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Mechanical Design 1

06 Assignment

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Mechanical Design 1

Class Section 01

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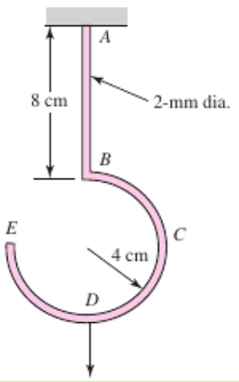
Problem 1

A hook is formed from a 2-mm-diameter steel wire and fixed firmly into the ceiling as shown. A 1-kg mass is hung from the hook at point *D*. Use Castigliano's theorem to determine the vertical deflection of point *D*



Solution:

For this question, we are asked to determine the vertical deflection of point D.

$$U = \int_0^L \frac{F^2}{2AE}dx + \int_0^\pi \frac{M^2 R}{2EI}d\theta$$

$$\delta = \frac{\partial U}{\partial F} = \int_0^L \frac{F\dfrac{\partial F}{\partial F}}{AE}dx + \int_0^\pi \frac{M\dfrac{\partial M}{\partial F}R}{EI}d\theta = \int_0^L \frac{F\dfrac{\partial F}{\partial F}}{AE}dx + \int_0^\pi \frac{FR\sin\theta\, R\sin\theta\, R}{EI}d\theta$$

$$= \int_0^L \frac{F}{AE}dx + \int_0^\pi \frac{FR^3\sin^2\theta}{EI}d\theta = \int_0^L \frac{F}{AE}dx + \int_0^\pi \frac{FR^3(1-\cos 2\theta)}{2EI}d\theta$$

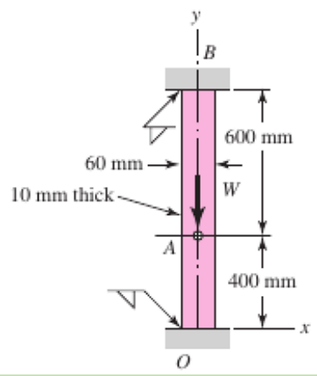
$$= \int_0^L \frac{F}{AE}dx + \int_0^\pi \frac{FR^3}{2EI}d\theta - \int_0^\pi \frac{FR^3\cos 2\theta}{2EI}d\theta = \frac{FL}{AE} + \frac{\pi FR^3}{2EI}$$

$$= \frac{(1\ \text{kg})\times(9.81\ \text{m/s}^2)\times(8\ \text{cm})}{\dfrac{\pi(2\ \text{mm})^2}{4}\times(207\ \text{GPa})}$$

$$+ \frac{\pi\times(1\ \text{kg})\times(9.81\ \text{m/s}^2)\times\left(\dfrac{8\ \text{cm}}{2}\right)^3}{2\times(207\ \text{GPa})\times\dfrac{\pi(2\ \text{mm})^4}{64}} = 6.067\times10^{-3}\ \text{m}$$

Problem 2

A rectangular aluminium bar 10 mm thick and 60 mm wide is welded to fixed supports at the ends, and the bar supports a load $W = 4$ kN, acting through a pin as shown. Find the reactions at the supports and the deflection of point A.



Solution:

For this question, we are asked to find the reactions at the supports and the deflection of point A.

$$R_B + R_O = W$$

$$\frac{R_B L_B}{EA} = \frac{R_O L_O}{EA} \Longrightarrow R_B L_B = R_O L_O$$

$$\Rightarrow \frac{L_O}{L_B} R_O + R_O = W$$

$$\Rightarrow \begin{cases} R_O = 2.4 \text{ kN} \\ R_B = 1.6 \text{ kN} \end{cases}$$

$$\delta_A = \frac{R_O L_O}{EA} = \frac{(2.4 \text{ kN}) \times (400 \text{ mm})}{(71.7 \text{ GPa}) \times (60 \text{ mm}) \times (10 \text{ mm})} = 2.23 \times 10^{-5} \text{ m}$$

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