量子少体模型在核反应研究中的挑战与前沿进展

金磊
同济大学

Outline

- STARS
- Three body Forces in Reaction Model
- Trojan Horse Mechanism
- Quantum Four Body Model

Shanghai Tongji Advanced Reaction Solver (STARS)

For a two−body structured projectile $a = b + x$ interacting with a target A, in the three−body model space the scattering wave function can be solved by

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STARS uses modern techniques such as emulator, Bayesian Analysis, and machine learning and to solve the linear equations of the quantum three-body problem.

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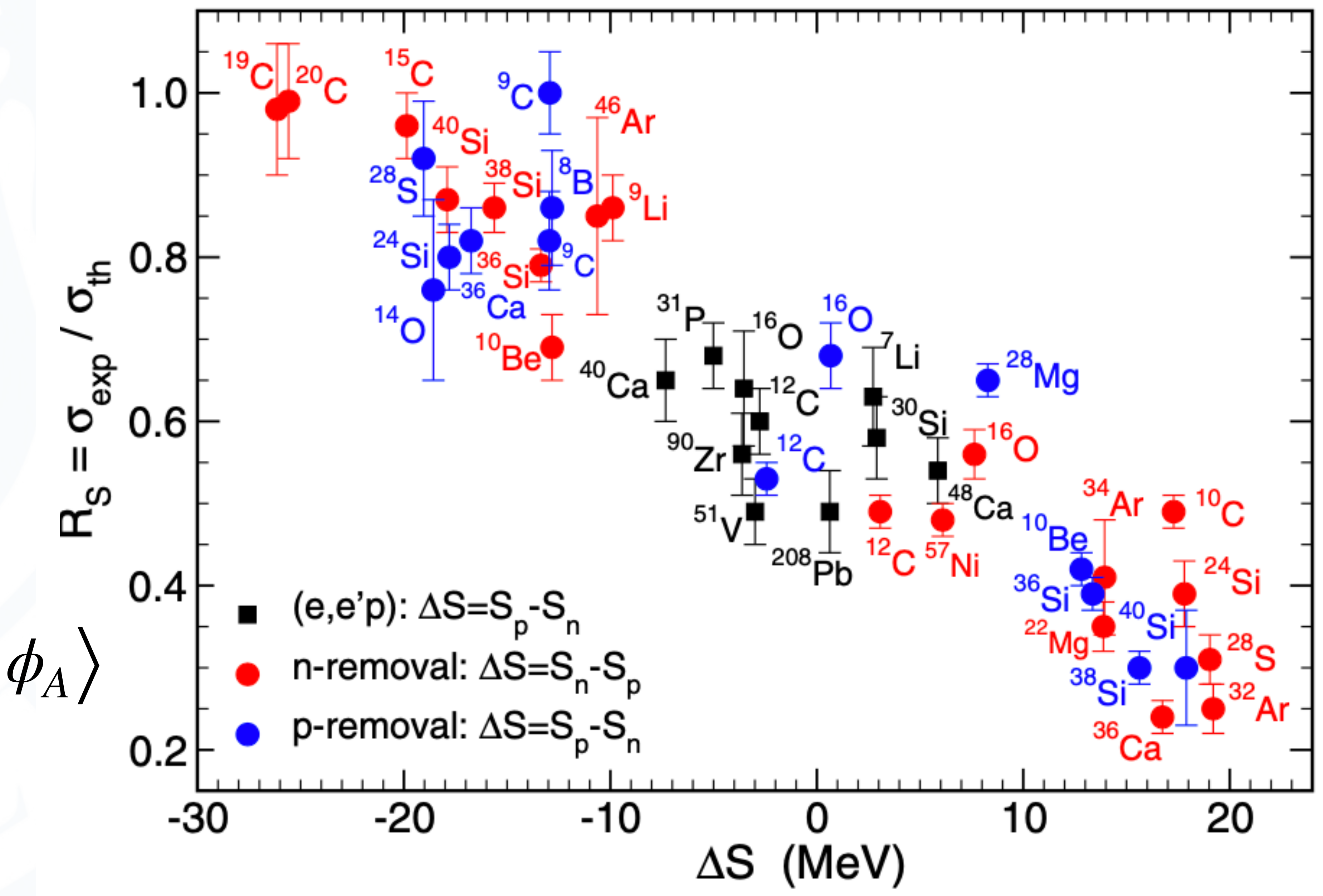

[Tostevin and Gade. PRC **90**, 057602 (2014)]

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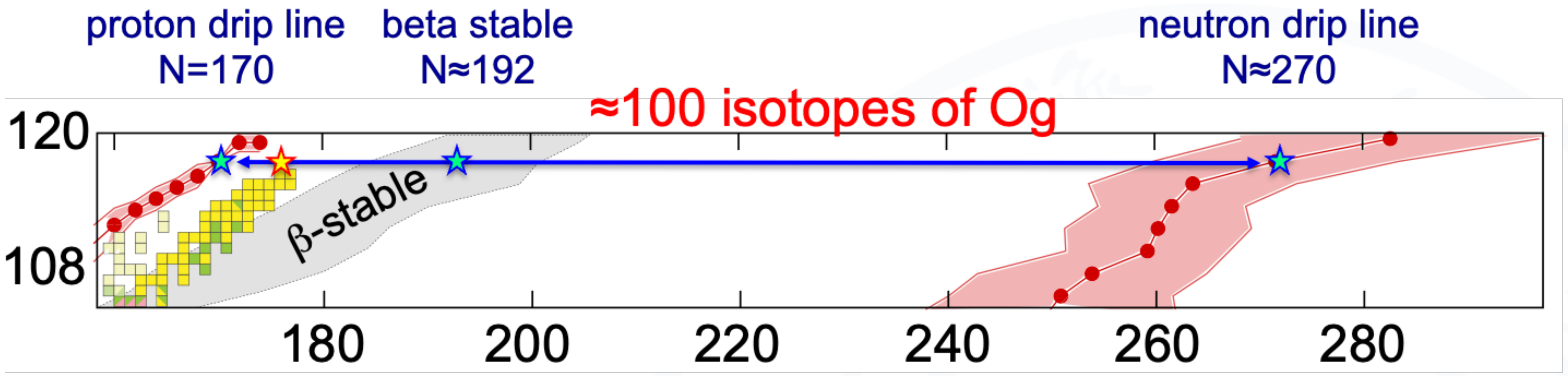
Trojan Horse Mechanism



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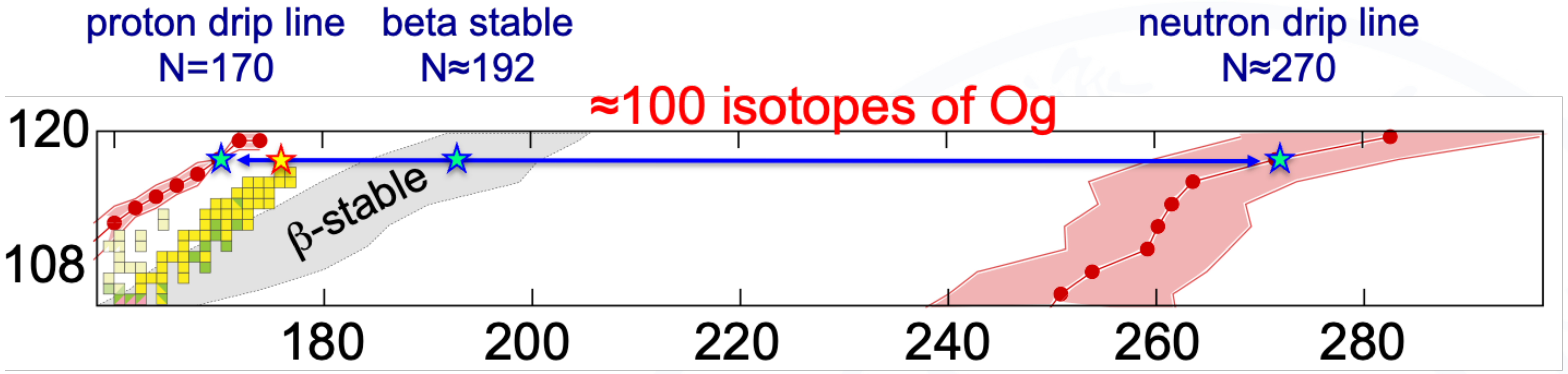

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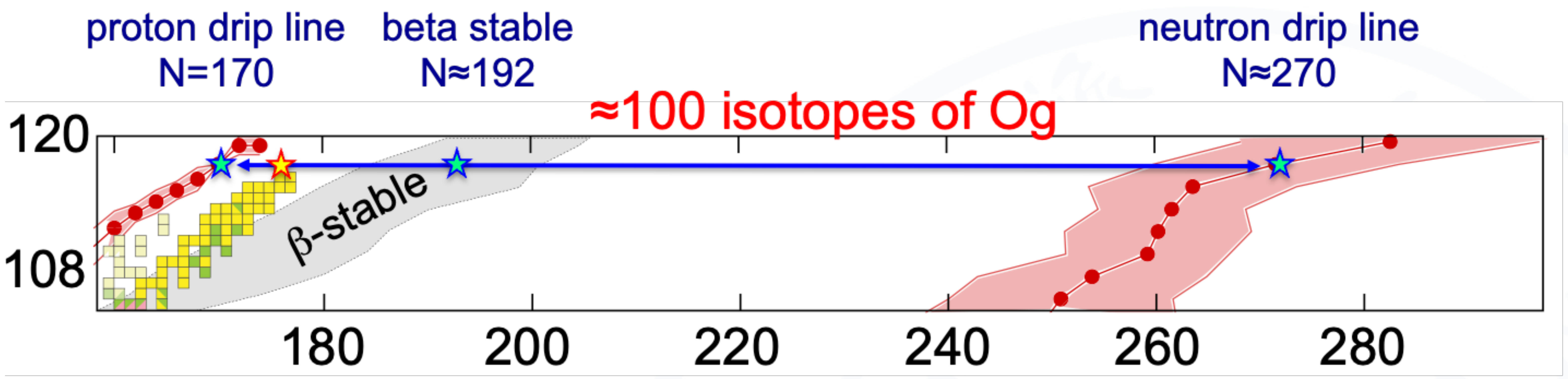

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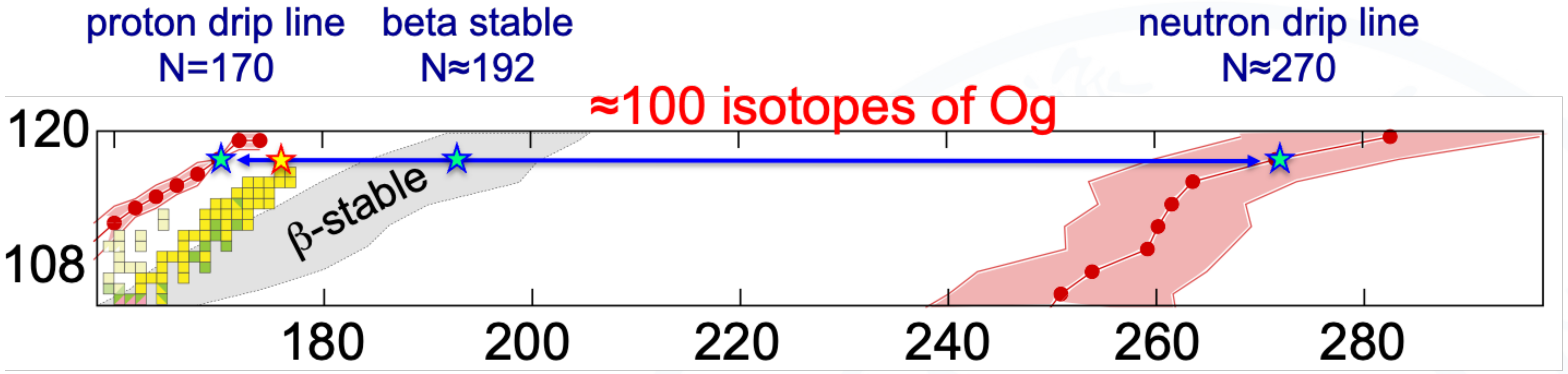

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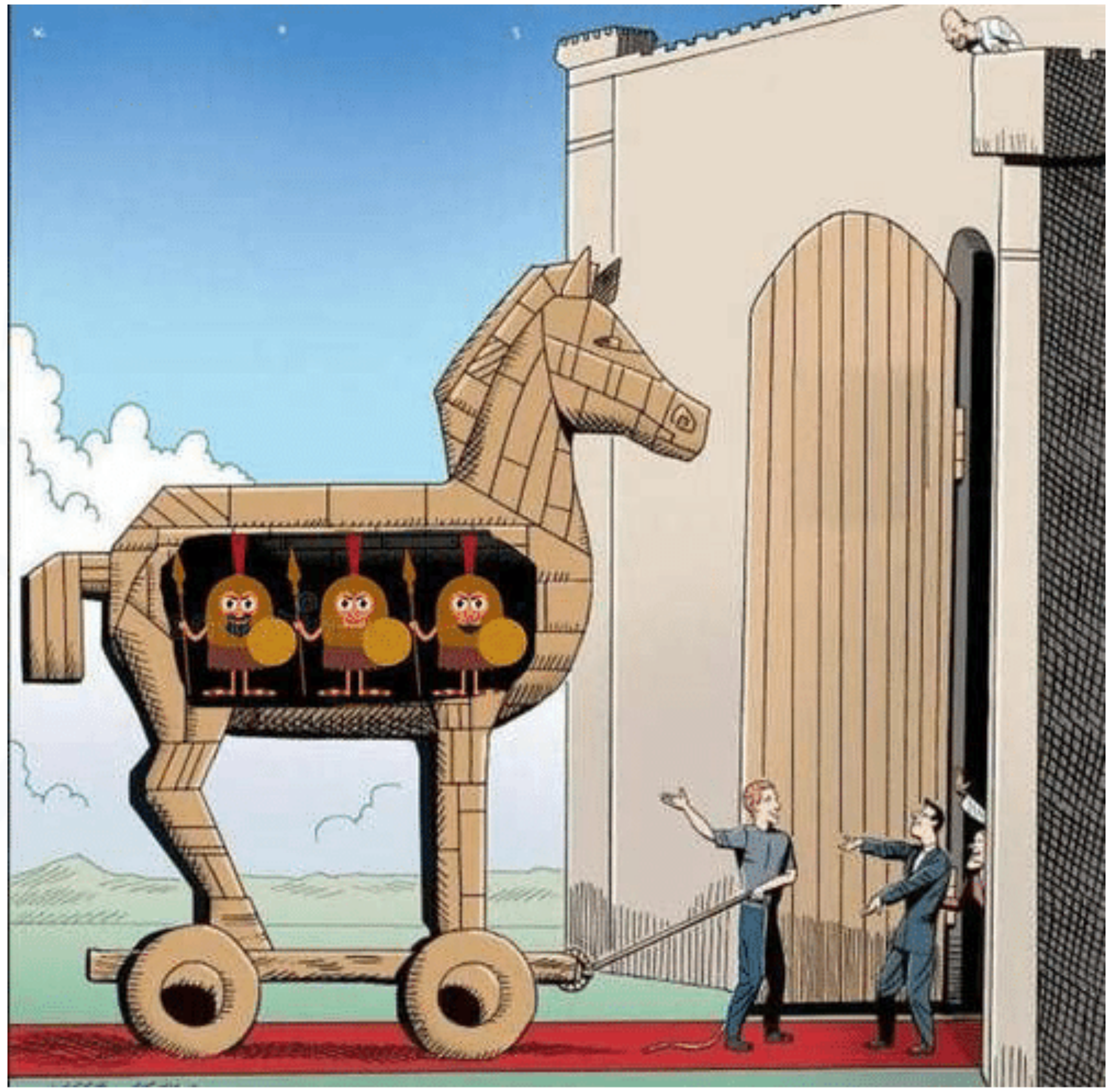

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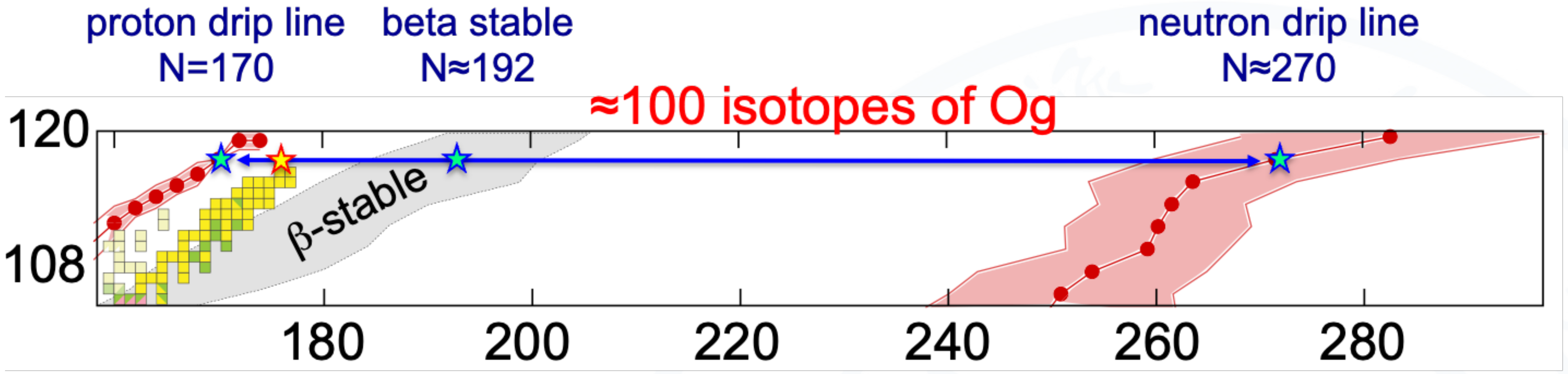

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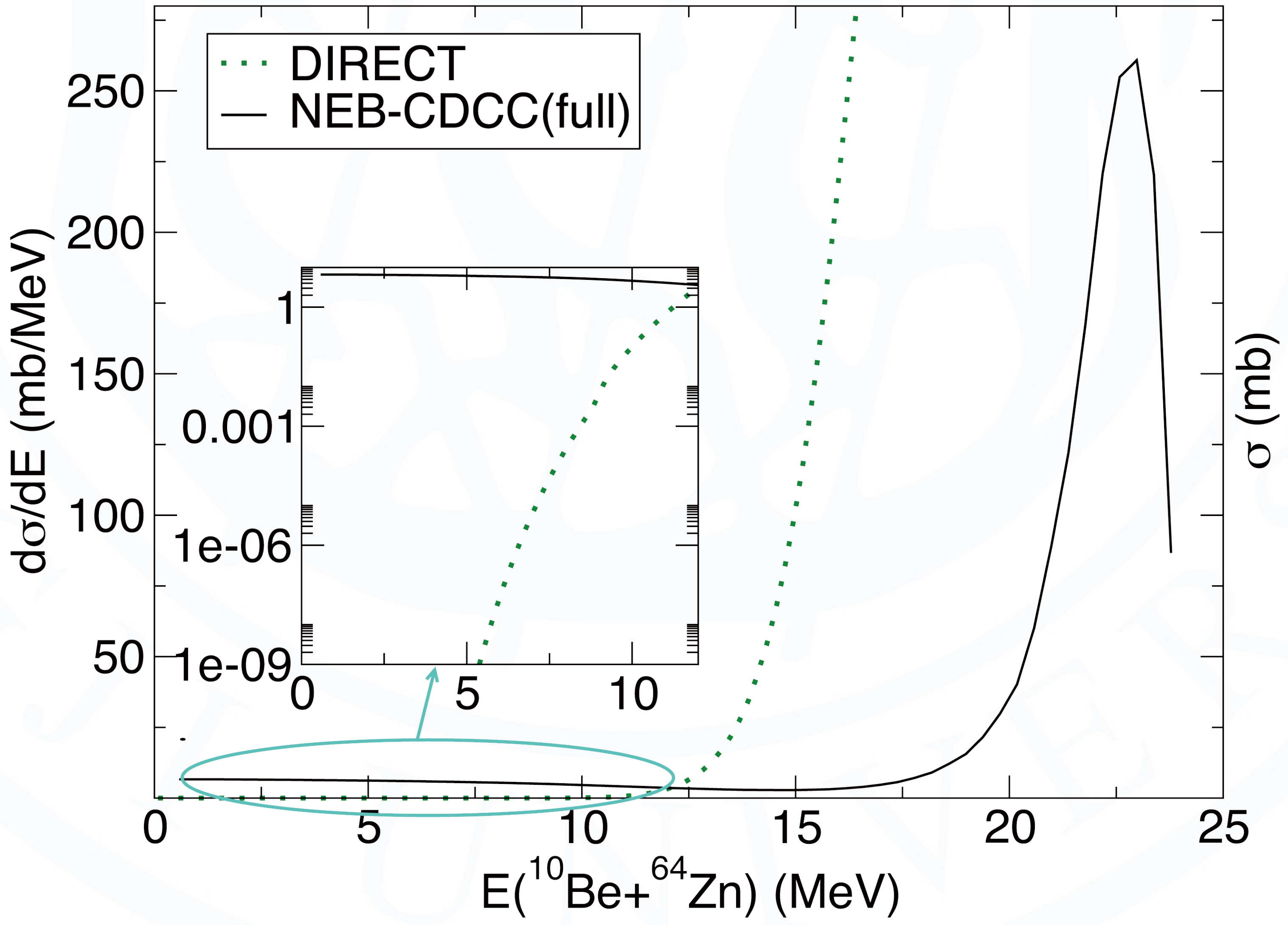

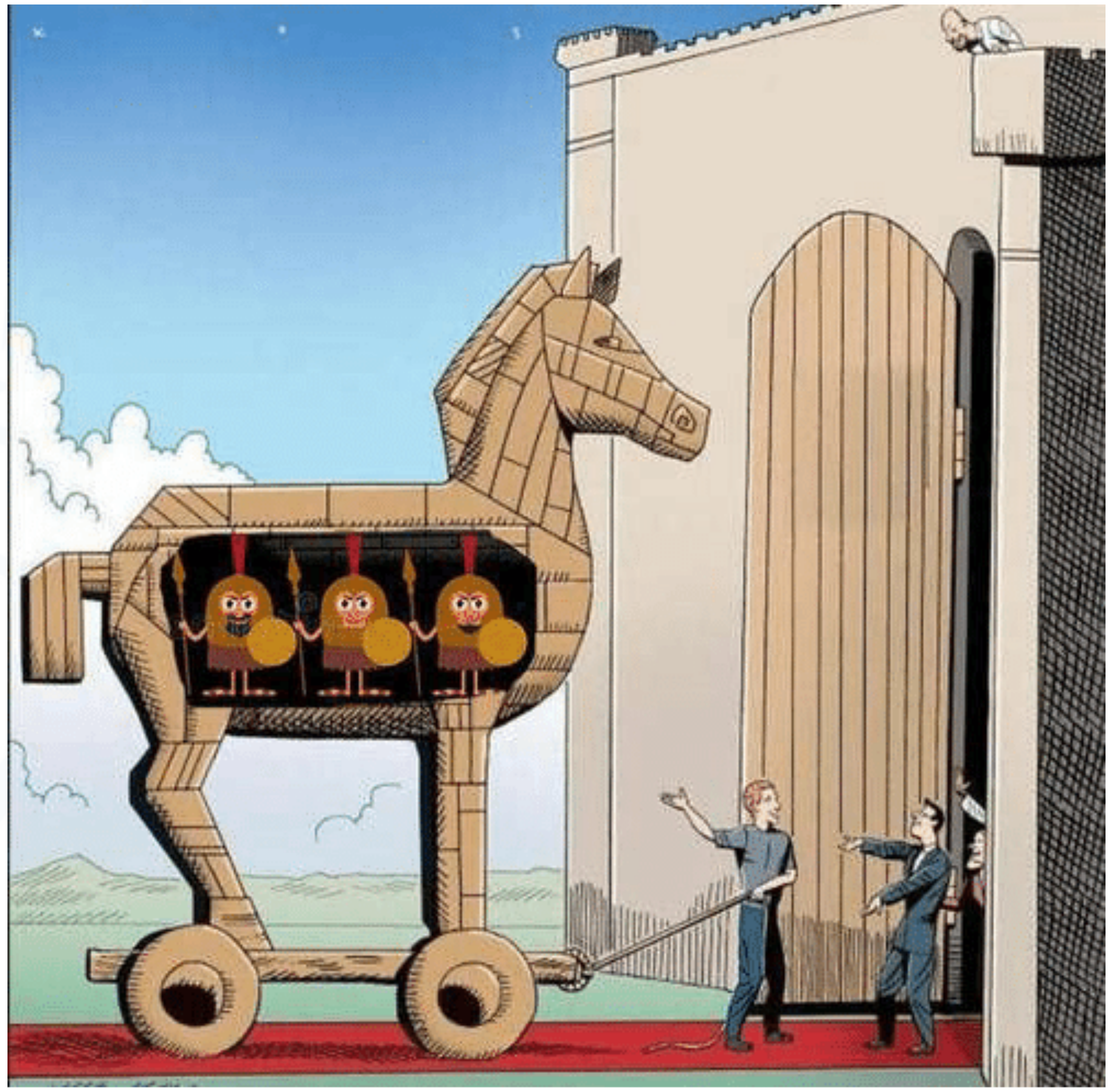

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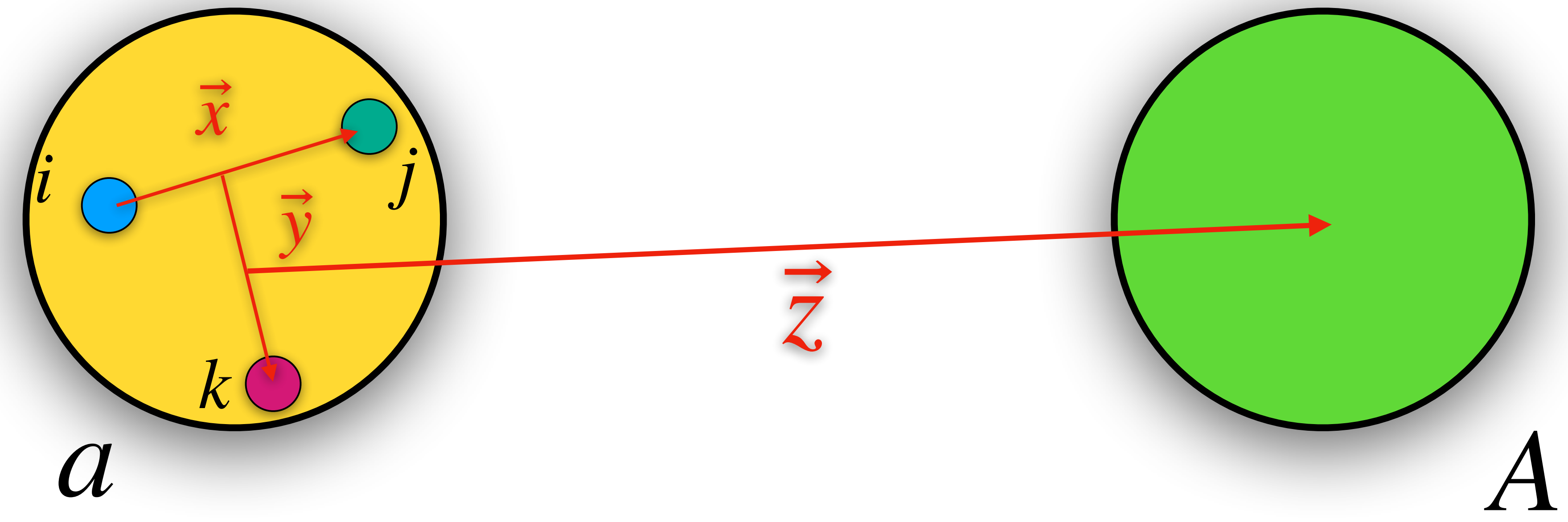

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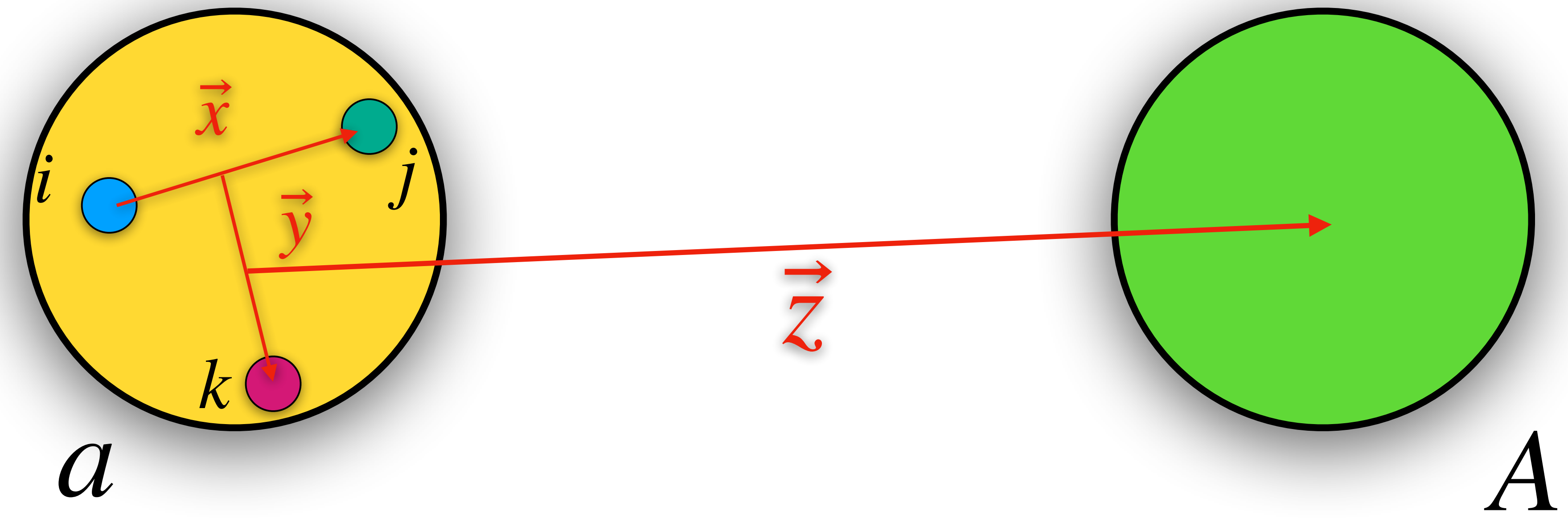

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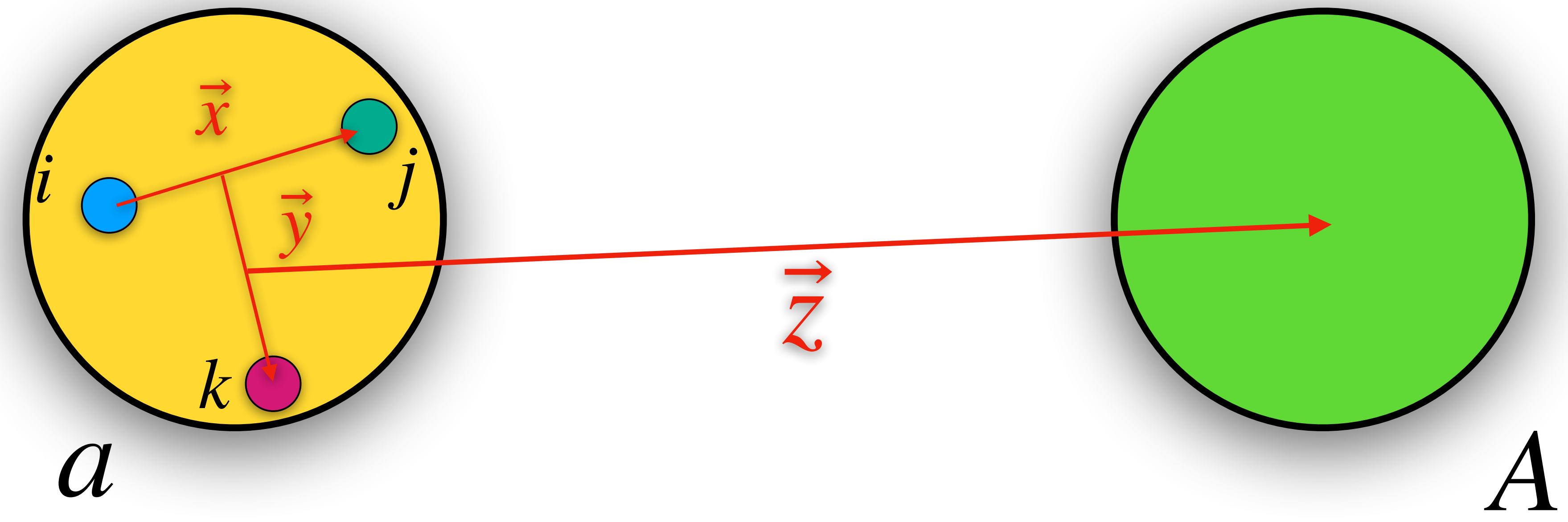

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Using the complex scaling technique, one can treat the three−body bound state, scattering states, and resonance states simultaneously