



Sichuan University - Pittsburgh Institute

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ME 1042

Dynamics/Controls Lab

*Fundamentals of Feedback  
Control*

Revised October 2021

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Mechanical Engineering Department

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**Goal:** The objective of this experiment is to study the speed control of a motor with different types of controls. Also, to investigate the effect of proportional controls and PI controls on the steady state error.

**Equipment Needed:**

CE110 Servo Trainer

CE120 Controller

PC with software

Note: In order to protect the equipment, reset the motor drive voltage to zero before running or stopping the program,

## 1 **Introduction and Basic Theory**

Control theory deals with the control of dynamical systems in engineered processes and machines. The objective is to develop a control model for controlling such systems using a control action in an optimum manner without delay or overshoot and ensuring control stability.

### 1.1 ***Overview of Control Systems***

A control system is an interconnection of components to have a system achieved a desired response. The two basic components of a control system are the plant (process), a mathematical model of the system (transfer function) and the controller. The objective is to have the plant output achieve certain goals (e.g. tracking a desired signal) and if the plant's output does not achieve the goals then one approach of achieving these goals is to use feedback control theory.

Feedback control achieves the desired goals by comparing the difference between the plant's output and the desired signal to create an error signal then the error signal is sent to the controller (another mathematical model accounting for the plant's deficiencies) to manipulate the plant's input to satisfy the goals. A graphical representation of a feedback system (closed-loop system) is called a block diagram as shown in Figure 1, where  $R(s)$  is the desired signal,  $E(s)$  is the error signal,  $U(s)$  is the control signal,  $G_c(s)$  is the controller, and  $G(s)$  is the plant. In the block diagram, the error signal is  $E(s) = R(s) - Y(s)$ , the control signal is  $U(s) = G_c(s)E(s)$ , and the plant's output is  $Y(s) = G(s)U(s)$ .

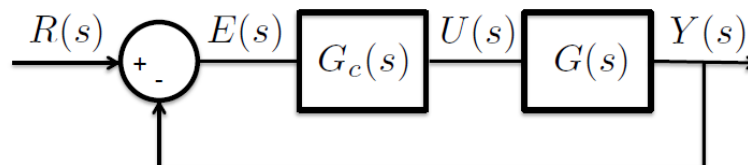


Figure 1: A block diagram of a feedback control system.

In tracking control problems, it is desired to have the plant's output track the desired signal, which is equivalent to having the error signal be zero. The plant's output and the error signal are expressed in the Laplace domain by

$$Y(s) = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)} R(s)$$

$$E(s) = \frac{1}{1 + G(s)G_c(s)} R(s)$$

One approach to examine if the controller design satisfied the goals is to apply the final-value theorem (FVT) to the first equation listed. For a constant desired signal,  $R(s) = r/s$  (where  $r$  is the constant value input), the final value of the plant's output is

$$y(\infty) = \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s)$$

and can only be applied to a closed-loop system if the roots of the denominator of  $Y(s)/R(s)$  have negative real parts. Note the only way the plant's output will track a constant desired signal is if zero is one of the roots of the denominator of  $G_c(s)G(s)$ .

## 2 Basic Testing

The objective of this section is to calibrate the circuits in the Servo Trainer, namely the motor circuit and speed and angular position sensors.

### 2.1 *Motor Calibration*

In this section of the experiment, you will vary the voltage to a motor and obtain a relationship between voltage input and motor angular velocity. To accomplish this, load the Circuit 1.1 from the lab folder and follow the following steps:

1. Using the fine potentiometer, increase the voltage until the motor starts to move at a constant speed, record this voltage input as the dead zone voltage.
2. Using the coarse potentiometer, step through voltages input from 1-10V in increments of 1V and record the displayed value for angular velocity (in Servo Motor bench top display)
3. Repeat for the range of -1 to -10V.

Table 1: Motor Drive Calibration

Motor Drive Voltage (V) (Positive)	Motor Speed (rpm)	Motor Drive Voltage (V) (Negative)	Motor Speed (rpm)
0	0	0	0
Dead-Zone Size =	0	Dead-Zone Size =	0
1		-1	
2		-2	
3		-3	
4		-4	
5		-5	
6		-6	
7		-7	
8		-8	
9		-9	
10		-10	

**Choose one of two options. Either continue to Section 2.2 or proceed to Section 3.1**

## 2.2 Speed Sensor Setting

Similarly, we are interested in having a relationship between the speed of the motor and the speed sensor voltage. To obtain a relationship, load Circuit1-2, then slowly increase the potentiometer voltage until the speed sensor reads 1V, and record the voltage input value in

Table 2. Repeat the process for the speed sensor voltage range -9V to 9V, in increments of 1V.

Table 2: Speed Sensor Calibration

Motor Speed (rpm) (Positive)	Speed Sensor Output (V)	Motor Speed (rpm) (Negative)	Speed Sensor Output (V)
	1		-1
	2		-2
	3		-3
	4		-4
	5		-5
	6		-6
	7		-7
	8		-8
	9		-9

## 3 System Response

The objective of this section is to determine the gain,  $G_1$ , and time constant,  $T$ , of the servo-motor assembly.

### 3.1 Motor Input to Speed Motor Gain

To obtain the Motor input to Speed Output gain, combine the results in Tables 1 and 2. Or, if you skipped Section 2.2, load Circuit2-1 and fill out Table 3. If you use Circuit 2-1, measure the speed sensor voltages for the different drive voltages shown in Table 3.

Table 3: Speed Sensor Calibration

Motor Drive Voltage (V) (Positive)	Speed Sensor Output (V)	Motor Drive Voltage (V) (Negative)	Speed Sensor Output (V)
1		-1	
2		-2	
3		-3	
4		-4	
5		-5	
6		-6	
7		-7	
8		-8	
9		-9	

### 3.2 Measurement of Time Constant

In order to measure the time constant, load circuit 2.2 and run the file. Go to the Graph tab, and select Draw, then use the “Find Time Constant Tool” and click and drag from the bottom of the

graph to the steady state value. Steps to find the time constant (this can only be done after recording data):

1. Record a few cycles of the trial run from the circuit.
2. Open recording in graph reader
3. Zoom into an area where the viewable window includes the motor just before an increase and just after the system hits a steady state.
4. You will see three lines appear on the screen. The very bottom line should be parallel with the flat line. The top line should be parallel to the steady state point and release when the crosshairs of the cursor are on the slope of the increasing line. The time constant will appear.

## 4 Proportional Speed Control

The objective of this experimental section is to implement a proportional controller of the Servo Trainer speed and to investigate the closed loop transient response.

### 4.1 *Steady State Controls*

To study the steady state error, load Circuit3-1. Then, we need to investigate the factors that have an impact on the steady state error. First, vary the reference speed ( $Y_r$ ) in steps of 1V from 2 volts to 10 volts with a fixed gain ( $K_p$ ) of 3. Calculate the steady state error and discuss the effect of the reference signal. Write the results in Table 4.

Table 4: Steady State Error for Various Reference Speeds.

Potentiometer Setting (Reference speed $y_r$ )(V)	Measured Steady State Error Signal (V)	Theoretical Steady State Error Signal (V)
2		
3		
4		
5		
6		
7		
8		
9		
10		

Then, to evaluate the effect of the controller gain, load Circuit 3.2 and vary the control gain from 1-10 in increments of 1 with a fixed reference  $Y_r$  of 5V. Record your results on Table 5 and calculate the theoretical steady state error.

Table 5: Steady State Error for Various Controller Gains.

Controller Gain	Measured Steady State Error Signal (V)	Theoretical Steady State Error Signal (V)
2		
3		
4		
5		
6		
7		
8		
9		
10		

$$e_{ss} = \frac{Y_r}{1 + k_p G}$$

where  $Y_r$  is reference speed,  $k_p$  is the control gain and  $G$  is the calibration gain

## 5 Proportional and Integral Control

The objective of this experimental section is to investigate the effect of proportional plus integral control upon the servo motor speed control loop in terms of steady state error.

To study the effect of the integral controller, load Circuit4-1. Initial conditions should be  $K_p = 1$ ,  $K_i = 0.1$  and switched open. Slowly increase the potentiometer output voltage to 4 V, and observe the steady state error. To observe the effect of the integral control, switch it on (top left corner) and click on the switch to include the integrator in the loop. (It is very important to click ON the integrator before clicking on the switch!!). Repeat the process for  $K_i = 0.5, 1, 2, 4, 6$ . Comment on the behavior for different integral gains (Use plots if necessary).

## 6 For the Report

The following topics should be addressed in the extended memo report:

- Basic Control Theory
- Determination of the Gain (Plot of Voltage Input vs. Speed Sensor Voltage)
- Time Constant Measurement (Print Screen of Time constant tool)
- Discuss the effect of proportional gain. Does the behavior is the one you expected? Support your answer with equations.
- What is the effect of the integrator controller? Is the observed behavior what you expect? Support your answer with equations.
- Is there an optimal combination of gains in the PI controller? Is it dependent on the system? Discuss.