CS3230
Tutorial 6

1. Consider the greedy algorithm for coin-change problem.

   Suppose the coin denominations are $d_1 > d_2 > \ldots > d_n = 1$.

   Suppose that d_{i+1} is a factor of d_i, for $1 \leq i < n$.

   Then, show that the greedy algorithm is optimal.

   Ans: (i) Due to the constraint given in the problem, in the optimal algorithm, one has $< d_i/d_{i+1}$ coins of denomination d_{i+1}.

   (ii) Fact (i) implies that sum of values of coins of denomination d_j, $j > i$, is $< d_i$. (This can be shown by induction)

   (iii) Using (ii), it follows that the greedy algorithm and optimal algorithm must have the same number of coins of each denomination.

2. (a) Suppose we modify the greedy algorithm for fractional knapsack problem to consider the objects in order of "non-increasing" value (rather than non-increasing ratio of value/weight as done in class).

   Is the modified algorithm still optimal? If so, give an argument for its optimality. If not, give a counterexample.

   Ans: No. Counterexample:

   item 1: value=5, weight = 10

   item 2: value=4, weight = 5

   item 3: value=4, weight = 5

   Total weight allowed $A = 10$

   Then, optimal takes items 2 and 3, whereas greedy method above takes item 1.

   (b) Suppose we modify the greedy algorithm to consider the objects in order of "non-decreasing" weight (rather than non-increasing ratio of value/weight as done in class).

   Is the modified algorithm still optimal? If so, give an argument for its optimality. If not, give a counterexample.

   Ans: No. Counterexample:

   item 1: value=1 weight=90

   item 2: value=5 weight=100

   Total weight allowed $A = 100$

Then optimal chooses item 2, whereas the above algorithm chooses item 1 and 1/10th of item 2 (of weight 10).

3. Using the algorithm done in class, give Huffman tree and code if the frequencies of the letters are as follows:

   $freq(a) = 25$, $freq(b) = 2$, $freq(c) = 5$, $freq(d) = 6$, $freq(e) = 6$, $freq(f) = 6$

   Ans:

   (i) Initially, b and c are combined to form bc with frequency 7.

   (ii) d and e are combined to form de with frequency 12

   (iii) bc and f are combined to form bcf with frequency 13

   (iv) bcf and de are combined to form $bcfde$ with frequency 25

   (v) a and $bcfde$ are combined to for $abcfde$ with frequency 50

   Gives code

   $a = 0$, $b = 1000$, $c = 1001$, $f = 101$, $d = 110$, $e = 111$

4. Suppose T is a Huffman coding tree for the frequencies $f_1, f_2, f_3, \ldots, f_n$, where f_1 and f_2 have the same parent. Consider the tree T' with f_1 and f_2 deleted, and the parent of f_1 and f_2 labeled with frequency $f_1 + f_2$.

   Consider the following conjecture: If T' is optimal for frequencies $f_1 + f_2, f_3, \ldots, f_n$ then T is optimal for $f_1, f_2, \ldots, f_n$.

   Either prove the conjecture to be true or give a counterexample.

   Ans: conjecture is false, as given by the frequencies,

   $f_1 = 5$, $f_2 = 6$, $f_3 = 1$.