CREATE WITH MATH

Worksheet Solutions

CREATE WITH

MATH

Foundation 8

Learning Outcome:
This lesson explores properties of exponents, use of the properties to generate equivalent expressions, and solving problems using relations.
8.EE.A.1

Your basic ideas of exponents helped you to bring success to Prochips.
Now practice some more challenges on exponents.

1 Find the missing numbers:

$\frac{3}{4}, \frac{5}{4}, \square, \frac{9}{4}, \square, \frac{13}{4}$

2 Arrange the given expressions in the ascending order: $-\frac{3^9}{3^6}$, $3^5 \times 3^{-5}$, $-\frac{3^3}{3^6}$, 3^7

3 How long would sunlight take to reach Earth, if Earth is 15×10^7 km away from the Sun and light travels through space at the speed of 3×10^5 km/sec? [Time = Distance/Speed]

4 Match the expressions given on the left side with its simplest form on the right.

$\frac{5^8}{5^{12}}$	5^{10}
$5^4 \times 5^6$	10^8
5^0	5^{-4}
$2^8 \times 5^8$	1

5 Find the area of a playground, whose length is 2^7 yards and width is 2^5 yards.

6 Find the simplest form of the expression $\frac{(m+n)^2}{(m^2-n^2)}$.

- [ ] $\frac{(m-n)}{(m+n)}$
- [ ] $\frac{(m+n)}{(m-n)}$
- [ ] $(m+n)(m-n)$
- [ ] $(m+n) + (m-n)$

Foundation

1 Find the missing numbers:

6

$\frac{3}{4}$, $\frac{5}{4}$, $\boxed{}$, $\frac{9}{4}$, $\boxed{}$, $\frac{13}{4}$

Answer: 7/4, 11/4

Given:
A series of fraction numbers in a series

Solution:
The denominators of all the fractions given are the same. Therefore, we can extend the same to the missing numbers too.
All denominators would hence be 4.

Moving to the numerators, we can see that the numerator increases by 2 in each fraction. Therefore, the numerators of the missing numbers are (5 + 2) = 7 and (9 + 2) = 11 respectively.

The missing fraction numbers are 7/4 and 11/4.

2 Arrange the given expressions in the ascending order: $\frac{3^9}{3^6}$, $3^5 \times 3^{-5}$, $\frac{3^3}{3^6}$, 3^7

Answer: $3^3/3^6 < 3^5 \times 3^{-5} < 3^9/3^6 < 3^7$

Given:

$3^9/3^6$, $3^5 \times 3^{-5}$, $3^3/3^6$, and 3^7

Solution:

- $3^9/3^6 = 3^{9-6} = 3^3$ (Since $a^m/a^n = a^{m-n}$)
- $3^5 \times 3^{-5} = 3^{5+(-5)} = 3^0 = 1$ (Since $a^m \times a^n = a^{m+n}$)
- $3^3/3^6 = 3^{(3-6)} = 3^{-3}$ (Since $a^m/a^n = a^{m-n}$)
- 3^7

Since all the expressions have the same base of 3, we can compare them by comparing their powers. We can hence write that

$-3 < 0 < 3 < 7$

Therefore, the ascending order of the expressions is:

$3^3/3^6$, $3^5 \times 3^{-5}$, $3^9/3^6$, and 3^7

3 How long would sunlight take to reach Earth, if Earth is 15×10^7 km away from the Sun and light travels through space at the speed of 3×10^5 km/sec? [Time = Distance/Speed]

Answer: 500 seconds

Given:

The distance between earth and sun = 15×10^7 km

Speed of light = 3×10^5 km/sec

Solution:

Time = Distance/Speed

Therefore, the time taken by sunlight to reach earth=

15×10^7 km / 3×10^5 km/sec

$= 5 \times 10^{7-5}$ sec

$= 5 \times 10^2$ sec

$= 5 \times 100$ sec

$= 500$ sec

Therefore, sunlight takes 500 seconds to reach earth.

4 Match the expressions given on the left side with its simplest form on the right.

$\frac{5^8}{5^{12}}$ ○	○ 5^{10}
$5^4 \times 5^6$ ○	○ 10^8
5^0 ○	○ 5^{-4}
$2^8 \times 5^8$ ○	○ 1

$5^8/5^{12}$	5^{10}
$5^4 \times 5^6$	10^8
5^0	5^{-4}
$2^8 \times 5^8$	1

Given:
Expressions:

Solution:

- $5^8/5^{12} = 5^{8-12} = 5^{-4}$
  (Since $a^m/a^n = a^{m-n}$)

- $5^4 \times 5^6 = 5^{4+6} = 5^{10}$
  (Since $a^m \times a^n = a^{m+n}$)

- $5^0 = 1$

- $2^8 \times 5^8 = (2 \times 5)^8 = 10^8$
  (Since $a^m \times b^m = (a \times b)^m$)

5 Find the area of a playground, whose length is 2^7 yards and width is 2^5 yards.

Answer: 2^{12} sq. yards

Given:
Length of a playground = 2^7 yards
Width of the playground = 2^5 yards

Solution:
Area = Length × Width
$= 2^7 \times 2^5$ sq. yards
$= 2^{7+5}$ sq. yards
$= 2^{12}$ sq. yards

Area of the playground is 2^{12} sq. yards

6 Find the simplest form of the expression $\frac{(m+n)^2}{(m^2-n^2)}$.

- [ ] $\frac{(m-n)}{(m+n)}$
- [ ] $\frac{(m+n)}{(m-n)}$
- [ ] $(m+n)(m-n)$
- [ ] $(m+n) + (m-n)$

Given:

Expression: $\frac{(m+n)^2}{(m^2-n^2)}$

Solution:

We know that $m^2 - n^2 = (m + n)(m - n)$

Hence the expression reduces to

$$\frac{(m+n)(m+n)}{(m+n)(m-n)}$$

$$= \frac{(m+n)}{(m-n)}$$

Answer: $\frac{(m+n)}{(m-n)}$

Application

CREATE WITH MATH

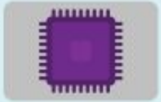
Application 8

Learning Outcome:
This lesson explores properties of exponents, use of the properties to generate equivalent expressions, and solving problems using relations.
8.EE.A.1

You have become an expert in using exponents. Your contribution to Prochips was awesome. With that wide knowledge, explore more on exponents by answering the following questions:

1. A submarine begins its descent from the surface of the water and it dives 20 feet for each minute. What would be the depth of the submarine after $\frac{2}{3}$ hours?

2. After research, it was decided that the weight of the chip shouldn't exceed 0.4×10^7g. If they used x transistors, each weighing 0.002 g to meet the requirement, find the maximum value of x.



3. A chip that can house 10^7 transistors per sq.mm is fabricated such that it covers an area of 1 sq cm. Find the total number of transistors that it'll have.

4. Match each value with the most appropriate measurement.

2.6×10^2 meters	Depth of bathtub
2.5×10^8 miles	Length of memory chip
1.6×10^1 inches	Distance between two asteroids
7.8×10^0 millimeters	Height of a skyscraper

5. While renovating the bathroom, it was observed that the amount of water that flowed from a faulty showerhead was 24^{-3} liters per second. If a person takes 20 min on an average to take a shower, how much water would be used during this time?

6. Chip A is stacked up with 2020^3 transistors. Chip B is stacked up with 2019^3 transistors. What would be the ratio of number in transistors in chip A to chip B?

1 A submarine begins its descent from the surface of the water and it dives 20 feet for each minute. What would be the depth of the submarine after $\frac{2}{3}$ hours?

Given:

Submarine dives 20 feet for each minute.

Time = $\frac{2}{3}$ hour

Solution:

Time = $\frac{2}{3}$ hour

= [$\frac{2}{3} \times 60$] minutes

= 40 minutes

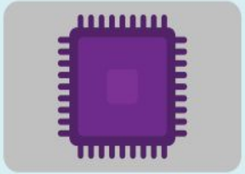
In 1 minute, the submarine dives = 20 feet

In 40 minutes, the submarine would dive 40 × 20 feet

= 800 feet

Answer: 800 feet

2 After research, it was decided that the weight of the chip shouldn't exceed 0.4 x 10^7g. If they used x transistors, each weighing 0.002 g to meet the requirement, find the maximum value of x.



Answer: 2×10^9

Given:

Maximum weight of a chip = 0.4×10^7 g

Weight of each transistors = 0.002 g

Number of transistors = x

Solution:

Number of transistors, x = Maximum weight of a chip/Weight of each transistor

$= 0.4 \times 10^7 / 0.002$

$= 4 \times 10^6 / 2 \times 10^{-3}$

$= 2 \times 10^{6 - (-3)}$

$= 2 \times 10^9$

Therefore, number of transistors, x = 2×10^9

3 A chip that can house 10^7 transistors per sq.mm is fabricated such that it covers an area of 1 sq cm. Find the total number of transistors that it'll have.

Given:

Number of transistors per sq. mm = 10^7

Maximum area = 1 sq. cm

Solution:

Maximum area = 1 sq. cm

= 1 cm × 1 cm

= 10 mm × 10 mm

= 100 sq. mm

Number of transistors = Number of transistors per sq. mm × Area

= $10^7 \times 100$

= $10^7 \times 10^2$

= 10^9

Total number of transistors = 10^9

Answer: 10^9 transistors

4 Match each value with the most appropriate measurement.

2.6×10^2 meters	Depth of bathtub
2.5×10^5 miles	Length of memory chip
1.6×10^1 inches	Distance between two asteroids
7.8×10^0 millimeters	Height of a skyscraper

2.6×10^2 meters	Depth of bathtub
2.5×10^5 miles	Length of memory chip
1.6×10^1 inches	Distance between two asteroids
7.8×10^0 millimeters	Height of skyscraper

Given:
Two columns of exponent numbers and lengths of a few things

Solution:
2.6×10^2 meters = 2.6×100 meters = 260 meters
Of the 4 given options, this is closest to the height of a skyscraper.

2.5×10^5 miles = 2.5×100000 miles
= 250000 miles
Of the remaining 3 options, this is closest to the distance between the two asteroids.

1.6×10^1 inches = 1.6×10 inches = 16 inches
Of the remaining 2 options, this is closest to the depth of a bathtub.

7.8×10^0 millimeters = 7.8×1 mm = 7.8 mm
This is the approx. length of a memory chip.

5 While renovating the bathroom, it was observed that the amount of water that flowed from a faulty showerhead was 24^{-1} liters per second. If a person takes 20 min on an average to take a shower, how much water would be used during this time?

Given:

Rate of flowing of water from shower = 24^{-1} liters per second

Time taken by a person to take a shower = 20 minutes

Solution:

Time taken by a person = 20 minutes

= 20 × 60 seconds

= 1200 seconds

Total amount of water = Water flow per second × Time that the shower is running for in seconds

= 24^{-1} liters per second × 1200 liters

= 1/24 × 1200 liters

= 50 liters

Answer: 50 liters

6 Chip A is stacked up with 2020^2 transistors. Chip B is stacked up with 2019^2 transistors. What would be the ratio of number in transistors in chip A to chip B?

Given:
Chip A consists of 2020^2 transistors.
Chip B consists of 2019^2 transistors.

Solution:
Chip A : Chip B = $2020^2 : 2019^2$
$= 2020^2/2019^2$
$= (2020/2019)^2$
{as $a^m/b^m = (a/b)^m$}

Answer: $(2020/2019)^2$

CREATE WITH

MATH

Create

GRADE 8

Learning Outcome:
This lesson explores properties of exponents, use of the properties to generate equivalent expressions, and solving problems using relations.
8.EE.A.1

We have sound coming from an external source up to the window of the room. At the time it reaches the window the level of the sound is about 20 dBA. We want to develop a soundproof window so that by the time the sound reaches inside the room it should be less than 2 dBA. When sound travels through air, its intensity reduces. The intensity is inversely proportional to the square of the distance from source.

For sound traveling through air, intensity $I = \frac{P \times 0.08}{r^2}$, where P is the original intensity of sound at source, I is the intensity of sound at the destination at a distance r from source.

We can use a sheet of wood, foam, and glass to develop the soundproof window. (Each has a different level of sound absorption.)

- Wood -> intensity $I_1 = \frac{P_1 \times 0.7}{r_1^2}$
- Foam -> intensity $I_2 = \frac{P_2 \times 0.05}{r_2^2}$
- Glass -> intensity $I_3 = \frac{P_3 \times 0.95}{r_3^2}$

You decide to have a sheet of wood, foam, and glass one after the other as shown below.

- P - Intensity outside and same as P_1
- Sound travels through wood, intensity reduces to I_1 - same as P_2
- Sound travels through foam, intensity reduces to I_2 - same as P_3
- Sound travels through glass, intensity reduces to I_3 - same as the intensity of sound inside the room

What should be the thickness (the value of r in each case in inches) of each layer made of wood, foam, and glass so that the level of the sound is reduced from 20 dBA to just 2 dBA?

Wood			Foam			Glass		
P_1	r_1	I_1	P_2	r_2	I_2	P_3	r_3	I_3
20								2

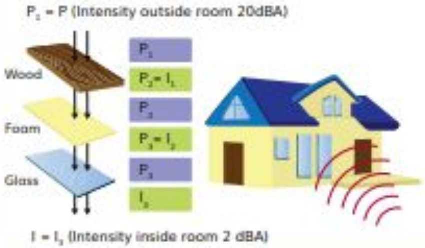
Create

We have sound coming from an external source up to the window of the room. At the time it reaches the window the level of the sound is about 20 dBA. We want to develop a soundproof window so that by the time the sound reaches inside the room it should be less than 2 dBA. When sound travels through air, its intensity reduces. The intensity is inversely proportional to the square of the distance from source.

For sound traveling through air, intensity $I = \frac{P \times 0.08}{r^2}$, where P is the original intensity of sound at source, I is the intensity of sound at the destination at a distance r from source.

We can use a sheet of wood, foam, and glass to develop the soundproof window. (Each has a different level of sound absorption.)

- Wood -> intensity $I_1 = \frac{P_1 \times 0.7}{r_1^2}$
- Foam -> intensity $I_2 = \frac{P_2 \times 0.05}{r_2^2}$
- Glass -> intensity $I_3 = \frac{P_3 \times 0.95}{r_3^2}$



- P - Intensity outside and same as P_1
- Sound travels through wood, intensity reduces to I_1 - same as P_2
- Sound travels through foam, intensity reduces to I_2 - same as P_3
- Sound travels through glass, intensity reduces to I_3 - same as the intensity of sound inside the room

Given:

Intensity in wood, $I_1 = (p_1 \times 0.7)/(r_1)^2$

Intensity in foam, $I_2 = (p_2 \times 0.05)/(r_2)^2$

Intensity in glass, $I_3 = (p_3 \times 0.95)/(r_3)^2$

P = Intensity outside and same as P_1= 20 dBA

Sound travels through wood, intensity reduces to I_1 = same as P_2

Sound travels through foam, intensity reduces to I_2 = same as P_3

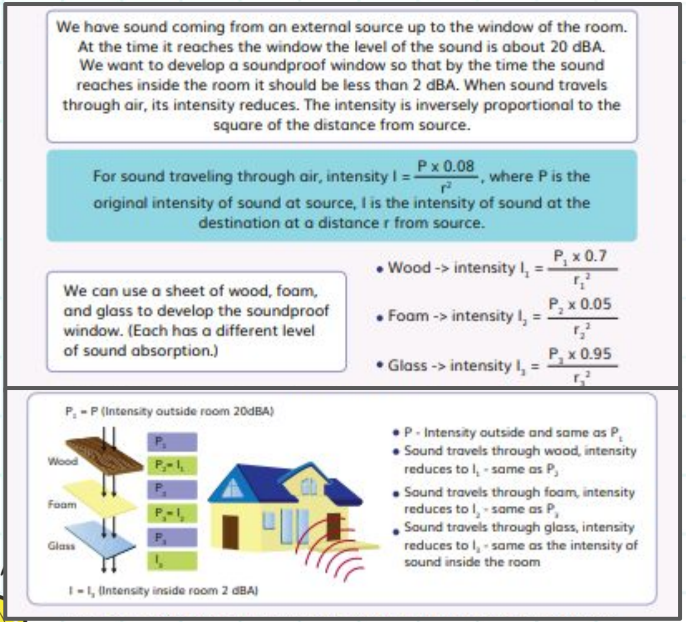
Sound travels through glass, intensity reduces to I_3 = same as the intensity of sound inside the room = 2 dBA

Solution:

$I_1 = (p_1 \times 0.7)/(r_1)^2$

$\Rightarrow I_1 = (20 \times 0.7)/(r_1)^2$

$\Rightarrow I_1 = 14/(r_1)^2$ ----------- 1



$I_2 = (P_2 \times 0.05)/r_2^2$

$\Rightarrow I_2 = (I_1 \times 0.05)/r_2^2$ {Since $P_2 = I_1$}

$\Rightarrow I_2 = (14 \times 0.05)/r_1^2 \times r_2^2$ {from 1}

$\Rightarrow I_2 = 0.7/r_1^2 \times r_2^2$ --------------- 2

$I_3 = (p_3 \times 0.95)/(r_3)^2$

$\Rightarrow I_3 = (I_2 \times 0.95)/ (r_3)^2$

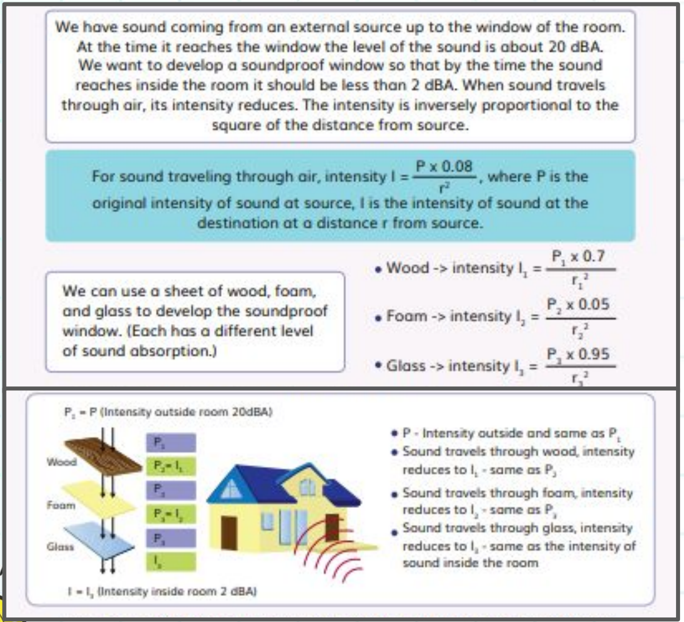
$\Rightarrow I_3 = (0.7 \times 0.95)/ (r_1)^2 \times (r_2)^2 \times (r_3)^2$ {from 2}

Since I_3 = 2 dB

$2 = 0.665/(r_1)^2 \times (r_2)^2 \times (r_3)^2$

$(r_1)^2 \times (r_2)^2 \times (r_3)^2 = 0.665/2 = 0.3325$

$r_1 \times r_2 \times r_3 = 0.57$ ------------- 3



We have sound coming from an external source up to the window of the room. At the time it reaches the window the level of the sound is about 20 dBA. We want to develop a soundproof window so that by the time the sound reaches inside the room it should be less than 2 dBA. When sound travels through air, its intensity reduces. The intensity is inversely proportional to the square of the distance from source.

For sound traveling through air, intensity $I = \frac{P \times 0.08}{r^2}$, where P is the original intensity of sound at source, I is the intensity of sound at the destination at a distance r from source.

We can use a sheet of wood, foam, and glass to develop the soundproof window. (Each has a different level of sound absorption.)

- Wood -> intensity $I_1 = \frac{P_1 \times 0.7}{r_1^2}$
- Foam -> intensity $I_2 = \frac{P_2 \times 0.05}{r_2^2}$
- Glass -> intensity $I_3 = \frac{P_3 \times 0.95}{r_3^2}$

We have,
$r_1 \times r_2 \times r_3 = 0.57$

One possible set of solutions:
Choose r_1 = 1 unit, r_2 = 0.5 units
Which gives us r_3 = 1.14 units.

Therefore,
$I_2 = 0.7 / (r_1)^2 \times (r_2)^2$ {from 2}
$I_2 = 0.7/(1)^2 \times (0.5)^2$
$I_2 = 2.8$

Since $I_2 = P_3 = 2.8$

$I_1 = 14/r_1^{\,2}$ {from 1}
$I_1 = 14/(1)^2$
$I_1 = 14$

Therefore, $I_1 = P_2 = 14$

Wood			Foam			Glass		
P_1	r_1	I_1	P_2	r_2	I_2	P_3	r_3	I_3
20	1	14	14	0.5	2.8	2.8	1.14	2