Appendix

Conditional Posterior distribution

The conditional posterior distributions for the common parameters are given as follows:

$$\begin{aligned}
\pi(\theta_k) &\propto \left[\prod_{K=1}^{N}\prod_{i=1}^{P}\mathbb{P}(Y_{k,i}=y_{k,i}|\boldsymbol{\Theta})\right]\times\left[N_{\theta_k}(0,\sigma^2)\right]\\
\pi(\beta_i) &\propto \left[\prod_{k=1}^{N}\prod_{i=1}^{P}\mathbb{P}(Y_{k,i}=y_{k,i}|\boldsymbol{\Theta})\right]\times\left[N_{\beta_i}(0,\tau_\beta^2)\right]\\
\pi(\gamma) &\propto \left[\prod_{k=1}^{N}\prod_{i=1}^{P}\mathbb{P}(Y_{k,i}=y_{k,i}|\boldsymbol{\Theta})\right]\times\left[\text{Log-normal}_\gamma(\mu_\gamma,\tau_\gamma^2)\right]\\
\pi(\boldsymbol{z_k}) &\propto \left[\prod_{k=1}^{N}\prod_{i=1}^{P}\mathbb{P}(Y_{k,i}=y_{k,i}|\boldsymbol{\Theta})\right]\times\left[\text{MVN}_{D,\boldsymbol{z_k}}(\boldsymbol{0},\boldsymbol{I_D})\right]\\
\pi(\boldsymbol{w_i}) &\propto \left[\prod_{k=1}^{N}\prod_{i=1}^{P}\mathbb{P}(Y_{k,i}=y_{k,i}|\boldsymbol{\Theta})\right]\times\left[\text{MVN}_{D,\boldsymbol{w_i}}(\boldsymbol{0},\boldsymbol{I_D})\right]\\
\pi(\sigma^2) &\propto \text{Inv-Gamma}\left(\left(\frac{N}{2}+a_\sigma\right),\frac{1}{2}\sum_{k=1}^{N}\theta_k^2+b_\sigma\right).
\end{aligned} \tag{1}$$

The conditional posterior distributions of α and σ_ϵ^2 for the 2PL LSIRM and the LSIRM-continuous, respectively, are given as follows:

$$\begin{aligned}
\pi(\alpha_i) &\propto \left[\prod_{k=1}^{N}\prod_{i=1}^{P}\mathbb{P}(Y_{k,i}=y_{k,i}|\theta_k,\alpha_i,\beta_i,\gamma,\boldsymbol{z_k},\boldsymbol{w_i})\right]\times\left[\text{Log-normal}_{\alpha_i}(0,\tau_\alpha^2)\right]\\
\pi(\sigma_\epsilon^2) &\propto \text{Inv-Gamma}\left(\left(\frac{NP}{2}+a_{\sigma_\epsilon}\right),\frac{1}{2}\sum_{k=1}^{N}\sum_{i=1}^{P}\left(y_{k,i}-(\theta_k+\beta_i-\gamma||\boldsymbol{z_k}-\boldsymbol{w_i}||)\right)^2+b_{\sigma_\epsilon}\right),
\end{aligned}$$

where $\boldsymbol{\Theta}=\{\theta_k,\beta_i,\gamma,\boldsymbol{z_k},\boldsymbol{w_i}\}$ for 1PL LSIRM, $\boldsymbol{\Theta}=\{\theta_k,\alpha_i,\beta_i,\gamma,\boldsymbol{z_k},\boldsymbol{w_i}\}$ for 2PL LSIRM, and $\boldsymbol{\Theta}=\{\theta_k,\beta_i,\gamma,\boldsymbol{z_k},\boldsymbol{w_i},\sigma_\epsilon^2\}$ for 1PL LSIRM-continuous.

Default value for the prior specification and the jumping rules

Parameter	Models	Arguments	Default value
θ	`lsirm1pl` `lsirm2pl`	`pr_mean_theta`	0
β	`lsirm1pl` `lsirm2pl`	`pr_mean_beta` `pr_sd_beta`	0 1
α	`lsirm2pl`	`pr_mean_alpha` `pr_sd_alpha`	0.5 1
$\log \gamma$	`lsirm1pl` `lsirm2pl`	`pr_mean_gamma` `pr_sd_gamma`	0.5 1
σ^2	`lsirm1pl` `lsirm2pl`	`pr_a_theta` `pr_b_theta`	0.001 0.001
σ_ϵ^2	`lsirm1pl` `lsirm2pl`	`pr_a_eps` `pr_b_eps`	0.001 0.001

Table 1: Model parameters, related models, corresponding arguments, and default values for priors. Note that for $\log \gamma$, the mode of its prior distribution is 0.61, with mean 2.72, and standard deviation 3.56.

Parmeter	Models	Arguments	Default value
θ	`lsirm1pl` `lsirm2pl`	`jump_theta`	1
β	`lsirm1pl` `lsirm2pl`	`jump_beta`	0.4
α	`lsirm2pl`	`jump_alpha`	1
$\log \gamma$	`lsirm1pl` `lsirm2pl`	`jump_gamma`	0.025
$\boldsymbol{z}$	`lsirm1pl` `lsirm2pl`	`jump_z`	0.5
$\boldsymbol{w}$	`lsirm1pl` `lsirm2pl`	`jump_w`	0.5

Table 2: Model parameters, related models, corresponding arguments, and default values for jumping rules.

List of package argument

Arguments	Description
data	Item response matrix to be analyzed.
ndim	Dimension of latent space.
niter	Number of iterations to run MCMC sampling.
nburn	Number of initial, pre-thinning, MCMC iterations to discard.
nthin	Number of thinning, MCMC iterations to discard.
nprint	MCMC samples are displayed.
pr_mean_theta	Mean of the normal prior distribution for $\boldsymbol{\theta}$.
pr_mean_beta	Mean of the normal prior distribution for $\boldsymbol{\beta}$.
pr_sd_beta	Standard deviation of the normal prior distribution for $\boldsymbol{\beta}$.
pr_mean_alpha	Mean of the normal prior distribution for α.
pr_sd_alpha	Standard deviation of the normal prior distribution for α.
pr_mean_gamma	Mean of the log-normal prior distribution for γ.
pr_sd_gamma	Standard deviation of the log-normal prior distribution for γ.
pr_a_theta	Shape parameter of the inverse gamma prior for the variance of $\boldsymbol{\theta}$, σ.
pr_b_theta	Scale parameter of the inverse gamma prior for the variance of $\boldsymbol{\theta}$, σ.
pr_a_eps	Shape parameter of the inverse gamma prior for the variance of data likelihood, σ_ϵ^2.
pr_b_eps	Scale parameter of the inverse gamma prior for the variance of data likelihood, σ_ϵ^2.
jump_theta	Jumping rule of the proposal density for $\boldsymbol{\theta}$.
jump_beta	Jumping rule of the proposal density for $\boldsymbol{\beta}$.
jump_alpha	Jumping rule of the proposal density for α.
jump_gamma	Jumping rule of the proposal density for γ.
jump_z	Jumping rule of the proposal density for z.
jump_w	Jumping rule of the proposal density for w.
missing	Replaced value for missing data.

Table 3: Arguments for prior distributions, jumping rules, and others.

Arguments	Description
data	Item response matrix to be analyzed.
bic	Bayesian information criterion value.
mcmc_inf	Number of MCMC iteration, burn-in periods, and thinning intervals.
map_inf	Log maximum a posterior.
beta_estimate	Posterior estimates of $\boldsymbol{\beta}$.
theta_estimate	Posterior estimates of $\boldsymbol{\theta}$.
sigma_theta_estimate	Posterior estimate of the standard deviation of $\boldsymbol{\theta}$, σ.
gamma_estimate	Posterior estimates of γ.
z_estimate	Posterior estimates of $\boldsymbol{z}$.
w_estimate	Posterior estimates of $\boldsymbol{w}$.
beta	Posterior samples of $\boldsymbol{\beta}$.
theta	Posterior samples of $\boldsymbol{\theta}$.
theta_sd	Posterior samples of the standard deviation of $\boldsymbol{\theta}$, σ.
gamma	Posterior samples of γ.
sigma	Posterior samples of σ_ϵ.
z	Posterior samples of $\boldsymbol{z}$.
w	Posterior samples of $\boldsymbol{w}$.
accept_beta	Accept ratios of $\boldsymbol{\beta}$.
accept_theta	Accept ratios of $\boldsymbol{\theta}$.
accept_z	Accept ratios of $\boldsymbol{z}$.
accept_w	Accept ratios of $\boldsymbol{w}$.
accept_gamma	Accept ratio of γ.

Table 4: Arguments for the result list returned by `lsirm1pl`.