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Mechanical Design 1

09 Assignment

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Mechanical Design 1

Class Section 01

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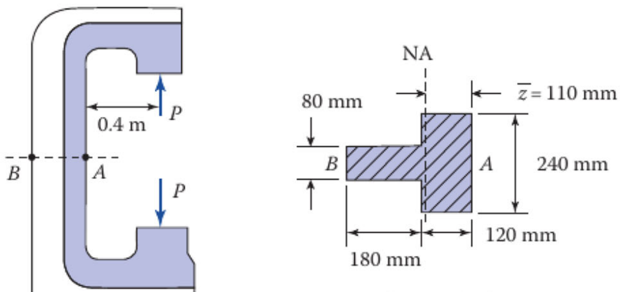
Problem 1

A punch press frame made of ASTM A-48 gray cast iron has ultimate strengths in tension and compression with values S_u = 170 MPa and S_{uc} = 650 MPa, respectively. Calculate the allowable load P using the Coulomb–Mohr theory and a factor of safety of $n = 2.5$.



Section A–B

Solution:

For this question, we are asked to calculate the allowable load P using the Coulomb-Mohr theory and a factor of safety of $n = 2.5$.

Since the load acts in a plane of symmetry, there are three independent equations of equilibrium.

The internal forces at the section are found using the equations of equilibrium as follows:

$$+\rightarrow \sum F_x = 0 \Rightarrow V = 0$$

$$+\uparrow \sum F_y = 0 \Rightarrow F - P = 0 \Rightarrow F = P$$

$$+\circlearrowleft \sum M_{A-B} = 0 \Rightarrow M - (0.4 + 0.11)P = 0 \Rightarrow M = 0.51P$$

Thus, there are two internal forces on the section, an axial force P, which produces a constant normal stress a $\sigma_1 = \frac{F}{A}$ over the section, and a bending moment M, which produces a linear variation of normal stress $\sigma_2 = -\frac{Mx}{I}$ over the section. The cross-sectional area A and the second moment I of the cross-sectional area with respect to the centroidal axis NA are

$$A = (80\text{ mm}) \times (180\text{ mm}) + (240\text{ mm}) \times (120\text{ mm}) = 0.0432\text{ m}^2$$

$$\begin{aligned} I = &\frac{1}{12} \times (240\text{ mm}) \times (120\text{ mm})^3 + (110\text{ mm} - 60\text{ mm})^2 \times (240\text{ mm}) \times (120\text{ mm}) \\ &+ \frac{1}{12} \times (80\text{ mm}) \times (180\text{ mm})^3 \\ &+ (90\text{ mm} + 10\text{ mm})^2 \times (80\text{ mm}) \times (180\text{ mm}) = 2.8944 \times 10^{-4}\text{ m}^4 \end{aligned}$$

The stresses σ_1 and σ_2 due to the internal forces are

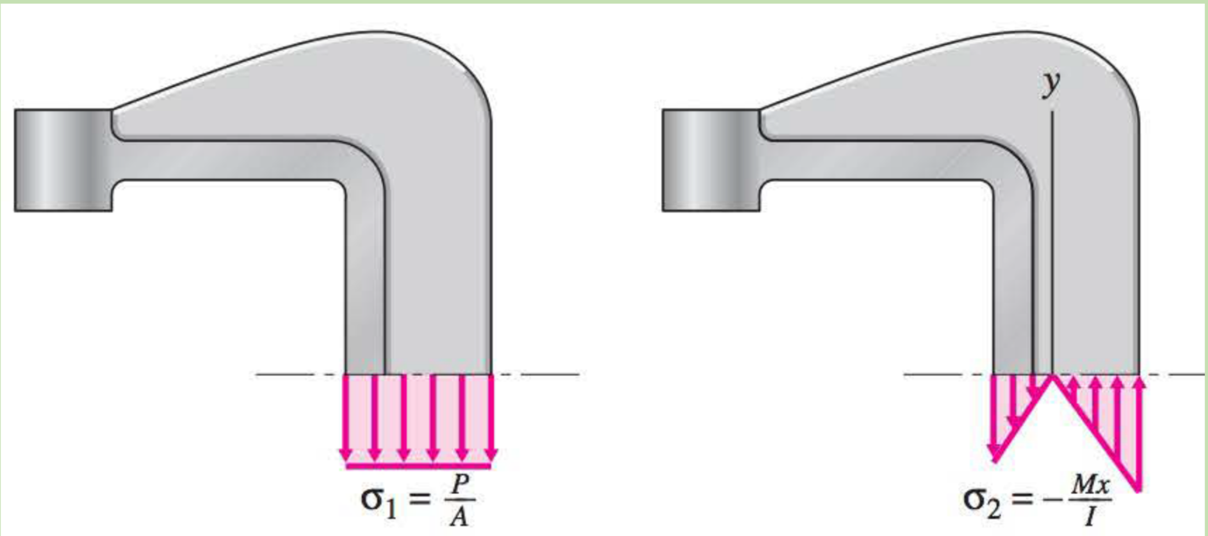
$$\sigma_1 = \frac{F}{A} = \frac{P}{0.0432\text{ m}^2}$$

The maximum tensile flexural stress occurs at the left edge of section NA (point A) and is

$$\sigma_{2A} = -\frac{Mx_A}{I} = -\frac{(0.51P) \times (-110\text{ mm})}{2.8944 \times 10^{-4}\text{ m}^4}$$

The maximum compressive flexural stress occurs at the right edge of section NA (point B) and is

$$\sigma_{2B} = -\frac{Mx_B}{I} = -\frac{(0.51P) \times (190\text{ mm})}{2.8944 \times 10^{-4}\text{ m}^4}$$



The distributions of stresses a 1 and a2 are shown in figures above, respectively. Superimposing the normal stresses at points A and B gives

$$\sigma_A = \sigma_1 + \sigma_{2A} = \frac{P}{0.0432\ \mathrm{m^2}} + \frac{(0.51P)\times(110\ \mathrm{mm})}{2.8944\times10^{-4}\ \mathrm{m^4}} = 216.971\ P\ (\mathrm{T})$$

$$\sigma_B = \sigma_1 + \sigma_{2B} = \frac{P}{0.0432\ \mathrm{m^2}} - \frac{(0.51P)\times(190\ \mathrm{mm})}{2.8944\times10^{-4}\ \mathrm{m^4}} = -311.64\ P = 311.64\ P\ (\mathrm{C})$$

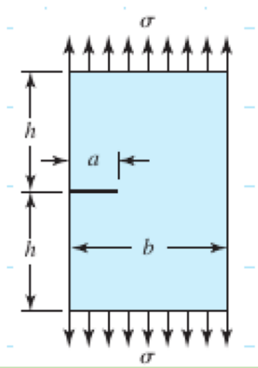
Quadrant condition	Failure criteria
$\sigma_A \geq \sigma_B \geq 0$	$\sigma_A = \frac{S_{ut}}{n}$
$\sigma_A \geq 0 \geq \sigma_B$ and $\left\lvert\frac{\sigma_B}{\sigma_A}\right\rvert \leq 1$	$\sigma_A = \frac{S_{ut}}{n}$
$\sigma_A \geq 0 \geq \sigma_B$ and $\left\lvert\frac{\sigma_B}{\sigma_A}\right\rvert > 1$	$\frac{(S_{uc}-S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$
$0 \geq \sigma_A \geq \sigma_B$	$\sigma_B = -\frac{S_{uc}}{n}$

$$\Rightarrow \begin{cases} \sigma_A \leq \frac{S_{ut}}{n} \\ \sigma_B \leq -\frac{S_{uc}}{n} \end{cases} \Rightarrow \begin{cases} 216.971\ P \leq \frac{170\ \mathrm{MPa}}{2.5} \\ -311.64\ P \leq \frac{650\ \mathrm{MPa}}{2.5} \end{cases} \Rightarrow \begin{cases} P \leq 313.4\ \mathrm{kN} \\ P \leq 834.3\ \mathrm{kN} \end{cases}$$

Therefore, the allowable load P using the Coulomb-Mohr theory and a factor of safety of $n = 2.5$ is equal to 313.4 kN.

Problem 2

A plate 100 mm wide, 200 mm long, and 12 mm thick is loaded in tension in the direction of the length. The plate contains a crack as shown with the crack length of 16 mm. The material is steel with $K_{Ic} = 80\ \text{MPa}\sqrt{\text{m}}$, and $S_y = 950$ MPa. Determine the maximum possible load that can be applied before the plate (a) yields, and (b) has uncontrollable crack growth



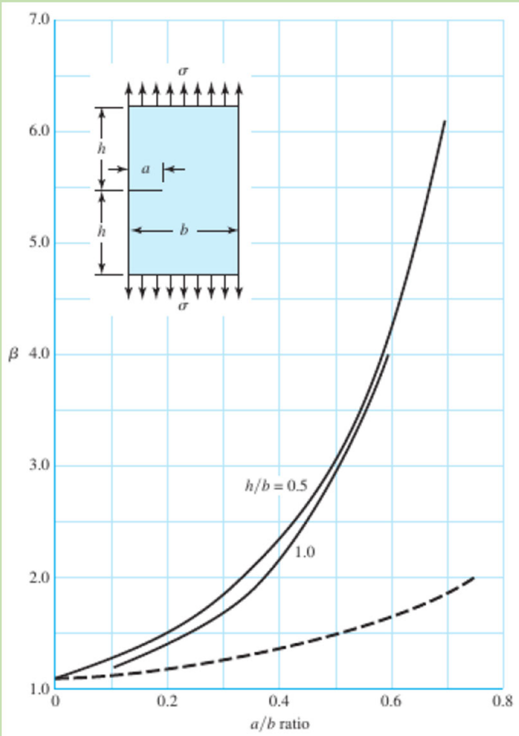
Solution:

For this question, we are asked to determine the maximum possible load that can be applied before the plate (a) yields, and (b) has uncontrollable crack growth.

(a)

$$F = \frac{S_y}{n} A = \frac{(950\ \text{MPa})}{1} \times (12\ \text{mm}) \times (100\ \text{mm}) = 1140\ \text{kN}$$

(b)



$$\begin{cases} \dfrac{a}{b} = \dfrac{16\ \text{mm}}{100\ \text{mm}} = 0.16 \\ \dfrac{h}{b} = \dfrac{100\ \text{mm}}{100\ \text{mm}} = 1 \end{cases} \Longrightarrow \beta = 1.3$$

$$K = K_{Ic} = \beta\sigma\sqrt{\pi a} = \frac{\beta F\sqrt{\pi a}}{A}$$

$$F = \frac{K_{Ic}A}{\beta\sqrt{\pi a}} = \frac{\left(80\ \text{MPa}\sqrt{\text{m}}\right) \times (12\ \text{mm}) \times (100\ \text{mm})}{1.3 \times \sqrt{\pi \times (16\ \text{mm})}} = 329.38\ \text{kN}$$

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