

### Task 3

$$L(p_w(\theta), \eta, k, \lambda) = \int p_w(\theta) g(\theta) d\theta + \eta (E - KL(p_w(\theta) || p_{old}(\theta))) + k (H(p_w(\theta)) - \beta) + \lambda (1 - \int p_w(\theta) d\theta)$$

$$= \eta E - k\beta + \lambda + \int p_w(\theta) g(\theta) d\theta - \eta \cdot KL(p_w(\theta) || p_{old}(\theta)) + k \cdot H(p_w(\theta)) - \lambda \int p_w(\theta) d\theta$$

$$= \eta E - k\beta + \lambda + \int p_w(\theta) (g(\theta) - \eta (\log p_w(\theta) - \log p_{old}(\theta)) - k \log p_w(\theta) - \lambda) d\theta$$

$$= \eta E - k\beta + \lambda + \int p_w(\theta) (g(\theta) - (\eta + k) \log p_w(\theta) + \eta \log p_{old}(\theta) - \lambda) d\theta$$

$$\frac{\partial L}{\partial p_w(\theta)} = \int \frac{\partial}{\partial p_w(\theta)} p_w(\theta) (g(\theta) - (\eta + k) \log p_w(\theta) + \eta \log p_{old}(\theta) - \lambda) d\theta$$

$$= \int \left( p_w(\theta) \frac{-(\eta + k)}{p_w(\theta)} + g(\theta) - (\eta + k) \log p_w(\theta) + \eta \log p_{old}(\theta) - \lambda \right) d\theta$$

$$= \int -(\eta + k + \lambda) + g(\theta) - (\eta + k) \log p_w(\theta) + \eta \log p_{old}(\theta) d\theta \stackrel{!}{=} 0$$

$$-(\eta + k + \lambda) + g(\theta) - (\eta + k) \log p_w(\theta) + \eta \log p_{old}(\theta) \stackrel{!}{=} 0$$

$$\Leftrightarrow -(\eta + k + \lambda) + g(\theta) + \eta \log p_{old}(\theta) = (\eta + k) \log p_w(\theta)$$

$$p_w(\theta) = \exp\left(-\frac{\eta + k + \lambda}{\eta + k}\right) \exp\left(\frac{g(\theta)}{\eta + k}\right) p_{old}(\theta)^{\frac{\eta}{\eta + k}}$$

$$\propto \exp\left(\frac{g(\theta)}{\eta + k}\right) p_{old}(\theta)^{\frac{\eta}{\eta + k}} = \exp\left(\frac{\eta \log p_{old}(\theta) + g(\theta)}{\eta + k}\right)$$