CS3230
Tutorial 10

1. Consider the following problem:

   > Input: Given a weighted graph G, two vertices u and v in G, and a value d.
   > Question: Is there a path from u to v of weight at most d?

   Is the above problem in NP? Could it be NP-complete?

2. In class we saw that it is open at present whether $P = NP$ or not. It is also open whether $NP = EXP$ or not. Is it possible that both $P = NP$ and $NP = EXP$ are true?

3. It can be shown that knapsack problem is NP-complete. Thus, if knapsack problem can be solved in polynomial time, then all problems in NP can be solved in polynomial time.

   Professor S claimed that he could solve the discrete knapsack problem in time proportional to $C * n$ (see the dynamic programming algorithm done in class), where C is the capacity of the knapsack and n is the number of objects in the problem. Thus the discrete knapsack problem is in P.

   Thus, Professor S claimed that he has shown P=NP. Could you find a flaw in his argument?

4. Show that testing whether a graph $G = (V, E)$ is a subgraph of graph $G' = (V', E')$ is in NP.

5. A coloring of a graph $G = (V, E)$ is assignment of colors to each vertex of a graph such that if (u, v) is an edge, then the color assigned to u and v are different. A graph is k-colorable, if one can color the graph (with above constraint) using k colors.

   Show that checking whether a graph G is k-colorable is in NP.

6. In class you were told the decision problem regarding satisfiability (SAT).

   Consider the corresponding function problem,

   Input: A set of variables V and a set of clauses $C = \{c_1, c_2, \ldots, c_n\}$.

   Output: An assignement to the variables $v(x) \in \{true, false\}$, such that if the set of clauses is satisfiable, then the assignment $v(x)$ makes all the clauses true.

   Show how you could solve the above problem, in polynomial time, if you are given a "subroutine" (as black box) to solve the SAT problem in linear time.

7. Consider the following decision problem:

Input: A set of variables V and a boolean formula F (which uses only variables from V).

Question: Is there a truth assignment to the variables which makes the formula true?

(a) Is the above problem in NP?

(b) Is the above problem NP-complete? Give reasoning for your answer.