



# ME 1071: Applied Fluids

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Spring 2021

# Outlines

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- Approaches for Fluid Mechanics
- Continuum Hypothesis
- Properties of Fluids
  - Viscous Flow
- Fluid Statics and Motion
- A Brief Review of Thermodynamics
- Some Aspects of Supersonic Flow: Shock Waves
- Compressible Flow and Hypersonic Flow

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# Outlines

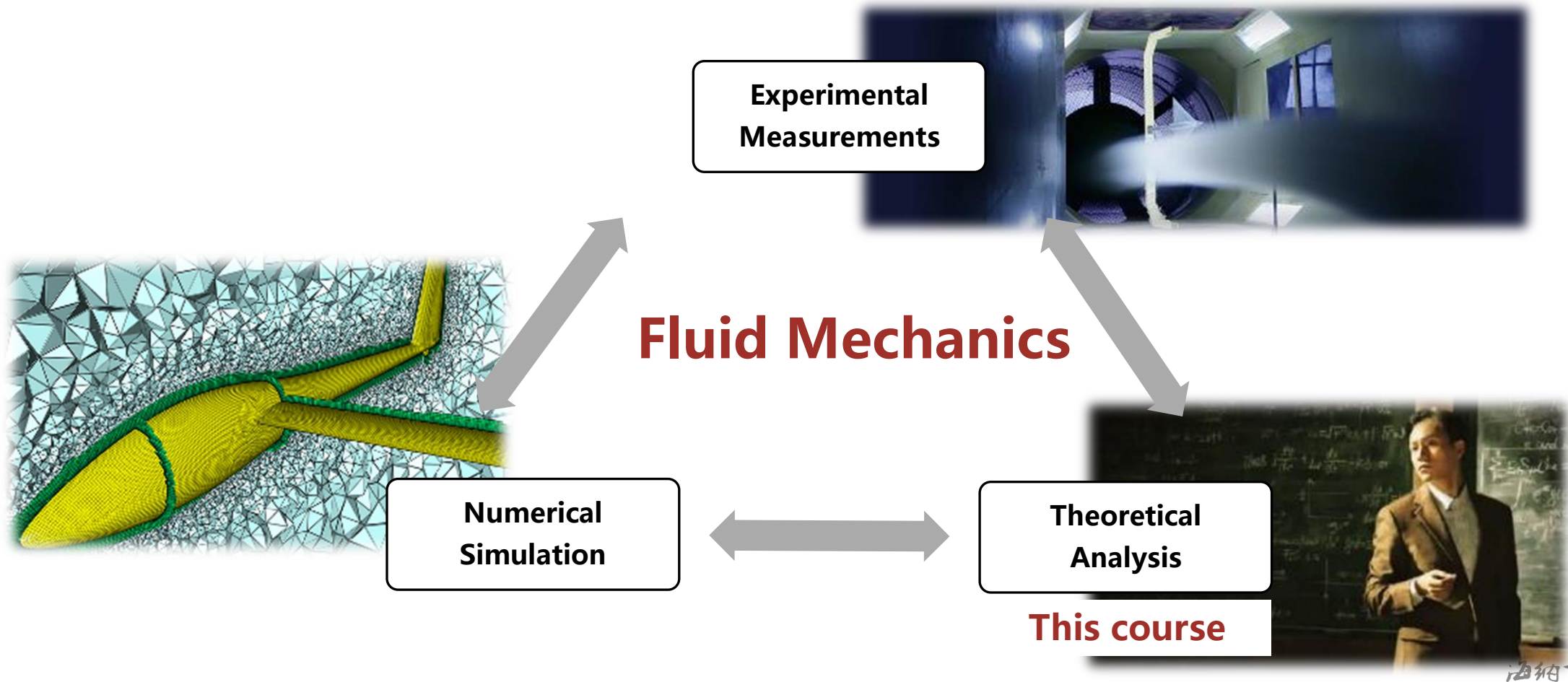
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- **Approaches for Fluid Mechanics**
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# Approaches



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# Approaches



## Theoretical Analysis

- **Navier-Stokes Equations (the full governing equations)**

$$\nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Momentum  
conservation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} + \frac{1}{3} \nu \nabla (\nabla \cdot \mathbf{u}) + \mathbf{f}$$

Energy  
conservation

$$\rho \frac{D}{Dt} \left( e + \frac{|\mathbf{u}|^2}{2} \right) = \rho \dot{q} - \nabla \cdot (-k \nabla T) - \nabla \cdot (p \mathbf{u}) + \nabla \cdot (\ddot{\mathbf{t}} \cdot \mathbf{u}) + \rho \mathbf{f} \cdot \mathbf{u}$$

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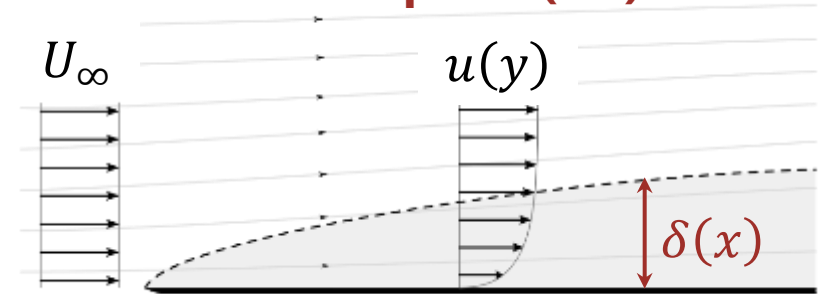
# Approaches



## Theoretical Analysis

- Model of the physical problem
  - Through appropriate simplification & approximation
- Governing Equations, Initial & Boundary conditions
- **Exact solution** by using mathematic tools

### Boundary layer flow over a flat plate (2D)



- ✓ All the physics involved in the problem
- ✓ Direct information on what are the important variables
- ✓ Simple tools for rapid calculations
- × **Only possible for a few simplified cases**

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial p}{\partial y} = 0 \end{array} \right. \quad \begin{array}{l} \\ \text{Momentum Equation} \\ \text{in } y \text{ direction} \end{array}$$

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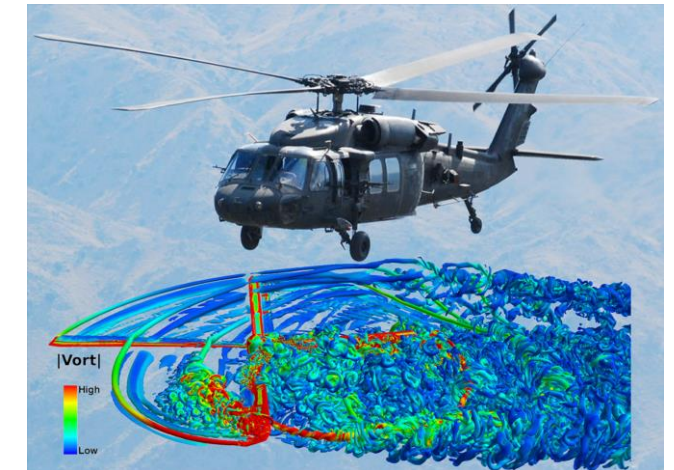
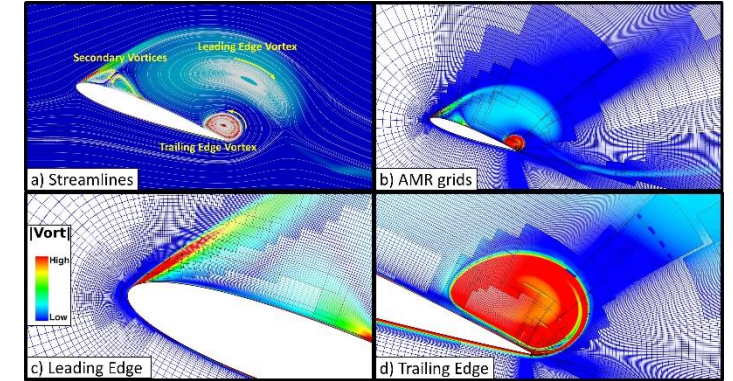


# Approaches



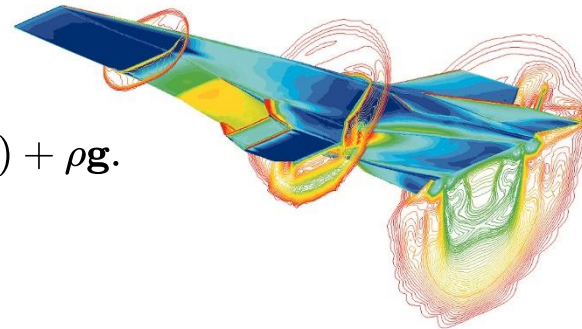
- **Computational Fluid Dynamics (计算流体力学, CFD)**

- A branch of fluid mechanics that uses numerical analysis and data structures to analyze and solve problems that involve fluid flows.
- CFD deals with computing the equations of fluid flows using numerical methods instead of partial differential equations, represent a continuous flow field by a series of distinct grid points/cells in space.
- **Finite volume method** (有限体积法, FVM, most common), **Finite element method** (有限元法, FEM), **Finite difference method** (有限差分法, FDM, historically important), **Spectral element method** (谱方法, SEM), **Lattice Boltzmann method** (格子玻尔兹曼, LBM), .....



$$\rho \frac{D\mathbf{u}}{Dt} = \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla \bar{p} + \mu \nabla^2 \mathbf{u} + \frac{1}{3} \mu \nabla (\nabla \cdot \mathbf{u}) + \rho \mathbf{g}.$$

**The Navier-Stokes Equation**



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# Approaches



## Experiment

- Ground test of scaled models in Experimental facilities
- Flight test of the prototype aircraft

- ✓ Experimental results are reliable and straightforward
- × Scaling issuing
- × Cost- and time-consuming



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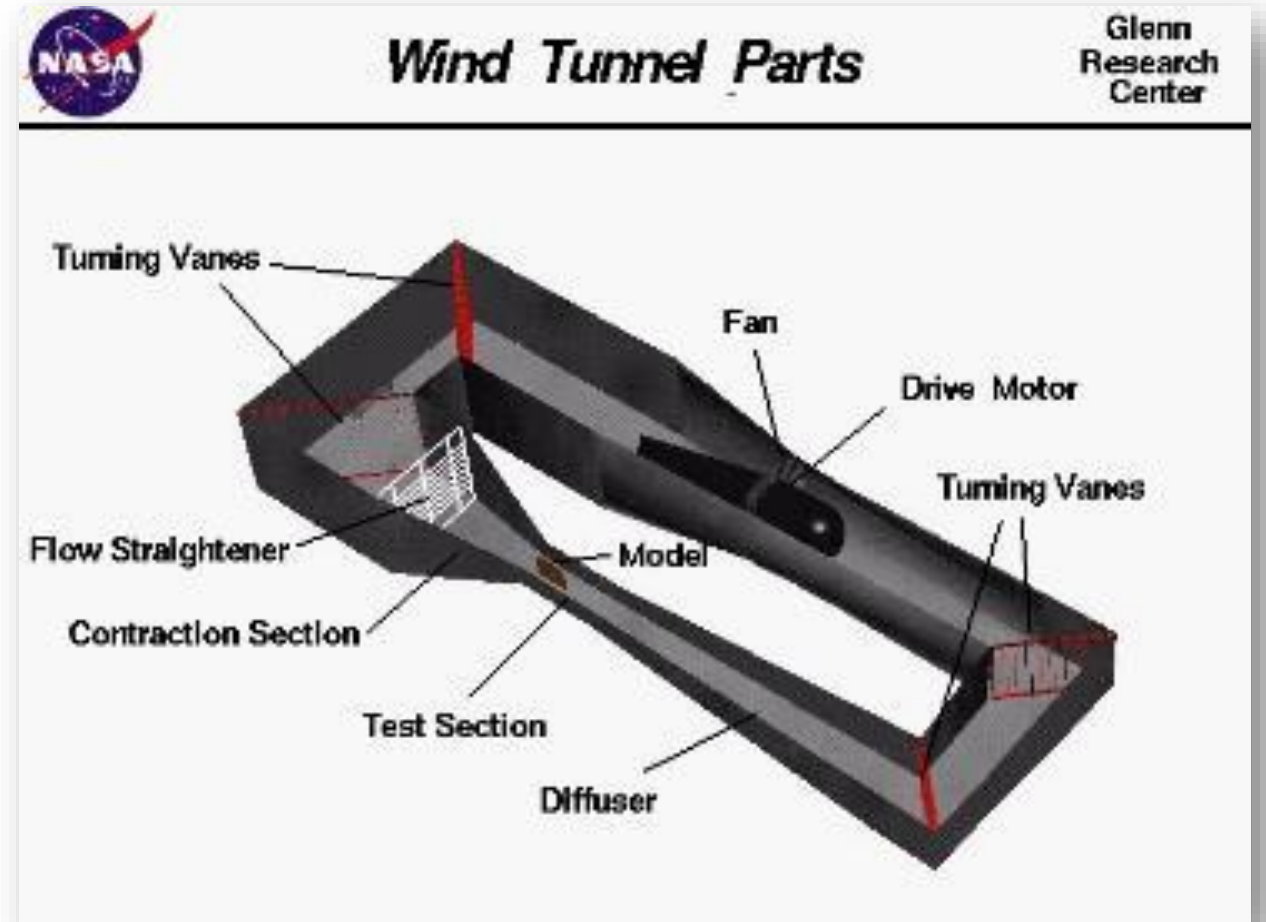
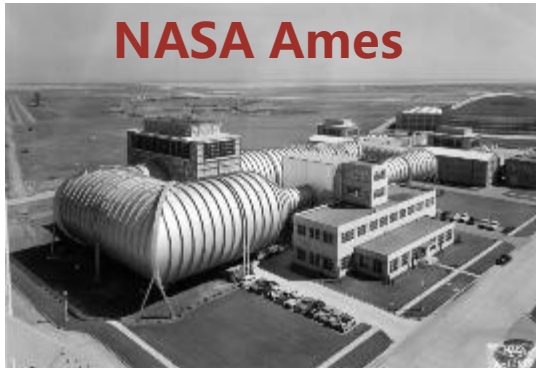


# Approaches



## Experiment

- Facilities-**Wind Tunnel**



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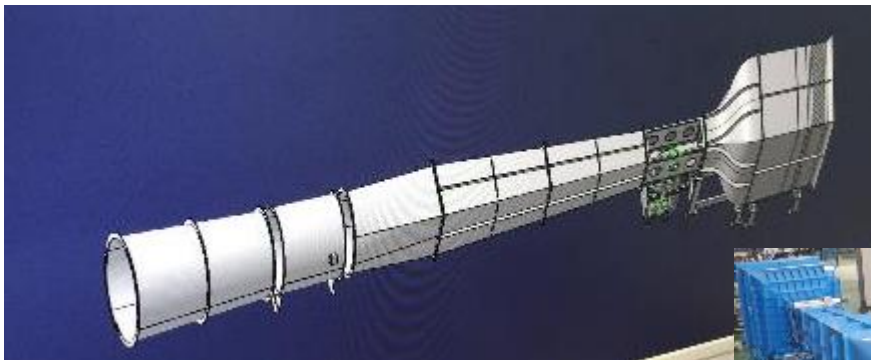
# Approaches



## Experiment

- Facilities-**Wind Tunnel**

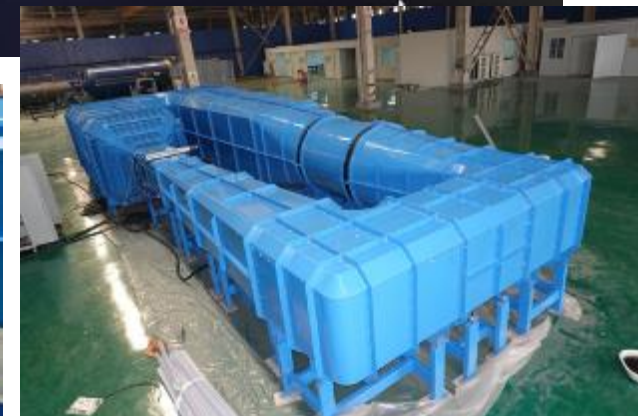
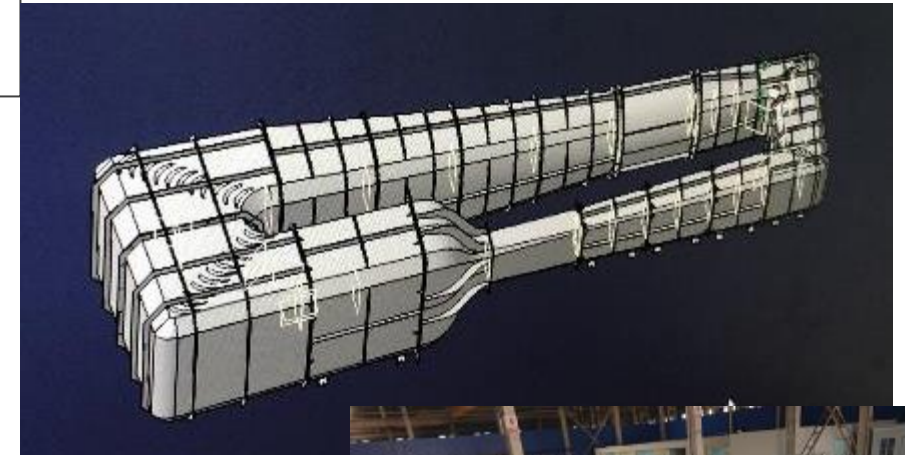
**School of Aeronautics  
& Astronautics,  
Sichuan University**



Open return wind tunnel  
(低速开口式风洞)



Closed return wind tunnel  
(低速闭口式风洞)



四川航空职业技术学院

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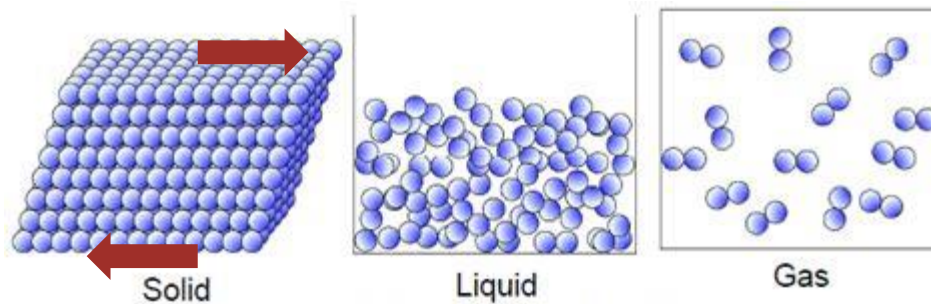
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# Continuum Hypothesis

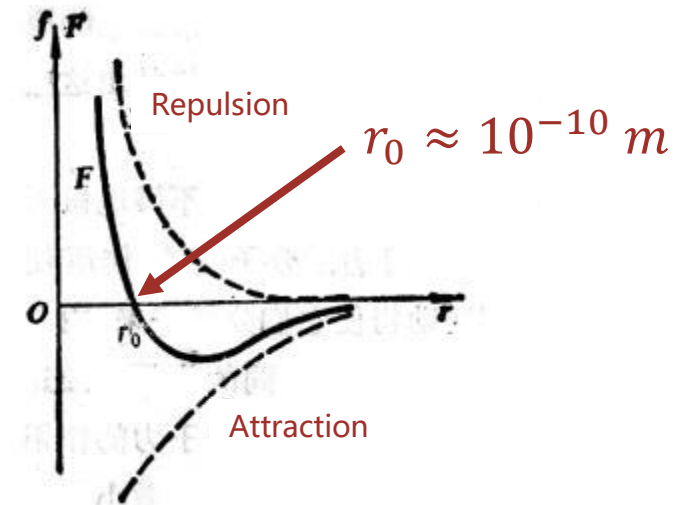


The defining property of fluids

- Continuously increasing deformation under the act of tangential shear stress
- the shear stress is proportional to the rate of change of the deformation



Sketch of the force exerted by one (un-ionized) simple molecule on another as a function of the distance  $r$  between their centers.



The properties of fluids are directly related to their molecular structure and to the nature of the forces between the molecules

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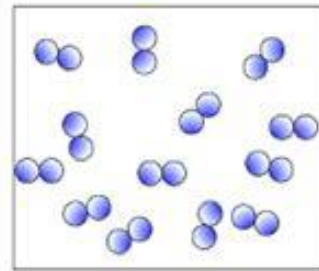
# Continuum Hypothesis



## On a microscopic scale

- Fluid consists of individual molecules
  - **Vacuous regions** between molecules of a gas – **Mean free path** (平均自由程)  $l$
  - In standard atmosphere,  $l = 6 \times 10^{-8}$  m for air
  - Diameter of air molecules  $d = 3.7 \times 10^{-10}$  m

$$\frac{l}{d} \approx 160$$



Gas

- Fluid substance is sparse (稀疏的)
- Its motion is **non-uniform** (不均匀的), **discrete** (离散的) and **random** (随机的)

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# Continuum Hypothesis



## The Continuum Hypothesis

- On a macroscopic scale, the fluid properties (e.g., velocity, pressure, density) are **uniform**, **continuous** and **definite**
- There is huge difference in the **length scales** of the problem
  - The length scale  $L$  in aerodynamics is much larger than the mean free path  $l$

$$\frac{l}{L} \ll 1$$

- The behavior of individual molecular **has no effect on** the observed average properties



# Continuum Hypothesis



## The Continuum Hypothesis

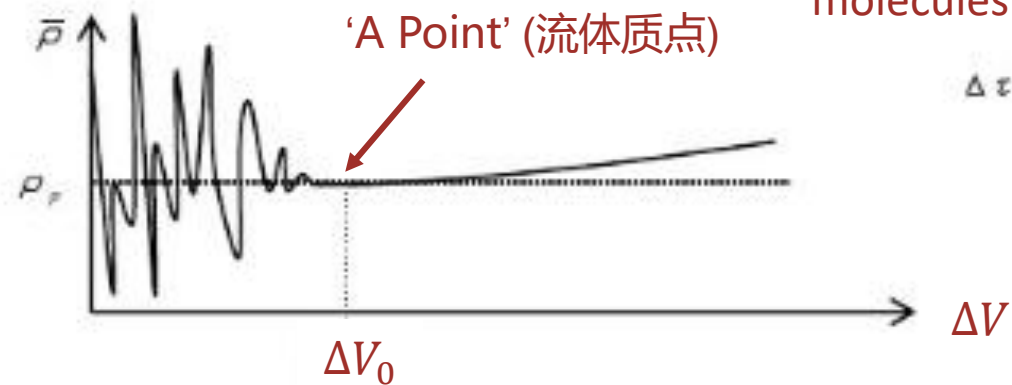
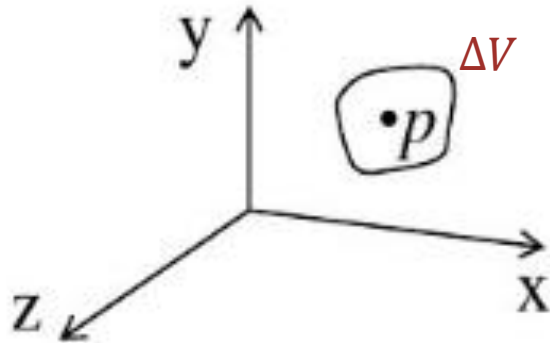
- Considering **the definition of density** as mass per unit volume

$$\rho_p = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V}$$

For Air:

$$\Delta V_0 = 10^{-9} m^3$$

With about  $3 \times 10^7$  molecules



A critical value  $\Delta V_0$ , below which molecular variations assume importance, and above which one finds a macroscopic variation of density within the region.

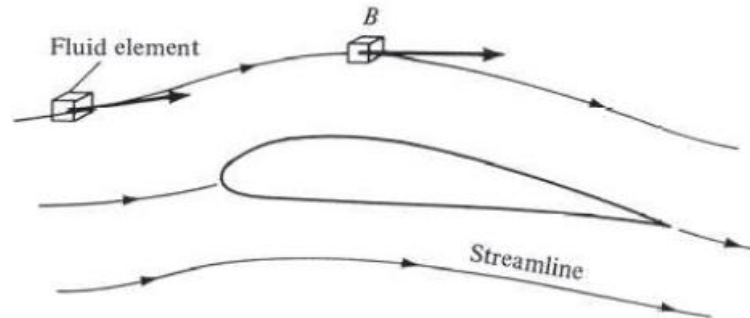
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# Continuum Hypothesis



## The Continuum Hypothesis

- **Fluid element** (流体微团): an infinitesimally small element of mass in the gas



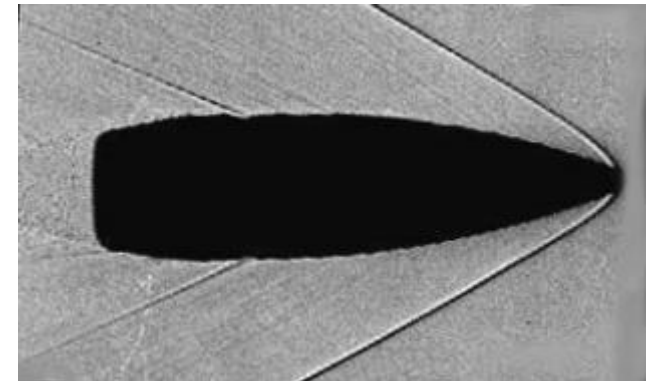
- The **velocity  $\mathbf{U}$**  of a flowing gas at any fixed-point  $B$  in space is the velocity of an infinitesimally small fluid element as it sweeps through  $B$
- $\mathbf{U}$  is a vector

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# Continuum Hypothesis



- **Knudsen number**  $Kn = \frac{l}{L}$ 
  - When  $Kn \ll 1$ , the continuum hypothesis is satisfied.
  - Named after Danish physicist **Martin Knudsen** (1871–1949).
- **Other extreme cases**
  - At greater altitudes,  $Kn \sim 1$ , rarefied gasdynamics
  - Shock wave is typically regarded as a discontinuous surface.



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# Continuum Hypothesis

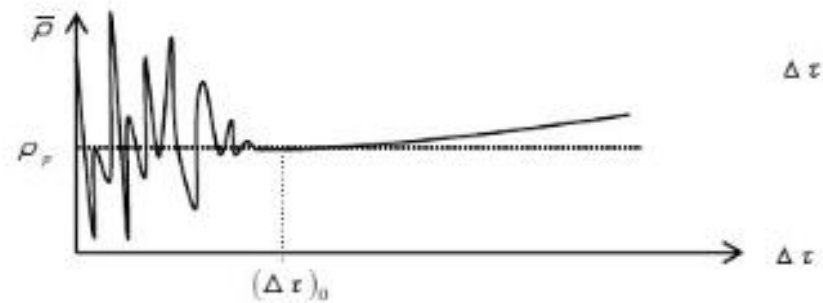
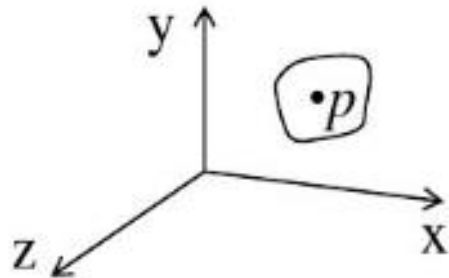


## Question

Under the continuum hypothesis, the **fluid density** is defined as

$$\rho_p = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V}$$

Please explain the physical and mathematical meanings of '0' in the limit  $\Delta V \rightarrow 0$ ?



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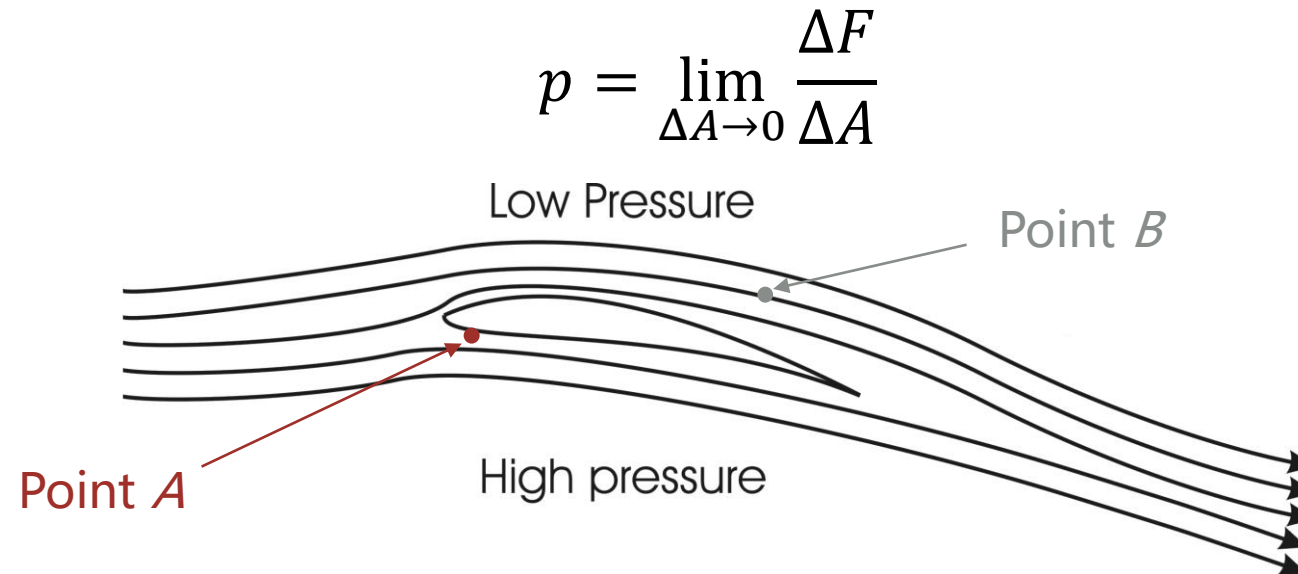
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# Properties of Fluids



## Pressure (压强)

- Normal force per unit area exerted on a surface due to the time rate of change of momentum of the gas molecules impacting on (or crossing) that surface



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# Properties of Fluids

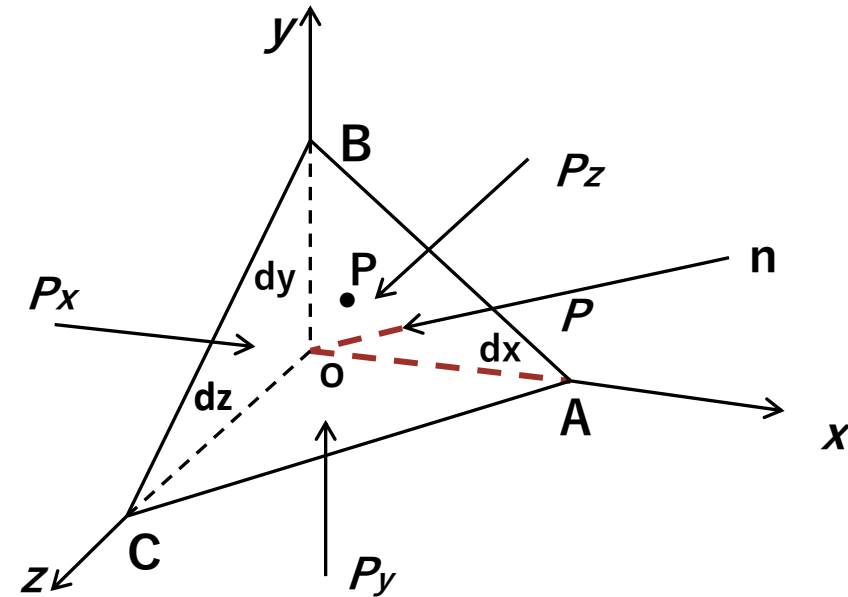


## Pressure (压强)

- Dimensions (量纲): **force per unit area**, or  $\{ML^{-1}T^{-2}\}$
- Direction: always acts **inward normal** to any surface



**Question:** Does the pressure depend on the direction of the surface on which it acts?



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# Properties of Fluids



## Pressure (压强)

- Dimensions (量纲): **force per unit area**, or  $\{ML^{-1}T^{-2}\}$
- Direction: always acts **inward normal** to any surface
  - The magnitude of pressure is **independent** of the orientation of the surface on which the pressure acts.
- Pressure is a **scalar** (point property).

$$p = p(t, x, y, z)$$

# Properties of Fluids



## Temperature $T$

- A measure of **the average kinetic energy** of the random motion of the molecules of the fluid at a point

$$KE = \frac{3}{2}kT$$

where  $KE$  is the mean molecular kinetic energy and  $k$  is **the Boltzmann constant**.

- Temperature is **a scalar quantity** (magnitude, no direction)
- Unit: **Kelvin (K)**, as an absolute measure of thermal energy

$$T_{(K)} = T_{(^{\circ}\text{C})} + 273.15$$

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# Properties of Fluids



## Ideal Gas Law

- The equation of state of a hypothetical ideal gas.

- $R$  is the specific gas constant  $p = \frac{\bar{R}}{M} \rho T = \rho R T$   $\bar{R}$ : Universal gas constant (8314 J/mol.K)  
 $M$ : Molecular weight

- The standard atmospheric condition at sea level

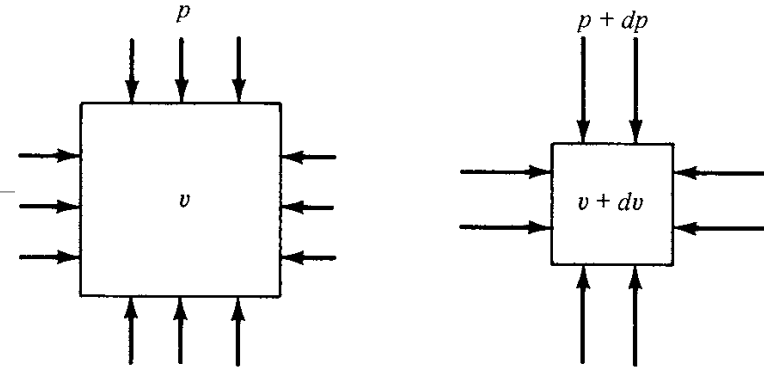
For air,  $R = 287.15 \text{ J}/(\text{kg} \cdot \text{K})$

$$p_s = 1.01325 \times 10^5 \text{ N m}^{-2} = 1\text{atm} = 0.1\text{MP}$$

$$\rho_s = 1.225 \text{ kg m}^{-3} \quad T_s = 288.15 \text{ K} = 15^\circ\text{C}$$

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# Properties of Fluids



## Compressibility (可压缩性)

- A measure of the **relative volume change** of a fluid as a response to a pressure (or mean stress) change.
- Specific Volume (比体积): the volume per unit mass

- **Bulk Modulus** (体积弹性模量) determines how compressible a fluid is

$$v = \frac{1}{\rho} \quad \longrightarrow \quad dv = -\frac{1}{\rho^2} d\rho$$

- **Higher** the value of Bulk Modulus indicates that it is **difficult** to compress the fluid

$$E_v = -\frac{dp}{dv/v} = \rho \boxed{\frac{dp}{d\rho}}$$

The speed of sound  $c^2 = \boxed{\frac{dp}{d\rho}}$

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# Properties of Fluids



## Compressibility (可压缩性)

$$E_v = \rho \frac{dp}{d\rho} \quad c^2 = \frac{dp}{d\rho}$$

- Bulk Modulus for different fluids

- Water:  $E_{v_w} = 2.16 \times 10^9 \text{ N/m}^2$
- Air:  $E_{v_a} = 1.01 \sim 1.42 \times 10^5 \text{ N/m}^2$

- Speed of sound for different fluids

- Water:  $c_w \approx 1481 \text{ m/s}$
- Air:  $c_a = 340.27 \text{ m/s}$

At standard atmospheric condition

- All real substances (fluids or solids) are compressible to some greater or lesser extent

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# Properties of Fluids



## Compressibility (可压缩性)

- Compressibility of a fluid **flow** (流动的可压缩性)

- Whether the density  $\rho$  can be regarded as **a constant** in the flow

$$E_v = \rho \frac{dp}{d\rho} \quad \longrightarrow \quad d\rho = \frac{1}{E_v} \rho dp$$

- For **liquid**,  $E_v$  is typically very large. Therefore,  $d\rho$  will be negligibly small

The flow of a liquid is **incompressible**

- For **gas**,  $E_v$  is typically small

- Mach Number:

$$Ma = \frac{U}{c}$$

High speed flow:  $dp$  is large,  $d\rho$  can be large

**compressible**

Low speed flow:  $dp$  is small,  $d\rho$  can be small

**incompressible**

$$Ma < 0.3$$

$$d\rho/\rho < 5\%$$

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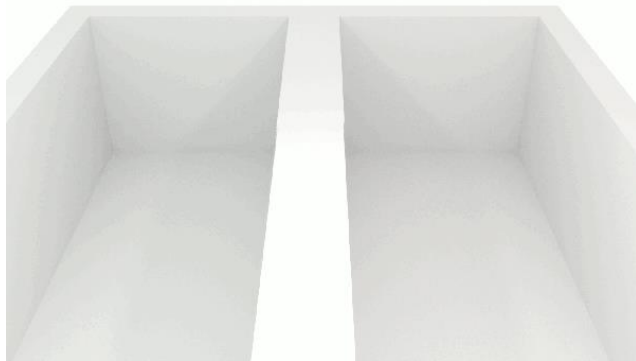
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# Introduction to Viscous Flow



- **Viscous Flow (黏性流动)**

- A flow where the effects of **viscosity**, **thermal conduction**, and **mass diffusion** are important, viscous flow are rotational flows.
- **Viscosity**: a measure of its resistance to deformation at a given rate.
- Thermal conduction: the transfer of internal energy by microscopic collisions of particles and movement of electrons within a body or between two bodies in contact.
- Mass diffusion: the transport of material due to concentration gradients (less considered in this course).



Low viscosity vs. high viscosity



High viscosity vs. low viscosity

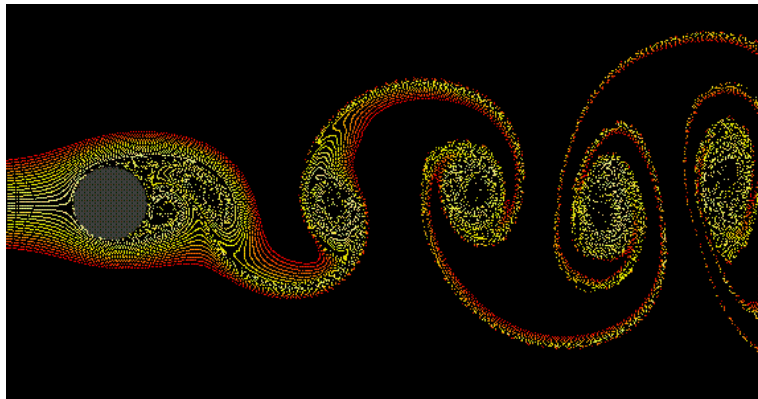
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# Introduction to Viscous Flow

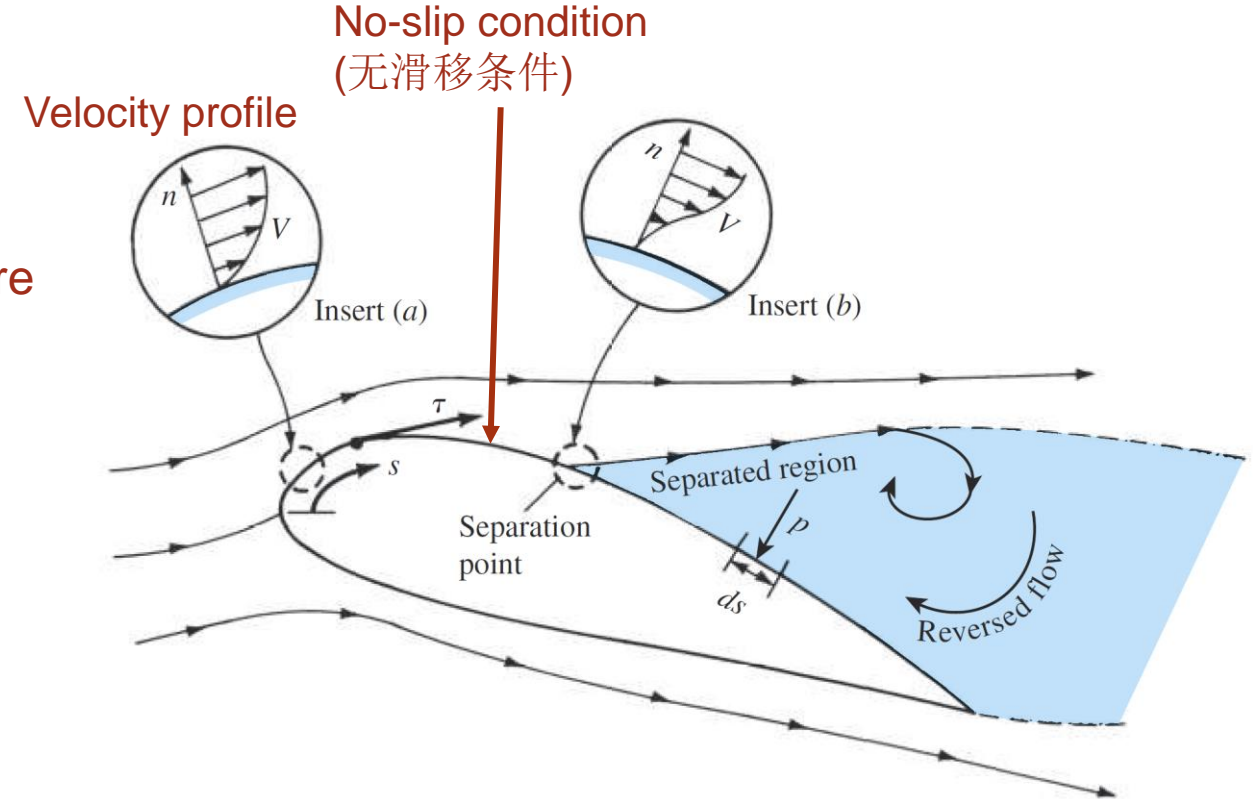


## • Influence of viscosity

- Shear stress  $\tau$ : tangential 'frictional' force per unit area.
- No-slip condition:  $V = 0$  at wall surface.
- Reverse flow cause by adverse pressure gradient.
- Separated flow: detachment of a boundary layer from a surface into a wake ( $\partial V / \partial n = 0$ ).



The Von Karman vortex street



Effect of viscosity on a body in a moving fluid

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# Introduction to Viscous Flow

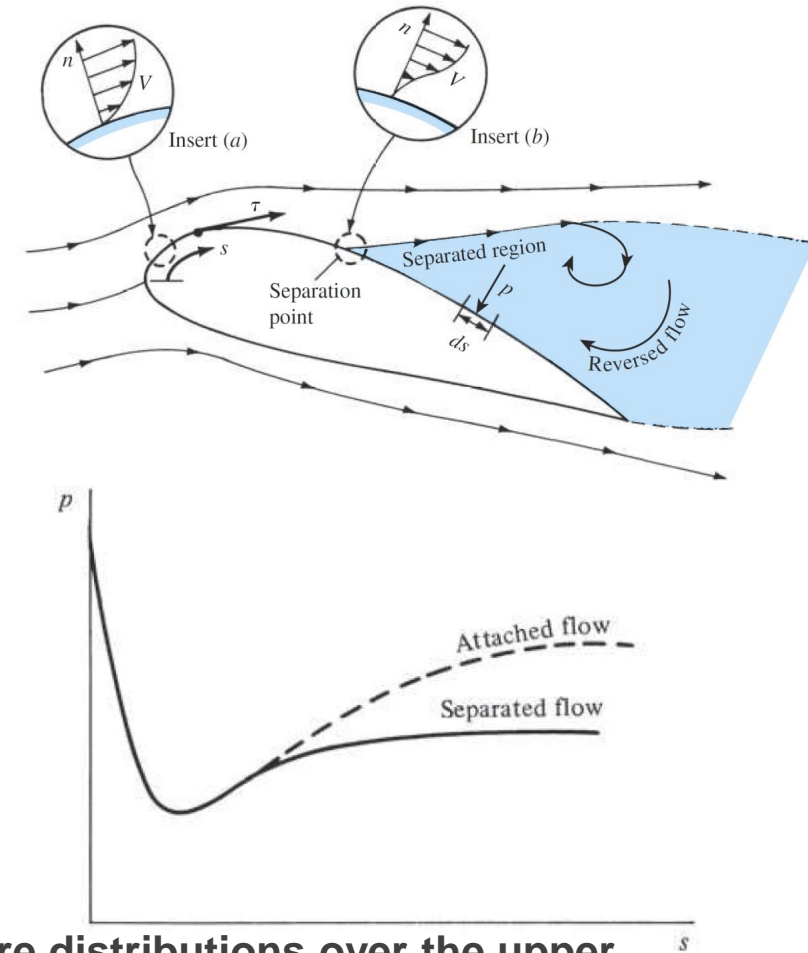


## • Influence of viscosity

- Skin friction drag  $D_f$ : the component in the drag direction of the integral of the shear stress  $\tau$  over the body.
- Pressure drag due to separation  $D_p$ : the component in the drag direction of the integral of the pressure distribution over the body.

$D_p \longrightarrow$  form drag (形状阻力)

$D_f + D_p \longrightarrow \left\{ \begin{array}{l} \text{2D: profile drag (型阻)} \\ \text{3D: parasite drag (寄生阻力)} \end{array} \right.$



The pressure distributions over the upper surface of the body

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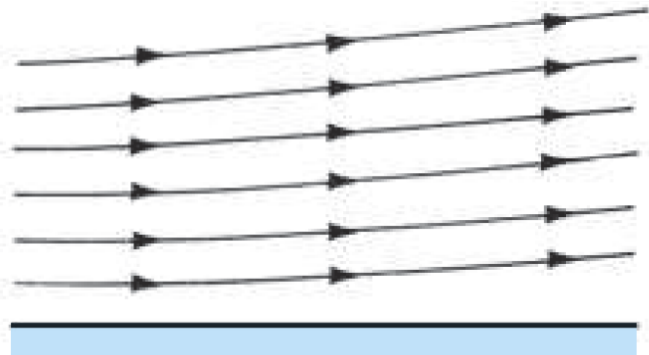
# Introduction to Viscous Flow



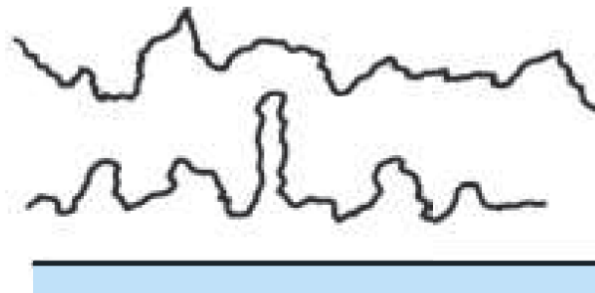
## • Flow State (流动状态)

- **Laminar flow**: the path lines of various fluid elements are smooth and regular.
- **Turbulent flow**: the motion of a fluid element is very irregular (不规则的) and tortuous (弯弯曲曲的).

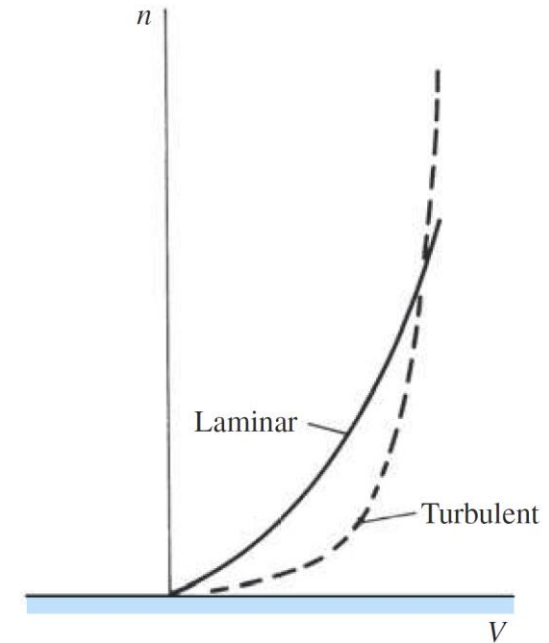
$$\left[ \left( \frac{\partial V}{\partial n} \right)_{n=0} \right]_{\text{turbulent}} > \left[ \left( \frac{\partial V}{\partial n} \right)_{n=0} \right]_{\text{laminar}}$$



(a) Laminar flow



(b) Turbulent flow



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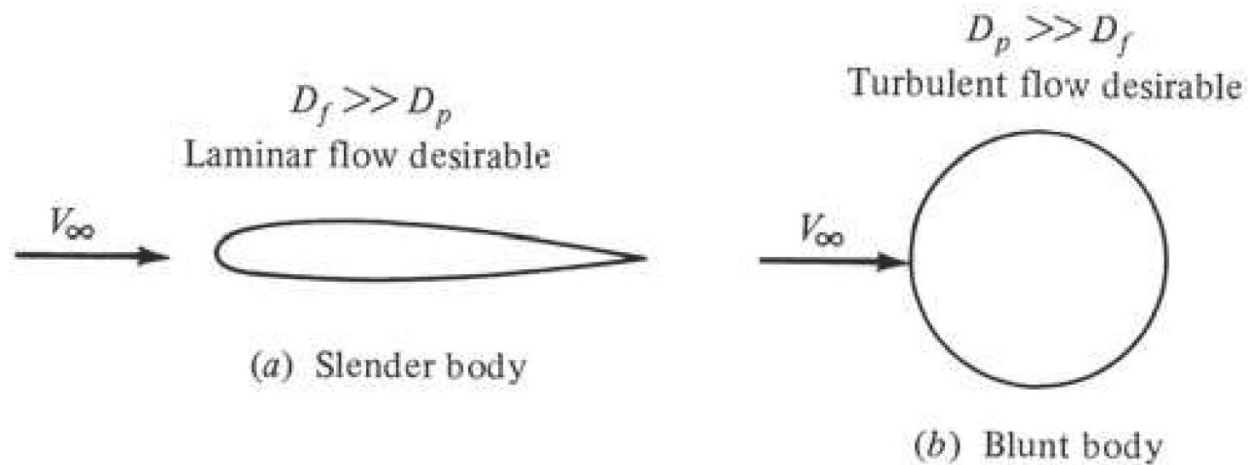
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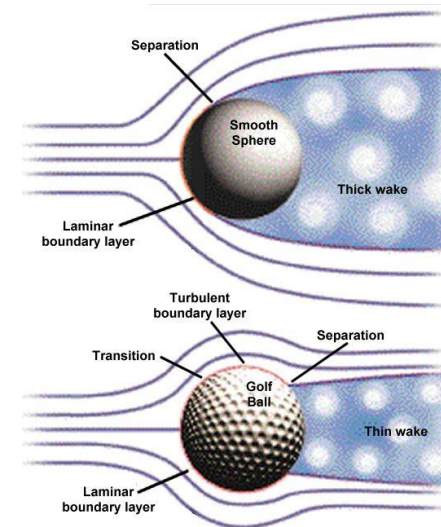
- **Viscosity flows over different bodies**

- Laminar flow is preferable for slender bodies (细长体).
- Turbulent flow is preferable for blunt bodies (钝体)

$$D = D_f + D_p$$



**Why there are dimples on the golf ball?**



**Answer: To delay the flow separation so to reduce the pressure drag**

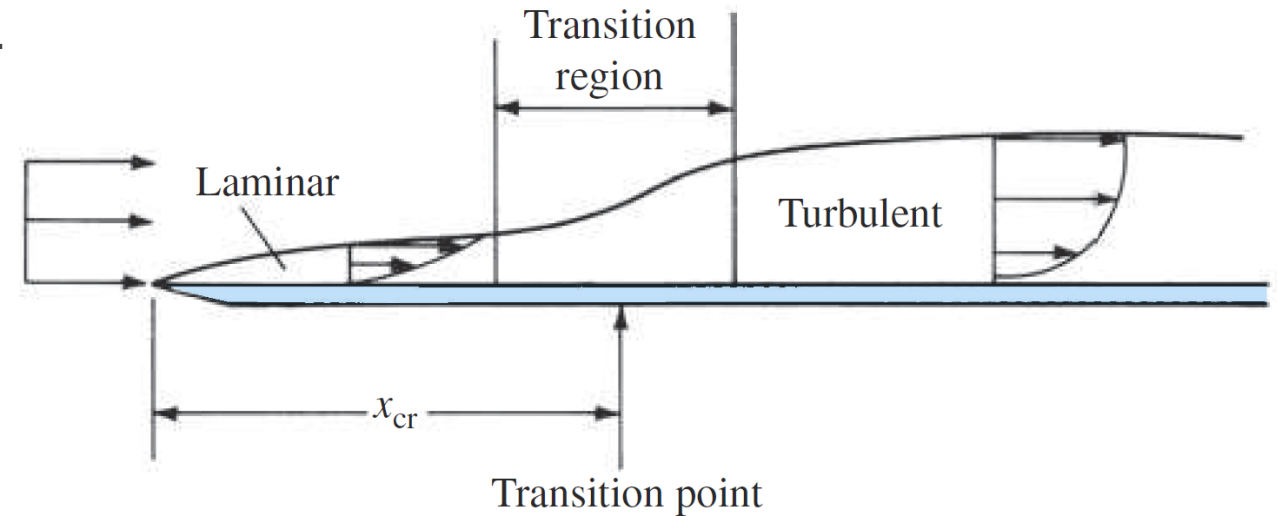
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# Introduction to Viscous Flow



- **Transition from laminar to turbulent flow (how to control?)**

- Increased surface roughness (表面粗糙度).
- Increased turbulence in the freestream.
- Adverse pressure gradients.
- Heating of the fluid by the surface.



**For practical aerodynamic applications,**

$$\text{Re}_{\text{cr}} = \frac{\rho_{\infty} V_{\infty} x_{\text{cr}}}{\mu_{\infty}} \longrightarrow \begin{cases} < 500,000: \text{laminar} \\ > 500,000: \text{turbulent} \end{cases}$$

**Critical Reynolds Number**

**For pipe flow (管流),**

$$\text{Re}_{\text{cr}} = \frac{\rho V D}{\mu} \longrightarrow \begin{cases} < 2,000: \text{laminar} \\ > 5,000: \text{turbulent} \end{cases}$$

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# Introduction

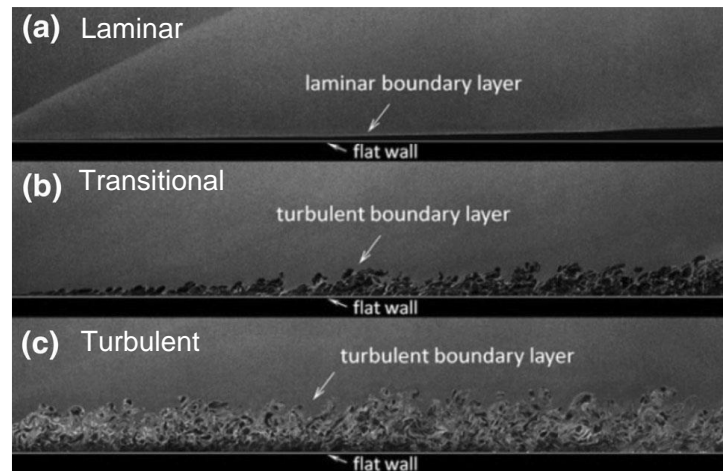


- **Boundary Layer**

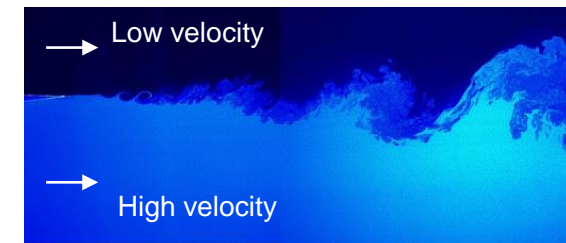
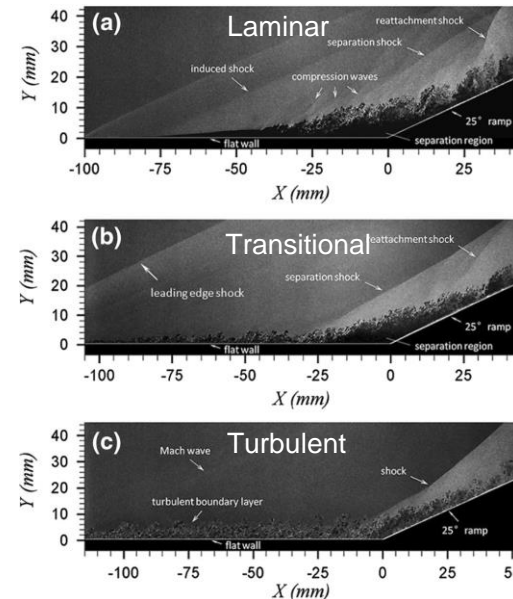
- The thin region of flow **adjacent to a surface**, where the flow is retarded by the influence of friction between a solid surface and the fluid.

- **Shear Layer**

- A layer of flow where a **shear or velocity gradient** exists. Boundary layer is also a form of shear layer, but shear layer is often used in **fluid-fluid interactions**.



Different boundary layers.



An example of shear layer.

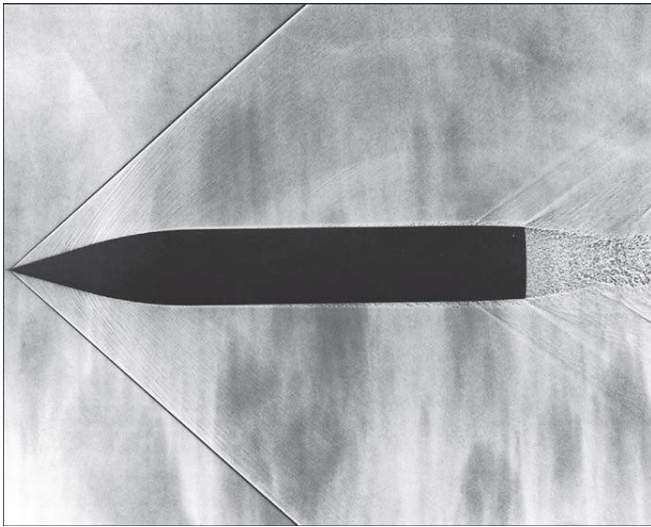
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# Introduction

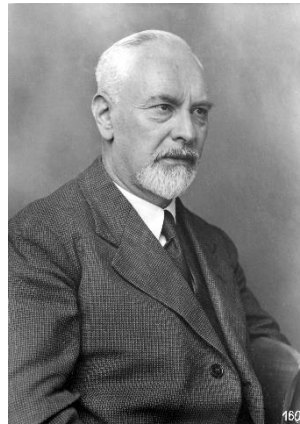


## • Boundary Layer

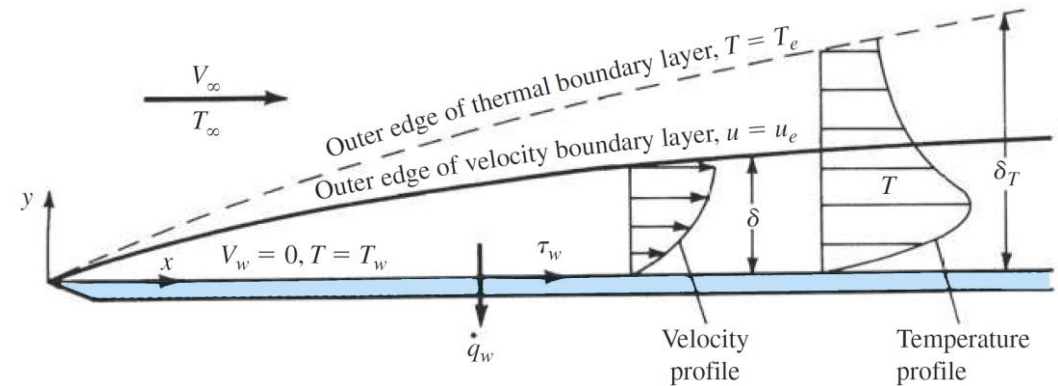
- Using Prandtl's concept of a boundary layer adjacent to an aerodynamic surface, the **Navier-Stokes equations can be reduced to** a more tractable (容易处理的) form called the **boundary-layer equations**.
- Prandtl's boundary-layer concept was an advancement in the science of fluid mechanics of the caliber of a **Nobel prize**, although he never received that honor.



Boundary layer on an aerodynamic body.



Ludwig Prandtl



Boundary layer on a flat plate.

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# Outlines

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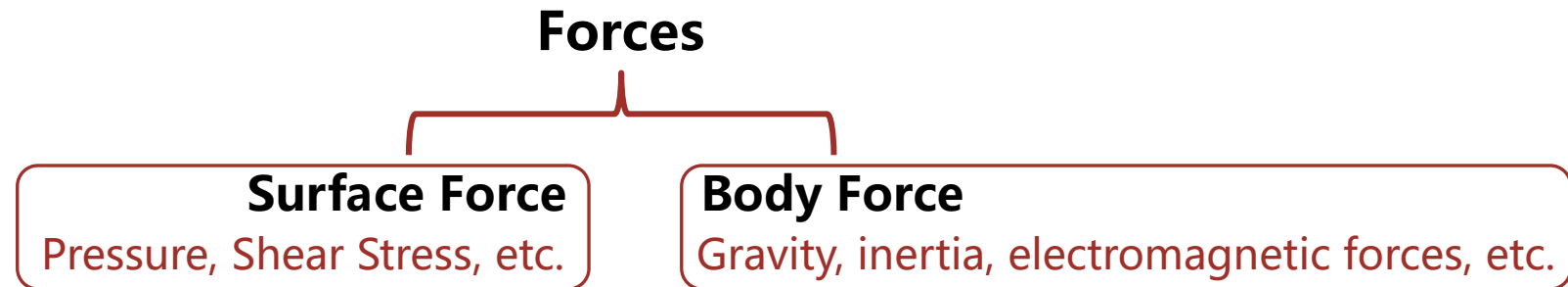
- Approaches for Fluid Mechanics
- Continuum Hypothesis
- Properties of Fluids
  - Viscous Flow
- **Fluid Statics and Motion**
- A Brief Review of Thermodynamics
- Some Aspects of Supersonic Flow: Shock Waves
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# Fluid Statics



## The type of forces

- The forces that could act on fluid elements are be divided into **two categories**



- **Surface forces** are exerted to the surface of an object
  - Brought about by **contact** of fluid with another fluid or a solid body
- **Body forces** are distributed on every portion of the body they act on
  - Depend upon the **mass** of the substance

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# Fluid Statics



## Hydrostatic equation

### ◦ Application: **U-Tube Manometer**

$$p + \rho gh = \text{Const.} \quad (1.54)$$

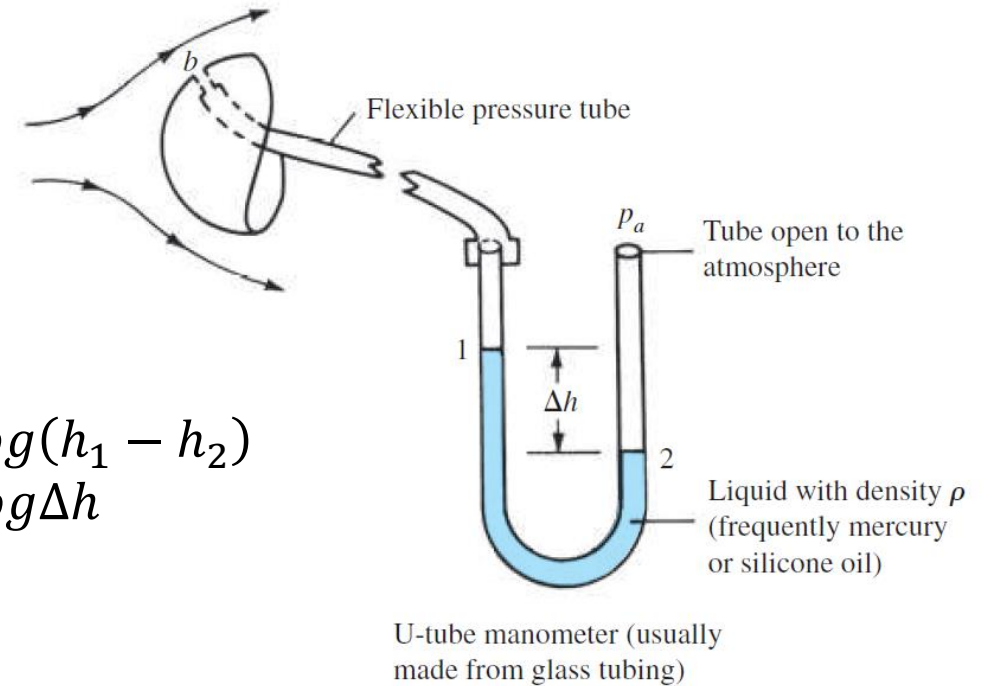
$$\star p_1 + \rho gh_1 = p_2 + \rho gh_2$$

$$\Rightarrow p_1 = p_2 - \rho g(h_1 - h_2)$$

$$\star p_1 \approx p_b; p_2 \approx p_a$$

(because  $\rho_{\text{air}} \ll \rho_{\text{liquid}}$ )

$$\begin{aligned} p_b &= p_a - \rho g(h_1 - h_2) \\ &= p_a - \rho g\Delta h \end{aligned}$$

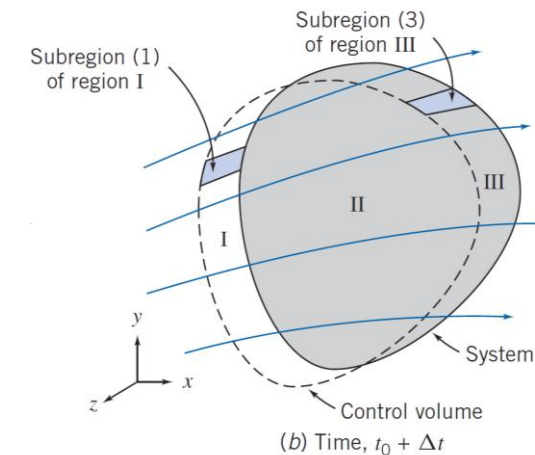
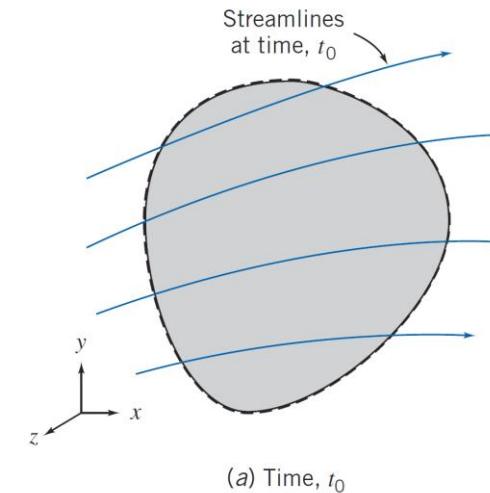


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# Fluid Motion



- Conservation of Mass
  - Uniform Flow Through a Control Volume
  - Non-uniform Flow Through a Control Volume
- Linear Momentum
  - Forces Acting on a Flow Through a Control Volume
  - Differential Analysis and the Bernoulli Equation
  - Constant Velocity Through a Control Volume
- Principle of Angular Momentum
  - Fixed Control Volume
- Moving Control Volume
  - Constant Velocity
  - Acceleration



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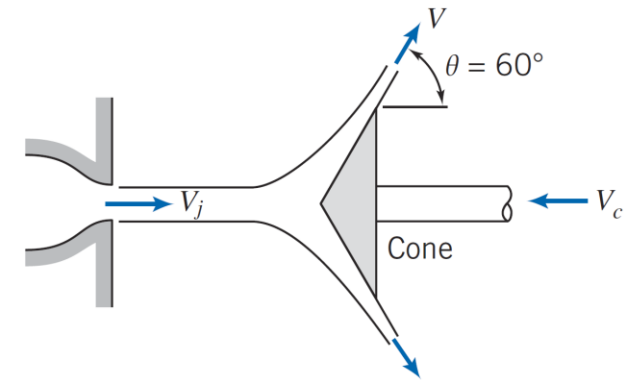
# Fluid Motion



## Question 1

Water moving at 30 m/s from a 100 mm jet is deflected by a cone moving at 14 m/s. Find the thickness of the jet at a distance of 230 mm from the center of the cone and the force to move the cone.

- Hint : mass conservation



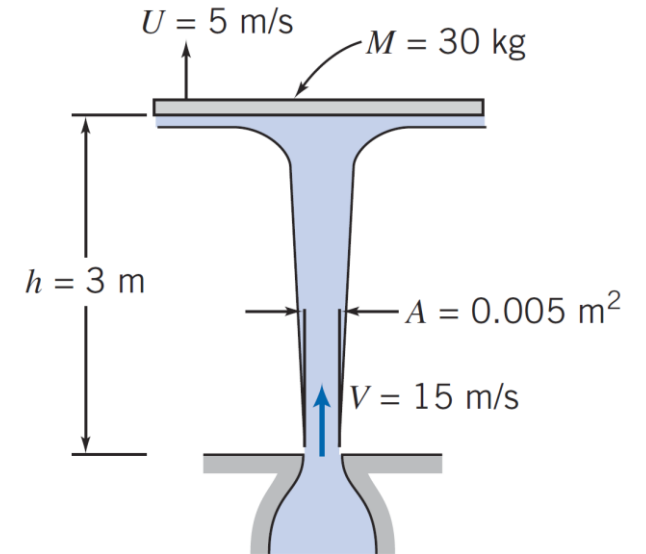
# Fluid Motion



## Question 2

Water from a jet hits a horizontal disk that will be moving upward at 5 m/s when it is 3 m above the nozzle exit. What is the vertical acceleration of the disk at this point?

- Hint 1: proper selection of control volume
- Hint 2: Bernoulli's Equation
- Hint 3: mass conservation
- Newton's second law and momentum equation



# Outlines

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- Approaches for Fluid Mechanics
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# A Brief Review of Thermodynamics



- Perfect gas

- A gas which the intermolecular forces are neglected
- Equation of state

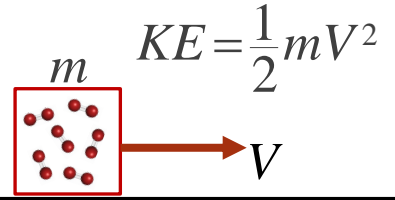
$$p = \rho RT, R = 287 \text{ J/(kg} \cdot \text{K)}$$

- Internal energy ( $e$ )

- Summation of the energies of all molecules (in random motions) in a system.

- Kinetic energy 动能

- Macro system motions



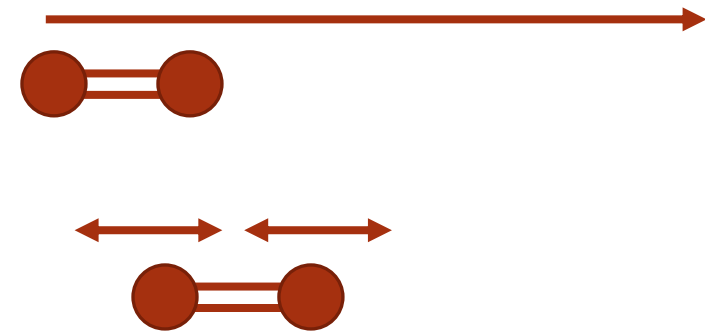
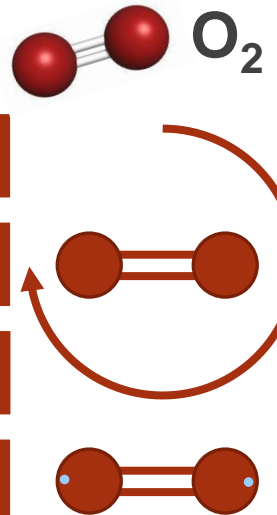
Internal energy

Translational 平移

Rotational 旋转

Vibrational 振动

Electronical 电子运动



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# A Brief Review of Thermodynamics



- Enthalpy
  - The sum of the system's internal energy and the product of its pressure and volume.

$$h = e + pv$$

- For a perfect gas, both  $e$  and  $h$  are functions of temperature only

$$e = e(T) = c_v T \quad h = h(T) = c_p T$$

- **Calorically perfect gas**: a perfect gas where  $c_v$  and  $c_p$  are constant.
- **Mayer's relation** for ideal gas:  $c_p - c_v = R$
- **Heat capacity ratio**:  $\gamma \equiv c_p / c_v$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

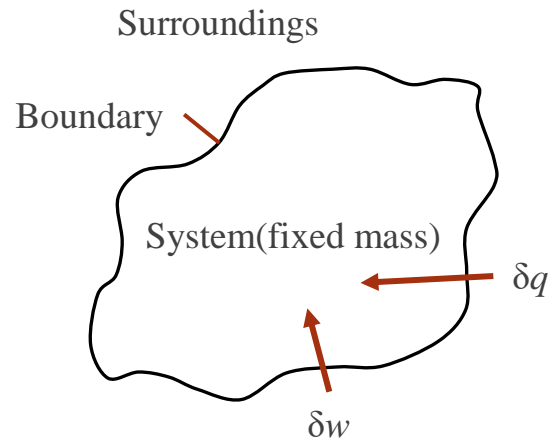
$$c_v = \frac{R}{\gamma - 1}$$

- For air at standard conditions (1 atm, 25 °C),  $\gamma = 1.4$ .

# A Brief Review of Thermodynamics



- **First law of thermodynamics**



$\delta q$  : An incremental amount of heat added to the system across the boundary

$\delta w$  : The work done on the system by the surroundings

$de$  : The change in energy of the system

$$de = \delta q + \delta w = \delta q - p dv$$

- **Adiabatic process**

- No heat is added to or taken away from the system.

- **Reversible process**

- No dissipative phenomena (effects of viscosity, thermal conductivity and mass diffusion) occur.

- **Isentropic process**

- Both adiabatic and reversible.

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# A Brief Review of Thermodynamics



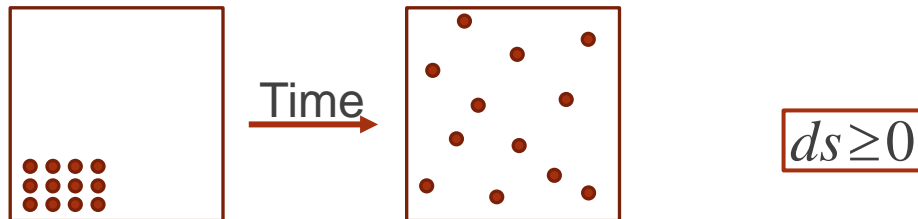
## • Entropy

- The degree of randomness
- The change in entropy (ideal)  $ds = \frac{\delta q_{\text{rev}}}{T}$
- $\delta q_{\text{rev}}$  requires the difference in temperature between the heat source and the system has to approach zero, i.e., the transfer would have to occur infinitely slowly (quasi-statically).

$$ds = \frac{\delta q}{T} + ds_{\text{irrev}}, \quad ds_{\text{irrev}} \geq 0$$

## • Second law of thermodynamics

- The total entropy of an isolated system can never decrease over time, and is constant if and only if all processes are reversible.



Disorder is more probable than order.

## • Third law of thermodynamics

- The entropy of a system approaches a constant value as its temperature approaches absolute zero.

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# A Brief Review of Thermodynamics



- **Isentropic relations**

- Isentropic process is a process that is both **adiabatic and reversible**.

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 0 \quad s_2 - s_1 = c_v \ln \frac{T_2}{T_1} - R \ln \frac{v_2}{v_1} = 0$$

$$\frac{p_2}{p_1} = \left( \frac{\rho_2}{\rho_1} \right)^\gamma = \left( \frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)}$$

- A large amount of practical compressible flow problems can be considered as isentropic.
  - The flow outside the boundary layer can be assumed to be isentropic.
  - Within the boundary layer, entropy increases due to strong **dissipative mechanisms of viscosity, thermal conduction and diffusion**.
- **Third law of thermodynamics**
    - The entropy of a system approaches a constant value as its temperature approaches absolute zero.

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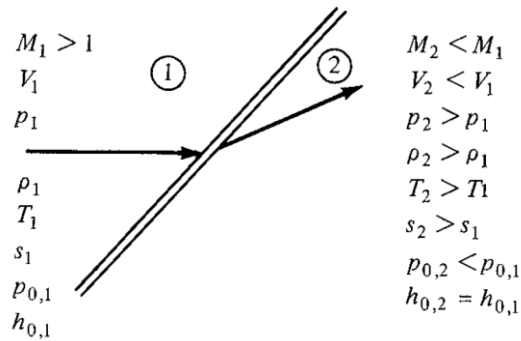
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# Some Aspects of Supersonic Flow: Shock Waves

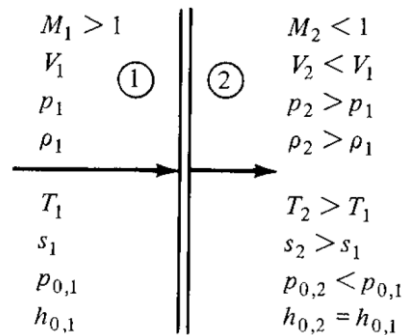
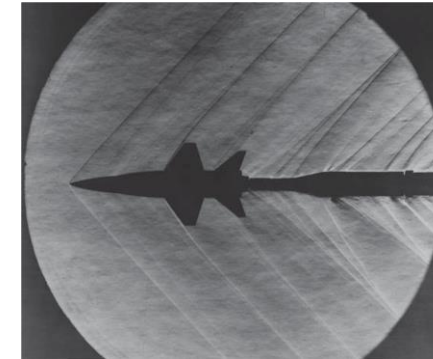
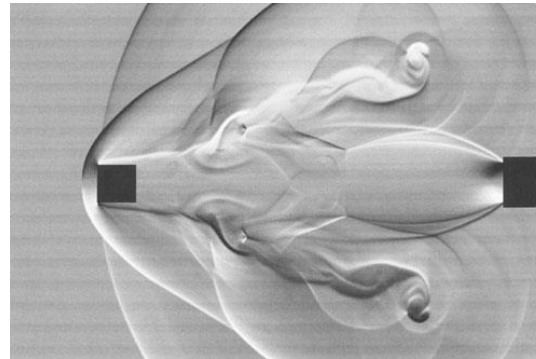


- Shock Wave

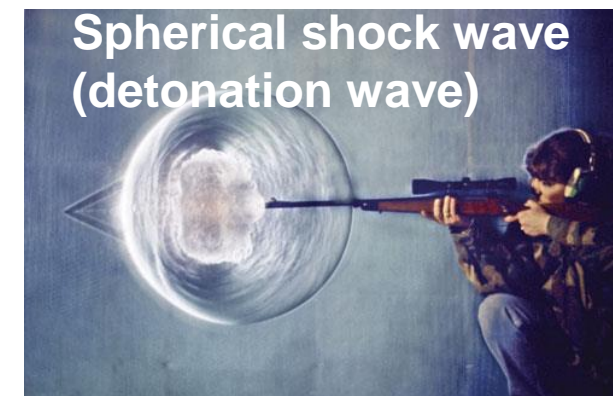
- A type of propagating disturbance that moves faster than the local speed of sound in the medium.



Oblique shock wave



Normal shock wave

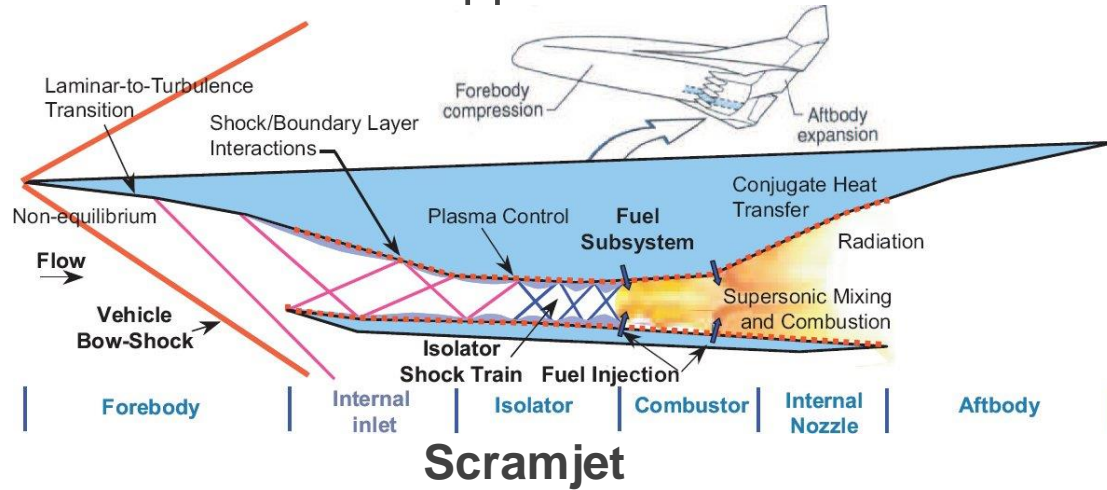


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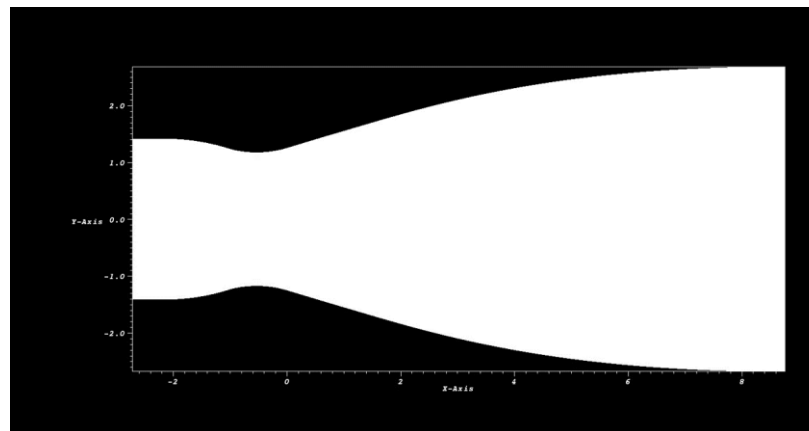
# Some Aspects of Supersonic Flow: Shock Waves



- Shock waves in applications



Shock diamond



Startup of a rocket engine

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# Outlines

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# Compressible Flow and Hypersonic Flow

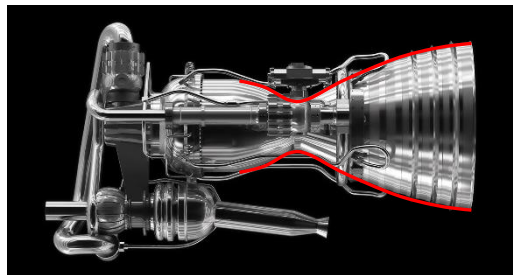


## • Compressible Flow Through Ducts (槽道)

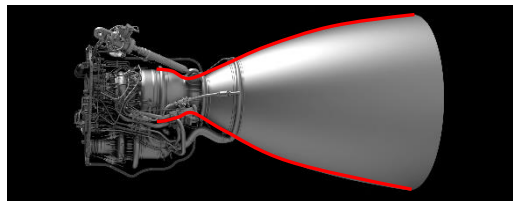
- Nozzle (喷管): A duct to increase the flow velocity in the expense of pressure or internal energy.
- Diffuser (扩压器): A duct to decrease the flow velocity.
- Wind tunnel (风洞): combination of nozzles and diffusers to provide uniform supersonic flow for testing.
- **Quasi-one-dimensional flow (准一维流动)**: one with varied cross-sectional area in which all variables vary primarily along one direction.



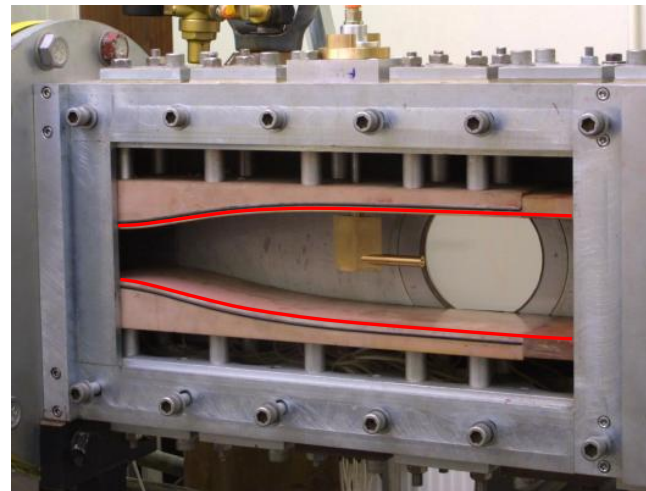
SpaceX's Merlin Engines in Falcon Heavy



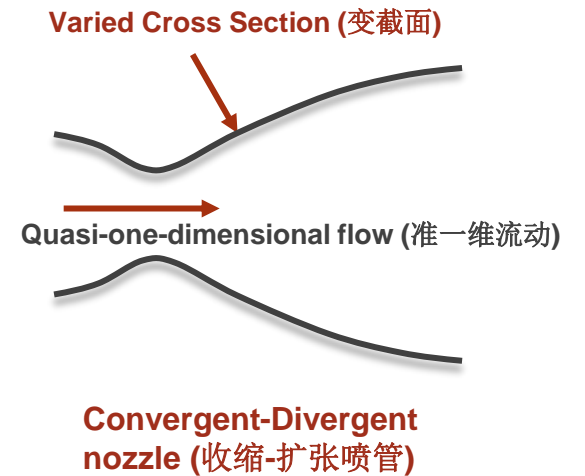
SpaceX's Merlin Engine



SpaceX's Raptor Engine



Supersonic Wind Tunnel of Imperial College London



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# Compressible Flow and Hypersonic Flow



## • Hypersonic Flow (高超音速流动)

- In aerodynamics, a hypersonic speed is one that greatly exceeds the speed of sound, often stated as starting at speeds of **Mach 5 and above**.
- Classified as **hypersonic aerodynamics**.
- Atmospheric reentry (to Earth, 地球大气再入), atmospheric entry (space exploration, 太空探索的大气进入), military applications
- New phenomena arise when Mach number is sufficiently high.



Supersonic Vehicle: Chengdu J20



Hypersonic Vehicles: DF-17 missiles and hypersonic transportation concepts

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# Compressible Flow and Hypersonic Flow

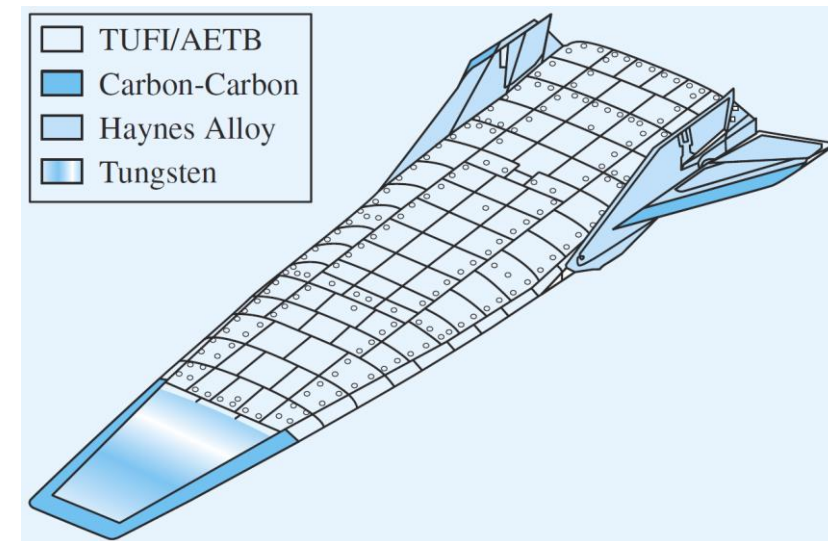


## • Hypersonic Vehicle: X-43 (NASA)

- An experimental unmanned (无人操纵的) hypersonic aircraft with multiple planned scale variations meant to test various aspects of hypersonic flight.
- It was part of the X-plane series and specifically of NASA's Hyper-X program.
- Rocket engine boosted and scramjet engine propelled.
- The X-43 is the fastest aircraft on record at approximately **Mach 9.6**.



Supersonic Vehicle: NASA X-43



氧化铝增强热障 1500°C  
碳碳复合材料 2600°C  
海恩斯合金 1200°C  
钨 1650°C

Thermal protection system of X-43

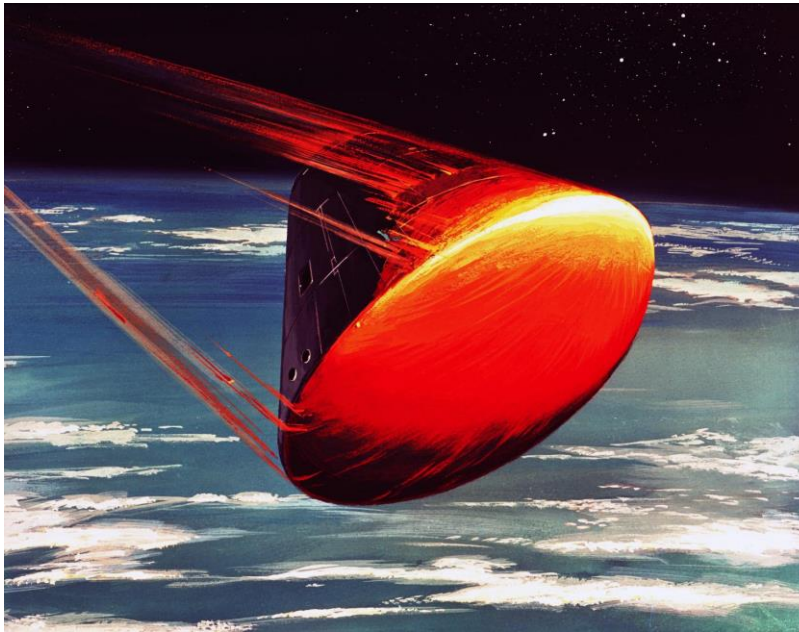
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# Compressible Flow and Hypersonic Flow



- **Hypersonic Vehicle: Atmospheric Reentry of Apollo 11**

- On July 24, 1969, Apollo 11 splashed down in Earth's oceans, successfully completing the return trip from moon with human beings (1969 crewed lunar mission, 载人登月任务).
- Reentering the atmosphere at **Mach 36**.



The reentry of Apollo 11 (artwork)

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# Compressible Flow and Hypersonic Flow



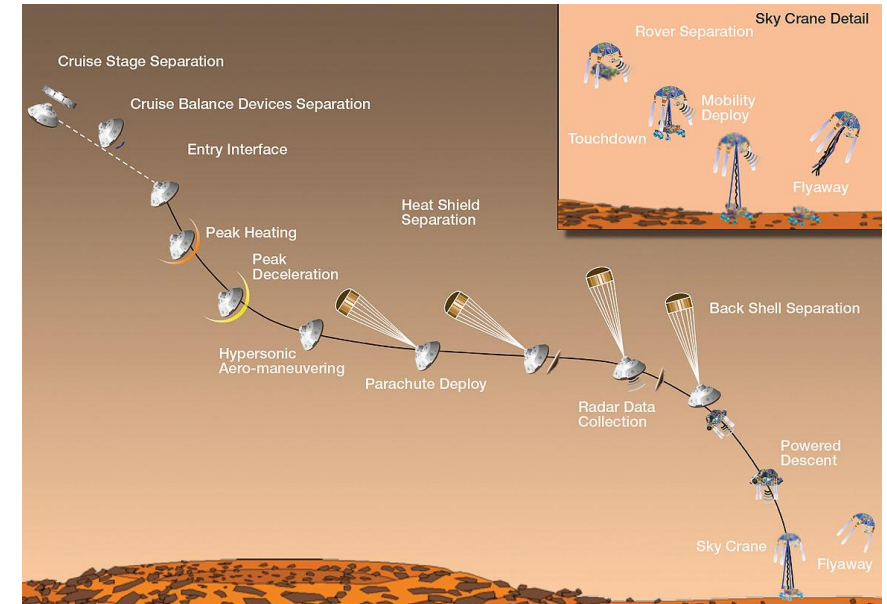
- **Hypersonic Vehicle: Atmospheric Entry of Space Exploration Probe/Rover (太空探索探测器/探测车)**
  - Curiosity Rover landed on Mars on August 2012.
  - Galileo Jupiter Atmospheric Probe entered Jupiter on December 1995.



The entry of Mars  $\text{CO}_2\text{-N}_2$  atmosphere (artwork)



The entry of Jupiter  $\text{H}_2\text{-Ne-CH}_4\text{-NH}_3$  atmosphere (artwork)



The entry of Curiosity Rover into Mars

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# Question 3

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Which would you prefer for our class?

1. More lecture, homework is assigned after class;
2. Less lecture, in-class homework.

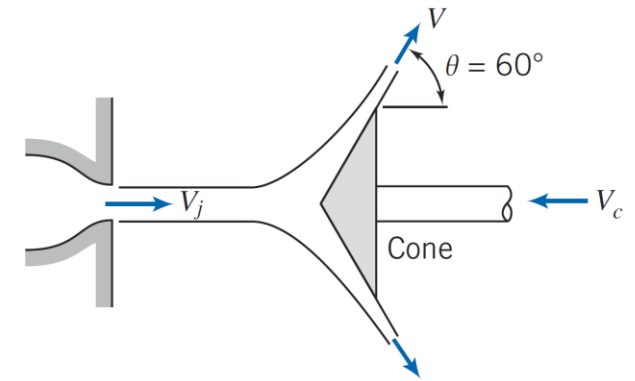
# Fluid Motion



## Question 1

Water moving at 30 m/s from a 100 mm jet is deflected by a cone moving at 14 m/s. Find the thickness of the jet at a distance of 230 mm from the center of the cone and the force to move the cone.

$$\rho V_1 A_1 = \rho V_2 A_2$$
$$\rho(V_j + V_c) \frac{\pi D_j^2}{4} = \rho(V_j + V_c) (2\pi R t)$$
$$t = \frac{D_j^2}{8R} = \frac{100}{8 \cdot 230} = 5.435 \text{ mm}$$



# Fluid Motion



## Question 2

Water from a jet hits a horizontal disk that will be moving upward at 5 m/s when it is 3 m above the nozzle exit. What is the vertical acceleration of the disk at this point?

$$\frac{p_0}{\rho} + \frac{V_0^2}{2} + gz_0 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \quad V_1 = 12.9 \text{ m/s}$$

momentum equation:  $\dot{m}_2 U - \dot{m}_1 V_1 = F = -Mg - Ma$

mass conservation:  $\dot{m}_1 = \dot{m}_2 = \rho(V_1 - U)A_1$ ,  $\rho V_1 A_1 = \rho V A$

$$a = \frac{\rho(V_1 - U)A_1(V_1 - U) - Mg}{M} = 2.29 \text{ m/s}^2$$

