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Mechanical Design 1

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Problem 1

A flat leaf spring has fluctuating stress of $\sigma_{\text{max}} = 360 \text{ MPa}$ and $\sigma_{\text{min}} = 160 \text{ MPa}$ applied for 80000 cycles. If the load changes to $\sigma_{\text{max}} = 320 \text{ MPa}$ and $\sigma_{\text{min}} = -200 \text{ MPa}$, how many cycles should the spring survive? The material is AISI 1020 CD and has a fully corrected endurance strength of $S_e = 175 \text{ MPa}$. Assume that f = 0.9. Use Miner's method.

Solution:

For this question, we are asked to determine how many cycles should the spring survive.

$$(\sigma_m)_1 = \frac{(360 \text{ MPa}) + (160 \text{ MPa})}{2} = 260 \text{ MPa}$$

$$(\sigma_a)_1 = \frac{(360 \text{ MPa}) - (160 \text{ MPa})}{2} = 100 \text{ MPa}$$

$$\sigma_{rev} = \frac{(\sigma_a)_1}{1 - \left[\frac{(\sigma_m)_1}{S_{ut}}\right]^2} = \frac{(100 \text{ MPa})}{1 - \left[\frac{(260 \text{ MPa})}{(470 \text{ MPa})}\right]^2} = 223.8 \text{ MPa} > S_e$$

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{[(0.9) \times (470 \text{ MPa})]^2}{(175 \text{ MPa})} = 1022.5 \text{ MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{(0.9) \times (470 \text{ MPa})}{(175 \text{ MPa})}\right] = -0.127767$$

$$N_1 = \left[\frac{(223.8 \text{ MPa})}{(1022.5 \text{ MPa})}\right]^{\frac{1}{-0.127767}} = 145920$$

$$(\sigma_m)_1 = \frac{(320 \text{ MPa}) + (-200 \text{ MPa})}{2} = 60 \text{ MPa}$$



$$(\sigma_a)_1 = \frac{(320 \text{ MPa}) - (-200 \text{ MPa})}{2} = 260 \text{ MPa}$$

$$\sigma_{rev} = \frac{(\sigma_a)_1}{1 - \left[\frac{(\sigma_m)_1}{S_{ut}}\right]^2} = \frac{(260 \text{ MPa})}{1 - \left[\frac{(60 \text{ MPa})}{(470 \text{ MPa})}\right]^2} = 298.0 \text{ MPa} > S_e$$

$$N_2 = \left(\frac{\sigma_{rev}/n}{a}\right)^{\frac{1}{b}} = \left[\frac{(298.0 \text{ MPa})}{(1022.5 \text{ MPa})}\right]^{\frac{1}{-0.127767}} = 15520$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \Rightarrow \frac{(80000)}{(145920)} + \frac{n_2}{(15520)} = 1 \Rightarrow n_2 = 7000$$

Problem 2

A 1-m-long cantilever spring is composed of eight graduated leaves. The leaves are 50 mm wide. A load of 2000 N at the end of the spring causes a deflection of 75 mm. The spring is made of steel with modulus of elasticity of 207 GPa. Determine the thickness of the leaves and the maximum bending stress

Solution:

For this question, we are asked to determine the thickness of the leaves and the maximum bending stress.

$$\delta = \frac{6PL^3}{Enbh^3}$$

$$\Rightarrow h = \sqrt[3]{\frac{6PL^3}{En\delta b}} = \sqrt[3]{\frac{6 \times (2000 \text{ N}) \times (1 \text{ m})^3}{(207 \text{ GPa}) \times (8) \times (75 \text{ mm}) \times (50 \text{ mm})}} = 12.46 \text{ mm}$$

$$\sigma_{max} = \frac{6PL}{nbh^2} = \frac{6 \times (2000 \text{ N}) \times (1 \text{ m})}{(8) \times (50 \text{ mm}) \times (12.46 \text{ mm})^2} = 193.37 \text{ MPa}$$



