DUE: Friday, April 20, 2018 at 4:00 PM

- Read Chapters 1 & 2 in Eldén.

- Read Lecture Notes 01–04

- Given the prerequisites for this class - a strong knowledge of Linear Algebra - and the material outlined in the course syllabus, the following concepts are important for you to understand. We will discuss *some* of these concepts in my lectures, but do not wait for these lectures. If your memory is vague on these subjects, review them yourself now!

  – Solving linear systems

  – LU decomposition

  – Equivalences of non-singular matrices

  – Rank and span

  – Domain & Codomain

  – Domain, range

  – Column space and row space

  – Nullspace (Kernel) and Left Nullspace,

  – Linear combination

  – Basis, Change of basis

  – Orthogonal vectors, Gram-Schmidt orthogonalization

  – Inner product (dot product) and outer product

  – Eigenvalues, eigenvectors, diagonalization, eigenbasis

Problem 01   This is a simple MATLAB exercise.

   (a) Download the data file: HW_01.mat to your working directory, and load it into your MATLAB session by:

```
>> load HW_01;
```

Then, draw the signal x in the data file using the following commands:

```
>> figure(1);
>> stem(x); hold on; plot(x); grid;
```

Note that this signal x consists of only 8 points, i.e., a very short signal (vector).

   (b) In a different figure window, draw the 8 basis vectors stored as column vectors of the matrix U as follows:

```
>> figure(2);
>> for k=1:8
       subplot(8,1,k);
       stem(U(:,k)); axis([0 9 -0.5 0.5]); axis off; hold on;
   end
>> for k=1:8
       subplot(8,1,k);
       plot(U(:,k));
   end
```

You may need to see the details of these 8 plots by enlarging the window to a full screen. Print this figure and attach it to your HW submission.

(c) Compute the expansion coefficients (i.e., the weights of the linear combination) of x with respect to the basis vectors $U(:,1),\ldots,U(:,8)$ via

```
>> a=U'*x;
```

(d) Check the values of the entries of the coefficient vector a and create a new vector a_2 of length 8 whose only nonzero entries are the two largest entries of a in terms of their absolute values.

(e) Construct an approximation $x2$ of x using $a2$. Then, plot $x2$ over Figure 1 as follows:

```
>> figure(1); stem(x2,'r*'); plot(x2,'r');
```

(f) Now, instead of a_2, let's construct a_4 of length 8 whose only nonzero entries are the four largest entries of a in terms of their absolute values. Then,

(g) Construct an approximation x_4 of x using a_4. Then, plot x_4 over Figure 1 as follows (note using the different color from x_2):

```
>> figure(1); stem(x4,'gx'); plot(x4,'g');
```

Then, print out Figure 1, and attach it to your HW submission.

(h) Consider now x_8, which is just a full reconstruction without throwing out any coefficients, i.e.,

```
>> x8=U*a;
```

Finally, compute the relative error of x_8 by

```
>> sqrt(sum((x-x8).^2)/sum(x.^2))
```

and report the result. Similarly compute the relative error of x_4 and x_2, and report the results.

(i) Write a detailed explanation of what this MATLAB program does.

Problem 02 Consider the following set of terms (words) and documents (or rather book titles):

Terms		Documents	
T1:	Book (Handbook, BOOK)	D1:	The Princeton Companion to Mathematics
T2:	Equation (Equations)	D2:	NIST Handbook of Mathematical Functions
T3:	Function (Functions)	D3:	Table of Integrals, Series, and Products
T4:	Integral (Integrals)	D4:	Linear Integral Equations
T5:	Linear	D5:	Proofs from THE BOOK
T6:	Mathematics (Mathematical)	D6:	The Book of Numbers
T7:	Number (Numbers)	D7:	Number Theory in Science and Communication
T8:	Series	D8:	Green's Functions and Boundary Value Problems
		D9:	Discourse on Fourier Series
		D10:	Basic Linear Partial Differential Equations
		D11:	Mathematical Physics, An Advanced Course

(a) Construct 8×11 term-document matrix.

(b) Suppose we want to query "Integral Equation." Construct the query vector $\boldsymbol{q}$.

(c) Find three closest documents for the query in (b).

Problem 03 (This is a review problem.) At the beginning of 2009, the population of California was 36,453,973. The population living in the United States but outside of California was 271,491,582. During that year, 458,682 people moved to California from another state. Similarly, 545,921 people moved from California to elsewhere in the United States. Set up a matrix vector multiplication problem whose solution shows the population changes in California and in the rest of the United States for 2009.

Problem 04 (This is another review problem.) Use the Gram-Schmidt Process to construct an orthonormal basis for $\mathbf{R}^4$ starting with the following set of vectors.

(a)

$$\boldsymbol{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \qquad \boldsymbol{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \boldsymbol{x}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(b)

$$\boldsymbol{y}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \boldsymbol{y}_2 = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \boldsymbol{y}_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \qquad \boldsymbol{y}_4 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Problem 05 (This is also a review problem.) Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) View this matrix as a linear transformation, T, between two vector spaces. What are the domain and the codomain of T?

(b) Identify (i.e., define) the column space, C(A), (or image) of this linear transformation.

(c) Find a basis for the column space, C(A), of T.

(d) Identify the nullspace, N(A), (also known as the kernel) of T.

(e) Write a basis for the nullspace, N(A), of T.

(f) What is the row space of this linear transformation?

(g) What is the left null space $N(A^T)$, of T.

(h) What is the rank of the matrix A?

Problem 06 Convert the following numbers to from base X to base Y as specified below.

(a) Convert the following binary numbers to decimal:

   (i) 11111

   (ii) 1000000

   (iii) 1001101101

   (iv) 10101010

   (v) 000011110000

(b) Convert the following decimal numbers to binary:

   (i) 73

   (ii) 127

   (iii) 402

   (iv) 512

   (v) 1000

   (vi) 32767