CS3230 Lecture 4 (revised)

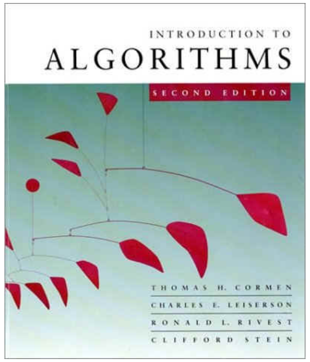
School of Computing

"A Review of Sorting, Quicksort Analysis, and Augmenting Data Structures"

- Lecture Topics and Readings
  - (Quick Review) of Sorting Methods [CLRS]-C?
  - Quicksort and Randomized QS [CLRS]-C7
  - Augmenting Data Structures [CLRS]-C14

Creative View of Sorting Methods
Quicksort (only 40% sub-optimal)
Be more aware of augmenting Data Structure

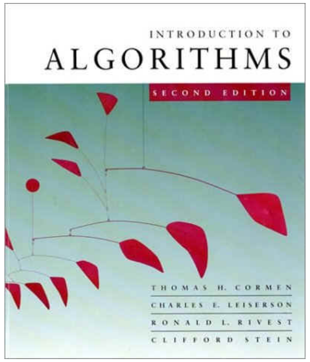
[CLRS]…



Sabbatical leave at NUS
Computer Science Dept 1995/96

[CLRS] & Charles Leiserson.

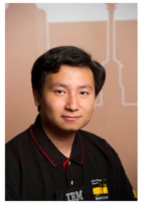
[CLRS] @500K



[CLRS]-90, [CLRS]-01, [CLRS]-09
Celebrating 500,000 copies sold

[HH2013]… 3rd edition

Steven Halim



Felix Halim

[HH13] *Competitive Programming*, (3rd edition)
by Steven Halim and Felix Halim, 2013.

Antony Hoare (1934 –)

Invented Quicksort (at age 26)

Developed Hoare's Logic (for program correctness)

Developed CSP (including dining philisophers' problem)

Quote: (about difficulties of creating software systems)

"There are two ways of constructing a software design: One way is to make it so simple that there are obviously no deficiencies, and the other way is to make it so complicated that there are no obvious deficiencies. The first method is far more difficult."

Tony Hoare, Singapore 2008 Computing in the 21st Century

- Turing Award, 1980
- Knighted, 2000

Thank you.

Q & A

School of Computing

CS3230 Lecture 4

School of Computing

"A Review of Sorting, Lower Bounds, and Sorting in Linear Time"

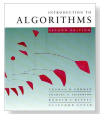
- **Lecture Topics and Readings**
  - (Quick Review) of Sorting Methods [CLRS]-C?
  - Quicksort and Randomized QS [CLRS]-C7
  - Lower Bound for Sorting [CLRS]-C8
  - Sorting in Linear Time [CLRS]-C8

Creative Review of Sorting,
Lower Bound and Optimal Sorting,
Busting the Lower Bound

A Creative Review of Sorting Algorithms

Sorting Animation: by Steven Halim & students

http://www.comp.nus.edu.sg/~stevenha/visualization/sorting.html



The problem of sorting

Input: sequence $\langle a_1, a_2, \ldots, a_n \rangle$ of numbers.

Output: permutation $\langle a'_1, a'_2, \ldots, a'_n \rangle$ such that $a'_1 \le a'_2 \le \cdots \le a'_n$.

Example: ***Input:*** 8 2 4 9 3 6

Output: 2 3 4 6 8 9

Sorting Animation: by Steven Halim & students

http://www.comp.nus.edu.sg/~stevenha/visualization/sorting.html

Sorting: Problem and Algorithms

Problem: Sorting

Given a list of n numbers, sort them

Algorithms:

- Selection Sort $\Theta(n^2)$
- Insertion Sort $\Theta(n^2)$
- Bubble Sort $\Theta(n^2)$
- Merge Sort $\Theta(n \lg n)$
- Quicksort $\Theta(n \lg n)$*

* *average case*

Start with Selection Sort

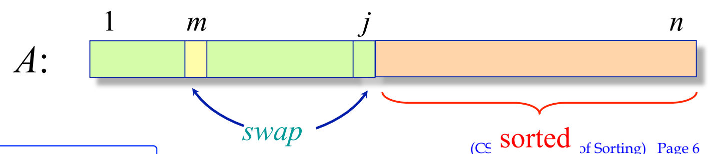
Selection Sort Algorithm

Recall from Lecture 2

```
SELECTION-SORT (A, n)      ▷ A[1 . . n]
  j ← n
  while j > 1
      do m ← Find-Max (A, j)
         Swap (A[m], A[j])
         j ← j – 1
```

Let's make this recursive.

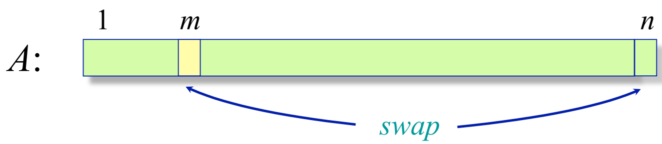
$A[m]$ is largest among $A[1..j]$



Recursive Selection Sort

SELECTION-SORT-R (A, n) ▷ $A[1\,..\,n]$
  if $n = 1$ **then return**
  $m \leftarrow$ Find-Max (A, n)
  Swap $(A[m], A[n])$
  SELECTION-SORT-R $(A, n-1)$

$A[m]$ is largest among $A[1..n]$

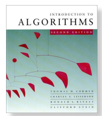


Recursive Selection Sort

SELECTION-SORT-R (A, n) ▷ $A[1\,..\,n]$
  if $n = 1$ **then return**
  $m \leftarrow$ Find-Max (A, n)
  Swap $(A[m], A[n])$
  SELECTION-SORT-R $(A, n-1)$

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ T(n-1) + \Theta(n) & \text{if } n > 1. \end{cases}$$

Next consider Insertion Sort

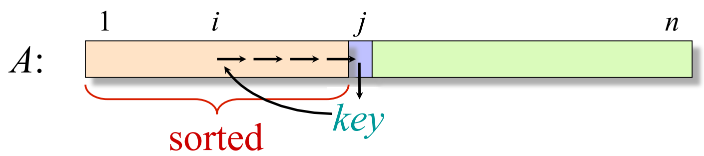


Insertion sort

Recall from Lecture 2

"pseudocode"

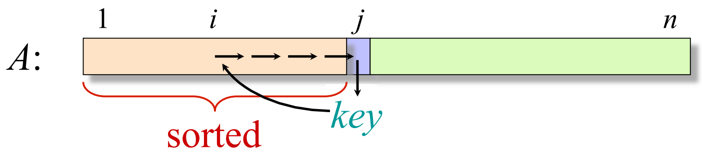
INSERTION-SORT (A, n) ▷ $A[1\,..\,n]$
  for $j \leftarrow 2$ **to** n
    do $key \leftarrow A[j]$
      $i \leftarrow j - 1$
      while $i > 0$ and $A[i] > key$
        do $A[i+1] \leftarrow A[i]$
          $i \leftarrow i - 1$
      $A[i+1] = key$



Insertion sort

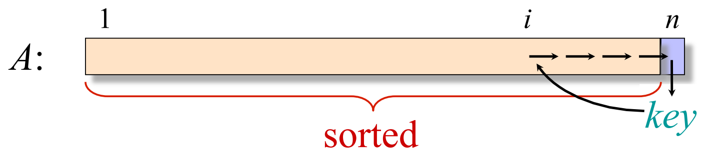
INSERTION-SORT (A, n) ▷ $A[1\,..\,n]$
for $j \leftarrow 2$ **to** n
  do $key \leftarrow A[j]$
    $i \leftarrow j - 1$
    while $i > 0$ and $A[i] > key$
      do $A[i+1] \leftarrow A[i]$
        $i \leftarrow i - 1$
    $A[i+1] = key$

"inserts" $A[j]$ into sorted $A[1\,..\,(j–1)]$



Recursive Insertion sort

INSERTION-SORT-R (A, n) ▷ $A[1\,..\,n]$
  if $n = 1$ **then return**
  INSERTION-SORT-R $(A, n–1)$
  insert $A[n]$ into sorted $A[1\,..\,n–1]$



Recursive Insertion sort

INSERTION-SORT-R (A, n) ▷ $A[1\,..\,n]$
  if $n = 1$ **then return**
  INSERTION-SORT-R $(A, n–1)$
  insert $A[n]$ into sorted $A[1\,..\,n–1]$

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ T(n-1) + \Theta(n) & \text{if } n > 1. \end{cases}$$

Recursive Bubble Sort

BUBBLE-SORT-R (A, n) $\triangleright$ $A[1 \,..\, n]$
  if $n = 1$ **then return**
  One bubble-phase on $A[1 \,..\, n]$
  BUBBLE-SORT-R $(A, n-1)$

Recursive Bubble Sort

BUBBLE-SORT-R (A, n) $\triangleright$ $A[1 \,..\, n]$
  if $n = 1$ **then return**
  One bubble-phase on $A[1 \,..\, n]$
  BUBBLE-SORT-R $(A, n-1)$

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ T(n-1) + \Theta(n) & \text{if } n > 1. \end{cases}$$

All have the *same* recurrence

Selection Sort, Insertion Sort, Bubble Sort

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ T(n-1) + \Theta(n) & \text{if } n > 1. \end{cases}$$

$(n-1), 0$

Extreme imbalance

All have the *same* recurrence

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ T(n-1) + \Theta(n) & \text{if } n > 1. \end{cases}$$

How to "solve" this recurrence

Answer: Use TELESCOPING

How to "solve" this recurrence

Answer: Use TELESCOPING

$$T(n) = cn + T(n-1)$$
$$= cn + c(n-1) + T(n-2)$$
$$= cn + c(n-1) + c(n-2) + T(n-3)$$
$$= cn + c(n-1) + c(n-2) + \ldots c2 + T(1)$$
$$= cn + c(n-1) + c(n-2) + \ldots c2 + c$$
$$= c\,(n + (n-1) + (n-2) + \ldots 2 + 1)$$

$\boxed{T(n) = \Theta(n^2)}$

Observation:

Selection Sort, Insertion Sort, Bubble Sort

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ T(n-1) + \Theta(n) & \text{if } n > 1. \end{cases}$$

They all have running time: $T(n) = \Theta(n^2)$

Imbalance in Divide &Conquer algorithms produces inefficient algorithms

How about Perfect Balance

Merge sort (Perfect balance)

MERGE-SORT $A[1 \,..\, n]$

1. If $n = 1$, done.
2. Recursively sort $A[\,1 \,..\, \lceil n/2 \rceil\,]$ and $A[\,\lceil n/2 \rceil + 1 \,..\, n\,]$.
3. "***Merge***" the 2 sorted lists.

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ 2T(n/2) + \Theta(n) & \text{if } n > 1. \end{cases}$$

M-Thm: $a = 2, b = 2 \;\Rightarrow\; n^{\log_b a} = n^{\log_2 2} = n$

$\Rightarrow$ CASE 2 $(k = 0) \;\Rightarrow\; T(n) = \Theta(n \lg n)$.

What about Heapsort, Quicksort

Heapsort $\Theta(n \lg n)$

builds a data structure – Heap $\Theta(n)$

sort efficiently using the Heap $\Theta(n \lg n)$

Quicksort

Partitions array about a pivot $\Theta(n)$

Recursively sort each partition $O(??)$

How balanced is QuickSort?
(We'll see in next section)

Thank you.

Q & A

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"A Review of Sorting, Lower Bounds, and Sorting in Linear Time"

❑ **Lecture Topics and Readings**

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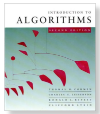
Creative Review of Sorting,
Lower Bound and Optimal Sorting,
Busting the Lower Bound



Quicksort

- Proposed by C.A.R. Hoare in 1962.
- Divide-and-conquer algorithm.
- Sorts "in place" (like insertion sort, but not like merge sort).
- Very practical (with tuning).

Divide and conquer

Quicksort an n-element array:

1. ***Divide:*** Partition the array into two subarrays around a ***pivot*** x such that elements in lower subarray $\leq x \leq$ elements in upper subarray.

$\leq x$	x	$\geq x$

2. ***Conquer:*** Recursively sort the two subarrays.
3. ***Combine:*** Trivial.

Key: *Linear-time partitioning subroutine.*

Partitioning subroutine

```
PARTITION(A, p, q)  ▷ A[p . . q]
    x ← A[p]        ▷ pivot = A[p]
    i ← p
    for j ← p + 1 to q
        do if A[j] ≤ x
              then  i ← i + 1
                    exchange A[i] ↔ A[j]
    exchange A[p] ↔ A[i]
    return i
```

Running time $= O(n)$ for n elements.

Invariant:

x	$\leq x$	$\geq x$	?
p	i	j	q

Example of partitioning

6	10	13	5	8	3	2	11
i	j						

Example of partitioning

6	10	13	5	8	3	2	11
i		j					

Example of partitioning

6	10	13	5	8	3	2	11
i			j				

Example of partitioning

6	10	13	5	8	3	2	11
6	5	13	10	8	3	2	11
	i		j				

Example of partitioning

6	10	13	5	8	3	2	11
6	5	13	10	8	3	2	11
	i			j			

Example of partitioning

6	10	13	5	8	3	2	11
6	5	13	10	8	3	2	11
	i				j		

Example of partitioning

6	10	13	5	8	3	2	11

6	5	13	10	8	3	2	11

6	5	3	10	8	13	2	11

$\longrightarrow i$ $\qquad j$

Example of partitioning

6	10	13	5	8	3	2	11

6	5	13	10	8	3	2	11

6	5	3	10	8	13	2	11

i $\qquad \longrightarrow j$

Example of partitioning

6	10	13	5	8	3	2	11

6	5	13	10	8	3	2	11

6	5	3	10	8	13	2	11

6	5	3	2	8	13	10	11

$\longrightarrow i$ $\qquad j$

Example of partitioning

6	10	13	5	8	3	2	11

6	5	13	10	8	3	2	11

6	5	3	10	8	13	2	11

6	5	3	2	8	13	10	11

i $\qquad \longrightarrow j$

Example of partitioning

6	10	13	5	8	3	2	11
6	5	13	10	8	3	2	11
6	5	3	10	8	13	2	11
6	5	3	2	8	13	10	11

i $\qquad\qquad \longrightarrow j$

Example of partitioning

6	10	13	5	8	3	2	11
6	5	13	10	8	3	2	11
6	5	3	10	8	13	2	11
6	5	3	2	8	13	10	11
2	5	3	6	8	13	10	11

i

Pseudocode for quicksort

```
QUICKSORT(A, p, r)
  if p < r
    then q ← PARTITION(A, p, r)
      QUICKSORT(A, p, q–1)
      QUICKSORT(A, q+1, r)
```

Initial call: QUICKSORT(A, 1, n)

Analysis of quicksort

- Assume all input elements are distinct.
- In practice, there are better partitioning algorithms for when duplicate input elements may exist.
- Let $T(n)$ = worst-case running time on an array of n elements.

Worst-case of quicksort

- Input sorted or reverse sorted.
- Partition around min or max element.
- One side of partition always has no elements.

$$\begin{aligned} T(n) &= T(0) + T(n-1) + \Theta(n) \\ &= \Theta(1) + T(n-1) + \Theta(n) \\ &= T(n-1) + \Theta(n) \\ &= \Theta(n^2) \qquad \textit{(arithmetic series)} \end{aligned}$$

Worst-case recursion tree

$T(n) = T(0) + T(n–1) + cn$

Worst-case recursion tree

$T(n) = T(0) + T(n–1) + cn$

$T(n)$

Worst-case recursion tree

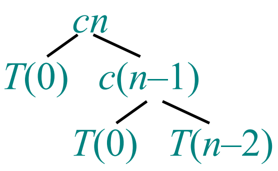
$T(n) = T(0) + T(n–1) + cn$

cn

$T(0)$ $T(n–1)$

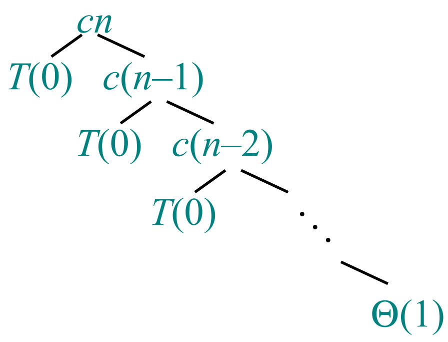
Worst-case recursion tree

$T(n) = T(0) + T(n-1) + cn$



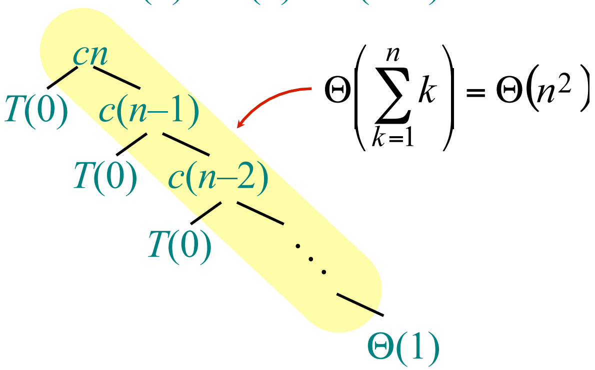
Worst-case recursion tree

$T(n) = T(0) + T(n-1) + cn$



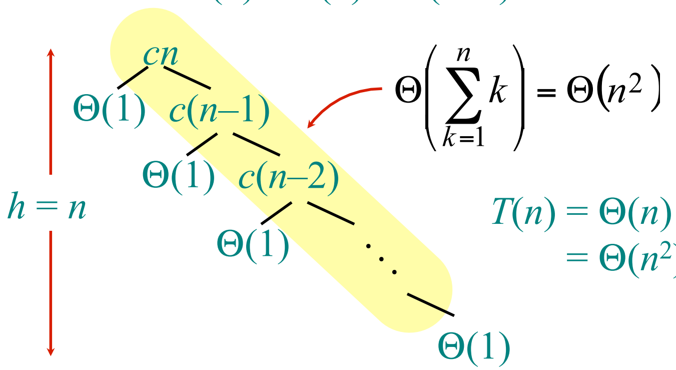
Worst-case recursion tree

$T(n) = T(0) + T(n-1) + cn$



Worst-case recursion tree

$T(n) = T(0) + T(n-1) + cn$



$$T(n) = \Theta(n) + \Theta(n^2) = \Theta(n^2)$$

Best-case analysis
(For intuition only!)

If we're lucky, PARTITION splits the array evenly:

$$\begin{aligned} T(n) &= 2T(n/2) + \Theta(n) \\ &= \Theta(n \lg n) \qquad \text{(same as merge sort)} \end{aligned}$$

What if the split is always $\frac{1}{10} : \frac{9}{10}$?

$$T(n) = T\left(\tfrac{1}{10}n\right) + T\left(\tfrac{9}{10}n\right) + \Theta(n)$$

What is the solution to this recurrence?

Analysis of "almost-best" case

$$T(n)$$

Analysis of "almost-best" case

$$cn$$

$$T\left(\tfrac{1}{10}n\right) \qquad T\left(\tfrac{9}{10}n\right)$$

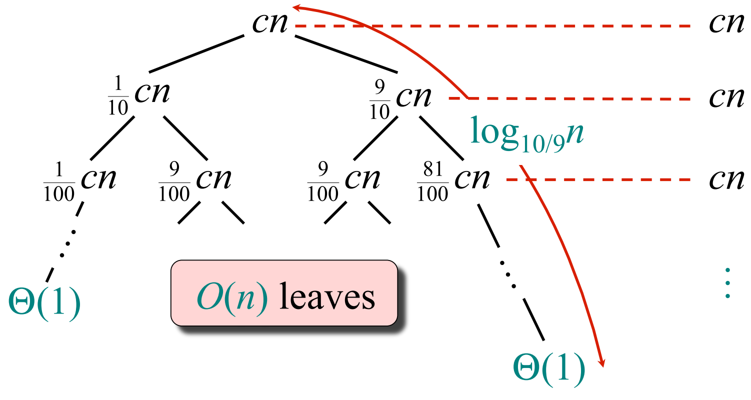
Analysis of "almost-best" case

$$cn$$

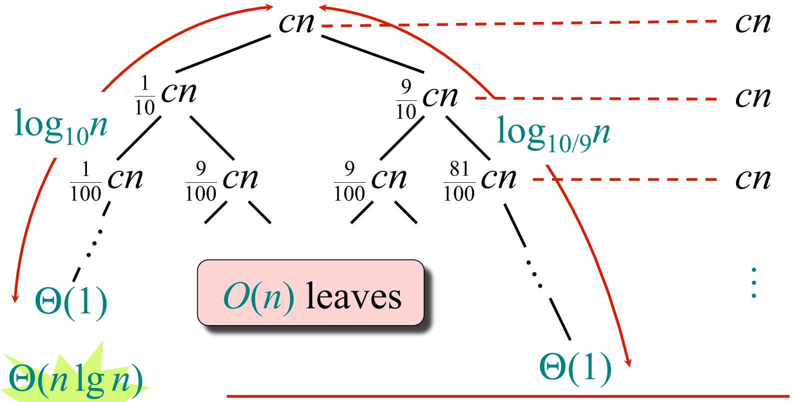
$$\tfrac{1}{10}cn \qquad \tfrac{9}{10}cn$$

$$T\left(\tfrac{1}{100}n\right) T\left(\tfrac{9}{100}n\right) \quad T\left(\tfrac{9}{100}n\right) T\left(\tfrac{81}{100}n\right)$$

Analysis of "almost-best" case



Analysis of "almost-best" case



$\Theta(n \lg n)$ ***Lucky!***

$cn \log_{10} n \leq T(n) \leq cn \log_{10/9} n + O(n)$

More intuition

Suppose we alternate lucky, unlucky, lucky, unlucky, lucky, ….

$L(n) = 2U(n/2) + \Theta(n)$ ***lucky***

$U(n) = L(n-1) + \Theta(n)$ ***unlucky***

Solving:

$$L(n) = 2(L(n/2-1) + \Theta(n/2)) + \Theta(n)$$
$$= 2L(n/2-1) + \Theta(n)$$
$$= \Theta(n \lg n)$$

Lucky!

How can we make sure we are usually lucky?

Randomized quicksort

IDEA: Partition around a ***random*** element.

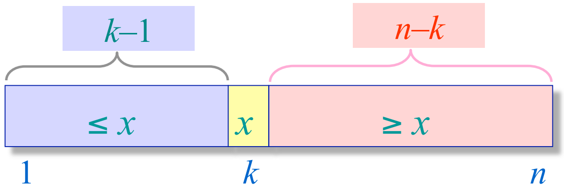
- Running time is independent of the input order.
- No assumptions need to be made about the input distribution.
- No specific input elicits the worst-case behavior.
- The worst case is determined only by the output of a random-number generator.

Analysis of Randomized Quicksort

Let $T(n)$ = the *average* time taken to sort an array of size n using Quicksort

If pivot x ends up in position k,

then $T(n) = T(k-1) + T(n-k) + (n+1)$



Prob(pivot is at pos k) = $1/n$ *for all k*

Analysis of Randomized Quicksort

Then, we have

$$T(n) = \begin{cases} T(0) + T(n-1) + (n+1) & \text{if } 0 : n-1 \text{ split} \\ T(1) + T(n-2) + (n+1) & \text{if } 1 : n-2 \text{ split} \\ T(2) + T(n-3) + (n+1) & \text{if } 2 : n-3 \text{ split} \\ \vdots & \vdots \\ T(n-1) + T(0) + (n+1) & \text{if } n-1 : 0 \text{ split} \end{cases}$$

Prob(pivot is at pos k) = $1/n$ *for all k*

$$T(n) = \sum_{k=1}^{n} \frac{1}{n} \cdot \left[T(k-1) + T(n-k) + (n+1)\right]$$

Analysis of Randomized Quicksort

Then, we have the following recurrence:

$$T(n) = \sum_{k=1}^{n} \frac{1}{n} \cdot \left[T(k-1) + T(n-k) + (n+1)\right]$$

→Expand the summations

Analysis of Randomized Quicksort

Then, we have the following recurrence:

$$T(n) = \sum_{k=1}^{n} \frac{1}{n} \cdot \left[T(k-1) + T(n-k) + (n+1)\right]$$

$$nT(n) = 2\sum_{k=0}^{n-1} T(k) + n(n+1)$$

$$nT(n) = 2\big(T(0) + T(1) + \ldots + T(n-1)\big) + n(n+1)$$

→Get rid of dependence on "full history"

Analysis of Randomized Quicksort

Then, we get rid of "full history":

$$T(n) = \sum_{k=1}^{n} \frac{1}{n} \cdot \left[T(k-1) + T(n-k) + (n+1)\right]$$

$$nT(n) = 2\left[T(0) + T(1) + \ldots + T(n-2) + T(n-1)\right] + n(n+1)$$

$$(n-1)T(n-1) = 2\left[T(0) + T(1) + \ldots + T(n-2)\right] + (n-1)n$$

$$nT(n) = (n+1)T(n-1) + 2n$$

→ ***Divide by n(n+1)…*** (make it telescopic)

Analysis of Randomized Quicksort

Divide by $n(n+1)$… (make it telescopic)

$$T(n) = \sum_{k=1}^{n} \frac{1}{n} \cdot \left[T(k-1) + T(n-k) + (n+1)\right]$$

$$nT(n) = (n+1)T(n-1) + 2n$$

$$\frac{T(n)}{n+1} = \frac{T(n-1)}{n} + \frac{2}{n+1}$$

→ ***Now "telescope"…***

Analysis of Randomized Quicksort

Now, telescope…

$$\frac{T(n)}{(n+1)} = \frac{2}{(n+1)} + \frac{T(n-1)}{(n)}$$

$$= \frac{2}{(n+1)} + \frac{2}{(n)} + \frac{T(n-2)}{(n-1)}$$

$$= \frac{2}{(n+1)} + \frac{2}{(n)} + \frac{2}{(n-1)} + \frac{T(n-3)}{(n-2)}$$

$$= \frac{2}{(n+1)} + \frac{2}{(n)} + \frac{2}{(n-1)} + \ldots + \frac{2}{(3)} + \frac{T(1)}{(2)}$$

$$\frac{T(n)}{(n+1)} = \frac{T(1)}{(2)} + 2\left[\frac{1}{(n+1)} + \frac{1}{(n)} + \frac{1}{(n-1)} + \ldots + \frac{1}{(3)}\right]$$

Analysis of Randomized Quicksort

Then, we have the following recurrence:

$$T(n) = \sum_{k=1}^{n} \frac{1}{n} \cdot \left[T(k-1) + T(n-k) + (n+1)\right]$$

$$\frac{T(n)}{(n+1)} = \frac{T(1)}{(2)} + 2\left[\frac{1}{(n+1)} + \frac{1}{(n)} + \frac{1}{(n-1)} + \ldots + \frac{1}{(3)}\right]$$

$$T(n) = 2(n+1)H(n+1) + O(n)$$

$$H(n) = \sum_{k=1}^{n} \frac{1}{k} \text{ is the Harmonic series}$$

Analysis of Randomized Quicksort

Avg running time of Randomized Quicksort:

$$T(n) = 2(n+1)H(n+1) + O(n)$$

$$H(n) = \ln n + O(1) \quad \text{[CLRS] - App.A}$$

$$T(n) = 2(n+1)\ln n + O(n)$$

$$T(n) = 1.386 n \lg n + O(n)$$

Randomized Quicksort is ***only* 38.6% *from optimal***.

Optimal sorting is $T^*(n) = (n \lg n)$ [See L.B. for Sorting]

Recap…

Beautiful analysis of Randomized Quicksort to get…

$$T(n) = 1.386 n \lg n + O(n)$$

Not that difficult, *right?*

Where are the key steps?

- Get rid of full history
- Telescope

Recap: The Key Steps

This recurrence depends on *full history*

$$n \cdot T(n) = 2\sum_{k=0}^{n} T(k) + n(n+1)$$

Step 1: *Get rid of full history*… to get

$$n \cdot T(n) = (n+1)T(n-1) + 2n$$

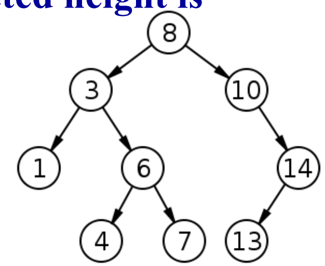
Step 2: Get to a form that can *telescope*…

$$\frac{T(n)}{(n+1)} = \frac{T(n-1)}{(n)} + \frac{2}{(n+1)}$$

Using the result…

Using a similar analysis, we can show…

- **For a randomly built *n*-node BST (binary search tree), the expected height is**
  - **1.386 lg *n***



- ***Try it out yourself…***

  Or read [CLRS]-C12.4

Randomized quicksort analysis [by CLRS]

[CLRS] uses a slightly different analysis …

Let $T(n)$ = the random variable for the running time of randomized quicksort on an input of size n, assuming random numbers are independent.

For $k = 0, 1, \ldots, n-1$, define the ***indicator random variable***

Quicksort in practice

- Quicksort is a great general-purpose sorting algorithm.
- Quicksort is typically over twice as fast as merge sort.
- Quicksort can benefit substantially from ***code tuning***.
- Quicksort behaves well even with caching and virtual memory.

Thank you.

Q & A

School of Computing

CS3230 Lecture 4 (revised)

School of Computing

"A Review of Sorting, Quicksort Analysis, and Augmenting Data Structures"

❑ **Lecture Topics and Readings**

- ❖ **(Quick Review) of Sorting Methods [CLRS]-C?**
- ❖ **Quicksort and Randomized QS [CLRS]-C7**
- ❖ **Augmenting Data Structures [CLRS]-C14**

Creative View of Sorting Methods
Quicksort (only 40% sub-optimal)
Be more aware of augmenting Data Structure

Augmenting Data Structures

- **Why augment a data structure?**
  - **When "standard" data structures are not adequate**
  - **Need to support *more* operations *efficiently***
- **Readings: [CLRS]-C14**

Note: For CS3230 Spring 2014, we use AVL tree (instead of the Red-Black tree) as our balanced BST.
So, when reading the notes and textbook, replace all references to "Red-Black tree" with "AVL tree".

Dynamic order statistics

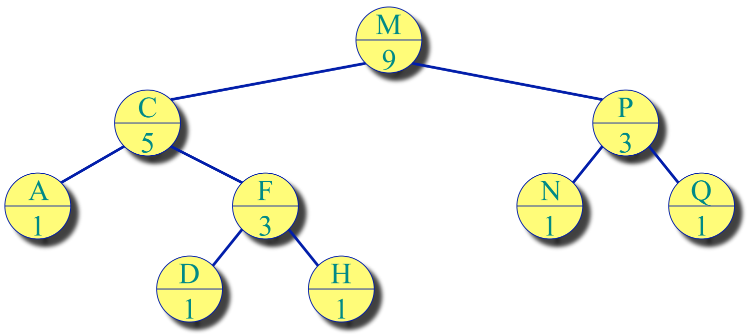
OS-SELECT(i, S): returns the ith smallest element in the dynamic set S.

OS-RANK(x, S): returns the rank of $x \in S$ in the sorted order of S's elements.

IDEA: Use an AVL tree for the set S, but keep subtree sizes in the nodes.

Notation for each node:
& *balance* (not shown)

Example of an OS-tree



$$size[x] = size[left[x]] + size[right[x]] + 1$$

Selection

Implementation trick: Use a ***sentinel*** (dummy record) for NIL such that $size[\text{NIL}] = 0$.

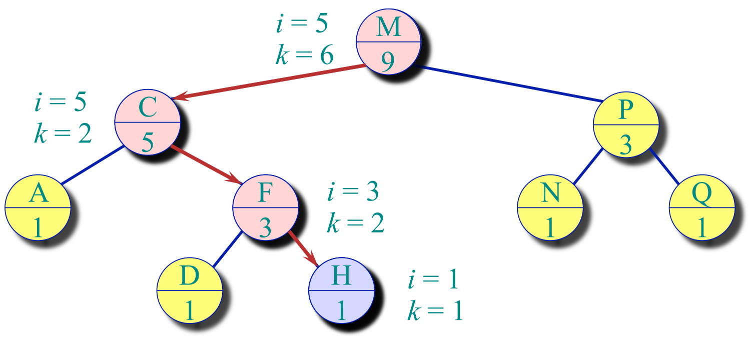
OS-SELECT(x, i) ▹ ith smallest element in the subtree rooted at x

$k \leftarrow size[left[x]] + 1$ ▹ $k = \text{rank}(x)$
if $i = k$ **then return** x
if $i < k$
 then return OS-SELECT($left[x]$, i)
 else return OS-SELECT($right[x]$, $i - k$)

(OS-RANK is in the textbook.)

Example

OS-SELECT(*root*, 5)



Running time = $O(h) = O(\lg n)$ for AVL trees.

Data structure maintenance

Q. Why not keep the ranks themselves in the nodes instead of subtree sizes?

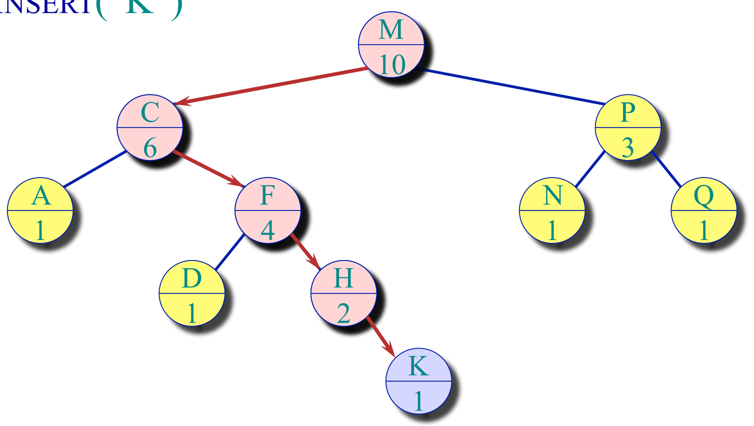
A. They are hard to maintain when the AVL tree is modified.

Modifying operations: INSERT and DELETE.

Strategy: Update subtree sizes when inserting or deleting.

Example of insertion

INSERT("K")

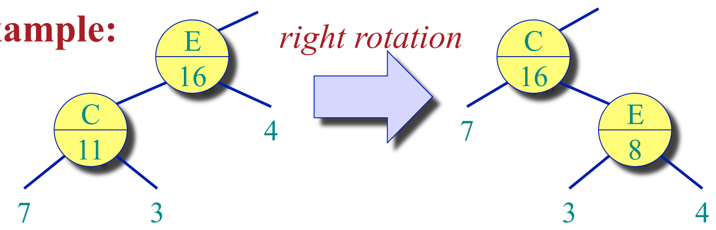


Handling rebalancing (updated)

Don't forget that AVL-INSERT and AVL-DELETE may also need to modify the AVL tree in order to maintain AVL tree *size* & *balance*.

- *Rotations*: fix up subtree sizes in $O(1)$ time & update *balance* of nodes affected

Example:



∴ AVL-INSERT & AVL-DELETE still run in $O(\lg n)$ time.

Data-structure augmentation

Methodology: (*e.g., order-statistics trees*)

1. Choose an underlying data structure (*AVL trees*).
2. Determine additional information to be stored in the data structure (*subtree sizes*).
3. Verify that this information can be maintained for modifying operations (*AVL-INSERT, AVL-DELETE – don't forget rotations*).
4. Develop new dynamic-set operations that use the information (*OS-SELECT and OS-RANK*).

These steps are *guidelines*, not rigid rules.

Augmenting Data Structures

- **Optional Readings:**
  - **Read up [CLRS] C14.3 Interval Trees**
- **Homework:**
  - **Try R-problem: [CLRS] Ex 14.1-1, 14.1-2**

Thank you.

Q & A

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