

A decorative graphic on the left side of the slide, consisting of a network of thin, light-blue lines and small circles, resembling a circuit board or a neural network. The lines are vertical and horizontal, with some diagonal connections, and the circles are placed at various points along these lines.

# UNSTEADY FEEDBACK CONTROL

LAB 4

# STABILITY: AN EXAMPLE

A **lifting body** is a fixed-wing aircraft or spacecraft configuration in which the body itself produces lift.



M2-F2

## Milestones in Flight History Dryden Flight Research Center



**M2-F2**

**Test Flight with F5D-1 and F-104N Escort**

**Circa 1967**

# STABILITY: AN EXAMPLE

## Milestones in Flight History Dryden Flight Research Center



**M2-F2**

**Experiencing Lateral Oscillations in Flight**

**Circa 1967**



Bruce Peterson



M2-F3



5



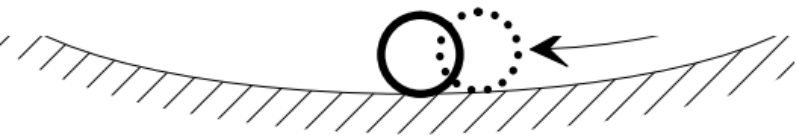
# UNSTEADY FEEDBACK CONTROL

- Stability
- Control strategy: cascade control
- Ball and beam modelling
- Phase advanced control
- Goal: to show how feedback control can be used to stabilize an unstable system

# STABILITY

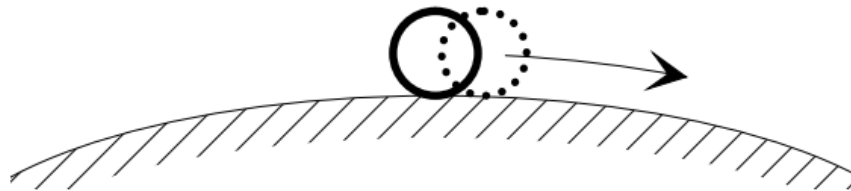
- Stability conditions

gravitational force acts to restore a displaced ball to its equilibrium position



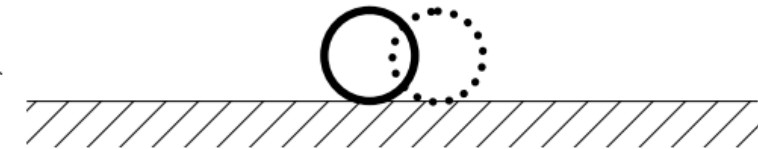
(a) a stable system

gravitational force acts to move a displaced ball away from its equilibrium position



(b) an unstable system

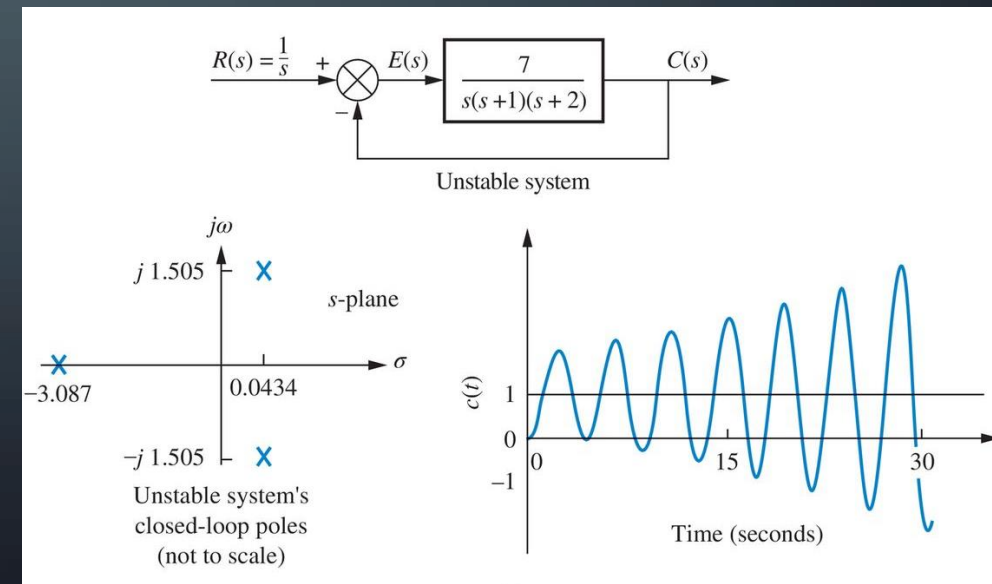
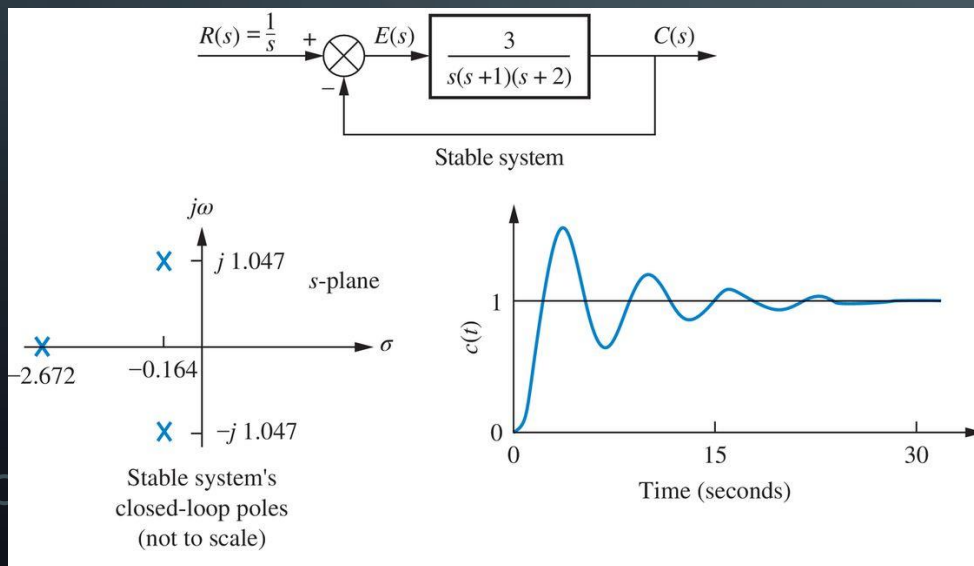
no forces act to move a displaced ball



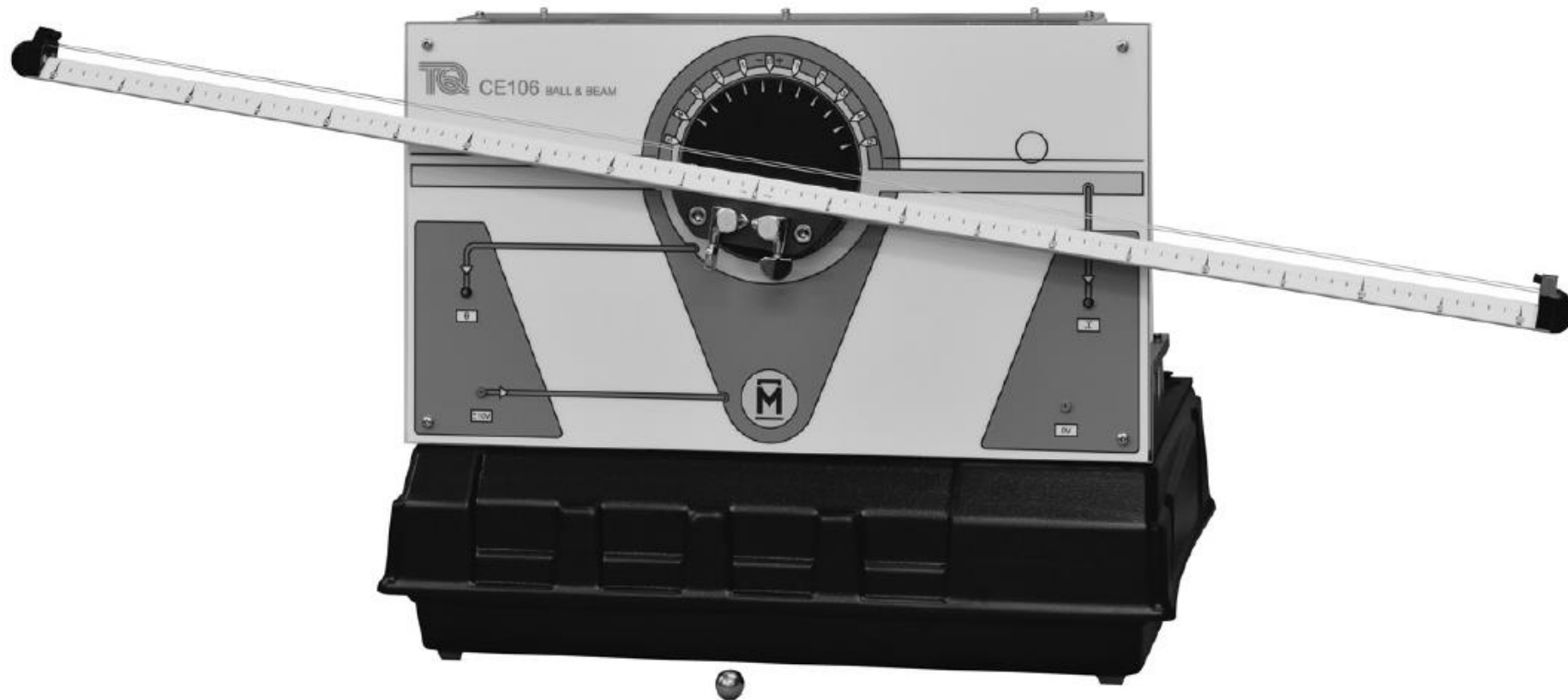
(c) a neutrally stable system

# STABILITY

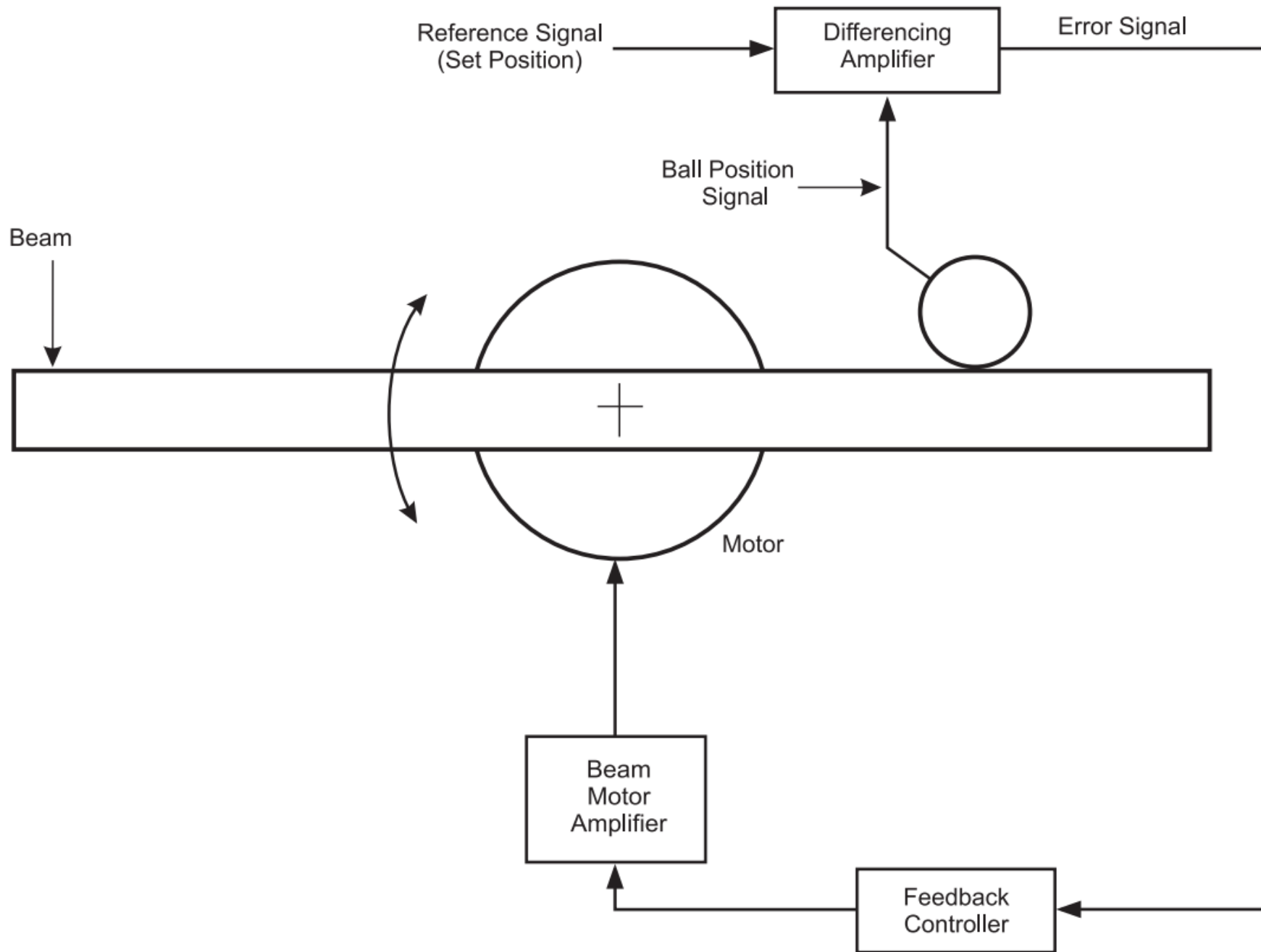
- Stable systems have closed-loop transfer functions with poles only in the **left half-plane**.
- Unstable systems have closed-loop transfer functions with at least one pole in the **right half-plane**.
- A linear system is defined to be stable only if all of its poles have **negative real parts**.



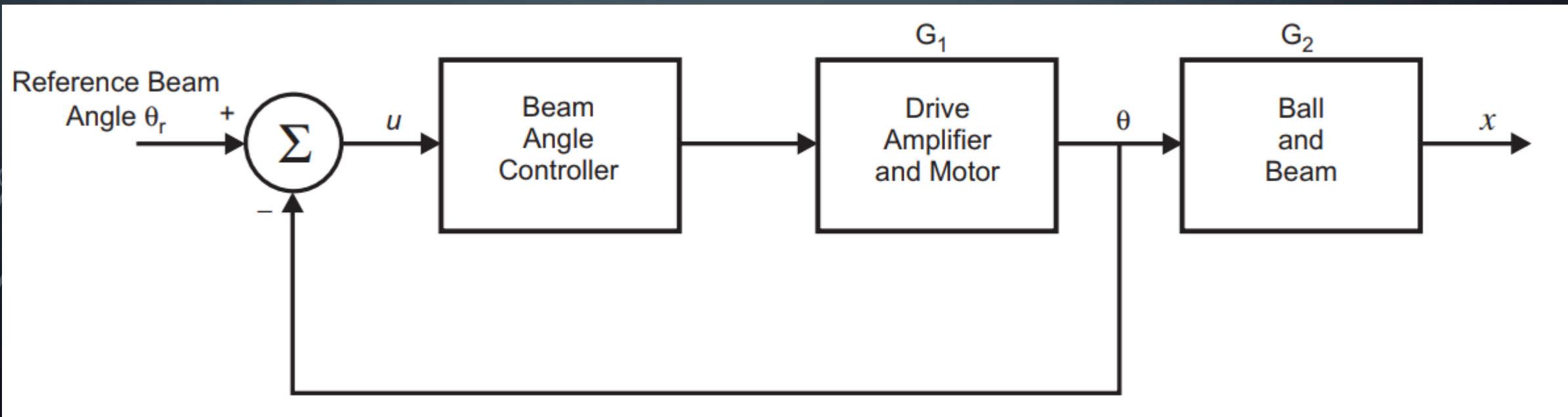
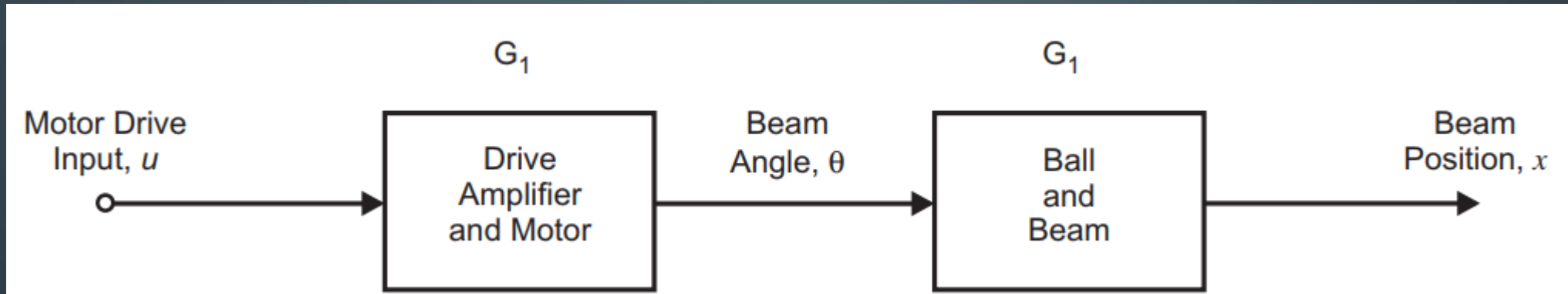
# BALL AND BEAM



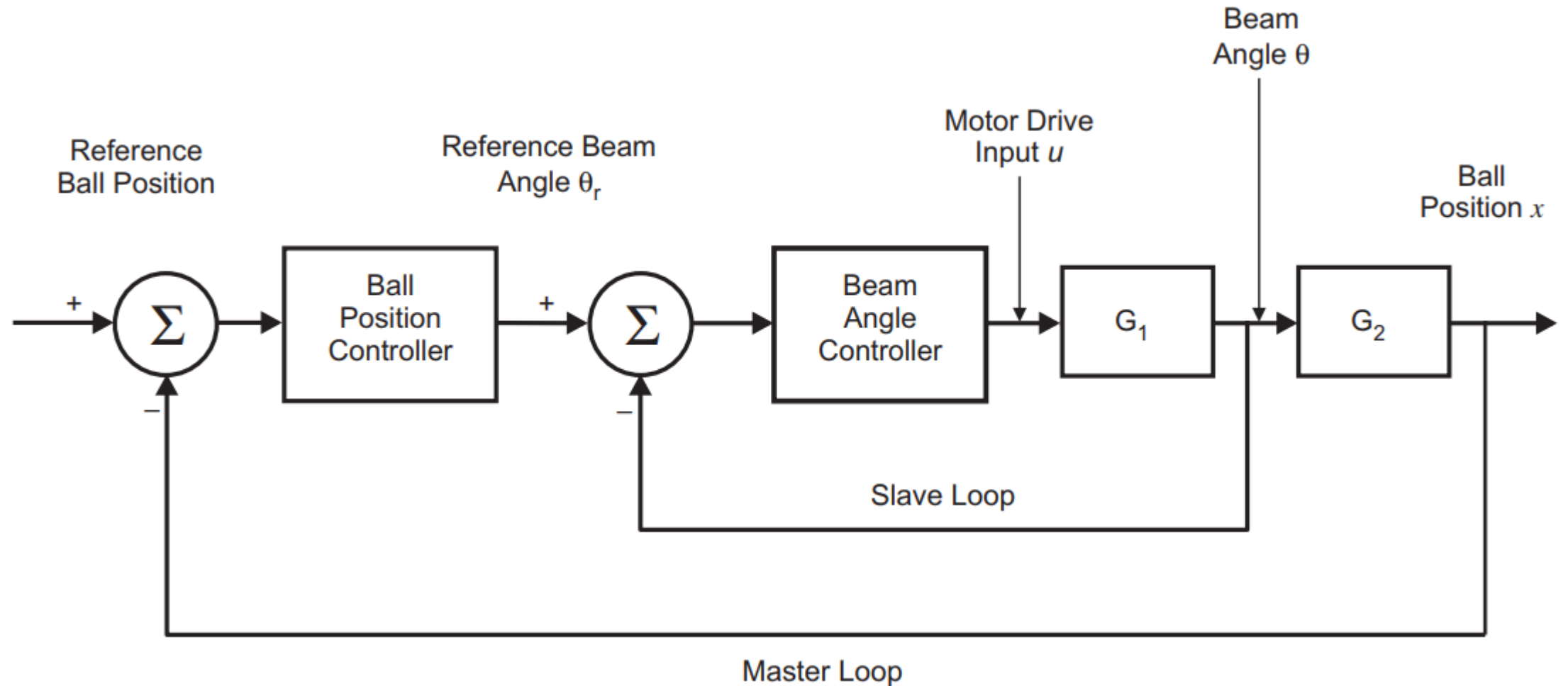




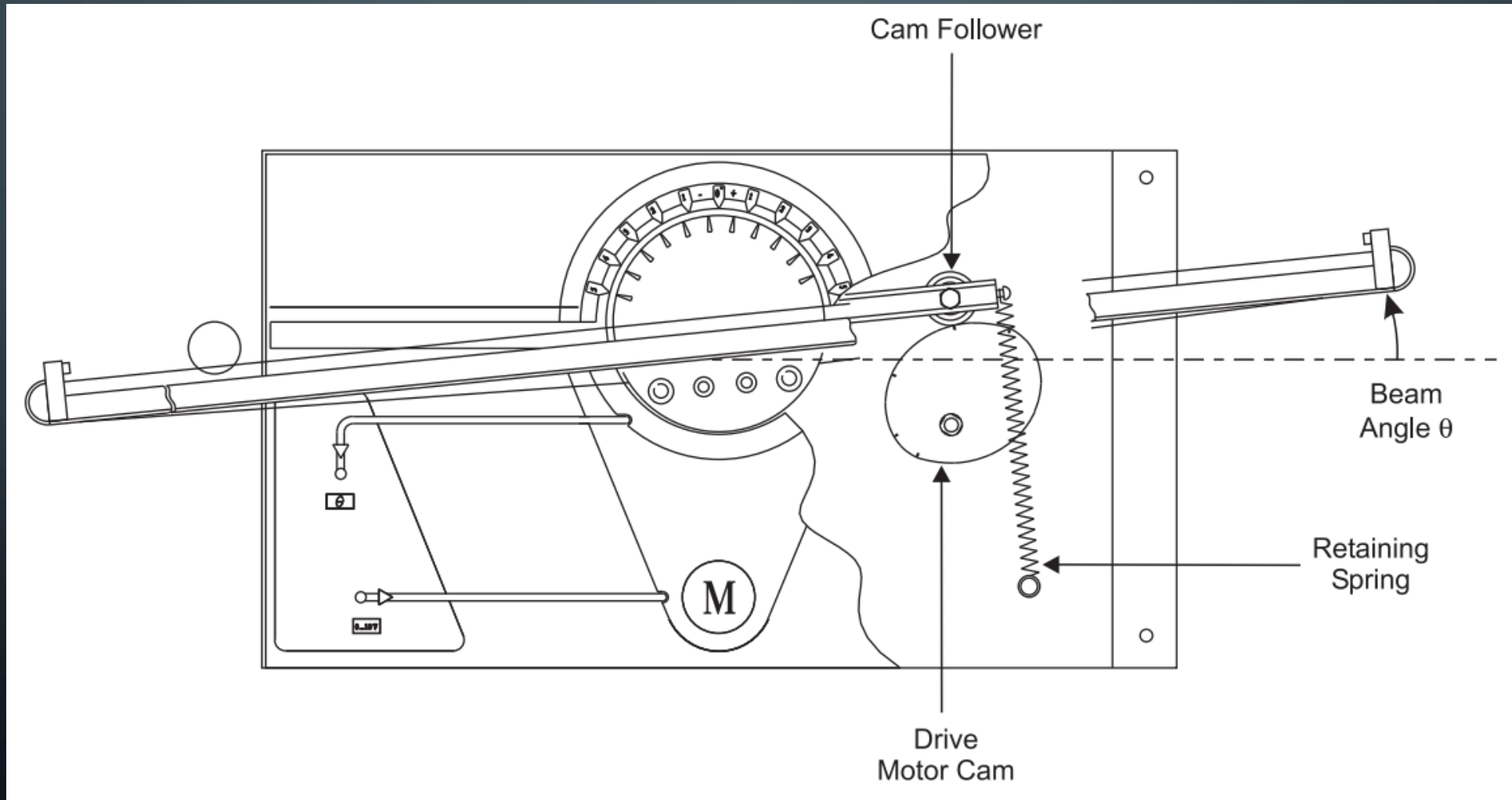
# CONTROL STRATEGY: CASCADE CONTROL



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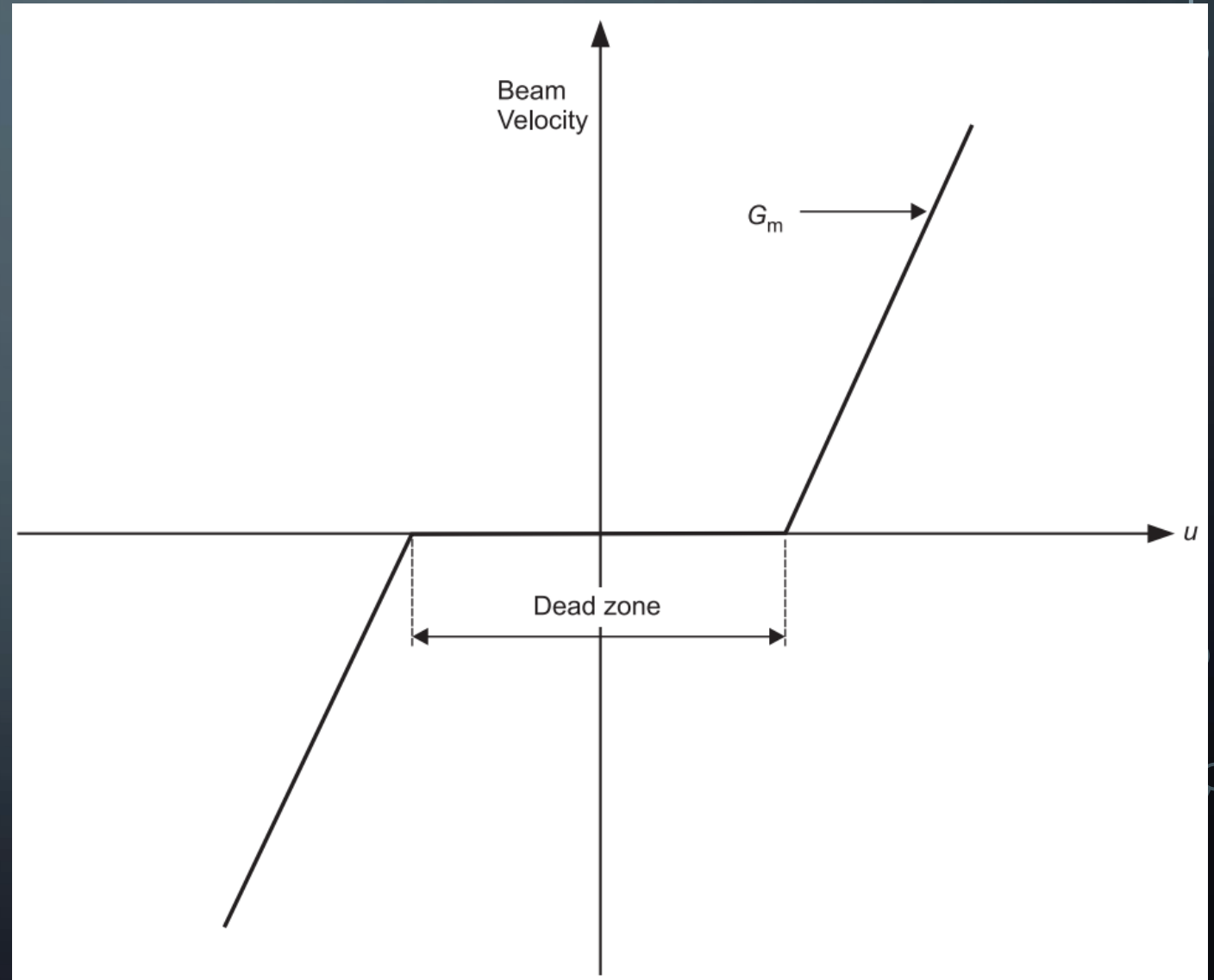


# MODELLING



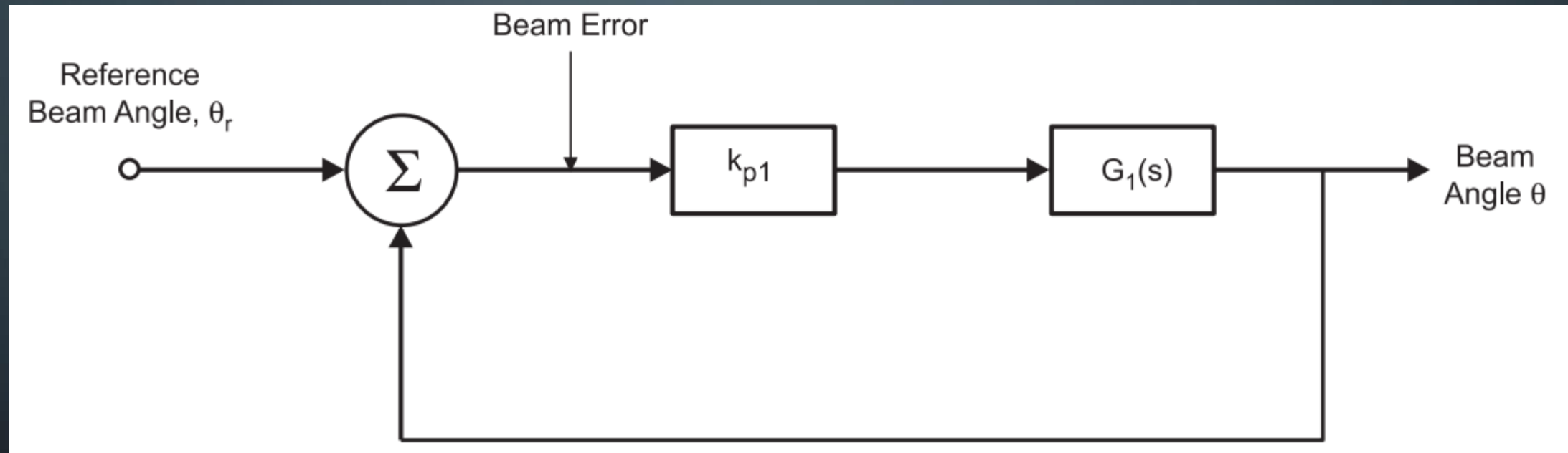
# DRIVE MOTOR AND BEAM: MODELLING

$$\frac{d\theta}{dt} = G_m u$$
$$\theta(s) = \frac{G_m}{s} u(s)$$



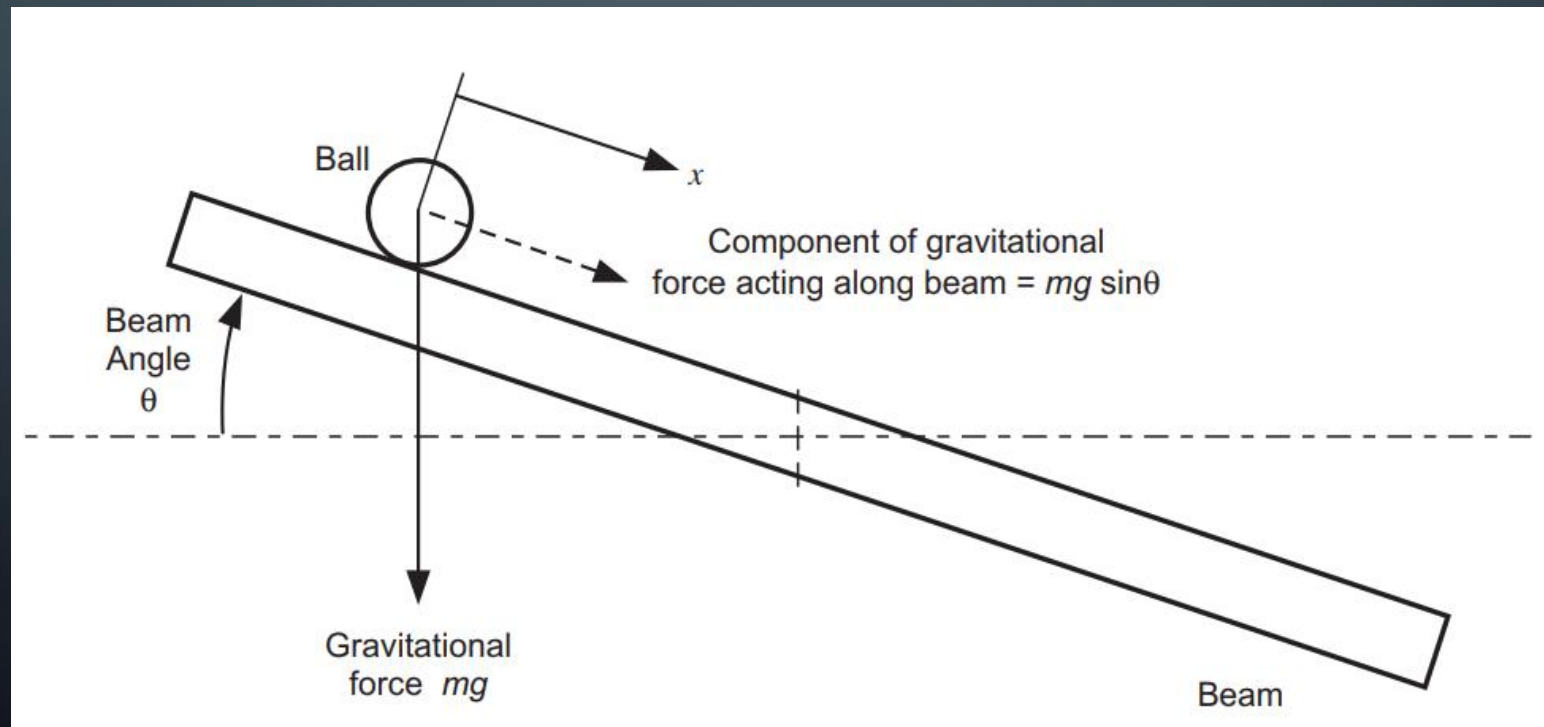
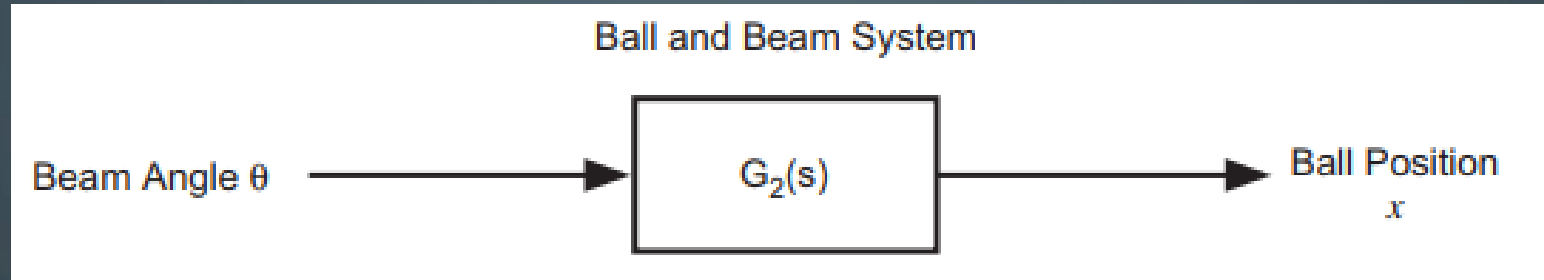


# DRIVE MOTOR AND BEAM: CONTROL



$$\theta(s) = \frac{k_{p1} G_1}{1 + k_p G_1} \theta_r(s)$$

# BALL AND BEAM SYSTEM: MODEL



# BALL AND BEAM SYSTEM: MODEL

- Assume no friction

$$mg\sin(\theta) = m\ddot{x}$$

- Consider small angle

$$\sin(\theta) = \theta$$

$$\ddot{x} = g\theta$$

- Laplace transform

$$x(s) = \frac{g}{s^2} \theta(s)$$

$$G_2(s) = \frac{g}{s^2}$$

# CONTROLLER TYPES

- Proportional control - multiplies the error signal by a constant gain  $k_p$ . This type of controller can be used to improve the steady-state error, increase the speed of the system's response, but can also cause the system to go unstable.

$$G_c(s) = k_p$$

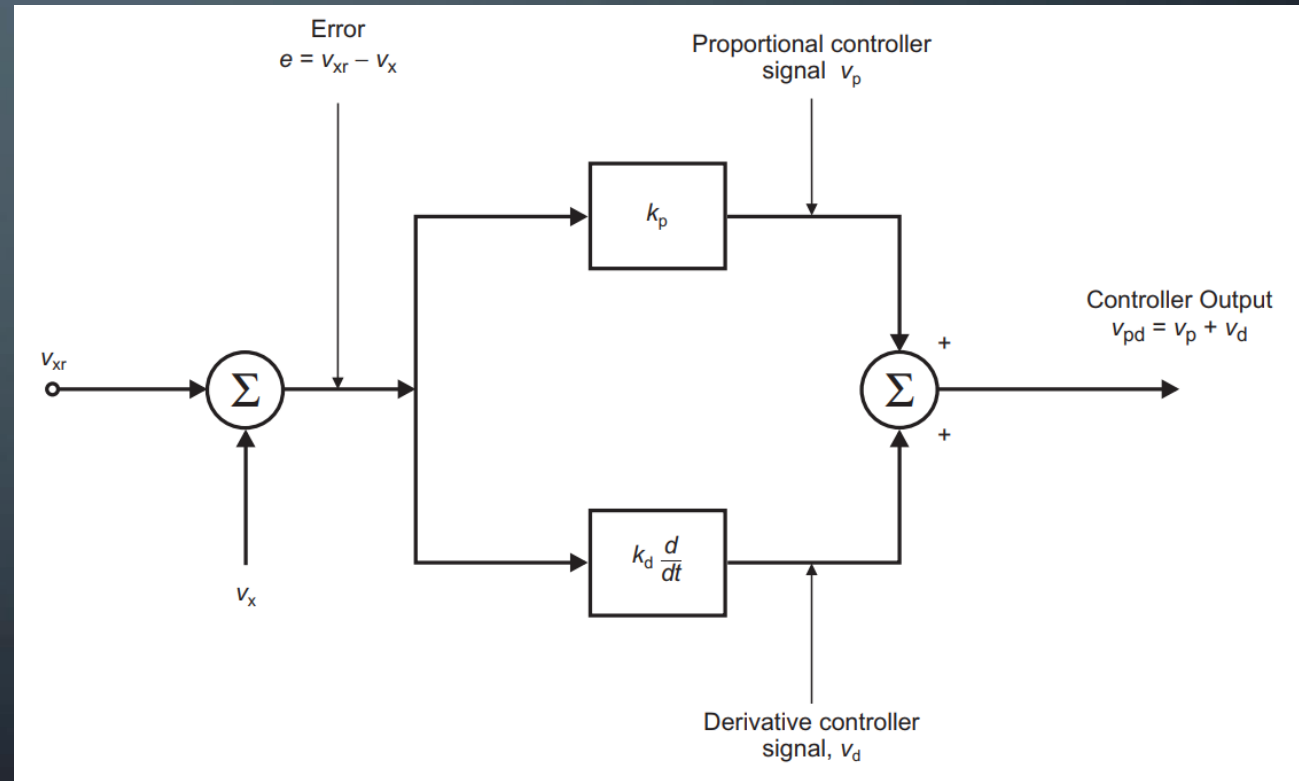
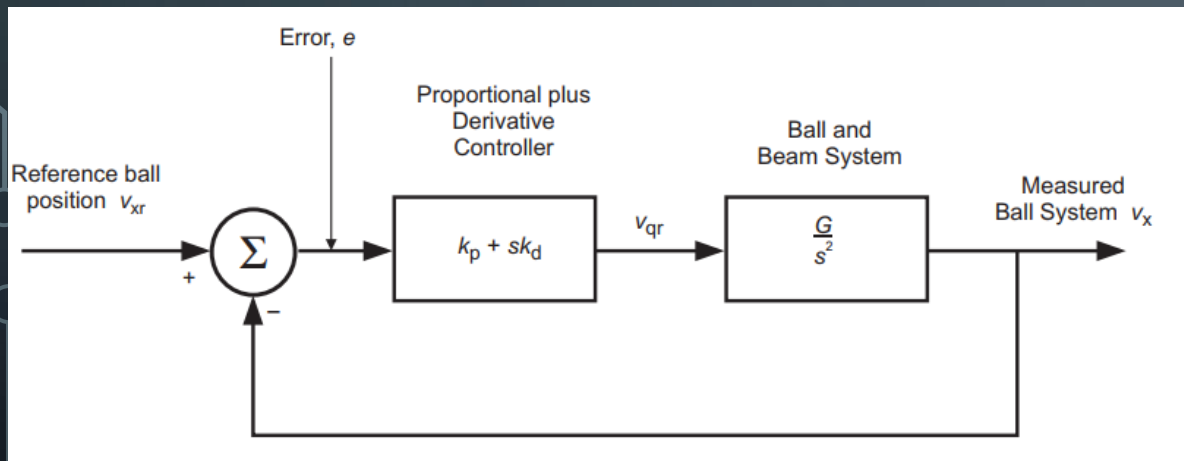
- Proportional and Integral control - multiplies the error signal by a constant gain  $k_p$  and multiplies the sum of all the past values of the error signal by a constant gain  $k_i$ . For constant desired signals, integral control makes the steady-state error zero.

$$G_c(s) = k_p + \frac{k_i}{s}$$

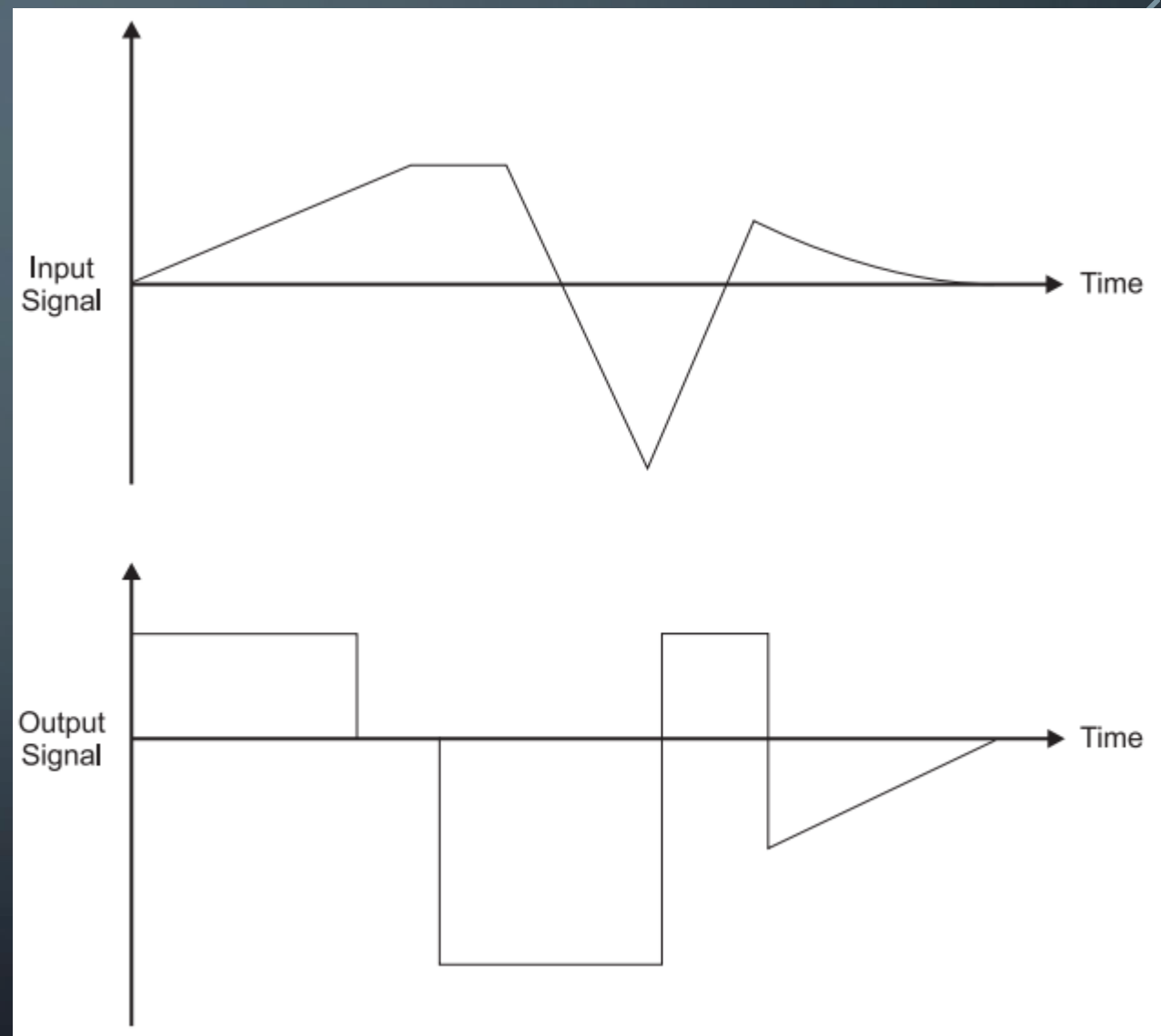
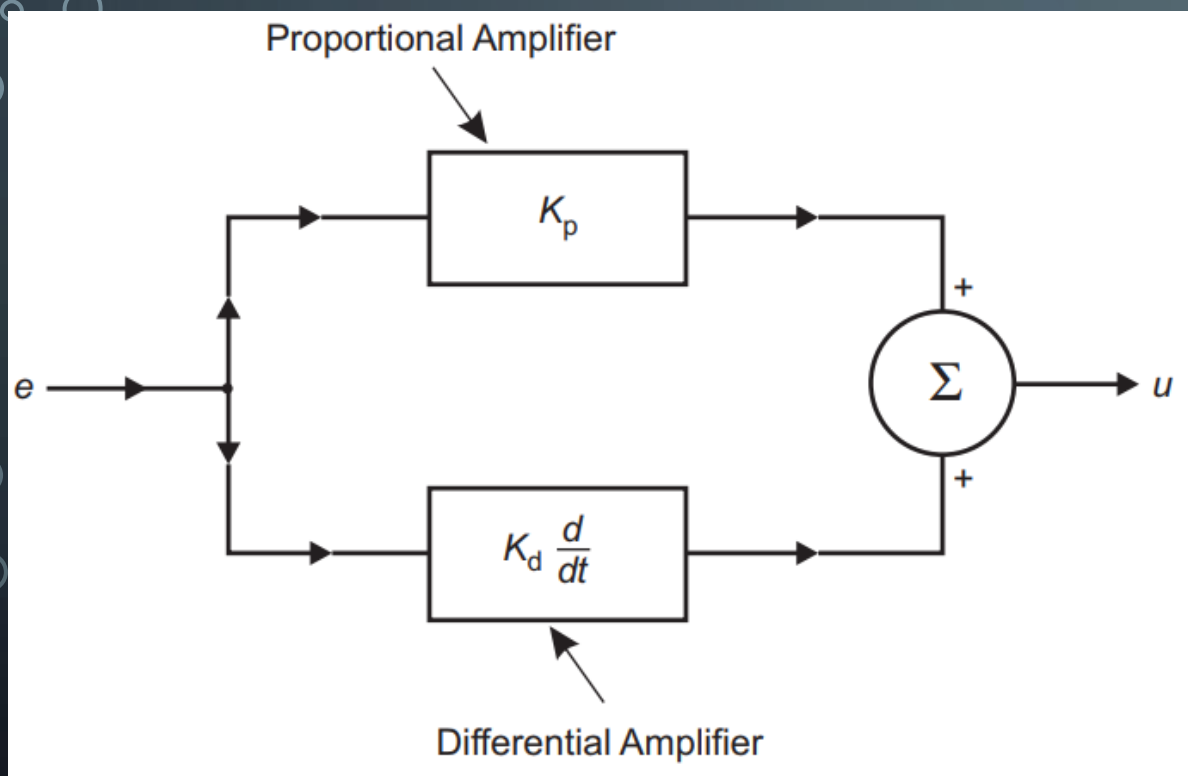
- Phase advance control (Lead compensation) - is used to speed up the system's response by adding anticipatory action to it. This anticipatory action is accounted for with adding positive phase to the system.

$$G_c(s) = \frac{Ts + 1}{\alpha Ts + 1} \quad \alpha < 1$$

# BALL AND BEAM SYSTEM: PD CONTROL

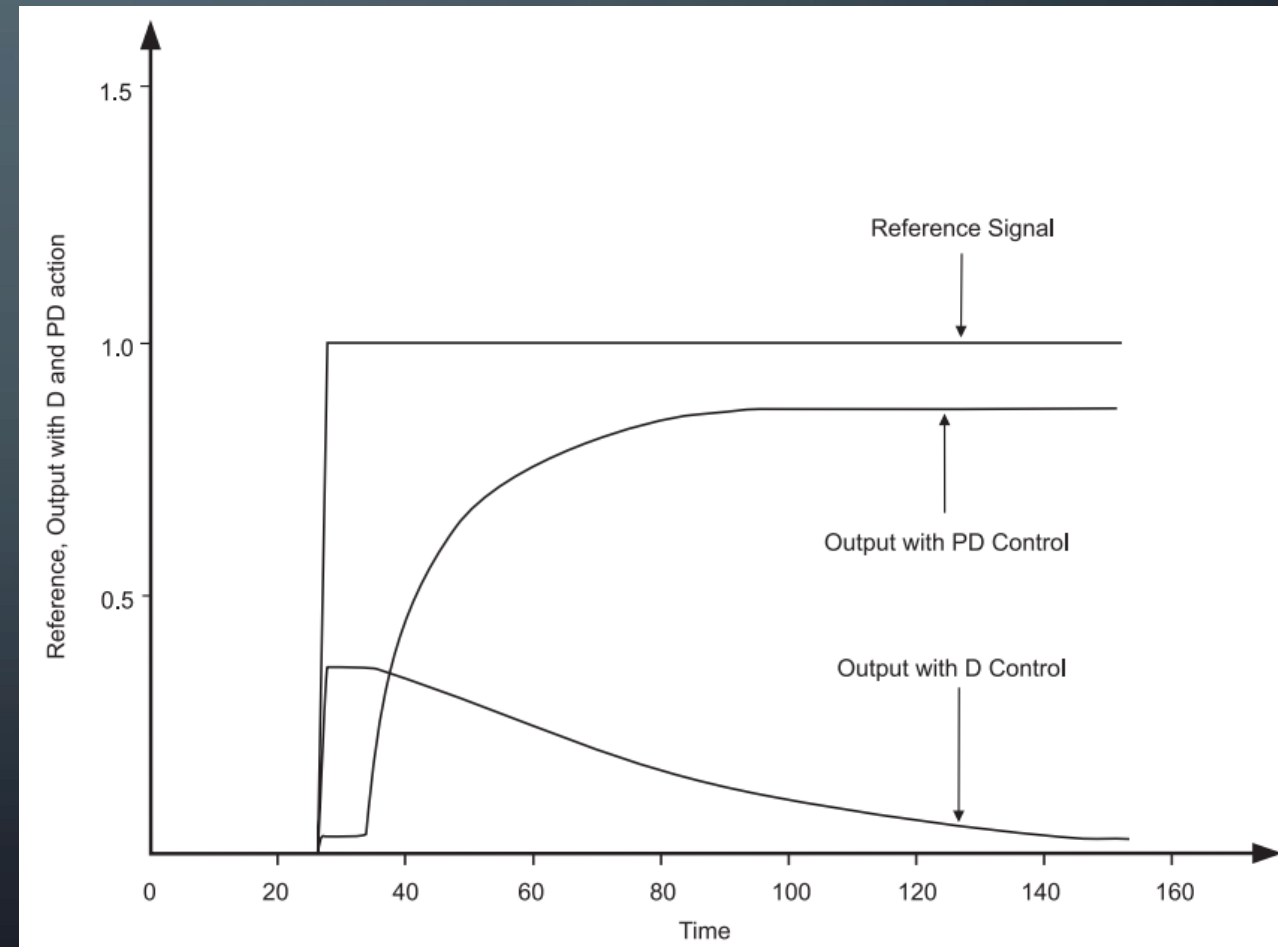
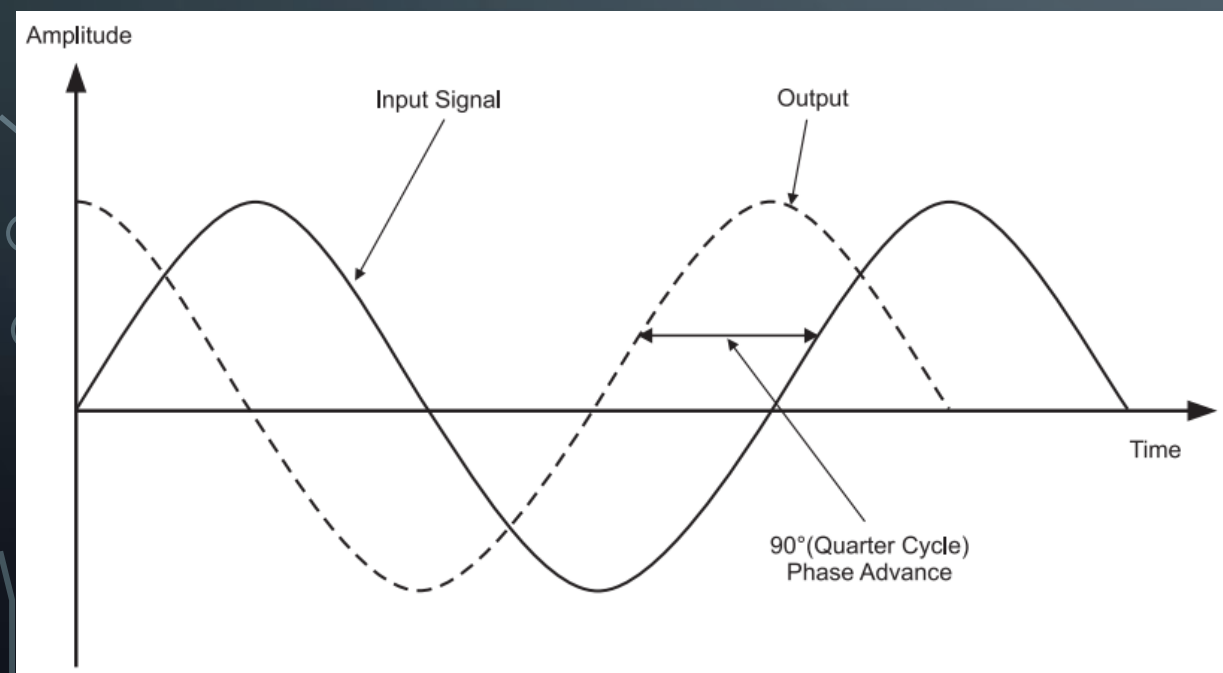






# PD CONTROLLER

- Predictive capability

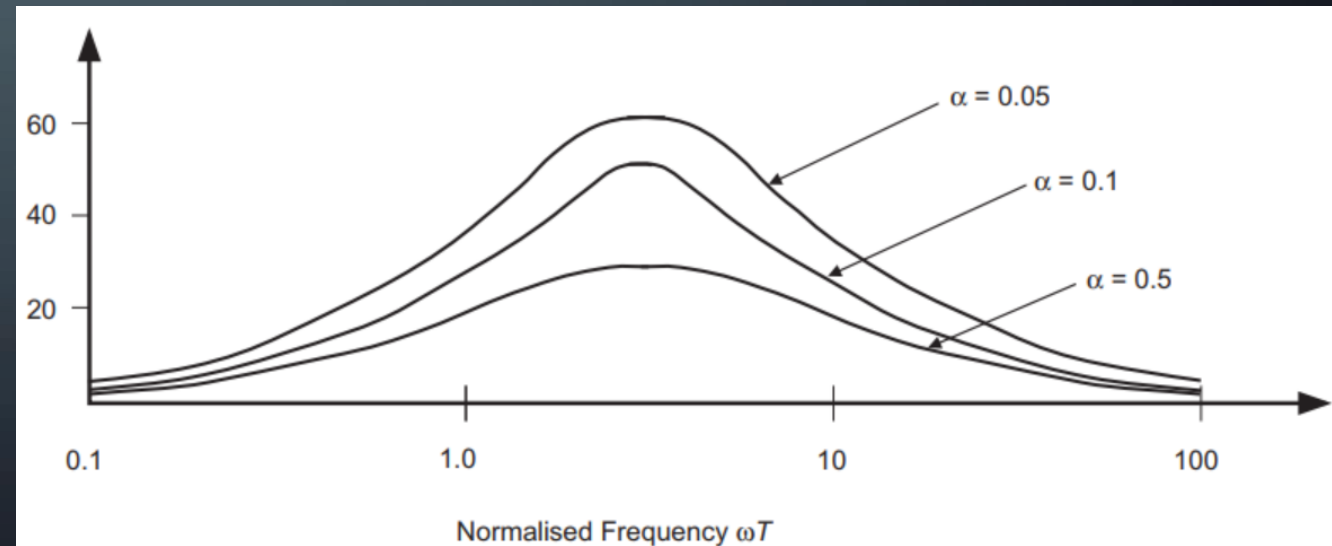
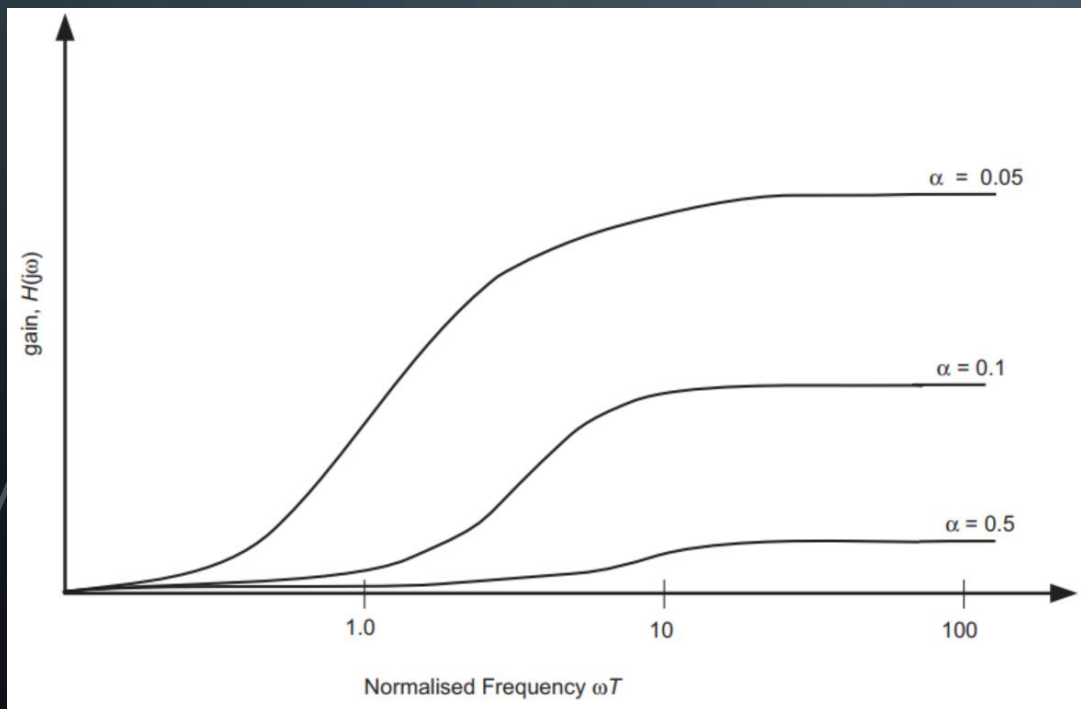


# BALL AND BEAM SYSTEM: PHASE ADVANCE CONTROL

- Phase advance control (lead compensator)

$$H(s) = \frac{1 + sT}{1 + s\alpha T} \quad \alpha < 1; \quad H(j\omega) = \frac{1 + j\omega T}{1 + j\omega\alpha T}$$

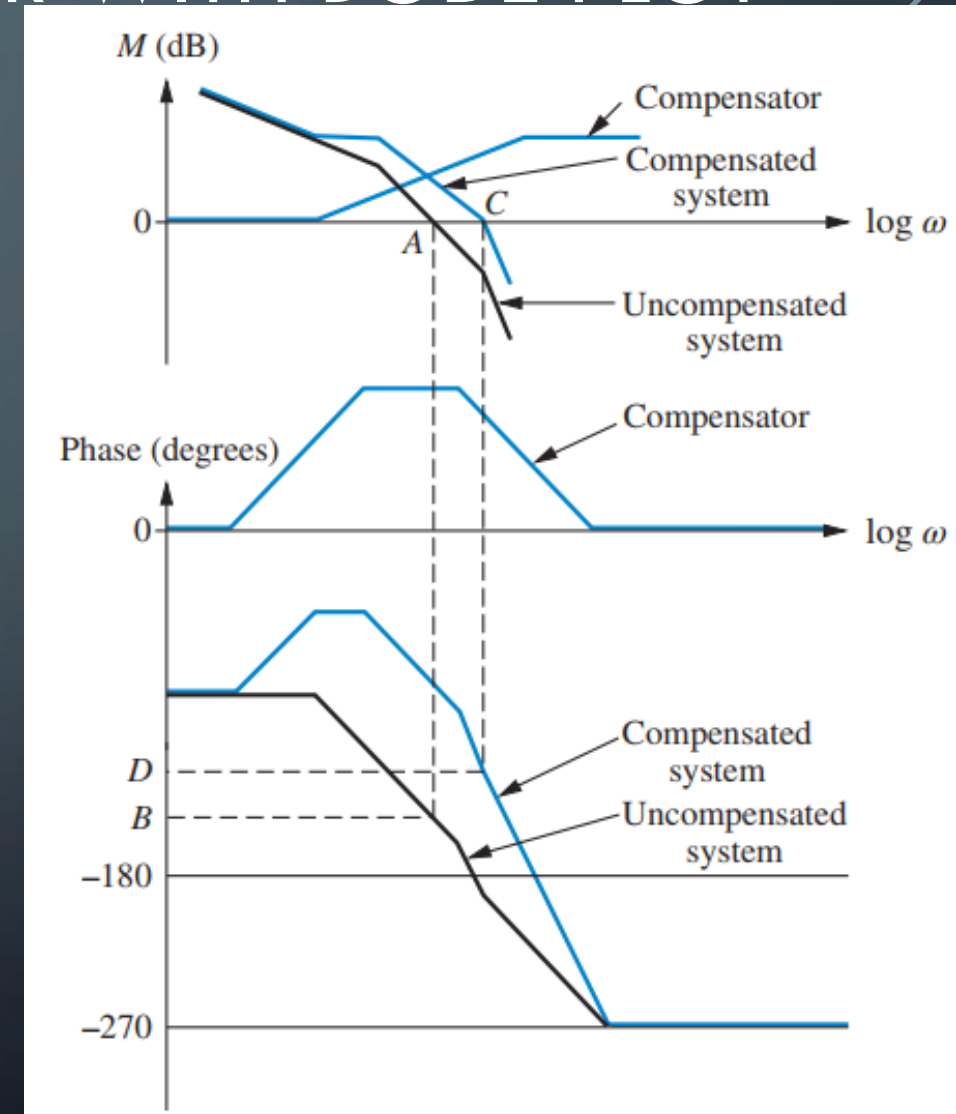
$$\phi_{\max} = \sin^{-1} \left( \frac{1 - \alpha}{1 + \alpha} \right); \quad \omega_{\max} = \frac{1}{T\sqrt{\alpha}}$$



# DESIGNING A LEAD COMPENSATOR WITH BODE PLOT

- Use Bode plots
- Common design criteria
  - Overshoot
  - Peak time

Design Criteria	Affecting Parameter
Overshoot	Phase margin
Peak time	Gain crossover frequency



# DESIGNING A LEAD COMPENSATOR WITH ROOT LOCUS

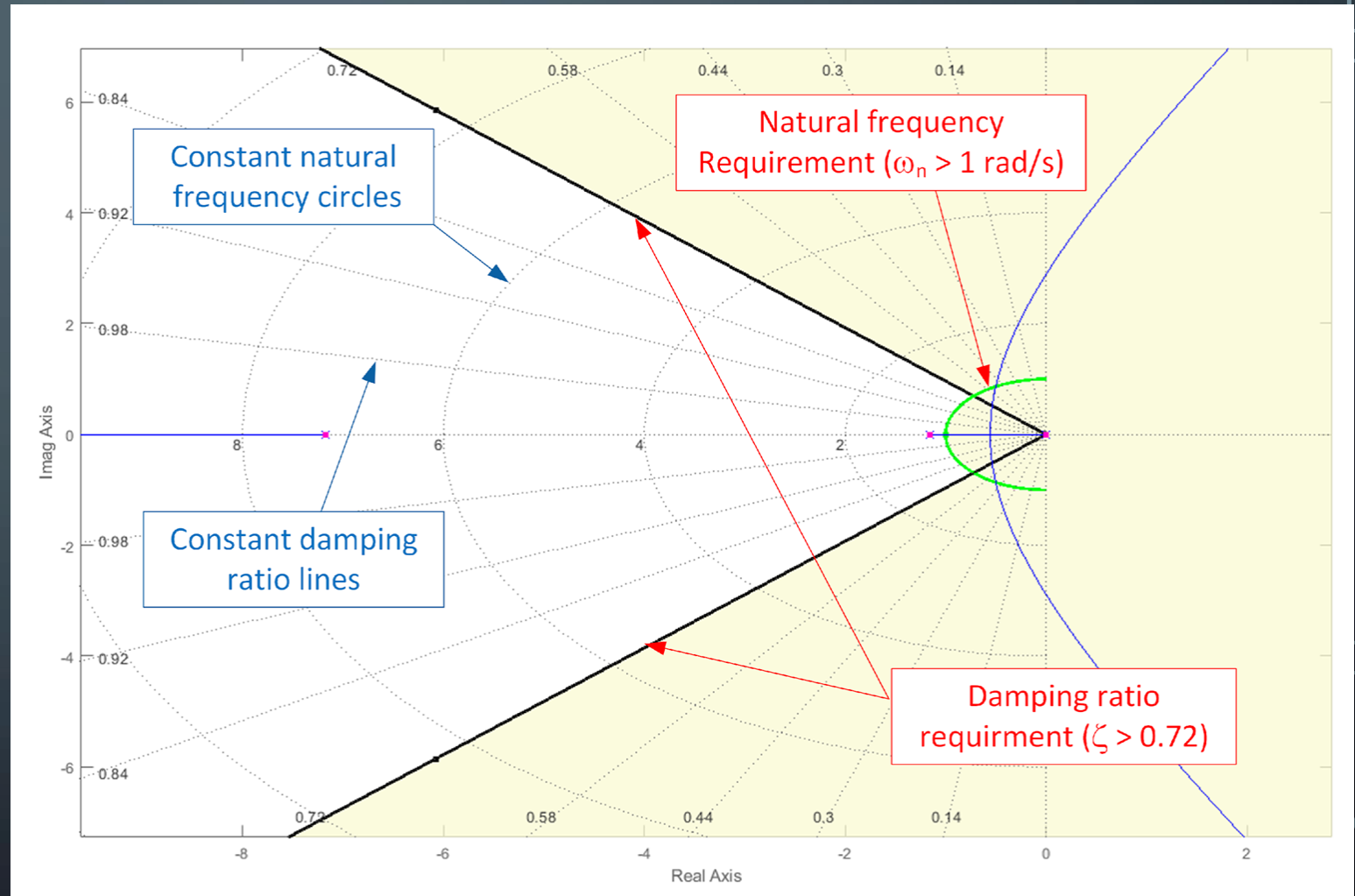
- Graphically superimpose the requirements on the root locus plot

$$P(s) = \frac{22.3}{0.12s^3 + s^2 + s}$$

- Requirements:

$$\omega_n > 1 \text{ rad/s}$$
$$\zeta > 0.72$$

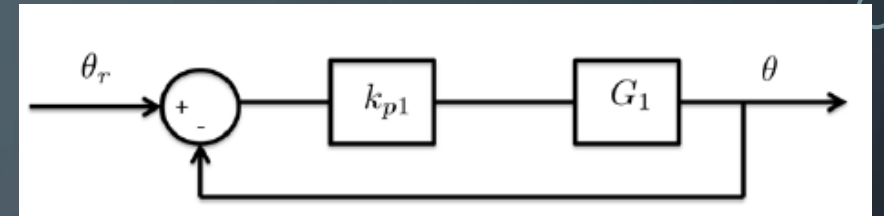
- Poles must fall inside the white intersecting area!
- Add a lead compensator to shift the locus inside the intersecting region



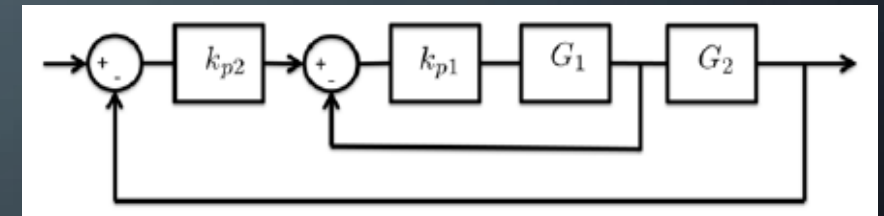


# SIMULINK TESTING

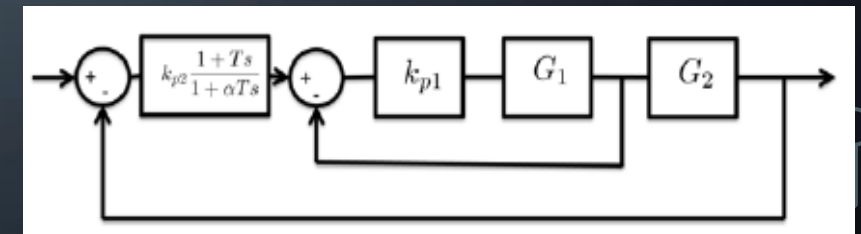
$$T_1(s) = \frac{k_{p1}G_m}{s + k_{p1}G_m}$$



$$T_2(s) = \frac{gk_{p1}k_{p2}G_m}{s^3 + k_{p1}G_ms^2 + gk_{p1}k_{p2}G_m}$$



$$T_3(s) = \frac{gk_{p1}k_{p2}G_m(Ts + 1)}{\alpha Ts^4 + (k_{p1}G_m\alpha T + 1)s^3 + k_{p1}G_ms^2 + gk_{p1}k_{p2}G_m(Ts + 1)}$$



# PHASE ADVANCED CONTROL EXPERIMENT

The image illustrates a phase-advanced control experiment. On the left is a schematic diagram of the control system. It features a motor (M) connected to a phase advance controller. The controller includes several gain blocks labeled  $k_{p1}$ ,  $k_{p2}$ ,  $k_i$ , and  $k_d$ , along with summing junctions and a feedback loop. The right side of the image shows a photograph of the physical experimental setup. It consists of a grey control unit with a circular scale and a black base. A ruler is placed across the top of the unit for scale.

