2 e^{-x^2} \, dx$ using $h = 0.25$. Use

a. Composite Trapezoidal rule.

b. Composite Simpson's rule.

c. Composite Midpoint rule.

9. Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$, and $f(0.25) = f(0.75) = \alpha$. Find α if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x) \, dx$.

10. The Midpoint rule for approximating $\int_{-1}^1 f(x) \, dx$ gives the value 12, the Composite Midpoint rule with $n = 2$ gives 5, and Composite Simpson's rule gives 6. Use the fact that $f(-1) = f(1)$ and $f(-0.5) = f(0.5) - 1$ to determine $f(-1)$, $f(-0.5)$, $f(0)$, $f(0.5)$, and $f(1)$.

11. Determine the values of n and h required to approximate

$$\int_0^2 e^{2x} \sin 3x \, dx$$

to within 10^{-4}. Use

a. Composite Trapezoidal rule.

b. Composite Simpson's rule.

c. Composite Midpoint rule.

12. Repeat Exercise 11 for the integral $\int_0^{\pi} x^2 \cos x \, dx$.

13. Determine the values of n and h required to approximate

$$\int_0^2 \frac{1}{x+4}\, dx$$

to within 10^{-5} and compute the approximation. Use

a. Composite Trapezoidal rule.

b. Composite Simpson's rule.

c. Composite Midpoint rule.

14. Repeat Exercise 13 for the integral $\int_1^2 x \ln x \, dx$.

15. Let f be defined by

$$f(x) = \begin{cases} x^3 + 1, & 0 \le x \le 0.1, \\ 1.001 + 0.03(x-0.1) + 0.3(x-0.1)^2 + 2(x-0.1)^3, & 0.1 \le x \le 0.2, \\ 1.009 + 0.15(x-0.2) + 0.9(x-0.2)^2 + 2(x-0.2)^3, & 0.2 \le x \le 0.3. \end{cases}$$

a. Investigate the continuity of the derivatives of f.

b. Use the Composite Trapezoidal rule with $n = 6$ to approximate $\int_0^{0.3} f(x)\, dx$, and estimate the error using the error bound.

c. Use the Composite Simpson's rule with $n = 6$ to approximate $\int_0^{0.3} f(x)\, dx$. Are the results more accurate than in part (b)?

16. Show that the error $E(f)$ for Composite Simpson's rule can be approximated by

$$-\frac{h^4}{180}[f'''(b) - f'''(a)].$$

[*Hint:* $\sum_{j=1}^{n/2} f^{(4)}(\xi_j)(2h)$ is a Riemann Sum for $\int_a^b f^{(4)}(x)\, dx$.]

17. **a.** Derive an estimate for $E(f)$ in the Composite Trapezoidal rule using the method in Exercise 16.

b. Repeat part (a) for the Composite Midpoint rule.

18. Use the error estimates of Exercises 16 and 17 to estimate the errors in Exercise 12.

19. Use the error estimates of Exercises 16 and 17 to estimate the errors in Exercise 14.

20. In multivariable calculus and in statistics courses it is shown that

$$\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)(x/\sigma)^2}\, dx = 1,$$

for any positive σ. The function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)(x/\sigma)^2}$$

is the *normal density function* with *mean* $\mu = 0$ and *standard deviation* σ. The probability that a randomly chosen value described by this distribution lies in $[a, b]$ is given by $\int_a^b f(x)\, dx$. Approximate to within 10^{-5} the probability that a randomly chosen value described by this distribution will lie in

a. $[-\sigma, \sigma]$ **b.** $[-2\sigma, 2\sigma]$ **c.** $[-3\sigma, 3\sigma]$

21. Determine to within 10^{-6} the length of the graph of the ellipse with equation $4x^2 + 9y^2 = 36$.

22. A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track?