Course 6: Theory for exploring nuclear reaction experiments

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Observables in Nuclear Collision[1]

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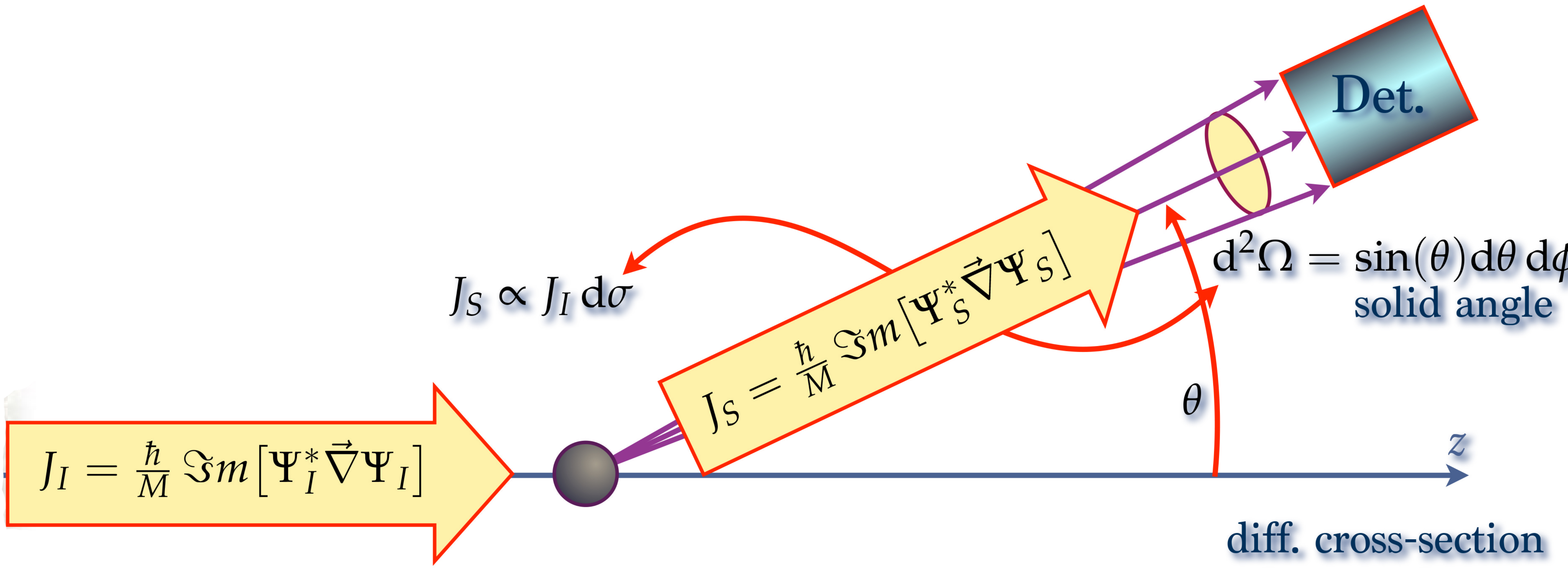
1 Some slides are taken from Tristan Hübsch, Quantum Mechanics II, Howard University

Cross sections



diff. cross-section

Determine J_S / J_I as a function of θ: $\frac{J_S}{J_I} = \frac{\mathrm{d}\sigma}{r^2 \mathrm{d}^2\Omega} =: \frac{1}{r^2} \underbrace{\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} := \frac{\mathrm{d}\sigma}{\mathrm{d}^2\Omega} \right]}_{:= \sigma(\theta, \phi)}$

Q M II

Quantum Scattering

General Theory

- Scattering types:
  - Elastic scattering: $A + X \rightarrow A + X$
    - Kinetic energy is conserved; linear & angular momenta are conserved
    - Ex.: billiard balls, marbles,
  - Inelastic scattering: $A + X \rightarrow A^* + X^*$
    - Kinetic energy is *not* conserved, *total* energy is; must include internal energy

Quantum Scattering

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    - Kinetic energy is *not* conserved, *total* energy is; must include internal energy
    - Ex.: traffic collisions; vehicles and people absorb some of the energy
  - Rearrangement collision: $A + X \rightarrow B + Y$
    - Parts of the subsystems A and X get rearranged / exchanged
    - Ex.: nuclear and chemical reactions
  - Particle production: $A + X \rightarrow B + C + D + \ldots + Y + Z$
    - The final state consists of more than two (n) separate particles
    - ($n - 2$) of which are therefore *produced* by the collision
    - Total energy-momentum being conserved, some of the kinetic energy of the colliding objects A and X is transformed into the masses of the created particles

In fact, total energy-momentum and angular momentum are always conserved

Quantum Scattering

General Theory

Special cases

- Focus on elastic and inelastic scattering / collisions
  - elastic scattering involves no exchange with internal energy
  - inelastic scattering involves some exchange with internal energy
- In- and out-states both involve two particles **work in CM frame**
  - For each of the different possible outcomes
  - …calculate a separate scattering amplitude and cross-section
  - The *total* cross-section is the the sum of these
  - The relative ratios are called branching ratios
- Set-up: $\widehat{H} = \underbrace{-\frac{\hbar^2}{2\mu}\vec{\nabla}^2 + \widehat{h}_1 + \widehat{h}_2}_{=\widehat{H}_0} + \widehat{W}$ $\qquad \underbrace{\big[\widehat{h}_1 + \widehat{h}_2\big]\chi_a = \epsilon_a \chi_a}_{\text{“internal”}}$
  - separation of variables:

$$\widehat{H}\,\Psi^{(+)}_{\mathbf{a}} = E_{\mathbf{a}}\,\Psi^{(+)}_{\mathbf{a}} \qquad \Psi^{(+)}_{\mathbf{a}} = \sum_{a'} \psi^{(+)}_{\mathbf{a},a'}(\vec{r})\,\chi_{a'} \qquad \mathbf{a} := (\vec{k}_a, a)$$

Quantum Scattering

General Theory

$$\Psi^{(+)}_{\mathbf{a}} = \sum_{a'} \psi^{(+)}_{\mathbf{a},a'}(\vec{r})\,\chi_{a'}$$

- The general solution is expected in the form

the incident state: *same* state

$$\psi^{(+)}_{\mathbf{a}}(\vec{r}) \sim A\Big\{ \underbrace{e^{i\vec{k}_a\cdot\vec{r}}}_{\text{incident}} + \underbrace{f^{(+)}_{\mathbf{a}\,a}(\Omega_r)\frac{e^{+ik_a r}}{r}}_{\textit{elastic}\text{-scattered}} \Big\}$$

any *other* state: $\chi_b \neq \chi_a$

$$\psi^{(+)}_{\mathbf{b}}(\vec{r}) \sim A\Big\{ \underbrace{f^{(+)}_{\mathbf{a},b}(\Omega_r)\frac{e^{+ik_b r}}{r}}_{\textit{inelastic}\text{-scattered}} \Big\}_{b\neq a}$$

out-going spherical wave

- The probability currents are

$$J_{I\,(\mathbf{a})} = |A|^2\frac{\hbar k_a}{\mu} \qquad J_{S\,(\mathbf{a})} = |A|^2\frac{\hbar k_a}{\mu}\frac{1}{r^2}\big|f^{(+)}_{\mathbf{a},a}(\Omega_r)\big|^2 \qquad J_{S\,(\mathbf{b})} = |A|^2\frac{\hbar k_b}{\mu}\frac{1}{r^2}\big|f^{(+)}_{\mathbf{a},b}(\Omega_r)\big|^2$$

- …and the cross-sections

$$\sigma_{\mathbf{a},a}(\theta_r) = \big|f^{(+)}_{\mathbf{a},a}(\Omega_r)\big|^2 \qquad\qquad \sigma_{\mathbf{a},b}(\theta_r) = \frac{k_b}{k_a}\big|f^{(+)}_{\mathbf{a},b}(\Omega_r)\big|^2$$

- Aim to compute the scattering amplitudes

Quantum Scattering

General Theory

- These wave-functions, after all, satisfy a Sturm-Liouville type 2nd order partial differential equation
  - …and there is always the “other” solution

$$\underbrace{\frac{e^{+ik_a r}}{r}}_{out} \leftrightarrow \underbrace{e^{ikx}}_{right} \quad \text{just as} \quad \underbrace{e^{-ikx}}_{left} \leftrightarrow \underbrace{\frac{e^{-ik_a r}}{r}}_{in}$$

$$e^{-i\omega t} e^{ikx} = \underbrace{e^{-i(\omega t - kx)}}_{\substack{\text{stationary} \\ @\, x = \frac{\omega}{k} t}}$$

as t grows, so does x

- So, include

$$\widehat{H}\,\Psi^{(-)}_{\mathbf{b}} = E_{\mathbf{b}}\,\Psi^{(-)}_{\mathbf{b}} \qquad \Psi^{(-)}_{\mathbf{b}} = \sum_{b'} \psi^{(-)}_{\mathbf{b},b'}(\vec{r})\,\chi_{b'} \qquad \mathbf{b} := (\vec{k}_b, b)$$

$$\psi^{(-)}_{\mathbf{b}}(\vec{r}) \sim A\Big\{ \underbrace{e^{i\vec{k}_b\cdot\vec{r}}}_{\text{plane}} + \underbrace{f^{(-)}_{\mathbf{b},b}(\Omega_r)\frac{e^{-ik_b r}}{r}}_{\text{captured}} \Big\}$$

$$\psi^{(-)}_{\mathbf{c}}(\vec{r}) \sim A\Big\{ \underbrace{f^{(-)}_{\mathbf{b},c}(\Omega_r)\frac{e^{-ik_c r}}{r}}_{\text{captured}} \Big\}_{c\neq b} \qquad \chi_c \neq \chi_b$$

in-coming spherical wave

Use

$$\{\Psi^{(+)}_{\mathbf{a}}\} \quad \& \quad \{\Psi^{(-)}_{\mathbf{b}}\}$$

Cross sections with T-matrix (general cases)

Relative transition probability with the T-matrix

$$w = 2\pi \sum_{\mathrm{b}} \delta(E_{\mathrm{b}} - E_{\mathrm{a}}) |\langle \mathrm{b}|\mathscr{T}|\mathrm{a}\rangle|^2$$

The cross section

$$\Delta\sigma = w/\Phi_{\mathrm{A}} \longrightarrow$$

relative flux of the incident particles with respect to the target

$$\Phi_{\mathrm{A}} = (2\pi)^{-3} v_{\mathrm{i}} = (2\pi)^{-3} |\boldsymbol{v}_{\mathrm{A}} - \boldsymbol{v}_{\mathrm{B}}|$$

Thus

$$\Delta\sigma = \sum_{\mathrm{b}} \delta(E_{\mathrm{b}} - E_{\mathrm{a}}) \delta(\mathbf{P}_{\mathrm{b}} - \mathbf{P}_{\mathrm{a}}) \frac{(2\pi)^4}{v_{\mathrm{i}}} |T_{\mathrm{ba}}|^2$$

If the collision has the type (breakup) $\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C}_1 + \mathrm{C}_2 + \cdots \mathrm{C}_n$

The cross section may rewritten as

$$\Delta\sigma = \sum_{\alpha_1,\alpha_2,,,,\alpha_n} \int \mathrm{d}\boldsymbol{k}_1\, \mathrm{d}\boldsymbol{k}_2 \ldots \mathrm{d}\boldsymbol{k}_n\, \delta(E_{\mathrm{b}} - E_{\mathrm{a}}) \delta(\mathbf{P}_{\mathrm{b}} - \mathbf{P}_{\mathrm{a}}) \frac{(2\pi)^4}{v_{\mathrm{i}}} |T_{\mathrm{ba}}|^2.$$

Cross sections with T-matrix (general cases)

Unpolarized- the average differential cross section

$$d\bar{\sigma} = \frac{1}{(2S_A + 1)(2S_B + 1)} \sum_{S_A, S_B} d\sigma^{(S_A, S_B)}$$

On-shell T-matrix

$$\mathcal{T}_{\beta\alpha} = \langle \Psi_\beta^{(-)} | V_\alpha | \phi_\alpha \rangle = \langle \phi_\beta | V_\beta | \Psi_\alpha^{(+)} \rangle.$$

Momentum space: compute T-matrix

two body

$$T = V + VG_0T$$

three body

$$U^{ij} = \bar{\delta}_{ij} G_0^{-1}(E) + \sum_k \bar{\delta}_{ik} t_k G_0(E) U^{kj}$$

$$U^{0j} = G_0^{-1}(E) + \sum_k t_k G_0 U^{kj}$$

$$\sigma_{i \leftarrow j} \propto |\langle \Phi_i | U^{ij} | \Phi_j \rangle|^2$$

Coordinate space: scattering wave function

Post-Prior equivalence

Gell-Mann-Goldberger Transformation

The system Hamiltonian $H = H_\alpha + T_\alpha + V_\alpha = H_\beta + T_\beta + V_\beta = \cdots$

If potential can divided into two term $V = U + \Delta V$

$$(E - H_\alpha - T_\alpha - V_\alpha)\Psi = 0,$$

$$(E - H_\alpha - T_\alpha - U_\alpha)\chi_\alpha = 0.$$

Then $(E - H_\alpha - T_\alpha - V_\alpha)\Psi_\alpha = (E - H_\alpha - T_\alpha - V_\alpha)\chi_\alpha + (V_\alpha - U_\alpha)\chi_\alpha,$

from which

$$\Psi_\alpha^{(+)} = \chi_\alpha^{(+)} + \frac{1}{E - H_\alpha - T_\alpha - V_\alpha + i\varepsilon}(V_\alpha - U_\alpha)\chi_\alpha^{(+)}$$

$$= \chi_\alpha^{(+)} + \frac{1}{E - H + i\varepsilon}(V_\alpha - U_\alpha)\chi_\alpha^{(+)}.$$

The T-matrix becomes $\mathscr{T}_{\beta\alpha} = \langle \phi_\beta | V_\beta | \Psi_\alpha^{(+)} \rangle$

$$= \langle \phi_\beta | V_\beta | \chi_\alpha^{(+)} \rangle + \left\langle \phi_\beta \middle| V_\beta \frac{1}{E - H + i\varepsilon}(V_\alpha - U_\alpha) \middle| \chi_\alpha^{(+)} \right\rangle$$

$$= \langle \phi_\beta | V_\beta - V_\alpha + U_\alpha | \chi_\alpha^{(+)} \rangle + \langle \Psi_\beta^{(-)} | V_\alpha - U_\alpha | \chi_\alpha^{(+)} \rangle,$$

Distorted wave Born approximation

The T-matrix
$$\mathscr{T}_{\beta\alpha} = \langle \phi_\beta | V_\beta - V_\alpha + U_\alpha | \chi_\alpha^{(+)} \rangle + \langle \Psi_\beta^{(-)} | V_\alpha - U_\alpha | \chi_\alpha^{(+)} \rangle,$$

where
$$\Psi_\beta^{(-)} = \chi_\beta^{(-)} + \frac{1}{E - H - i\varepsilon}(V_\beta - U_\beta^\dagger)\chi_\beta^{(-)},$$

then
$$\mathscr{T}_{\beta\alpha} = \langle \phi_\alpha | U_\alpha | \chi_\alpha^{(+)} \rangle \delta_{\alpha\beta}$$
$$+ \langle \chi_\beta^{(-)} | (V_\alpha - U_\alpha) \boxed{+ (V_\beta - U_\beta) \frac{1}{E - H + i\varepsilon}(V_\alpha - U_\alpha)} | \chi_\alpha^{(+)} \rangle.$$

The distorted wave Born approximation

$$\mathscr{T}^{\mathrm{DWBA}}_{\beta\alpha(\mathrm{prior})} = \langle \phi_\alpha | U_\alpha | \chi_\alpha^{(+)} \rangle \delta_{\alpha\beta} + \langle \chi_\beta^{(-)} | V_\alpha - U_\alpha | \chi_\alpha^{(+)} \rangle.$$

Similar we have

$$\mathscr{T}^{\mathrm{DWBA}}_{\beta\alpha(\mathrm{post})} = \langle \chi_\alpha^{(-)} | U_\alpha | \phi_\alpha \rangle \delta_{\alpha\beta} + \langle \chi_\beta^{(-)} | V_\beta - U_\beta | \chi_\alpha^{(+)} \rangle,$$

Connecting with transfer reaction

If the reaction has the form $\underbrace{a(=b+x)+A}_{\alpha} \rightarrow \underbrace{b+B(=x+A)}_{\beta}$

The three body Hamiltonian $H = T + V_{bx} + V_{xA} + U_{bA}$

Compare with $H = H_\alpha + T_\alpha + V_\alpha = H_\beta + T_\beta + V_\beta = \cdots$

We have
$$H_\alpha = T_{bx} + V_{bx} \qquad H_\beta = T_{xA} + V_{xA}$$
and
$$V_\alpha = V_{xA} + U_{bA} \qquad V_\beta = V_{bx} + U_{bA}$$

Then the DWBA T-matrix

$$\mathcal{T}^{\mathrm{DWBA}}_{\beta\alpha(\mathrm{prior})} = \langle \phi_\alpha | U_\alpha | \chi^{(+)}_\alpha \rangle \, \delta_{\alpha\beta} + \langle \chi^{(-)}_\beta | V_\alpha - U_\alpha | \chi^{(+)}_\alpha \rangle.$$

$$\mathcal{T}^{\mathrm{DWBA}}_{\beta\alpha(\mathrm{post})} = \langle \chi^{(-)}_\alpha | U_\alpha | \phi_\alpha \rangle \, \delta_{\alpha\beta} + \langle \chi^{(-)}_\beta | V_\beta - U_\beta | \chi^{(+)}_\alpha \rangle,$$

Becomes

$$T_{prior} = \langle \phi^b_{xA} \chi^{scatt}_{bB} \,|\, V_{xA} + U_{bA} - U_{aA} \,|\, \phi^b_{bx} \chi^{scatt}_{aA} \rangle$$

$$T_{post} = \langle \phi^b_{xA} \chi^{scatt}_{bB} \,|\, V_{bx} + \boxed{U_{bA} - U_{bB}} \,|\, \phi^b_{bx} \chi^{scatt}_{aA} \rangle$$

remnant term, when A is heavy ~0

Numerical implementations

Solve the equation in partial waves

$$T = \langle \phi^b_{xA} \boxed{\chi^{scatt}_{bB} | V | \phi^b_{bx} \chi^{scatt}_{aA}} \rangle$$

V: post/prior

Project on the partial wave basis

$V_{\alpha\alpha'}(R,R')$

$$S_\alpha(R) = \int_0^\infty \langle (LJ_p)J, J_t; J_T | \mathbf{V} | (L'J'_p)J', J'_t; J_T \rangle \, f^{J_T}_{(L'J'_p)J',J'_t}(R')dR'$$

Re-coupling the angular momentum basis

$$\langle (LJ_p)J, J_t; J_T | \mathbf{V} | (L'J'_p)J', J'_t; J_T \rangle = \sum_{\Lambda F} (-1)^{s+J'_p-F} \hat{J}\hat{J}'_t\hat{j}\hat{F}\hat{J}_p\hat{\Lambda} \begin{Bmatrix} L' & J'_p & J' \\ \ell' & s' & j' \\ \Lambda & F & J \end{Bmatrix}$$
$$\times W(J_t j' J_T J';\, J'_t J) W(ls J_p J'_p;\, jF) W(L\ell JF;\, \Lambda J_p) \langle \ell L; \Lambda | \mathbf{V} | \ell' L'; \Lambda \rangle,$$

where

$$\mathbf{X}^{\Lambda}_{\ell L:\ell' L'}(R,R') = \frac{|b|^3}{2} \sum_{nn'} c(\ell n) c(\ell' n') RR' (aR)^{\ell-n} (bR')^n (a'R)^{\ell'-n'} (b'R')^{n'}$$
$$\times \sum_T \mathbf{q}^T_{\ell,\ell'}(R,R')(2T+1)(-1)^{\Lambda+T+L+L'} \,\hat{\ell}\hat{\ell}'\, (\widehat{\ell - n})\, (\widehat{\ell' - n'}) \hat{n}\hat{n}'\hat{L}\hat{L}'$$
$$\times \sum_{KK'} (2K+1)(2K'+1) \begin{pmatrix} \ell-n & n' & K \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell'-n' & n & K' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} K & L & T \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} K' & L' & T \\ 0 & 0 & 0 \end{pmatrix}$$
$$\times \sum_Q (2Q+1) W(\ell L \ell' L'; \Lambda Q) W(KLK'L'; TQ) \begin{pmatrix} \ell' & Q & \ell \\ n' & K & \ell-n \\ \ell'-n' & K' & n \end{pmatrix}$$

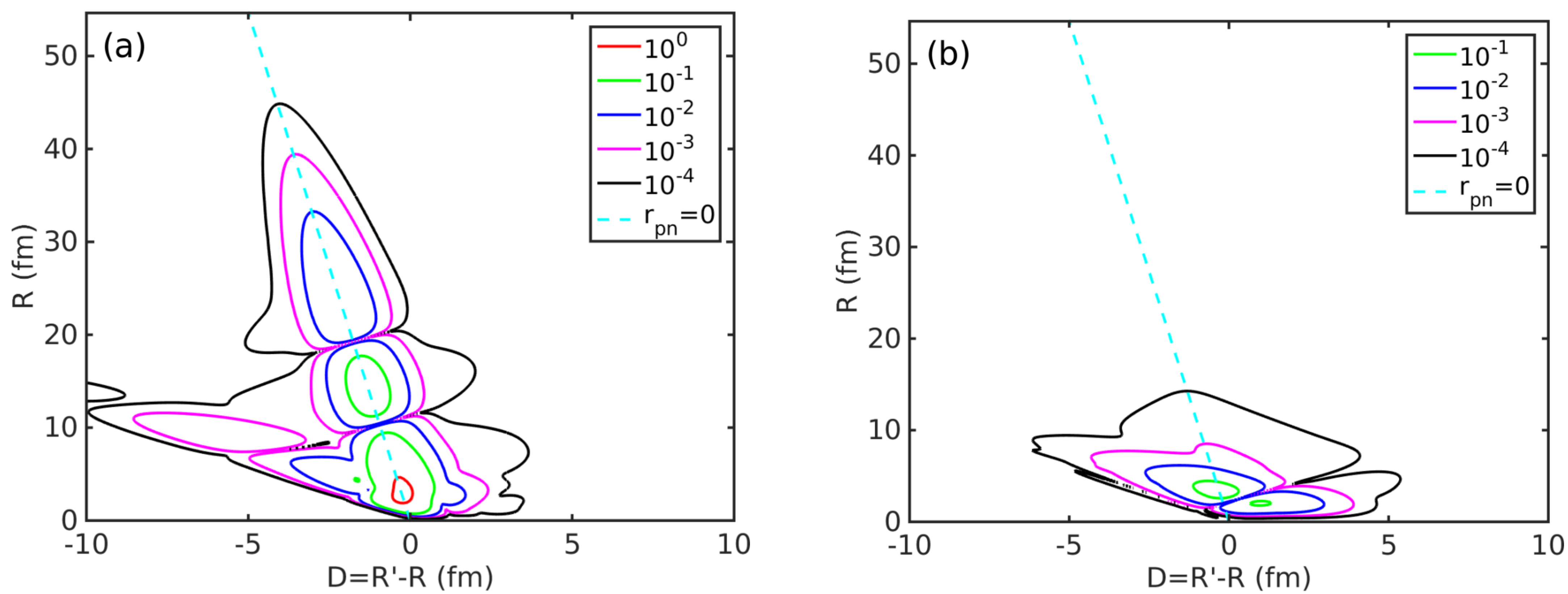
Calculate transfer using fresco

Numerical tricks:

The nonlocal kernels for single-particle transfers are calculated first at a much smaller number of interpolation points, and then expanded when necessary to calculate the source terms by integrating

$$S(R_f) = \int_0^{R_{match}} K_{fi}(R_f, R_i) u(R_i) dR_i$$

where RMATCH and HCM, the step size, are given on card 1. Since the kernel function $K_{fi}(R_f, R_i)$ is usually rapidly varying with $D_{fi} = R_f - R_i$ (especially with heavy-ion reactions), and only slowly varying with R_f (if D_{fi} is constant), FRESCO calculates and stores the function $K'_{fi}(R_f, D_{fi})$ at intervals of RINTP (card 1) in R_f, and intervals of HNL in D_{fi}. The D_{fi} range considered is CENTRE-RNL/2 to CENTRE+RNL/2, i.e. range of RNL centred at CENTRE, and FRESCO later suggests improved values for RNL & CENTRE. The HNL reflects to physical variation of K'_{fi} with D_{fi}, and



M. Gómez-Ramos et al., Phys. Rev. C **92**, 014613 (2015)

Potentials need for (d,p) transfer

Relative two body interactions

n-p interaction
(support bound state)

$$V_{pn}(r) = -72.15\ \mathrm{MeV}\exp[-(r/1.484\ \mathrm{fm})^2]$$

n-A interaction
(support bound state)

Assume it has Gaussian form

Adjust the potential depth to reproduce the binding energy

p-A interaction
complex interaction at half incoming energy

Wood-Saxon form
Global potentials

e.g.:

KD02 A.J.Koning, J.P.Delaroche, Nucl. Phys. A713, 231 (2003)

CH89 R. L. Varner, et al., Phys. Rep. 201, 57 (1991).

Potentials need for (d,p) transfer

Potentials used in DWBA

$$T_{prior} = \langle \phi^{b}_{xA} \chi^{scatt}_{bB} | V_{xA} + U_{bA} - \boxed{U_{aA}} | \phi^{b}_{bx} \chi^{scatt}_{aA} \rangle$$

$$T_{post} = \langle \phi^{b}_{xA} \chi^{scatt}_{bB} | V_{bx} + U_{bA} - \boxed{U_{bB}} | \phi^{b}_{bx} \chi^{scatt}_{aA} \rangle$$

$$\left.\begin{array}{l} U_{aA} \rightarrow U_{dA} \\ U_{bB} \rightarrow U_{p(A+n)} \end{array}\right\}$$

Some global potentials can be found in RIPL-3

Introduction | MASSES | LEVELS | RESONANCES | **OPTICAL** | DENSITIES | GAMMA | FISSION | CODES | Contacts

Optical Model Parameter Segment

Data required for preparing inputs for optical model calculations and, in addition, one FORTRAN code for microscopic calculation of optical model parameters. See README File (2.9kB) for further information.

Optical Model Potential (OMP) Parameters

Phenomenological OMP Library

The library contains all phenomenological optical model potentials that have been compiled, with a Users File that is a subset of the archival file with all single-energy potentials removed and an index file. See also the References.

Index of Users File ordered by Lib. No. (42kB)
Index of Users File ordered by Z-Range (42kB)
Users File (2.2MB)
References (11kB) README File (15.3kB)

Retrieval of OMP Index

Atomic number (Z) of Target
Mass number (A) of Target
Incident Particle n
retrieve reset

Retrieval of OMP Data

Atomic number (Z) of Target
Mass number (A) of Target
OMP Index
Minimum Incident Energy [MeV] 0.1
Maximum Incident Energy [MeV] 150
retrieve reset

fresco input example

```
n14(f17,ne18)c13 @ 170 MeV;
NAMELIST
 &FRESCO hcm=0.03 rmatch=40 rintp=0.20 hnl=0.1 rnl=5.00 centre=0.0
    jtmin=0.0  jtmax=120 absend=-1.0
    thmin=0.00 thmax=40.00 thinc=0.10
    iter=1 nnu=36
    chans=1 xstabl=1
    elab=170.0  /
```

```
 &PARTITION namep='f17'  massp=17. zp=9 namet='n14'  masst=14. zt=7 nex=1  /
 &STATES jp=2.5 bandp=1 ep=0.0 cpot=1 jt=1.0 bandt=1 et=0.0000  /

 &PARTITION namep='ne18' massp=18. zp=10 namet='c13'  masst=13. zt=6
qval=3.6286 nex=1  /
 &STATES jp=0. bandp=1 ep=0.0  cpot=2 jt=0.5 bandt=1 et=0.0000  /
 &partition /
```

fresco input example

```
 &POT kp=1 ap=17.000 at=14.000 rc=1.3  /
 &POT kp=1 type=1 p1=37.2 p2=1.2 p3=0.6  p4=21.6 p5=1.2 p6=0.69  /

 &POT kp=2 ap=18.000 at=13.000 rc=1.3  /
 &POT kp=2 type=1 p1=37.2 p2=1.2 p3=0.6  p4=21.6 p5=1.2 p6=0.69  /

 &POT kp=3 at=17 rc=1.2  /
 &POT kp=3 type=1 p1=50.00 p2=1.2 p3=0.65   /
 &POT kp=3 type=3 p1=6.00  p2=1.2 p3=0.65   /

 &POT kp=4 at=13 rc=1.2  /
 &POT kp=4 type=1 p1=50.00 p2=1.2 p3=0.65   /
 &POT kp=4 type=3 p1=6.00  p2=1.2 p3=0.65   /

 &POT kp=5 ap=17.000 at=14.000 rc=1.3  /
 &POT kp=5 type=1 p1=37.2 p2=1.2 p3=0.6  p4=21.6 p5=1.2 p6=0.69  /
 &pot /
```

```
 &Overlap kn1=1 ic1=1 ic2=2 in=1 kind=0 nn=1 l=2 sn=0.5 j=2.5 kbpot=3
be=3.922  isc=1 ipc=0 /
 &Overlap kn1=2 ic1=2 ic2=1 in=2 kind=3 nn=1 l=1 sn=0.5 ia=1 ib=1
j=1.0 kbpot=4 be=7.5506 isc=1 ipc=0 /
 &overlap /
```

fresco input example

```
&Coupling icto=-2 icfrom=1 kind=7 ip1=0 ip2=-1 ip3=5 /
&CFP in=1 ib=1 ia=1 kn=1 a=1.00  /
&CFP in=2 ib=1 ia=1 kn=2 a=1.00  /
&CFP /
&coupling /
```

More details can be found at http://www.fresco.org.uk/input2.9/html/index.html