e. $\int_0^{\pi/4} e^{3x} \sin 2x \, dx$

f. $\int_1^{1.6} \frac{2x}{x^2-4} \, dx$

g. $\int_3^{3.5} \frac{x}{\sqrt{x^2-4}} \, dx$

h. $\int_0^{\pi/4} (\cos x)^2 \, dx$

2. Use Romberg integration to compute $R_{3,3}$ for the following integrals.

a. $\int_{-1}^{1} (\cos x)^2 \, dx$

b. $\int_{-0.75}^{0.75} x \ln(x+1) \, dx$

c. $\int_1^4 \left((\sin x)^2 - 2x \sin x + 1\right) \, dx$

d. $\int_e^{2e} \frac{1}{x \ln x} \, dx$

3. Calculate $R_{4,4}$ for the integrals in Exercise 1.

4. Calculate $R_{4,4}$ for the integrals in Exercise 2.

5. Use Romberg integration to approximate the integrals in Exercise 1 to within 10^{-6}. Compute the Romberg table until either $|R_{n-1,n-1} - R_{n,n}| < 10^{-6}$, or $n = 10$. Compare your results to the exact values of the integrals.

6. Use Romberg integration to approximate the integrals in Exercise 2 to within 10^{-6}. Compute the Romberg table until either $|R_{n-1,n-1} - R_{n,n}| < 10^{-6}$, or $n = 10$. Compare your results to the exact values of the integrals.

7. Use the following data to approximate $\int_1^5 f(x)\, dx$ as accurately as possible.

x	1	2	3	4	5
$f(x)$	2.4142	2.6734	2.8974	3.0976	3.2804

8. Romberg integration is used to approximate

$$\int_0^1 \frac{x^2}{1+x^3} \, dx.$$

If $R_{11} = 0.250$ and $R_{22} = 0.2315$, what is R_{21}?

9. Romberg integration is used to approximate

$$\int_2^3 f(x) \, dx.$$

If $f(2) = 0.51342$, $f(3) = 0.36788$, $R_{31} = 0.43687$, and $R_{33} = 0.43662$, find $f(2.5)$.

10. Romberg integration for approximating $\int_0^1 f(x)\, dx$ gives $R_{11} = 4$ and $R_{22} = 5$. Find $f(1/2)$.

11. Romberg integration for approximating $\int_a^b f(x)\, dx$ gives $R_{11} = 8$, $R_{22} = 16/3$, and $R_{33} = 208/45$. Find R_{31}.

12. Use Romberg integration to compute the following approximations to

$$\int_0^{48} \sqrt{1 + (\cos x)^2} \, dx.$$

[*Note:* The results in this exercise are most interesting if you are using a device with between seven- and nine-digit arithmetic.]

a. Determine $R_{1,1}$, $R_{2,1}$, $R_{3,1}$, $R_{4,1}$, and $R_{5,1}$, and use these approximations to predict the value of the integral.

b. Determine $R_{2,2}$, $R_{3,3}$, $R_{4,4}$, and $R_{5,5}$, and modify your prediction.

c. Determine $R_{6,1}$, $R_{6,2}$, $R_{6,3}$, $R_{6,4}$, $R_{6,5}$, and $R_{6,6}$, and modify your prediction.

d. Determine $R_{7,7}$, $R_{8,8}$, $R_{9,9}$, and $R_{10,10}$, and make a final prediction.

e. Explain why this integral causes difficulty with Romberg integration and how it can be reformulated to more easily determine an accurate approximation.

13. Show that the approximation obtained from $R_{k,2}$ is the same as that given by the Composite Simpson's rule described in Theorem 4.4 with $h = h_k$.