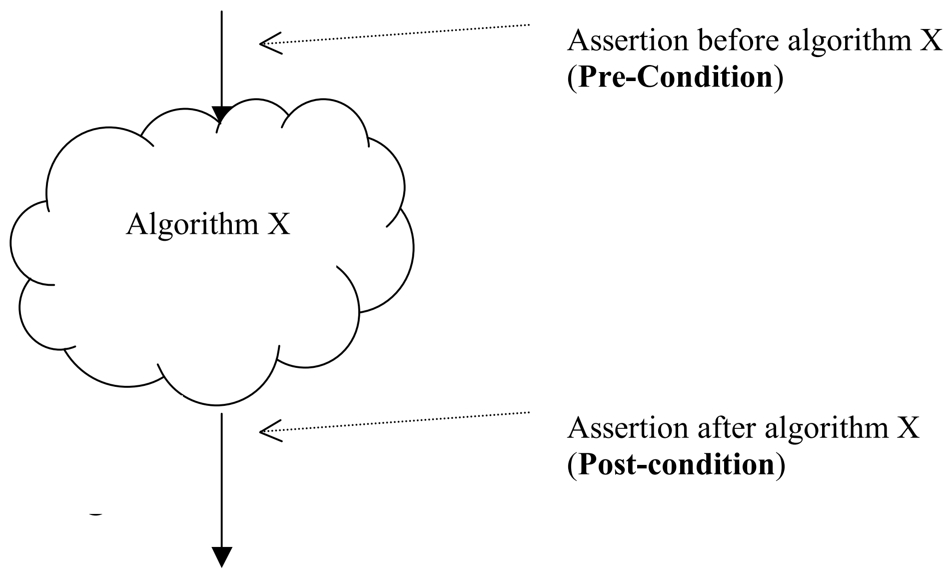
Algorithm Correctness

(or how to prove programs are correct)

Pre-condition of the algorithm: a predicate that describes the initial state (before execution)

Post-condition of the algorithm: a predicate that describes the final state (after execution)

> ***The algorithm is correct if it can be proved that if the pre-condition is true, the post-condition must be true.***
>
> **Pre-condition $\Rightarrow$ Post-condition**



ex 1:

```
program mystery
   begin
      .
      .
   end
```

Pre-condition: a = 3   b = 5

Post-condition: a = 5   b = 3

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ex 2:

Pre-condition: y = 6

```
x = 2
z = x + y
if ( y > 0 )
    z = z + 1
else
    z = 0
```

Post-condition: z = 9

Programs may be sliced into sequential parts and assertions inserted

program mystery2
begin

Segment 1

Segment 2

Segment 3

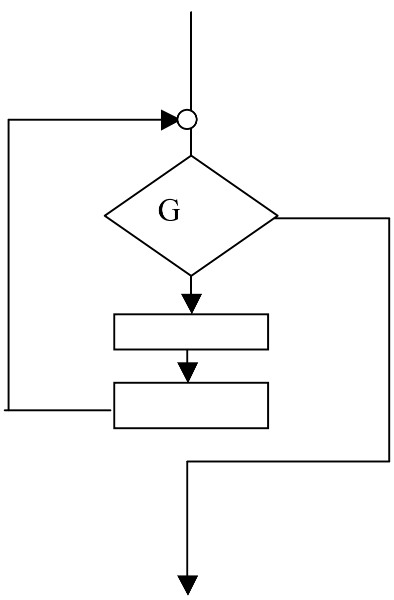
.
:

Segment n

end

The loop invariant I(n) is a predicate where the variable, n, represents the number of times the loop has iterated.

What about loops?
Consider a while loop with Pre-Condition and Post-Condition and guard G.



(ex: while (i < 6))

To prove that the loop is "correct" do the following:

1. Write a **loop invariant** that "captures" the purpose of the loop; call it I(n). (*Difficult!!*) I(n) is a predicate that is true after n iterations of the loop.

2. Show that the pre-condition implies the loop invariant I(0). (*Basis Step*)

3. Show that if I(k) and guard G are true, then I(k+1) is true. (*Inductive Step*)
   In other words,
   if I(n) is true for n = k and the guard G is still true, then it follows that I(n) is true for n = k+1.

4. Show that guard G eventually becomes false (looping stops).

5. If the loop iterates a maximum of N times, show that I(N) implies the post-condition.

Example 1

Pre-condition: m is a non-negative integer, x is real, Count = 0, Product = 0

```
while ( Count != m)
   {   Product = Product + x;
       Count = Count + 1;
   }
```

What's guard G?

Post-condition: Count = m and Product = m · x

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1. Write loop invariant I(n): (Remember that n represents the number of times the loop has iterated)

The loop invariant can be a conjunction (AND)

2. Show that the pre-condition implies that I(0) is true.

3. Inductive Step: Show that if I(k) and G are true, I(k+1) is true.

Assume G: and I(k): Product = k · x and Count = k are true.
(these are Product_{old} and Count_{old})

Show I(k+1): Product = (k+1) · x and Count = k + 1 is true.
(these are Product_{new} and Count_{new})

4. Show that G will eventually be false.

5. Show that the post-condition follows when the loop stops.

Example 2.

Pre-condition: Max = A[0]; Count = 0

```
while ( Count < 10)
 { Count = Count + 1;
   if ( A[Count] > Max )
          Max = A[Count];
  }
```

Post-condition: Max contains the largest value in A[0]..A[10]; Count = 10

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1) Write loop invariant I(n):

2) Show that the pre-condition implies that I(0) is true.

3) Show I(k) and G imply I(k+1).

I(k):

I(k+1):

4) Show that G will eventually be false.

5) Show the post-condition follows when the loop stops