MAT167: Homework 3
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Problem 1

(a)Since $\overline{\lambda}$ is an eigenvalue of A with $\overline{x}$

$$\overline{Ax} = \overline{\lambda x} \Rightarrow A\overline{x} = \overline{\lambda}\overline{x}.$$

Since $A^T = A$,

$$Ax = \lambda x \Rightarrow \overline{x}^T Ax = \overline{x}^T A^T x = (A\overline{x})^T x = \overline{\lambda}\overline{x}^T x = \lambda \overline{x}^T x.$$

Since

$$\overline{x} = \begin{bmatrix} a_1 - b_1 i \\ a_2 - b_2 i \\ \vdots \\ a_m - b_m i \end{bmatrix}, \text{and } x = \begin{bmatrix} a_1 + b_1 i \\ a_2 + b_2 i \\ \vdots \\ a_m + b_m i \end{bmatrix},$$

$$\overline{x}^T x = (a_1^2 + b_1^2) + (a_2^2 + b_2^2) + \cdots + (a_m^2 + b_m^2) \geq 0.$$

Since x is nonzero vector, $\overline{\lambda} = \lambda$.

By the property of complex numbers, λ is real.

□

(b)We know for any real matrix A,

$$A(x_1 \cdot x_2) = x_1 \cdot (A^T x_2).$$

Since A is sysmetric,

$$\lambda_1(x_1 \cdot x_2) = (\lambda_1 x_1) \cdot x_2 = (Ax_1) \cdot x_2 = x_1 \cdot (A^T x_2) = x_1 \cdot (Ax_2) = x_1 \cdot (\lambda_2 x_2) = \lambda_2(x_1 \cdot x_2).$$

$$\Rightarrow \lambda_1(x_1 \cdot x_2) - \lambda_2(x_1 \cdot x_2) = (\lambda_1 - \lambda_2)(x_1 \cdot x_2) = 0.$$

Since $\lambda_1 - \lambda_2 \neq 0$, $x_1 \cdot x_2 = 0$. $\Rightarrow x_1$ is orthogonal to x_2.

□

Problem 2

(a)

$$r_{11} = \|a_1\| = \sqrt{1+\epsilon^2} \approx 1,\ q_1 = \frac{a_1}{r_{11}} = \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix}.$$

$$r_{12} = q_1^T a_2 = \begin{bmatrix} 1 & \epsilon & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \epsilon \end{bmatrix} = 1.$$

$$q_2 = \frac{a_2 - r_{12} q_1}{r_{22}} = \frac{\begin{bmatrix} 1 \\ 0 \\ \epsilon \end{bmatrix} - \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix}}{r_{22}} = \frac{1}{r_{22}} \begin{bmatrix} 0 \\ -\epsilon \\ \epsilon \end{bmatrix}.$$

$$r_{22} = \|a_2 - r_{12} q_1\| = \left\| \begin{bmatrix} 1 \\ 0 \\ \epsilon \end{bmatrix} - \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 0 \\ -\epsilon \\ \epsilon \end{bmatrix} \right\| = \sqrt{2\epsilon^2} = \sqrt{2}\epsilon.$$

$$q_2 = \begin{bmatrix} 0 \\ \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}.$$

$$\Rightarrow A = \hat{Q}\hat{R} = \begin{bmatrix} 1 & 0 \\ \epsilon & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & \sqrt{2}\epsilon \end{bmatrix}.$$

(b)

$$v_1^{(1)} = \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix}, \, v_2^{(1)} = \begin{bmatrix} 1 \\ 0 \\ \epsilon \end{bmatrix}.$$

$$r_{11} = \|v_1^{(1)}\| = \sqrt{1+\epsilon^2} \approx 1, \, q_1 = \frac{v_1^{(1)}}{r_{11}} = \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix}.$$

$$r_{12} = q_1^T v_2^{(1)} = \begin{bmatrix} 1 & \epsilon & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \epsilon \end{bmatrix} = 1.$$

$$v_2^{(2)} = v_2^{(1)} - r_{12}q_1 = \begin{bmatrix} 1 \\ 0 \\ \epsilon \end{bmatrix} - \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -\epsilon \\ \epsilon \end{bmatrix}$$

$$r_{22} = \|a_2 - r_{12}q_1\| = \left\| \begin{bmatrix} 1 \\ 0 \\ \epsilon \end{bmatrix} - \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 0 \\ -\epsilon \\ \epsilon \end{bmatrix} \right\| = \sqrt{2\epsilon^2} = \sqrt{2}\epsilon.$$

$$q_2 = \frac{v_2^{(2)}}{r_{22}} = \frac{1}{\sqrt{2}\epsilon} \begin{bmatrix} 0 \\ -\epsilon \\ \epsilon \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}.$$

$$\Rightarrow A = \hat{Q}\hat{R} = \begin{bmatrix} 1 & 0 \\ \epsilon & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & \sqrt{2}\epsilon \end{bmatrix}.$$

(c)

$$x_1 = \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix}, \, v_1 = -sign(x_{11}) \|x_1\| e_1 - x_1 = -e_1 - \begin{bmatrix} 1 \\ \epsilon \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -\epsilon \\ 0 \end{bmatrix}.$$

$$v_1^T = \begin{bmatrix} -2 & -\epsilon & 0 \end{bmatrix} \Rightarrow F_1 = I - 2\frac{v_1 v_1^T}{v_1^T v_1} = I - \begin{bmatrix} 2 & \epsilon & 0 \\ \epsilon & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -\epsilon & 0 \\ -\epsilon & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = Q_1.$$

$$Q_1 A = \begin{bmatrix} -1 & -1 \\ 0 & -\epsilon \\ 0 & \epsilon \end{bmatrix}.$$

$$x_2 = \begin{bmatrix} -\epsilon \\ \epsilon \end{bmatrix}, \, v_2 = sign(x_{21}) \|x_2\| e_1 + x_2 = \begin{bmatrix} -\sqrt{2}\epsilon - \epsilon \\ \epsilon \end{bmatrix}.$$

$$v_2^T = \begin{bmatrix} -\sqrt{2}\epsilon - \epsilon & \epsilon \end{bmatrix} \Rightarrow F_2 = I - 2\frac{v_2 v_2^T}{v_2^T v_2} = \frac{1}{2+\sqrt{2}} \begin{bmatrix} -1-\sqrt{2} & 1+\sqrt{2} \\ 1+\sqrt{2} & 1+\sqrt{2} \end{bmatrix} = \begin{bmatrix} \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}.$$

$$\Rightarrow Q_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \Rightarrow R = \begin{bmatrix} -1 & -1 \\ 0 & \sqrt{2}\epsilon \\ 0 & 0 \end{bmatrix},$$

$$Q = (Q_2Q_1)^T = \begin{bmatrix} -1 & \frac{\sqrt{2}\epsilon}{2} & \frac{-\sqrt{2}\epsilon}{2} \\ -\epsilon & \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}.$$

$$\Rightarrow A = QR = \begin{bmatrix} -1 & \frac{\sqrt{2}\epsilon}{2} & \frac{-\sqrt{2}\epsilon}{2} \\ -\epsilon & \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & \sqrt{2}\epsilon \\ 0 & 0 \end{bmatrix}.$$

(d)

$$\text{CGS\&MGS: } \left\|\hat{Q}^T\hat{Q} - I\right\|_F = \left\| \begin{bmatrix} 1 & \epsilon & 0 \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \epsilon & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} - I \right\|_F = \left\| \begin{bmatrix} 0 & \frac{-\epsilon}{\sqrt{2}} \\ \frac{-\epsilon}{\sqrt{2}} & 0 \end{bmatrix} \right\|_F = \epsilon.$$

$$\text{Householder: } \left\|Q^TQ - I\right\|_F = \left\| \begin{bmatrix} -1 & -\epsilon & 0 \\ \frac{\sqrt{2}\epsilon}{2} & \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}\epsilon}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} -1 & \frac{\sqrt{2}\epsilon}{2} & \frac{-\sqrt{2}\epsilon}{2} \\ -\epsilon & \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} - I \right\|_F = 0.$$

Householder triangularizaiton does not have loss of orthogonality while CGS and MGS do.

Problem 3

(a)

$$F \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} x_m \\ \vdots \\ x_1 \end{bmatrix} \Rightarrow F = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & & \iddots & 0 \\ 0 & \iddots & & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}.$$

$$EEx = E^2x = \frac{E(x+Fx)}{2} = \frac{Ex + EFx}{2} = \frac{\frac{x+Fx}{2} + \frac{F(x+Fx)}{2}}{2} = \frac{1}{4}(x+Fx+Fx+FFx) = \frac{1}{4}(2x+2Fx) = \frac{x+Fx}{2} = Ex.$$

$$\Rightarrow E^2 = E.$$

$\Rightarrow$ E is a projector.

$$(Ex)^T = x^TE^T = \frac{(x+Fx)^T}{2} \Rightarrow E^T = \frac{I+F}{2} \Rightarrow E^Tx = \frac{x+Fx}{2} = Ex.$$

$$\Rightarrow E^T = E.$$

E is an orthogonal projector.

(b)

$$E = \frac{I+F}{2}.$$

In the case that m is even,

$$E = \frac{1}{2} \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 & 1 \\ 0 & \ddots & & & \iddots & 0 \\ \vdots & & 1 & 1 & & \vdots \\ \vdots & & 1 & 1 & & \vdots \\ 0 & \iddots & & & \ddots & 0 \\ 1 & 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}.$$

In the case that m is odd,

$$E = \frac{1}{2}\begin{bmatrix} 1 & 0 & \cdots & 0 & 1 \\ 0 & \ddots & & \iddots & 0 \\ \vdots & & 2 & & \vdots \\ 0 & \iddots & & \ddots & 0 \\ 1 & 0 & \cdots & 0 & 1 \end{bmatrix}.$$

Problem 4

(a) $A =$

$$\begin{pmatrix}
1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.0 & 0.0204 & 4.16\,10^{-4} & 8.5\,10^{-6} & 1.73\,10^{-7} & 3.54\,10^{-9} & 7.22\,10^{-11} & 1.47\,10^{-12} & 3.01\,10^{-14} & 6.14\,10^{-16} & 1.25\,10^{-17} & 2.56\,10^{-19} \\
1.0 & 0.0408 & 0.00167 & 6.8\,10^{-5} & 2.78\,10^{-6} & 1.13\,10^{-7} & 4.62\,10^{-9} & 1.89\,10^{-10} & 7.7\,10^{-12} & 3.14\,10^{-13} & 1.28\,10^{-14} & 5.24\,10^{-16} \\
1.0 & 0.0612 & 0.00375 & 2.29\,10^{-4} & 1.41\,10^{-5} & 8.6\,10^{-7} & 5.27\,10^{-8} & 3.22\,10^{-9} & 1.97\,10^{-10} & 1.21\,10^{-11} & 7.4\,10^{-13} & 4.53\,10^{-14} \\
1.0 & 0.0816 & 0.00666 & 5.44\,10^{-4} & 4.44\,10^{-5} & 3.63\,10^{-6} & 2.96\,10^{-7} & 2.42\,10^{-8} & 1.97\,10^{-9} & 1.61\,10^{-10} & 1.31\,10^{-11} & 1.07\,10^{-12} \\
1.0 & 0.102 & 0.0104 & 0.00106 & 1.08\,10^{-4} & 1.11\,10^{-5} & 1.13\,10^{-6} & 1.15\,10^{-7} & 1.18\,10^{-8} & 1.2\,10^{-9} & 1.22\,10^{-10} & 1.25\,10^{-11} \\
1.0 & 0.122 & 0.015 & 0.00184 & 2.25\,10^{-4} & 2.75\,10^{-5} & 3.37\,10^{-6} & 4.13\,10^{-7} & 5.05\,10^{-8} & 6.19\,10^{-9} & 7.58\,10^{-10} & 9.28\,10^{-11} \\
1.0 & 0.143 & 0.0204 & 0.00292 & 4.16\,10^{-4} & 5.95\,10^{-5} & 8.5\,10^{-6} & 1.21\,10^{-6} & 1.73\,10^{-7} & 2.48\,10^{-8} & 3.54\,10^{-9} & 5.06\,10^{-10} \\
1.0 & 0.163 & 0.0267 & 0.00435 & 7.11\,10^{-4} & 1.16\,10^{-4} & 1.89\,10^{-5} & 3.09\,10^{-6} & 5.05\,10^{-7} & 8.24\,10^{-8} & 1.35\,10^{-8} & 2.2\,10^{-9} \\
1.0 & 0.184 & 0.0337 & 0.0062 & 0.00114 & 2.09\,10^{-4} & 3.84\,10^{-5} & 7.05\,10^{-6} & 1.3\,10^{-6} & 2.38\,10^{-7} & 4.37\,10^{-8} & 8.03\,10^{-9} \\
1.0 & 0.204 & 0.0416 & 0.0085 & 0.00173 & 3.54\,10^{-4} & 7.22\,10^{-5} & 1.47\,10^{-5} & 3.01\,10^{-6} & 6.14\,10^{-7} & 1.25\,10^{-7} & 2.56\,10^{-8} \\
1.0 & 0.224 & 0.0504 & 0.0113 & 0.00254 & 5.7\,10^{-4} & 1.28\,10^{-4} & 2.87\,10^{-5} & 6.45\,10^{-6} & 1.45\,10^{-6} & 3.25\,10^{-7} & 7.3\,10^{-8} \\
1.0 & 0.245 & 0.06 & 0.0147 & 0.0036 & 8.81\,10^{-4} & 2.16\,10^{-4} & 5.28\,10^{-5} & 1.29\,10^{-5} & 3.17\,10^{-6} & 7.76\,10^{-7} & 1.9\,10^{-7} \\
1.0 & 0.265 & 0.0704 & 0.0187 & 0.00495 & 0.00131 & 3.49\,10^{-4} & 9.25\,10^{-5} & 2.45\,10^{-5} & 6.51\,10^{-6} & 1.73\,10^{-6} & 4.58\,10^{-7} \\
1.0 & 0.286 & 0.0816 & 0.0233 & 0.00666 & 0.0019 & 5.44\,10^{-4} & 1.55\,10^{-4} & 4.44\,10^{-5} & 1.27\,10^{-5} & 3.63\,10^{-6} & 1.04\,10^{-6} \\
1.0 & 0.306 & 0.0937 & 0.0287 & 0.00878 & 0.00269 & 8.23\,10^{-4} & 2.52\,10^{-4} & 7.71\,10^{-5} & 2.36\,10^{-5} & 7.23\,10^{-6} & 2.21\,10^{-6} \\
1.0 & 0.327 & 0.107 & 0.0348 & 0.0114 & 0.00371 & 0.00121 & 3.96\,10^{-4} & 1.29\,10^{-4} & 4.22\,10^{-5} & 1.38\,10^{-5} & 4.5\,10^{-6} \\
1.0 & 0.347 & 0.12 & 0.0418 & 0.0145 & 0.00503 & 0.00174 & 6.05\,10^{-4} & 2.1\,10^{-4} & 7.28\,10^{-5} & 2.53\,10^{-5} & 8.77\,10^{-6} \\
1.0 & 0.367 & 0.135 & 0.0496 & 0.0182 & 0.00669 & 0.00246 & 9.03\,10^{-4} & 3.32\,10^{-4} & 1.22\,10^{-4} & 4.47\,10^{-5} & 1.64\,10^{-5} \\
1.0 & 0.388 & 0.15 & 0.0583 & 0.0226 & 0.00877 & 0.0034 & 0.00132 & 5.11\,10^{-4} & 1.98\,10^{-4} & 7.68\,10^{-5} & 2.98\,10^{-5} \\
1.0 & 0.408 & 0.167 & 0.068 & 0.0278 & 0.0113 & 0.00462 & 0.00189 & 7.7\,10^{-4} & 3.14\,10^{-4} & 1.28\,10^{-4} & 5.24\,10^{-5} \\
1.0 & 0.429 & 0.184 & 0.0787 & 0.0337 & 0.0145 & 0.0062 & 0.00266 & 0.00114 & 4.88\,10^{-4} & 2.09\,10^{-4} & 8.96\,10^{-5} \\
1.0 & 0.449 & 0.202 & 0.0905 & 0.0406 & 0.0182 & 0.00819 & 0.00368 & 0.00165 & 7.41\,10^{-4} & 3.33\,10^{-4} & 1.49\,10^{-4} \\
1.0 & 0.469 & 0.22 & 0.103 & 0.0485 & 0.0228 & 0.0107 & 0.00502 & 0.00236 & 0.00111 & 5.19\,10^{-4} & 2.44\,10^{-4} \\
1.0 & 0.49 & 0.24 & 0.118 & 0.0576 & 0.0282 & 0.0138 & 0.00676 & 0.00331 & 0.00162 & 7.95\,10^{-4} & 3.89\,10^{-4} \\
1.0 & 0.51 & 0.26 & 0.133 & 0.0678 & 0.0346 & 0.0176 & 0.009 & 0.00459 & 0.00234 & 0.0012 & 6.1\,10^{-4} \\
1.0 & 0.531 & 0.282 & 0.149 & 0.0793 & 0.0421 & 0.0223 & 0.0118 & 0.00628 & 0.00333 & 0.00177 & 9.39\,10^{-4} \\
1.0 & 0.551 & 0.304 & 0.167 & 0.0922 & 0.0508 & 0.028 & 0.0154 & 0.0085 & 0.00468 & 0.00258 & 0.00142 \\
1.0 & 0.571 & 0.327 & 0.187 & 0.107 & 0.0609 & 0.0348 & 0.0199 & 0.0114 & 0.0065 & 0.00371 & 0.00212 \\
1.0 & 0.592 & 0.35 & 0.207 & 0.123 & 0.0726 & 0.043 & 0.0254 & 0.0151 & 0.00891 & 0.00527 & 0.00312 \\
1.0 & 0.612 & 0.375 & 0.229 & 0.141 & 0.086 & 0.0527 & 0.0322 & 0.0197 & 0.0121 & 0.0074 & 0.00453 \\
1.0 & 0.633 & 0.4 & 0.253 & 0.16 & 0.101 & 0.0641 & 0.0406 & 0.0257 & 0.0162 & 0.0103 & 0.0065 \\
1.0 & 0.653 & 0.426 & 0.279 & 0.182 & 0.119 & 0.0776 & 0.0507 & 0.0331 & 0.0216 & 0.0141 & 0.00921 \\
1.0 & 0.673 & 0.454 & 0.305 & 0.206 & 0.139 & 0.0933 & 0.0628 & 0.0423 & 0.0285 & 0.0192 & 0.0129 \\
1.0 & 0.694 & 0.481 & 0.334 & 0.232 & 0.161 & 0.112 & 0.0774 & 0.0537 & 0.0373 & 0.0259 & 0.018 \\
1.0 & 0.714 & 0.51 & 0.364 & 0.26 & 0.186 & 0.133 & 0.0949 & 0.0678 & 0.0484 & 0.0346 & 0.0247 \\
1.0 & 0.735 & 0.54 & 0.397 & 0.291 & 0.214 & 0.157 & 0.116 & 0.0849 & 0.0624 & 0.0458 & 0.0337 \\
1.0 & 0.755 & 0.57 & 0.431 & 0.325 & 0.245 & 0.185 & 0.14 & 0.106 & 0.0798 & 0.0603 & 0.0455 \\
1.0 & 0.776 & 0.601 & 0.466 & 0.362 & 0.281 & 0.218 & 0.169 & 0.131 & 0.101 & 0.0787 & 0.061 \\
1.0 & 0.796 & 0.633 & 0.504 & 0.401 & 0.319 & 0.254 & 0.202 & 0.161 & 0.128 & 0.102 & 0.0812 \\
1.0 & 0.816 & 0.666 & 0.544 & 0.444 & 0.363 & 0.296 & 0.242 & 0.197 & 0.161 & 0.131 & 0.107 \\
1.0 & 0.837 & 0.7 & 0.586 & 0.49 & 0.41 & 0.343 & 0.287 & 0.24 & 0.201 & 0.168 & 0.141 \\
1.0 & 0.857 & 0.735 & 0.63 & 0.54 & 0.463 & 0.397 & 0.34 & 0.291 & 0.25 & 0.214 & 0.183 \\
1.0 & 0.878 & 0.77 & 0.676 & 0.593 & 0.52 & 0.457 & 0.401 & 0.352 & 0.309 & 0.271 & 0.238 \\
1.0 & 0.898 & 0.806 & 0.724 & 0.65 & 0.584 & 0.524 & 0.471 & 0.423 & 0.38 & 0.341 & 0.306 \\
1.0 & 0.918 & 0.843 & 0.775 & 0.711 & 0.653 & 0.6 & 0.551 & 0.506 & 0.465 & 0.427 & 0.392 \\
1.0 & 0.939 & 0.881 & 0.827 & 0.777 & 0.729 & 0.684 & 0.643 & 0.603 & 0.566 & 0.532 & 0.499 \\
1.0 & 0.959 & 0.92 & 0.882 & 0.846 & 0.812 & 0.779 & 0.747 & 0.716 & 0.687 & 0.659 & 0.632 \\
1.0 & 0.98 & 0.96 & 0.94 & 0.921 & 0.902 & 0.884 & 0.866 & 0.848 & 0.831 & 0.814 & 0.797 \\
1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0
\end{pmatrix}$$

$(A^TA)^{-1} =$

$$\begin{pmatrix}
0.944 & -59.8 & 1300.0 & -1.39\,10^{4} & 8.59\,10^{4} & -3.31\,10^{5} & 8.31\,10^{5} & -1.38\,10^{6} & 1.51\,10^{6} & -1.04\,10^{6} & 4.09\,10^{5} & -7.03\,10^{4} \\
-59.8 & 8677.0 & -2.51\,10^{5} & 3.14\,10^{6} & -2.13\,10^{7} & 8.78\,10^{7} & -2.32\,10^{8} & 4.0\,10^{8} & -4.49\,10^{8} & 3.17\,10^{8} & -1.28\,10^{8} & 2.23\,10^{7} \\
1300.0 & -2.51\,10^{5} & 8.02\,10^{6} & -1.06\,10^{8} & 7.47\,10^{8} & -3.16\,10^{9} & 8.5\,10^{9} & -1.49\,10^{10} & 1.7\,10^{10} & -1.21\,10^{10} & 4.91\,10^{9} & -8.65\,10^{8} \\
-1.39\,10^{4} & 3.14\,10^{6} & -1.06\,10^{8} & 1.45\,10^{9} & -1.05\,10^{10} & 4.51\,10^{10} & -1.23\,10^{11} & 2.18\,10^{11} & -2.51\,10^{11} & 1.8\,10^{11} & -7.37\,10^{10} & 1.31\,10^{10} \\
8.59\,10^{4} & -2.13\,10^{7} & 7.47\,10^{8} & -1.05\,10^{10} & 7.7\,10^{10} & -3.37\,10^{11} & 9.29\,10^{11} & -1.66\,10^{12} & 1.93\,10^{12} & -1.39\,10^{12} & 5.73\,10^{11} & -1.02\,10^{11} \\
-3.31\,10^{5} & 8.78\,10^{7} & -3.16\,10^{9} & 4.51\,10^{10} & -3.37\,10^{11} & 1.49\,10^{12} & -4.14\,10^{12} & 7.47\,10^{12} & -8.71\,10^{12} & 6.34\,10^{12} & -2.62\,10^{12} & 4.68\,10^{11} \\
8.31\,10^{5} & -2.32\,10^{8} & 8.5\,10^{9} & -1.23\,10^{11} & 9.29\,10^{11} & -4.14\,10^{12} & 1.16\,10^{13} & -2.11\,10^{13} & 2.47\,10^{13} & -1.81\,10^{13} & 7.5\,10^{12} & -1.35\,10^{12} \\
-1.38\,10^{6} & 4.0\,10^{8} & -1.49\,10^{10} & 2.18\,10^{11} & -1.66\,10^{12} & 7.47\,10^{12} & -2.11\,10^{13} & 3.85\,10^{13} & -4.54\,10^{13} & 3.33\,10^{13} & -1.39\,10^{13} & 2.5\,10^{12} \\
1.51\,10^{6} & -4.49\,10^{8} & 1.7\,10^{10} & -2.51\,10^{11} & 1.93\,10^{12} & -8.71\,10^{12} & 2.47\,10^{13} & -4.54\,10^{13} & 5.37\,10^{13} & -3.96\,10^{13} & 1.65\,10^{13} & -2.98\,10^{12} \\
-1.04\,10^{6} & 3.17\,10^{8} & -1.21\,10^{10} & 1.8\,10^{11} & -1.39\,10^{12} & 6.34\,10^{12} & -1.81\,10^{13} & 3.33\,10^{13} & -3.96\,10^{13} & 2.93\,10^{13} & -1.23\,10^{13} & 2.22\,10^{12} \\
4.09\,10^{5} & -1.28\,10^{8} & 4.91\,10^{9} & -7.37\,10^{10} & 5.73\,10^{11} & -2.62\,10^{12} & 7.5\,10^{12} & -1.39\,10^{13} & 1.65\,10^{13} & -1.23\,10^{13} & 5.14\,10^{12} & -9.34\,10^{11} \\
-7.03\,10^{4} & 2.23\,10^{7} & -8.65\,10^{8} & 1.31\,10^{10} & -1.02\,10^{11} & 4.68\,10^{11} & -1.35\,10^{12} & 2.5\,10^{12} & -2.98\,10^{12} & 2.22\,10^{12} & -9.34\,10^{11} & 1.7\,10^{11}
\end{pmatrix}$$

$$x = \begin{bmatrix} 0.9933423296986708 \\ -0.008364150130270295 \\ -8.013302248819301 \\ 0.00200986234429084 \\ 10.65643945285954 \\ 0.03819355536709001 \\ -5.80195355819774 \\ 0.2002810836995605 \\ 1.387178827091387 \\ 0.2863476227457973 \\ -0.4511725901700139 \\ 0.1082040508358794 \end{bmatrix}$$

(b)

$$x = \begin{bmatrix} 0.9999980095417428 \\ 0.0005240487129328648 \\ -8.017781214722481 \\ 0.2317776652525206 \\ 9.133006608074849 \\ 5.824451075252701 \\ -19.12705214142045 \\ 19.11574452219357 \\ -14.6971553676225 \\ 7.621060716492418 \\ -1.771882814554435 \\ 0.0336602669258937 \end{bmatrix}$$

(c)

$$x = \begin{bmatrix} 0.9999999983863181 \\ 0.00000003612607411363169 \\ -8.00001090901282 \\ 0.0001209718519636858 \\ 10.66604887259279 \\ 0.001499873032841239 \\ -5.690652240820811 \\ 0.00470900127440044 \\ 1.598825773422054 \\ 0.07594885807110009 \\ -0.4034554134780703 \\ 0.09332123577732984 \end{bmatrix}$$

(d)

$$x = \begin{bmatrix} 0.9999999878508699 \\ 0.000003240106772508905 \\ -8.000088938990734 \\ 0.002073496357689155 \\ 10.65146262654028 \\ 0.05422419780233391 \\ -5.843045370682141 \\ 0.3148140300789312 \\ 1.213327954716196 \\ 0.2561167534590414 \\ -0.5100932056145835 \\ 0.1133117050781178 \end{bmatrix}$$

(e)

$$x = \begin{bmatrix} 1.000000000996608 \\ -0.000000422743334476907 \\ -7.999981235677272 \\ -0.0003187633315562077 \\ 10.6694307965398 \\ -0.01382029050032569 \\ -5.647075621036682 \\ -0.07531603476122548 \\ 1.693606974784582 \\ 0.006032101044458006 \\ -0.3742417004342318 \\ 0.08804057555741888 \end{bmatrix}$$

Householder traiangularization is the most stable or accurate method for QR decomposation. And based on observation, the normal equations exhibit very close to stable.