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Mechanical Design 1

05 Assignment

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Mechanical Design 1

Class Section 01

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Problem 1

A 20-mm-diameter steel bar is to be used as a torsion spring. If the torsional stress in the bar is not to exceed 110 MPa when one end is twisted through an angle of 15°, what must be the length of the bar?

Solution:

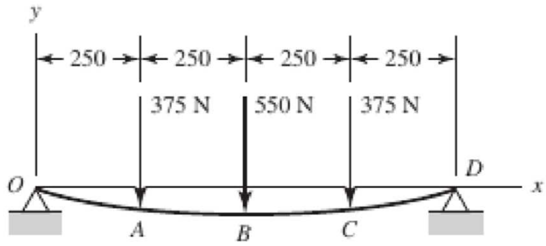
For this question, we are asked to determine what the length of the bar must be.

$$\theta = \frac{TL}{GJ} = \frac{\frac{\tau J}{r} L}{GJ} = \frac{\tau L}{Gr}$$

$$\Rightarrow L = \frac{Gr\theta}{\tau} = \frac{(79.3\ \ \mathrm{GPa}) \times (10\ \ \mathrm{mm}) \times \left(\frac{15°}{180°}\pi\right)}{(110\ \ \mathrm{MPa})} = 1.887\ \ \mathrm{m}$$

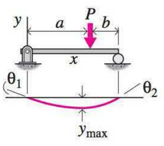
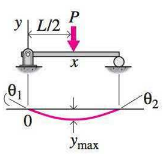
Problem 2

Using superposition for the bar shown, determine the minimum diameter of a steel shaft for which the maximum deflection is 2 mm (all dimensions in mm and E = 207GPa)



Solution:

For this question, we are asked to, determine the minimum diameter of a steel shaft for which the maximum deflection is 2 mm (all dimensions in mm and E = 207GPa) using superposition for the bar shown.

5	 $b < a$	$\theta_1 = -\frac{Pb(L^2-b^2)}{6LEI}$ at $x = 0$ $\theta_2 = +\frac{Pa(L^2-a^2)}{6LEI}$ at $x = L$	$y_{max} = -\frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}LEI}$ at $x = \sqrt{(L^2-b^2)/3}$ $y_{\substack{center \\ not\ max}} = -\frac{Pb(3L^2-4b^2)}{48EI}$	$y = -\frac{Pbx}{6EIL}(L^2-b^2-x^2)$ $0 \le x \le a$
6		$\theta_1 = -\frac{PL^2}{16EI}$ at $x = 0$ $\theta_2 = +\frac{PL^2}{16EI}$ at $x = L$	$y_{max} = -\frac{PL^3}{48EI}$ at $x = L/2$	$y = -\frac{Px}{48EI}(3L^2-4x^2)$ $0 \le x \le \frac{L}{2}$

From the picture above, I can know that

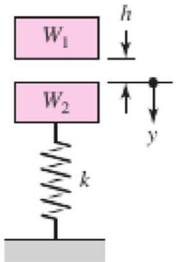
$$-\frac{Pb(3L^2-4b^2)}{48EI}\times 2-\frac{PL^3}{48EI}=y_{max}$$

$$-\frac{(375\ \ \mathrm{N})\times(250\ \ \mathrm{mm})\times[3\times(1000\ \ \mathrm{mm})^2-4\times(250\ \ \mathrm{mm})^2]}{48\times(207\ \ \mathrm{GPa})\times\frac{\pi D^4}{64}}\times 2$$

$$-\frac{(550\ \ \mathrm{N})\times(1000\ \ \mathrm{mm})^3}{48\times(207\ \ \mathrm{GPa})\times\frac{\pi D^4}{64}}=-2\ \ \mathrm{mm}\Longrightarrow D=32.3\ \ \mathrm{mm}$$

Problem 3

As shown in the figure below, the weight W_1 strikes W_2 from a height h. If $W_1 = 40$ N, $W_2 = 400$ N, $h = 200$ mm, and $k = 32$ kN/m, find the maximum values of the spring force and the deflection of W_2. Assume that the impact between W_1 and W_2 is inelastic, ignore the mass of the spring, and solve using energy conservation



Solution:

For this question, we are asked to find the maximum values of the spring force and the deflection of W_2.

$$\frac{1}{2}\frac{W_1}{g}v_1^2 = W_1 h$$

$$\Rightarrow v_1 = \sqrt{2gh}$$

$$\frac{W_1}{g}\sqrt{2gh} = \frac{W_1 + W_2}{g}v_2$$

$$\Rightarrow v_2 = \frac{W_1}{W_1 + W_2}\sqrt{2gh}$$

$$\frac{1}{2}\frac{W_1 + W_2}{g}\left(\frac{W_1}{W_1 + W_2}\sqrt{2gh}\right)^2 + (W_1 + W_2)x = \frac{1}{2}kx^2$$

$$\frac{1}{2} \times \frac{(40\ \mathrm{N}) + (400\ \mathrm{N})}{(9.81\ \mathrm{m/s^2})} \times \left(\frac{(40\ \mathrm{N})}{(40\ \mathrm{N}) + (400\ \mathrm{N})}\sqrt{2 \times (9.81\ \mathrm{m/s^2}) \times (200\ \mathrm{mm})}\right)^2$$

$$+ [(40\ \mathrm{N}) + (400\ \mathrm{N})]x = \frac{1}{2} \times (32\ \mathrm{kN/m})x^2$$

$$\Rightarrow x = 29.06\ \mathrm{mm}$$

$$\Longrightarrow F = kx = (32\ \mathrm{kN/m}) \times (29.06\ \mathrm{mm}) = 930.0\ \mathrm{N}$$

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