Metode Numerik EE221

Bab 3. Akar-akar Persamaan (Lanjutan)

Dirangkum dan diterjemahkan dari Thomson Brooks Chapra, Steven and Raymond Canale. 2009. Numerical Methods for Engineers 6th Edition, **Chapter 6**

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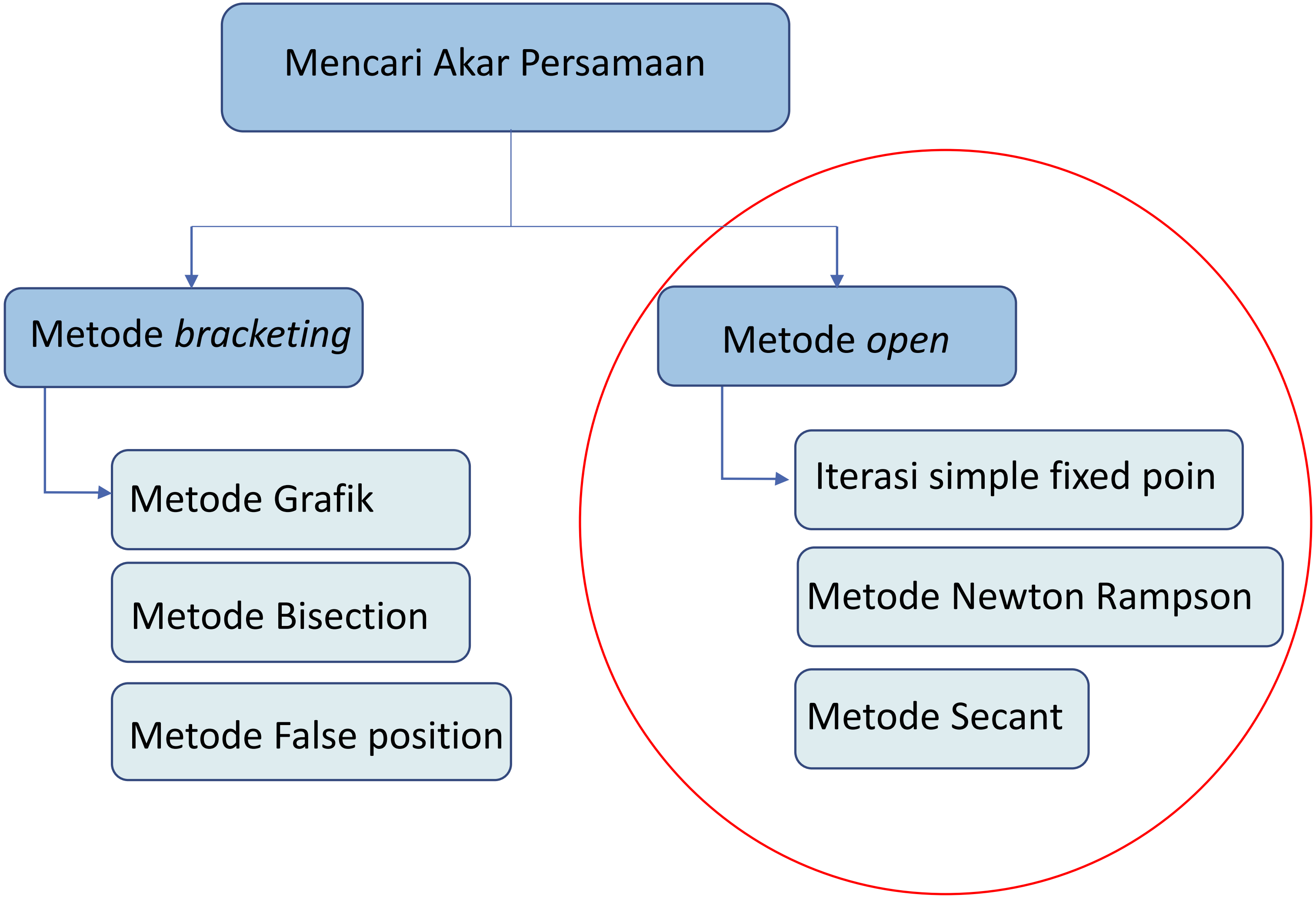
Sub Bahasan

Metode open

- Iterasi simple fixed poin
- Newton-Raphson
- Secant

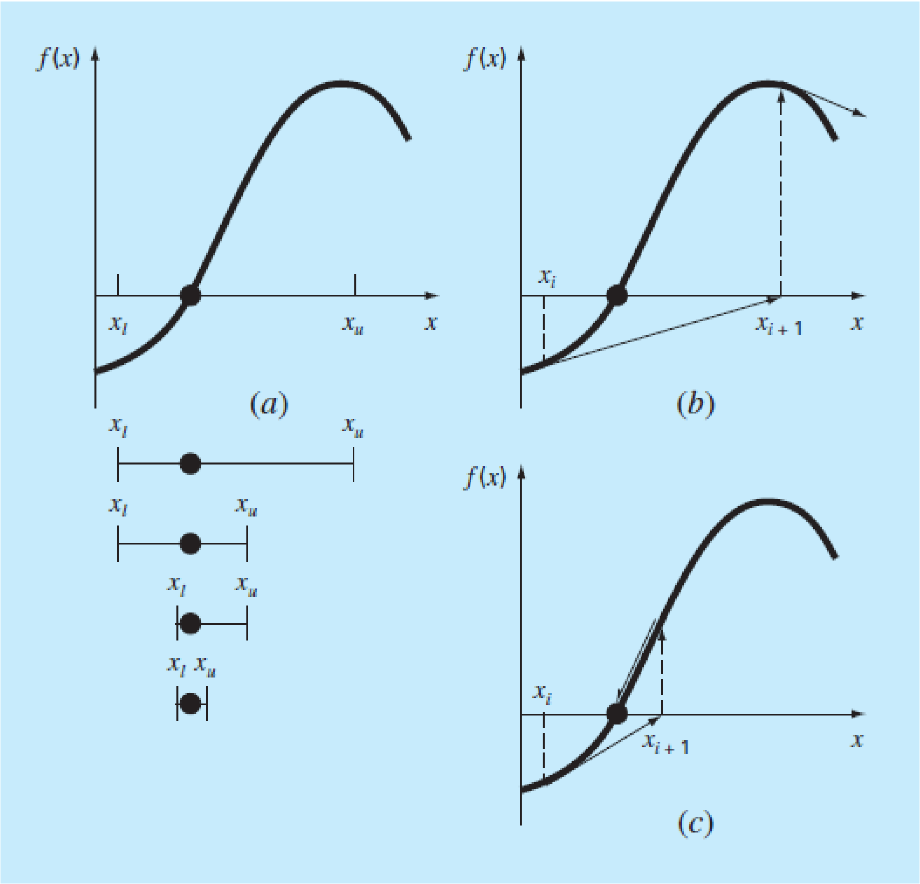
Mencari Akar-akar Persamaan



Metode Open

Pada metode open, hanya dibutuhkan satu nilai pada saat permulaan atau 2 nilai tanpa *bracket*

FIGURE 6.1
Graphical depiction of the fundamental difference between the (*a*) bracketing and (*b*) and (*c*) open methods for root location. In (*a*), which is the bisection method, the root is constrained within the interval prescribed by x_l and x_u. In contrast, for the open method depicted in (*b*) and (*c*), a formula is used to project from x_i to x_{i+1} in an iterative fashion. Thus, the method can either (*b*) diverge or (*c*) converge rapidly, depending on the value of the initial guess.



Metode Open

Metode Open

- Iterasi simple fixed poin
- Metode Newton Rampson
- Metode Secant

Iterasi Simple Fixed Poin

Iterasi Simple Fixed Poin

Iterasi simple fixed point disebut juga sebagai iterasi satu kali atau substitusi

Contoh

$$x = g(x)$$

$$x^2 - 2x + 3 = 0 \longrightarrow x = \frac{x^2 + 3}{2}$$

$$\sin x = 0 \longrightarrow x = \sin x + x$$

$$\boxed{x_{i+1} = g(x_i)}$$

$$\boxed{\varepsilon_a = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| 100\%}$$

Iterasi Simple Fixed Poin

Contoh 1.

Gunakan iterasi simple fixed point untuk menentukan akar persamaan dari fungsi $f(x) = e^{-x} - x$.

$$x_{i+1} = e^{-x_i}$$

i	x_i	ε_a (%)	ε_t (%)
0	0		100.0
1	1.000000	100.0	76.3
2	0.367879	171.8	35.1
3	0.692201	46.9	22.1
4	0.500473	38.3	11.8
5	0.606244	17.4	6.89
6	0.545396	11.2	3.83
7	0.579612	5.90	2.20
8	0.560115	3.48	1.24
9	0.571143	1.93	0.705
10	0.564879	1.11	0.399

Iterasi Simple Fixed Poin

Konvergensi

- Pada contoh 1 dapat dilihat bahwa presentase *true error* nya proporsional sebesar 0.5-0.6
- Hal tersebut menunjukkan bahwa hasil tersebut memiliki karakteristik konvergen linier
- Konsep konvergensi dan divergensi dapat ditentukan dengan metode grafik

$$f_1(x) = f_2(x)$$

$$y_1 = f_1(x)$$

$$y_2 = f_2(x)$$

Iterasi Simple Fixed Poin

Contoh 2.

Dengan metode grafik dua kurva, tentukan akar persamaan dari fungsi $f(x) = e^{-x} - x.$

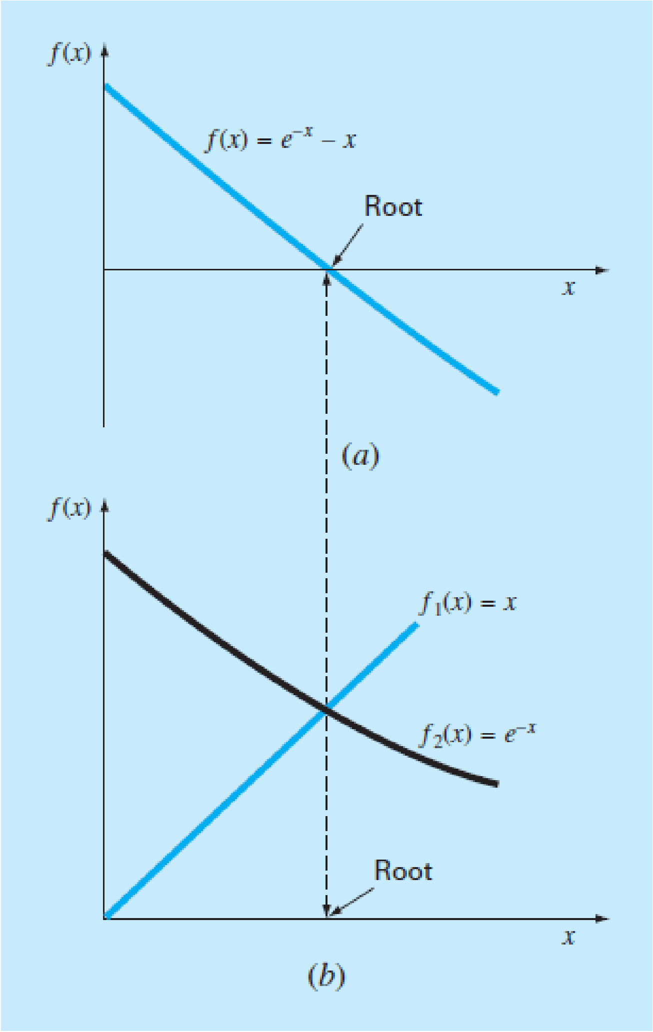
$$y_1 = x$$

$$y_2 = e^{-x}.$$

x	**y_1**	**y_2**
0.0	0.0	1.000
0.2	0.2	0.819
0.4	0.4	0.670
0.6	0.6	0.549
0.8	0.8	0.449
1.0	1.0	0.368

Iterasi Simple Fixed Poin

FIGURE 6.2
Two alternative graphical methods for determining the root of $f(x) = e^{-x} - x$. (*a*) Root at the point where it crosses the *x* axis; (*b*) root at the intersection of the component functions.



Iterasi Simple Fixed Poin

Pseudocode untuk iterasi simple fixed poin

```
FUNCTION Fixpt(x0, es, imax, iter, ea)
  xr = x0
  iter = 0
  DO
    xrold = xr
    xr = g(xrold)
    iter = iter + 1
    IF xr ≠ 0 THEN
      ea = |(xr − xrold) / xr| · 100
    END IF
    IF ea < es OR iter ≥ imax EXIT
  END DO
  Fixpt = xr
END Fixpt
```

Metode Newton-Rampson

Metode Newton-Rampson

- Metode newton-rampson merupakan metode yang paling banyak digunakan
- Jika nilai awal untuk akar persamaan adalah x_i, maka garis singgungnya dapat kita peroleh yaitu $[x_i, f(x_i)]$
- Titik dimana garis singgungnya memotong sumbu x biasanya merepresentasikan estimasi akar persamaan

$$f'(x_i) = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Metode Newton-Rampson

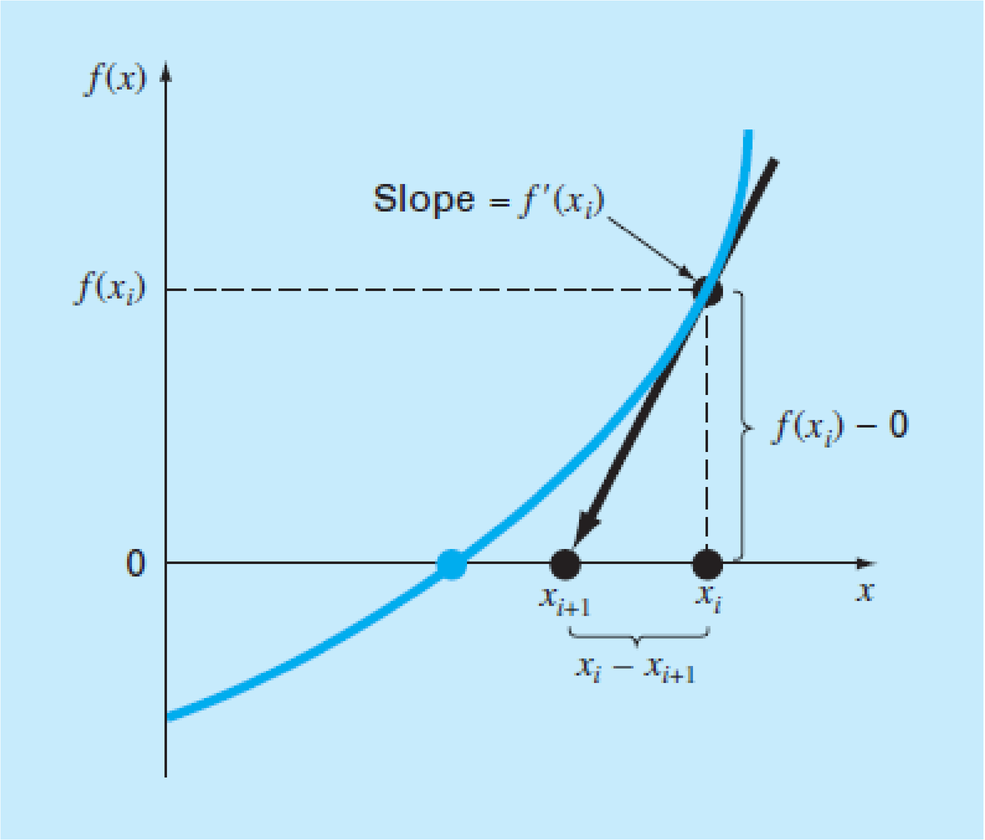


FIGURE 6.5
Graphical depiction of the Newton-Raphson method. A tangent to the function of x_i [that is, $f'(x_i)$] is extrapolated down to the x axis to provide an estimate of the root at x_{i+1}.

Metode Newton-Rampson

Contoh 3.

Tentukan akar persamaan untuk soal pada contoh 1 dengan menggunakan metode newton-rampson

$$f'(x) = -e^{-x} - 1$$

$$x_{i+1} = x_i - \frac{e^{-x_i} - x_i}{-e^{-x_i} - 1}$$

i	x_i	ε_t (%)
0	0	100
1	0.500000000	11.8
2	0.566311003	0.147
3	0.567143165	0.0000220
4	0.567143290	$< 10^{-8}$

Metode Newton-Rampson

Kekurangan metode newton-rampson

Contoh 4.

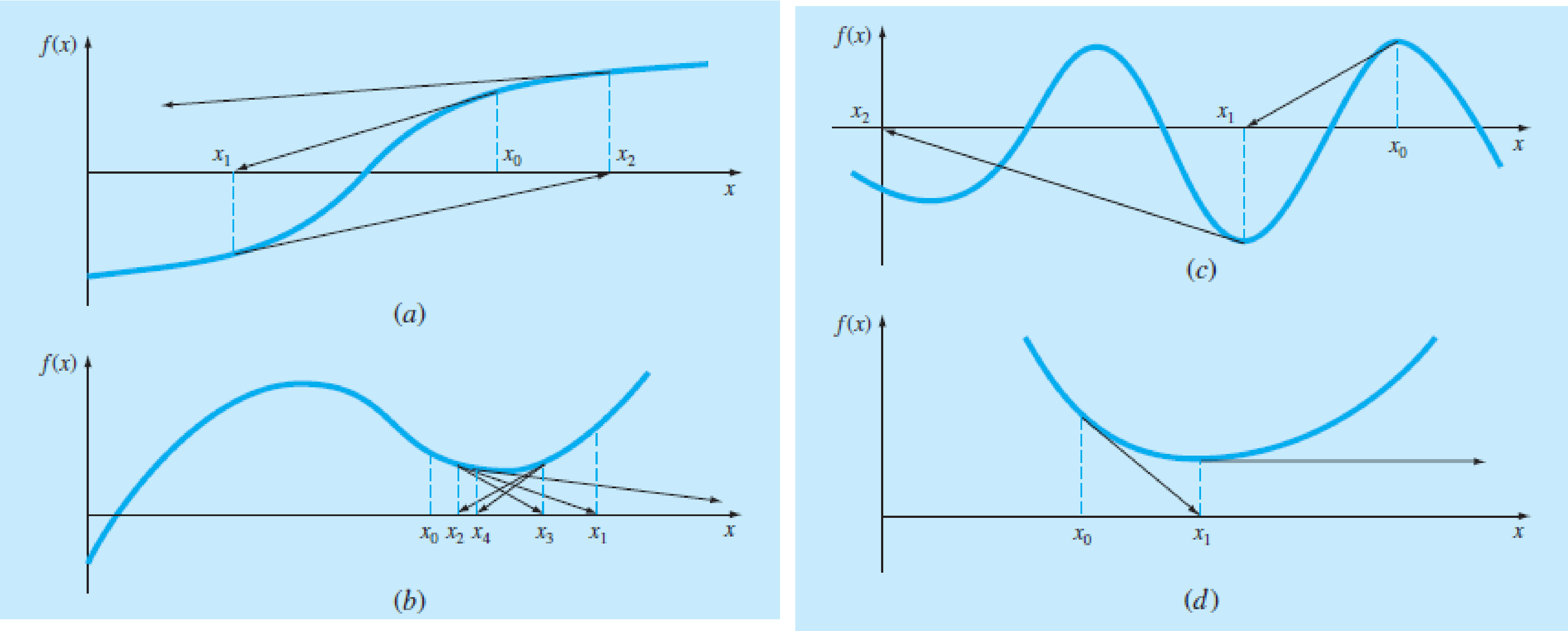
Tentukan akar persamaan dari fungsi $f(x) = x^{10} - 1$ dengan metode Newton-Rampson dengan nilai inisial $x = 0.5$

$$x_{i+1} = x_i - \frac{x_i^{10} - 1}{10x_i^9}$$

Iteration	x
0	0.5
1	51.65
2	46.485
3	41.8365
4	37.65285
5	33.887565
.	
.	
.	
∞	1.0000000

Metode Newton-Rampson

Contoh metode newton-rampson yang memiliki konvergensi yang kurang baik



Metode Secant

Metode Secant

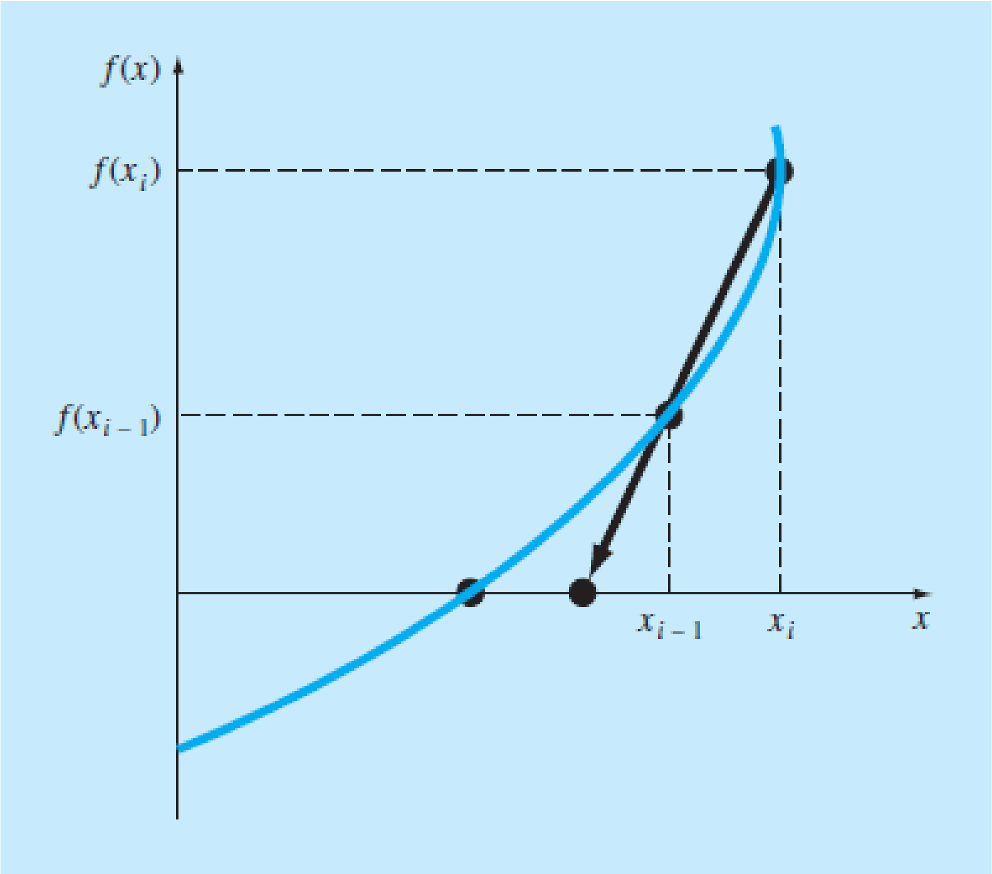
- Kesulitan dalam implementasi metode newton-rampson adalah dalam hal penurunan fungsinya
- Ada beberapa fungsi yang sulit diturunkan
- Untuk fungsi-fungsi yang sulit, dapat digunakan *backward finite divided difference*

$$f'(x_i) \cong \frac{f(x_{i-1}) - f(x_i)}{x_{i-1} - x_i}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

Metode Secant

FIGURE 6.7
Graphical depiction of the secant method. This technique is similar to the Newton-Raphson technique (Fig. 6.5) in the sense that an estimate of the root is predicted by extrapolating a tangent of the function to the x axis. However, the secant method uses a difference rather than a derivative to estimate the slope.



Metode Secant

Contoh 5

Tentukan akar persamaan untuk fungsi pada contoh 1 dengan metode secant. Gunakan nilai inisial awal $x_{-1} = 0$ dan $x_1 = 0$

true root = $0.56714329\ldots$

Iterasi pertama

$$x_{-1} = 0 \qquad f(x_{-1}) = 1.00000$$

$$x_0 = 1 \qquad f(x_0) = -0.63212$$

$$x_1 = 1 - \frac{-0.63212(0 - 1)}{1 - (-0.63212)} = 0.61270 \qquad \varepsilon_t = 8.0\%$$

Metode Secant

Iterasi kedua

$$x_0 = 1 \qquad f(x_0) = -0.63212$$

$$x_1 = 0.61270 \qquad f(x_1) = -0.07081$$

$$x_2 = 0.61270 - \frac{-0.07081(1 - 0.61270)}{-0.63212 - (-0.07081)} = 0.56384 \qquad \varepsilon_t = 0.58\%$$

Iterasi ketiga

$$x_1 = 0.61270 \qquad f(x_1) = -0.07081$$

$$x_2 = 0.56384 \qquad f(x_2) = 0.00518$$

$$x_3 = 0.56384 - \frac{0.00518(0.61270 - 0.56384)}{-0.07081 - (-0.00518)} = 0.56717 \qquad \varepsilon_t = 0.0048\%$$

Metode Secant

Perbedaan metode secant dengan false position

Contoh 6.

Gunakan metode secant dan false position untuk menentukan akar persamaan fungsi $f(x) = \ln x$. Mulai perhitungan dengan nilai inisial $x_l = x_{i-1} = 0.5$ dan $x_u = x_i = 5.0$.

Iteration	x_l	x_u	x_r
1	0.5	5.0	1.8546
2	0.5	1.8546	1.2163
3	0.5	1.2163	1.0585

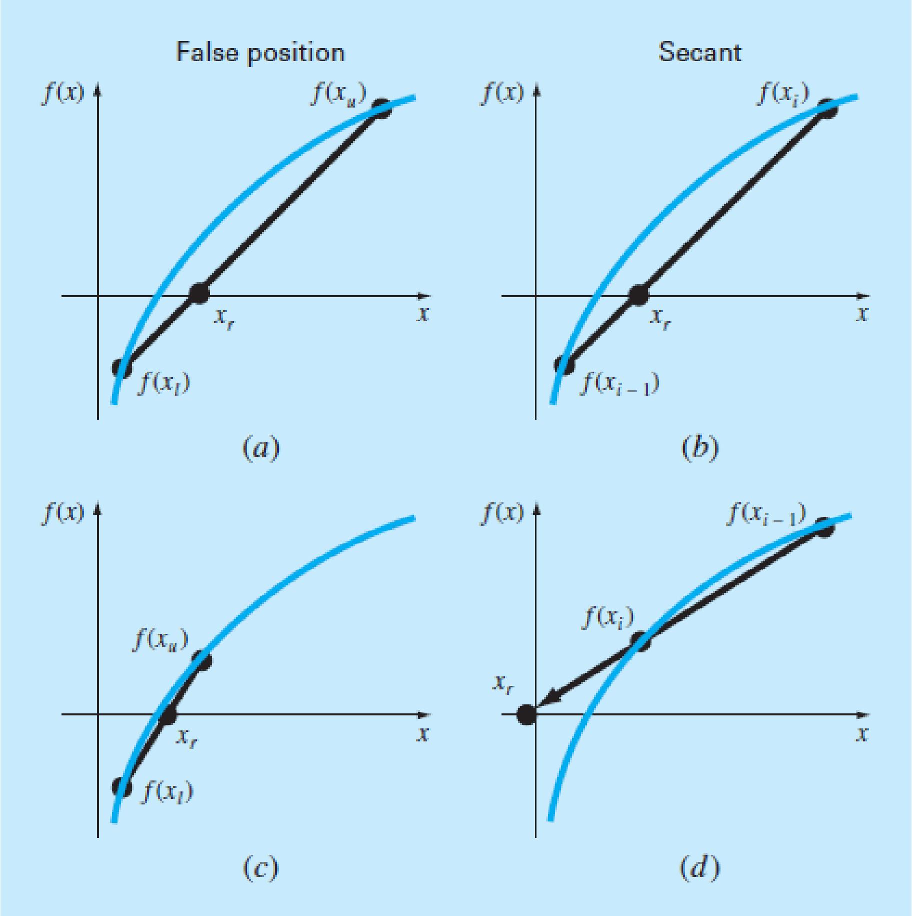
→ Metode false position

Iteration	x_{i-1}	x_i	x_{i+1}
1	0.5	5.0	1.8546
2	5.0	1.8546	−0.10438

→ Metode secant

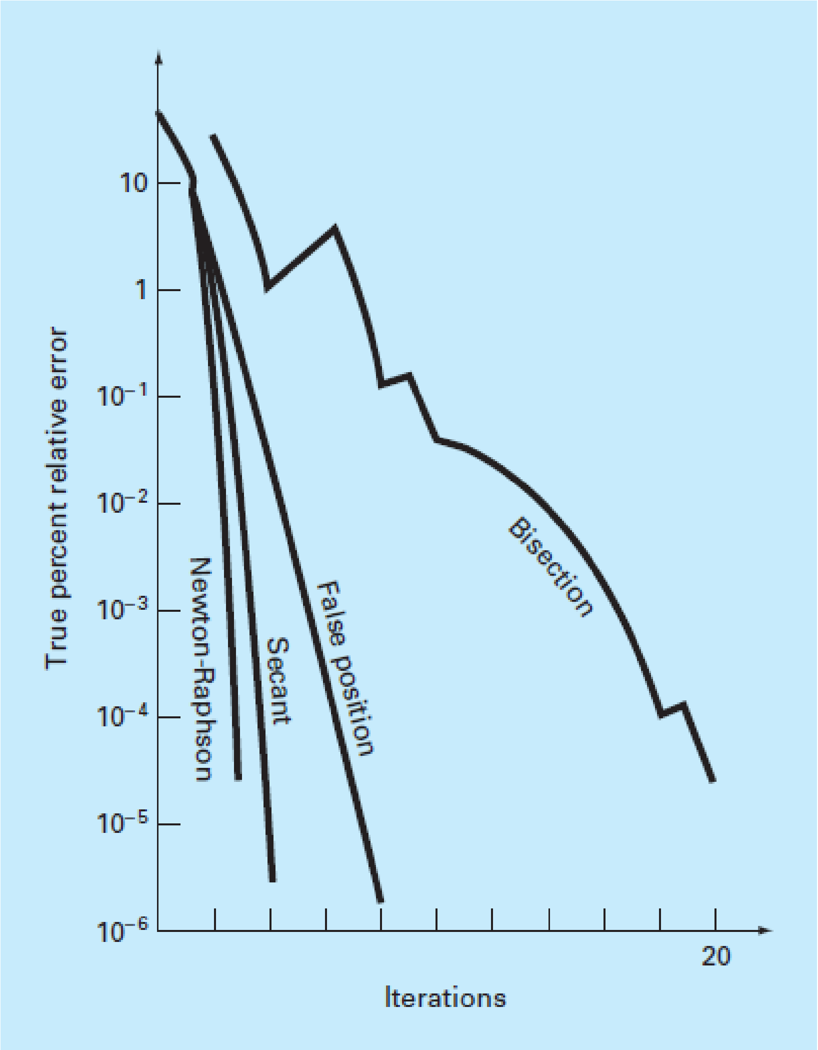
Metode Secant

FIGURE 6.8
Comparison of the false-position and the secant methods. The first iterations (*a*) and (*b*) for both techniques are identical. However, for the second iterations (*c*) and (*d*), the points used differ. As a consequence, the secant method can diverge, as indicated in (*d*).

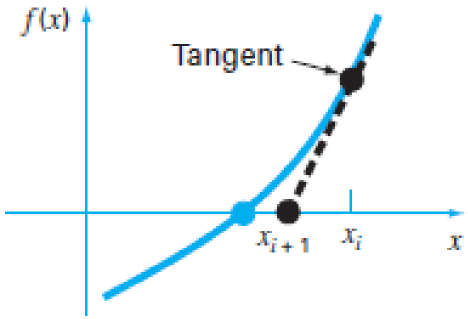
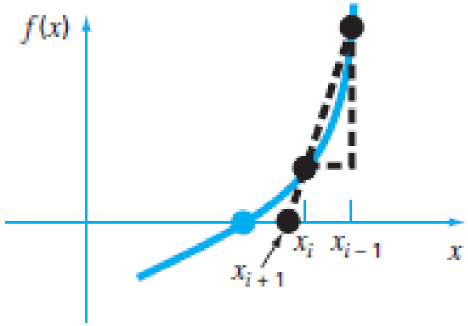


Metode Secant

FIGURE 6.9
Comparison of the true percent relative errors ε_t for the methods to determine the roots of $f(x) = e^{-x} - x$.



Rangkuman

Method	Formulation	Graphical Interpretation	Errors and Stopping Criteria
Newton-Raphson	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$	 	Stopping criterion: $\left\lvert \frac{x_{i+1} - x_i}{x_{i+1}} \right\rvert 100\% \le \epsilon_s$ Error: $E_{i+1} = O(E_i^2)$
Secant	$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$	 	Stopping criterion: $\left\lvert \frac{x_{i+1} - x_i}{x_{i+1}} \right\rvert 100\% \le \epsilon_s$

Latihan

Problems Chapter 6

Nomor 6.2 , 6.3, 6.4