Task 3

$$\mathcal{L}(p_w(\theta), \eta, \kappa, \lambda) = \int p_w(\theta)\, g(\theta)\, d\theta + \eta\,(\varepsilon - KL(p_w(\theta) \mid p_{old}(\theta))) + \kappa\,(H(p_w(\theta)) - \beta) + \lambda\,(1 - \int p_w(\theta)\, d\theta)$$

$$= \eta\varepsilon - \kappa\beta + \lambda + \int p_w(\theta)\, g(\theta)\, d\theta - \eta \cdot KL(p_w(\theta) \mid p_{old}(\theta)) + \kappa \cdot H(p_w(\theta)) - \lambda \int p_w(\theta)\, d\theta$$

$$= \eta\varepsilon - \kappa\beta + \lambda + \int p_w(\theta)\,(g(\theta) - \eta(\log p_w(\theta) - \log p_{old}(\theta)) - \kappa \log p_w(\theta) - \lambda)\, d\theta$$

$$= \eta\varepsilon - \kappa\beta + \lambda + \int p_w(\theta)\,(g(\theta) - (\eta + \kappa)\log p_w(\theta) + \eta \log p_{old}(\theta) - \lambda)\, d\theta$$

$$\frac{\partial \mathcal{L}}{\partial p_w(\theta)} = \int \frac{\partial}{\partial p_w(\theta)}\, p_w(\theta)\,(g(\theta) - (\eta + \kappa)\log p_w(\theta) + \eta \log p_{old}(\theta) - \lambda)\, d\theta$$

$$= \int \left(p_w(\theta)\, \frac{-(\eta + \kappa)}{p_w(\theta)} + g(\theta) - (\eta + \kappa)\log p_w(\theta) + \eta \log p_{old}(\theta) - \lambda\right) d\theta$$

$$= \int -(\eta + \kappa + \lambda) + g(\theta) - (\eta + \kappa)\log p_w(\theta) + \eta \log p_{old}(\theta)\, d\theta \overset{!}{=} 0$$

$$-(\eta + \kappa + \lambda) + g(\theta) - (\eta + \kappa)\log p_w(\theta) + \eta \log p_{old}(\theta) \overset{!}{=} 0$$

$$\Leftrightarrow -(\eta + \kappa + \lambda) + g(\theta) + \eta \log p_{old}(\theta) = (\eta + \kappa)\log p_w(\theta)$$

$$p_w(\theta) = \exp\left(-\frac{\eta + \kappa + \lambda}{\eta + \kappa}\right) \exp\left(\frac{g(\theta)}{\eta + \kappa}\right) p_{old}(\theta)^{\frac{\eta}{\eta + \kappa}}$$

$$\propto \exp\left(\frac{g(\theta)}{\eta + \kappa}\right) p_{old}(\theta)^{\frac{\eta}{\eta + \kappa}} = \exp\left(\frac{\eta \log p_{old}(\theta) + g(\theta)}{\eta + \kappa}\right)$$