Task 4

$$L(g, \eta) = g^T \nabla_\theta J + \eta(\varepsilon - g^T F g)$$

$$\frac{\partial L(g,\eta)}{\partial g} = \nabla_\theta J - 2\eta F g \overset{!}{=} 0 \quad \Leftrightarrow \quad \nabla_\theta J = 2\eta F g$$

$$g = (2\eta F)^{-1} \nabla_\theta J = \frac{1}{2} \eta^{-1} F^{-1} \nabla_\theta J$$

$$g(g^*, \eta) = \left(\frac{1}{2}\eta^{-1} F^{-1} \nabla_\theta J\right)^T \nabla_\theta J + \eta\left(\varepsilon - \left(\frac{1}{2}\eta^{-1} F^{-1} \nabla_\theta J\right)^T F \left(\frac{1}{2}\eta^{-1} F^{-1} \nabla_\theta J\right)\right)$$

$$= \frac{1}{2}\eta^{-1} \nabla_\theta J^T F^{-T} \nabla_\theta J + \eta\varepsilon - \eta \frac{1}{2}\eta^{-1} \nabla_\theta J^T F^{-T} F \frac{1}{2}\eta^{-1} F^{-1} \nabla_\theta J$$

$$= \frac{1}{4}\eta^{-1} \nabla_\theta J^T F^{-T} \nabla_\theta J + \eta\varepsilon$$

$$\frac{\partial g(g^*, \eta)}{\partial \eta} = -\frac{1}{4}\eta^{-2} \nabla_\theta J^T F^{-T} \nabla_\theta J + \varepsilon \overset{!}{=} 0$$

$$\rightarrow \eta^{-1} = \sqrt{\frac{4\varepsilon}{\nabla_\theta J^T F^{-1} \nabla_\theta J}}$$