Methodology

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NA handling

For research direction 2, whenever NA is simulated in Y_t due to large λ in $rpois$, we disgard it together with corresponding X_t.

Laplace implementation

In (todo:equation), we aim to have the $\hat{\theta}$ that maximize $l_{Lap}(\theta|Y)$. In addition, in (todo:equation), we need the value of X_θ for $\hat{\theta}$, this X_θ and θ have to be optimized simultanously; we use Newton-Raphson method to in Laplace approximation to achieve this.

Also, we compare the results of our self-implementated Laplace approximation with TMB built-in Laplace approximation. Despite of similar accuracy as demostrated in Appendix, the built-in Laplace implementatnion is much faster than ours. Thus, we decide to work with the built-in implementation.

Simulation for research direction 1

Simulation for research direction 2

To start, we simulate from OU process. We perform the experiment 4 times for a fixed set of $\beta_0, \beta_1, \gamma, \mu$. 200 $X_t's$ and $Y_t's$ without NA are generated in each experiment. We observe that for small values of σ_{OU}, for example $0.00000001, 0.001, 0.01, 0.1, 1$, AIC picks Brownian Motion model after paramter inferences are performed. The threshold for AIC to pick the correct model for σ_{OU} lies in somewhere between 1 and $\sqrt{2}$.

Similary, simulate from Brownian Motion process. We perform the experiment 4 times for a fixed set of β_0, β_1. 200 $X_t's$ and $Y_t's$ without NA are generated in each experiment. We observe that for small values of σ_{BM}, for example 0.00000001, AIC picks OU model after paramter inferences are performed. The threshold for AIC to pick the correct model for σ_{BM} lies in somewhere between 0.00000001 and 0.001.

The above observations match with our intuition on this research direction. As $\sigma_{OU} \to 0, \tau \to 0$, (todo:equation) becomes $X_{t+\Delta t}|X_t \sim N(\mu + \omega_\Delta t(X_t - \mu))$, which has the same form as (todo: equation) with a scalar $\omega_\Delta t$ and a shift $\mu - \omega_\Delta t\mu$ applied to X_t. Thus it is X_t generated from (todo:equation) can also be interpreted as generated from (todo:equation). Similarily, As $\sigma_{BM} \to 0$, (todo:equation) becomes $X_{t+\Delta t}|X_t \sim N(X_t, 0)$, which has the same from as (todo:equation) with $\tau \to 0$ or $\gamma \to 1$. Thus it is X_t generated from (todo:equation) can also be interpreted as generated from (todo:equation).