

Logistic Regression Model



Logistic Regression

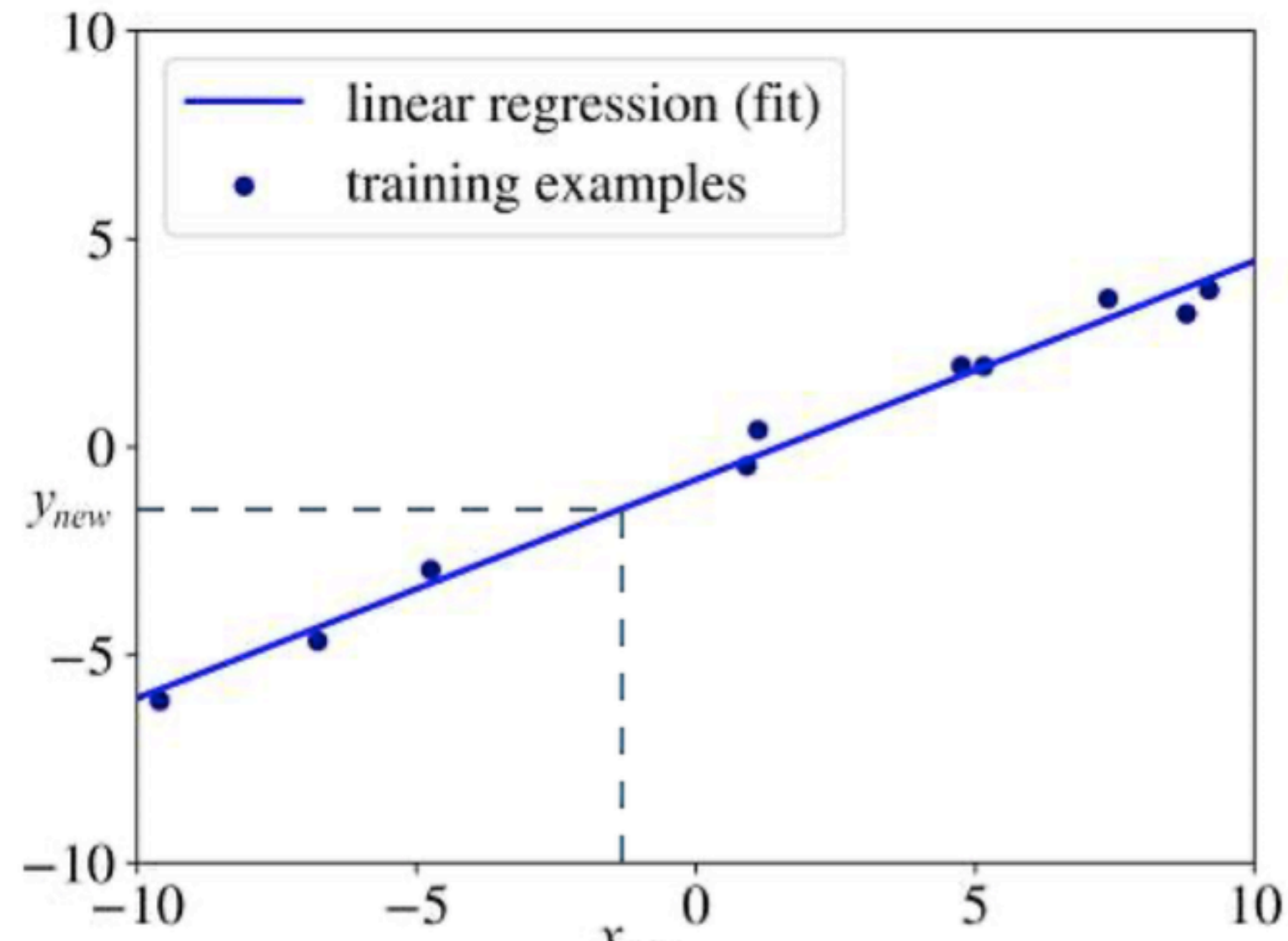
Jin Hyun Kim

Today

- Logistic Regression

Review

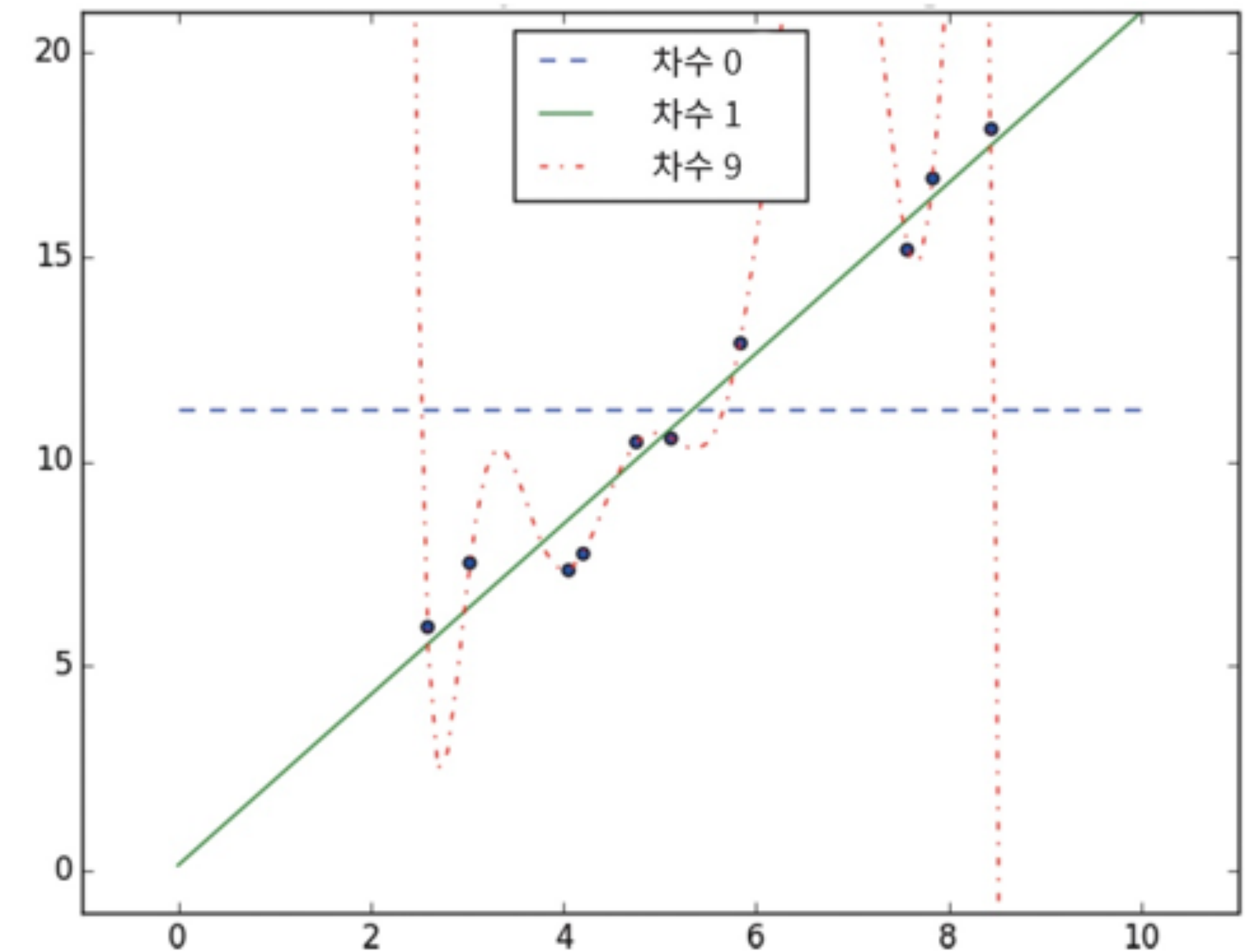
- Linear Regression



- Cost function 덧가 함수
- Gradient Descent Algorithm 경사 하강법

Overfitting vs Underfitting

- 과잉 적합(overfitting)
 - 학습하는 데이터에서는 성능이 뛰어나지만 새로운 데이터(일반화)에 대해서는 성능이 잘 나오지 않는 모델을 생성하는 것
- 과소 적합(underfitting)
 - 학습 데이터에서도 성능이 좋지 않은 경우
 - 이 경우에는 모델 자체가 적합하지 않은 경우, 따라서 더 나은 모델을 찾아야 함



Logistic Regression

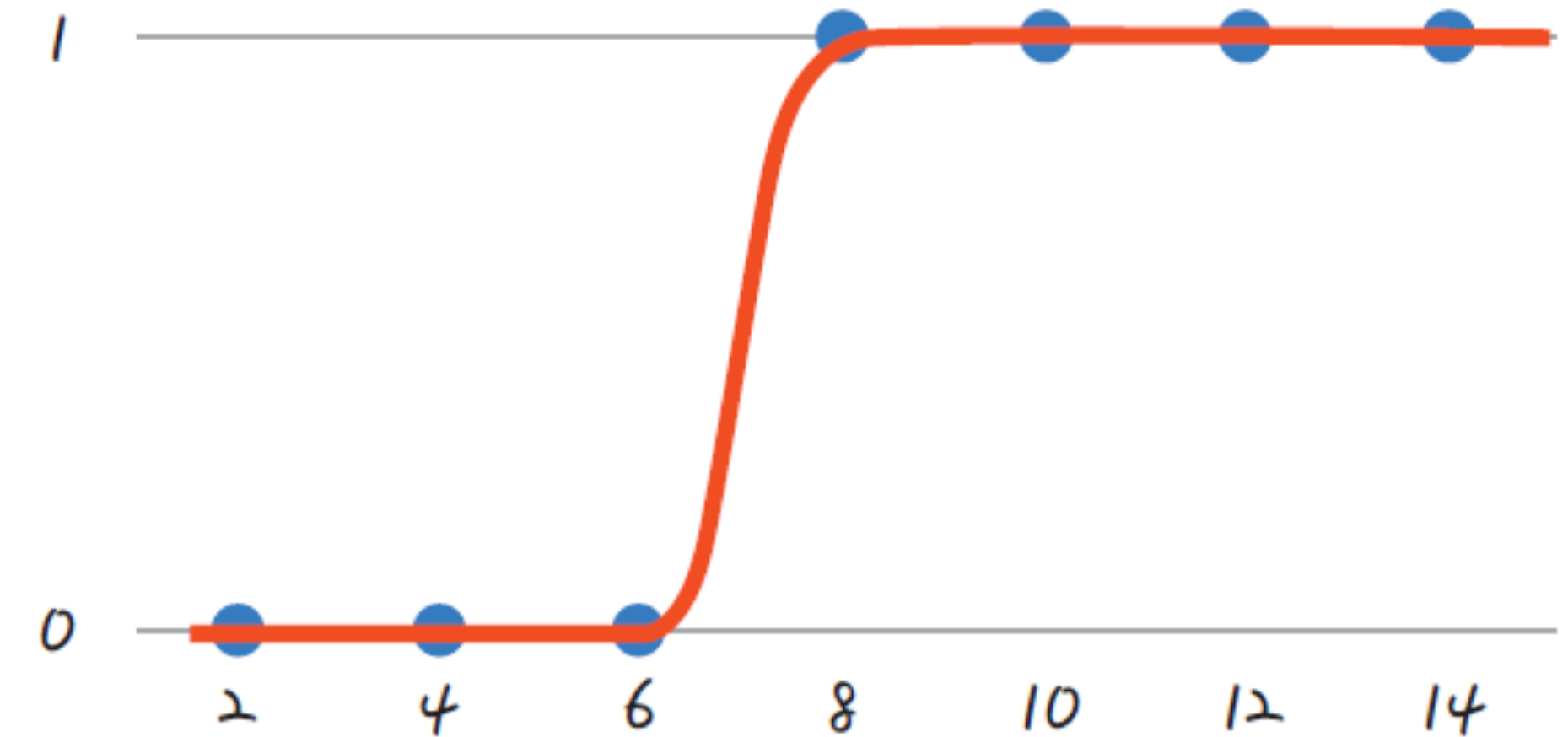
- A classification learning algorithm
- Problem
- Model y as a linear function of x , with a **binary** y

공부한 시간	2	4	6	8	10	12	14
합격 여부	불합격	불합격	불합격	합격	합격	합격	합격

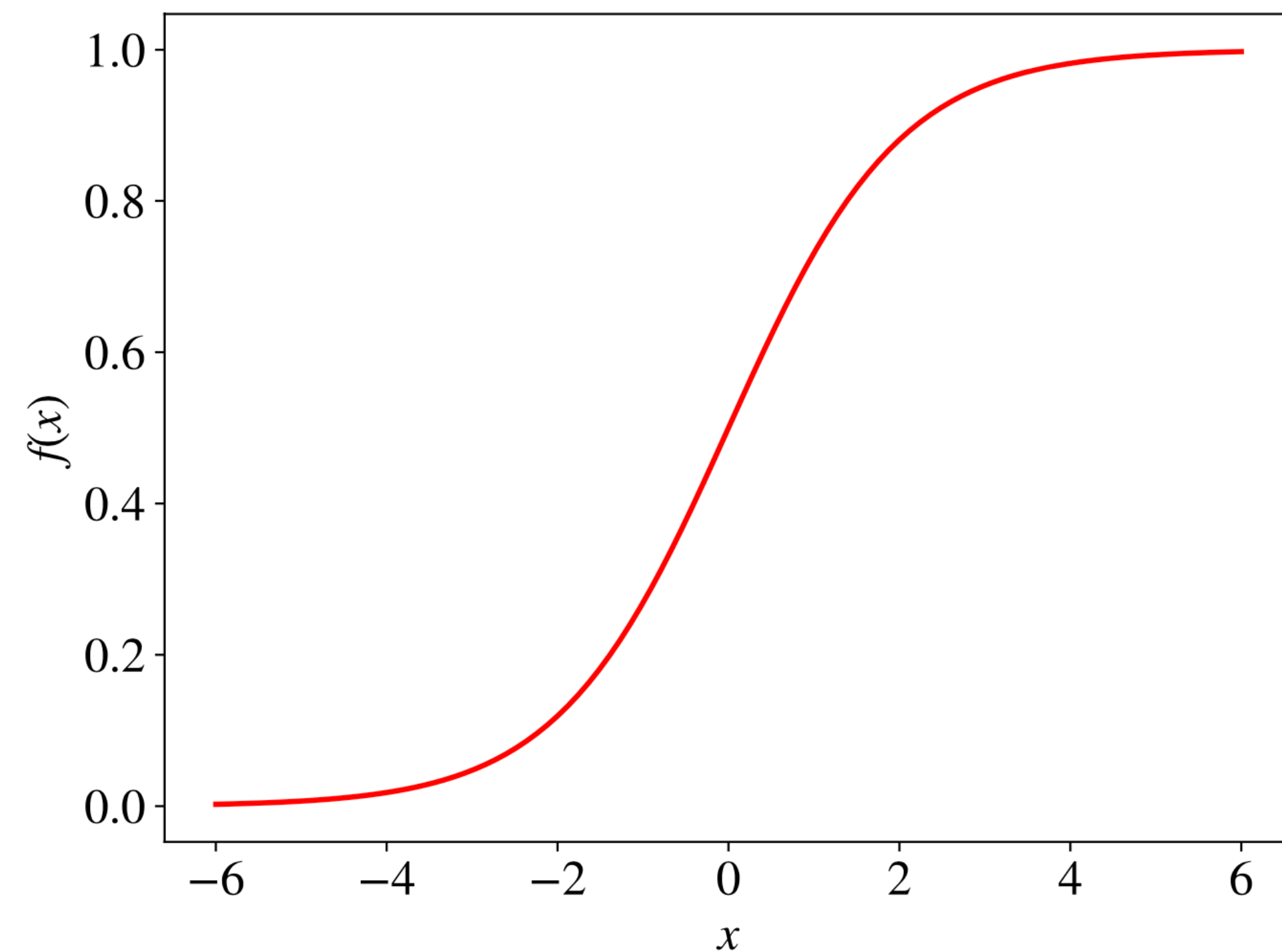
Logistic Regressin

- Using Logistic function, classify a linear values of x into binary y

공부한 시간	2	4	6	8	10	12	14
합격 여부	불합격	불합격	불합격	합격	합격	합격	합격



Logistic Function

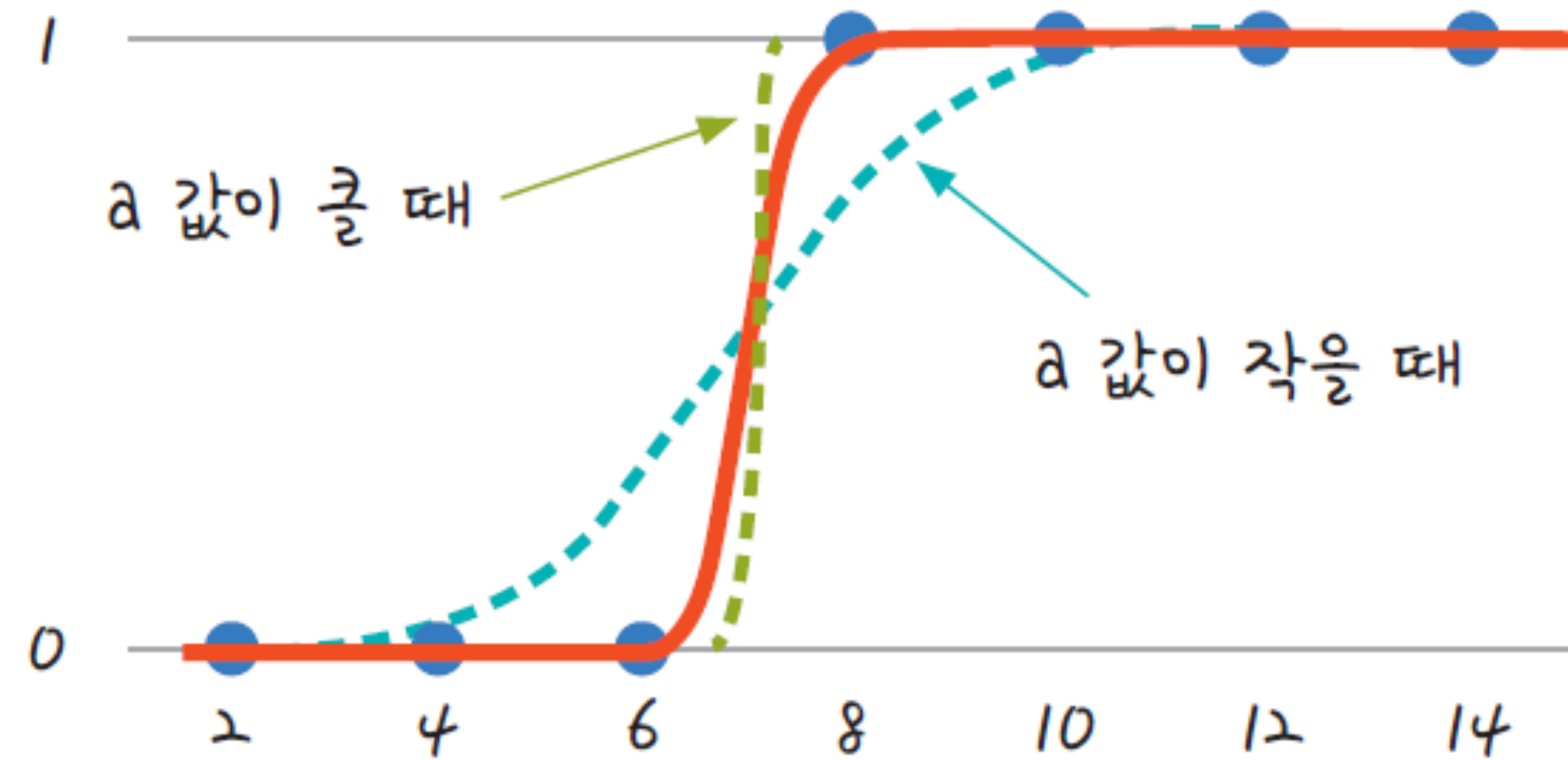


Sigmoid Function

$$y = \frac{1}{1 + e^{-(ax+b)}}$$

<https://www.desmos.com/calculator/kn9tpwdan5>

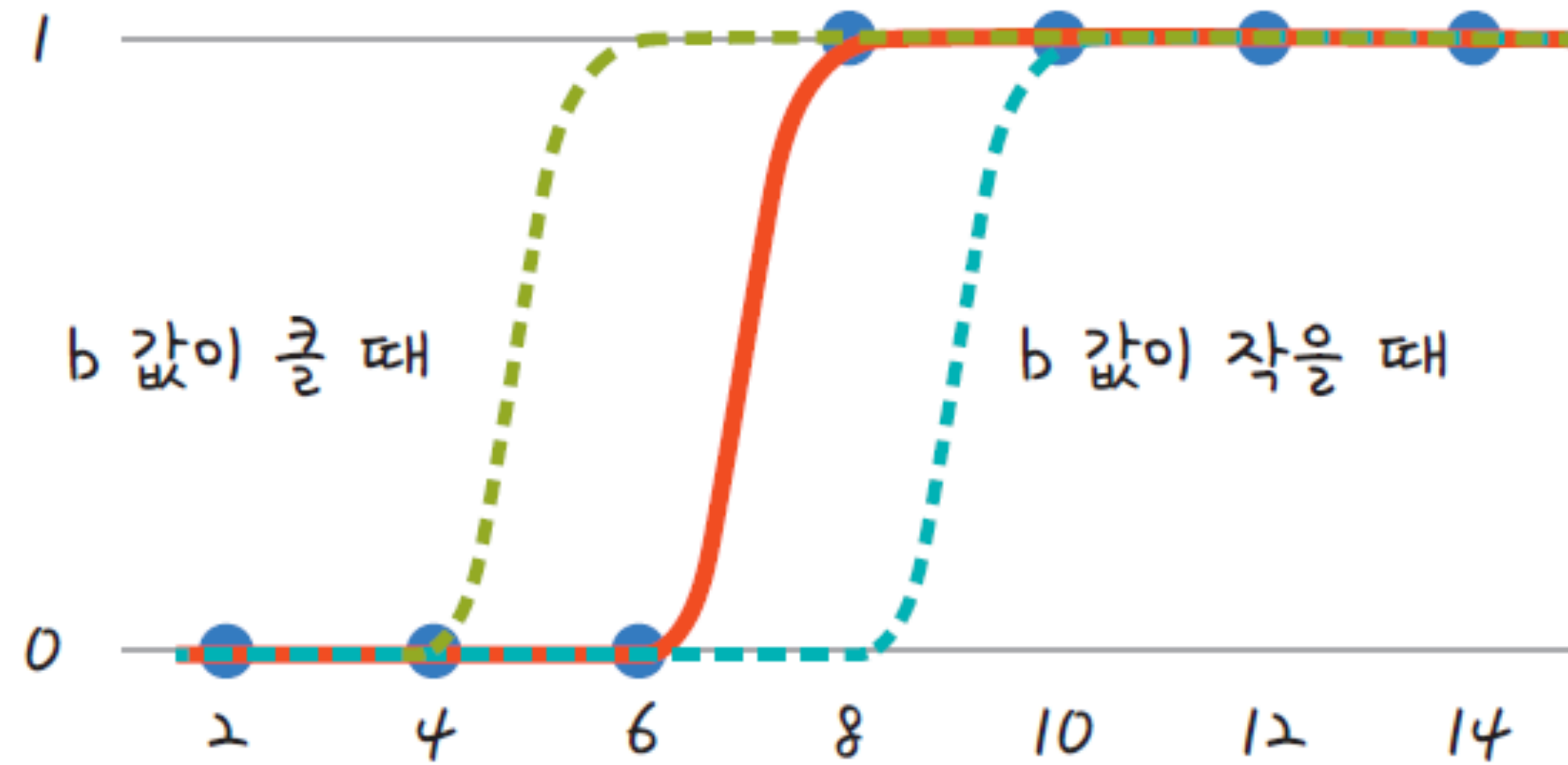
Sigmoid Function



Sigmoid Function

$$y = \frac{1}{1 + e^{-(ax+b)}}$$

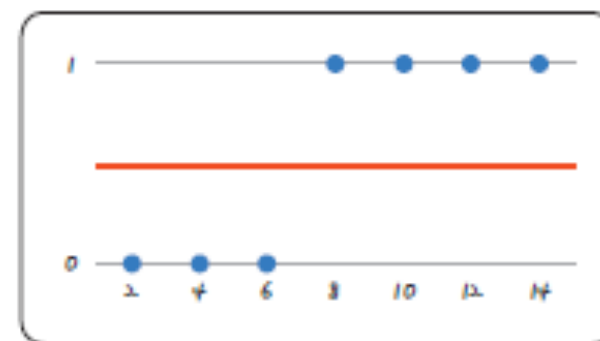
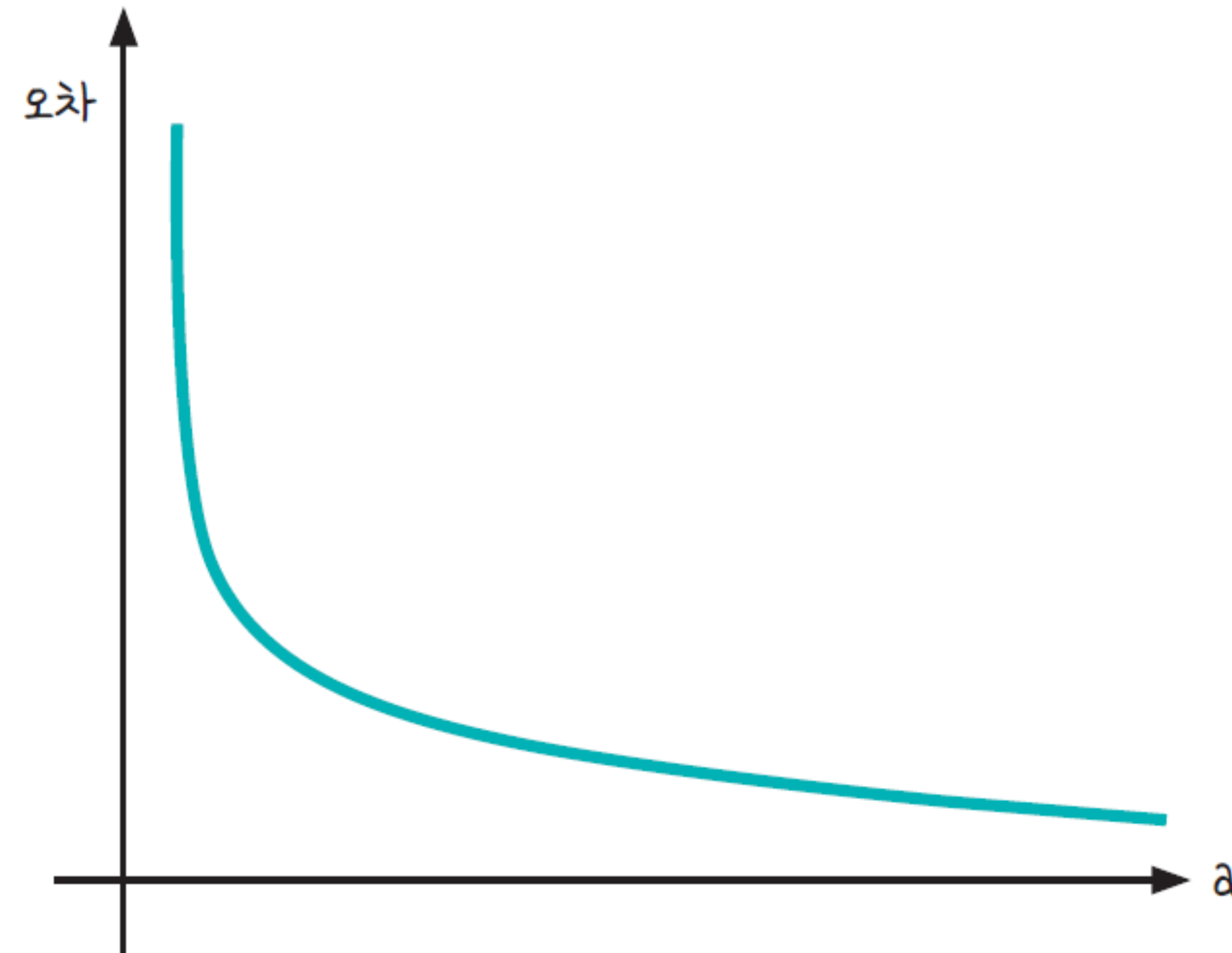
Sigmoid Function



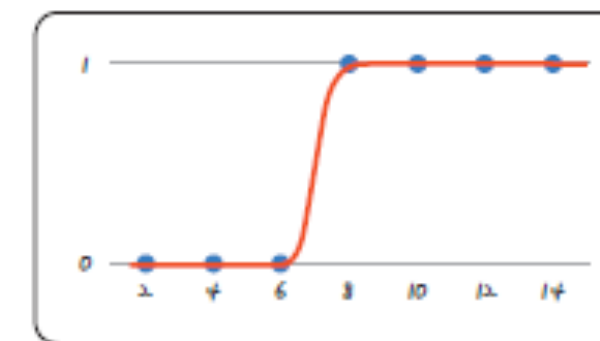
Sigmoid Function

$$y = \frac{1}{1 + e^{-(ax+b)}}$$

Sigmoid Function

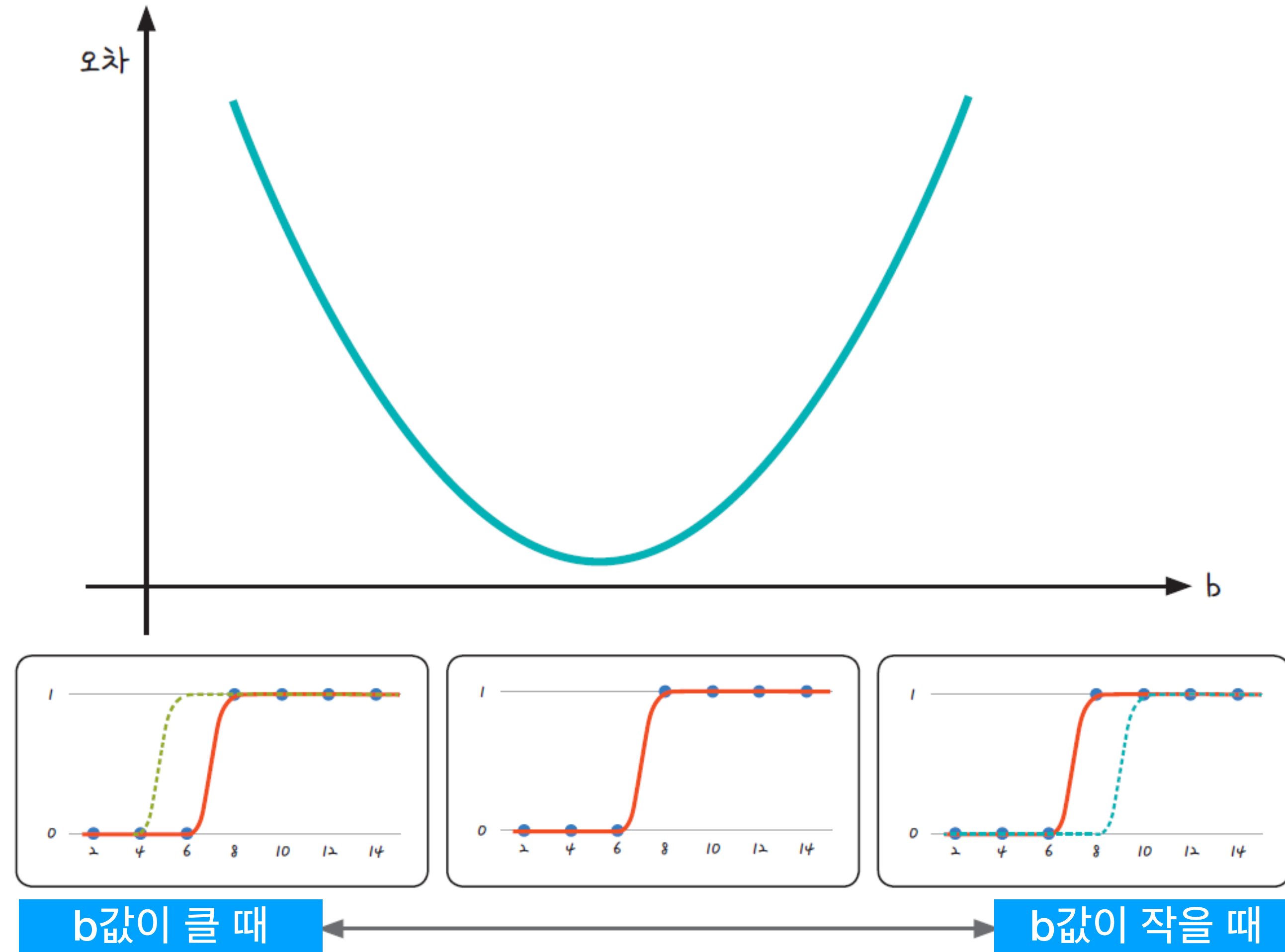


a 값이 작을 때
(0에 가까워질 때)



a 값이 클 때

Sigmoid Function



Cost Function

- Can we use the cost function of linear regression for logistic regression?

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$

$$H(x) = Wx + b$$

Linear Regression

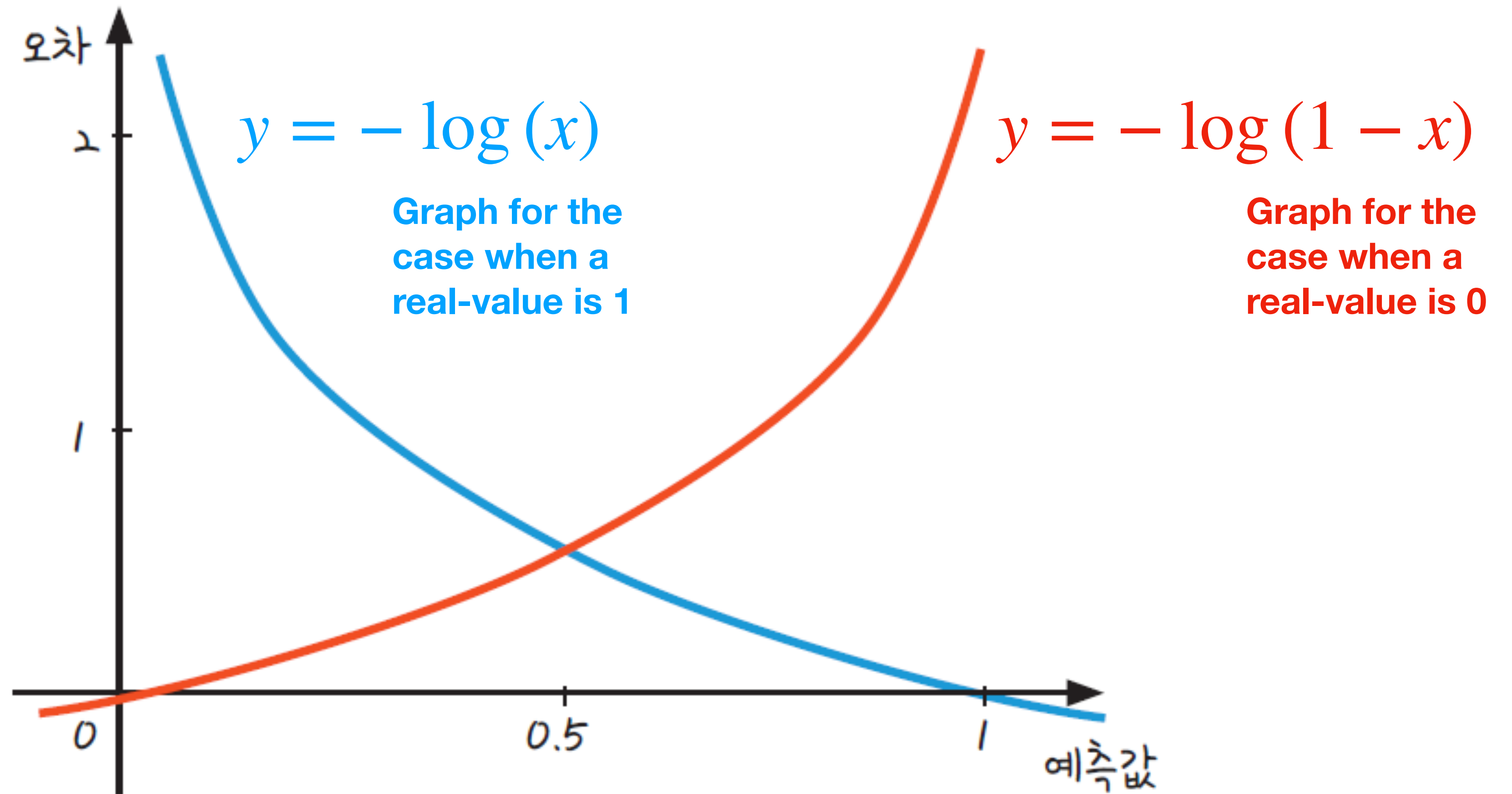
$$H(x) = \frac{1}{1 + e^{-WX}}$$

Logistic Regression

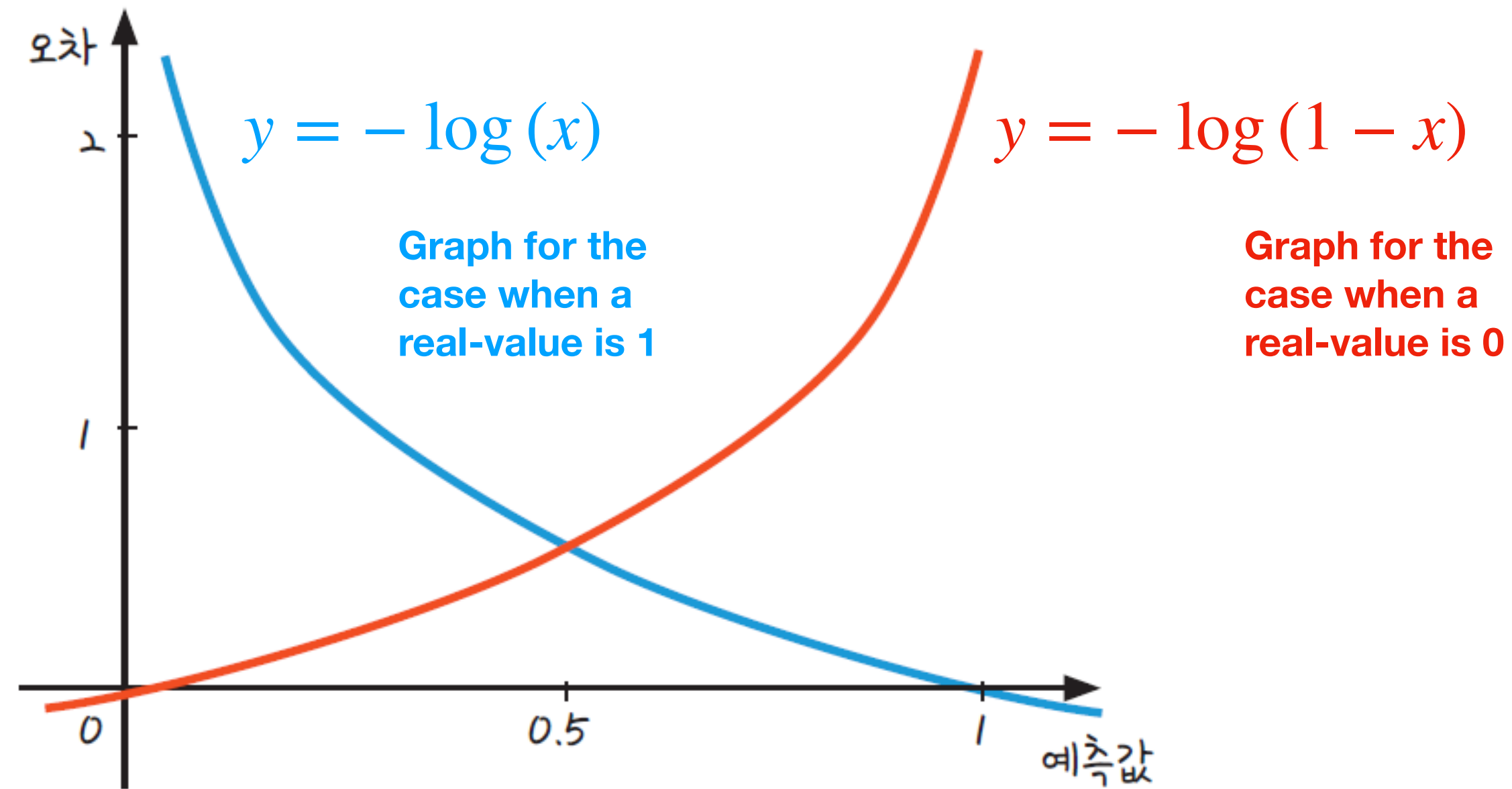
Cost Function

- A cost function we need for logistic regression is
 - If the predicted is closing to 0 when the real value is 1, then the cost function should return a bigger cost
 - If the predicted is closing to 1 when the real value is 0, then the cost function should return a bigger cost

Cost Function



Cost Function



$$\text{cost}(H(x)) = \frac{1}{N} \sum c(H(x), y) \quad \textbf{where} \quad c(H(x), y) = \begin{cases} -\log(H(x)) & \textbf{if } y = 1 \\ -\log(1 - H(x)) & \textbf{if } y = 0 \end{cases}$$

Cost Function

$$\text{cost}(H(x)) = \frac{1}{m} \sum c(H(x), y) \quad \text{where} \quad c(H(x), y) = \begin{cases} -\log(H(x)) & \text{if } y = 1 \\ -\log(1 - H(x)) & \text{if } y = 0 \end{cases}$$

Equivalent to

$$c(H(x), y) = -y \log H(x) - (1 - y) \log(1 - H(x))$$

New cost function for logistic regression

$$\text{cost}(H(x)) = \frac{1}{N} \sum -y \log H(x) - (1 - y) \log(1 - H(x))$$

Gradient Descent Algorithm

$$cost(H(x)) = \frac{1}{N} \sum -y \log H(x) - (1 - y) \log(1 - H(x))$$

$$cost(W, b) = \frac{1}{N} \sum -y \log\left(\frac{1}{1 + e^{Wx - b}}\right) - (1 - y) \log\left(1 - \left(\frac{1}{1 + e^{Wx - b}}\right)\right)$$

$$W = W - \alpha \frac{\partial}{\partial W} cost(W, b)$$

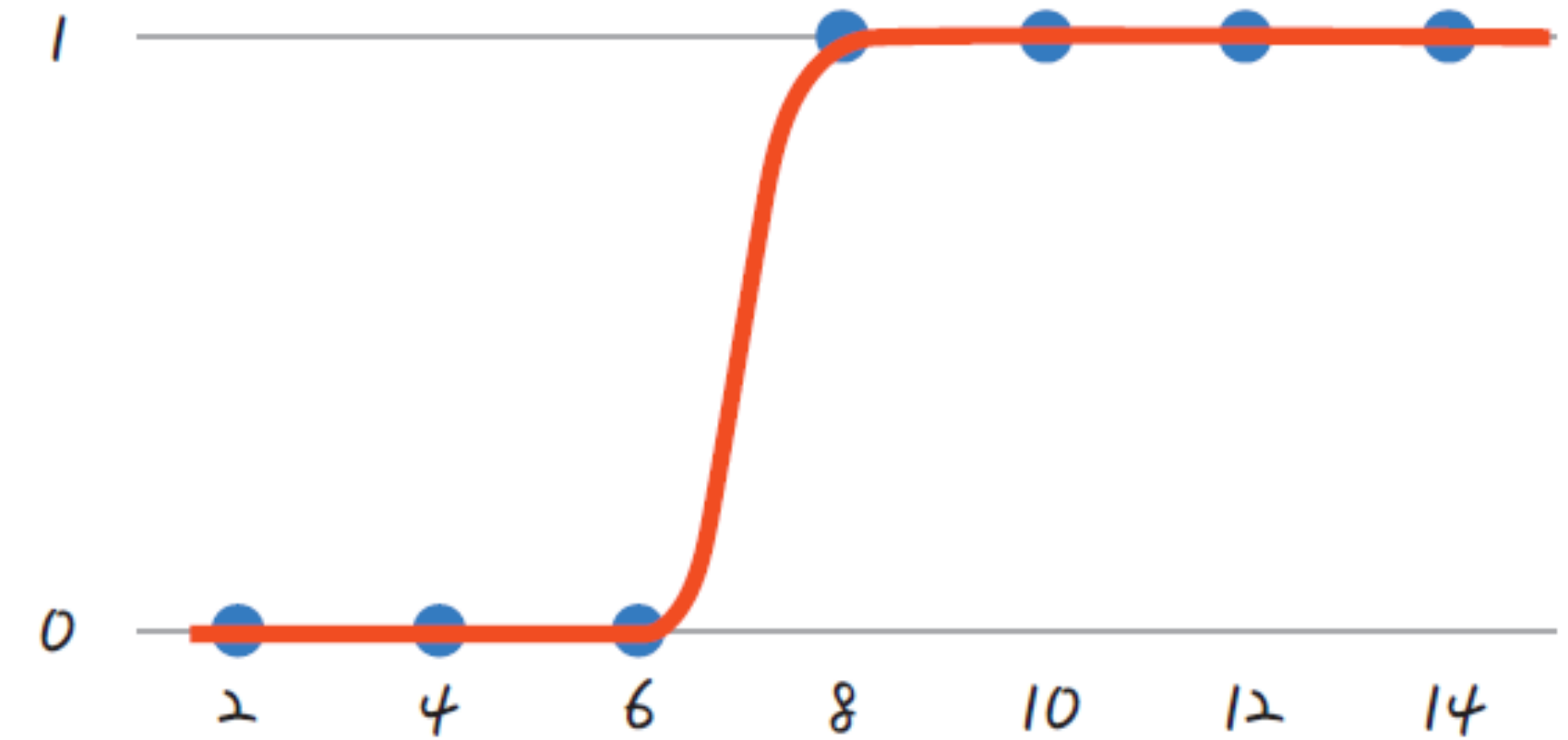
$$b = b - \alpha \frac{\partial}{\partial b} cost(W, b)$$

Next

- Logistic Regression Lab

Logistic Regression

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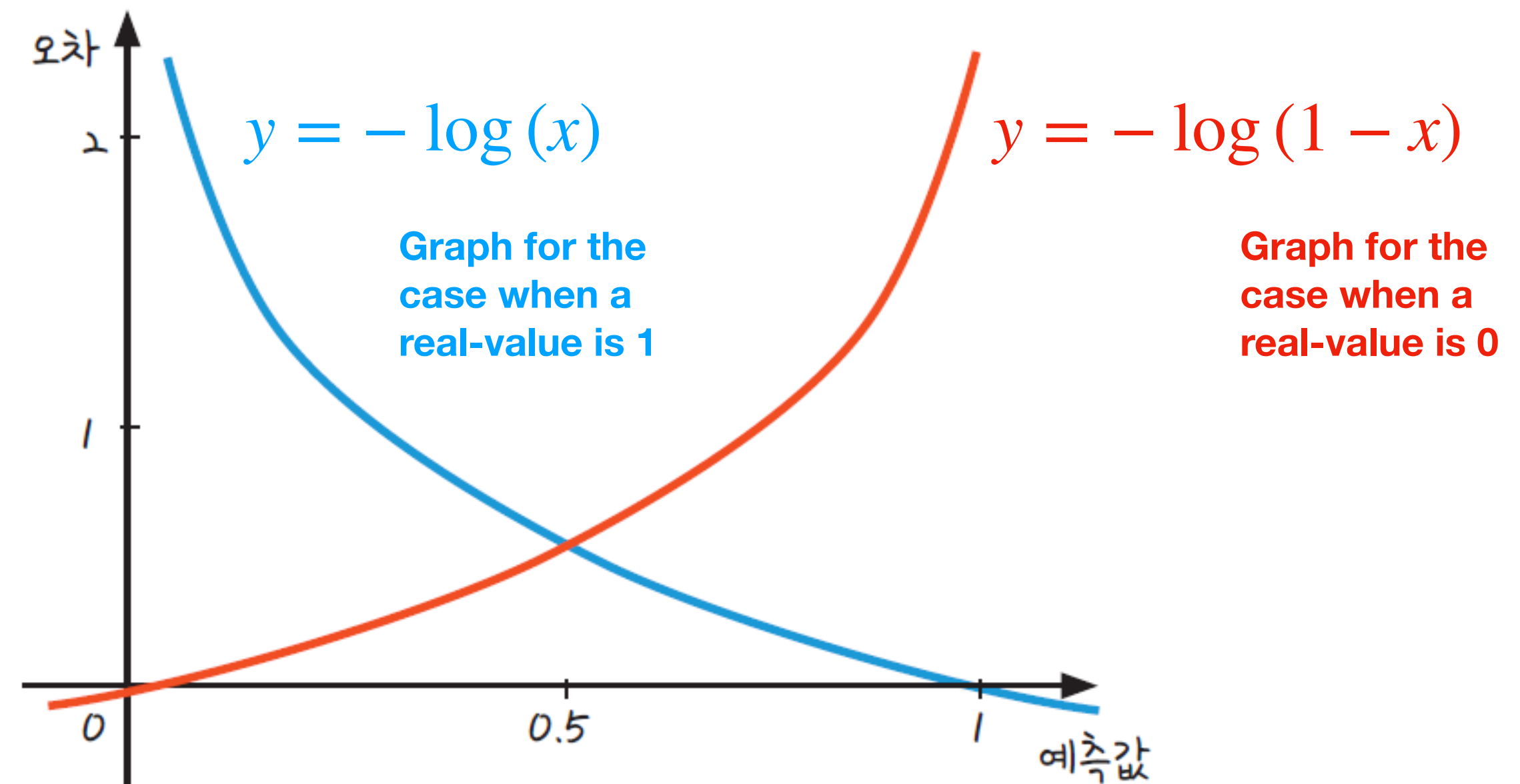


Logistic Regression

$$cost(W, b) = \frac{1}{N} \sum -y \log\left(\frac{1}{1 + e^{Wx-b}}\right) - (1 - y) \log\left(1 - \left(\frac{1}{1 + e^{Wx-b}}\right)\right)$$

$$W = W - \alpha \frac{\partial}{\partial W} cost(W, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} cost(W, b)$$



Logistic Regression

$$W = W - \alpha \frac{\partial}{\partial W} cost(W, b)$$

$$\frac{\partial}{\partial W} cost(W, b) = x \frac{1}{e^{-(Wx+b)}} - y$$

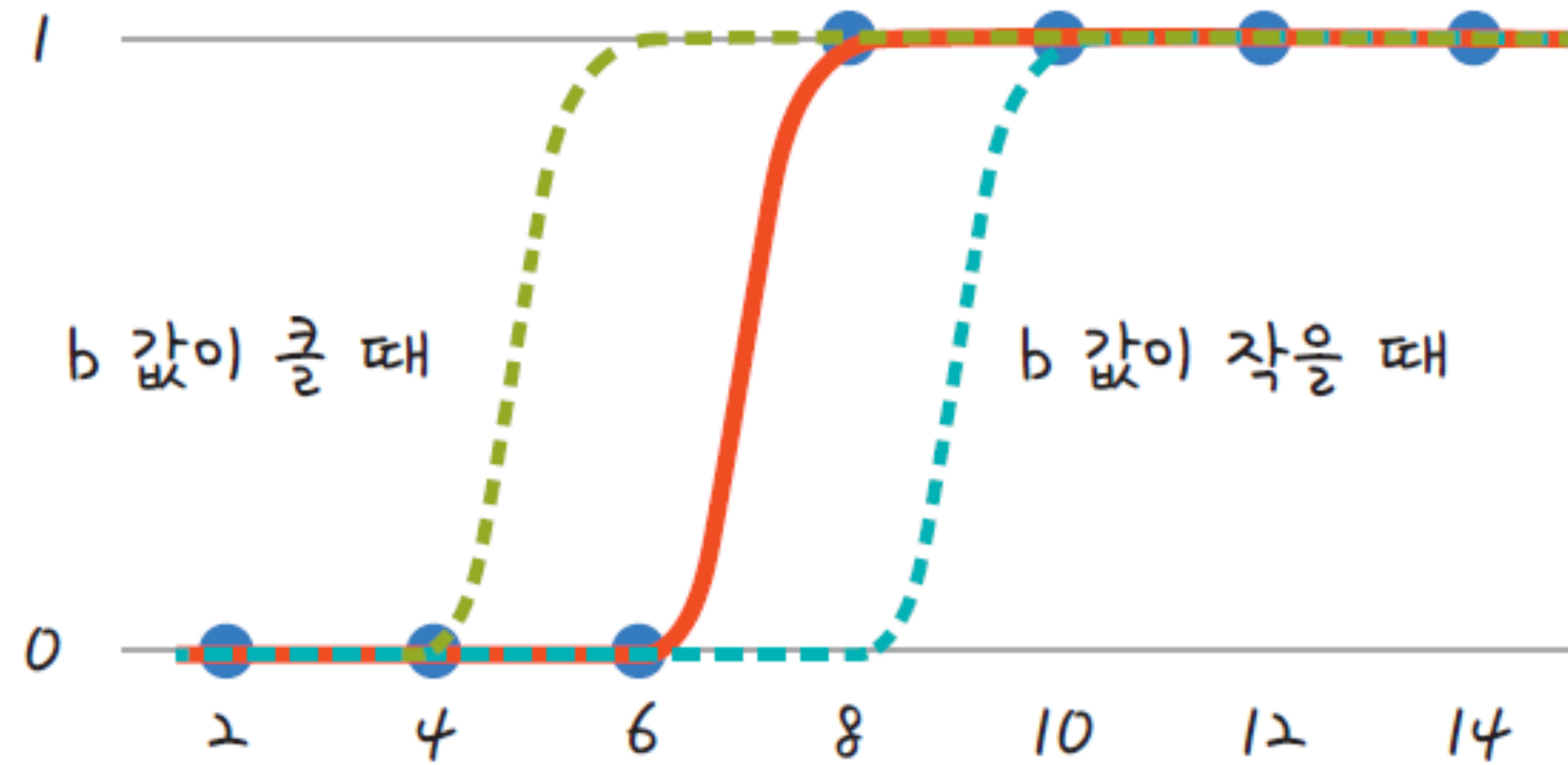
$$b = b - \alpha \frac{\partial}{\partial b} cost(W, b)$$

$$\frac{\partial}{\partial b} cost(W, b) = \frac{1}{e^{-(wx+b)}} - y$$

Lab

- Logistic Regression Lab #1 - Python Raw
- Logistic Regression Lab #2 - Tensorflow

Sigmoid Function



Sigmoid Function

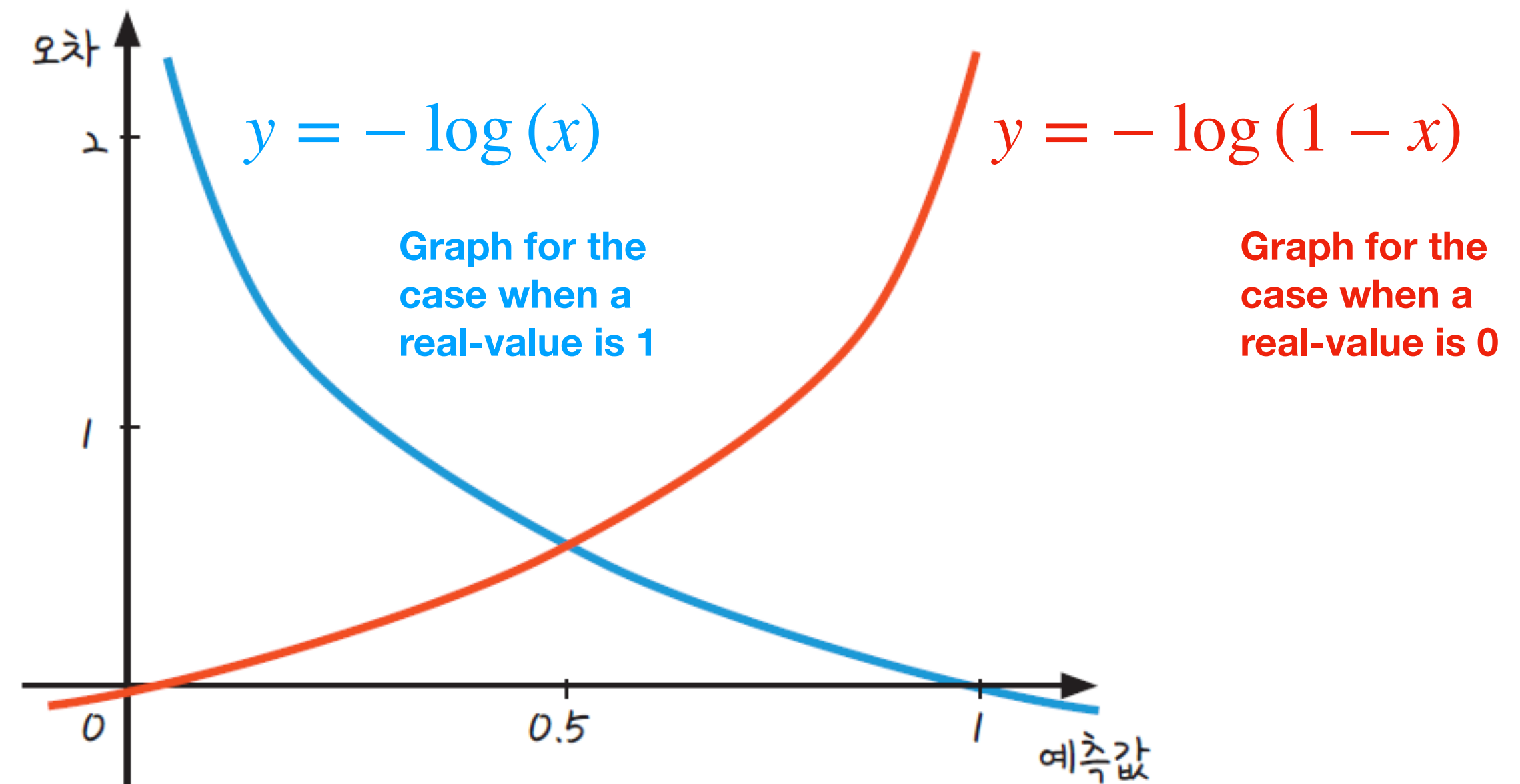
$$y = \frac{1}{1 + e^{-(ax+b)}}$$

Logistic Regression

$$cost(W, b) = \frac{1}{N} \sum -y \log\left(\frac{1}{1 + e^{Wx-b}}\right) - (1 - y) \log\left(1 - \left(\frac{1}{1 + e^{Wx-b}}\right)\right)$$

$$W = W - \alpha \frac{\partial}{\partial W} cost(W, b)$$

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Logistic Regression

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