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Underwater Image Enhancement Methods Based on CNN-PDE

Rui Wang [a,*], Guoyu Wang [a], Guoning Lan [b], Xue Yang [a,c]

[a] *College of Information Science & Engineering, Ocean University of China, Qingdao 266100, China*

[b] *China Unicom Company, Qingdao 266000, China*

[c] *Qingdao Agricultural University, Qingdao 266109, China*

Abstract

This paper compares Cellular Neural Network (CNN) applications of several different Partial Differential Equations (PDEs) for underwater image enhancement, including thermal diffusion equation, Laplace equation, Poisson equation, the P-M model and the TV model. The results show that the TV model can effectively preserve edge detail, and the Poisson equation is closer to the water degradation model; both of that can get a sharper image than other partial differential equations.

Keywords: Underwater Image; Different Partial Differential Equation (PDE); Cellular Neural Network (CNN); Image Enhancement

1 Introduction

Image processing methods were used to aiming at statistics and geometry at the beginning; researchers obtained the statistical properties from the information such as the signal or the noise in an image and then according to which they could modify the histogram, adjust the contrast, and design different image filters to implement image enhancement and denoising. Underwater image degraded badly because of the scattering and absorption. However the backscattering noise is mainly distributed at the low frequency component so that the conventional linear filter cannot remove the low-frequency noise and enhance the high-frequency components at the same time. Hummel [1] and Koenderink [2] proposed the diffusion equation model for image processing. Perona and Malik proposed an improved anisotropic diffusion equation [3] instead of the Gaussian filter; the P-M model laid the theoretical foundation of partial differential equations (PDEs) in image processing. Osher, Rudin, and Fatemi researched on the total variation model [4] which was fully demonstrated the great potential of PDEs in the field of image processing. PDEs can describe an image in the continuous domain and can be associated with the earlier approaches, so that it became one of the most popular image processing methods. PDE has the advantages of

*Corresponding author.
Email address: oucwangrui@163.com (Rui Wang).

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flexibility, accuracy and stability in image processing, but it also has the problem of huge amount of calculation and low speed during the solving process, thus it cost of hardware resources much and cannot meet the real-time requirement.

Cellular Neural Network (CNN) [5, 6] was proposed in 1988, which is a large scale non-linear analog system. This system is made up of local connected cells, each cell is capable of real-time signal processing, thus it can be applied in large scale integrated circuit and realize high-speed parallel processing.

2 Cellular Neural Networks

2.1 Model and Concept

Each cell in the neural networks has direct action only with its neighboring cells, and has indirect action with other cells. Fig. 1 shows an $M \times N$ cellular neural network, which consists of M rows and N columns, $C(i, j)$ denote the cell on row i and column j.

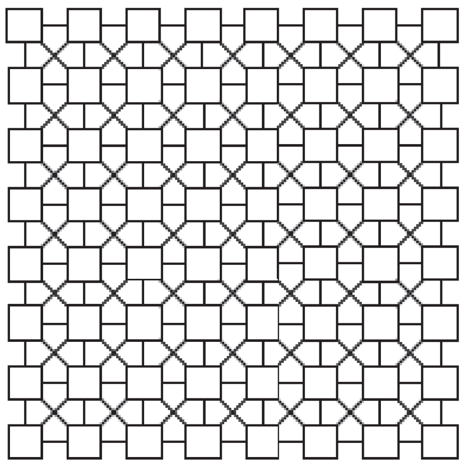


Fig. 1: $M \times N$ cellular neural network

Each cell performs a circuit, including capacitors, resistors, linear and nonlinear control, and independent power supply. Fig. 2 shows the circuit:

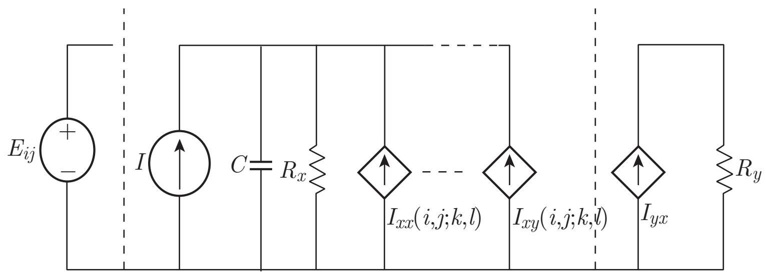


Fig. 2: Circuit of cellular neural network

The neighborhood r of cell $C(i, j)$ is defined as:

$$N_r(i, j) = \{C(k, l)\} : \max\{|k - i|, |l - j| \leq r, 1 \leq k \leq M, 1 \leq l \leq N\} \tag{1}$$

The status equation of cell $C(i,j)$ is:

$$C\frac{dv_{xij}(t)}{dt}=-\frac{1}{R_x}v_{xij}+\sum_{C(k,l)\in N_r(i,j)} A_{ij,kl}v_{ykl}(t)+\sum_{C(k,l)\in N_r(i,j)} B_{ij,kl}v_{ukl}(t)+z_{ij}, 1\le i\le M, 1\le j\le N \tag{2}$$

where $v_{xij}(t)$, $v_{yij}(t)$, $v_{uij}(t)$ are the status, output, input of $C(i,j)$ respectively, and z_{ij} is the threshold. C is linear capacitor, R_x is linear resistance, $A_{ij,kl}$, $B_{ij,kl}$ are the feedback operator and the control operator respectively.

Since digital image is discrete in both space and time domain, to process the digital image using CNN, the status equation must be discrete [6].

$$x_{ij}(n+1)=x_{ij}(n)+\frac{1}{C}\left[-\frac{1}{R_x}x_{ij}(n)+\sum_{kl\in N_{ij}(r)} A_{kl}y_{kl}(n)+\sum_{kl\in N_{ij}(r)} B_{kl}u_{kl}+I_{ij}\right] \tag{3}$$

When $t=nh$ in (2), h is the step length of t and is usually defined as 1, then the derivative of $x_{ij}(t)$ will be discrete.

The status equation of CNNs can be regarded as a two-dimensional nonlinear filter. Its output equation is $v_{uij}(t)=f(v_{xij}), 1\le i\le M, 1\le j\le N$, input equation is $u_{ij}=E_{ij}$ where $|u_{ij}|\le 1, |x_{ij}(0)|\le 1$, $C>0, R_x>0$

In order to ensure a meaningful state, the boundary cells must be considered. In this paper we selected the fixed boundary conditions (Dirichlet boundary conditions), which sets the cells outside the boundary as constant zero.

Each cell in CNNs interacted with neighbor cells via input variables, the initial state and the threshold. The dynamic properties of the entire system are related with the feedback operator and control operator of the status equation; they are arranged in accordance with the position of the cells to make up the feedback template A and the control template B. Typically feedback template and control template are spatially invariant.

$$A=\begin{matrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{matrix}, \qquad B=\begin{matrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{matrix} \tag{4}$$

We use the vector equation to express the status of CNNs:

$$\dot{x}_{i,j}=-x_{i,j}+A(y_{i,j})+B(u_{i,j})+z_{i,j} \tag{5}$$

where x, y, u are the status variable, output variable and input variable respectively; Z is the threshold.

2.2 Image Processing Methods Based on CNNs

Image processing using CNNs mapped the input image to the output image according to the status equation. We use two-dimensional CNNs because image is two-dimensional signal. In order to satisfy the constraint conditions of CNNs, for gray level image, image gray value is mapped from {0, 1, 2…255} to [−1.0, +1.0]:

$$u_{ij}=(1-2g_{ij}/255)\in[-1.0,+1.0] \tag{6}$$

3 Underwater Image Enhancement Based on CNN-PDE

According to the CNNs energy theorem, the energy of the cell network is decreasing and will finally be stable at the minimum value point, which means $\lim_{t\to\infty} E_{CNN}(t) = C$

When $A_{ij} > \frac{1}{R_x}$, then $\lim_{t\to\infty} |x_{ij}(t)| > 1$, $\lim_{t\to\infty} y_{ij}(t) = \pm 1$.

For a gray image $\Phi_0(x, y)$, $\Phi_0 : R^2 \to R$, its PDE can be described as:

$$\frac{\partial \Phi}{\partial t} = F[\Phi(x, y, t)] \tag{7}$$

F is the mapping method of image processing, generally function F is relied on the first-order derivative, the second-order derivative and the original image. The processed image can be viewed as the solution of the differential equation. The solution of differential equations can be obtained by minimizing the energy function:

$$\arg\{Min_\Phi E(\Phi)\} \tag{8}$$

This paper uses the CT-CNN method, where the input $U(t) = 0$, the initial status $X(0)$ is the original image, and the boundary condition is the fixed boundary condition. When we set the templates parameters of A and B according to different PDEs, the CNN will run to the minimum energy direction, the processed image is the stable output $Y(\infty) = (y_{i,j}(\infty))_{M\times N}$

(1) Thermal diffusion equation [7]

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x_1}\left(k_1 \frac{\partial u}{\partial x_1}\right) + \cdots + \frac{\partial}{\partial x_n}\left(k_1 \frac{\partial u}{\partial x_n}\right) + F(x, t) \tag{9}$$

where u denotes the concentration, k denotes the diffusion coefficient.

When $k_1 = k_2 = \ldots k_n = a (a > 0)$,

$$\frac{\partial u}{\partial t} = a\Delta u + F(x, t) \tag{10}$$

where $\Delta = \dfrac{\partial^2}{\partial x_1^2} + \dfrac{\partial^2}{\partial x_2^2} + \cdots + \dfrac{\partial^2}{\partial x_n^2}$ is the Laplacian.

The initial status of thermal diffusion equation is

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u \\ u|_{t=0} = u_0 \end{cases} \tag{11}$$

The parameters of A, B and Z is [8]:

$$A: \begin{matrix} 0.1 & 0.15 & 0.1 \\ 0.15 & 0 & 0.15 \\ 0.1 & 0.15 & 0.1 \end{matrix} \qquad B: \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \qquad Z = 0$$

(2) Poisson equation [9]:

$$\Delta u = -f(x) \tag{12}$$

The parameters of A, B and Z is [9]:

$$A: \begin{matrix} 0 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & 0 \end{matrix} \qquad B: \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \qquad Z = -f(x)$$

(3) Laplace equation [7]

$$\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \ldots + \frac{\partial^2 u}{\partial x_n^2} = 0 \tag{13}$$

The parameters of A, B and Z is [7]:

$$A: \begin{matrix} 0 & \nu/h^2 & 0 \\ \nu/h^2 & -4 * \nu/h^2 + 1/R & \nu/h^2 \\ 0 & \nu/h^2 & 0 \end{matrix},$$

ν is the diffusion coefficient, h is the step length, R is the cell resistance, where $h = 1$, $R = 1$. $\nu = 0.1$ according to the power spectrum of pure water in our previous work [10];

$$B: \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \qquad Z = 0$$

(4) P-M model [7]

$$\frac{\partial u}{\partial t} = div(g(|\nabla u|).\nabla u), g(s) = \frac{1}{1 + \lambda s^2} \tag{14}$$

Local coordinate expression is:

$$\frac{\partial u}{\partial t} = \frac{1 - \lambda^2 |\nabla u|^2}{(1 + \lambda^2 |\nabla u|^2)}.u_{\xi\xi} + \frac{1}{1 + \lambda^2 |\nabla u|^2}.u_{\eta\eta} \tag{15}$$

The parameters of A, B and Z is [11]:

$$A = \begin{bmatrix} 0 & a & 0 \\ a & 1 & a \\ 0 & a & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad Z = 0$$

where $a = \lambda f(y_{ij} - y_{kl}) |y_{ij} - y_{kl}|$, $b = 2\alpha(x_{ij} - u_{ij})$.

Function f is called the stopping function; the diffusion speed is reduced at the edges:

$$f(x) = \exp\left(-\left(\frac{s}{k}\right)^2\right) \tag{16}$$

(5) TV model

According to equation (6), we can combine two different image processing progresses:

$$\frac{\partial \Phi}{\partial t} = F_1[\Phi(x, y, t)], \quad \frac{\partial \Phi}{\partial t} = F_2[\Phi(x, y, t)] \tag{17}$$

whose PDE is described as:

$$\frac{\partial \Phi}{\partial t} = F_1 + \lambda F_2 \tag{18}$$

Similarly, the solution of differential equations can be obtained by minimizing the energy function $E = E_1 + \lambda E_2$.

Image degradation is through a linear model as the following form:

$$y = H \cdot \hat{x} + N \tag{19}$$

where y is the degraded image, $\hat{x}$ is the original image, H is a linear degrade function, and N is the additive noise .

Image recovery can be realized by using minimum error function according to (18):

$$E = \|y - H \cdot x\|^2 /2 + \lambda \|D \cdot x\|^2 /2 \tag{20}$$

where $\| \ \|$ denotes L_2 norm, x is the estimate of $\hat{x}$ and D denotes smooth operator.

The energy function E is defined in [12, 13] as:

$$E(\Phi, G) = \iint \|\nabla \Phi\|^2 \, dxdy + \lambda |G| \tag{21}$$

It needs to meet the following conditions [14] when using CNN to solve the TV PDE.

$$\text{(a)} \quad \left| \sum_{C_{kl} \in N_r} A_{ij,kl} y_{kl} \right| \leq 1 \tag{22}$$

This condition ensures that the output of the neural network is between -1 and $+1$, otherwise it means each cell did not reach the saturation level.

$$\text{(b)} \quad \lim_{t \to \infty} x_{ij}(t) = x_{ij}^*, \quad \left. \frac{dx_{ij}}{dt} \right|_{x_{ij} = x_{ij}^*} = 0 \tag{23}$$

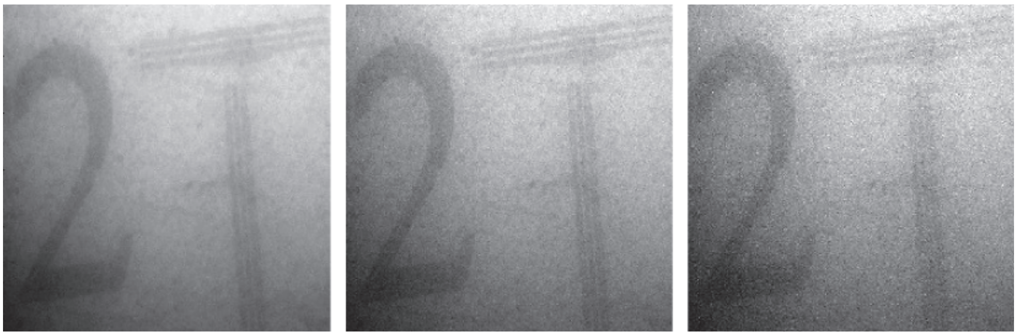
This condition ensures that CNN is stable so that we can calculate the cellular neural network template parameters.

$$\sum_{C_{kl} \in N_r} A_{ij,kl} y_{kl} = x_{i,j} = y_{i,j}, 1 \leq i \leq M, 1 \leq j \leq N \tag{24}$$

The parameters of A, B and Z is [14]:

$$A = \begin{bmatrix} 0 & 0.25 & 0 \\ 0.25 & 0 & 0.25 \\ 0 & 0.25 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -\lambda & 0 \\ -\lambda & 4 * \lambda & -\lambda \\ 0 & -\lambda & 0 \end{bmatrix} \quad Z = 0$$

If $\lambda < 1$ the TV model plays a main part in smoothing, if $\lambda > 1$ the TV model plays a main part in enhancement, else when $\lambda = 1$ the TV model comes to a better result due to the balance between smoothing and enhancement. Fig. 3 compares the results by TV models with different λ values.

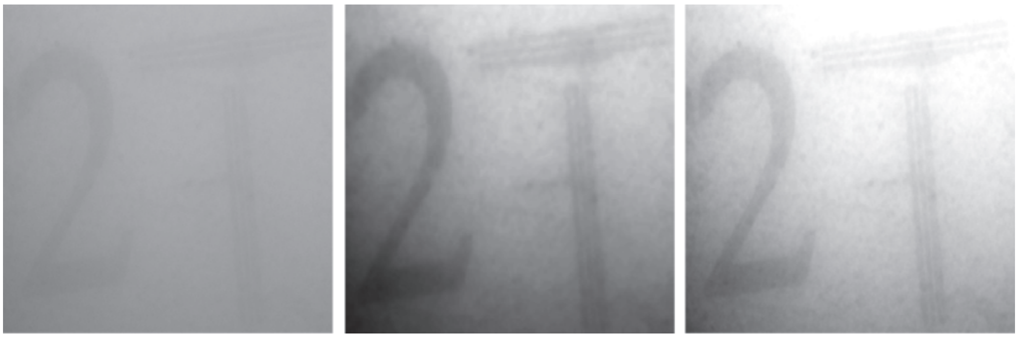


l=0.5 l=1.0 l=1.5

Fig. 3: Results with different λ

4 Results

We use CNN-PDEs of Thermal diffusion equation, Poisson equation, Laplace equation P-M model and TV model (when $\lambda = 1$) introduced respectively in chapter 3 to enhance the underwater image, the enhancement results by various methods is shown in Fig. 4.



Original image Thermal diffusion equation Poisson equation

Laplace equation P-M model TV model (λ=1)

Fig. 4: Results of different PDEs

The experimental results show that the thermal diffusion equation and Laplace equation both suppressed the noise, but blurred the edges. The P-M model is often used to determine the edge by detecting the gradient, while underwater image has low contrast, the strong backscattering make the gradient detection unreliable, that led to the failure of edge information diffusion by P-M model. TV model performed well but it needs to adjust λ value. The result shows that Poisson equation brought a better result since it is similar to the formation mechanism of underwater backscattering.

5 Conclusions

In this paper, the PDE based underwater image processing has been studied, we used CNN to get the solutions of different PDE, and the solutions are the results of image enhancement. The experiment compared the performances of five kinds of PDE model including thermal diffusion equation, Laplace equation, Poisson equation, the P-M model and the TV model in underwater image enhancement and confirmed that underwater image enhancement method based on CNN-PDE is feasible.

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