



ME 1071: Applied Fluids

# **Lecture 3 Internal Incompressible Viscous Flow**

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Spring 2021

# Weekly Study Plan



Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
<b>5</b>	<b>Apr. 6</b>	<b>Exam I</b>
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 27	Chapter 9: External Incompressible Viscous Flow
9	May. 4	Chapter 11: Flow in Open Channels
10	May. 11	Chapter 11/Exam II Review
<b>11</b>	<b>May. 18</b>	<b>Exam II</b>
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review

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# Outlines

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- **Multiple-Path Systems**
- **Restriction Flow Meters for Internal Flows**

# Outlines

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- **Multiple-Path Systems**
- **Restriction Flow Meters for Internal Flows**

# Multiple-Path Systems



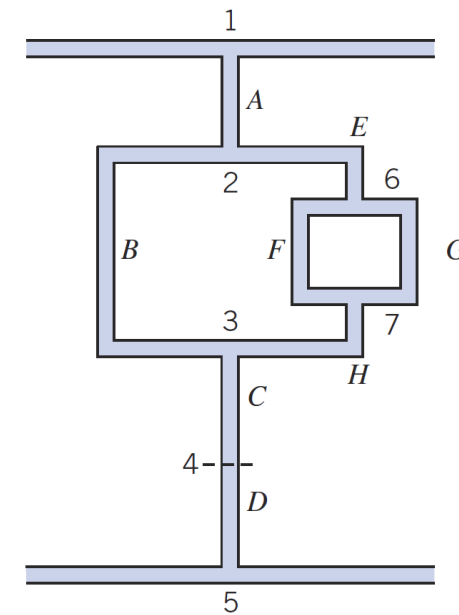
## Additional Rules (comparing to one-path system)

- The net flow out of any node (junction) is zero
- Each node has a **unique** pressure head (HGL), sometimes the gravity effect can be neglected

For a pipe that branches out into two (or more) parallel pipes and then rejoins at a junction downstream, the total flow rate is the sum of the flow rates in the individual pipes. The pressure drop (or head loss) in each individual pipe connected in parallel must be the same.

## To Solve the Multiple-Path Flow Problem

- Manually iterate energy equation and friction factor for each branch to satisfy all constraints, or
- Directly solve, simultaneously, complete set of equations using (for example) Excel, MATLAB (coding)



A:	$L = 10 \text{ ft}$ , $D = 1.5 \text{ in.}$
B:	$L = 20 \text{ ft}$ , $D = 1.5 \text{ in.}$
C:	$L = 10 \text{ ft}$ , $D = 2 \text{ in.}$
D:	$L = 10 \text{ ft}$ , $D = 1.5 \text{ in.}$
E:	$L = 5 \text{ ft}$ , $D = 1.5 \text{ in.}$
F:	$L = 10 \text{ ft}$ , $D = 1 \text{ in.}$
G:	$L = 10 \text{ ft}$ , $D = 1.5 \text{ in.}$
H:	$L = 5 \text{ ft}$ , $D = 2 \text{ in.}$

Schematic of part of a pipe network

# Multiple-Path Systems



For the cast-iron water pipe network shown, the pressure head at point 1 is 30 m of water and point 5 is open to the atmosphere. Find the flow rates in each pipe.

**Assumption:** 1. No gravity effects, 2. No minor losses.

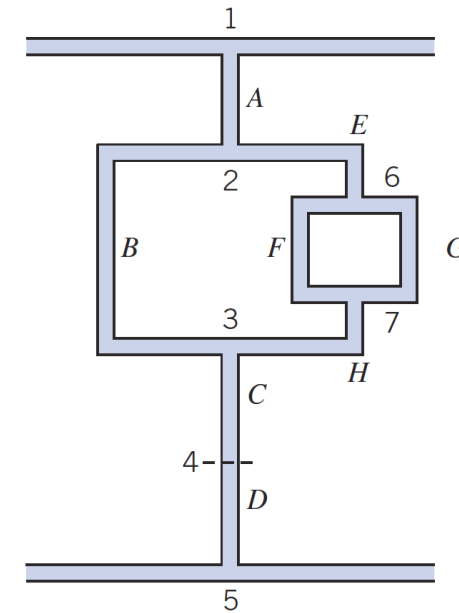
$$\text{Node 2: } Q_A = Q_B + Q_E \quad h_{1-5}: h = h_A + 2h_B + h_C + h_D$$

$$\text{Node 6: } Q_E = Q_F + Q_G \quad h_{2-3}: h_B = h_E + h_F + h_H$$

$$Q_A = Q_C = Q_D \quad h_{6-7}: h_F = h_G$$

$$Q_E = Q_H$$

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2} \quad \frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{e/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right)$$



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Schematic of part of a pipe network

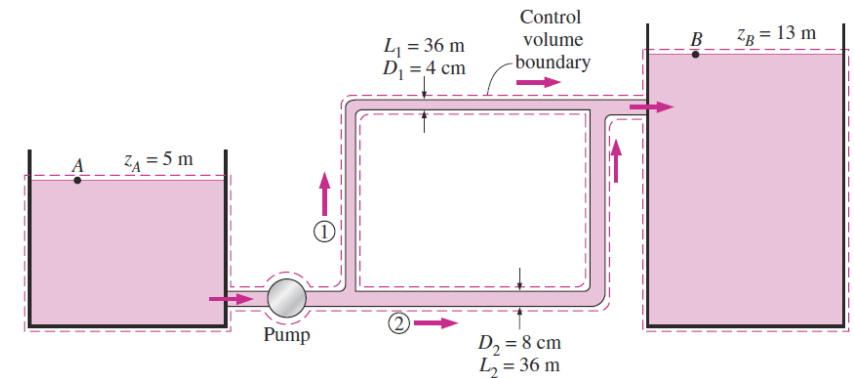
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# Multiple-Path Systems



Water at 20° C is to be pumped from a reservoir ( $z_A = 5$  m) to another reservoir at a higher elevation ( $z_B = 13$  m) through two 36-m-long pipes connected in parallel. The pipes are made of commercial steel, and the diameters of the two pipes are 4 and 8 cm. Water is to be pumped by a 70% efficient motor–pump combination that draws 8 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.

**Assumptions** 1 The flow is steady and incompressible. 2 The entrance effects are negligible, and thus the flow is fully developed. 3 The elevations of the reservoirs remain constant. 4 The minor losses and the head loss in pipes other than the parallel pipes are said to be negligible. 5 Flows through both pipes are turbulent (to be verified).



$$h_{l,1} = f_1 \frac{L_1}{D_1} \frac{\bar{V}_1^2}{2} = h_{l,2} = f_2 \frac{L_2}{D_2} \frac{\bar{V}_2^2}{2}$$

$$f_1 = 0.0221, f_2 = 0.0182$$

$$Q_1 = 0.0041 \text{ m}^3/\text{s}, Q_2 = 0.0259 \text{ m}^3/\text{s}$$

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# Outlines

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- **Multiple-Path Systems**
- **Restriction Flow Meters for Internal Flows**

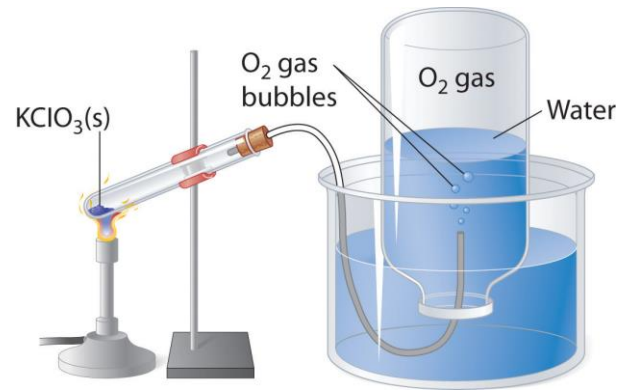


# Restriction Flow Meters for Internal Flows



## Direct Method

- Accumulation in a container
- Positive displacement flow meter



## Restriction Flow Meters for Internal Flows

- Orifice Plate; Flow Nozzle; Venturi; Laminar Flow Element

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# Restriction Flow Meters for Internal Flows



## Restriction Flow Meters for Internal Flows

- From Bernoulli and the mass equations

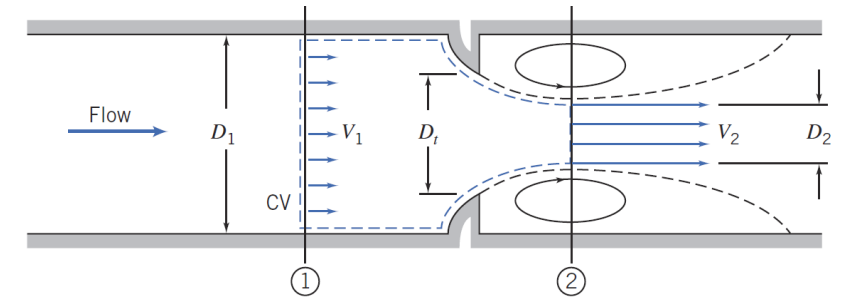
$$p_1 - p_2 = \frac{\rho}{2} (V_2^2 - V_1^2) = \frac{\rho V_2^2}{2} \left[ 1 - \left( \frac{V_1}{V_2} \right)^2 \right]$$

$$(-\rho V_1 A_1) + (\rho V_2 A_2) = 0$$

$$p_1 - p_2 = \frac{\rho V_2^2}{2} \left[ 1 - \left( \frac{A_2}{A_1} \right)^2 \right]$$

$$\dot{m}_{\text{theoretical}} = \frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2\rho(p_1 - p_2)}$$

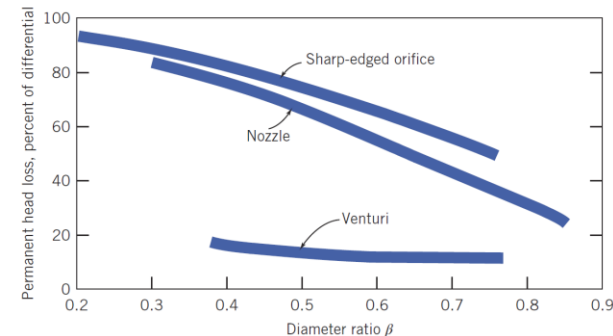
$$\dot{m}_{\text{actual}} = \frac{CA_t}{\sqrt{1 - (A_t/A_1)^2}} \sqrt{2\rho(p_1 - p_2)}$$



**Table 8.6**

Characteristics of Orifice, Flow Nozzle, and Venturi Flow Meters

Flow Meter Type	Diagram	Head Loss	Initial Cost
Orifice		High	Low
Flow Nozzle		Intermediate	Intermediate
Venturi		Low	High



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# Restriction Flow Meters for Internal Flows



## Linear Flow Meters

- Float Meter (Rotameter); Turbine; Vortex; Electromagnetic; Magnetic; Ultrasonic

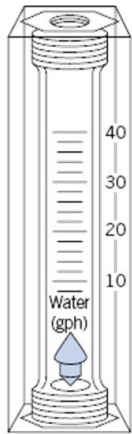


Fig. 8.25 Float-type variable-area flow meter. (Courtesy of Dwyer Instrument Co., Michigan City, Indiana.)

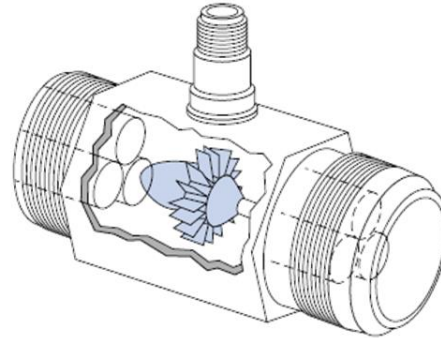
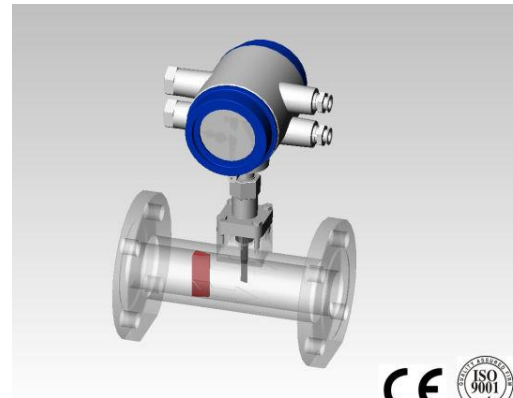


Fig. 8.26 Turbine flow meter. (Courtesy of Potter Aeronautical Corp., Union, New Jersey.)



Vortex flow meter



Ultrasonic flow meter

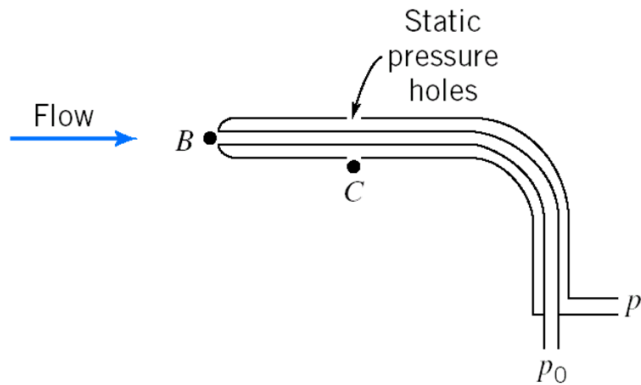
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# Restriction Flow Meters for Internal Flows



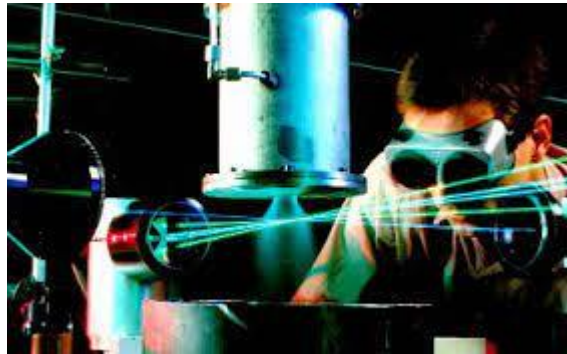
## Traversing Methods

- Pitot tube; Laser Doppler Anemometer; Hot-wire Anemometer

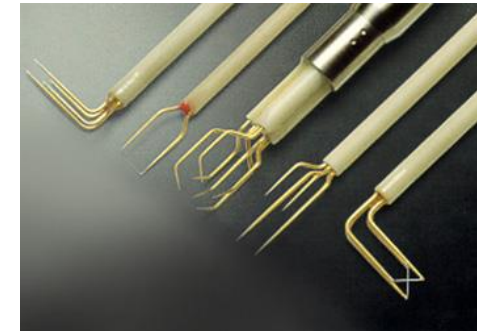
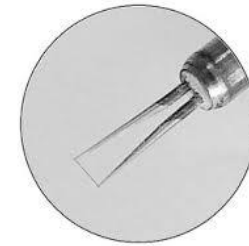


$$V = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

Pitot tube



Laser Doppler Anemometer



Hot-wire Anemometry

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# Exam 1

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**Tuesday April 6th from 1:50pm-4:25pm**

Closed-book, calculators are allowed but no smart phones or tablets.

Work must be done by hand on the solution form provided, scanned and uploaded to Black Board by 4:25pm

Each page of the solution form must be signed with name and student ID.

Will cover all of chapter 8 on internal viscous flow.

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# Homework

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This assignment is due by **6pm on March 30<sup>th</sup> (Today)**

- Upload your solution to BB.
- You can either type your solution out using a word editor like Microsoft Word or clearly hand write the information and then copy/scan your solution to a digital file.

# Homework



**Problem 8.68** A pipe of length  $10^3$  m and diameter 18 cm is laid at a slope of 1 in 150. Oil is pumped at the rate of 25 L/s. The specific gravity of the oil is 1.1 and viscosity is  $0.18 \text{ Ns/m}^2$ . Determine the head lost due to friction and also calculate the power required to pump the oil.

$$L = 1000 \text{ m}$$

$$D = 0.18 \text{ m}$$

$$Q = 0.025 \text{ m}^3/\text{s}$$

$$\mu = 0.18 \text{ N} \frac{\text{s}}{\text{m}^2}$$

$$\rho = 1100 \frac{\text{kg}}{\text{m}^3}$$

$$Q \rightarrow Re \rightarrow f \rightarrow h_{l_T} \rightarrow \Delta p$$

$$f = \frac{64}{Re} \quad (Re < 2300)$$

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$h_{l_T} - \Delta h_{pump} = \sum h_l + \sum h_{l_m} - \Delta h_{pump} = \left( \frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1 \right) - \left( \frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2 \right)$$

$$h_{pump} = h_l + gz_2 = h_l + g(1000) \left( \frac{1}{150} \right)$$

$$\dot{W}_{pump} = Q \rho h_{pump}$$

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# Homework



**Problem 8.75** Water flows through a 50 mm diameter tube that suddenly contracts to 25 mm diameter. The pressure drop across the contraction is 3.45 kPa. Determine the volume flow rate.

$$\Delta p \rightarrow h_{l_T} \rightarrow \bar{V} \rightarrow Q$$

$$h_{l_m} = \left( \frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1 \right) - \left( \frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2 \right) = \frac{KV_2^2}{2}$$

$$V_1 = V_2 \frac{A_2}{A_1}$$

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho \left( 1 - \left[ \frac{A_2}{A_1} \right]^2 + K \right)}}$$

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# Homework



**Problem 8.108** Two open standpipes are connected by a straight tube, as shown. Water flows by gravity from one standpipe to the other. For the instant shown, estimate the rate of change of water level in the left standpipe.

$\Delta p \rightarrow h_{l_T} \rightarrow \text{Guess } f \rightarrow \bar{V} \rightarrow Re \rightarrow f \rightarrow \text{if equal} \rightarrow \bar{V} \rightarrow Q$

$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left( \frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1 \right) - \left( \frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2 \right)$$

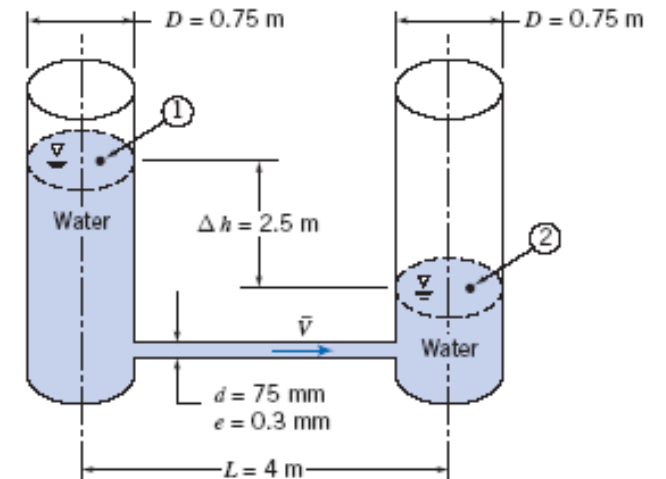
$$\left[ \frac{f(L - D)}{d} + K_{ent} + K_{exit} \right] \frac{V^2}{2} = g\Delta h$$

$$V = \sqrt{\frac{2g\Delta h}{\frac{f(L - D)}{d} + K_{ent} + K_{exit}}}$$

$\frac{e}{d} = 0.004$ , Figure 8.13 in fully rough zone

Guess  $f \rightarrow \bar{V} \rightarrow Re \rightarrow f \rightarrow \text{if equal} \rightarrow \bar{V}$

$$V_{p1} = \frac{A_t}{A_{p1}} V$$



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