



ME 1071: Applied Fluids

# **Lecture 8 Exam II Review**

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Spring 2021

# Weekly Study Plan



Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
5	<b>Apr. 6</b>	<b>Exam I</b>
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 25	Chapter 11: Flow in Open Channels
9	Apr. 27	Chapter 11: Flow in Open Channels
10	May. 11	Exam II Review
<b>11</b>	<b>May. 18</b>	<b>Exam II</b>
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review ( <b>Final Exam at Jun. 26</b> )

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# Exam 2



**Tuesday May 18th from 1:50pm to 4:25pm**

**5 long answer questions are included! No hints but all the important equations and tables will be provided as supplemental materials.**

Closed-book, calculators are allowed but no smart phones or tablets.

Work must be done by hand on the solution form provided, scanned and uploaded to Black Board by 4:25pm

Each page of the solution form must be signed with name and student ID.

Will cover all of Lectures 4 to 7.

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# Outlines

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## ➤ External Flow

- Laminar Flow Over Flat Plate
- Turbulent Flow Over Flat Plate
- Skin Friction
- Drag
- Lift

## ➤ Open Channel Flow

- Froude Number
- Critical Flow
- Area Change
- Hydraulic Jump

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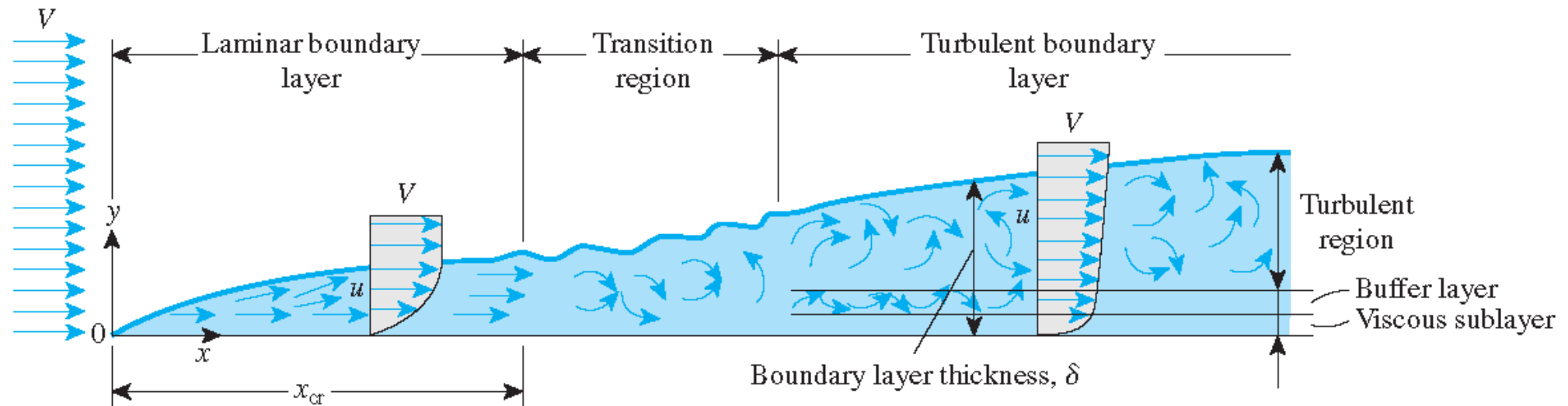
# Boundary Layer



## Regions for Boundary Layer Development over a Flat Plate

- The simplest possible boundary layer
- constant pressure field and zero pressure gradient
- Laminar from the leading edge and transits to turbulent downstream

$$\text{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} \approx 500,000$$



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# Boundary Layer



## Disturbance Thickness $\delta$

- $\delta$  is defined as the distance above the wall where  $u = 0.99U$

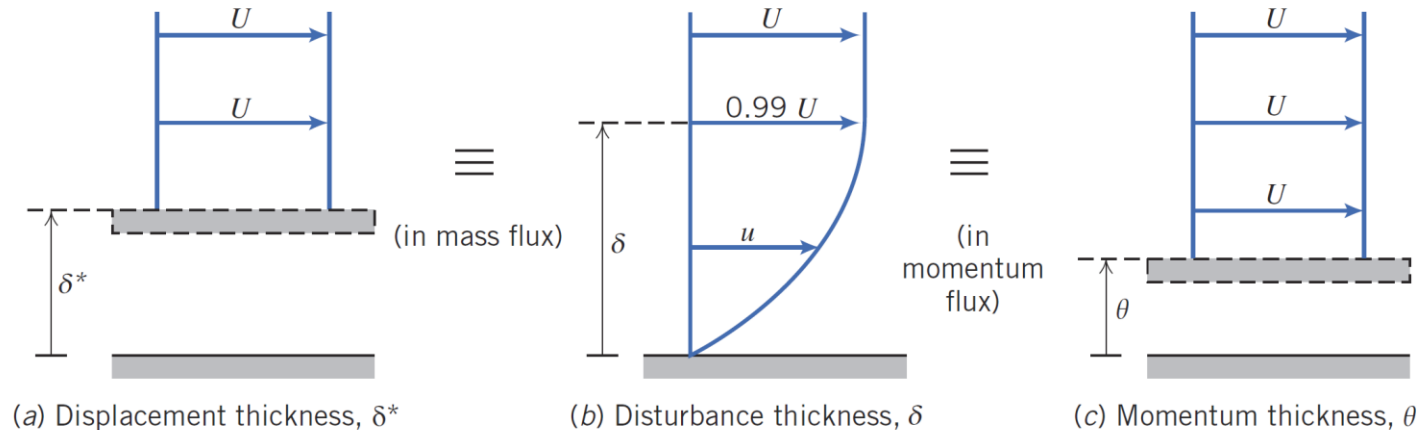
$$u = 0.99U$$

## Displacement Thickness $\delta^*$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

## Momentum Thickness $\theta$

$$\theta = \int_0^\infty \frac{u}{U} \left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$



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# Momentum Integral Equation



## Flow with Zero Gradient

$$\tau_w = \frac{\partial}{\partial x} U^2 \int_0^\delta \rho \frac{u}{U} \left(1 - \frac{u}{U}\right) dy + U \frac{dU}{dx} \int_0^\delta \rho \left(1 - \frac{u}{U}\right) dy \xrightarrow{U(x) = U = \text{constant}} \tau_w = \rho U^2 \frac{d}{dx} \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$

## Laminar Flow

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \quad \tau_w = \frac{2\mu U}{\delta} \quad \frac{\delta}{x} = \sqrt{\frac{30\mu}{\rho U x}} = \frac{5.48}{\sqrt{\text{Re}_x}} \quad C_f = \frac{0.730}{\sqrt{\text{Re}_x}}$$

## Turbulent Flow

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} \quad \frac{\delta}{x} = 0.382 \left(\frac{\nu}{Ux}\right)^{1/5} = \frac{0.382}{\text{Re}_x^{1/5}} \quad C_f = \frac{0.0594}{\text{Re}_x^{1/5}}$$

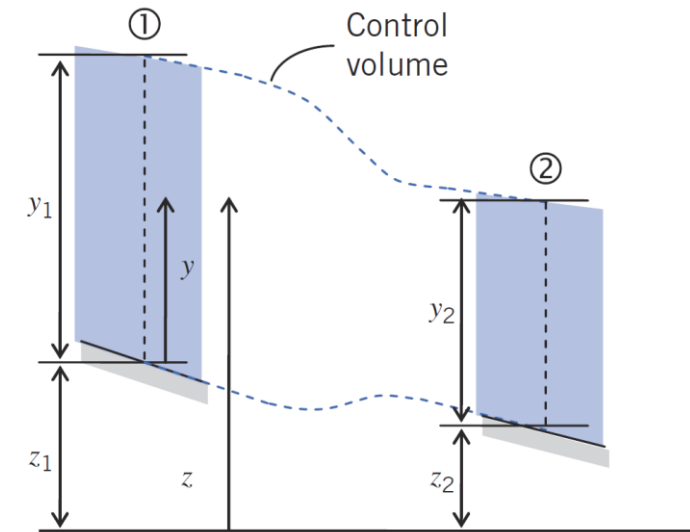
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# Energy Equation for Open-Channel Flows



## Assumptions

1. Steady flow.
2. Incompressible flow.
3. Uniform velocity at a section.
4. Gradually varying depth so that pressure distribution is hydrostatic.
5. Small bed slope.
6.  $W_s = W_{\text{shear}} = W_{\text{other}} = 0$ .



## Energy Equation for Open-Channel Flow

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + H_l$$

## Total Head or Energy Head

$$H = \frac{V^2}{2g} + y + z$$

$$H_1 - H_2 = H_l$$

## Specific Energy

$$E = \frac{V^2}{2g} + y$$

$$E_1 - E_2 + z_1 - z_2 = H_l$$

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# Drag



## Drag Coefficient

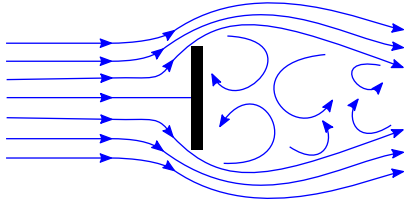
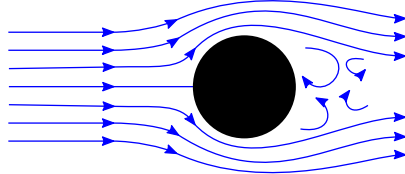
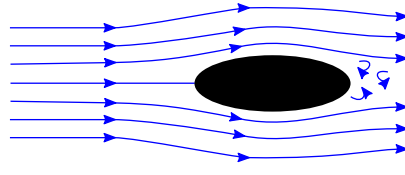
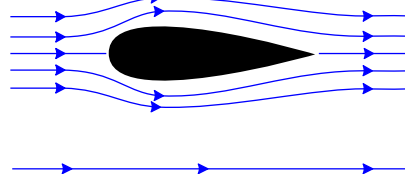

A dimensionless quantity that is used to quantify the drag or resistance of an object in a fluid environment.

Drag Force  $F_D = \text{Pressure Drag} + \text{Friction Drag} + \text{Induced Drag}$

$$C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A}$$

The specific area  $A$  depends on type of  $C_D$

- Car - projected frontal area of the vehicle
- Airfoil – the nominal wing area

Shape and Flow	Pressure Drag	Skin Friction
	≈ 100%	≈ 0%
	≈ 90%	≈ 10%
	≈ 60%	≈ 40%
	≈ 10%	≈ 90%
	≈ 0%	≈ 100%

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# Drag



## Pure Friction Drag: Flow over a Flat Plate

### Laminar Flow

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}}$$

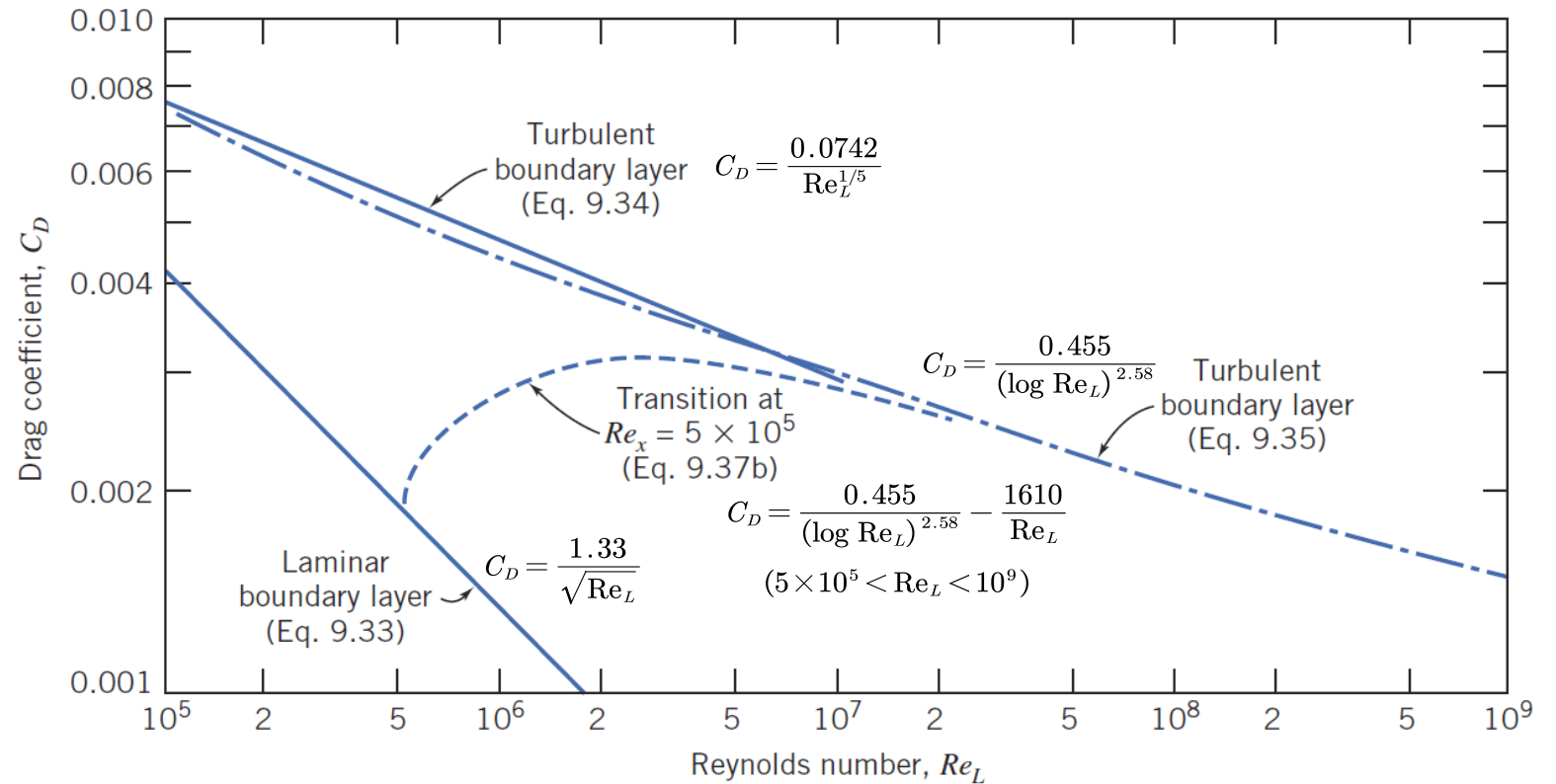
$$C_D = \frac{1.33}{\sqrt{Re_L}}$$

### Turbulent Flow

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{Re_x^{1/5}}$$

$$C_D = \frac{0.0742}{Re_L^{1/5}}$$

$$(5 \times 10^5 < Re_L < 10^7)$$



**Fig. 9.8** Variation of drag coefficient with Reynolds number for a smooth flat plate parallel to the flow.

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# Drag



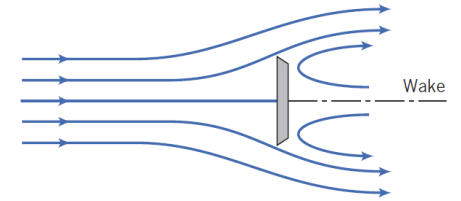
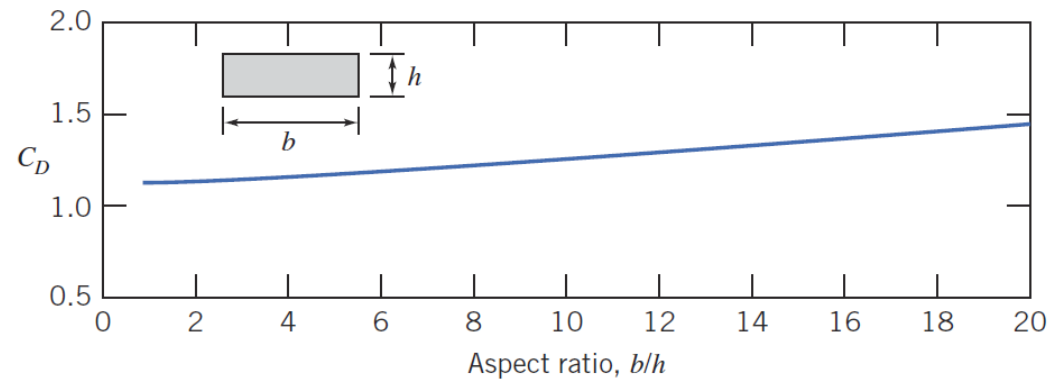
## Pure Pressure Drag : Flow Normal to a Flat Plate

### Pressure Drag Force

$$F_D = \int_{\text{plate surface}} p dA$$

### Drag Coefficient (empirical results only)

$C_D \approx 1.18$  for aspect ratio  $b/h = 1$ ,  $Re \gtrsim 1000$



**Table 9.3**

Drag Coefficient Data for Selected Objects ( $Re \gtrsim 10^3$ )<sup>4</sup>

Object	Diagram	$C_D(Re \gtrsim 10^3)$
Square prism		$b/h = \infty$ 2.05
		$b/h = 1$ 1.05
Disk		1.17
Ring		$1.20^b$
Hemisphere (open end facing flow)		1.42
Hemisphere (open end facing downstream)		0.38
C-section (open side facing flow)		2.30
C-section (open side facing downstream)		1.20

<sup>4</sup> Data from Hoerner [16].

<sup>b</sup> Based on ring area.

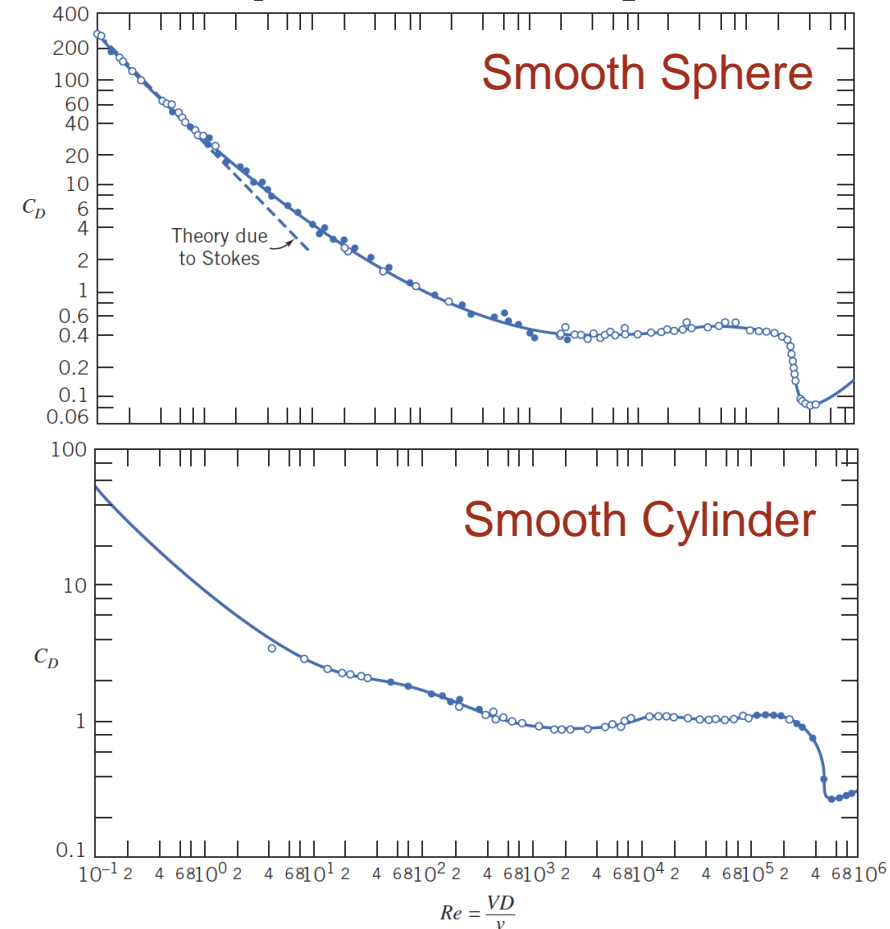
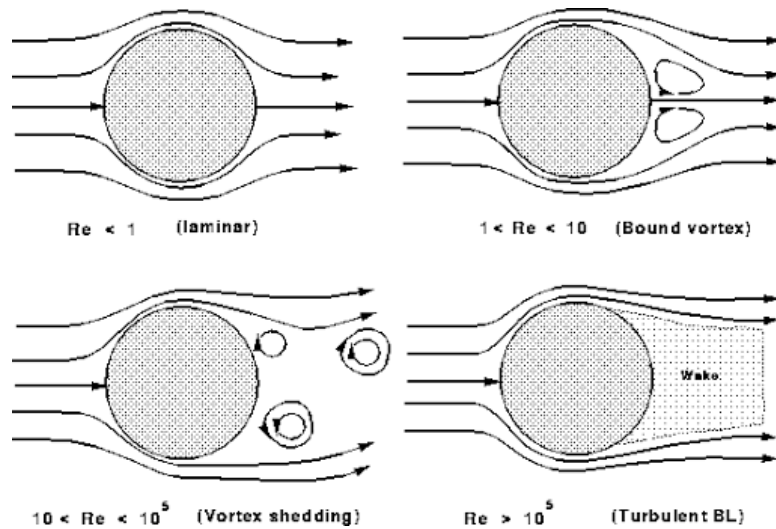
# Drag



## Friction and Pressure Drag: Flow over a Sphere and Cylinder

Laminar Case ( $Re < 1$ )

$$F_D = 3\pi\mu Vd \quad C_D = \frac{24}{Re}$$



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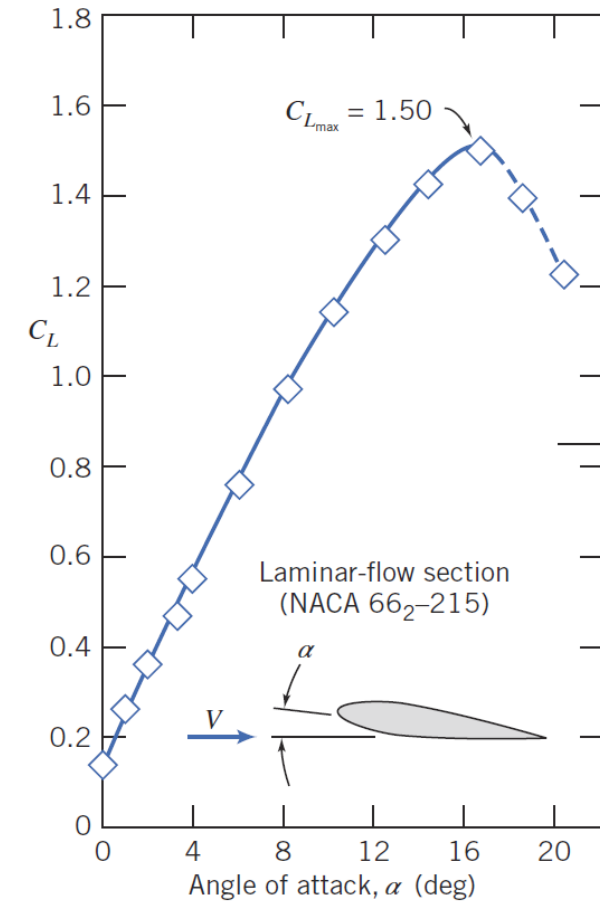
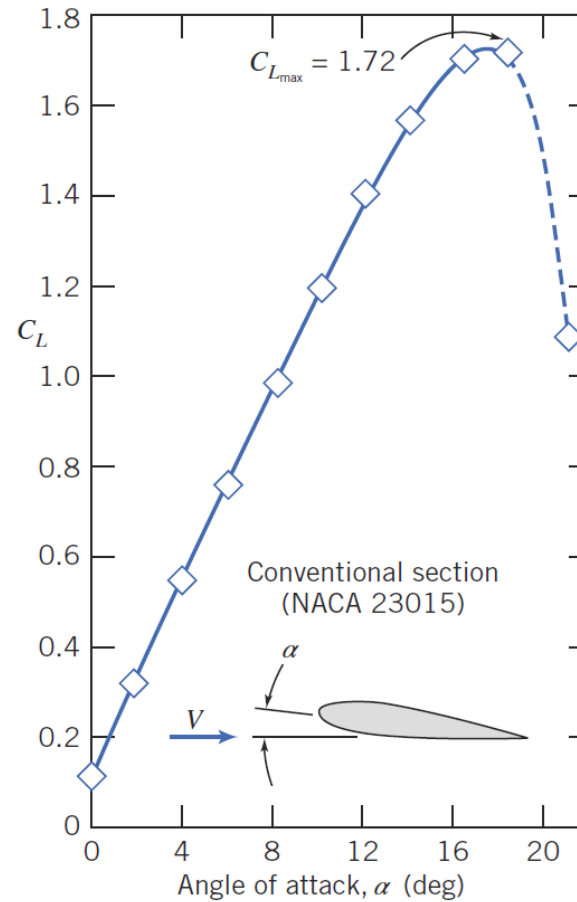
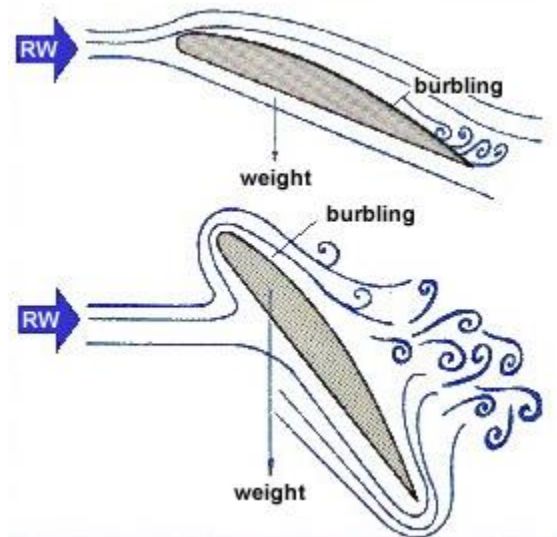
# Lift



## Lift vs Angle of Attack Stall

- $C_L$  decreases as  $\alpha$  increases

$$C_L = \frac{F_L}{\frac{1}{2} \rho V^2 A}$$



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# Basic Concepts and Definitions



## Speed of Surface Waves and the Froude Number

### The Froude number

$$\text{Rectangular channels: } Fr = \frac{V}{\sqrt{gy}}$$

$$\text{Nonrectangular channels: } Fr = \frac{V}{\sqrt{gy_h}}$$

- **$Fr < 1$**  Flow is *subcritical*, *tranquil*, or *streaming*.

Disturbances can travel upstream; downstream conditions can affect the flow upstream. The flow can gradually adjust to the disturbance.

- **$Fr = 1$**  Flow is *critical*.
- **$Fr > 1$**  Flow is *supercritical*, *rapid*, or *shooting*.

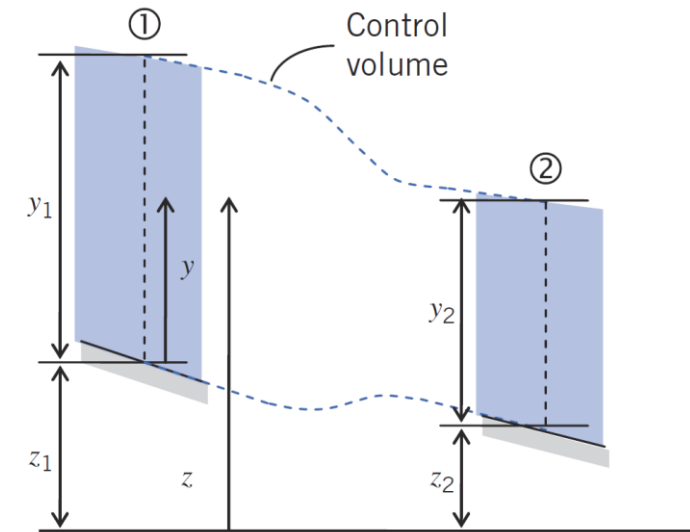
No disturbance can travel upstream; downstream conditions cannot be felt upstream. The flow may “violently” respond to the disturbance because the flow has no chance to adjust to the disturbance before encountering it.

# Energy Equation for Open-Channel Flows



## Assumptions

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## Energy Equation for Open-Channel Flow

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + H_l$$

## Total Head or Energy Head

$$H = \frac{V^2}{2g} + y + z$$

$$H_1 - H_2 = H_l$$

## Specific Energy

$$E = \frac{V^2}{2g} + y$$

$$E_1 - E_2 + z_1 - z_2 = H_l$$

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# Energy Equation for Open-Channel Flows



## The Specific Energy

- $E$  indicates actual energy (kinetic plus potential/pressure per unit mass flow rate) being carried by the flow

$$E = \frac{V^2}{2g} + y$$

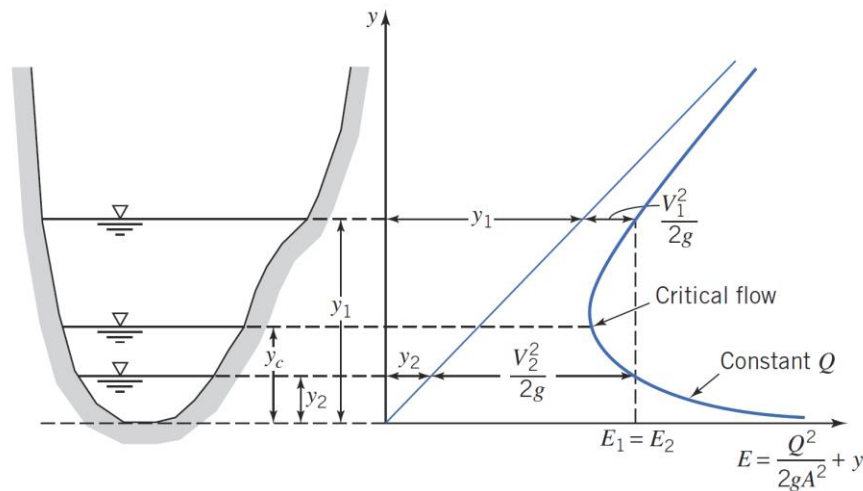


Fig. 11.7 Specific energy curve for a given flow rate.

## Critical Depth ( $Fr = 1$ )

$$Q^2 = \frac{gA_c^3}{b_{sc}}$$

$$V_c = \sqrt{gy_{hc}}$$

## Minimum Specific Energy

- the specific energy is at its minimum at critical conditions, i.e.,  $Fr = 1$ .

$$y_c = \left[ \frac{Q^2}{gb^2} \right]^{1/3} \quad E_{\min} = \frac{3}{2}y_c \quad (\text{Rectangular channel})$$

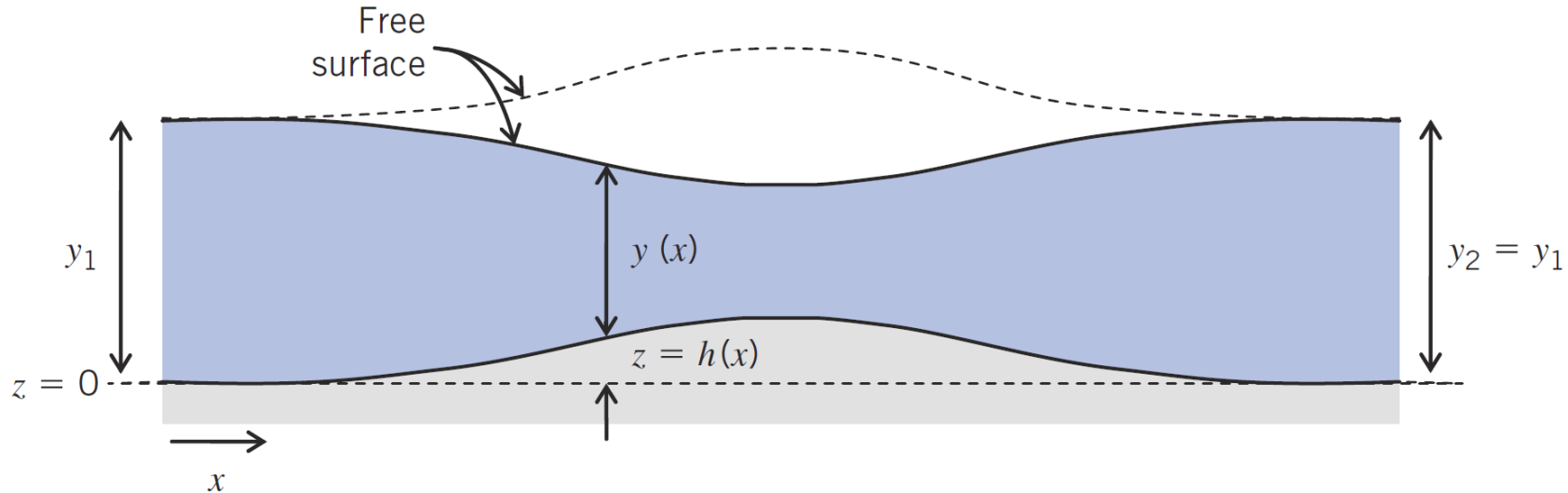
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# Localized Effect of Area Change



## Flow over a Bump

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 = \frac{V^2}{2g} + y + z = \text{const}$$



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# The Hydraulic Jump



## Governing Equations for Hydraulic Jump

$$\text{Continuity} \quad V_1 y_1 = V_2 y_2$$

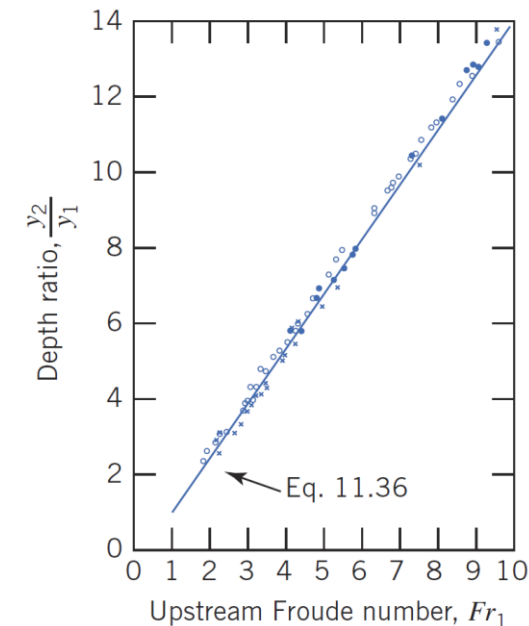
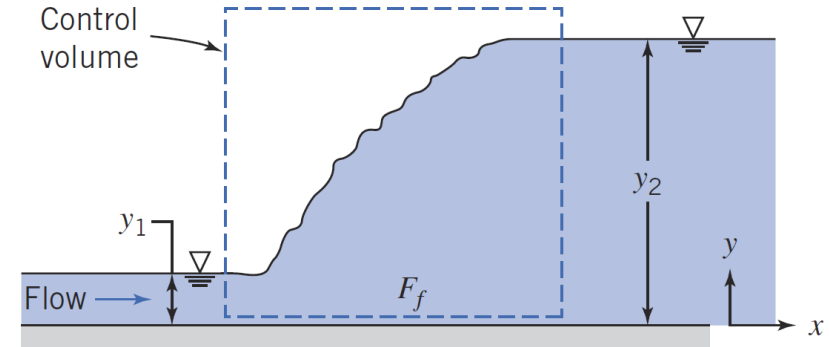
$$\text{Momentum} \quad \frac{V_1^2 y_1}{g} + \frac{y_1^2}{2} = \frac{V_2^2 y_2}{g} + \frac{y_2^2}{2}$$

$$\text{Energy} \quad E_1 = \frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + H_l = E_2 + H_l$$

## Depth Increase Across a Hydraulic Jump

- The ratio of downstream to upstream depths across a hydraulic jump is only a function of the upstream Froude number.

$$\frac{y_2}{y_1} = \frac{1}{2} [\sqrt{1 + 8Fr_1^2} - 1], \quad Fr_1 > 1$$



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# The Hydraulic Jump



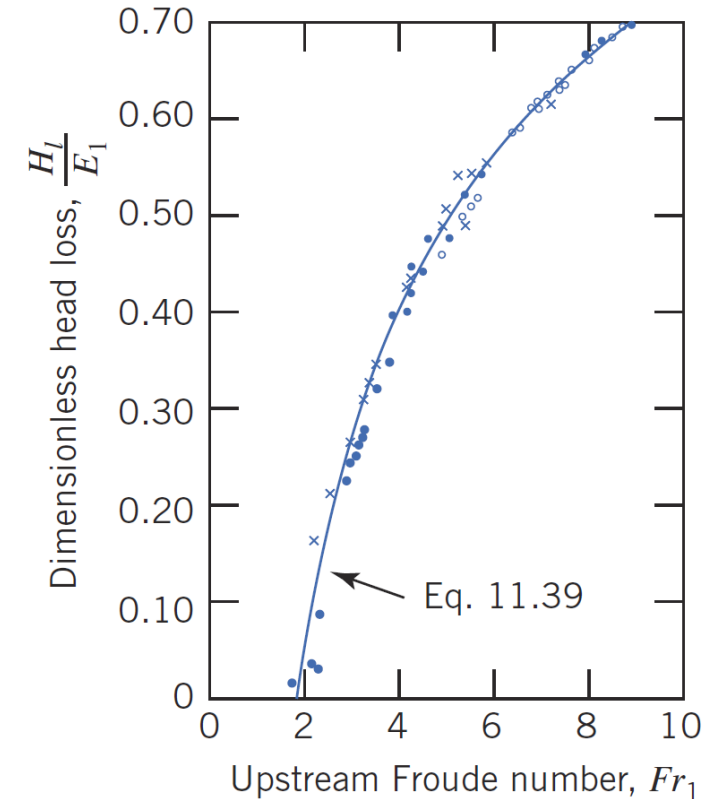
## Head Loss Across a Hydraulic Jump

- The head loss is only a function of the upstream Froude number.
- Hydraulic jump can occur only in supercritical flow.
- Flow downstream from a jump always is subcritical.

$$H_l = \frac{[y_2 - y_1]^3}{4y_1y_2}, \quad y_2 > y_1$$

$$\frac{H_l}{E_1} = \frac{[\sqrt{1 + 8Fr_1^2} - 3]^3}{8[\sqrt{1 + 8Fr_1^2} - 1][Fr_1^2 + 2]}, \quad Fr_1 > 1$$

**Energy Dissipation Ratio**



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# Example

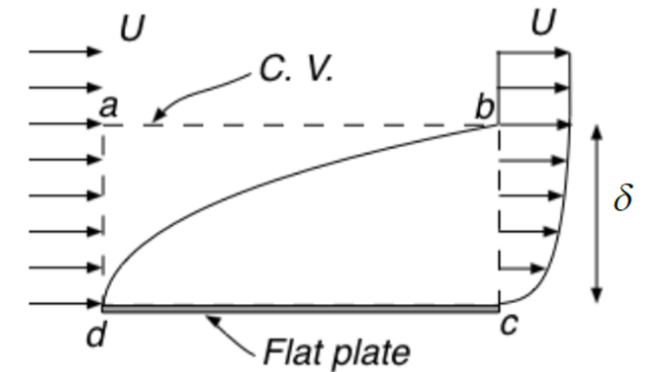


## Example 1

Consider the following boundary layer over a flat plate with a width of  $b$ . Assume that the velocity profile at the trailing edge is:

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

The boundary layer thickness is  $\delta$  and the flow is two-dimensional. Using the control volume  $abcd$ , shown by the dashed lines in the Figure below, determine the mass flow rate on control surface  $a-b$  using the mass conservation relationship.



# Example



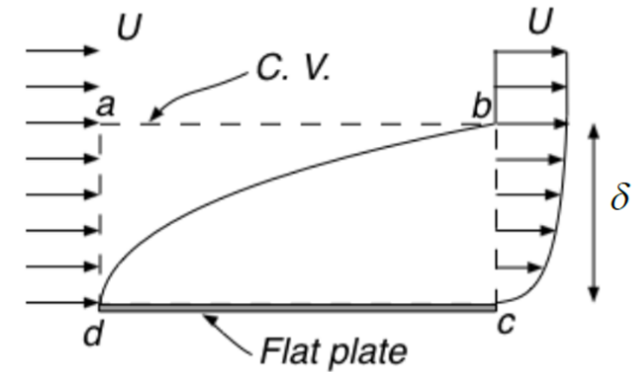
## Example 1

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$0 = \frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho (\vec{V} \cdot d\vec{A})$$

$$\int_{CS} \rho (\vec{V} \cdot d\vec{A}) = 0 = (-\rho U b \delta)_{da} + \dot{m}_{ab} + \left( \int_0^\delta \rho u b dy \right) + (0)_{cd}$$

$$\dot{m}_{ab} = \frac{\rho U b \delta}{3}$$

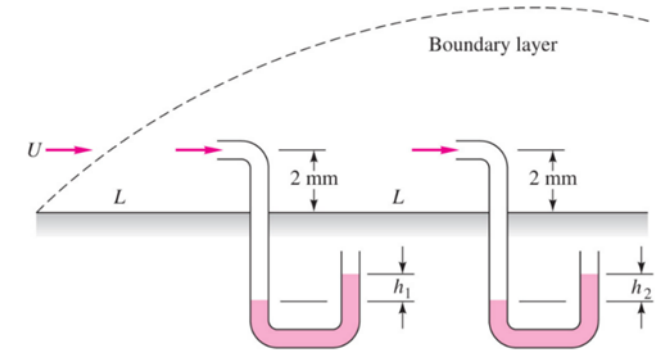


# Example



## Example 2

Air at 20°C and 1 atm flows past the flat plate as shown under laminar conditions. There are two equally spaced pitot stagnation tubes, each placed 2 mm from the wall. The manometer fluid is water at 20°C. If  $U = 15$  m/s and  $L = 50$  cm, determine the values of the manometer readings  $h_1$  and  $h_2$  in mm.



$$\frac{\delta}{x} = \sqrt{\frac{30\mu}{\rho U x}} = \frac{5.48}{\sqrt{Re_x}}$$
$$\delta_1 = x_1 \sqrt{\frac{30\mu}{\rho U x_1}} = 0.5 \sqrt{\frac{30(1.81 \times 10^{-5})}{1.21(15)(0.5)}} = 0.00387 \text{ m}$$
$$\delta_2 = x_2 \sqrt{\frac{30\mu}{\rho U x_2}} = 1.0 \sqrt{\frac{30(1.81 \times 10^{-5})}{1.21(15)(1.0)}} = 0.00547 \text{ m}$$

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# Example



## Example 2

$$\frac{u}{U} = 2 \left( \frac{y}{\delta} \right) - \left( \frac{y}{\delta} \right)^2$$

$$u_1 = U \left( 2 \left( \frac{y_1}{\delta_1} \right) - \left( \frac{y_1}{\delta_1} \right)^2 \right) = 15 \left[ 2 \left( \frac{0.002}{0.00387} \right) - \left( \frac{0.002}{0.00387} \right)^2 \right] = 11.5 \text{ m/s}$$

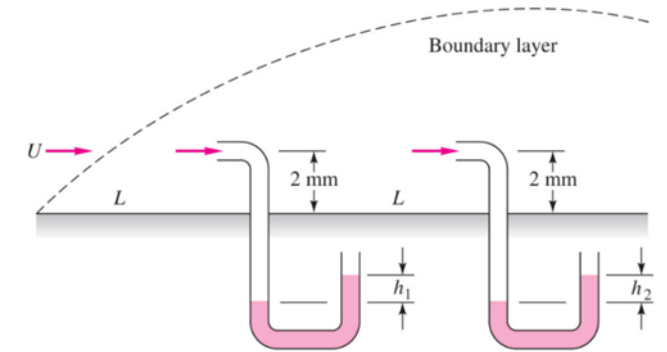
$$u_2 = U \left( 2 \left( \frac{y_2}{\delta_2} \right) - \left( \frac{y_2}{\delta_2} \right)^2 \right) = 15 \left[ 2 \left( \frac{0.002}{0.00547} \right) - \left( \frac{0.002}{0.00547} \right)^2 \right] = 8.96 \text{ m/s}$$

$$P = \rho_w g h = \frac{1}{2} \rho u^2$$

$$h_1 = \frac{1}{2} \frac{\rho}{\rho_w} \frac{u_1^2}{g} = \frac{1}{2} \left( \frac{1.21}{998} \right) \left( \frac{11.5^2}{9.81} \right) = 0.00817 \text{ m}$$

$$h_2 = \frac{1}{2} \frac{\rho}{\rho_w} \frac{u_2^2}{g} = \frac{1}{2} \left( \frac{1.21}{998} \right) \left( \frac{8.96^2}{9.81} \right) = 0.00496 \text{ m}$$

$$h_1 = 8.17 \text{ mm} \quad h_2 = 4.96 \text{ mm}$$



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# Example

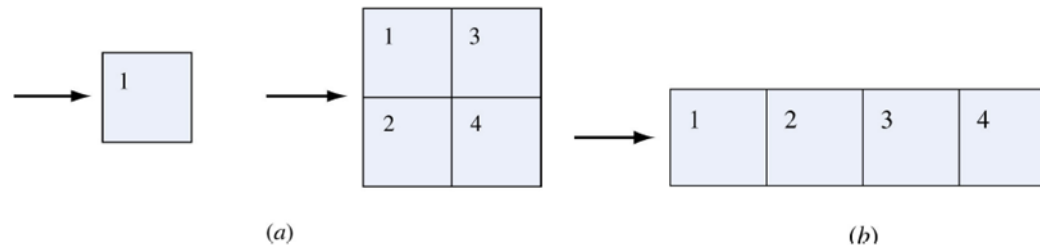


## Example 3

Consider the following square plate arrangements (top view) with boundary layer flow. Compared to the friction drag of a single plate 1:

(a) how much larger is the drag of four plates together as in configuration (a) and (b)? Assume that the boundary layer flow is laminar in the entire surface.

(b) What if the boundary layer is turbulent in the entire surface?



# Example



## Example 3

$$C_D = \frac{F_D}{1/2 \rho u^2 A} = \frac{0.0742}{Re_L^{1/5}}$$

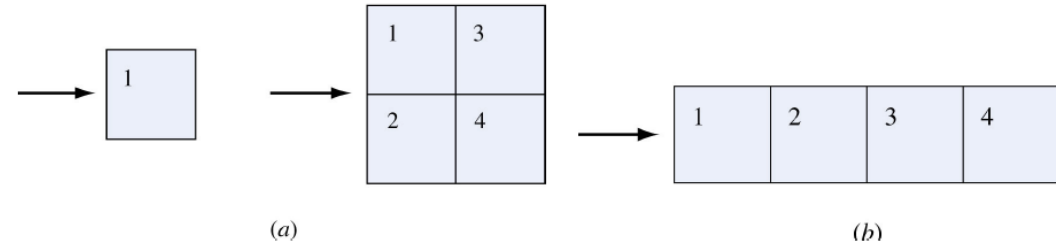
$$F_{D,1} = \frac{1}{2} \rho u^2 (L^2) \left[ \frac{0.0742}{\left(\frac{uL}{\nu}\right)^{1/5}} \right] = \frac{L^2}{L^{1/5}} \left[ \frac{\frac{1}{2} \rho u^2 (0.0742)}{\left(\frac{u}{\nu}\right)^{1/5}} \right]$$

$$F_{D,a} = \frac{1}{2} \rho u^2 (4L^2) \left[ \frac{0.0742}{\left(\frac{u2L}{\nu}\right)^{1/5}} \right] = \frac{4L^2}{(2L)^{1/5}} \left[ \frac{\frac{1}{2} \rho u^2 (0.0742)}{\left(\frac{u}{\nu}\right)^{1/5}} \right]$$

$$\frac{F_{D,a}}{F_{D,1}} = \frac{\frac{4L^2}{(2L)^{1/5}}}{\frac{L^2}{L^{1/5}}} = \frac{4}{2^{1/5}} = 2^{9/5}$$

$$F_{D,b} = \frac{1}{2} \rho u^2 (4L^2) \left[ \frac{0.0742}{\left(\frac{u4L}{\nu}\right)^{1/5}} \right] = \frac{4L^2}{(4L)^{1/5}} \left[ \frac{\frac{1}{2} \rho u^2 (0.0742)}{\left(\frac{u}{\nu}\right)^{1/5}} \right]$$

$$\frac{F_{D,b}}{F_{D,1}} = \frac{\frac{4L^2}{(4L)^{1/5}}}{\frac{L^2}{L^{1/5}}} = \frac{4}{4^{1/5}} = 4^{4/5}$$



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# Example



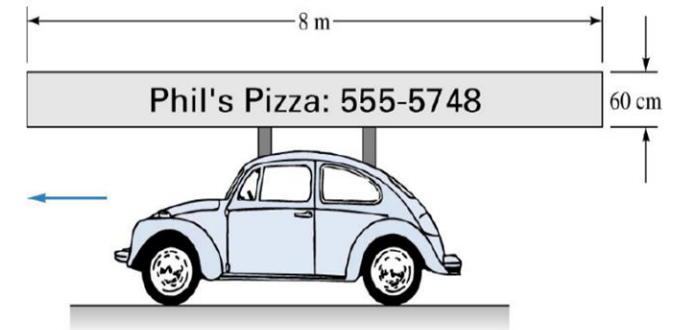
## Example 4

A delivery vehicle carries a long sign on top that has a square cross-section, as shown in Figure below. If the sign is very thin and the vehicle moves at 30 m/s. Assume that the sign is perfectly aligned with the airflow for problems (a-c).

**(a)** What is the boundary layer thickness at the trailing edge of the sign if the boundary layer remains laminar?

**(b)** What is the drag force on the sign if the boundary layer remains laminar?

**(c)** Answer questions **(a)** and **(b)** but with assumption that the boundary layer is turbulent from the leading edge of the sign.



$$\frac{\delta}{x} = \sqrt{\frac{30\mu}{\rho U x}} = \frac{5.48}{\sqrt{Re_x}}$$

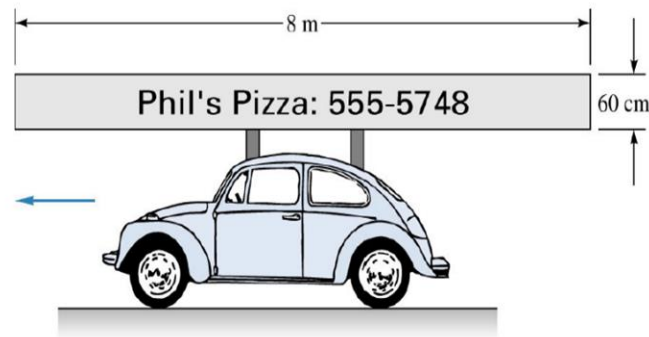
$$\delta = x \sqrt{\frac{30\mu}{\rho U x}} = 8 \sqrt{\frac{30(1.81 \times 10^{-5})}{1.21(30)(8)}} = 0.0109 \text{ m}$$

$\delta = 10.9 \text{ cm}$

# Example



## Example 4



$$C_D = \frac{F_D}{1/2 \rho V^2 A} = \frac{1.33}{\sqrt{Re_L}}$$

$$F_D = \frac{1}{2} \rho V^2 A \left[ \frac{1.33}{\sqrt{\frac{VL}{v}}} \right] = \frac{1}{2} (1.21) (30^2) (8) (0.60) \left[ \frac{1.33}{\sqrt{\frac{30(8)}{1.50 \times 10^{-5}}}} \right]$$

**$F_D = 0.869 N$**

$$\frac{\delta}{x} = 0.382 \left( \frac{v}{Ux} \right)^{1/5} = \frac{0.382}{Re_x^{1/5}}$$

$$\delta = \frac{x(0.382)}{Re_x^{1/5}} = \frac{8(0.382)}{\left( \frac{(30)(8)}{1.5 \times 10^{-5}} \right)^{1/5}} = 0.1107 m$$

**$\delta = 11.1 cm$**

$$C_D = \frac{F_D}{1/2 \rho V^2 A} = \frac{0.455}{(\log Re_L)^{2.58}}$$

$$F_D = \frac{1}{2} \rho V^2 A \left[ \frac{0.455}{\left( \log \frac{VL}{v} \right)^{2.58}} \right] = \frac{1}{2} (1.21) (30^2) (8) (0.60) \left[ \frac{0.455}{\left( \log \frac{(30)(8)}{1.5 \times 10^{-5}} \right)^{2.58}} \right]$$

**$F_D = 7.29 N$**

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# Example



## Example 5

A paratrooper and his 8 m diameter parachute weigh 950 N. Taking the average air density to be  $1.2 \text{ kg/m}^3$ , determine the terminal velocity of the paratrooper.

$$\sum F_y = 0 = F_D - mg$$

$$F_D = mg = \frac{1}{2} \rho V^2 C_D A$$

$C_D \rightarrow \text{Table 9.3}$



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# Example



## Example 6

An airplane with an effective lift area of 25 m<sup>2</sup> is fitted with airfoils of NACA 23012. The maximum flap setting that can be used at takeoff corresponds to the configuration of double slotted. The maximum gross mass possible for the airplane is 10,000 kg.

- Determine the minimum takeoff speed required for this gross mass at the sea level.
- Determine the minimum takeoff speed required for this gross mass if the airplane is instead taking off from an elevation of 1.6 km.

From Table A.3 when  $GA = 0$ ,  $\rho = \rho_{SL} = 1.2250 \text{ kg/m}^3$

$$\sum F_y = 0 = F_L - mg$$

$$F_L = mg = \frac{1}{2} \rho V^2 C_L A$$

$$C_{L,max} \rightarrow \text{Figure 9.23}$$

From Table A.3 when  $GA = 1600 \text{ m} \rightarrow \rho/\rho_{SL}$

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# Example



## Example 7

At a section of a 3 m wide rectangular channel, the depth is 0.09 m for a discharge of 0.57 m<sup>3</sup>/s. A smooth bump of 0.03 m high is placed on the floor of the channel. Determine the local change in the depth caused by the bump.

$$V_1 = \frac{Q}{A_1} = \frac{0.57}{(3)(0.09)} = 2.11 \text{ m/s}$$
$$V_2 = \frac{Q}{A_2} = \frac{0.57}{(3)y_2} = \frac{0.19}{y_2}$$
$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + h$$

*Guess*  $\rightarrow y_2$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}}$$
$$Fr_1 < 1 \quad y_2 < y_1$$
$$Fr_2 > 1 \quad y_2 > y_1$$

$$Fr_2 = \frac{V_2}{\sqrt{gy_2}}$$

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# Example



## Example 8

A hydraulic jump occurs in a rectangular channel. The flow rate is  $6.5 \text{ m}^3/\text{s}$ , and the depth before the jump is  $0.4 \text{ m}$ . Determine the depth behind the jump and the head loss, if the channel is  $1 \text{ m}$  wide.

$$V_1 = \frac{Q}{A_1}$$
$$Fr_1 = \frac{V_1}{\sqrt{gy_1}}$$
$$\frac{y_2}{y_1} = \frac{1}{2} \left[ \sqrt{1 + 8Fr_1^2} - 1 \right]$$

$$V_2 = \frac{Q}{A_2}$$
$$Fr_2 = \frac{V_2}{\sqrt{gy_2}}$$

$$H_l = \frac{V_1^2}{2g} + y_1 - \frac{V_2^2}{2g} - y_2 = \frac{[y_2 - y_1]^3}{4y_1y_2}$$

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