MEMS1045 Automatic control

Lecture 10

Root Locus 2

Objectives

- Describe the common industrial controllers and their transfer functions
- Explain controller design process using root locus
- Design controller using root locus to meet given time response specifications

Design specifications

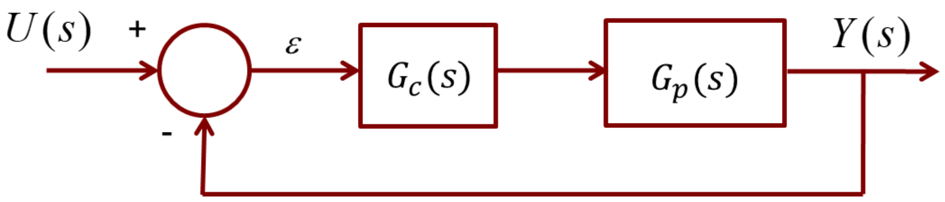
For practical applications, the system time response needs to meet certain design specifications, which can be classified into:

- ❖ Steady state response specifications such as steady state errors
- ❖ Transient response specifications such as percentage overshoot (or damping ratio), settling time, peak time, rise time, etc.

If the response does not meet these specifications, possible solutions include:

- ❖ modification of the plant dynamics, which may not be possible in many practical situations because the plant may be fixed and not modifiable
- ❖ Use a proportional controller and adjust the gain to change the location of the closed-loop poles using the root locus techniques. In many cases, the root-locus plot of a system may indicate that the desired performance cannot be achieved just by the adjustment of gain. In some cases, the system may not be stable for all values of gain
- ❖ Use other controllers that modify the root locus by adding poles and zeros

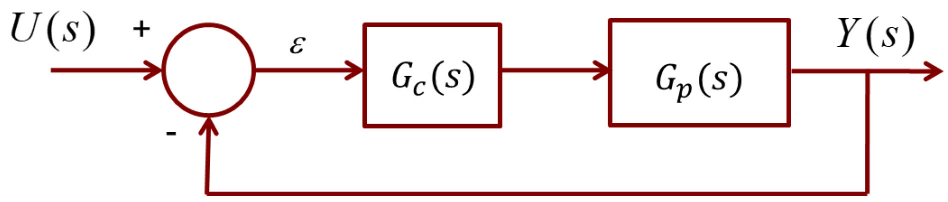
Common industrial controllers



Commonly used industrial controllers include the following:

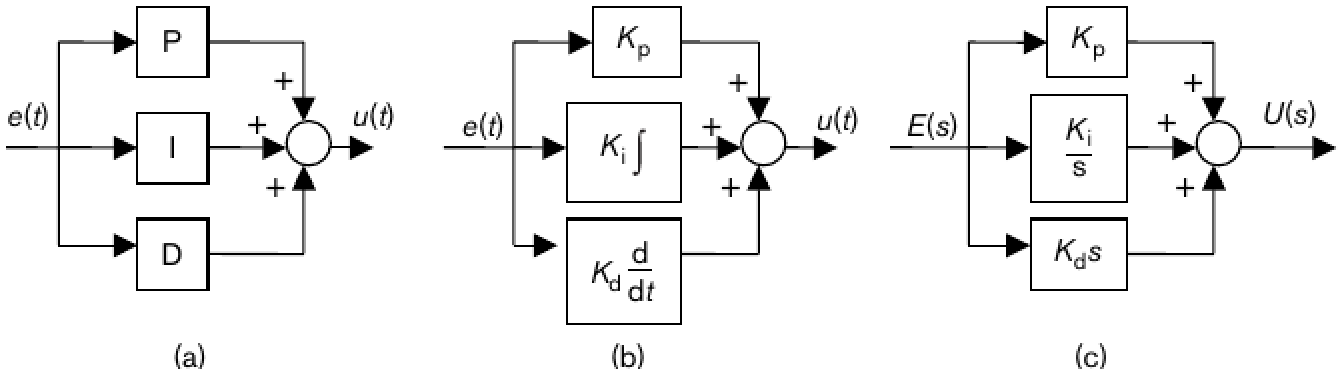
a) On-off controller; (not considered in this course)
b) Proportional controller (P); (this was discussed in last lecture)
c) Proportional and integral controller (PI);
d) Proportional and derivative controller (PD);
e) Proportional, integral, and derivative controller (PID);
f) Lead compensator;
g) Lag compensator;
h) Lead-lag compensator

Design by root locus



- The design by the root-locus method is based on reshaping the root locus of the system by adding poles and zeros to the system's open-loop transfer function and forcing the root loci to pass through desired closed-loop poles in the s plane
- The concept is based on the using the dominant closed-loop poles to approximate the response characteristics
- P controller transfer function: $G_c(s) = k_p$ does not change the open-loop poles or zeros and confined to the path of the root locus
- The controllers we will discussed add open-loop poles and zeros to reshape the root locus

Controller transfer functions



- P controller transfer function: $G_c(s) = k_p$
- PI controller transfer function: $G_c(s) = k_p + \frac{k_I}{s} = \frac{k_p s + k_I}{s}$
- PD controller transfer function: $G_c(s) = k_p + k_D s$
- PID controller transfer function: $G_c(s) = k_p + \frac{k_I}{s} + k_D s = \frac{k_D s^2 + k_p s + k_I}{s}$
- Lead compensator: $G_c(s) = \frac{(s+z)}{(s+p)}$ where $|z| < |p|$
- Lag compensator: $G_c(s) = \frac{(s+z)}{(s+p)}$ where $|z| > |p|$

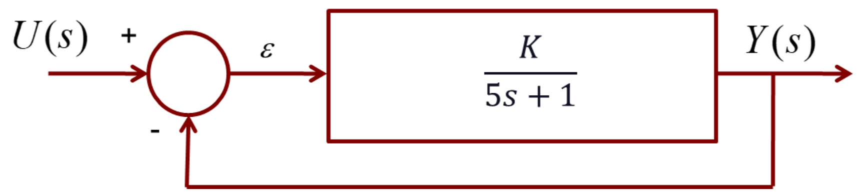
Typical controller characteristics

- ❖ Proportional control: speeds up response but results in steady state error
- ❖ Integral control: eliminates steady state error but produces oscillating response; It increases the order of the closed-loop system and can introduce stability concerns
- ❖ Derivative control: adds damping and improves the transient response
- ❖ Lead compensator: It tends to shift the root locus toward the left half plane. This results in an improvement in the system's stability and an increase in the response speed
- ❖ Lag compensator: It tends to shift the root locus to the right, which is undesirable. For this reason, the pole and zero of a lag compensator must be placed close together (usually near the origin) so they do not appreciably change the transient response or stability characteristics of the system
- ❖ Lead-lag compensator: first design the lead compensator to achieve the desired transient response and stability, and then add on a lag compensator to improve the steady-state response

Example 1

Given the system shown, design a controller with the following specifications:

- Zero steady-state error to a unit step input
- Settling time < 4sec.; percent overshoot < 25%



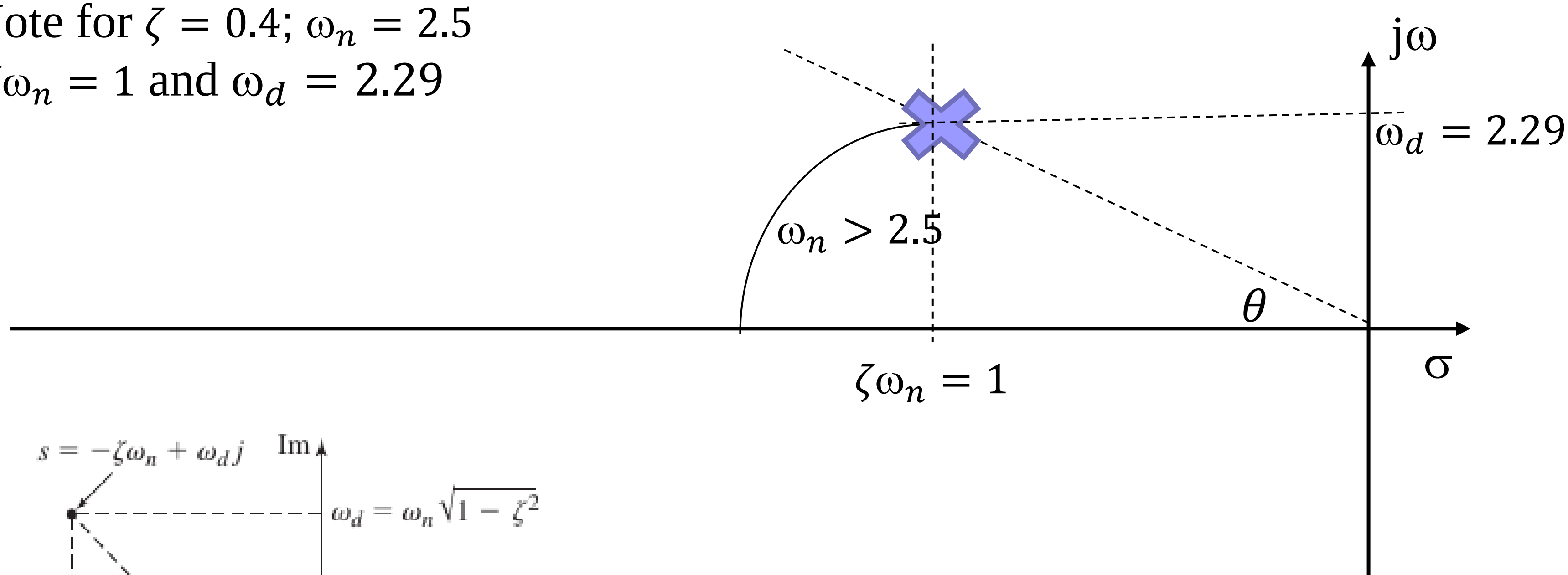
Note: system is type 0 and will not meet steady error specs (must change to type 1)

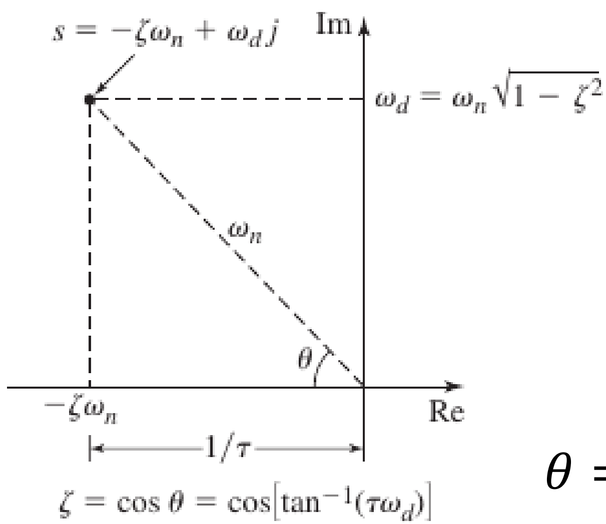
- For percent overshoot < 25%, $\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}} > 0.4$
- For settling time < 4 sec., $T_s = \frac{4}{\zeta\omega_n}$ or $\omega_n > 2.5$
- The original closed loop system is 1st order and would meet the OS% specs
- The closed-loop pole is located at $s = \frac{-1-k}{5}$; $K > 4$ to meet T_s specs

Example 1

Note for $\zeta = 0.4$; $\omega_n = 2.5$

$\zeta\omega_n = 1$ and $\omega_d = 2.29$





$$\theta = \cos^{-1}\zeta = 66.4^0$$

Example 1

Plot root locus of original system using open loop transfer function

$$G(s) = \frac{K}{5s + 1}$$

Step 1: Locate the open-loop poles and zeros in s-plane

- ❖ Note: $n = 1$ branch; starts from $K = 0$ at open-loop pole $s = -0.2$
- ❖ Note: $m = 0$ (there are $(n - m = 1)$ asymptote as $K \rightarrow \infty$ on real axis

Step 2: Determine the root loci occupying the real axis

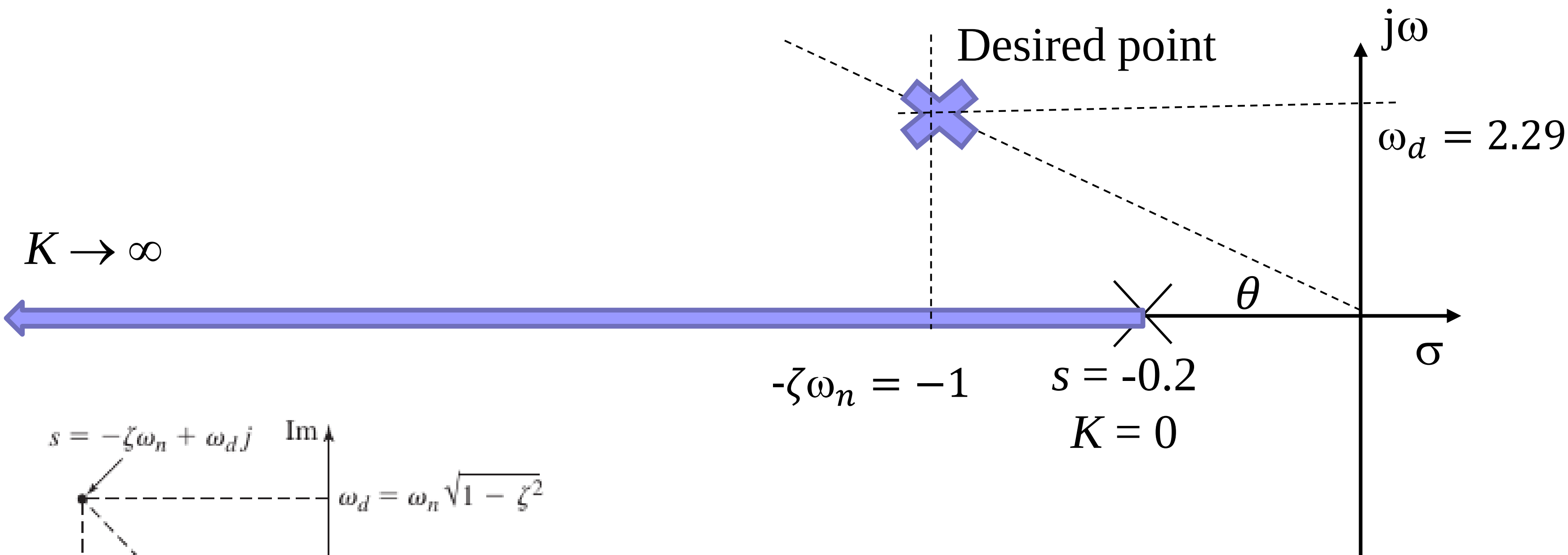
- ❖ It will occupy the real axis from -0.2 to - ∞)

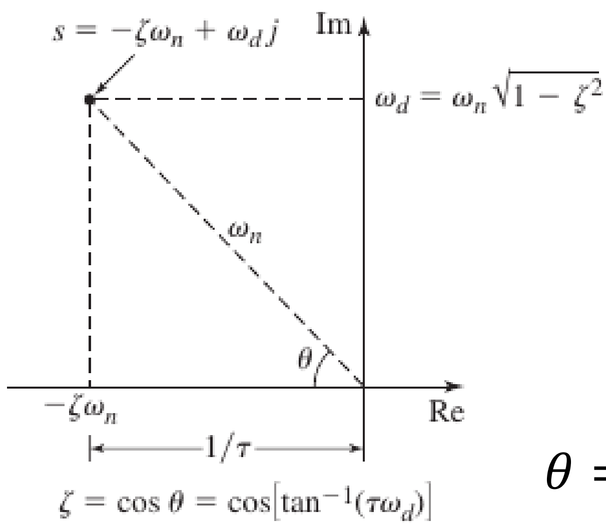
Note: characteristics equation is $5s + 1 + K = 0$

Step 3: Determine the asymptotes of the root loci (if any) – no need

Step 4: Locate the breakaway or break-in points (if any) – no need

Example 1





How to reshape the root locus to pass through the desired point?

$$\theta = \cos^{-1}\zeta = 66.4^0$$

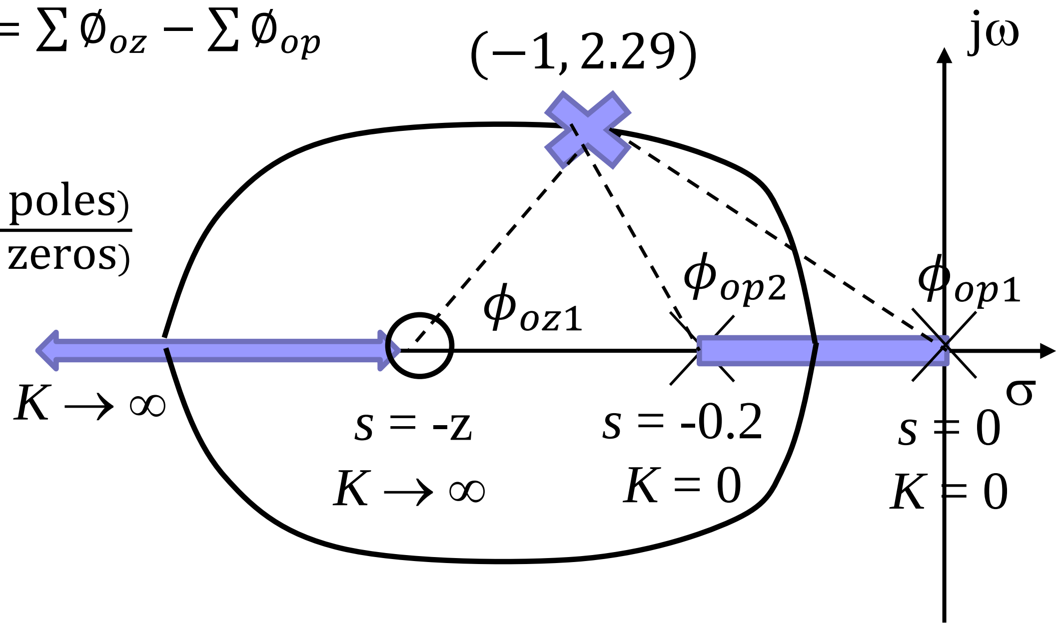
Example 1

Angle criterion: $180^0(2k+1) = \sum \emptyset_{oz} - \sum \emptyset_{op}$

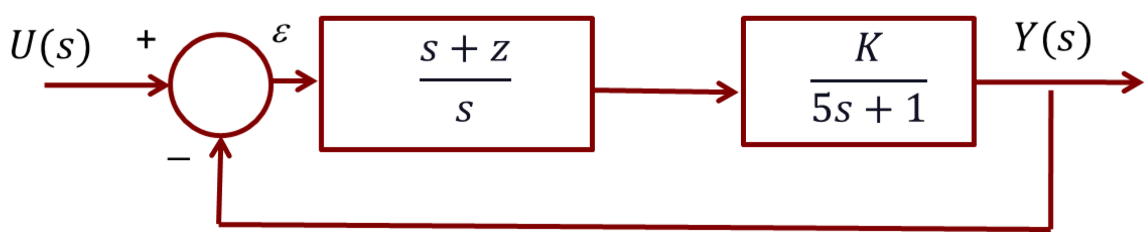
Where $k = 0, \pm 1, \pm 2, \pm 3, \ldots$

Magnitude criterion:

$$K = \frac{\prod(\text{lengths from } s_0 \text{ to OLTF poles})}{\prod(\text{lengths from } s_0 \text{ to OLTF zeros})}$$



Add a PI (one pole at origin and another zero and then adjust the gain K)



Example 1

❖ Use angle criterion to find zero for branch to pass s = -1+j2.29
$180^0(2k+1) = \sum \emptyset_{oz} - \sum \emptyset_{op} = (\emptyset_{oz1} + \cdots + \emptyset_{ozm}) - \left(\emptyset_{op1} \ldots + \emptyset_{ozn}\right)$ where $k = 0, \pm 1, \pm 2, \pm 3, \ldots$

❖ $\phi_{op1} = 180^0 - \tan^{-1}(2.29/1) = 113.6^0$

❖ $\phi_{op2} = \tan^{-1}(2.29/(1-0.2) = 109.26^0$

❖ Hence $\emptyset_{oz1} - (\emptyset_{op1} + \phi_{op2}) = 180^0(2k+1)$

❖ $\emptyset_{oz1} - 222.86^0 = 180^0$; hence $\emptyset_{oz1} = 42.86^0$

❖ Hence $\tan(42.86^0) = 2.29/(z-1)$, or z =3.47

The controller is (s+3.47)

To find the gain K, we use the magnitude criterion:

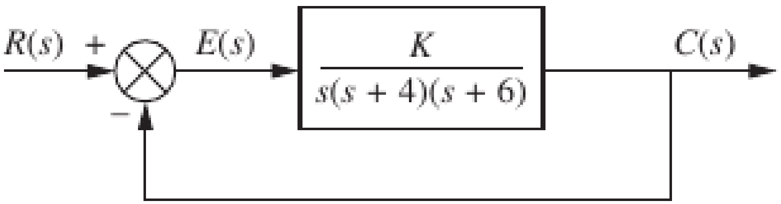
$$K = \frac{\prod(\text{lengths from } s_0 \text{ to OLTF poles})}{\prod(\text{lengths from } s_0 \text{ to OLTF zeros})}$$

Example 1

- The point is at s_0 = -1+j2.29
- $L_{op1} = \sqrt{1^2 + 2.29^2} = 2.5$
- $L_{op2} = \sqrt{0.8^2 + 2.29^2} = 2.43$
- $L_{oz1} = \sqrt{2.47^2 + 2.29^2} = 3.37$
- $K = \dfrac{\prod(\text{lengths from } s_0 \text{ to OLTF poles})}{\prod(\text{lengths from } s_0 \text{ to OLTF zeros})} = 1.8$
- The response of the compensated system should be checked and refined if needed for the compensated system

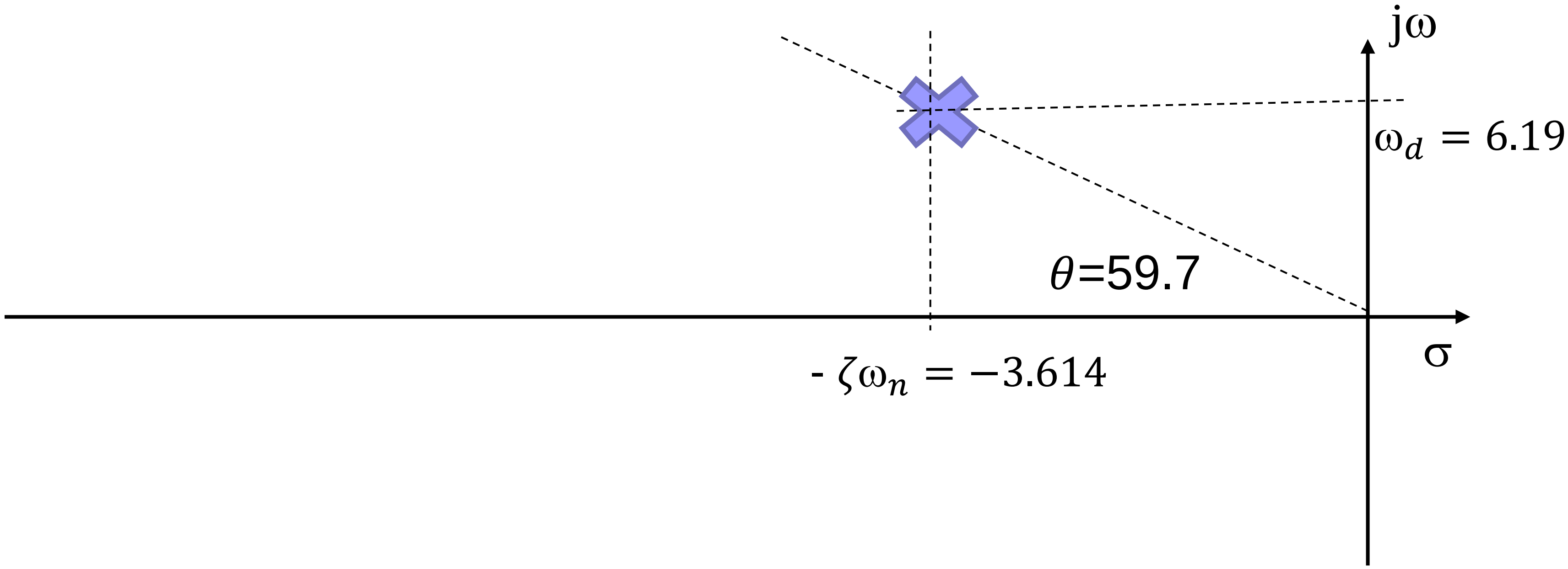
Example 2

Given the system shown, the gain K is tuned to 43.35 so that the settling time is 3.32 sec. with 16% overshoot. Design a controller so that settling time is reduced by threefold with percent overshoot remaining at 16%



- ❖ For percent overshoot = 16%, $\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}} = 0.504$
- ❖ $\theta = \cos^{-1}\zeta = 59.7^0$
- ❖ For settling time = 3.32 sec., $T_s = \frac{4}{\zeta\omega_n}$ or $\zeta\omega_n = 1.205$
- ❖ To reduce T_s to 1.1067 sec., new $\zeta\omega_n = 3.614$

Example 2



To meet the specs, the new root locus must pass through the point:

- New $\zeta\omega_n = 3.614$ = real part with $\zeta = 0.504$ and $\omega_n = 7.17$
- New $\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.19$

Example 2

Plot root locus of original system using open loop transfer function

$$G(s) = \frac{K}{s(s+4)(s+6)}$$

Step 1: Locate the open-loop poles and zeros in s-plane

- Note: n = 3 branches; start from K = 0 at open-loop poles $s = 0$, $s = -2$ and $s = -6$
- Note: m = 0 (there are $(n - m = 3)$ asymptotes as $K \rightarrow \infty$ with 1 on real axis (need to determine 2 more asymptotes)

Step 2: Determine the root loci occupying the real axis

- It will occupy the real axis from 0 to -2 and from -6 to -∞

Note: characteristics equation is $s^3 + 10s^2 + 24s + K = 0$

Step 3: Determine the asymptotes of the root loci

- $\sum P_O = -10, \sum Z_O = 0, \bar{x} = \frac{\sum P_O - \sum Z_O}{n-m} = -3.33, \theta = \frac{(2b+1)180^0}{n-m} = \pm 60^0$

Example 2

Step 4: Locate the breakaway or break-in points

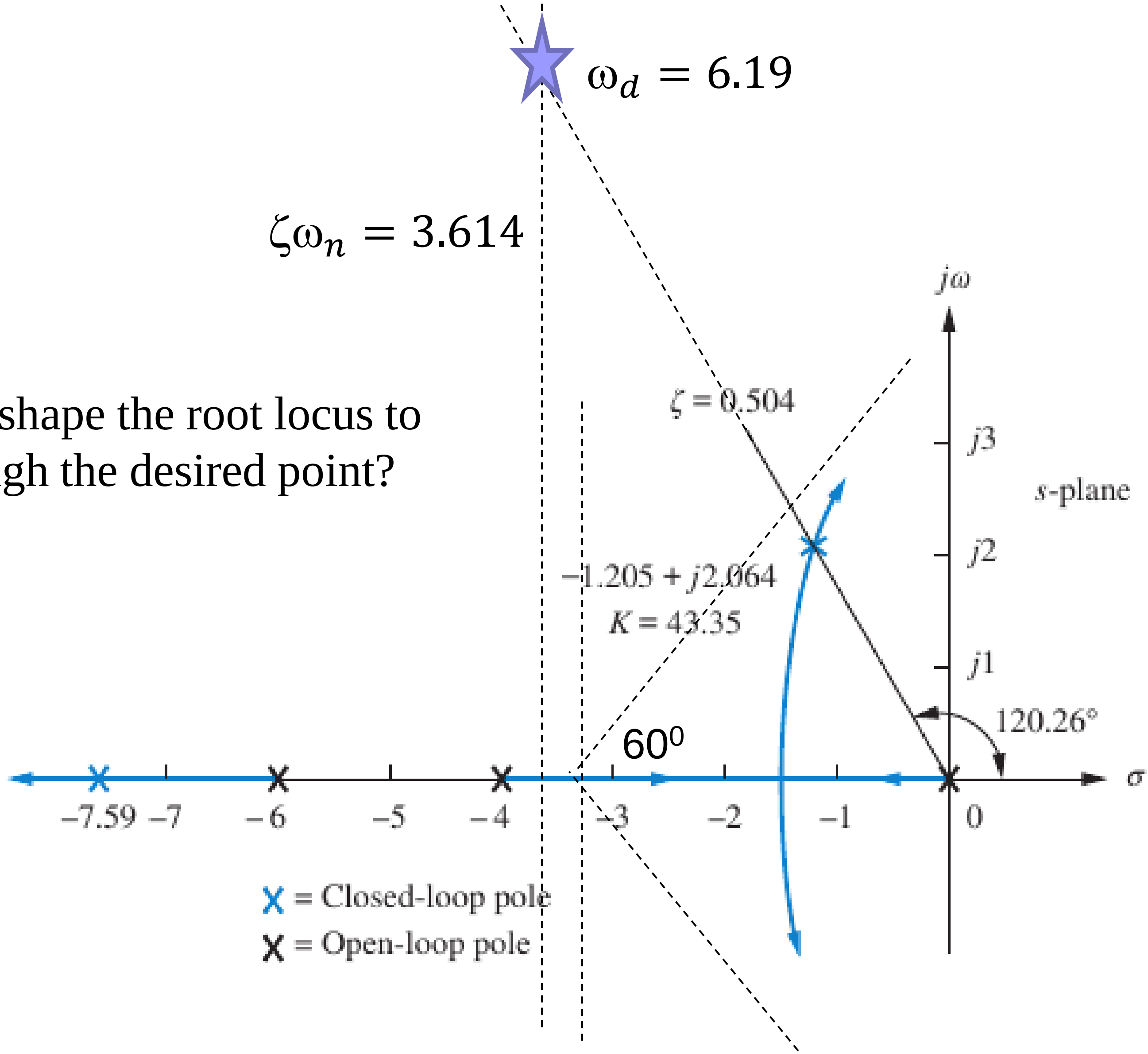
- ❖ breakaway will occur between 0 and -4
- ❖ Get characteristic equation, differentiate, equate to zero to solve for *s*:

$\frac{\partial K}{\partial s} = \frac{\partial}{\partial s} f(s) = \frac{\partial}{\partial s}(-s^3 - 10s^2 - 24s) = -2s^2 - 20s - 24 = 0$ or at $s = -1.39$; Note no break-n or breakaway at $s = -8.605$

Note: when gain is tuned to *K*=43.35, the settling time = 3.32 sec with

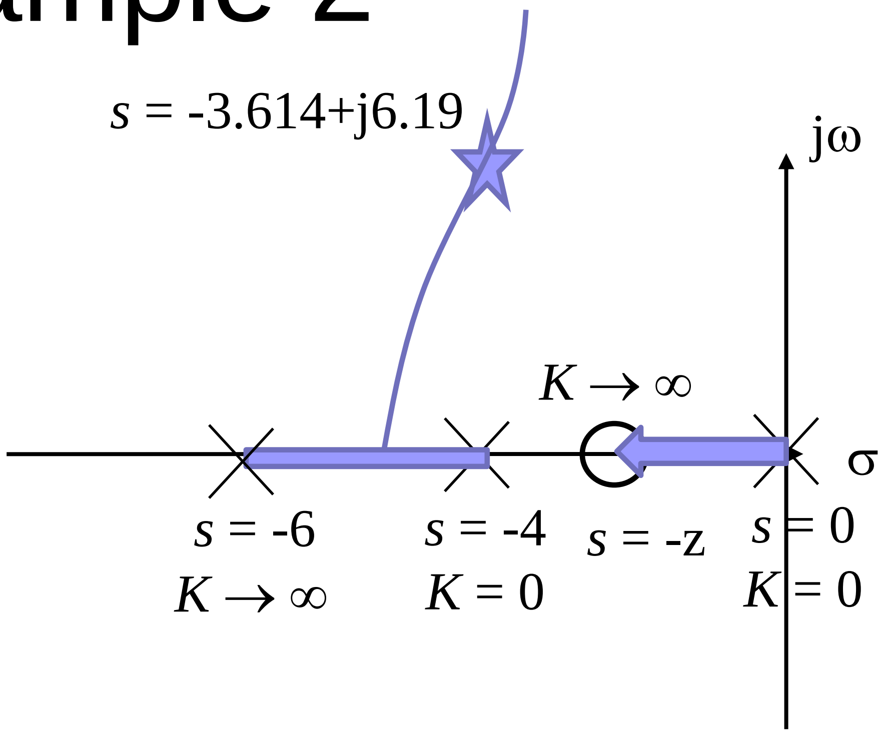
- ❖ $\zeta\omega_n = 1.205$ = real part with $\zeta = 0.504$ and $\omega_n = 2.39$
- ❖ $\omega_d = \omega_n\sqrt{1 - \zeta^2} = 2.064$

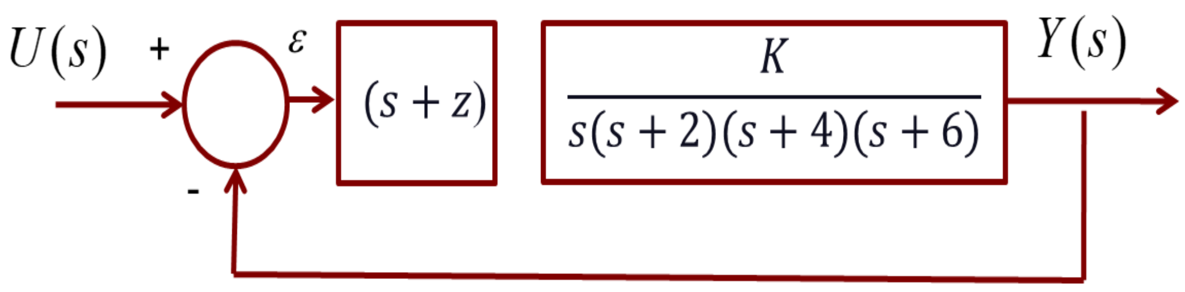
How to reshape the root locus to pass through the desired point?



Example 2

Add a PD. How to determine zero location so that the root locus to pass through the desired point?





Example 2

❖ Use angle criterion to find zero for branch to pass s = -3.614+j6.19
$180^0(2k+1) = \sum \emptyset_{oz} - \sum \emptyset_{op} = (\emptyset_{oz1} + \cdots + \emptyset_{ozm}) - \left(\emptyset_{op1} \ldots + \emptyset_{ozn}\right)$ where $k = 0, \pm 1, \pm 2, \pm 3, \ldots$

❖ $\phi_{op1} = 180^0 - \tan^{-1}(6.19/3.614) = 120.28^0$

❖ $\phi_{op2} = \tan^{-1}(6.19/(4-3.614)) = 86.43^0$

❖ $\phi_{op3} = \tan^{-1}(6.19/(6-3.614)) = 68.92^0$

❖ Hence $\emptyset_{oz1} - (\emptyset_{op1} + \phi_{op2} + \phi_{op3}) = 180^0(2k+1)$

❖ $\emptyset_{oz1} - 275.63^0 = 180^0$; hence $\emptyset_{oz1} = 95.63^0$

❖ Hence $\tan(180 - 95.63) = 6.19/(3.614 - z)$, or z =-3

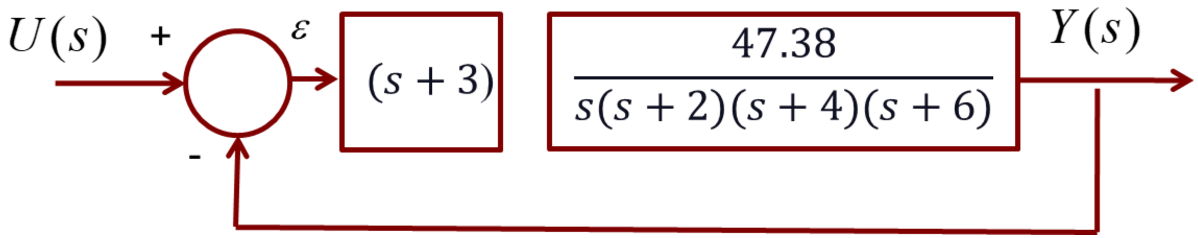
Hence the controller is (s+3)

To find the gain K, we use the magnitude criterion:

$$K = \frac{\prod(\text{lengths from } s_0 \text{ to OLTF poles})}{\prod(\text{lengths from } s_0 \text{ to OLTF zeros})}$$

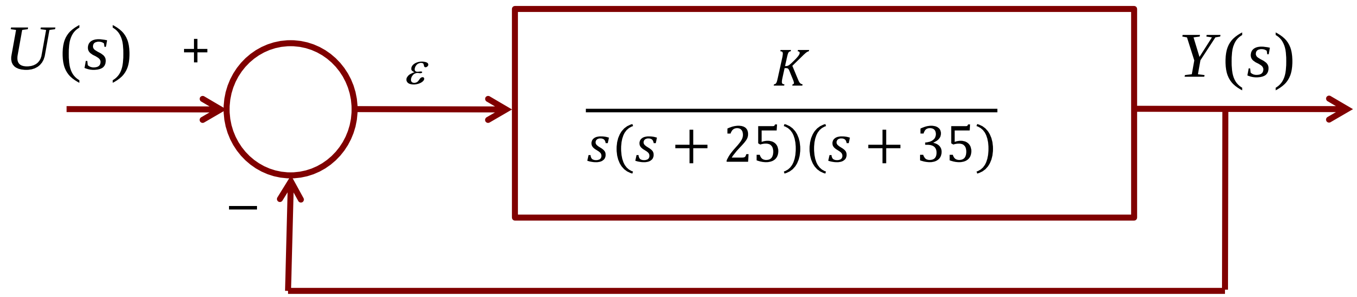
Example 2

- The point is at s_0 = -3.614+j6.19
- $L_{op1} = \sqrt{3.614^2 + 6.19^2} = 7.17$
- $L_{op2} = \sqrt{0.386^2 + 6.19^2} = 6.20$
- $L_{op3} = \sqrt{2.386^2 + 6.19^2} = 6.63$
- $L_{oz1} = \sqrt{0.614^2 + 6.19^2} = 6.22$
- $K = \dfrac{\prod(\text{lengths from } s_0 \text{ to OLTF poles})}{\prod(\text{lengths from } s_0 \text{ to OLTF zeros})} = 47.38$
- The response of the compensated system should be checked and refined if needed



Example 3

Given the system shown. Sketch the root locus of the system. Describe without calculation how you would reshape the root locus with a PID controller so that the output response has zero steady state error to the ramp input and damping of 0.7071 with settling time of less than 1 sec.



Specifications:

Zero steady state error to ramp: need type 2 (need to add one pole at s = 0

$T_s = \frac{4}{\zeta\omega_n}$ or $\zeta\omega_n = 4$

$\zeta = 0.707$ or $\theta = \cos^{-1}\zeta = 45^0$