CS3230: Design and Analysis of Algorithms (Fall 2014)
Tutorial Set #8
[For discussion during Week 10]
S-Problems are due (outside Prof. Leong's office): Friday, 17-Oct, before noon.

OUT: 10-Oct-2014 **Tutorials:** Tue & Wed, 21, 22 Oct 2014

IMPORTANT: Read "Remarks about Homework".

Submit solutions to S-Problem(s) by deadline given above.
Prepare your answers to all the D-Problems in every tutorial set.

When preparing to present your answers,
- Think of a CLEAR EXPLANATION
- Illustrate with a good worked example;
- Describe the main ideas,
- Can you sketch why the solution works;
- Give analysis of running time, if appropriate
- Can you think of other (perhaps simpler) solutions?

Please note that everywhere below we are asking for Karp reductions unless otherwise specified.

Please take the definitions of the problems below from the lecture notes.

Routine Practice Problems *-- do not turn these in -- but make sure you know how to do them.*

R1. Is P inside co-NP?

Answer. Yes. Let L $\in$ P , then $\overline{L}$ $\in$ P (since P is closed under complementation). This implies, $\overline{L}$ $\in$ NP (since P is inside NP). This implies L $\in$ co-NP (by definition of co-NP) and hence P $\subseteq$ co-NP.

(Please note that co-NP is not included in the syllabus for exams).

R2. TAUTOLOGY, MST (Minimum Spanning Tree), FACTORIZE, and SUBSET-SUM belong to which class? Choice: P/NP/co-NP/NP-complete ?

Answer. TAUTOLOGY $\in$ co-NP ; MST $\in$ P ; FACTORIZE $\in$ NP ; SUBSET-SUM $\in$ NP-complete.

R3. Is this statement true: "A polynomial time solution does not exists for SUBSET-SUM?"

Answer. No. We do not know if P is equal to NP or not.

R4. Show that a graph is 2-colorable if and only if it is bipartite.

Answer. (if) Let G be two colorable with colors red and blue. Let V_1 be the set of vertices colored red and V_2 be the set of vertices colored blue. This means that there are no edges inside V_1 and no edges inside V_2. Hence G is a bipartite graph.

(only if) Let G be a bipartite graph. Color the vertices on the left blue and color the vertices on the right red. This is a valid 2 coloring.

S-Problems: (To do and submit by due date given in page 1)
Solve this S-problem(s) and submit for grading.

IMPORTANT: Write your NAME, Matric No, Tutorial Group in your Answer Sheet.

S1. [Exhibiting NP-completeness]

Show that PARTITION is NP-complete.

Hint: May reduce from SUBSET-SUM.

Answer. Please refer to page 13 in T8-Fall-2014-SS1.pdf .

D-Problems: Solve these D-problems and prepare to discuss them in tutorial class. You may be called upon to present your solution *or your best attempt at a solution*. Your solution presentation does NOT need to be fully correct, given your best attempt. The TA will help clarify and correct any issues or errors.

D1. [Reduction by simple equivalence]

Show that VERTEX-COVER $\equiv_P$ INDEPENDENT-SET

Answer. Please refer to page 1-4 in T8-Fall-2014-SS1.pdf .

D2. [Reduction from special case to general case]

Show that VERTEX-COVER $\leq_P$ SET-COVER

Answer. Please refer to pages 5-6 in T8-Fall-2014-SS1.pdf .

D3. [Reductions using gadgets]

a) Show that DIR-HAM-CYCLE $\leq_P$ HAM-CYCLE
Answer. Please refer to pages 8-10 in T8-Fall-2014-SS1.pdf .

b) Show that 3-SAT $\leq_P$ LONGEST-PATH
Answer. Please refer to page 11 in T8-Fall-2014-SS1.pdf .

c) Show that HAM-CYCLE $\leq_P$ TSP
Answer. Please refer to page 12 in T8-Fall-2014-SS1.pdf .

D4. [Self reducibility]

VERTEX-COVER = Does there exist a vertex cover of size $\leq$ k?
VERTEX-COVER-SEARCH = Find vertex cover of minimum cardinality.

Show (via Cook reductions) that VERTEX-COVER $\equiv_P$ VERTEX-COVER-SEARCH

Answer. Please refer to page 7 in T8-Fall-2014-SS1.pdf .