CS3230
Tutorial 4

Q3. In merge sort algorithm done in class, for merging two arrays of size m and n respectively, we used one extra array of size $m+n$.

Can you give an algorithm to do the merge which uses an extra array B of size only $\min(m,n)$? Complexity of your algorithm should still be linear in the size of the two arrays.

Ans: Below is the main algorithm. Details of why this works was done in the tutorial class.

Merge(A, i, k, j)

Assumption: $i \leq k < j$; $A[i:k]$ and $A[k+1:j]$ are sorted; and $k-i \leq j-k-1$ (that is $A[i:k]$ is smaller in size than $A[k+1:j]$)

1. For $r = i$ to k do { $B[r] = A[r]$ }
2. $p1 = i$; $p2 = k+1$; $p3 = i$
3. While $p1 \leq k$, $p2 \leq j$ do {

    3.1 If $B[p1] \leq A[p2]$, then $A[p3] = B[p1]$, $p1 = p1 + 1$
    Else $A[p3] = A[p2]$, $p2 = p2 + 1$; Endif

    3.2 $p3 = p3 + 1$;

    }
4. While $p1 \leq k$, { $A[p3] = A[p1]$; $p1 = p1 + 1$; $p3 = p3 + 1$; }

    (* Note: we don't need to do the part

    While $p2 \leq j$ { $A[p3] = A[p2]$; $p2 = p2 + 1$; $p3 = p3 + 1$; },

    as at this point $p2 = p3$. *)

End

Q4. (a) Show that any board of size $2j \times 3k$ (without any missing squares) can be tiled using trominoes.

(b) Show that a 5×5 board, with a corner missing, can be tiled using trominos.

(c) Show that any 7×7 board with any one square missing can be tiled using trominos.

(This is lengthy, and done by considering several possible cases of missing square. So if you can't find an answer just assume that this can be done).

(d) Show that a 11×11 board, with any one square missing can be tiled using trominos.

Hint: Divide the board into a sub-board of size 7×7 (which contains that missing square), a sub-board of size 5×5 with a corner missing, and two sub-boards of size 4×6.

(e) By induction show that, for $n > 11$, any $n \times n$ board with one square missing can be tiled using trominoes as long as n is odd and not a multiple of 3.

Hint: Divide the board into four parts, One of size $(n-m) \times (n-m)$ containing the missing square (which can be solved using induction), two boards of sizes $(n-m-1) \times m$, which can be solved using part (a), and one board of size $(m+1) \times (m+1)$, with a missing corner.

Ans:

(a) Note that two trominoes put together can cover a 2×3 square. By using this repeatedly, we can cover any $2j \times 3k$ board.

(b) See Tiling 1.

	1	1	2	2
4	1	3	3	2
4	4	3	8	8
5	6	6	8	7
5	5	6	7	7

Tiling 1

(c) Consider the two partial tilings given in Tiling 2 and Tiling 3, where the missing square may be one of ?. Note that all possible places for missing squares are symmetrical to one of these cases. Furthermore, in each of these cases it is easy to tile the remaining left-over squares once the position missing square is known.

1	1	5	5	7	7	8
1	2	5	6	7	8	8
2	2	6	6	10	10	11
3	3	4	9	10	11	11
3	4	4	9	9	12	12
?	?		?	?	12	13
?	?		?	?	13	13

Tiling 2

1	1	2	10	10	7	7
1	2	2	9	10	7	8
3	3	9	9	11	8	8
3	4	?	?	11	11	12
4	4	?	?	14	12	12
5	6	6	15	14	14	13
5	5	6	15	15	13	13

Tiling 3

(d) Suppose the original board was $A[1:11, 1:11]$.

Suppose without loss of generality that the missing square belongs to $A[1:7, 1:7]$ (any other missing square can be handled in a symmetric way).

Then tile $A[1:7, 1:7]$ using part (c). Tile $A[7:11, 7:11]$, with missing square $A[7,7]$ as in part (b). Tile, $A[1:6, 8:11]$ and $A[8:11, 1:6]$ using part (a).

(e) Suppose the original board was $A[1:n, 1:n]$.

Suppose without loss of generality that the missing square belongs to $A[1:n-6, 1:n-6]$.

Consider the sub-boards $A[1:n-6, 1:n-6]$ (which is tiled inductively), $A[1:n-7, n-5:n]$, $A[n-5:n, 1:n-7]$ (which can be tiled using part (a)), $A[n-6:n, n-6:n]$ (with missing corner, which can be tiled using part (c)).