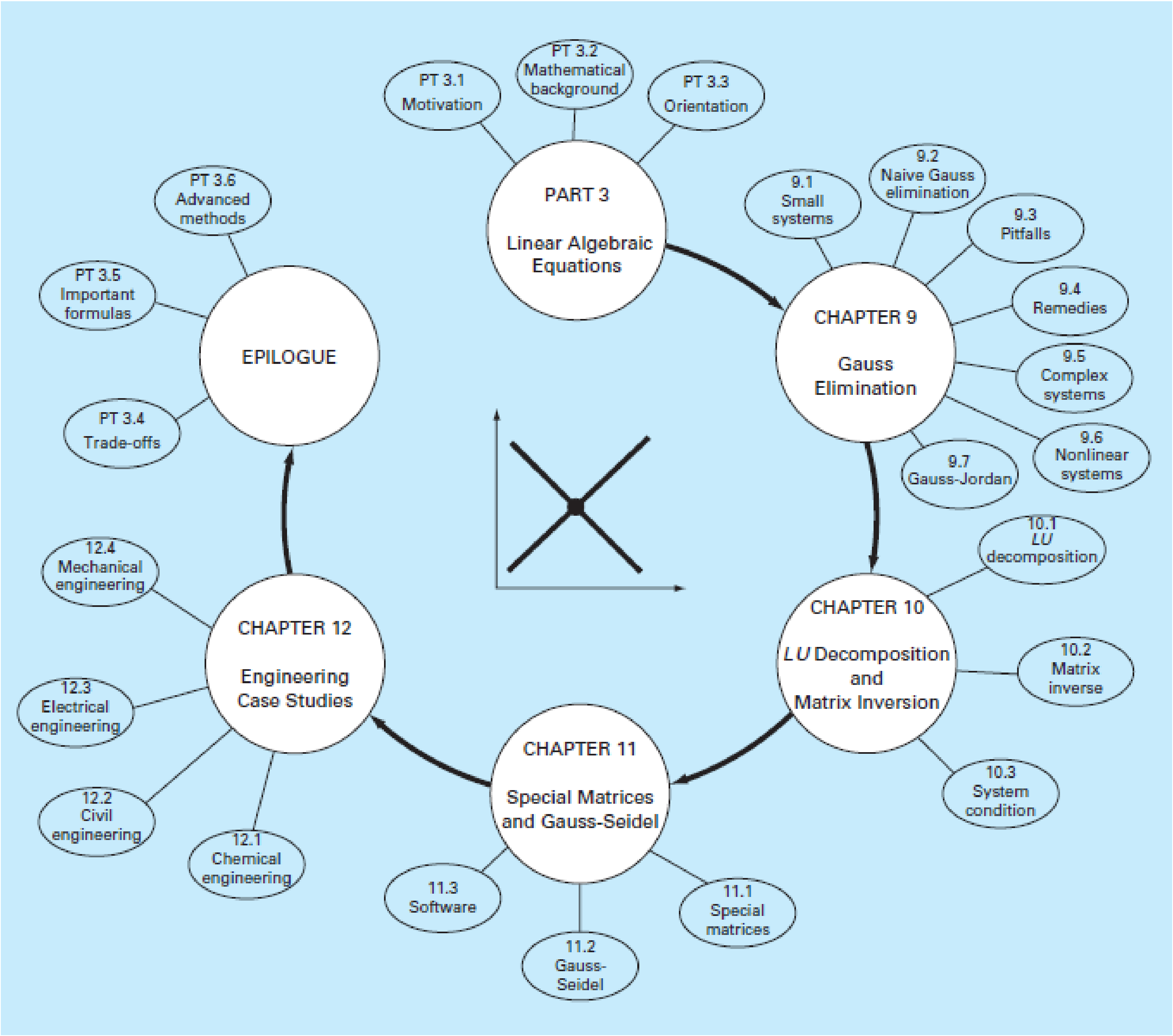
Metode Numerik
EE221

Bab 5. Solusi Sistem Persamaan Linier (Lanjutan)

Dirangkum dan diterjemahkan dari :
- Thomson Brooks Chapra, Steven and Raymond Canale. 2009. Numerical Methods for Engineers 6th Edition, **Chapter 10-11-12**

Nabila Husna Shabrina
Fakultas Teknik dan Informatika, Universitas Multimedia Nusantara

nabila.husna@umn.ac.id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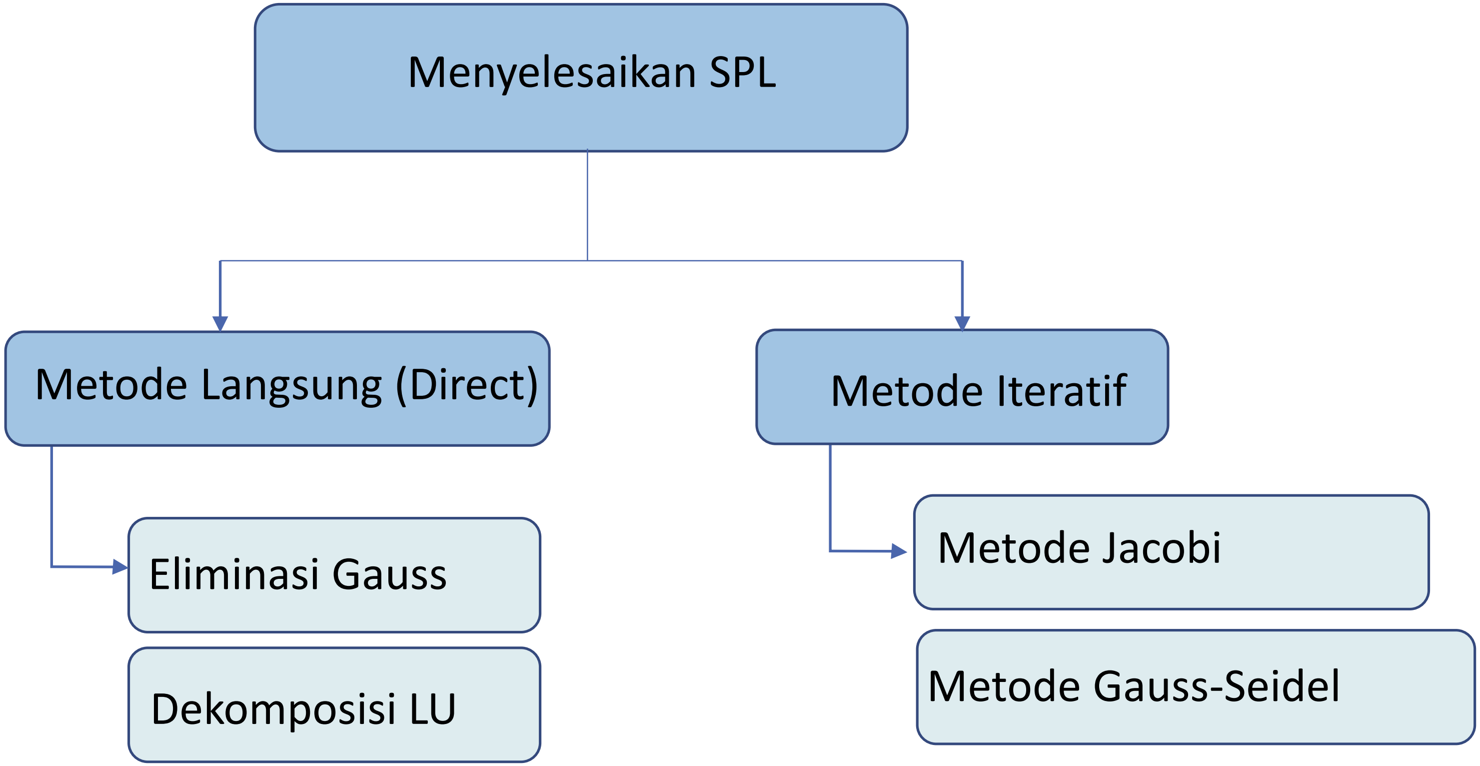
Menyelesaikan SPL



Sub Bahasan :

- Dekomposisi LU
- Metode iterative
  - Jacobi
  - Gauss Seidel

Dekomposisi LU

Dekomposisi LU

Menyatakan SPL dalam matrik

$$[A]\{X\} = \{B\} \longrightarrow [A]\{X\} - \{B\} = 0$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} \longrightarrow [U]\{X\} - \{D\} = 0$$

Asumsi : $[L] = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$

$$[L]\{[U]\{X\} - \{D\}\} = [A]\{X\} - \{B\}$$

Sehingga $[L][U] = [A]$

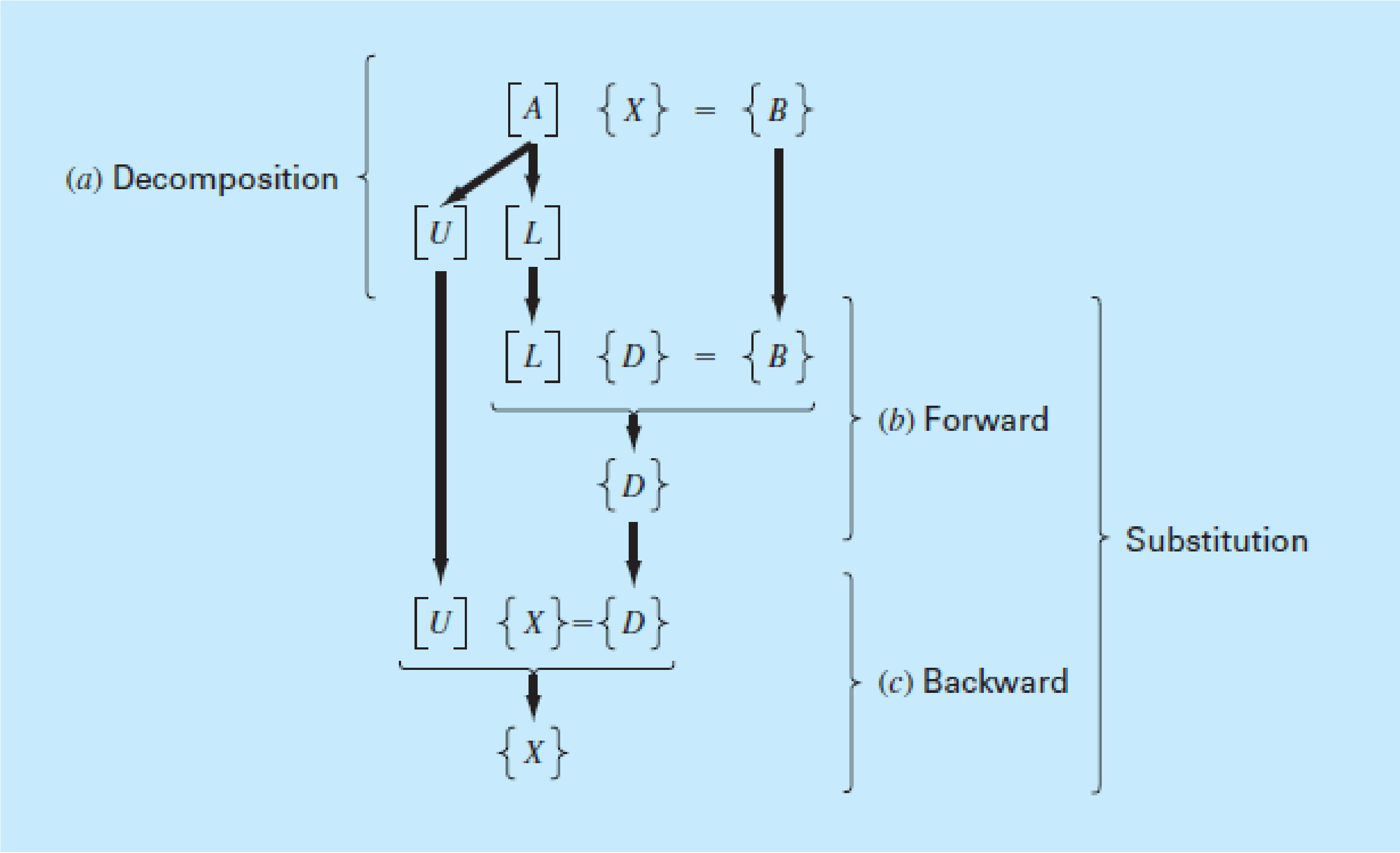
$$[L]\{D\} = \{B\}$$

Dekomposisi LU

Step

1. Dekomposisi [A] menjadi lower [L] dan upper [U]
2. Dengan persamaan $[L]\{D\} = \{B\}$, tentukan D
3. Dengan persamaan $[U]\{X\} - \{D\} = 0$, tentukan X

Dekomposisi LU



Dekomposisi LU

Dekomposisi LU dengan eliminasi gaussian

$$[U] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

$$f_{21} = \frac{a_{21}}{a_{11}} \qquad f_{31} = \frac{a_{31}}{a_{11}} \qquad f_{32} = \frac{a'_{32}}{a'_{22}}$$

Dekomposisi LU

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ f_{21} & a'_{22} & a'_{23} \\ f_{31} & f_{32} & a''_{33} \end{bmatrix}$$

$$[A] \rightarrow [L][U]$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ f_{21} & 1 & 0 \\ f_{31} & f_{32} & 1 \end{bmatrix} \quad [U] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix}$$

Dekomposisi LU

Contoh 1.

Dengan dekomposisi LU dengan versi eliminasi gauss, selesaikan SPL berikut

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

$$[A] = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}$$

Dekomposisi LU

Dengan metode eliminasi gauss didapatkan

$$[U] = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00333 & -0.293333 \\ 0 & 0 & 10.0120 \end{bmatrix}$$

$$f_{21} = \frac{0.1}{3} = 0.03333333 \qquad f_{31} = \frac{0.3}{3} = 0.1000000 \qquad f_{32} = \frac{-0.19}{7.00333} = -0.0271300$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ 0.0333333 & 1 & 0 \\ 0.100000 & -0.0271300 & 1 \end{bmatrix}$$

Dekomposisi LU

$$[A] = [L][U] = \begin{bmatrix} 1 & 0 & 0 \\ 0.0333333 & 1 & 0 \\ 0.100000 & -0.0271300 & 1 \end{bmatrix} \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00333 & -0.293333 \\ 0 & 0 & 10.0120 \end{bmatrix}$$

$$[L][U] = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.0999999 & 7 & -0.3 \\ 0.3 & -0.2 & 9.99996 \end{bmatrix}$$

Dekomposisi LU

Pseudocode

```
SUB Decompose (a, n)
  DOFOR k = 1, n − 1
    DOFOR i = k + 1, n
      factor = a_{i,k}/a_{k,k}
      a_{i,k} = factor
      DOFOR j = k + 1, n
        a_{i,j} = a_{i,j} − factor * a_{k,j}
      END DO
    END DO
  END DO
END Decompose
```

Dekomposisi LU

Contoh 2.

Kerjakan soal contoh 1 dengan metode substitusi LU

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{Bmatrix}$$

Dengan forward substitution didapatkan

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00333 & -0.293333 \\ 0 & 0 & 10.0120 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.5617 \\ 70.0843 \end{Bmatrix}$$

$$[L][U] = [A] \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0.0333333 & 1 & 0 \\ 0.100000 & -0.0271300 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{Bmatrix}$$

Dekomposisi LU

$$\begin{aligned} d_1 \quad\quad\quad\quad\quad\quad &= 7.85 \\ 0.0333333d_1 + \quad d_2 \quad\quad &= -19.3 \\ 0.1d_1 - 0.02713d_2 + d_3 &= 71.4 \end{aligned}$$

$$d_1 = 7.85$$

$$d_2 = -19.3 - 0.0333333(7.85) = -19.5617$$

$$d_3 = 71.4 - 0.1(7.85) + 0.02713(-19.5617) = 70.0843$$

Sehingga didapatkan $\{D\} = \begin{Bmatrix} 7.85 \\ -19.5617 \\ 70.0843 \end{Bmatrix}$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00333 & -0.293333 \\ 0 & 0 & 10.0120 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 7.85 \\ -19.5617 \\ 70.0843 \end{Bmatrix} \longrightarrow \{X\} = \begin{Bmatrix} 3 \\ -2.5 \\ 7.00003 \end{Bmatrix}$$

Dekomposisi LU

Contoh 3.

Dengan dekomposisi LU tentukan matrik inverse dari matrik berikut.

$$[A] = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00333 & -0.293333 \\ 0 & 0 & 10.0120 \end{bmatrix} \quad [L] = \begin{bmatrix} 1 & 0 & 0 \\ 0.0333333 & 1 & 0 \\ 0.100000 & -0.0271300 & 1 \end{bmatrix}$$

Solusi

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.0333333 & 1 & 0 \\ 0.100000 & -0.0271300 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

Dekomposisi LU

$$\{D\}^T = \lfloor 1 \quad -0.03333 \quad -0.1009 \rfloor.$$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00333 & -0.293333 \\ 0 & 0 & 10.0120 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.03333 \\ -0.1009 \end{Bmatrix}$$

$$\{X\}^T = \lfloor 0.33249 \quad -0.00518 \quad -0.01008 \rfloor,$$

$$[A]^{-1} = \begin{bmatrix} 0.33249 & 0 & 0 \\ -0.00518 & 0 & 0 \\ -0.01008 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.0333333 & 1 & 0 \\ 0.100000 & -0.0271300 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

Dekomposisi LU

$$\{X\}^T = \lfloor 0.004944 \quad 0.142903 \quad 0.00271 \rfloor,$$

$$[A]^{-1} = \begin{bmatrix} 0.33249 & 0.004944 & 0 \\ -0.00518 & 0.142903 & 0 \\ -0.01008 & 0.00271 & 0 \end{bmatrix}$$

$$\{X\}^T = \lfloor 0.006798 \quad 0.004183 \quad 0.09988 \rfloor,$$

$$[A]^{-1} = \begin{bmatrix} 0.33249 & 0.004944 & 0.006798 \\ -0.00518 & 0.142903 & 0.004183 \\ -0.01008 & 0.00271 & 0.09988 \end{bmatrix}$$

Dekomposisi LU

Kompleksitas menghitung inverse matrik dengan dekomposisi LU

$$\underbrace{\frac{n^3}{3} - \frac{n}{3}}_{\text{decomposition}} + \underbrace{n(n^2)}_{n \times \text{substitutions}} = \frac{4n^3}{3} - \frac{n}{4}$$

Dekomposisi LU

Latihan

Problems Chapter 10

Nomor 10.3, 10.4, 10.9

Metode Iteratif

Metode Iteratif

- **Metode Gauss Seidel & Jacobi**

$$x_1 = \frac{b_1 - a_{12}x_2 - a_{13}x_3}{a_{11}}$$

$$x_2 = \frac{b_2 - a_{21}x_1 - a_{23}x_3}{a_{22}}$$

$$x_3 = \frac{b_3 - a_{31}x_1 - a_{32}x_2}{a_{33}}$$

$$|\varepsilon_{a,i}| = \left|\frac{x_i^j - x_i^{j-1}}{x_i^j}\right| 100\% < \varepsilon_s$$

Metode Iteratif

First Iteration

$$x_1 = (b_1 - a_{12}x_2 - a_{13}x_3)/a_{11}$$
$$x_2 = (b_2 - a_{21}x_1 - a_{23}x_3)/a_{22}$$
$$x_3 = (b_3 - a_{31}x_1 - a_{32}x_2)/a_{33}$$

$$x_1 = (b_1 - a_{12}x_2 - a_{13}x_3)/a_{11}$$
$$x_2 = (b_2 - a_{21}x_1 - a_{23}x_3)/a_{22}$$
$$x_3 = (b_3 - a_{31}x_1 - a_{32}x_2)/a_{33}$$

Second Interation

$$x_1 = (b_1 - a_{12}x_2 - a_{13}x_3)/a_{11}$$
$$x_2 = (b_2 - a_{21}x_1 - a_{23}x_3)/a_{22}$$
$$x_3 = (b_3 - a_{31}x_1 - a_{32}x_2)/a_{33}$$

$$x_1 = (b_1 - a_{12}x_2 - a_{13}x_3)/a_{11}$$
$$x_2 = (b_2 - a_{21}x_1 - a_{23}x_3)/a_{22}$$
$$x_3 = (b_3 - a_{31}x_1 - a_{32}x_2)/a_{33}$$

(a) Metode Gauss Seidel

(b) Metode Jacobi

Metode Iteratif

Contoh 4.

Tentukan solusi dari SPL berikut dengan Metode Gauss Seidel.

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 \end{aligned}$$

Bandingkan hasil yang diperoleh dengan solusi yang sebenarnya yaitu

$x_1 = 3, x_2 = -2.5$ dan $x_3 = 7$

Metode Iteratif

$$x_1 = \frac{7.85 + 0.1x_2 + 0.2x_3}{3}$$

$$x_2 = \frac{-19.3 - 0.1x_1 + 0.3x_3}{7}$$

$$x_3 = \frac{71.4 - 0.3x_1 + 0.2x_2}{10}$$

Iterasi pertama

$$x_1 = \frac{7.85 + 0 + 0}{3} = 2.616667$$

$$x_2 = \frac{-19.3 - 0.1(2.616667) + 0}{7} = -2.794524$$

$$x_3 = \frac{71.4 - 0.3(2.616667) + 0.2(-2.794524)}{10} = 7.005610$$

Metode Iteratif

Iterasi kedua

$$x_1 = \frac{7.85 + 0.1(-2.794524) + 0.2(7.005610)}{3} = 2.990557 \qquad |\varepsilon_t| = 0.31\%$$

$$x_2 = \frac{-19.3 - 0.1(2.990557) + 0.3(7.005610)}{7} = -2.499625 \qquad |\varepsilon_t| = 0.015\%$$

$$x_3 = \frac{71.4 - 0.3(2.990557) + 0.2(-2.499625)}{10} = 7.000291 \qquad |\varepsilon_t| = 0.0042\%$$

Metode Iteratif

- **Pelajari penggunaan software untuk menyelesaikan SPL pada chapter 11.3**
- **Latihan**

Problems Chapter 11

Nomor 11.10 , 11.11

Contoh dalam Teknik Elektro

Analisis rangkaian listrik di bawah. Tentukan arus pada titik 1, 2, 3, 4, 5 dan 6.