Classical Scattering.

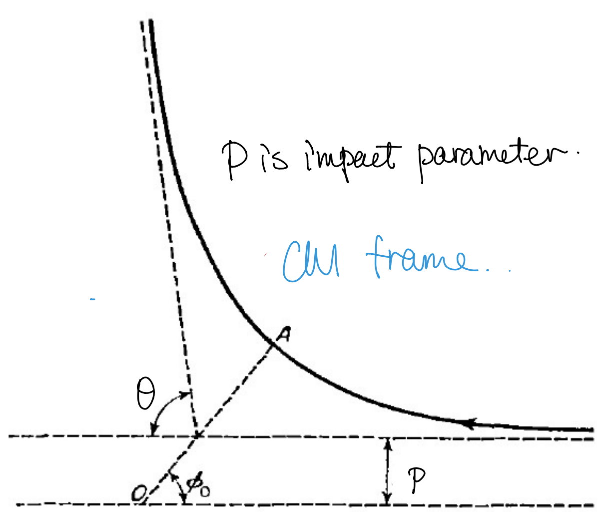
taken from Landau's book mechanics

CM frame.

In physical applications, we are concerned the scattering of a beam of identical particles incident with uniform velocity V_{in}. on the scattering centre. The different particles in the beam have different impact parameters and are therefore scattered through different angles θ. Let dN be the number of particles scattered per unit time through angles between θ and $\theta + d\theta$. This number itself is not suitable for describing the scattering process, since it is proportional to the density of the incident beam. We therefore use the ratio

$$d\sigma = dN/n \quad (1)$$

where n is the number of particles passing in unit time through unit area of the beam. This ratio has the dimensions of area and is called the effective scattering cross section.



The path of a particle in a central field is symmetrical about a line from the centre to the nearest point in the orbit (OA). Hence the two asymptotes to the orbit make equal angles (ϕ_0, say) with this line. The angle θ through which the particle is deflected is.

$$\theta = |\pi - 2\phi_0| . \quad (2)$$

The scattering problem can be modeled as a particle scatt in a central potential $U(r)$, then the total energy can be written as

$$E = \frac{1}{2}(m\dot{r}^2 + r^2\dot{\phi}^2) + U(r) = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}P_L^2/mr^2 + U(r). \quad ③$$

where the angular momentum P_L is defined by.

$$P_L = mr^2\dot{\phi} \quad ④$$

Hence.

$$\dot{r} = \frac{dr}{dt} = \sqrt{\frac{2}{m}[E - U(r)] - \frac{P_L^2}{m^2r^2}}. \quad ⑤$$

From the defination of angular momentum, we have $d\phi = P_L dt/mr^2$; substituting dt from eq. ⑤ and integrating, we find.

$$\phi = \int_{r_{min}}^{\infty} \frac{P_L\, dr/r^2}{\sqrt{2m(E-U(r)) - P_L^2/r^2}}. \quad ⑥$$

r_{min} is a zero of the radicand.

It is convenient to use instead of constants E and P_L, the velocity V_{in} of the particle at infinity and the impact parameter P. The energy and angular momentum are given in terms of these quantities by

$$E = \frac{1}{2}mv_{in}^2, \quad P_L = mPv_{in}. \quad ⑦$$

and formula ⑥ becomes.

$$\phi = \int_{r_{min}}^{\infty} \frac{(P^2/r^2)\, dr}{\sqrt{1 - (P^2/r^2) - (2U/mv_{in}^2)}}. \quad ⑧$$

Together with eq ②, this gives θ as a function of P.

We shall suppose that the relation between θ and P is one to one; In that case, only those particles whose impact parameters lie between $P(\theta)$ and $P(\theta) + dP(\theta)$ are scatted at angles between θ and $\theta + d\theta$. The number of such particles is equal to the product of n and the area between two circles of radii P and $P+dp$. i.e. $dN = 2\pi P dp \cdot n$ The effective cross section is therefore

$$d\sigma = 2\pi P dp. \qquad (9)$$

In order to find the dependence of $d\sigma$ on the angle of scattering, we need to rewrite eq. (9) as

$$d\sigma = 2\pi P(\theta) \left| dP(\theta)/d\theta \right| d\theta.$$

Often $d\sigma$ is referred to the solid angle element $d\Omega$ instead of the plane angle element $d\theta$. Hence we have

$$d\sigma = \frac{P(\theta)}{\sin\theta} \left| \frac{dP}{d\theta} \right| d\Omega.$$

Classical trajectory

taken from Angela's thesis.

If one assume the classical motion is happened in one plane, the classical trajectory is then given by the solution of.

$$V_x = \frac{dx}{dt}.$$

$$\mu \dot{V}_x = -\frac{\partial U(r)}{\partial x}$$

$$V_y = \frac{dy}{dt}$$

$$\mu \dot{V}_y = -\frac{\partial U(r)}{\partial y}$$