First of all, carefully reread Chapter 2, especially the sections on vector norms and operator norms.

Problem 01 Using MATLAB, do the following procedure.

(**a**) Download the data file

HW_02.mat

from Canvas to your working directory, and load it into your MATLAB session. Check what variables (i.e., arrays) are defined in this data file by running

```
>> whos
```

(**b**) Plot the data by typing

```
>> plot(x,y); grid;
```

(**c**) Create the Vandermonde matrix for polynomials of degree 1; (i.e., lines) by typing

```
>> A=[x.^0 x.^1];
```

(**d**) Compute the least squares line over the given data by typing

```
>> sol = inv(A'*A)*A'*y;
```

Then, overlay the least squares line over the current plot by typing

```
>> hold on; plot(x, sol(1)+sol(2)*x, '--');
```

Create the title and axis labels by typing

```
>> title('Least Squares Linear Fit'); xlabel('x'); ylabel('y');
```

Print out this plot and include a the PDF copy of the plot in you HW PDF file also with a *carefully written description* of how you obtained the plot and what it is.

(**e**) Finally, create a MATLAB file HW_02.m from the above commands with appropriate comments. Use the MATLAB listing in the Solutions to HW_01 as an example of good commenting practice.

(**f**) Write a detailed explanation of what this MATLAB program does and put it in your PDF file.

Problem 02 Prove that for a square matrix A, null(A) = $\{\mathbf{0}\}$ implies A is invertible.

Problem 03 Find the minimum value of $\|\mathbf{x}\|_1$ subject to $\|\mathbf{x}\|_2 = 1$ in $\mathbb{R}^2$. Which $\mathbf{x}$ achieves such minimum?
[Hint: set $\mathbf{x} = [\cos\theta, \sin\theta]^T$, $0 \le \theta \le 2\pi$.]

Problem 04 Let $\|\cdot\|$ denote any norm on $\mathbb{R}^m$ and also the induced matrix norm on $\mathbb{R}^{m\times m}$. Let $\rho(\mathrm{A})$ be the *spectral radius* of A; i.e., $\rho(\mathrm{A}) \stackrel{\text{def}}{=} \max_{1\le i\le m} |\lambda_i(\mathrm{A})|$, where $\lambda_i(\mathrm{A})$ is the ith eigenvalue of A. Prove $\rho(\mathrm{A}) \le \|\mathrm{A}\|$.

Problem 05 Let $\mathrm{A} = \boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}$ where $\boldsymbol{u} \in \mathbb{R}^m$ and $\boldsymbol{v} \in \mathbb{R}^n$. Prove $\|\mathrm{A}\|_2 = \|\boldsymbol{u}\|_2 \|\boldsymbol{v}\|_2$.

Problem 06 (**a**) Define the following matrix

$$\mathrm{A} = \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 1 & 3 \end{bmatrix},$$

in MATLAB. Then, compute the 2-norm by the `norm` function, and report the result in a long format (16 digits) via

```
>> format long
>> norm(A)
```

(**b**) Compute the 2-norm explicitly using the largest eigenvalue of $\mathrm{A}^{\mathrm{T}}\mathrm{A}$ using the `eig` function, i.e.,

```
>> sqrt(max(eig(A'*A)))
```

Then, compare the result with that of Part (a). What is the relative error between the norm computed in Part (a) and that in Part (b)?

(**c**) Compute the 1-norm, ∞-norm, and Frobenius norm of A by hand using the formulas derived in the class. Then, using the `norm` function, compare the MATLAB outputs with your hand-computed results. You should check how to use the `norm` function using the `help` utility:

```
>> help norm
```

(**d**)] Let's load the MATLAB data file

```
>> load HW_01.mat
```

that you used for HW 01 again. It's located on both Piazza and Canvas Then, compute first the coefficient vector by

```
>> a = U'*x;
```

Now, compute $\|\mathbf{x}\|_p$ and $\|\mathbf{a}\|_p$, $p = 1, 2, \infty$, using the `norm` function, and report the results. Which value of p, you got $\|\mathbf{x}\|_p = \|\mathbf{a}\|_p$?

(**e**) Now, compute the matrix norms, $\|\mathrm{U}\|_p$, $p = 1, 2, \infty$ as well as $\|\mathrm{U}\|_{\mathrm{F}}$ using the `norm` function, then report the results.

Problem 07 **Linear Least Squares:** You are meant to do this problem by hand calculation as you would on a test.

(**a**) Set up the *normal equation* for the linear least squares approximation for the data $(1, -1)$, $(2, 3)$, and $(3, 1)$.

(**b**) Solve for the least squares approximation from **Problem 07 (a)**.