CS3230
Tutorial 6

1. Consider the greedy algorithm for coin-change problem.

   Suppose the coin denominations are $d_1 > d_2 > \ldots > d_n = 1$.

   Suppose that d_{i+1} is a factor of d_i, for $1 \leq i < n$.

   Then, show that the greedy algorithm is optimal.

2. (a) Suppose we modify the greedy algorithm for fractional knapsack problem to consider the objects in order of "non-increasing" value (rather than non-increasing ratio of value/weight as done in class).

   Is the modified algorithm still optimal? If so, give an argument for its optimality. If not, give a counterexample.

   (b) Suppose we modify the greedy algorithm to consider the objects in order of "non-decreasing" weight (rather than non-increasing ratio of value/weight as done in class).

   Is the modified algorithm still optimal? If so, give an argument for its optimality. If not, give a counterexample.

3. Using the algorithm done in class, give Huffman tree and code if the frequencies of the letters are as follows:

   $freq(a) = 25$, $freq(b) = 2$, $freq(c) = 5$, $freq(d) = 6$, $freq(e) = 6$, $freq(f) = 6$

4. Suppose T is a Huffman coding tree for the frequencies $f_1, f_2, f_3, \ldots, f_n$, where f_1 and f_2 have the same parent. Consider the tree T' with f_1 and f_2 deleted, and the parent of f_1 and f_2 labeled with frequency $f_1 + f_2$.

   Consider the following conjecture: If T' is optimal for frequencies $f_1 + f_2, f_3, \ldots, f_n$ then T is optimal for $f_1, f_2, \ldots, f_n$.

   Either prove the conjecture to be true or give a counterexample.

5. Consider the following undirected graph.

   $G = (V, E)$, where the set of vertices is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ and edges and their weights are given as follows:

   $wt(1,2) = 3$, $wt(1,5) = 2$, $wt(1,4) = 10$, $wt(2,3) = 4$, $wt(2,5) = 9$, $wt(3,5) = 6$,

   $wt(3,6) = 5$, $wt(4,5) = 4$, $wt(4,7) = 4$, $wt(5,6) = 3$, $wt(5,7) = 6$, $wt(5,8) = 2$,

   $wt(5,9) = 6$, $wt(6,9) = 6$, $wt(7,8) = 8$, $wt(7,10) = 3$, $wt(8,9) = 8$, $wt(8,10) = 3$,

   $wt(8,11) = 5$, $wt(8,12) = 7$, $wt(9,12) = 4$, $wt(10,11) = 5$, $wt(11,12) = 2$

   Use Dijkstra's algorithm to find the shortest path to all the nodes from node 8.