Metode Numerik
EE221

Bab 2. Akar-akar Persamaan

Dirangkum dan diterjemahkan dari Thomson Brooks Chapra, Steven and Raymond Canale. 2009. Numerical Methods for Engineers 6th Edition, **Chapter 5**

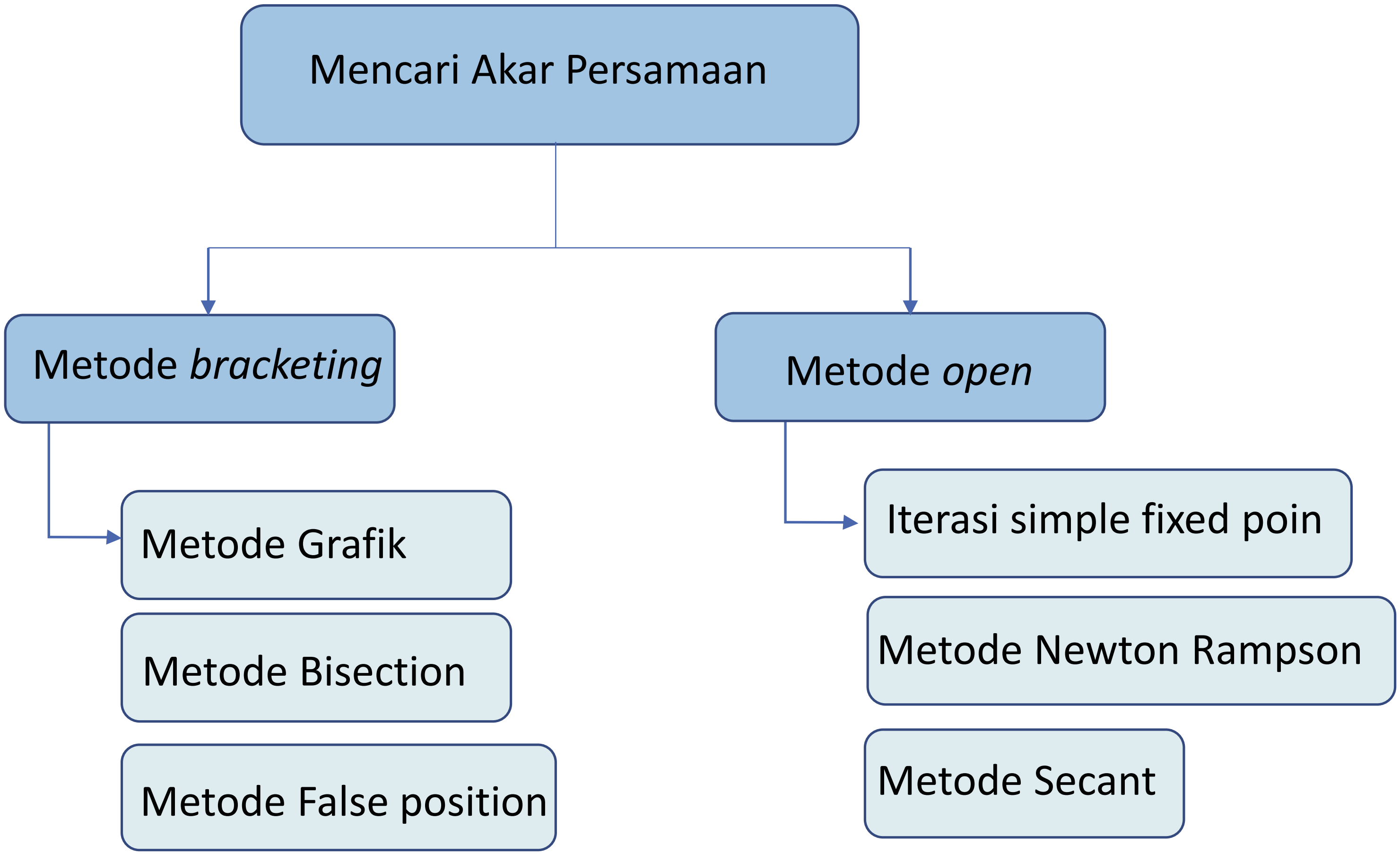
Nabila Husna Shabrina
Fakultas Teknik dan Informatika, Universitas Multimedia Nusantara

nabila.husna@umn.ac.id

Sub Bahasan :

- Metode Bisection
- Metode False position

Mencari Akar-akar Persamaan



Mencari Akar-akar Persamaan

TABLE PT2.3 Comparison of the characteristics of alternative methods for finding roots of algebraic and transcendental equations. The comparisons are based on general experience and do not account for the behavior of specific functions.

Method	Type	Guesses	Convergence	Stability	Programming	Comments
Direct	Analytical	—	—	—		
Graphical	Visual	—	—	—	—	Imprecise
Bisection	Bracketing	2	Slow	Always	Easy	
False-position	Bracketing	2	Slow/medium	Always	Easy	
Modified FP	Bracketing	2	Medium	Always	Easy	
Fixed-point iteration	Open	1	Slow	Possibly divergent	Easy	
Newton-Raphson	Open	1	Fast	Possibly divergent	Easy	Requires evaluation of $f'(x)$
Modified Newton-Raphson	Open	1	Fast (multiple), medium (single)	Possibly divergent	Easy	Requires evaluation of $f'(x)$ and $f''(x)$
Secant	Open	2	Medium/fast	Possibly divergent	Easy	Initial guesses do not have to bracket the root
Modified secant	Open	1	Medium/fast	Possibly divergent	Easy	
Brent	Hybrid	1 or 2	Medium	Always (for 2 guesses)	Moderate	Robust
Müller	Polynomials	2	Medium/fast	Possibly divergent	Moderate	
Bairstow	Polynomials	2	Fast	Possibly divergent	Moderate	

Metode Grafik

Salah satu cara untuk mencari akar-akar dari suatu persamaan adalah dengan grafik

Contoh 1. Dengan aproksimasi grafik, tentukan koefisien c yang dibutuhkan oleh parasut dengan massa 68.1 kg dan kecepatan 40 m/s setelah jatuh bebas selama 10 detik. Percepatan gravitasi sebesar $9.81\ \mathrm{m}/s^2$

$$f(c) = \frac{9.81(68.1)}{c}\left(1 - e^{-(c/68.1)10}\right) - 40$$

Atau

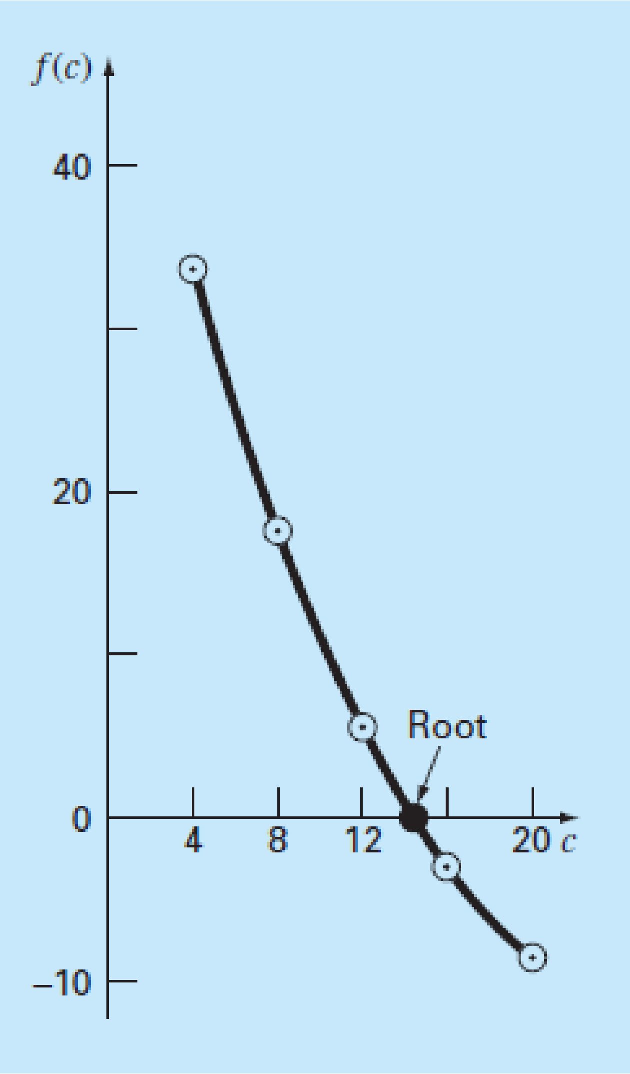
$$f(c) = \frac{668.06}{c}\left(1 - e^{-0.146843c}\right) - 40$$

Metode Grafik

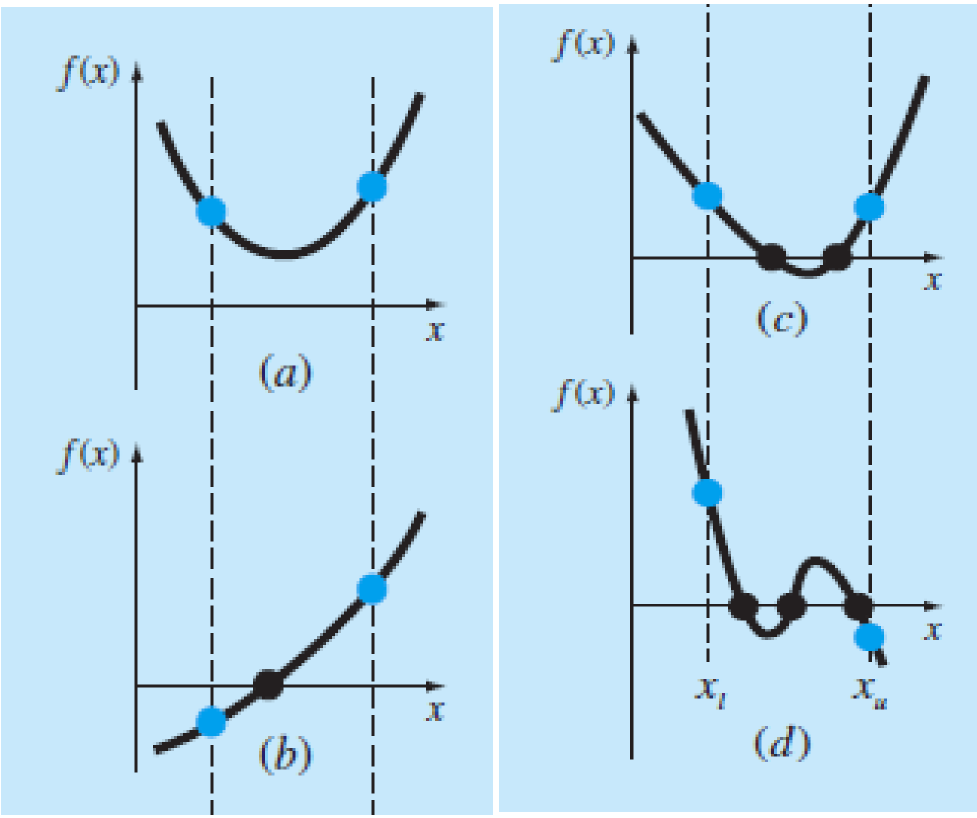
c	f(c)
4	34.190
8	17.712
12	6.114
16	−2.230
20	−8.368

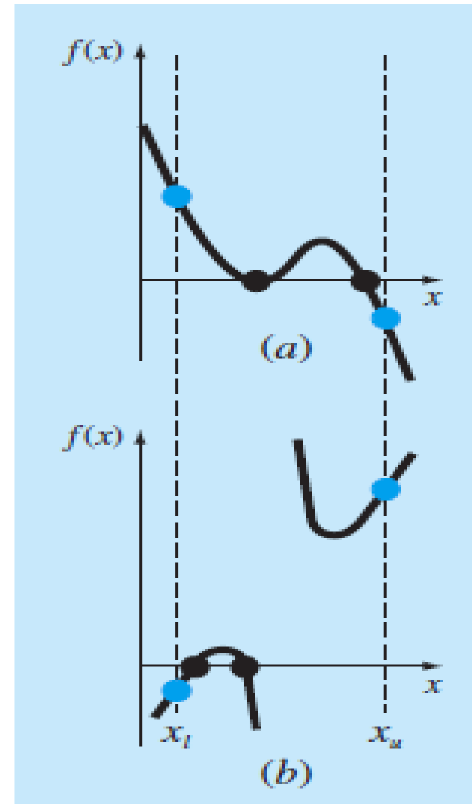
$$f(14.75) = \frac{668.06}{14.75}(1 - e^{-0.146843(14.75)}) - 40 = 0.100$$

$$v = \frac{9.81(68.1)}{14.75}(1 - e^{-(14.75/68.1)10}) = 40.100$$



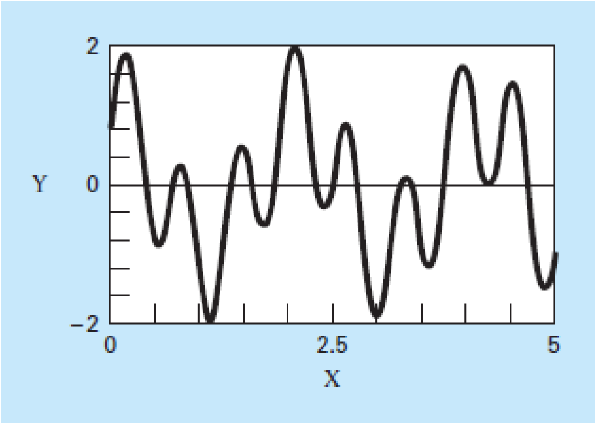
Metode Grafik



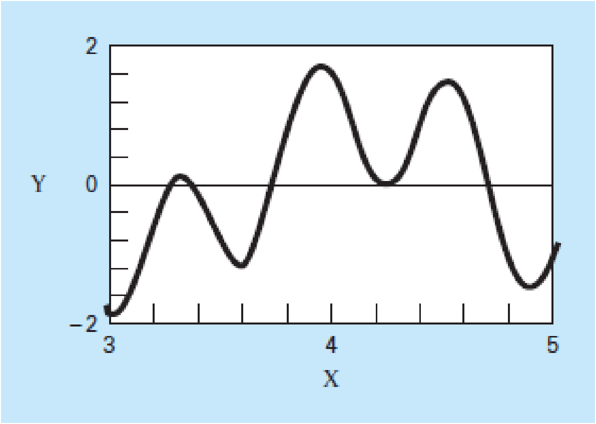


Metode Grafik

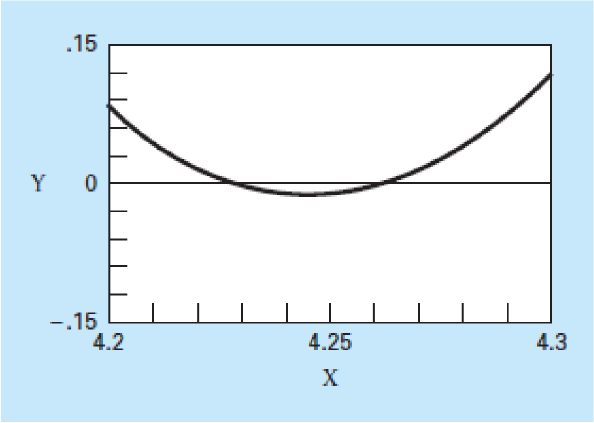
Contoh dengan MATLAB



(a)



(b)



(c)

Metode Bisection

Metode Bisection

Step

1. Tentukan batas bawah x_l dan batas atas x_u untuk akar persamaan yang menyebabkan fungsi tersebut berubah nilainya pada batas interval tersebut
2. Gunakan estimasi akar → $$x_r = \frac{x_l + x_u}{2}$$
3. Evaluasi dengan cara sebagai berikut

   a. Jika $f(x_l)f(x_r) < 0$ maka akar nya ada di interval bawah, sehingga kita dapat mengubah $x_u = x_r$ dan kembali pada step 2

   b. Jika $f(x_l)f(x_r) > 0$ maka akar nya ada di interval atas, sehingga kita dapat mengubah $x_l = x_r$ dan kembali pada step 2

   c. Jika $f(x_l)f(x_r) = 0$ maka akar persamaan nya adalah x_r

Metode Bisection

Contoh 2.

Selesaikan contoh 1 dengan pendekatan bisection

$$x_r = \frac{12 + 16}{2} = 14 \qquad \varepsilon_t = 5.3\%$$

$$f(12)f(14) = 6.114(1.611) = 9.850$$

$$x_r = \frac{14 + 16}{2} = 15 \qquad \varepsilon_t = 1.3\%.$$

Jika dilakukan iterasi

$$f(14)f(15) = 1.611(-0.384) = -0.619$$

$$x_r = \frac{14 + 15}{2} = 14.5 \quad \varepsilon_t = 2.0\%.$$

Metode Bisection

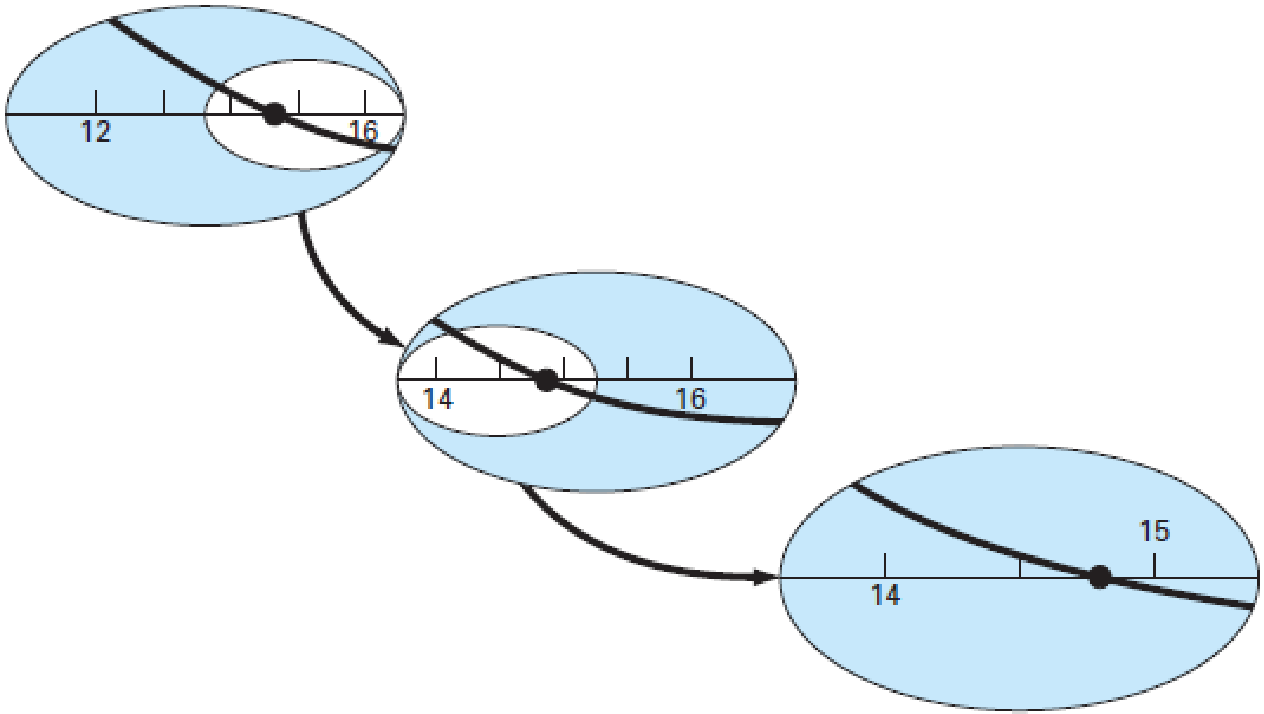


FIGURE 5.6
A graphical depiction of the bisection method. This plot conforms to the first three iterations from Example 5.3.

Metode Bisection

Estimasi error

$$\varepsilon_a = \left| \frac{x_r^{\text{new}} - x_r^{\text{old}}}{x_r^{\text{new}}} \right| 100\%$$

Contoh 3

Lanjutkan iterasi untuk contoh 2 sampai didapatkan error $\varepsilon_s = 0.5\%$.

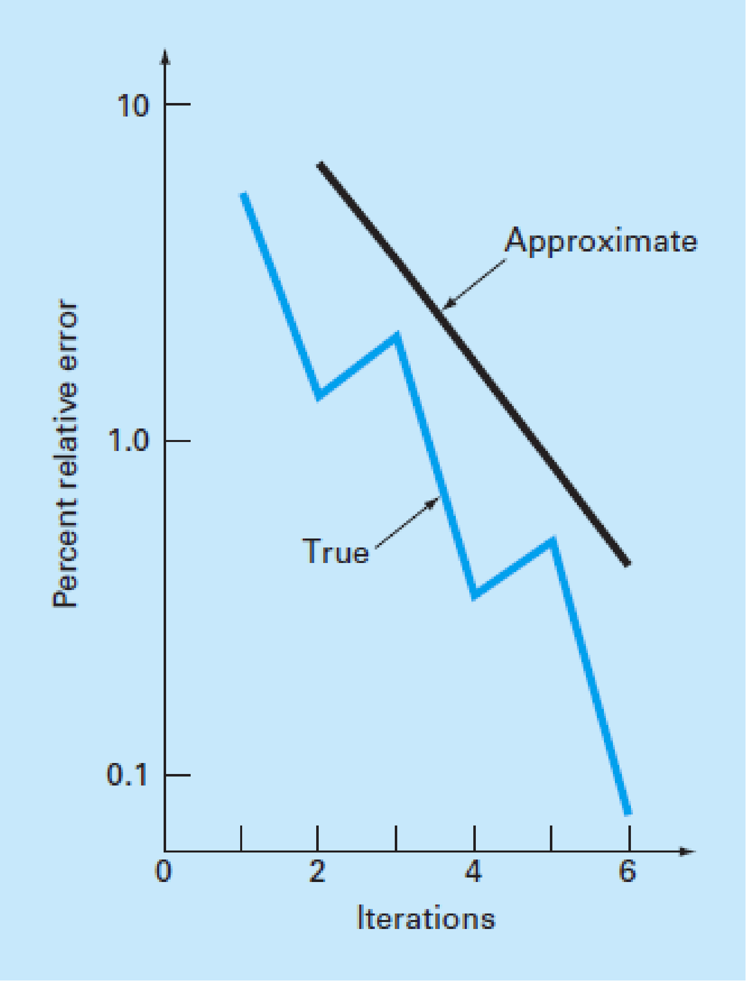
$$|\varepsilon_a| = \left| \frac{15 - 14}{15} \right| 100\%$$

$$= 6.667\%$$

Iteration	x_l	x_u	x_r	ε_a (%)	ε_t (%)
1	12	16	14		5.413
2	14	16	15	6.667	1.344
3	14	15	14.5	3.448	2.035
4	14.5	15	14.75	1.695	0.345
5	14.75	15	14.875	0.840	0.499
6	14.75	14.875	14.8125	0.422	0.077

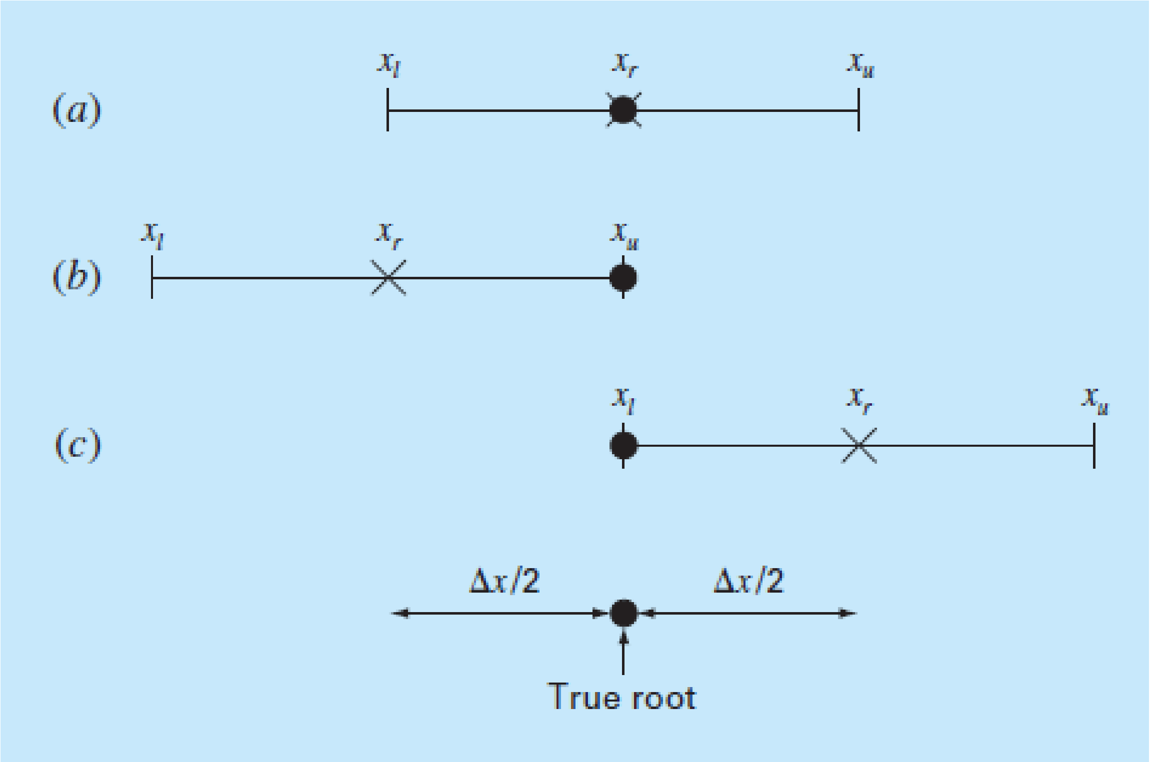
Metode Bisection

FIGURE 5.7
Errors for the bisection method. True and estimated errors are plotted versus the number of iterations.



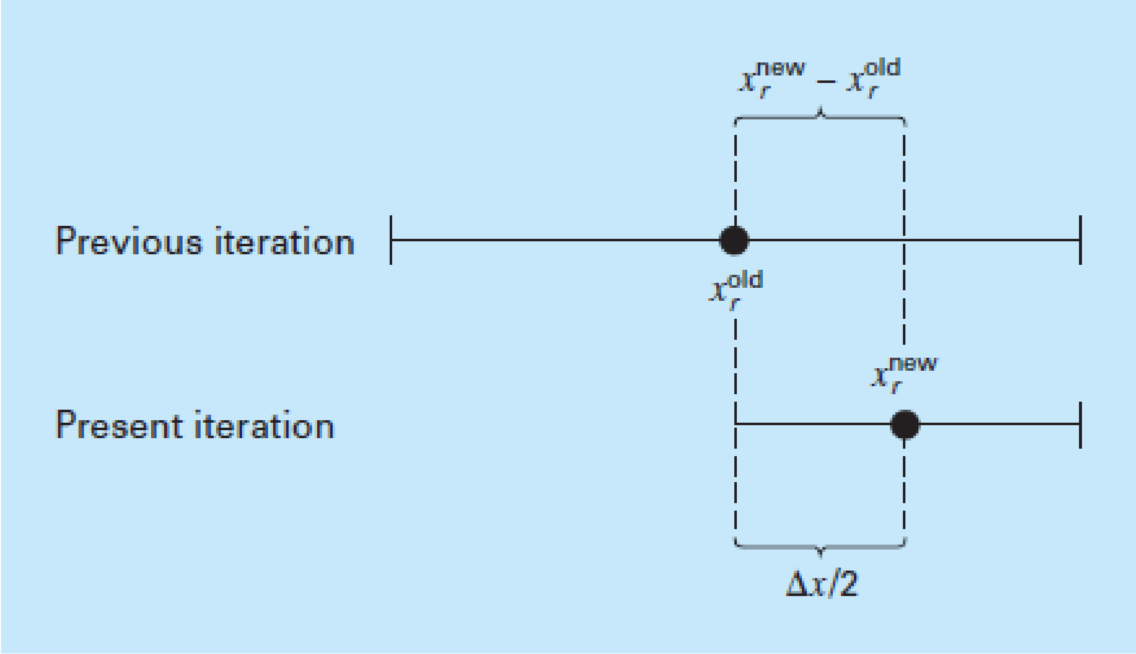
Metode Bisection

FIGURE 5.8
Three ways in which the interval may bracket the root. In (*a*) the true value lies at the center of the interval, whereas in (*b*) and (*c*) the true value lies near the extreme. Notice that the discrepancy between the true value and the midpoint of the interval never exceeds half the interval length, or $\Delta x/2$.



Metode Bisection

FIGURE 5.9
Graphical depiction of why the error estimate for bisection ($\Delta x/2$) is equivalent to the root estimate for the present iteration (x_r^{new}) minus the root estimate for the previous iteration (x_r^{old}).



Metode Bisection

Rumus-rumus penting

$$x_r^{\text{new}} - x_r^{\text{old}} = \frac{x_u - x_l}{2}$$

$$x_r^{\text{new}} = \frac{x_l + x_u}{2}$$

$$\varepsilon_a = \left| \frac{x_u - x_l}{x_u + x_l} \right| 100\%$$

Metode Bisection

Pseudocode untuk metode bisection

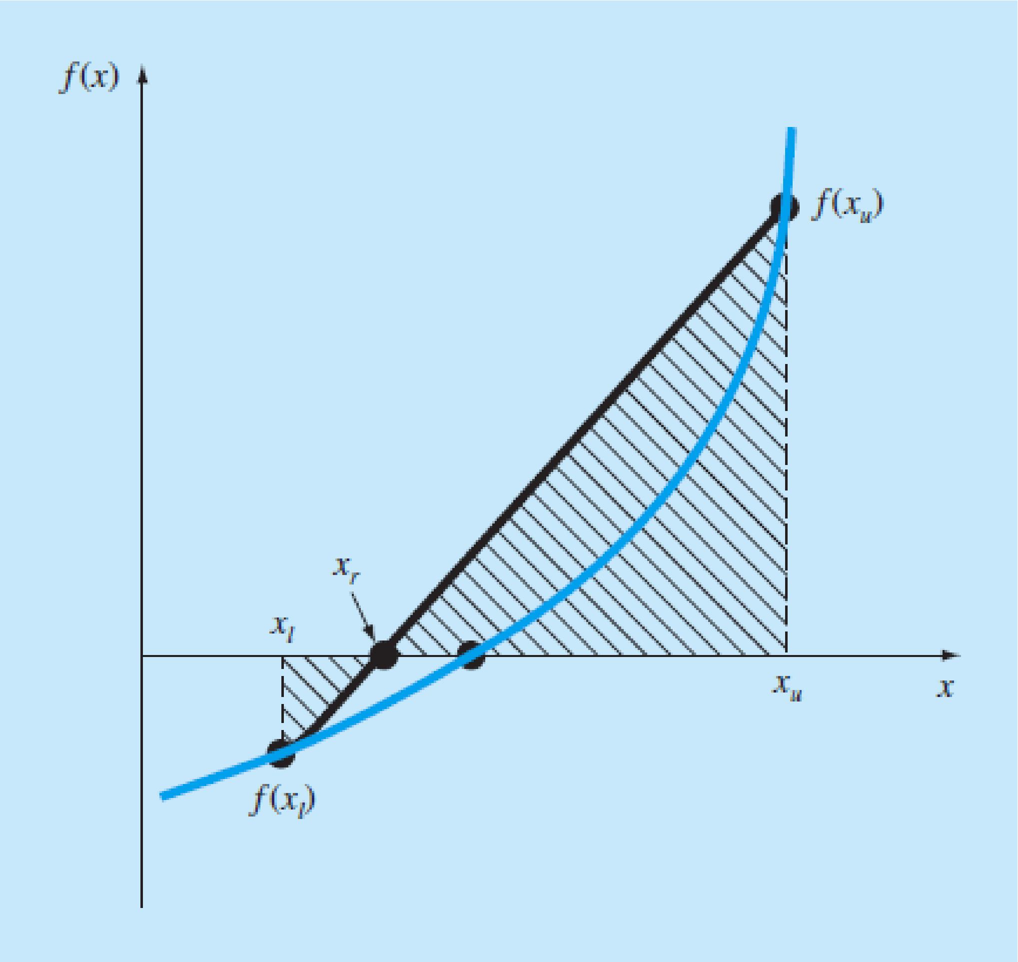
```
FUNCTION Bisect(xl, xu, es, imax, xr, iter, ea)
  iter = 0
  DO
    xrold = xr
    xr = (xl + xu) / 2
    iter = iter + 1
    IF xr ≠ 0 THEN
       ea = ABS((xr - xrold) / xr) * 100
    END IF
    test = f(xl) * f(xr)
    IF test < 0 THEN
       xu = xr
    ELSE IF test > 0 THEN
      xl = xr
    ELSE
      ea = 0
    END IF
     IF ea < es OR iter ≥ imax EXIT
  END DO
  Bisect = xr
END Bisect
```

Metode False Position

Metode False Position

FIGURE 5.12
A graphical depiction of the method of false position. Similar triangles used to derive the formula for the method are shaded.

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$



Metode False Position

Box 5.1 Derivation of the Method of False Position

Cross-multiply Eq. (5.6) to yield

$$f(x_l)(x_r - x_u) = f(x_u)(x_r - x_l)$$

Collect terms and rearrange:

$$x_r [f(x_l) - f(x_u)] = x_u f(x_l) - x_l f(x_u)$$

Divide by $f(x_l) - f(x_u)$:

$$x_r = \frac{x_u f(x_l) - x_l f(x_u)}{f(x_l) - f(x_u)} \qquad \text{(B5.1.1)}$$

This is one form of the method of false position. Note that it allows the computation of the root x_r as a function of the lower and upper guesses x_l and x_u. It can be put in an alternative form by expanding it:

$$x_r = \frac{x_u f(x_l)}{f(x_l) - f(x_u)} - \frac{x_l f(x_u)}{f(x_l) - f(x_u)}$$

then adding and subtracting x_u on the right-hand side:

$$x_r = x_u + \frac{x_u f(x_l)}{f(x_l) - f(x_u)} - x_u - \frac{x_l f(x_u)}{f(x_l) - f(x_u)}$$

Collecting terms yields

$$x_r = x_u + \frac{x_u f(x_u)}{f(x_l) - f(x_u)} - \frac{x_l f(x_u)}{f(x_l) - f(x_u)}$$

or

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

which is the same as Eq. (5.7). We use this form because it involves one less function evaluation and one less multiplication than Eq. (B5.1.1). In addition, it is directly comparable with the secant method, which will be discussed in Chap. 6.

Metode False Position

Contoh 4

Dengan metode false position, tentukan solusi untuk contoh 1

Iterasi pertama

$$x_l = 12 \qquad f(x_l) = 6.1139$$
$$x_u = 16 \qquad f(x_u) = -2.2303$$
$$x_r = 16 - \frac{-2.2303(12 - 16)}{6.1139 - (-2.2303)} = 14.309$$

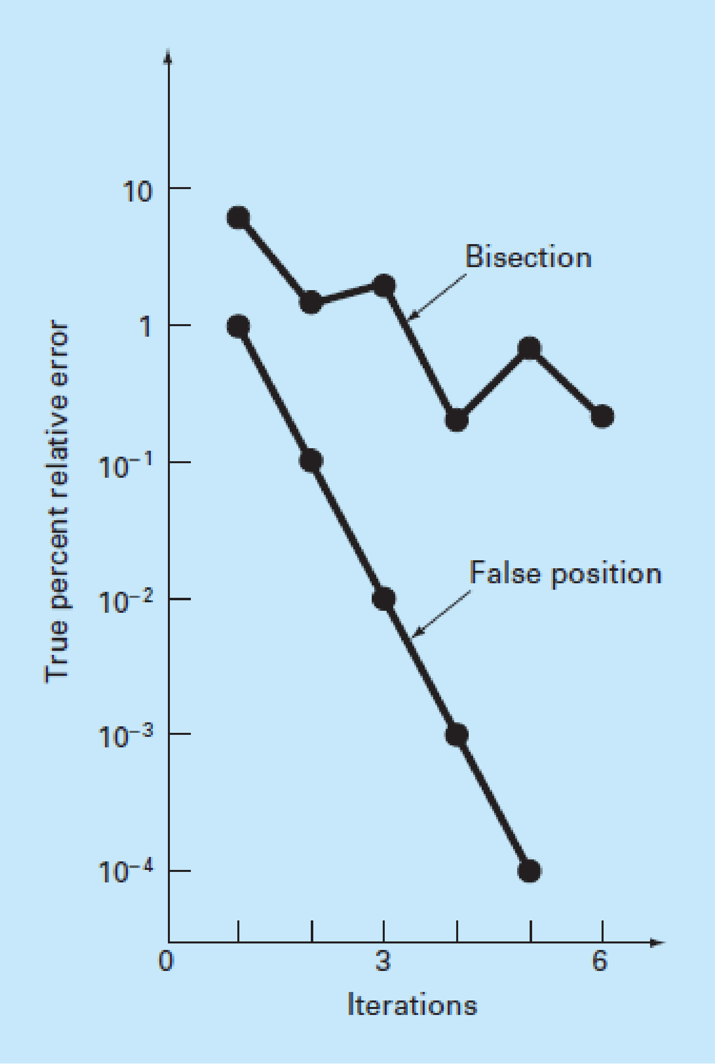
Iterasi kedua

$$f(x_l)f(x_r) = -1.5376$$

$$x_l = 12 \qquad f(x_l) = 6.1139$$
$$x_u = 14.9309 \qquad f(x_u) = -0.2515$$
$$x_r = 14.9309 - \frac{-0.2515(12 - 14.9309)}{6.1139 - (-0.2515)} = 14.8151$$

Metode False Position

FIGURE 5.13
Comparison of the relative errors of the bisection and the false-position methods.



Metode False Position

Contoh 5.

Tentukan solusi untuk $f(x) = x^{10} - 1$ untuk nilai x diantara 0 dan 1.3

Dengan bisection

Iteration	x_l	x_u	x_r	ε_a (%)	ε_t (%)
1	0	1.3	0.65	100.0	35
2	0.65	1.3	0.975	33.3	2.5
3	0.975	1.3	1.1375	14.3	13.8
4	0.975	1.1375	1.05625	7.7	5.6
5	0.975	1.05625	1.015625	4.0	1.6

Metode False Position

Dengan false position

Iteration	x_l	x_u	x_r	ε_a (%)	ε_t (%)
1	0	1.3	0.09430		90.6
2	0.09430	1.3	0.18176	48.1	81.8
3	0.18176	1.3	0.26287	30.9	73.7
4	0.26287	1.3	0.33811	22.3	66.2
5	0.33811	1.3	0.40788	17.1	59.2

Metode False Position

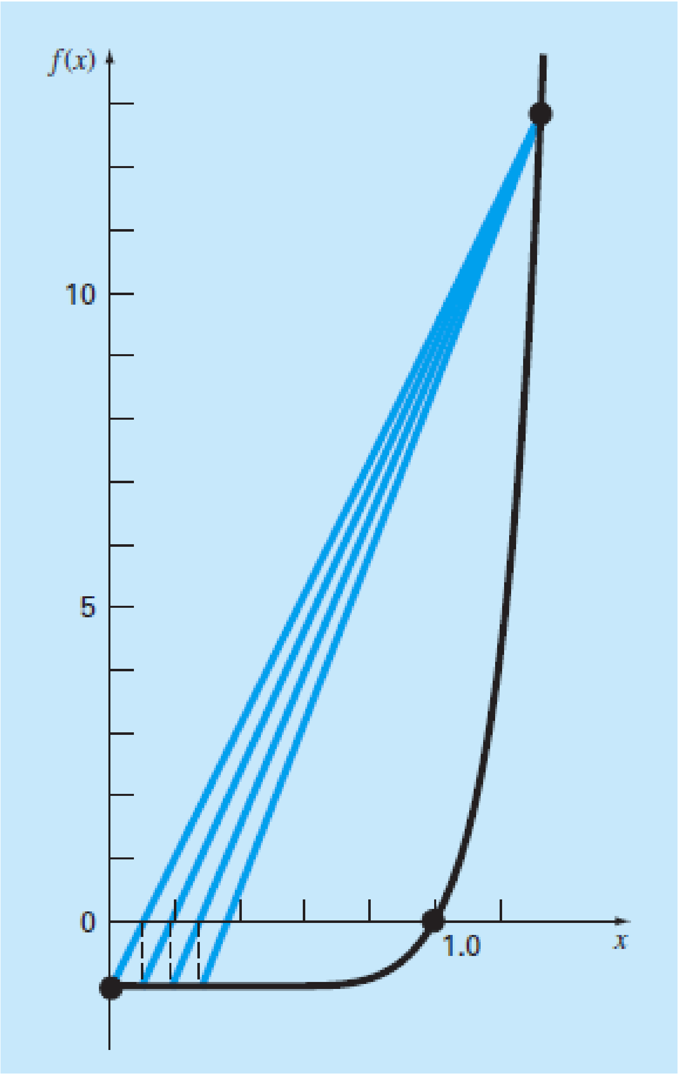
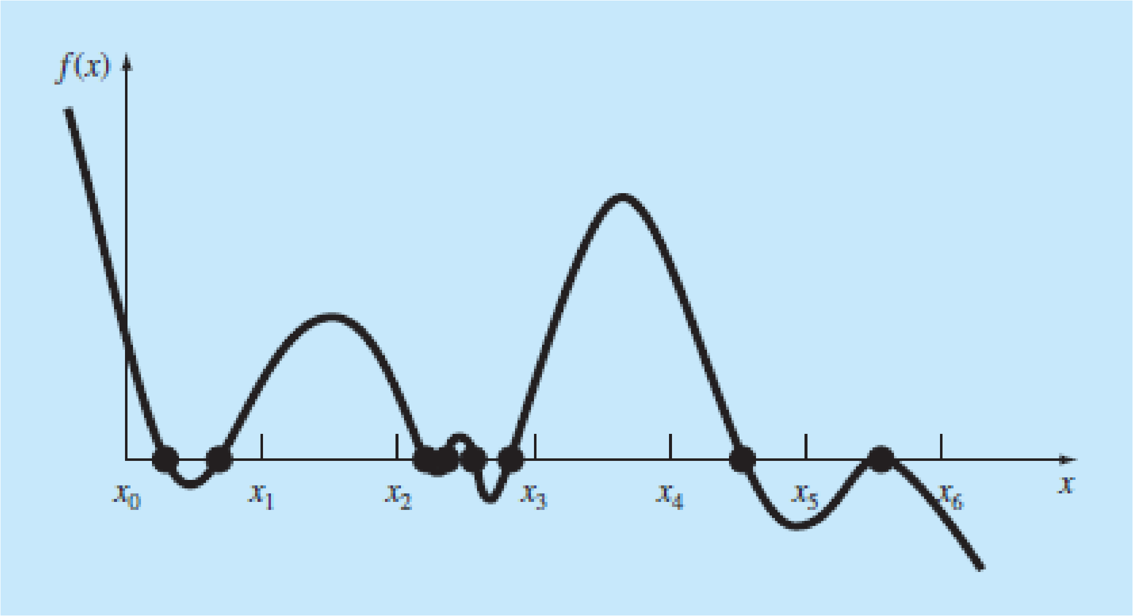


FIGURE 5.14
Plot of $f(x) = x^{10} - 1$, illustrating slow convergence of the false-position method.

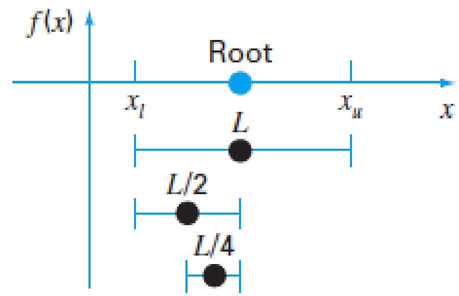
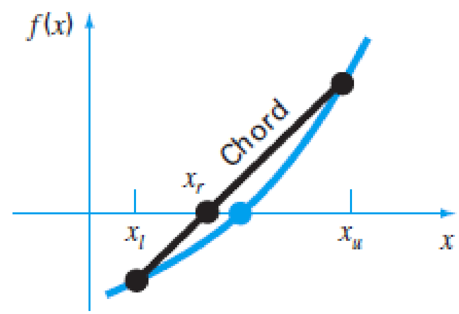
Metode False Position

FIGURE 5.16
Cases where roots could be missed because the increment length of the search procedure is too large. Note that the last root on the right is multiple and would be missed regardless of increment length.



Kesimpulan

TABLE PT2.4 Summary of important information presented in Part Two.

Method	Formulation	Graphical Interpretation	Errors and Stopping Criteria
		Bracketing methods:	
Bisection	$x_r = \frac{x_l + x_u}{2}$ If $f(x_l)f(x_r) < 0, \quad x_u = x_r$ $f(x_l)f(x_r) > 0, \quad x_l = x_r$		Stopping criterion: $\left\lvert \frac{x_r^{\text{new}} - x_r^{\text{old}}}{x_r^{\text{new}}} \right\rvert 100\% \le \epsilon_s$
False position	$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$ If $f(x_l)f(x_r) < 0, \quad x_u = x_r$ $f(x_l)f(x_r) > 0, \quad x_l = x_r$		Stopping criterion: $\left\lvert \frac{x_r^{\text{new}} - x_r^{\text{old}}}{x_r^{\text{new}}} \right\rvert 100\% \le \epsilon_s$

Latihan

Problems Chapter 5

Nomer 5.3 b dan c

Nomer 5.6