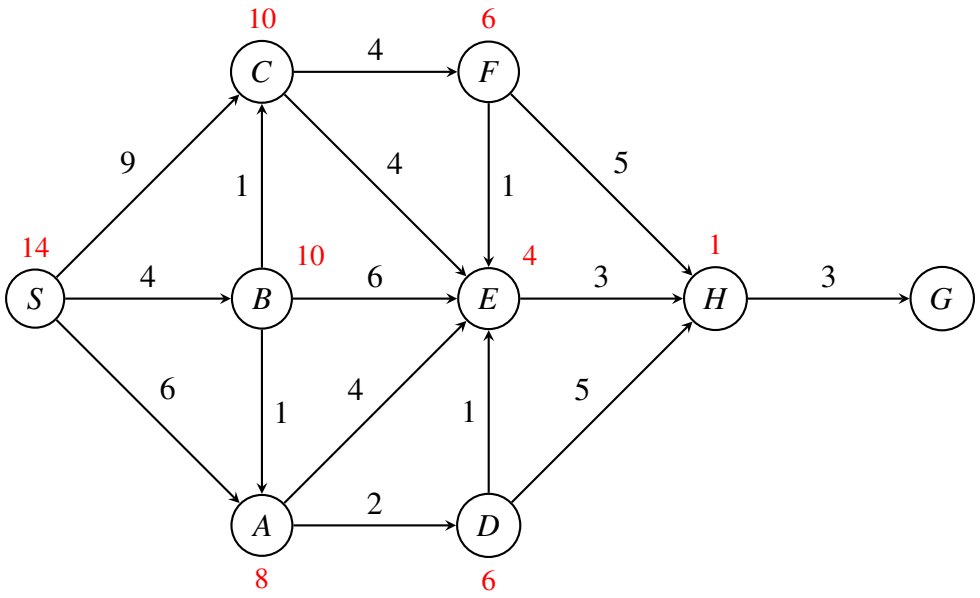
Introduction to AI

Assignment 2

March 29, 2023

1. For[1] the below graph (heuristic values are provided in red color and actual costs are in black color), please provide answers to the following answers:
   Please note that S is start and G is the goal state.



   (a) Is the heuristic admissible? Provide justification. Please fill in the intermediate table below to answer this question.

Node n	Min cost to reach G from n	$h(n)$
S	14	14
A	9	8
B	10	10
C	10	10
D	7	6
E	6	4
F	7	6
H	3	1

   Table 1: Comparison of min cost to goal and heuristic for every node

   Since $\forall n,\ h(n) \leq Mincost(n)$, the heuristic h is admissible.

   (b) Is the heuristic consistent? Provide justification.

   A heuristic is consistent if $\forall n$ and $\forall p,\ h(n) \leq c(n,p)+h(p)$. For the given graph, the heuristic h is not consistent. This can be proven by counter-example.

[1]Source code on https://github.com/jinhanloh2021/AI_AS2

Take node B and A. Given

$$\begin{aligned} h(B) &= 10 \\ c(B, A) &= 1 \\ h(A) &= 8 \end{aligned}$$

Then

$$\begin{aligned} h(B) &> c(B, A) + h(A) \\ 10 &> 8 + 1 = 9 \end{aligned}$$

Hence, h is not consistent.

(c) Provide the search steps (as discussed in class) with vanilla **Breadth First Search (BFS)**. Show the **final solution path** and the **cost of that solution** for each algorithm. Also compute the search steps for **A* search** with the following heuristic:

Specify for each algorithm if the open list is queue, stack or priority queue. As a simplifying assumption, let index zero (i.e first element) in the open list be the top of the stack or front of the (priority) queue, as appropriate for the corresponding algorithm. Break ties by alphabetical order.
Please use the following tables for your working. Open list contains nodes that are to be explored, and "nodes to add" are the successors of the node that is recently popped or dequeued.

Breadth First Search algorithm

Step #	Open Queue	Dequeue	Nodes to add
1	S	S	A, B, C
2	A^S, B^S, C^S	A	D, E
3	B^S, C^S, D^A, E^A	B	
4	C^S, D^A, E^A	C	F
5	D^A, E^A, F^C	D	H
6	E^A, F^C, H^D	E	
7	F^C, H^D	F	
8	H^D	H	G
9	G^H	G	

Table 2: BFS priority queue

Path taken is

$$S \rightarrow A \rightarrow D \rightarrow H \rightarrow G$$

$$cost = 6 + 2 + 5 + 3$$
$$= 16$$

A* Search algorithm

Let X_f^n be a node X, where n is the parent state and $f = g(X) + h(X)$ as defined by the algorithm.

Step #	Open Priority Queue	Dequeue	Nodes to add
1	S	S	A, B, C
2	$A_{14}^S, B_{14}^S, C_{19}^S$	A	D, E
3	$B_{14}^S, D_{14}^A, E_{14}^A, C_{19}^S$	B	A, C
4	$A_{13}^B, D_{14}^A, E_{14}^A, C_{15}^B$	A	D, E
5	$D_{13}^A, E_{13}^A, C_{15}^B$	D	E, H
6	$E_{12}^D, H_{13}^D, C_{15}^B$	E	H
7	H_{12}^E, C_{15}^B	H	G
8	G_{14}^H, C_{15}^B	G	

Table 3: A* search priority queue

Path taken is

$$S \rightarrow A \rightarrow D \rightarrow E \rightarrow H \rightarrow G$$

$$cost = 6 + 2 + 1 + 3 + 3$$
$$= 15$$

2. You are hired by a moving company to move N boxes from a corner of a room to the opposite corner of the room. The room can be roughly divided into four corners, and there is an indoor pond in the middle of the room preventing any direct diagonal crossing. The three boxes have different sizes and can only be stacked in such a way that larger boxes can never be placed on top of smaller ones. Please answer the following questions:

   (a) Given N boxes (stacked appropriately in corner A with the smallest box on the top), formulate this problem as a search problem. Indicate the

   - States (describe specifically what a state is in this problem, how you would store it in a computer using a data structure, and justify the correctness of your representation)
   - Actions
   - Cost of different actions
   - Successor state for each action and give one example with all of its successor states listed
   - The objective

   I made some assumptions for this problem:

   - I can only move one box at a time
   - I can only remove a box from the top of a stack
   - I can only add a box to the top of a stack
   - I cannot sort a stack in place
   - Each room can only contain one stack
   - All boxes have unique sizes
   - All stacks must be appropriately placed, smallest box at top largest at bottom

   With the rules of the game clear, we can define the state unambiguously.
   Let the set of rooms be $R = \{a, b, c, d\}$, a box B with size i be $b_i \in B = \{b_1, b_2, \ldots, b_n\}$, a stack G in room r be $g(r) \subseteq B$ and a box b_i in a stack $g(r)$ be $g(r, i) \in B$, the number of boxes be N, and the position of the mover M in the room r be $M = m_r$.
   We can then represent the start state as with $N = 3$ as

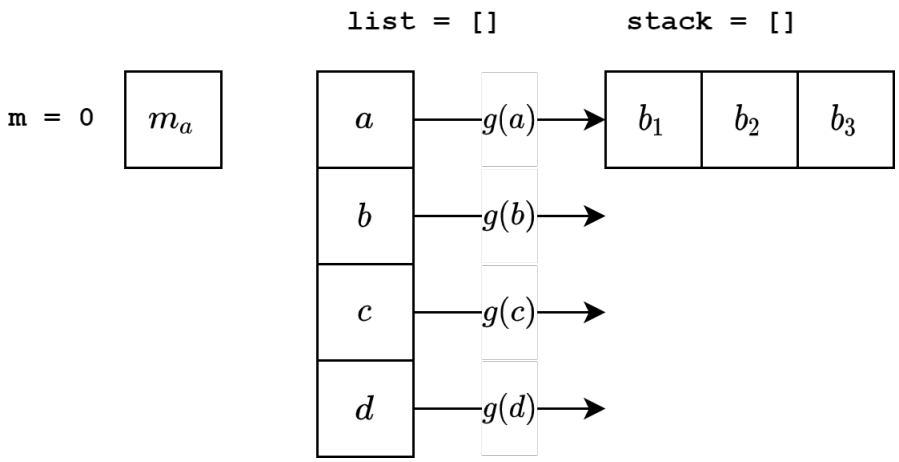
$$\begin{aligned} g(a) &= \{b_1, b_2, b_3\} \\ g(b) &= \{\emptyset\} \\ g(c) &= \{\emptyset\} \\ g(d) &= \{\emptyset\} \\ M &= m_a \end{aligned}$$

   And the goal state as the state where all boxes are in $g(d)$

$$\begin{aligned} g(a) &= \{\emptyset\} \\ g(b) &= \{\emptyset\} \\ g(c) &= \{\emptyset\} \\ g(d) &= \{b_1, b_2, b_3\} \\ M &= m_d \end{aligned}$$

   This represents 4 stacks $g(r)$ in each room $R = \{a, b, c, d\}$, with a set of boxes. Each stack must be ordered from smallest to largest. And M represents the position of the mover.

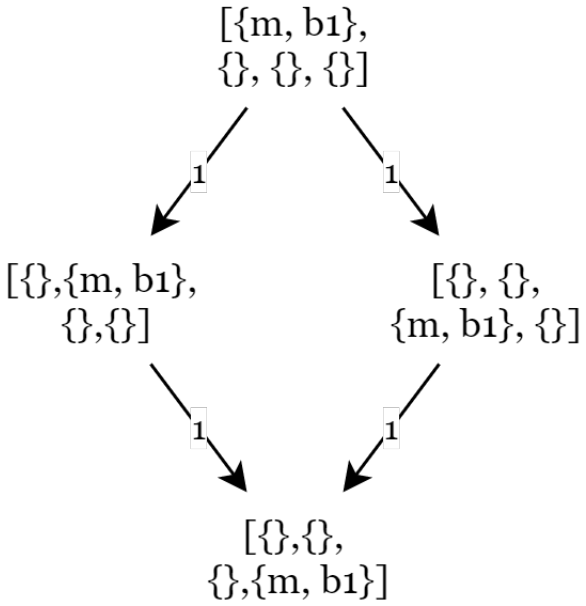
To represent the state as a data structure in Python, we can use a list containing 4 stacks. Each stack represents the stack of boxes in a room. We can assign each room to each stack respectively in increasing order. The stack is implemented as an integer list and each integer represents the size of a box. The mover can be represented as a separate integer variable containing the index of the stack he is in. Here is a diagram to illustrate this model.



This data structure is correct because it contains all possible positions of the boxes in every room. It also contains all possible rooms the mover can be in. It does not enforce the constraints such as the ordering or the diagonal constraint. These will be enforced by the methods we are allowed to perform on the data structure.
The base action in our system is to move a box from one room to an adjacent room, subject to the system rules. This action has a cost 1. We further simplify the actions by assuming that:

- The size of the box doesn't affect the cost
- Moving without a box has 0 cost
- Appending and popping from a stack (picking up and putting down a box) costs 0
- Mover must be in the same room as a box to pick up or put down a box

For example, for an action $C(a, b), C(b, d)$ represents popping a box from the top of the stack $g(a)$ and carrying it to room b, then carrying it to room d and appending it to the top of the stack $g(d)$. The popping and appending costs 0. The moving has a cost of 1 + 1 = 2 as we travelled from $a \rightarrow b$ and from $b \rightarrow d$. In the implementation, there must be checks to ensure that the ordering rule is respected when appending to a stack.
Here is an example of a state with $N = 1$ and all successor states.



This represents a start state where one box b_1 is in room a and the mover is also in room a. The possible actions from the start state are to move the box to rooms b or c, then to d.
We can see that we have reached the goal state by completing action in two steps as all the boxes are in the room d.

(b) For using heuristic search methods such as A*, provide an admissible heuristic and justify why it is admissible.

An admissible heuristic for some state n would be the number of boxes not in room d.

$$h(n) = N - |g(d)|$$

Given a base case where we have a single box in room b, the least cost path is 1, which is to take the box directly to $C(b, d)$. But for any other case where we have more than 1 box or if our box is located diagonally from the d, the cost would be greater as we cannot directly append the boxes to stack $g(d)$ as it will violate the ordering rule. We might also have to travel more than one room to reach d.
This will always result in a cost greater than the heuristic.Thus the heuristic admissible.

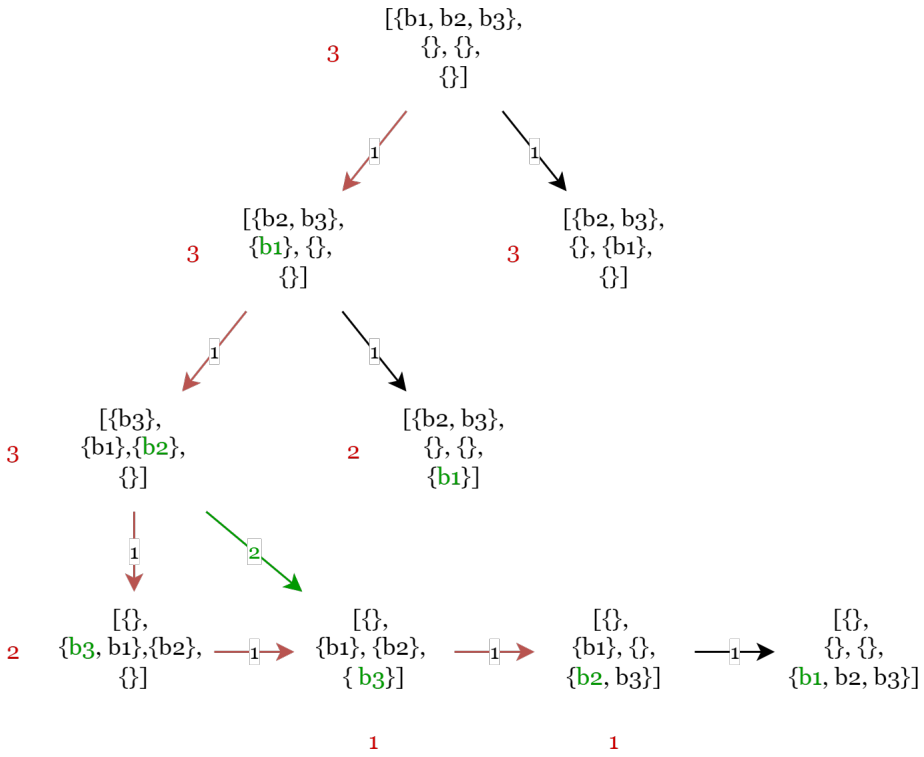
(c) For the case where $N = 3$, compute the first three steps of DFS search. You can use the table structure as in question 1c to show different steps.

Step #	Stacks
1	$[\{m, b_1, b_2, b_3\}, \{\}, \{\}, \{\}]$
2	$[\{b_2, b_3\}, \{m, b_1\}, \{\}, \{\}]$
3	$[\{m, b_2, b_3\}, \{b_1\}, \{\}, \{\}]$

Table 4: States of stacks for first three steps of DFS

(d) Give a solution to the problem for $N = 3$. You can achieve it either using some methods from the class or through guessing. I am ignoring the position of the mover m in the state here as

his movement without a box costs 0. To transition from a state $[\{b_2, b_3\}, \{m, b_1\}, \{\}, \{\}]$ to $[\{m, b_2, b_3\}, \{b_1\}, \{\}, \{\}]$ costs 0, so I will omit it for brevity's sake.



The solution is shown by the red arrows. The cost of each action is shown in the arrow as black text, and the red text represents the heuristic of each state. The green elements highlights the box that is currently held by the mover. There is a green arrow which represents a "shortcut" as the mover moves from a to d through c. Note that in $[\{\}, \{b_3, b_1\}, \{b_2\}, \{\}]$ the order rule appears to be violated as b_3 is above b_1. But since the box b_3 is being held by the mover in room b, it is not violated.

(e) How would the state representation change in part A if the following modifications are applied to the problem:

- The cost to move one box is now proportional to the size of the box (cost of 1 for moving the smallest box, cost of 3 for moving the largest one).
- If the moving company employee moves from one corner to the other without carrying any box, it also incurs a cost of 0.5

For my state representation, the box representation will still work as my boxes are labelled as b_i where i is the weight. Similarly, the position of the mover is still the same with the new rule that movement from one corner to another without a box will cost 0.5.

The changes can be seen in the **actions** rather than in the state. Now we can redefine an action to $C(a, b, b_i)$, where the mover moves a box b_i from room a to b. Previously the cost was 1 for any b_i but the new cost will be i.
Another change we have to make is to represent the position of the mover. Previously I ignored his position in our state representation as his movement cost without a box was 0. But now, we can have a new action $C(a, b, b_0)$ which costs 0.5, when a mover moves from room a to

room b without a box. Now we cannot ignore the position of the mover, and we require an additional variable in our state m to represent the room the mover is currently in.

3. Please recall the robot runner in the grid world example with tiger and food cells on right last column. The states are grid cells, i.e., $(1,1),(1,2),\ldots$. First component of a state is the row number, second component is the column number. The actions available are North, East, West, South. Rewards are given as follows:

   - $R(*,*,*) = -0.1$ (Penalty of movement, assuming result state is not food or tiger)
   - $R(*,*,(3,4)) = +2$ (Moving to food cell)
   - $R(*,*,(2,4)) = -1$ (Moving to tiger cell)

   Transition probability is 0.5 to the state in the direction of the action and 0.25 in states perpendicular to the direction of the action. If agent hits a wall, the agent moves back to its original location. Terminating states are when agent is in the food cell or in the tiger cell. Discount factor is 0.95. For value iteration method, the values of different states $V^t(.)$ at an iteration **t** are given by:

3	0.653	1.059	1.381	Food
2	0.400		0.434	Tiger
1	0.082	−0.11	−0.00	−0.35
	1	2	3	4

   (a) Compute the values of $V^{t+1}(.)$ for different states at iteration $t+1$. The V^{t+1} values for some states are given in the table below. You need to compute the values for states with "?" entry. Write numerical answers for such cells in the table below.

3	0.653	1.059	1.381	Food
2	0.400		0.434	Tiger
1	0.082	?	?	?
	1	2	3	4

   Please show analytical expressions denoting all computations without using any python code.

   For state $S = (1,3)$,

$$\begin{aligned} Q^{t+1}((1,3),N) &= \sum_{s'} P(s'|(1,3),N)[R((1,3),N,s') + \gamma V^t(s')] \\ &= .5(-.1 + .95(.434)) + .25(-.1 + .95(-.11)) + .25(-.1 + .95(-.35)) \\ &= -0.00435 \\ Q^{t+1}((1,3),W) &= \sum_{s'} P(s'|(1,3),W)[R((1,3),W,s') + \gamma V^t(s')] \\ &= .5(-.1 + .95(-.11)) + .25(-.1 + .95(.434)) + .25(-.1 + .95(0)) \\ &= -0.0875 \end{aligned}$$

   Taking action S and E are trivial, as S is worse than N because the successful action is greater for N than S and the unsuccessful action are the same. By the same logic, action W is worse

than E. Hence,

$$V^{t+1}(1,3) = -0.00435$$

For state $S = (1,4)$,

$$\begin{aligned} Q^{t+1}((1,4),W) &= \sum_{s'} P(s'|(1,4),W)[R((1,4),W,s') + \gamma V^t(s')] \\ &= .5(-.1 + .95(0)) + .25(-1) + .25(-.1 + .95(-.35)) \\ &= -0.4275 \\ Q^{t+1}((1,4),S) &= \sum_{s'} P(s'|(1,4),S)[R((1,4),S,s') + \gamma V^t(s')] \\ &= .5(-.1 + .95(-.35)) + .25(-.1 + .95(0)) + .25(-.1 + .95(-.35)) \\ &= -0.2075 \end{aligned}$$

Taking action N and E are trivial, as N is the worst move as it goes to the Tiger. Since going W is worst than S, we can also conclude that E is worse than S as E has a chance of meeting the Tiger which automatically decreases our utility by 0.25. Hence,

$$V^{t+1}(1,4) = -0.2075$$

For state $S = (1,2)$,

$$\begin{aligned} Q^{t+1}((1,2),W) &= \sum_{s'} P(s'|(1,2),W)[R((1,2),W,s') + \gamma V^t(s')] \\ &= .5(-.1 + .95(0.082)) + .25(-.1 + .95(-.11)) + .25(-.1 + .95(-.11)) \\ &= -0.1236 \end{aligned}$$

Taking action W is the only action with a positive successful state. The other actions have negative successful states.

$$V^{t+1}(1,2) = -0.1236$$

Hence, at V^{t+1}

3				Food
2				Tiger
1		−0.1236	−0.00435	−0.2075
	1	2	3	4

(b) What is the policy for each state as per the function $Q^{t+1}(..)$. Note it down below for each entry with "?" mark.

3	E	E	E	Food
2	N		N	Tiger
1	N	W	N	S
	1	2	3	4

From the Q^{t+1} I've calculated in the previous part, the optimal policy is shown in the diagram above.

4. In this question, you will employ Singular Value Decomposition to obtain word embeddings and compare the generated word embeddings with the word embeddings generated using word2vec. The corpus to be considered is a set of tweets posted about the Covid- 19 pandemic on Twitter (`Corona_Tweets.csv`), which is a real-life dataset. You need to do the following (code template provided is `Q4_template_tweets.ipynb` you are free to use helper functions if required):

   (a) Update the `load_data` function from the `Q4_template_tweets.ipynb` to preprocess words: remove non-letters, convert words into the lower case, and remove the stop words. You can employ some functions from the `NLP-pipeline-example.ipynb` example and may use regular expressions from `Word2Vec.ipynb`. **[2 marks]**

```
def load_data():
  """ Read tweets from the file.
      Return:
          list of lists (list_words), with words from each of the
              processed tweets
  """
  tweets = pd.read_csv('/content/drive/MyDrive/Colab
      Notebooks/AI_AS2/Corona_Tweets.csv', names=['text'])
  list_words = []
  ### iterate over all tweets from the dataset
  for i in tweets.index:
    ### remove URLs
    text = re.sub("https?://\S+|www\.\S+", " ", tweets.loc[i, 'text'])
    ### remove non-letter.
    text = re.sub("[^a-zA-Z]"," ",text)
    ### tokenize
    words = text.split()

    new_words = []
    ### iterate over all words of a tweet
    for w in words:
      ## TODO: remove the stop words and convert a word (w) to the lower
          case
      stops = set(stopwords.words("english"))
      if w not in stops:
        new_words.append(w.lower())

    list_words.append(new_words)
  return list_words
```

   (b) Create the co-occurrence matrix for all the remaining words (after stop words are eliminated), where the window of co-occurrence is 5 on either side of the word. What is the size of your vocabulary (i.e., how many unique words you end up with)? **[4 marks]**

```
def distinct_words(corpus):
  """ get a list of distinct words for the corpus.
      Params:
          corpus (list of list of strings): corpus of documents
      Return:
```

```
            corpus_words (list of strings): list of distinct words across
                the corpus, sorted (using python 'sorted' function)
            num_corpus_words (integer): number of distinct words across the
                corpus
    """
    corpus_words = set()
    for tweet in corpus:
      for word in tweet:
        corpus_words.add(word)
    corpus_words = sorted(list(corpus_words))
    num_corpus_words = len(corpus_words)
    return corpus_words, num_corpus_words

words, num_words = distinct_words(twitter_corpus) # 11454 unique words
```

```
def compute_co_occurrence_matrix(corpus, window_size=5):
    """ Compute co-occurrence matrix for the given corpus and window_size
        (default of 5).
        Params:
            corpus (list of list of strings): corpus of documents
            window_size (int): size of context window
        Return:
            M (numpy matrix of shape = [number of corpus words x number of
                corpus words]):
                Co-occurence matrix of word counts.
                The ordering of the words in the rows/columns should be the
                    same as the ordering of the words given by the
                    distinct_words function.
            word2Ind (dict): dictionary that maps word to index (i.e.
                row/column number) for matrix M.
    """
    M = np.zeros((num_words, num_words), dtype=int)
    word2Ind = {}
    for i, w in enumerate(words):
      word2Ind[w] = i
    for tweet in corpus:
      for i, w in enumerate(tweet):
        w_idx = word2Ind[w]
        start = i - 5
        end = i + 5 + 1 #exclusive
        for j in range(start, end):
          if(i != j and j >= 0 and j < len(tweet)):
            c_idx = word2Ind[tweet[j]]
            M[w_idx, c_idx] += 1
            M[c_idx, w_idx] += 1
    return M, word2Ind

M, word2Ind = compute_co_occurrence_matrix(twitter_corpus)
```

The number of unique vocabulary is 11454.

(c) Apply SVD and obtain word embeddings of size 75. **[2 marks]**

```
# ------------------------------
# Run SVD
# Note: This may take several minutes (~20-30 minutes)
# ------------------------------
la = np.linalg
U, s, Vh = la.svd(M, full_matrices=False)

# Compute SVD embeddings
embedding_size = 75
SVD_embeddings = np.dot(U[:,:embedding_size], np.diag(s[:embedding_size]))
```

(d) Then, please generate word embeddings of size 75 using `Word2Vec.ipynb` (uploaded in class lecture material) on the same dataset. Please show comparison on few examples to understand which method works better. You may use the `svd.most_similar` function from the template to perform the comparisons. Note your observations in your solution. You can use words like "covid", "grocery" etc to compare the two models. **[2 marks]**

```
# Creating the word2vec model and setting values for the various
    parameters
# Initializing the train model.
num_features = 75 # Word vector dimensionality
min_word_count = 0 # Minimum word count. You can change it also.
num_workers = 4  # Number of parallel threads, can be changed
context = 5      # Context window size
downsampling = 1e-3 # (0.001) Downsample setting for frequent words, can
    be changed
# Initializing the train model
print("Training Word2Vec model....")
model = word2vec.Word2Vec(twitter_corpus,
                          workers=num_workers,
                          vector_size=num_features, # API Change to
                              vector_size
                          min_count=min_word_count,
                          window=context,
                          sample=downsampling)

# To make the model memory efficient
model.init_sims(replace=True)
```

```
def svd_most_similar(query_word, n=10):
    """ return 'n' most similar words of a query word using the SVD word
        embeddings similar to word2vec's most_similar
        Params:
            query_word (strings): a query word
        Return:
            most_similar (list of strings): the list of 'n' most similar
                words
    """
    # get index of a query_word
```

```
    query_word_idx = word2Ind[query_word]
    # get word embedding for a query_word
    word = SVD_embeddings[query_word_idx]
    #cosine similarity matrix
    cos_similarity = cosine_similarity(SVD_embeddings, word.reshape(1, -1))
    most_similar = []
    # model.wv.most_similar(query_word)
    '''
        Write additional code to compute the list most_similar. Each entry in the list is a tuple (w, cos)
        where w is one of the most similar word to query_word and cos is cosine similarity of w with query_word
    '''
    # get index of top n most similar words
    similar_i = np.argsort(-cos_similarity.flatten())[1:n+1]

    # get similar words and cos_sim score
    for i in similar_i:
      word = list(word2Ind.keys())[i]
      cos_sim = cos_similarity[i][0]
      most_similar.append((word, cos_sim))

    # sort decreasing based on second item in tuple
    most_similar.sort(key=lambda x: x[1], reverse=True)

    return most_similar
```

```
svd_most_similar("covid")
model.wv.most_similar("covid") #this word2vec trained model on tweets
svd_most_similar("grocery")
model.wv.most_similar("grocery")
```

	covid		**grocery**	
Rank	SVD	word2vec	SVD	word2vec
1	outbreak	panicbuying	mailing	went
2	pandemic	coronaoutbreak	mall	shelves
3	new	coronavirus	liquor	empty
4	check	pandemic	ht	local
5	fear	coronavirusoutbreak	accusations	today
6	due	lockdown	elys	no
7	toiletpaper	corona	llama	retail
8	change	coronapocalypse	dollargeneral	bread
9	probably	uk	pajama	packs
10	news	due	quarterly	pasta

Table 5: Similar words for covid and grocery for each model