MAT167: Homework 2
Hangshi Jin
913142686
E. G. Puckett
Due on May 7

Problem 1

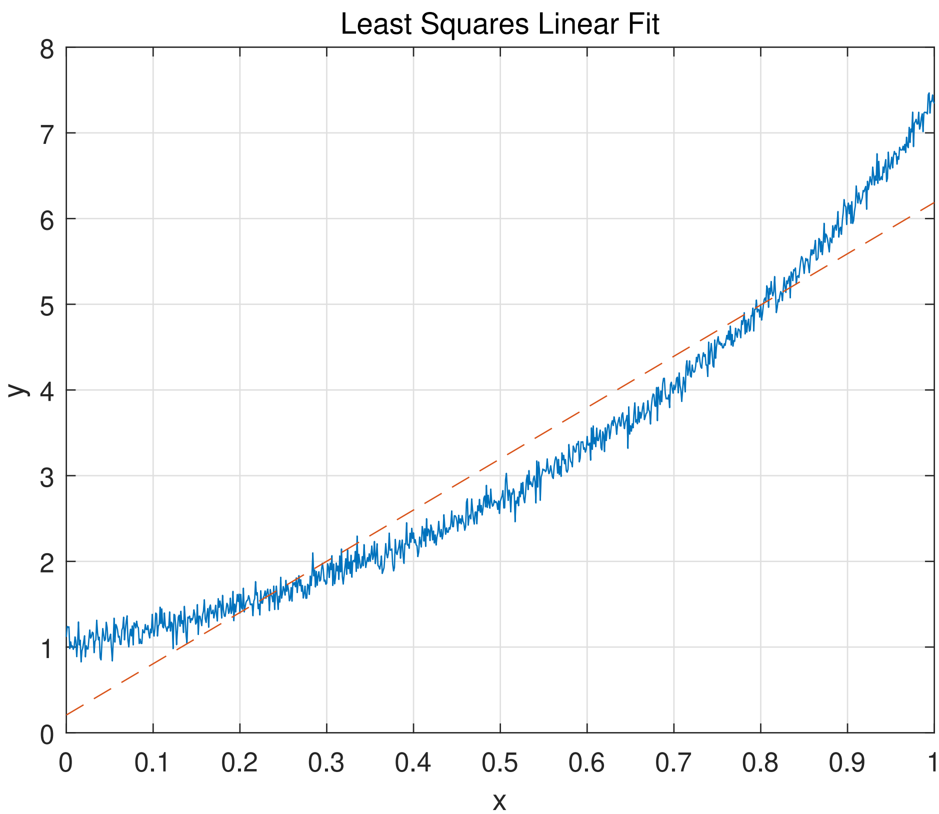

Figure1: By setting the entries in vector x in the input data as the x coordinates and the entries in vector y as the y coordinates of the points of the line, and connect these points, the curvy line is obtained. Let A be the matrix formed with the first column being each entry of x with power 0, and the second column being each entry of x with power 1. Then use the least square method to compute the solution for the least square line with the normal equation $sol = inv(A^A)A^Ty$. The $'--'$ line is obtained by setting the entries in vector x in the input data as the x coordinates and the entries of the linear combination $b = sol(1)x_1 + sol(2)x_2$ as the y coordinates of the $'--'$ line.

(f)Given vectors x and y both with n entries as inputs which form a linear function $e + kx = y$ where e and k are constants, this program solves e and k by the least squares method by first creating a Vendermonde matrix A with the first column being each entry of x with power 0, and with the second column being each entry of x with power 1. Then by using the normal equation, the program computes the solution. The result is performed on Figure1 as the least square line $'--'$ over the plot of vectors x and y where the entries of x is on the horizontal axis and that of y is on the vertical axis.

Problem 2

$null(A) = 0$ gives that Ax=0 only has the trivial solution.

$\Rightarrow$ The vectors $\{a_1\, a_2 \dots a_n\}$ in a square matrix A with n×n are linearly independent.

Hangshi Jin
913142686

$\Rightarrow$The dimension of span$\{a_1, a_2, ..., a_n\}$ is n.

$\Rightarrow$rank(A)=dim(range(A))=n$\Rightarrow$A has full rank.

$\Rightarrow$Since A is square and has full rank, it is invertible.

$\square$

Problem 3

Set $x = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \in R^2$ for $0 \le \theta \le 2\pi$, then $\|x\|_2 = 1$.

Now we find, for $0 \le \theta \le 2\pi$

$$\min\|x\|_1 = \min\{|\cos\theta| + |\sin\theta|\} = 1.$$

$\Rightarrow x = \begin{bmatrix} \cos\frac{a\pi}{2} \\ \sin\frac{a\pi}{2} \end{bmatrix}$ achieves such minimum, where $a \in I$.

Problem 4

By the definition of the induced matrix norm on $R^{m\times m}$,

$$\|A\| \ge \frac{\|Ax_i\|}{\|x_i\|}$$

for $1 \le i \le m$.

By the definition of eigenvalue,

$$\frac{\|Ax_i\|}{\|x_i\|} = \frac{\|\lambda_i x_i\|}{\|x_i\|} = |\lambda_i|\frac{\|x_i\|}{\|x_i\|} = |\lambda_i|,$$

$\Rightarrow$

$$\|A\| \ge \frac{\|Ax_i\|}{\|x_i\|} = |\lambda_i|.$$

for $1 \le i \le m$.

Thus, we have

$$\|A\| \ge \max_{1\le i\le m} |\lambda_i(A)| = \rho(A).$$

$\square$

Problem 5

$$\|u\|_2\|v\|_2 = \sqrt{u_1^2 + \cdots + u_m^2} \cdot \sqrt{v_1^2 + \cdots + v_n^2} = \sqrt{(u_1^2 + \cdots + u_m^2)(v_1^2 + \cdots + v_n^2)}$$

$$= \sqrt{u_1^2v_1^2 + \cdots + u_1^2v_n^2 + u_2^2v_n^2 + \cdots + u_m^2v_n^2} = \sqrt{(u_1v_1)^2 + \cdots (u_1v_n)^2 + (u_2v_n)^2 + \cdots + (u_mv_n)^2}$$

$$= \|uv^T\|_2 = \|A\|_2$$

$\square$

Problem 6

(a)

$$norm(A) = 4.302775637731994$$

(b)

$$sqrt(max(eig(A' * A))) = 4.302775637731995$$

$$\text{the relative error} = \frac{sqrt(max(eig(A' * A))) - norm(A)}{norm(A)} = 2.064198774185416e^{-16} \approx 0$$

(c)

$$\|A\|_1 = \max_{1\leq j\leq 2} \sum_{i=1}^{3} |a_{ij}| = \sum_{i=1}^{3} |a_{i2}| = 2+2+3 = 7$$

$$\|A\|_\infty = \max_{1\leq i\leq 3} \sum_{j=1}^{2} |a_{ij}| = \sum_{j=1}^{2} |a_{3j}| = 1+3 = 4$$

$$\|A\|_F = \sqrt{\sum_{i=1}^{3}\sum_{j=1}^{2} a_{ij}^2} = \sqrt{(1+4)+(0+4)+(1+9)} = \sqrt{19}$$

(d)

$$\|x\|_1 = 7.895242360521543$$

$$\|x\|_2 = 3.408079473961677$$

$$\|x\|_\infty = 2.258846861003648$$

$$\|a\|_1 = 7.895048063565293$$

$$\|a\|_2 = 3.408079473961677$$

$$\|a\|_\infty = 2.351901738628052$$

$\Rightarrow$

$$\|x\|_2 = \|a\|_2$$

(e)

$$\|U\|_1 = 2.828427124746190$$

$$\|U\|_2 = 1.000000000000000$$

$$\|U\|_\infty = 2.641845987495489$$

$$\|U\|_F = 2.828427124746190$$

Problem 7

(a)

$$\begin{aligned} e+k &= -1 \quad \text{for (1,-1),} \\ e+2k &= 3 \quad \text{for (2,3),} \\ e+3k &= 1 \quad \text{for (3,1).} \end{aligned}$$

$\Rightarrow$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} e \\ k \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

$\Rightarrow$

$$\text{normal equation} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} e \\ k \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} e \\ k \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} e \\ k \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

$$x = \begin{bmatrix} e \\ k \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$