Homework 3, ME 498/599, WI 2017

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Problem 2:

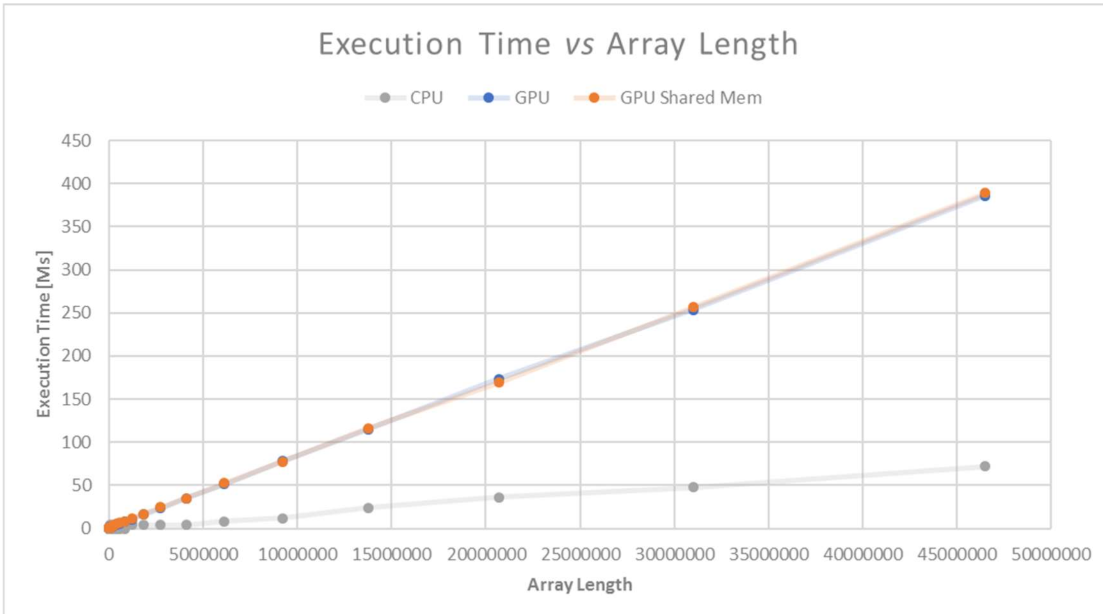

Figure 2.1. Execution Time versus Array Length.

Problem 3:

d. i.

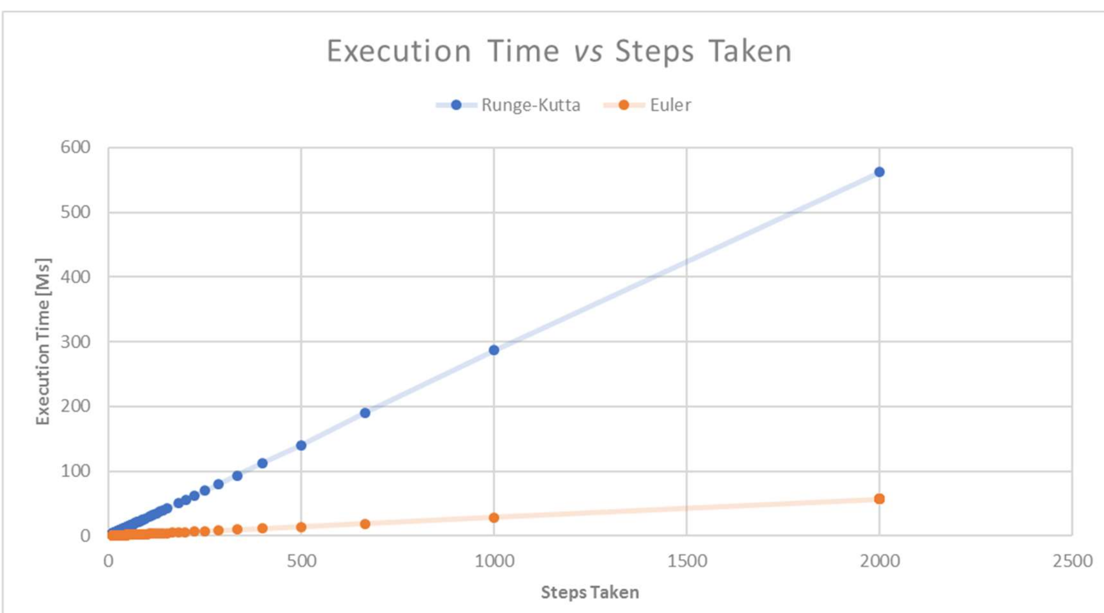

Figure 3.1. Execution Time versus Steps Taken.

d. ii.

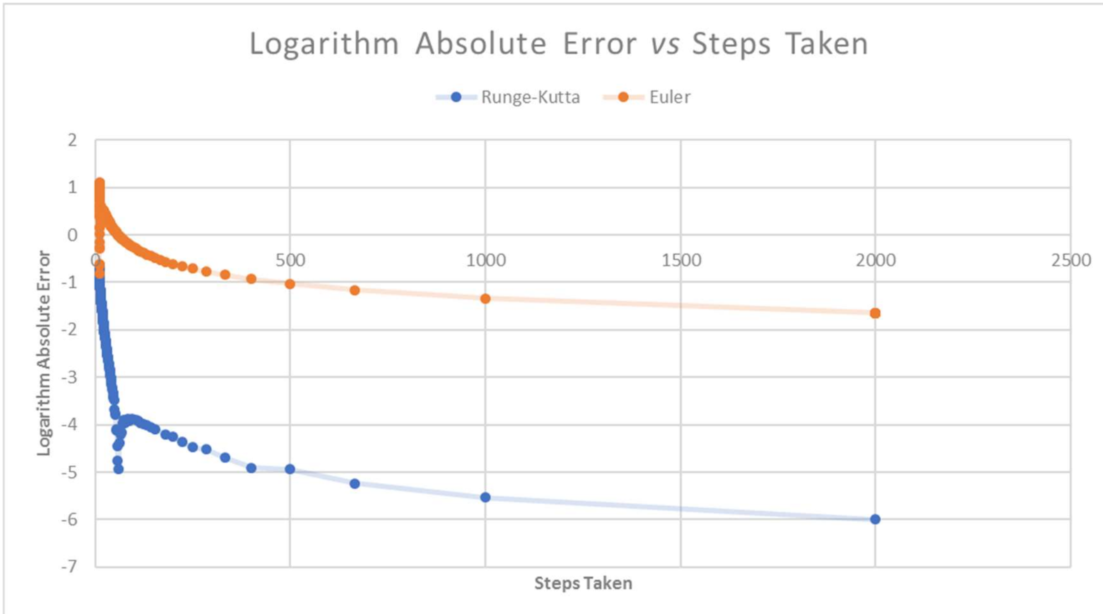

Figure 3.2. Logarithm Absolute Error versus Steps Taken.

d.iii.

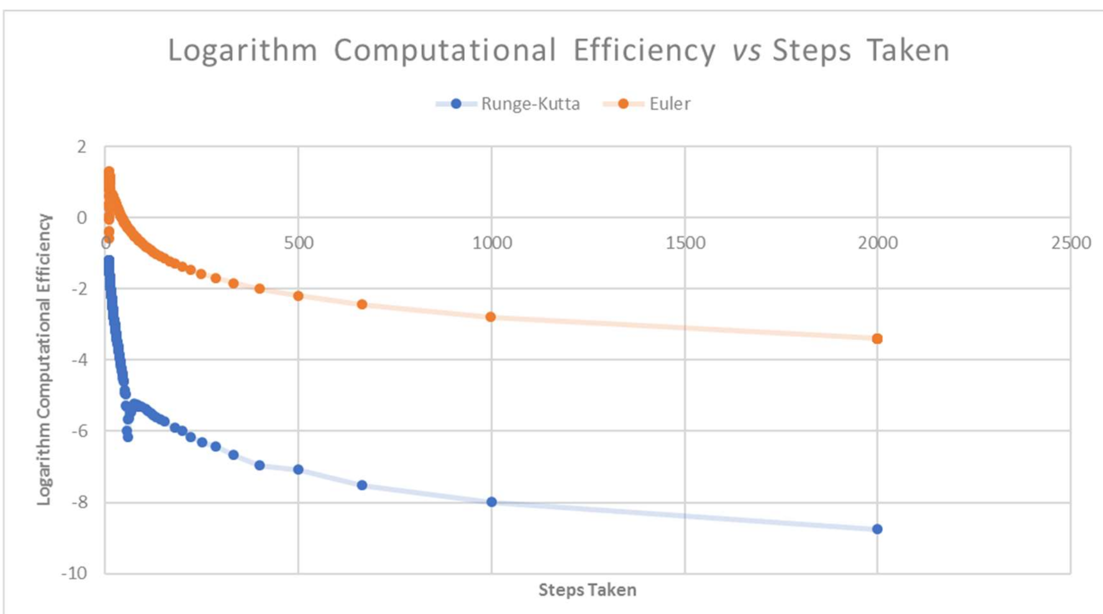

Figure 3.3. Logarithm Computational Efficiency versus Steps Taken.

e. For the absolute error smaller than 10%, the comparison between the Euler's method and the Runge-Kutta's method is as shown in **Table 3.1**.

Table 3.1. Comparison Between Runge-Kutta and Euler's Methods.

	Error	Time Step	Kernel Time [Ms]
Runge-Kutta	9.00%	0.830	3.854
Euler	22.70%	0.005	57.03

Problem 4:

a. The estimation of Newton's method is as shown in **Figure 4.1**.

According to the figure, we can estimate the expression of the line $f'(x_k)$ is

$$f'(x_k) = \frac{f(x_k) - 0}{x_k - x_{k+1}}$$

Thus,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

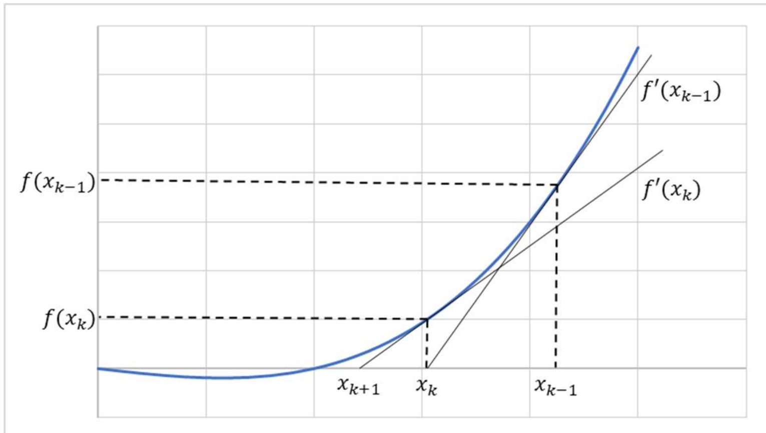

Figure 4.1. Estimation of Newton's Method.

Running a program for two iterations using Newton's method for $f(x) = x^3 - x$ gives

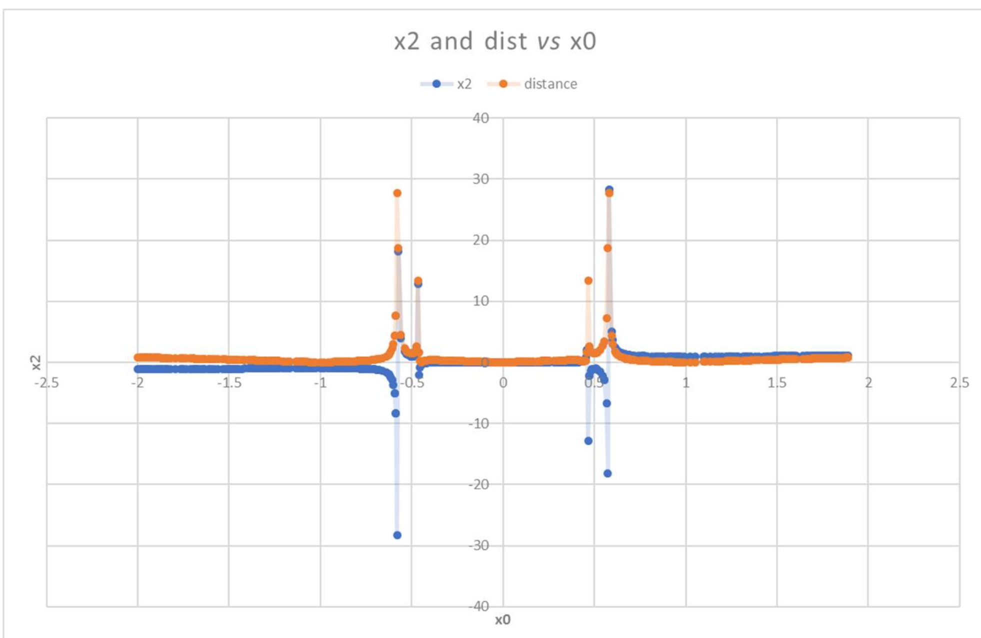

Figure 4.2. Two Iteration of Newton's Method for 1D Function.

Based on the figure, the points near 0 and ± 0.7 have iterated x_2 close to x_0. However, at the points near ± 0.6, the $f'(x_k)$ or $f'(x_{k+1})$ are zero and therefore, the estimated points are infinity. Thus, there are two poles where the distance between x_2 and x_0 are theoretically infinity.

b. The app developed for a complex function $f(z) = z^3 - 1 = 0$ is as shown in **Figure 4.3**.

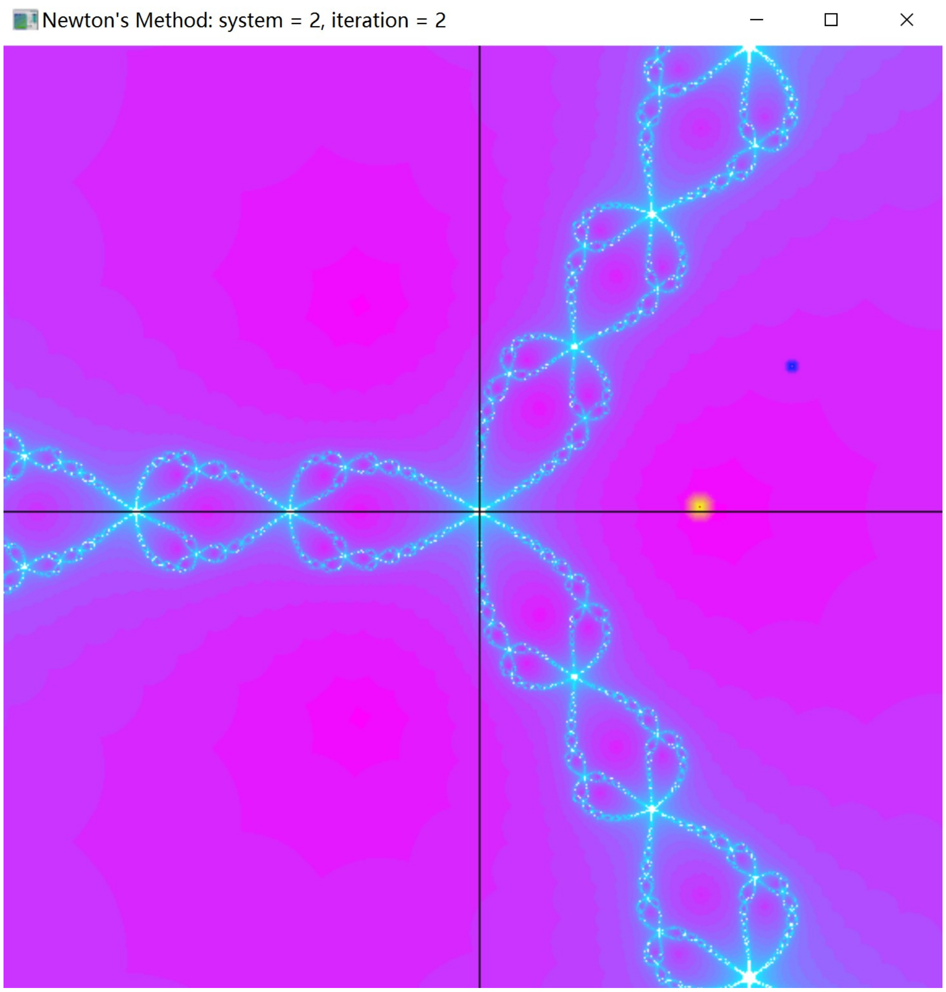

Figure 4.3. App for Newton's Method for the Complex Function.

In the figure, the white-blue color refers to more iteration and pink refers to less iteration required to get to the roods. The dark blue point shows the position of mouse and the yellow point shows the two-times-iterated result with Newton's method.

According to the figure, three roots (colored with light pink) lie on the circle with radius of 1 and the angle between each two roods is $120°$. Correspondingly, there are three asymptotic axes (shown in white-blue) on which all points around the axes will never get to the roots.

Though the UI of the app "newton" is not beautiful, you may still play with it according to the instruction displayed in the command window.