MEMS1045 Automatic control

Lecture 4

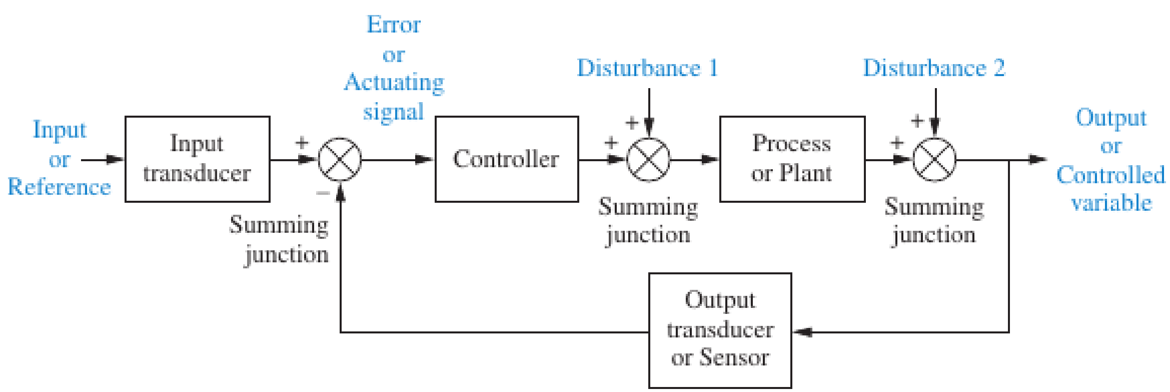
Block diagrams

Objectives

- Represent the equations of motion in block diagram
- Reduce a block diagram to the transfer function

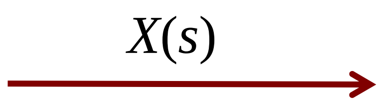
Block diagrams

- A complicated system can consist of many components (or subsystems)
- Each component can be represented by a transfer function with its input and output
- We can treat each component as a block and connect them to form the system
- The block diagram, which shows the interconnection of blocks, can then be used to graphically represent the mathematical relationship between variables in a system

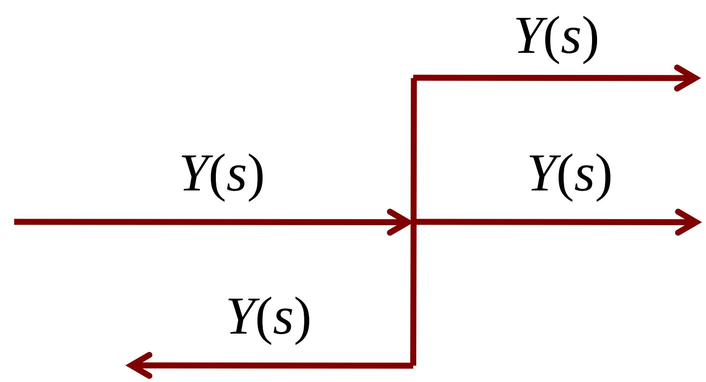


Block diagram elements

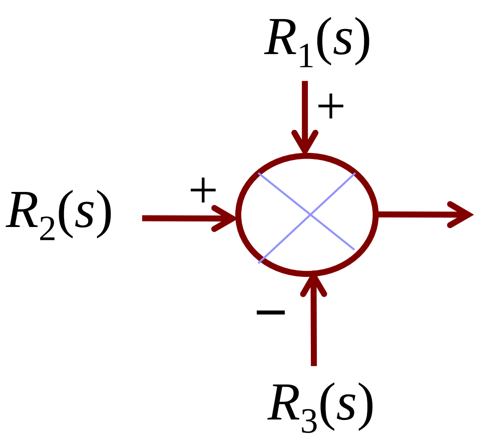
Signal



Pickoff point



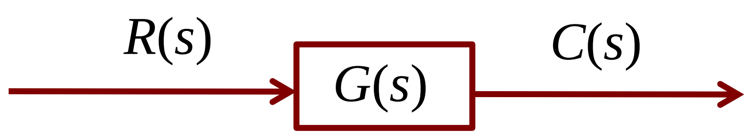
Summing junction



$$Y(s) = R_1(s) + R_2(s) - R_3(s)$$

Block diagram elements

System or component



$$\frac{C(s)}{R(s)} = G(s)$$

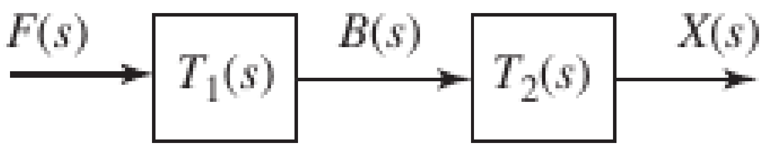
- When $G(s) = K$= constant
  It is called a "Gain"
- When $G(s) = 1/s$
  It is called an integrator

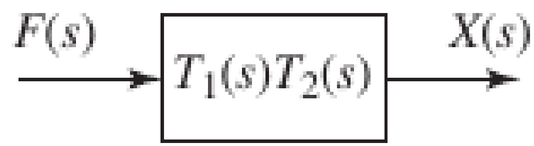
Block diagram simplification

Blocks in series:

$B(s) = F(s) \times T_1(s)$

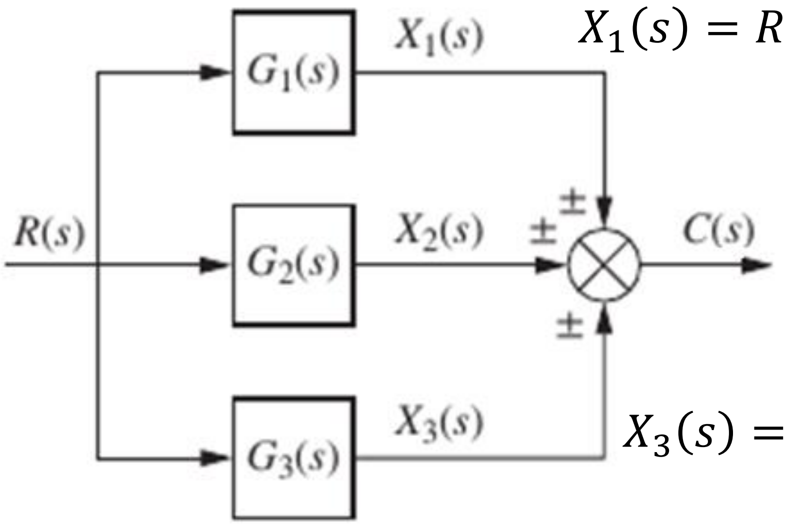
$X(s) = B(s) \times T_2(s) = F(s) \times T_1(s) \times T_2(s)$



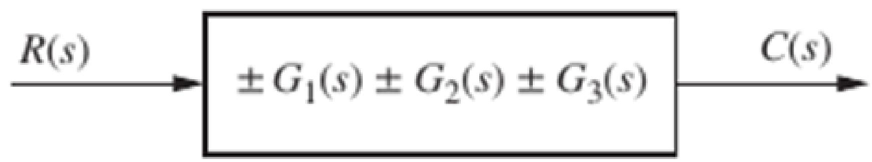


Parallel form:

$X_1(s) = R(s) \times G_1(s)$



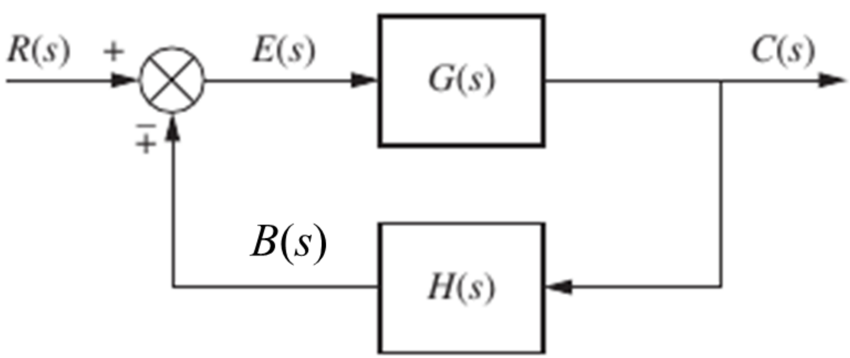
$C(s) = X_1(s) \pm X_2(s) \pm X_{13}(s)$

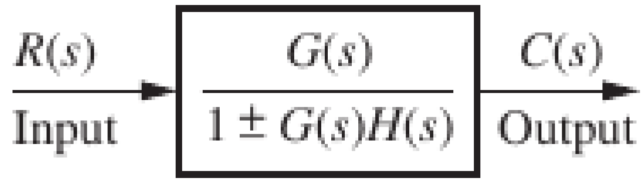


$X_3(s) = R(s) \times G_3(s)$

Block diagram simplification

Blocks in feedback loop (basis of feedback control):





Closed-loop transfer function

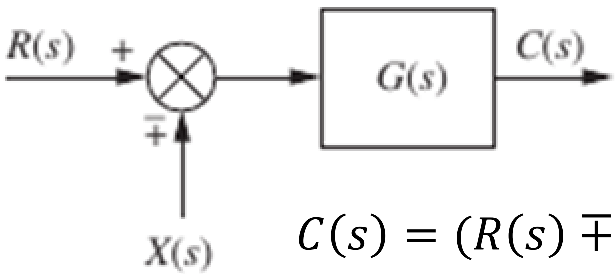
Terminology:

Open-loop transfer function: $\frac{B(s)}{E(s)} = G(s)H(s)$

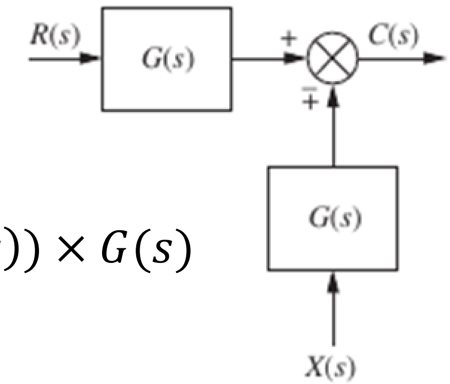
Feedforward transfer function: $\frac{C(s)}{E(s)} = G(s)$

Block diagram simplification

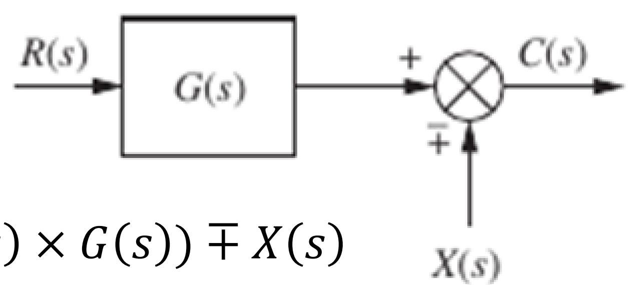
Rearranging blocks after summer to before summer:



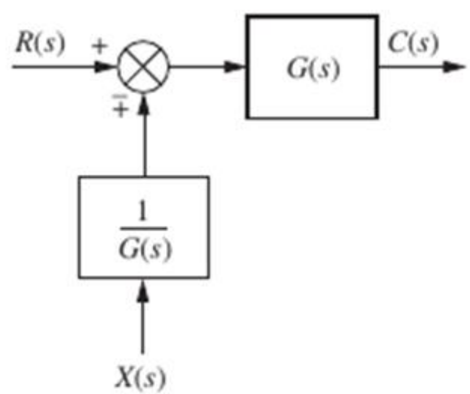
$$C(s) = (R(s) \mp X(s)) \times G(s)$$



Rearranging blocks before summer to after summer:

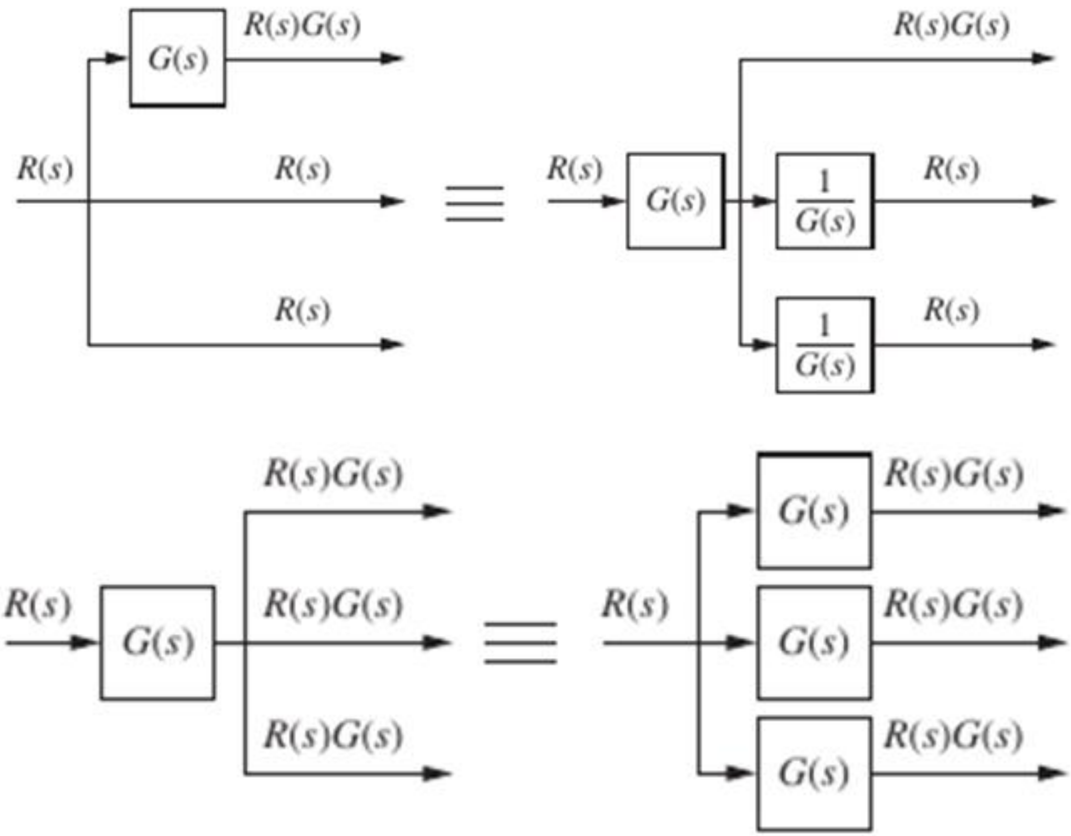


$$C(s) = (R(s) \times G(s)) \mp X(s)$$

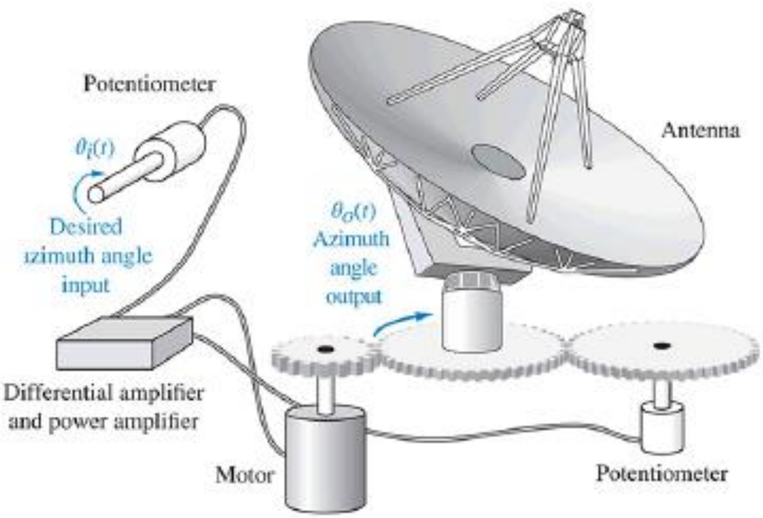


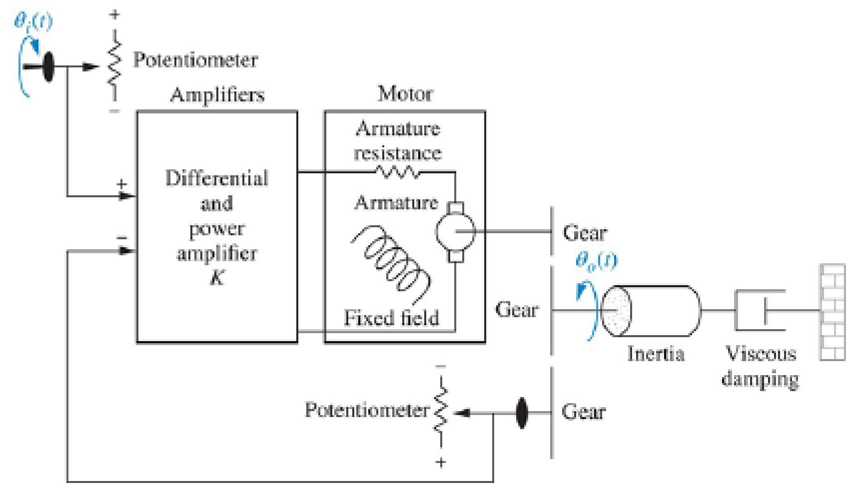
Block diagram simplification

Rearranging blocks around pickoff points:

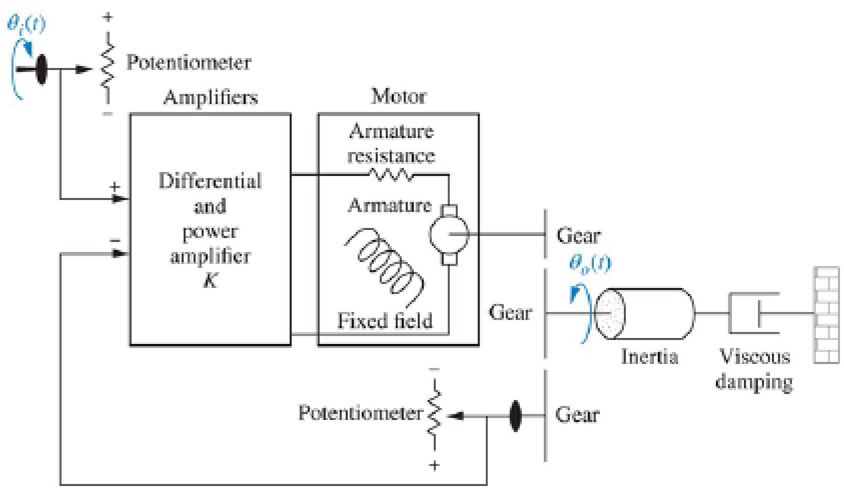


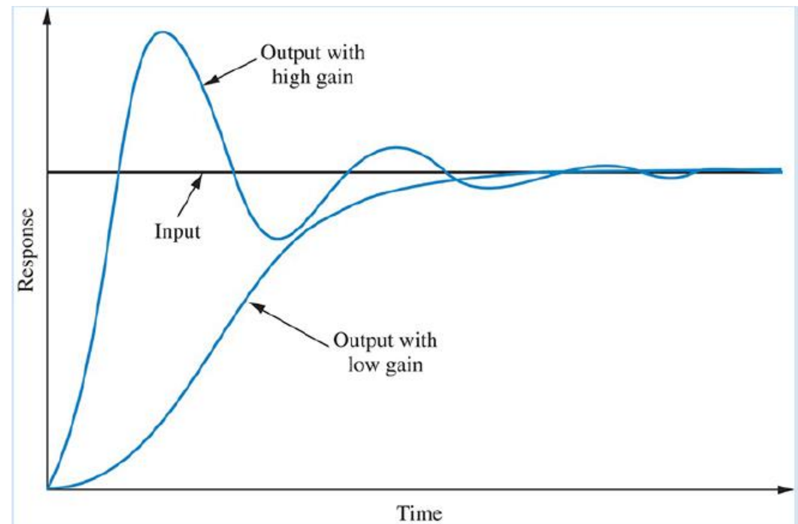
Putting it together

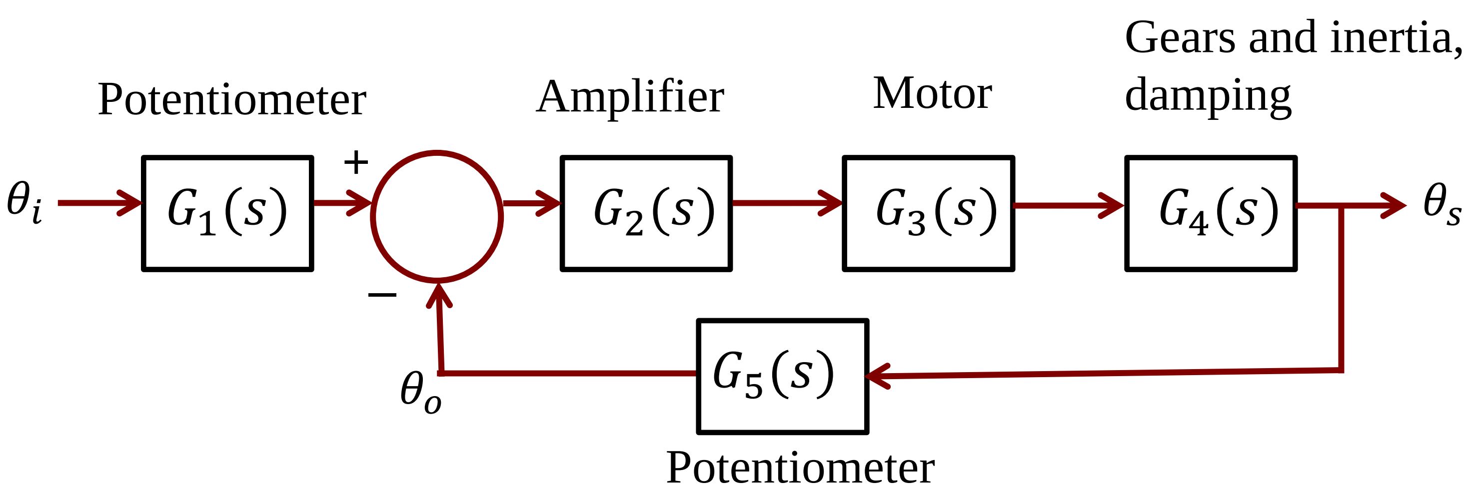




Putting it together

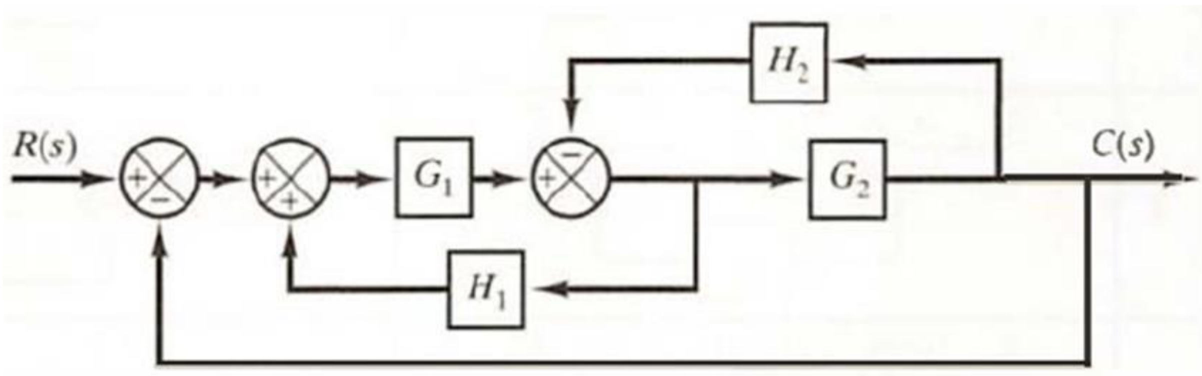


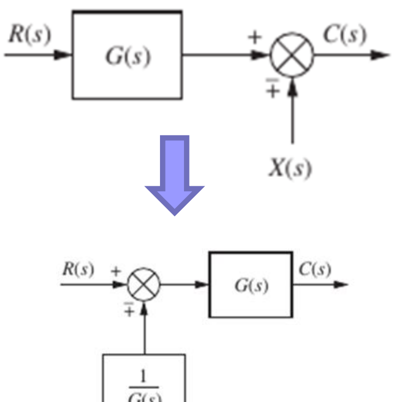


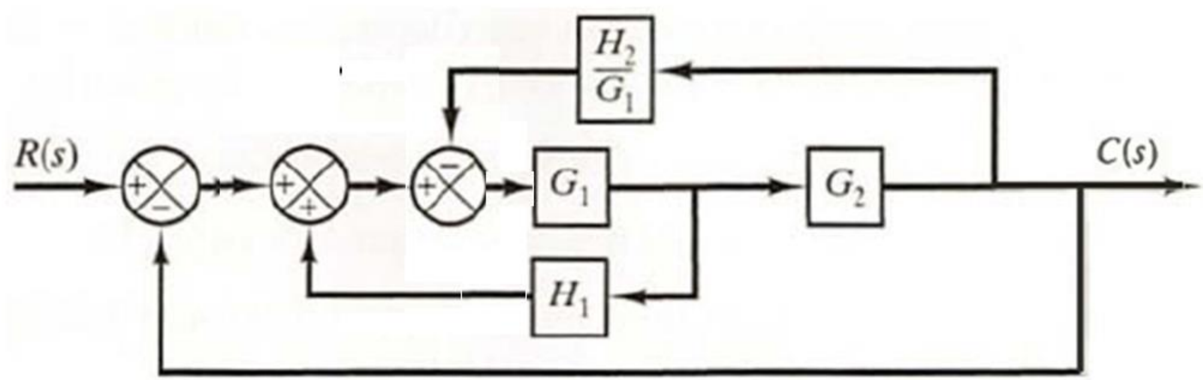


Example 1

Reduce the block diagram shown to a single transfer function

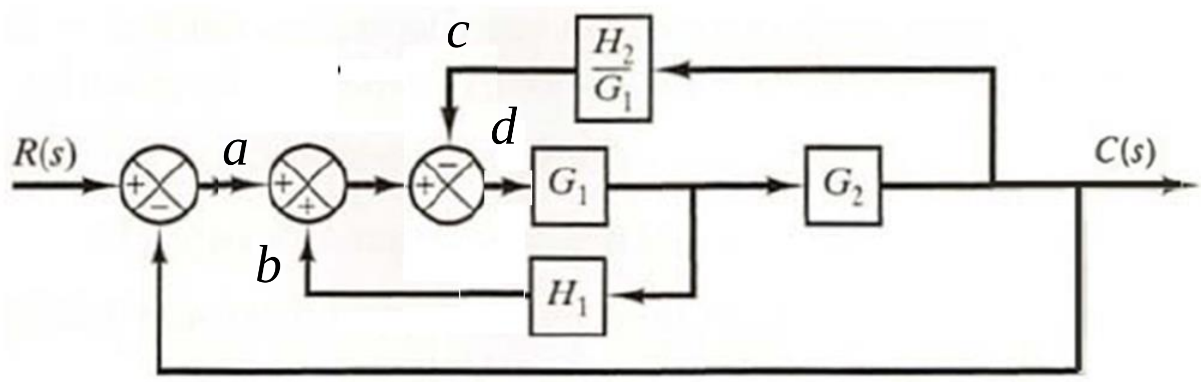






Example 1

$d = a + b - c$



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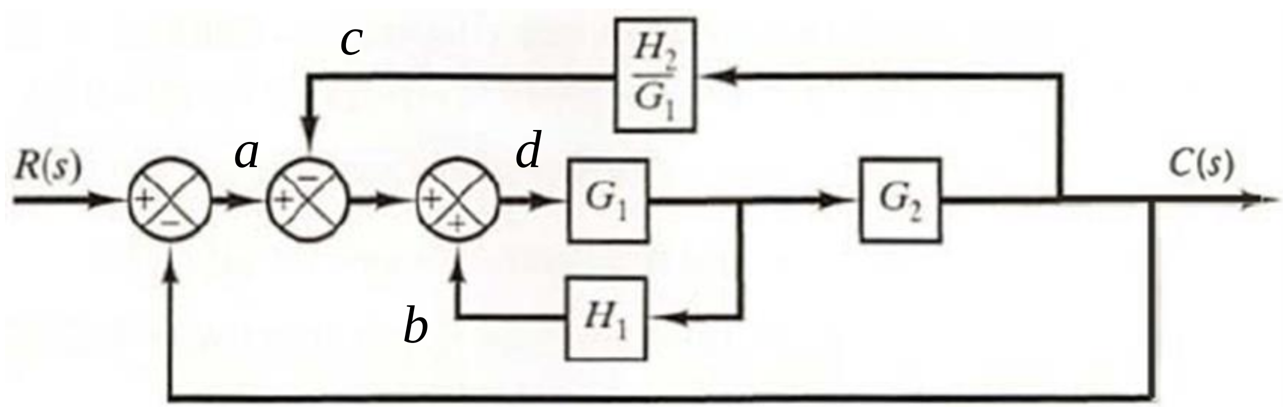
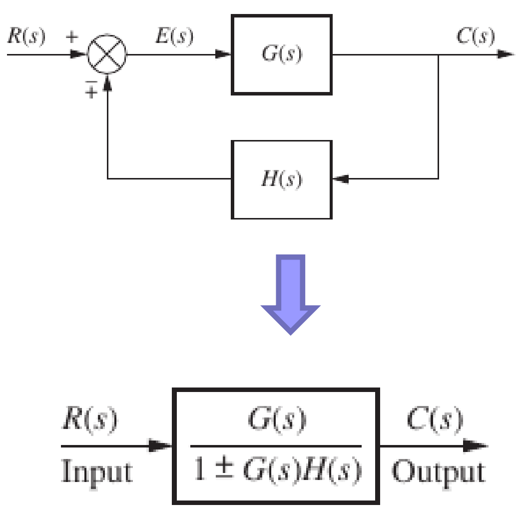
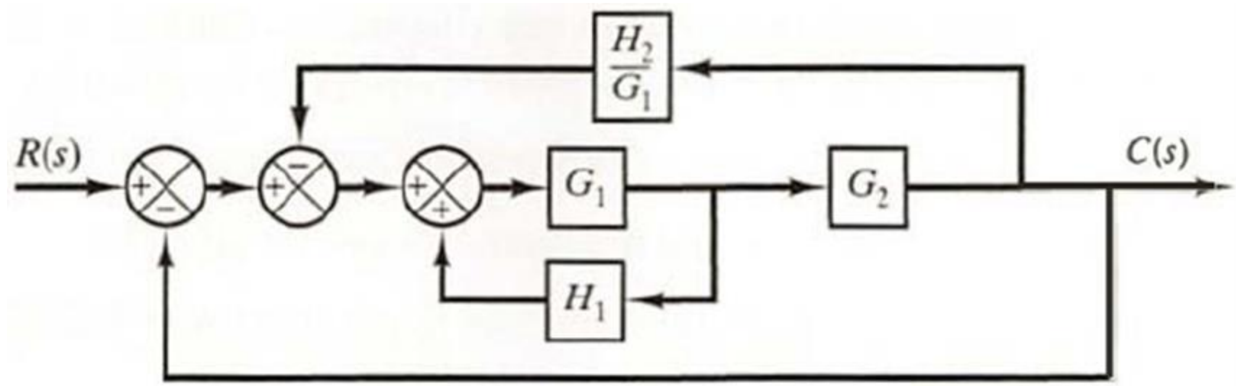
$d = a - c + b$



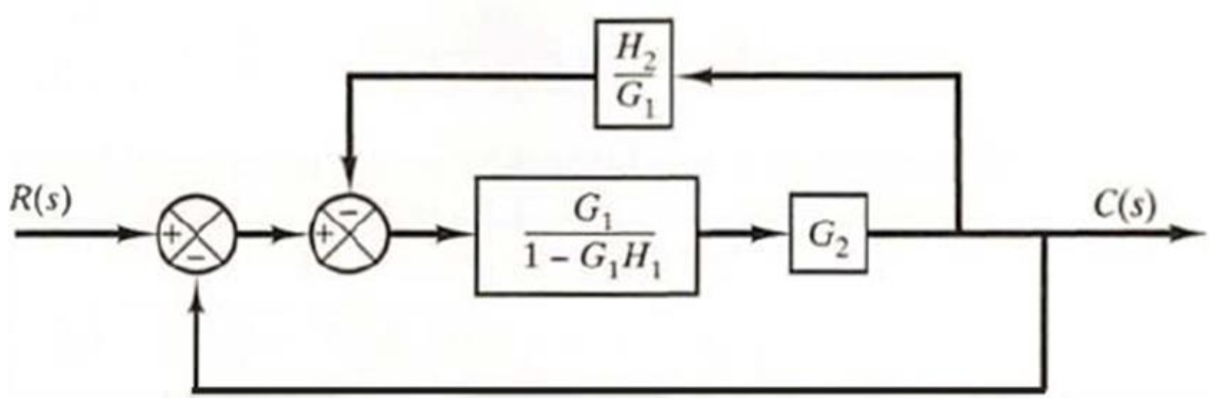
Example 1



R(s)
+
E(s)
G(s)
C(s)
∓
H(s)
R(s)
Input
G(s) / 1 ± G(s)H(s)
C(s)
Output

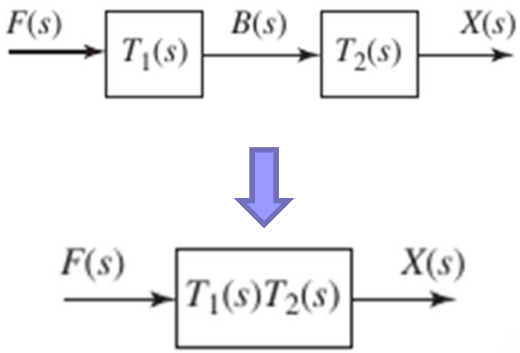


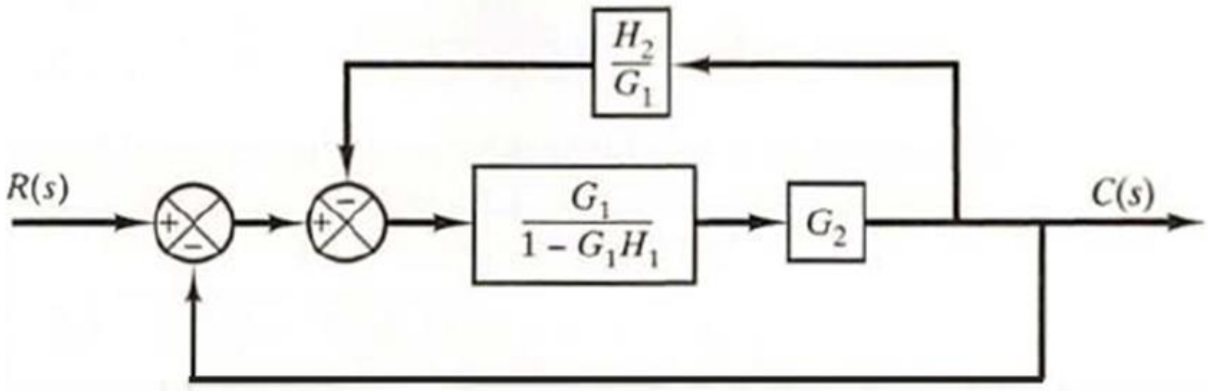
H2 / G1
R(s)
G1
G2
C(s)
H1

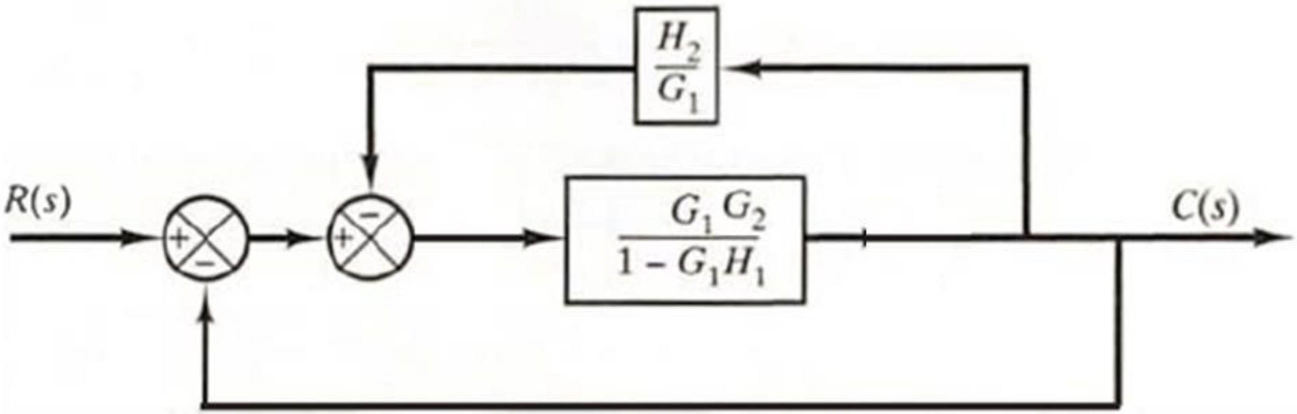


H2 / G1
R(s)
G1 / 1 − G1H1
G2
C(s)

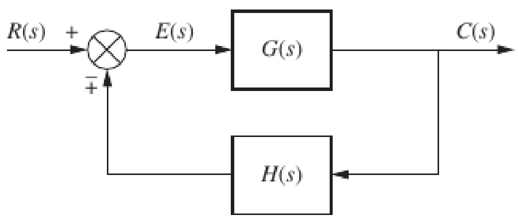
Example 1

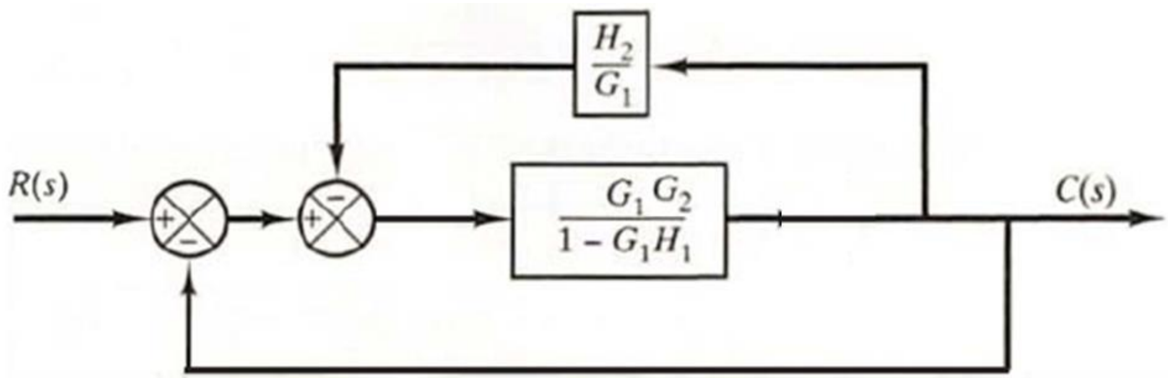

F(s)
T1(s)
B(s)
T2(s)
X(s)
F(s)
T1(s)T2(s)
X(s)

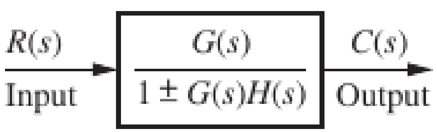

H2/G1
R(s)
+
−
+
−
G1/(1 − G1H1)
G2
C(s)


H2/G1
R(s)
+
−
+
−
G1G2/(1 − G1H1)
C(s)

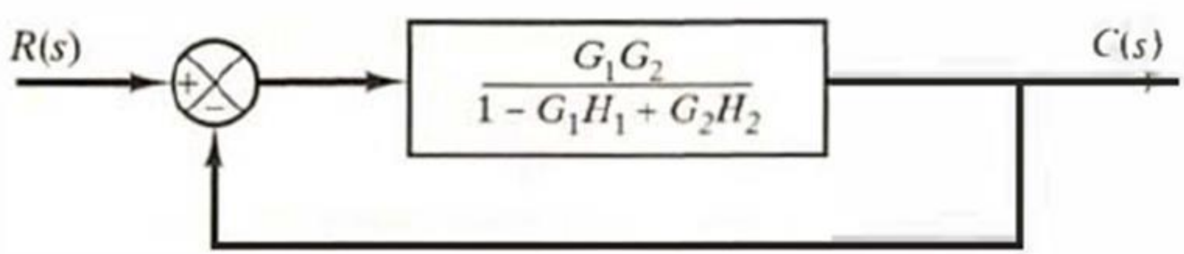
Example 1



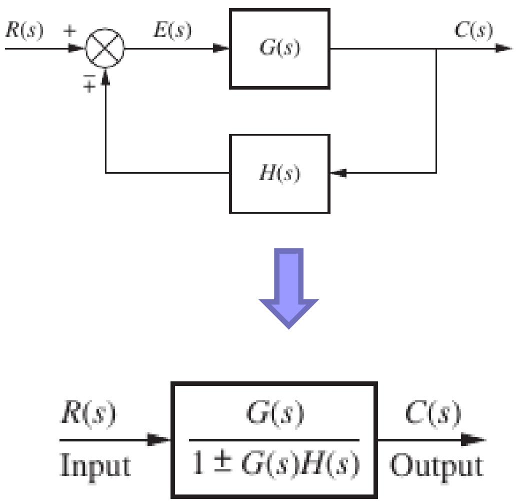


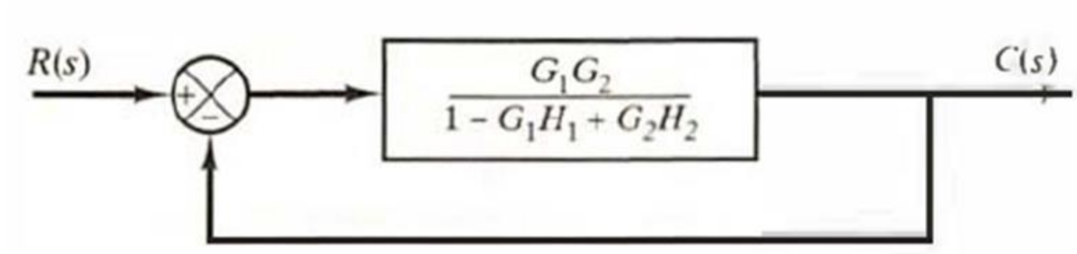


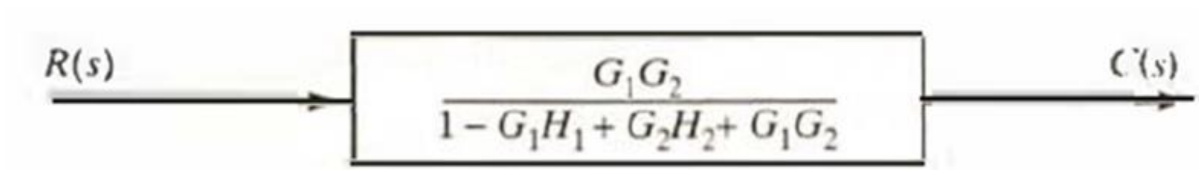
$$\frac{\dfrac{G_1G_2}{1-G_1H_1}}{1+\dfrac{G_1G_2}{1-G_1H_1}\left(\dfrac{H_2}{G_1}\right)}$$



Example 1



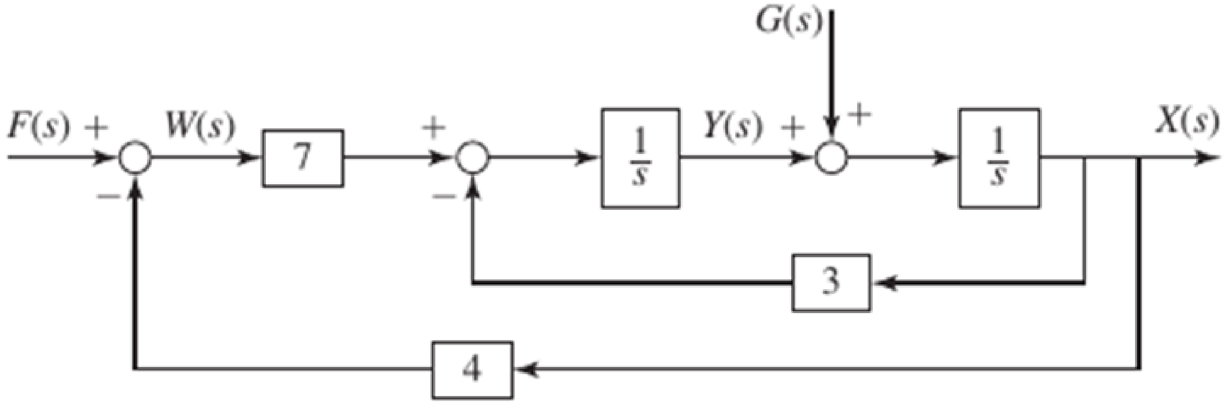


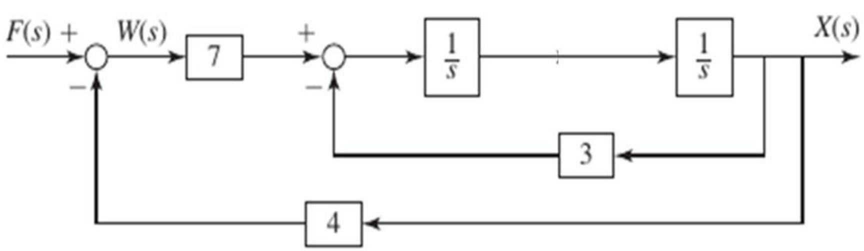


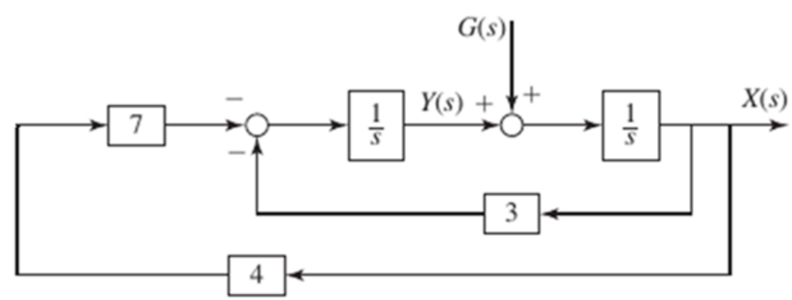
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2}{1-G_1H_1+G_2H_2}}{1+\dfrac{G_1G_2}{1-G_1H_1+G_2H_2}(1)}$$

Example 2

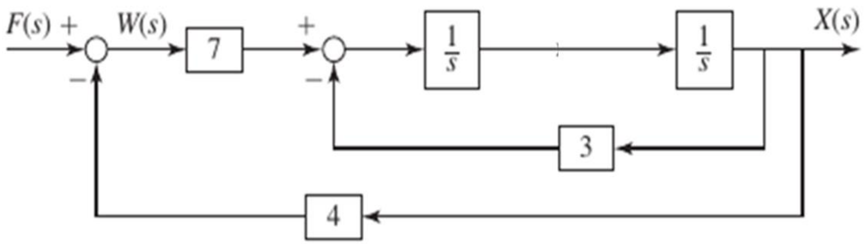
Reduce the block diagram shown to two transfer functions. Determine the state space representation for the system

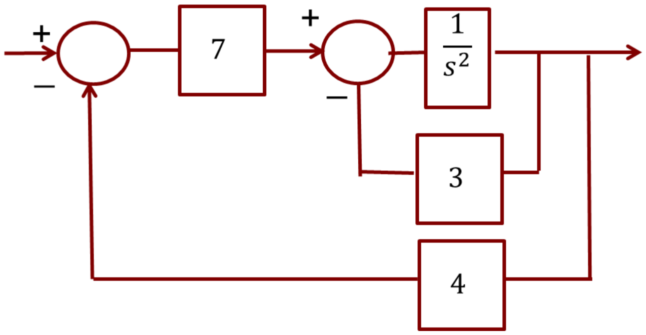


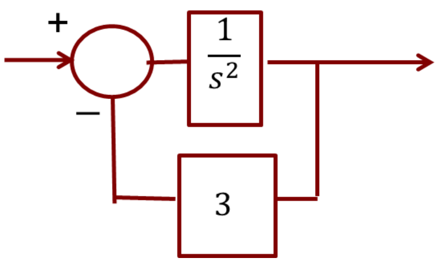


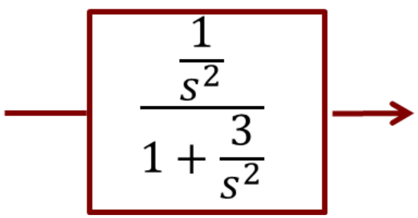


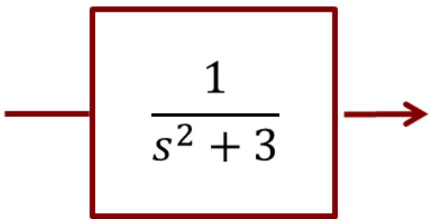
Example 2a

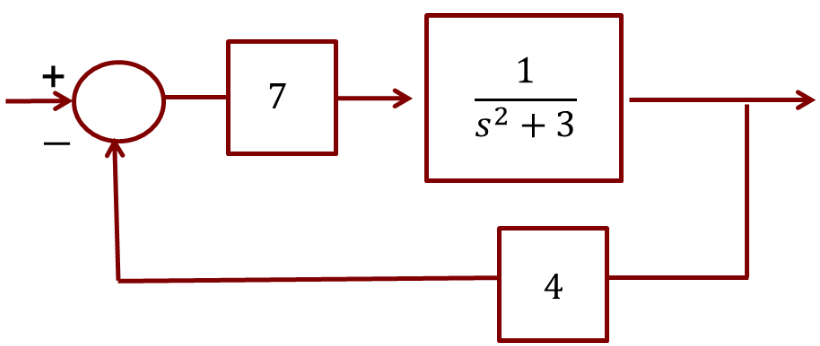






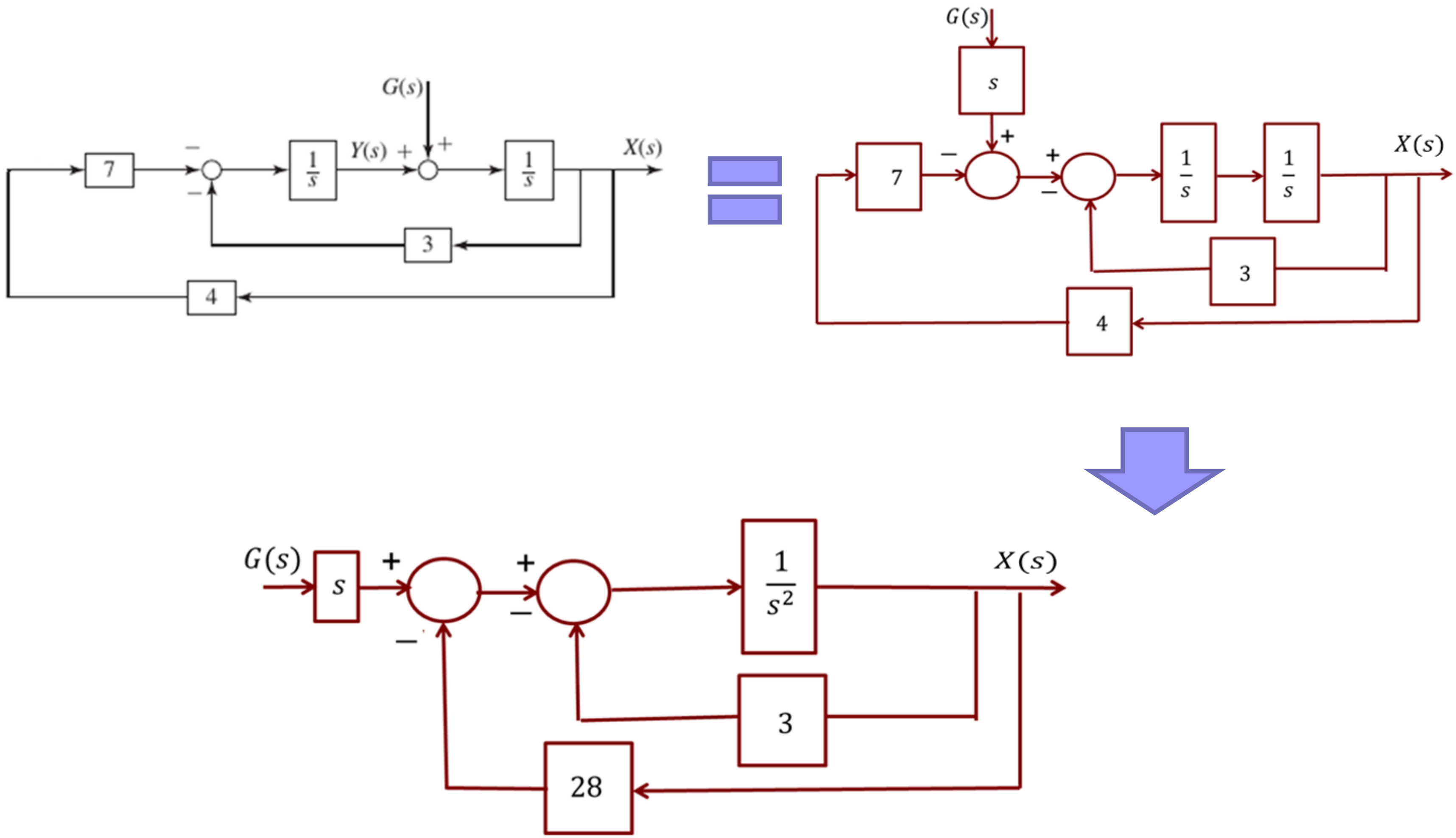




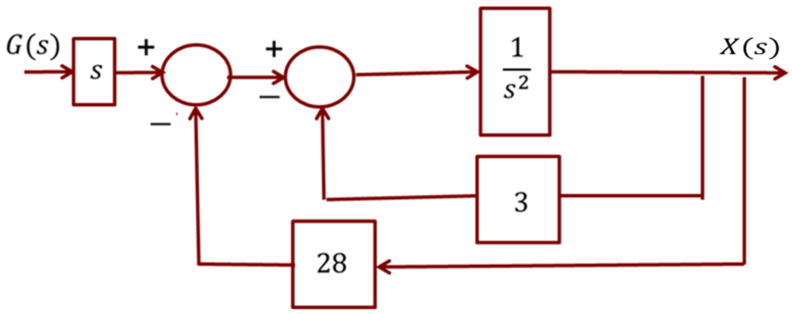


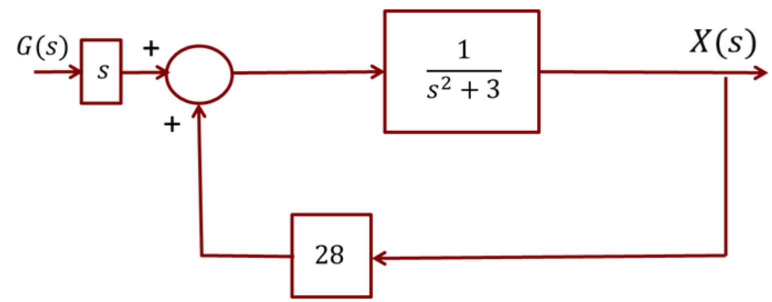
$$\frac{X(s)}{F(s)} = \frac{\frac{7}{s^2+3}}{1+(4)\frac{7}{s^2+3}} = \frac{7}{s^2+31}$$

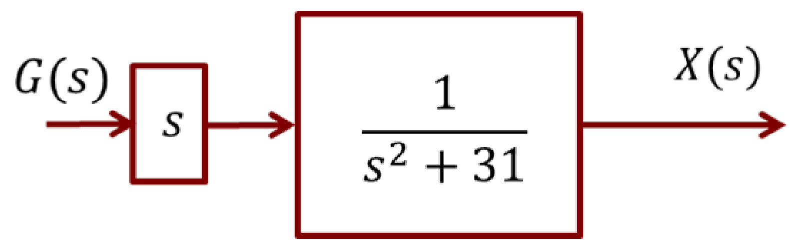
Example 2b



Example 2b







$$\frac{X(s)}{G(s)} = \frac{s}{s^2 + 31}$$

Example 2

$\frac{X(s)}{F(s)} = \frac{7}{s^2+31}$ and $\frac{X(s)}{G(s)} = \frac{s}{s^2+31}$

$\ddot{x} + 31x = 7f(t)$ and $\ddot{x} + 31x = \dot{g}(t)$

Define state variables as $x_1 = x$ and $x_2 = \frac{dx}{dt}$

Derivatives of state variables $\dot{x}_1 = \frac{dx}{dt} = x_2$

and $\dot{x}_2 = \frac{d^2x}{dt^2} = 7f - 31x = 7f - 31x_1$

Also $\dot{x}_2 = \frac{d^2x}{dt^2} = \dot{g} - 31x = \dot{g} - 31x_1$

Define $u_1 = f$ and $u_2 = \dot{g}$

State equation:

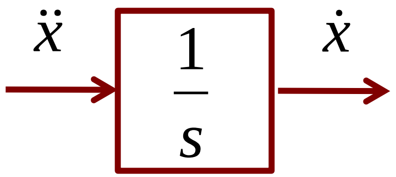
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -31 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Output equation:

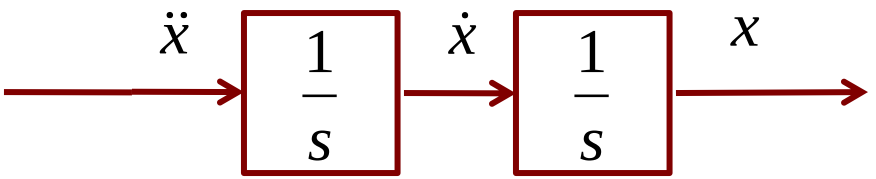
$$[y] = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0 \quad 0] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Example 3

Construct the block diagram for: $m\ddot{x} + b\dot{x} + cx = f$
Hint: Integrate $\ddot{x}$ will get $\dot{x}$ and integrate $\dot{x}$ will get x
The integration can be accomplished with a "integrator" component shown



Generate $\dot{x}$ and x from $\ddot{x}$



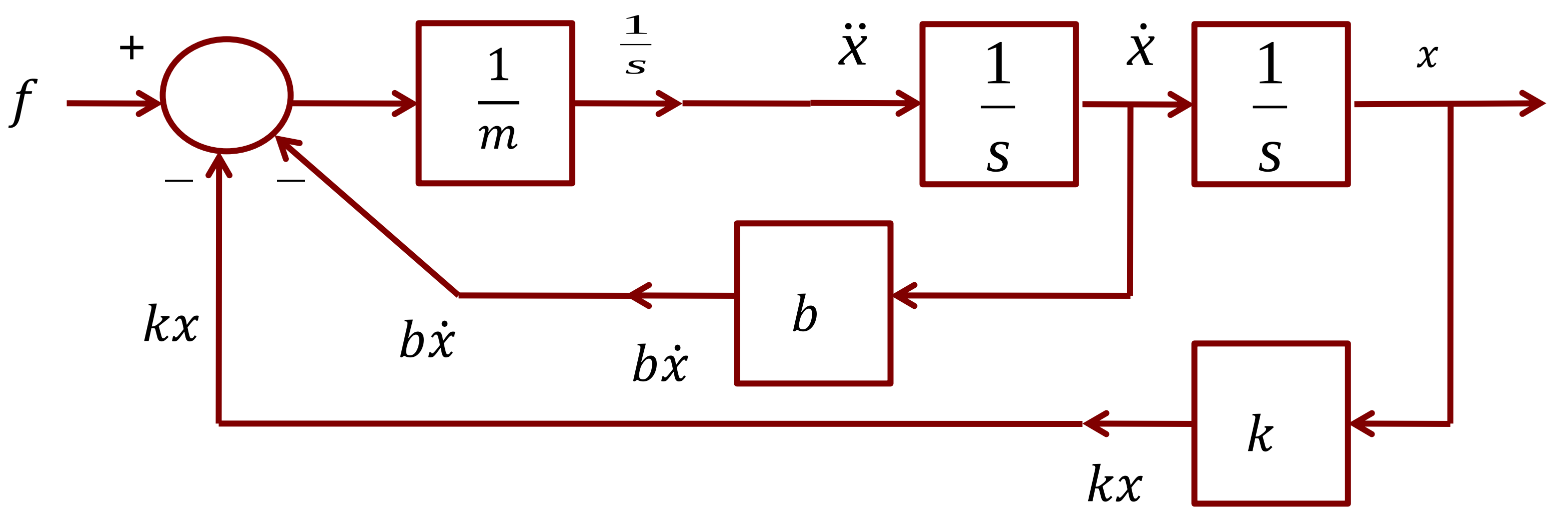
Rearrange the EOM to:

$$\ddot{x} = \frac{1}{m}(f(t) - b\dot{x} - kx)$$

How to generate the $\ddot{x}$ from the above components?

Example 3

$$\ddot{x} = \frac{1}{m}(f(t) - b\dot{x} - kx)$$



Reduce the block diagram to confirm the answer!