MAT167: Homework 1
Hangshi Jin 913142686
E. G. Puckett
Due on April 23

Problem 1

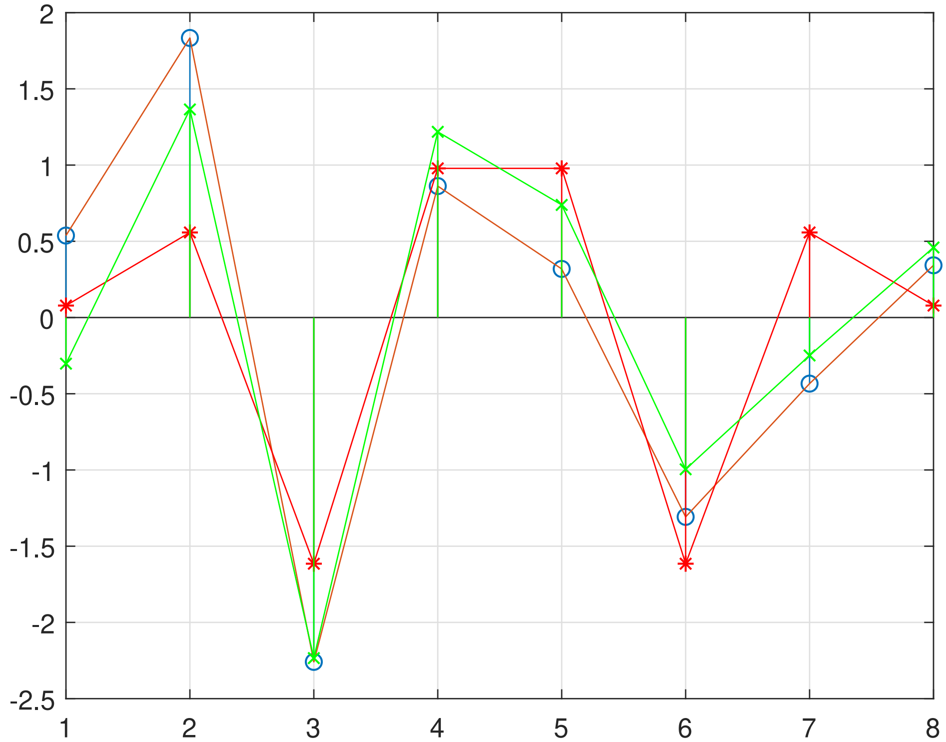

Figure 1: plot of x(o points), x2(* points), and x4(x points)

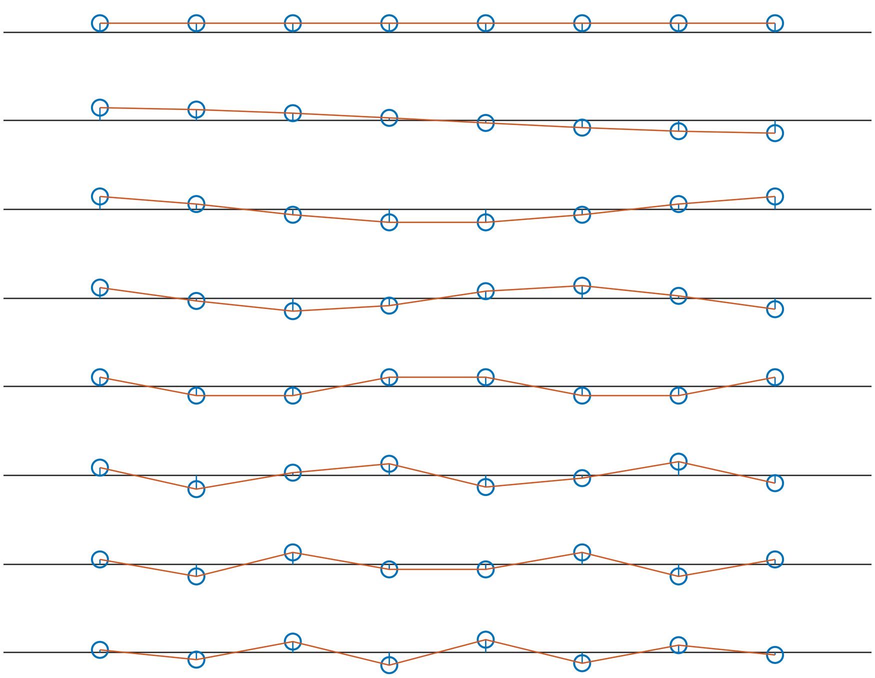

Figure 2: 8 basis vectors of U represented by their entry values

Hangshi Jin
913142686

(i)the program calculates the weights of the linear combination of the 8 basis vectors of U on x, and by using this program, we can decide which basis vectors catch the most important features of the graph of x, and then cut off the rest by setting the corresponding entries in a to 0. The decision will be made based on how much contribution each entry value does to the a in terms of its absolute value. On Figure 1, we can see how accurate the approximation has to be made in order to have a relatively similar shape to the graph of x. Numerically, we can set a threshold based on the value of the error of each approximation to decide how accurate we want the approximation to be.

Problem 2

(a)

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11
T1	0	1	0	0	1	1	0	0	0	0	0
T2	0	0	0	1	0	0	0	0	0	1	0
T3	0	1	0	0	0	0	0	1	0	0	0
T4	0	0	1	1	0	0	0	0	0	0	0
T5	0	0	0	1	0	0	0	0	0	1	0
T6	1	1	0	0	0	0	0	0	0	0	0
T7	0	0	0	0	0	1	1	0	0	0	1
T8	0	0	1	0	0	0	0	0	1	0	0

(b)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \vec{q} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(c)Three closest documents for q are D3,D4, and D10.

Problem 3

$P_{C\to U}$ =Population moved from California to elsewhere in the United States, $P_{U\to C}$ =Population moved to California from another state.

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} P_{C\to U} \\ P_{U\to C} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 545921 \\ 458682 \end{bmatrix} = \begin{bmatrix} 87239 \\ -87239 \end{bmatrix}$$

Hangshi Jin
913142686

Problem 4

(a)

Let $v_1 = x_1 = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}$, then

$$v_2 = x_2 - proj_{w_1} x_2 = \begin{bmatrix}0\\1\\0\\-1\end{bmatrix} - \frac{\begin{bmatrix}1\\0\\1\\0\end{bmatrix} \cdot \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}}{\left\|\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\right\|^2} \begin{bmatrix}1\\0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}$$

where w_1 is the space spanned by v_1.

$$v_3 = x_3 - proj_{w_2} x_3 = \begin{bmatrix}1\\0\\0\\1\end{bmatrix} - \frac{\begin{bmatrix}1\\0\\0\\1\end{bmatrix} \cdot \begin{bmatrix}1\\0\\1\\0\end{bmatrix}}{\left\|\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\right\|^2} \begin{bmatrix}1\\0\\1\\0\end{bmatrix} - \frac{\begin{bmatrix}1\\0\\0\\1\end{bmatrix} \cdot \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}}{\left\|\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}\right\|^2} \begin{bmatrix}0\\1\\0\\-1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\1\\-1\\1\end{bmatrix}$$

where w_2 is the space spanned by v_1 and v_2.

$$v_4 = x_4 - proj_{w_3} x_4 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} - \frac{\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}1\\0\\1\\0\end{bmatrix}}{\left\|\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\right\|^2} \begin{bmatrix}1\\0\\1\\0\end{bmatrix} - \frac{\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}}{\left\|\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}\right\|^2} \begin{bmatrix}0\\1\\0\\-1\end{bmatrix} - \frac{1}{4}\frac{\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}1\\1\\-1\\1\end{bmatrix}}{\left\|\frac{1}{2}\begin{bmatrix}1\\1\\-1\\1\end{bmatrix}\right\|^2} \begin{bmatrix}1\\1\\-1\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-1\\1\\1\\1\end{bmatrix}$$

where w_3 is the space spanned by v_1, v_2, and v_3.

$\Rightarrow$The orthogonal basis is $\left\{\begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}, \begin{bmatrix}1\\1\\-1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\1\\1\end{bmatrix}\right\}$. $\Rightarrow$The orthonormal basis is $\left\{\frac{\sqrt{2}}{2}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \frac{\sqrt{2}}{2}\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}, \frac{1}{2}\begin{bmatrix}1\\1\\-1\\1\end{bmatrix}, \frac{1}{2}\begin{bmatrix}-1\\1\\1\\1\end{bmatrix}\right.$

(b)

Let $u_1 = y_1 = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}$, then

$$u_2 = y_2 - proj_{z_1} y_2 = \begin{bmatrix}4\\1\\0\\0\end{bmatrix} - \frac{\begin{bmatrix}4\\1\\0\\0\end{bmatrix} \cdot \begin{bmatrix}1\\0\\0\\1\end{bmatrix}}{\left\| \begin{bmatrix}1\\0\\0\\1\end{bmatrix} \right\|^2} \begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \begin{bmatrix}2\\1\\0\\-2\end{bmatrix}$$

where z_1 is the space spanned by u_1.

$$u_3 = y_3 - proj_{z_2} y_3 = \begin{bmatrix}1\\0\\2\\1\end{bmatrix} - \frac{\begin{bmatrix}1\\0\\2\\1\end{bmatrix} \cdot \begin{bmatrix}1\\0\\0\\1\end{bmatrix}}{\left\| \begin{bmatrix}1\\0\\0\\1\end{bmatrix} \right\|^2} \begin{bmatrix}1\\0\\0\\1\end{bmatrix} - \frac{\begin{bmatrix}1\\0\\2\\1\end{bmatrix} \cdot \begin{bmatrix}2\\1\\0\\-2\end{bmatrix}}{\left\| \begin{bmatrix}2\\1\\0\\-2\end{bmatrix} \right\|^2} \begin{bmatrix}2\\1\\0\\-2\end{bmatrix} = \begin{bmatrix}0\\0\\2\\0\end{bmatrix}$$

where z_2 is the space spanned by u_1 and u_2.

$$u_4 = y_4 - proj_{z_3} y_4 = \begin{bmatrix}0\\2\\0\\1\end{bmatrix} - \frac{\begin{bmatrix}0\\2\\0\\1\end{bmatrix} \cdot \begin{bmatrix}1\\0\\0\\1\end{bmatrix}}{\left\| \begin{bmatrix}1\\0\\0\\1\end{bmatrix} \right\|^2} \begin{bmatrix}1\\0\\0\\1\end{bmatrix} - \frac{\begin{bmatrix}0\\2\\0\\1\end{bmatrix} \cdot \begin{bmatrix}2\\1\\0\\-2\end{bmatrix}}{\left\| \begin{bmatrix}2\\1\\0\\-2\end{bmatrix} \right\|^2} \begin{bmatrix}2\\1\\0\\-2\end{bmatrix} - \frac{\begin{bmatrix}0\\2\\0\\1\end{bmatrix} \cdot \begin{bmatrix}0\\0\\2\\0\end{bmatrix}}{\left\| \frac{1}{2} \begin{bmatrix}0\\0\\2\\0\end{bmatrix} \right\|^2} \begin{bmatrix}0\\0\\2\\0\end{bmatrix} = \frac{1}{2} \begin{bmatrix}-1\\4\\0\\1\end{bmatrix}$$

where z_3 is the space spanned by u_1, u_2, and u_3.

$\Rightarrow$The orthogonal basis is $\left\{ \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\0\\-2\end{bmatrix}, \begin{bmatrix}0\\0\\2\\0\end{bmatrix}, \begin{bmatrix}-1\\4\\0\\1\end{bmatrix} \right\}$.$\Rightarrow$The orthonormal basis is $\left\{ \frac{\sqrt{2}}{2} \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \frac{1}{3} \begin{bmatrix}2\\1\\0\\-2\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \frac{\sqrt{2}}{6} \begin{bmatrix}-1\\4\\0\\1\end{bmatrix} \right\}$.

Hangshi Jin
913142686

Problem 5

(a)Domain of T is R^4, and codomain of T is R^7.

(b)

$$C(A) = span\left\{ \begin{bmatrix}1\\0\\0\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}3\\5\\1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}4\\7\\6\\1\\0\\0\\0\end{bmatrix} \right\}$$

(c)

$$A = \begin{bmatrix}1&2&3&4\\0&1&5&7\\0&0&1&6\\0&0&0&1\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \xrightarrow{r_1-2r_2} \begin{bmatrix}1&0&-7&-10\\0&1&5&7\\0&0&1&6\\0&0&0&1\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \xrightarrow[r_2-5r_3]{r_1+7r_3} \begin{bmatrix}1&0&0&32\\0&1&0&-23\\0&0&1&6\\0&0&0&1\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \xrightarrow{\substack{r_1-32r_4\\ r_2+23r_4\\ r_3-6r_4}} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} = rref(A)$$

$$\Rightarrow B_{C(A)} = \left\{ \begin{bmatrix}1\\0\\0\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}3\\5\\1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}4\\7\\6\\1\\0\\0\\0\end{bmatrix} \right\}$$

(d)

$$N(A) = span \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \vec{0}$$

(e)

$$B_{N(A)} = \left\{\vec{0}\right\}$$

(f)

$$row(A) = span\left\{ \begin{bmatrix}1&2&3&4\end{bmatrix}, \begin{bmatrix}0&1&5&7\end{bmatrix}, \begin{bmatrix}0&0&1&6\end{bmatrix}, \begin{bmatrix}0&0&0&1\end{bmatrix} \right\}$$

Hangshi Jin
913142686

(g)

$$A^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 3 & 5 & 1 & 0 & 0 & 0 & 0 \\ 4 & 7 & 6 & 1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow N(A^T) = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(h)

$$rank(A) = 4$$

Problem 5

(a)(i)$11111 = (1 \cdot 2^4) + (1 \cdot 2^3) + (1 \cdot 2^2) + (1 \cdot 2^1) + (1 \cdot 2^0) = 31$

(ii)$1000000 = (1 \cdot 2^6) + (0 \cdot 2^5) + (0 \cdot 2^4) + (0 \cdot 2^3) + (0 \cdot 2^2) + (0 \cdot 2^1) + (0 \cdot 2^0) = 64$

(iii)$1001101101 = (1 \cdot 2^9) + (0 \cdot 2^8) + (0 \cdot 2^7) + (1 \cdot 2^6) + (1 \cdot 2^5) + (0 \cdot 2^4) + (1 \cdot 2^3) + (1 \cdot 2^2) + (0 \cdot 2^1) + (1 \cdot 2^0) = 621$

(iv)$10101010 = (1 \cdot 2^7) + (0 \cdot 2^6) + (1 \cdot 2^5) + (0 \cdot 2^4) + (1 \cdot 2^3) + (0 \cdot 2^2) + (1 \cdot 2^1) + (0 \cdot 2^0) = 170$

(v)$000011110000 = (0 \cdot 2^{11}) + (0 \cdot 2^{10}) + (0 \cdot 2^9) + (0 \cdot 2^8) + (1 \cdot 2^7) + (1 \cdot 2^6) + (1 \cdot 2^5)$

$+ (1 \cdot 2^4) + (0 \cdot 2^3) + (0 \cdot 2^2) + (0 \cdot 2^1) + (0 \cdot 2^0) = 240$

(b)(i)

Division by 2	Quotient	Remainder	Bit Number
73/2	36	1	0
36/2	18	0	1
18/2	9	0	2
9/2	4	1	3
4/2	2	0	4
2/2	1	0	5
1/2	0	1	6

$\Rightarrow$1001001

(b)(ii)

Division by 2	Quotient	Remainder	Bit Number
127/2	63	1	0
63/2	31	1	1
31/2	15	1	2
15/2	7	1	3
7/2	3	1	4
3/2	1	1	5
1/2	0	1	6

$\Rightarrow$1111111

Hangshi Jin
913142686

(b)(iii)

Division by 2	Quotient	Remainder	Bit Number
402/2	201	0	0
201/2	100	1	1
100/2	50	0	2
50/2	25	0	3
25/2	12	1	4
12/2	6	0	5
6/2	3	0	6
3/2	1	1	6
1/2	0	1	6

⇒110010010

(b)(iv)

Division by 2	Quotient	Remainder	Bit Number
512/2	256	0	0
256/2	128	0	1
128/2	64	0	2
64/2	32	0	3
32/2	16	0	4
16/2	8	0	5
8/2	4	0	6
4/2	2	0	6
2/2	1	0	6

⇒1000000000

(b)(v)

Division by 2	Quotient	Remainder	Bit Number
1000/2	500	0	0
500/2	250	0	1
250/2	125	0	2
125/2	62	1	3
62/2	31	0	4
31/2	15	1	5
15/2	7	1	6
7/2	3	1	6
3/2	1	1	6
1/2	0	1	7

⇒1111101000

Hangshi Jin
913142686

(b)(vi)

Division by 2	Quotient	Remainder	Bit Number
32767/2	16383	1	0
16383/2	8191	1	1
8191/2	4095	1	2
4095/2	2047	1	3
2047/2	1023	1	4
1023/2	511	1	5
511/2	255	1	6
255/2	127	1	6
127/2	63	1	6
63/2	31	1	6
31/2	15	1	6
15/2	7	1	6
7/2	3	1	6
3/2	1	1	6
1/2	0	1	6

$\Rightarrow$111111111111111