



Second edition

Mechanical DESIGN

Peter Childs

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Mechanical Design

This book is dedicated to Hazel, Tabitha and Angus

Mechanical Design

Second edition

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PREFACE

The overall aims of this book are to introduce the subject of total design and the design and selection of various common mechanical engineering components and machine elements. These provide ‘building blocks’ with which the designer and engineer can practise their art.

The approach adopted for defining design follows that developed by the SEED (Sharing Experience in Engineering Design) programme where design is viewed as the ‘the total activity necessary to provide a product or process to meet a market need’. Within this framework the text is concentrated on developing detailed mechanical design skills in the areas of bearings, shafts, gears, seals, belt and chain drives, clutches and brakes, springs and fasteners. Where standard components are available from manufacturers, the steps necessary for their specification and selection are developed.

The framework used within the text has been to provide descriptive and illustrative information to introduce principles and individual components and to expose the reader to the detailed methods and calculations necessary to specify and design or select a component. To provide the reader with sufficient information to develop the necessary skills to repeat calculations and selection processes, detailed examples and worked solutions are supplied throughout the text.

This book is principally a year/level 1 and 2 undergraduate text. Prerequisite skills include some year 1 undergraduate mathematics, fluid mechanics and heat transfer, principles of materials, statics and dynamics. However, as the subjects are introduced in a descriptive and illustrative format and as full worked solutions are provided, it is possible for readers without this formal level of education to benefit from this book. The text is specifically

aimed at automotive and mechanical engineering degree programmes and would be of value for modules in design, mechanical engineering design, design and manufacture, design studies, automotive power-train and transmission and tribology, as well as modules and project work incorporating a design element requiring knowledge about any of the content described.

The aims and objectives described are achieved by short introductory chapters on total design, mechanical engineering and machine elements followed by nine chapters on machine elements covering: bearings, shafts, gears, seals, chain and belt drives, clutches and brakes, springs and fasteners. Chapters 13 and 14 introduce casings and enclosures and sensors and actuators, key features of most forms of mechanical technology. The subject of tolerancing from a component to a process level is introduced in Chapter 15. The last chapter serves to present the subject of design management and an integrated design using the detailed design aspects covered within the book. The design methods where appropriate are developed to national and international standards.

The first edition of this text served to introduce a variety of machine elements as building blocks with which design of mechanical devices can be undertaken. The material covered by this first edition is still valid and the approach adopted of introducing and explaining the aspects of technology by means of text, photographs, diagrams and step-by-step procedures has been well received. A number of important machine elements were omitted in the first edition such as fasteners, springs, sensors and actuators. They are included here. Reworking of aspects of the original text includes the introductory chapters covering total

design, the scope of mechanical engineering and machine elements. A new chapter is included on casings and enclosures and the final chapter has been rewritten to provide an integrated approach. The use of multiple worked examples

and completed solutions has been maintained as these proved to be both useful and popular in the first edition.

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1

DESIGN

The aims of this book are to present an overview of the design process and design methodology, and to introduce the technology and selection of a number of specific machine elements that are fundamental to a wide range of engineering. This chapter introduces the design process from an inventor's perspective to a more formal model called 'total design'.

LEARNING OBJECTIVES

At the end of this chapter you should:

- be able to recognize the iterative steps associated with the design process;
- be aware of the process involved in developing a product design specification from a market brief;
- be able to develop a morphological design chart to aid the development of conceptual solutions;
- be able to participate in brainstorming activities;
- recognize the scope of available technologies to the designer.

1.1 Introduction

The term 'design' is popularly used to refer to an object's aesthetic appearance with specific reference to its form or outward appearance, as well as its function. For example, we often refer to designer clothes, design icons and beautiful cars, examples of which are given in Figures 1.1 and 1.2. In these examples it is both visual impact, appealing to our



Figure 1.1 Piaggio's Vespa launched in 1946 (Piaggio Museo). The Vespa was an early example of monocoque construction where the skin and frame are combined as a single construction to provide appropriate rigidity and mounting for the vehicle's components and riders.



Figure 1.2 The Audi TT, originally launched in 1998, and a contender for the most attractive sports car of the 20th century. (Courtesy of Audi.)

visual perception, and the concept of function, that the product will fulfil a range of requirements, which are important in defining so-called 'good design'.

The word ‘design’ comes from the Latin ‘designare’, which means to designate or mark out. Design can be taken to mean all the processes of conception, invention, visualization, calculation, refinement and specification of details that determine the form of a product. Design generally begins with either a need or requirement or, alternatively, an idea. It ends with a set of drawings or computer representations and other information that enables a product to be manufactured and utilized.

In this book, design is defined as ‘the total activity necessary to provide a product or process to meet a market need’. This definition comes from the SEED (Sharing Experience in Engineering Design, now DESIG the Design Education Special Interest Group of the Design Society) model, see Pugh (1990).

1.2 The design process

Probably from your own experience you will know that design can consist of examining a design need and working on the problem by means of sketches, models, brain-storming, calculations as necessary, development of styling as appropriate, making sure the product fits together and can be manufactured, and calculation of the costs. The process of design can be represented schematically to levels of increasing formality and complexity. Figure 1.3 represents the traditional approach associated with lone inventors. This model comprises the generation of the ‘bright idea’, drawings and calculations giving form or shape to the idea, judgement of the design and re-evaluation if necessary, resulting in the generation of the end product. The process of evaluation and reworking an idea is common in design and is represented in the model by the iteration arrow taking the design activity back a step so that the design can be improved. Figure 1.4 illustrates the possible results from this process for a helmet providing peripheral and reverse vision.

Figure 1.5 shows a more formal description of the design process that might be associated with

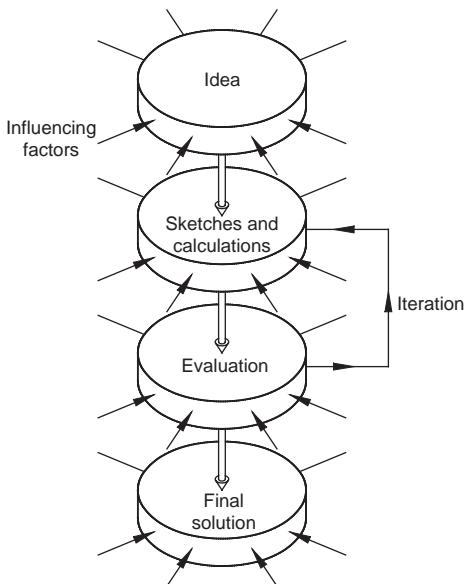


Figure 1.3 The traditional and familiar ‘inventor’s’ approach to design.

engineers operating within a formal company management structure. The various terms used in Figure 1.5 are described in the following text.

- **Recognition of need.** Often design begins when an individual or company recognizes a need, or identifies a potential market, for a product, device or process. Alternatively ‘need’ can be defined as when a company decides to re-engineer one of its existing products (for example, producing a new car model). The statement of need is sometimes referred to as the brief or market brief.
- **Definition of problem.** This involves all the specifications of the product or process to be designed. For example, this could include inputs and outputs, characteristics, dimensions and limitations on quantities.
- **Synthesis.** This is the process of combining the ideas developed into a form or concept, which offers a potential solution to the design requirement. The term synthesis may be familiar from its use in chemistry where it is used to describe the process of producing a

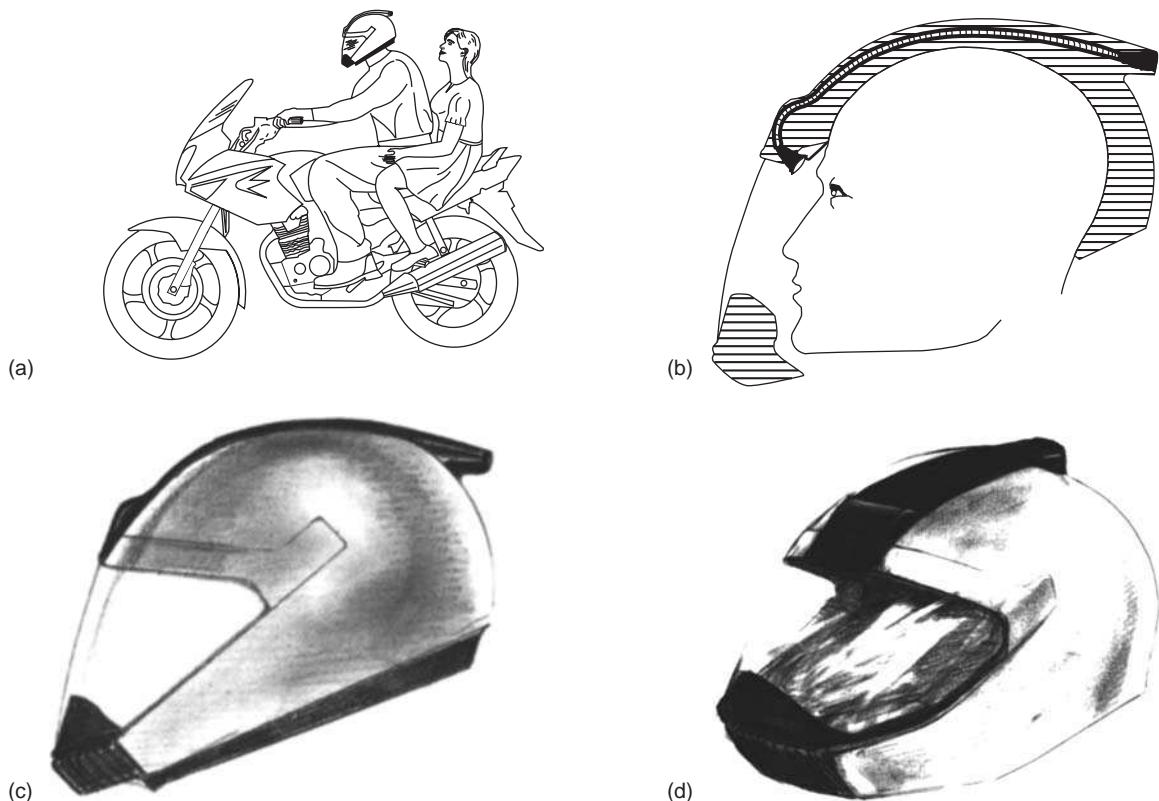


Figure 1.4 Panoramic helmet by Alberto Meda and Denis Santachiara (Manzini, 1989). (a) The need: to be able to view behind you. (b) The idea: an optical link using fibre optics and lenses. (c) & (d) Practical sketches showing the concept.

compound by a series of reactions of other substances.

- **Analysis.** This involves the application of engineering science: subjects explored extensively in traditional engineering courses, such as statics and dynamics, mechanics of materials, fluid flow and heat transfer. These engineering ‘tools’ and techniques can be used to examine the design to give quantitative information such as whether it is strong enough or will operate at an acceptable temperature. Analysis and synthesis invariably go together. Synthesis means putting something together and analysis means resolving something into its constituent parts or taking it to pieces. Designers have to synthesize something before it can be analysed. The famous chicken and the egg scenario!

When a product is analysed some kind of deficiency or inadequacy may be identified requiring the synthesis of a new solution prior to re-analysis and repetition of the process until an adequate solution is obtained.

- **Optimization.** This is the process of repetitively refining a set of often-conflicting criteria to achieve the best compromise.
- **Evaluation.** This is the process of identifying whether the design satisfies the original requirements. It may involve assessment of the analysis, prototype testing and market research.

Although Figures 1.3 and 1.5 at first sight show design occurring in a sequential fashion, with one task following another, the design process may actually occur in a step forward, step back fashion.

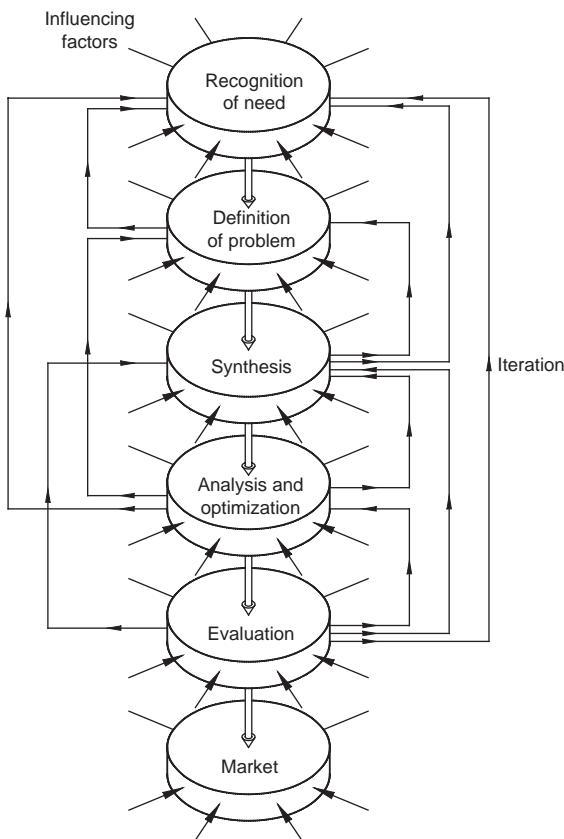


Figure 1.5 The design process illustrating some of the iterative steps associated with the process.

For instance, you may propose a solution to the design need and then perform some calculations or judgements, which indicate that the proposal is inappropriate. A new solution will need to be put forward and further assessments made. This is known as the iterative process of design and forms an essential part of refining and improving the product proposal.

Note that the flow charts shown in Figures 1.3 and 1.5 do not represent a method of design, but rather a description of what actually occurs within the process of design. The method of design used is often unique to the engineer or design team. Design methodology is not an exact science and there are indeed no guaranteed methods of design. Some designers work in a progressive fashion, others work on several aspects simultaneously.

1.2.1 Case study

The process identified in Figure 1.5 can be illustrated by example. Following some initial market assessments the board of a plant machinery company has decided to proceed with the design of a new product for transporting pallets around factories. The board has in mind a forklift truck, but do not wish to constrain the design team to this concept alone. The process of the design can be viewed in terms of the labels used in Figure 1.5 as listed below.

- **Recognition of need (or market brief).** The company has identified a potential market for a new pallet-moving device.
- **Definition of problem.** A full specification of the product desired by the company should be written. This allows the design team to identify whether their design proposals meet the original request. Here a long list of information needs to be developed and clarified before design can proceed. For example for the pallet-moving device being explored here this would likely include aspects for consideration such as:
 - What sizes of pallet are to be moved?
 - What is the maximum mass on the pallet?
 - What is the maximum size of the load on the pallet?
 - What range of materials are to be moved and are they packaged?
 - What maximum height must the pallet be lifted?
 - What terrain must the pallet-moving device operate on?
 - What range is required for the pallet-moving device?
 - Is a particular energy source/fuel to be used?
 - What lifetime is required?
 - Are there manufacturing constraints to be considered?
 - What is the target sales price?
 - How many units can the market sustain?

- Is the device to be automatic or manned?
- What legal constraints need to be considered?

This list is not exhaustive and would require further consideration. The next step is to quantify each of the criteria. For instance the specification may yield that standard size pallets, see Figure 1.6, are involved, the maximum load to be moved is 1000 kg, the maximum volume of load is 2 m³, the reach must be up to 3 m, use on factory floor and asphalt surfaces, the pallet-moving device must be capable of moving a single pallet 100 m and must be able to repeat this task at least 300 times before refuelling if necessary, electricity, gas or diesel fuel, 7-year lifetime, production in an East European country, target selling price 9000 Euros, 12 000 units per year, manned use, design to ISO (International Organisation for Standardisation) and target country national standards (e.g. see BS ISO 509, BS ISO 6780, BS EN ISO 445, BS EN 1726-1, BS EN 13545, 99/705213 DC, ISO 18334, 99/712554 DC, BS 3726, BS 5639-1 and BS ISO 2330).

- **Synthesis.** This is often identified as the formative and creative stage of design. Some initial ideas must be proposed or generated in order for them to be assessed and improved. Concepts can be generated by imagination, experience or by the use of design techniques, such as morphological charts. Some evaluation should be made at this stage to reduce the number of

concepts requiring further work. Various techniques are available for this including merit and adequacy assessments.

- **Analysis.** Once a concept has been proposed it can then be analysed to determine whether constituent components can meet the demands placed on them in terms of performance, manufacture, cost and any other specified criteria. Alternatively, analysis techniques can be used to determine what size components need to be to meet the required functions.
- **Optimization.** Inevitably there are conflicts between requirements. In the case of the forklift truck, size, manoeuvrability, cost, aesthetic appeal, ease of use, stability and speed are not necessarily all in accordance with each other. Cost minimization may call for compromises on material usage and manufacturing methods. These considerations form part of the optimization of the product producing the best or most acceptable compromise between the desired criteria.
- **Evaluation.** Once a concept has been proposed and selected and the details of component sizes, materials, manufacture, costs and performance worked out, it is then necessary to evaluate it. Does the proposed design fulfil the specification? If it appears to, then further evaluation by potential customers and use of prototype demonstrators may be appropriate to confirm the functionality of the design, judge customer reaction and provide information of whether any aspects of the design need to be reworked or refined.

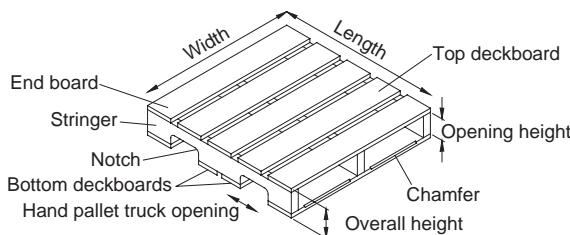


Figure 1.6 Pallet dimensions and terminology (see BS ISO 509, 99/712554 DC and 99/712555 DC).

1.3 Total design

The process of design has been the focus of research for many years and a number of design models and methodologies are available. Design methodology is a framework within which the designer can practise with thoroughness. One such approach called 'total design' has been proposed by the SEED programme (1985) and Pugh (1990)

and is illustrated schematically in Figure 1.7. This shows the core activities of design: marketing, specification, conceptual design, detailed design and marketing/selling. As in Figures 1.3 and 1.5 the iterative nature of design is accounted for where work on a design results in the need to go back and redo previous work in order to produce a better overall design to meet the requirements. Indeed it is sometimes necessary to go back several levels. An example might be the discovery at manufacture that an item cannot be made as envisaged and a new concept altogether is required. Ideally such a discovery should not occur, as every other level of the design process illustrated in Figure 1.7 should be considered at each stage. Each of the

design activities illustrated in Figure 1.7 is described in more detail in the following text. As it is the same process being described these descriptions are similar to those described for Figure 1.5.

- **Market.** The market phase refers to the assessment of sales opportunities or perceived need to update an existing product resulting in a statement sometimes called the market brief, design brief, brief or statement of need.
- **Specification.** Specification involves the formal statement of the required functions, features and performance of the product or process to be designed. Recommended practice from the outset of design work is to produce a product design specification that should be formulated from the statement of need. The product design specification is the formal specification of the product to be designed. It acts as the control for the total design activity because it sets the boundaries for the subsequent design. Further details of the product design specification are described in Section 1.4.
- **Conceptual design.** The early stages of design where the major decisions are to be made is sometimes called ‘conceptual design’. During this phase a rough idea is developed as to how a product will function and what it will look like. The process of conceptual design can also be described as the definition of the product’s morphology, how it is made up and its layout. Some methods such as brain-storming, morphological analysis and function trees used to aid the generation of concepts are described in Section 1.5.
- **Detailed design.** The detailed design phase consists of the determination of the specific shape and size of individual components, what materials should be used, how they fit together and the method of manufacture. Detailed design makes use of the many skills acquired and developed by engineers in the areas of analysis. It is the detailed design phase that can take up the bulk of the time spent on a design. However, as implied earlier it is wise to only

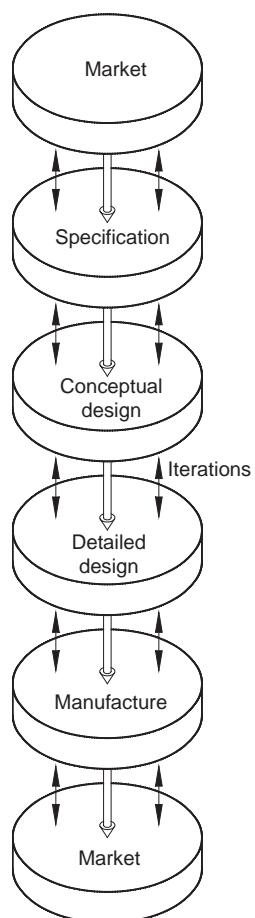


Figure 1.7 The total design core (after Pugh, 1990).

spend time on details once a sensible concept has been selected.

- **Manufacturing.** The manufacture phase, although identified as distinct within the structure, is typical of other phases in that it influences all the others. The design of any item must be such that it is feasible to manufacture it! The materials selected must be compatible with the manufacturing facilities and skills available and at acceptable costs to match marketing requirements. Manufacturing is so important that design strategies to reinforce its importance have been developed, such as design for assembly DFA and design for manufacture DFM (see for instance Boothroyd (1997)). More recently, the concept of concurrent engineering has been popular which is a systematic approach that encourages the developer from the outset to consider all the elements of a product life

cycle or process from concept to disposal, including quality control, scheduling and user requirements.

- **Marketing/sales.** The last phase, selling, is of course essential and should match the expectations of the initial market phase. This phase too has an impact on other phases within the design core. Information such as customer reaction to the product, any component failures or wear, damage during packaging and transportation should be fed back to influence the design of product revisions and future designs.

The double arrows shown in Figure 1.7 represent the flow of information and control from one activity to another, as well as the iterative nature of the process. For instance detailed design activity may indicate that an aspect of the conceptual

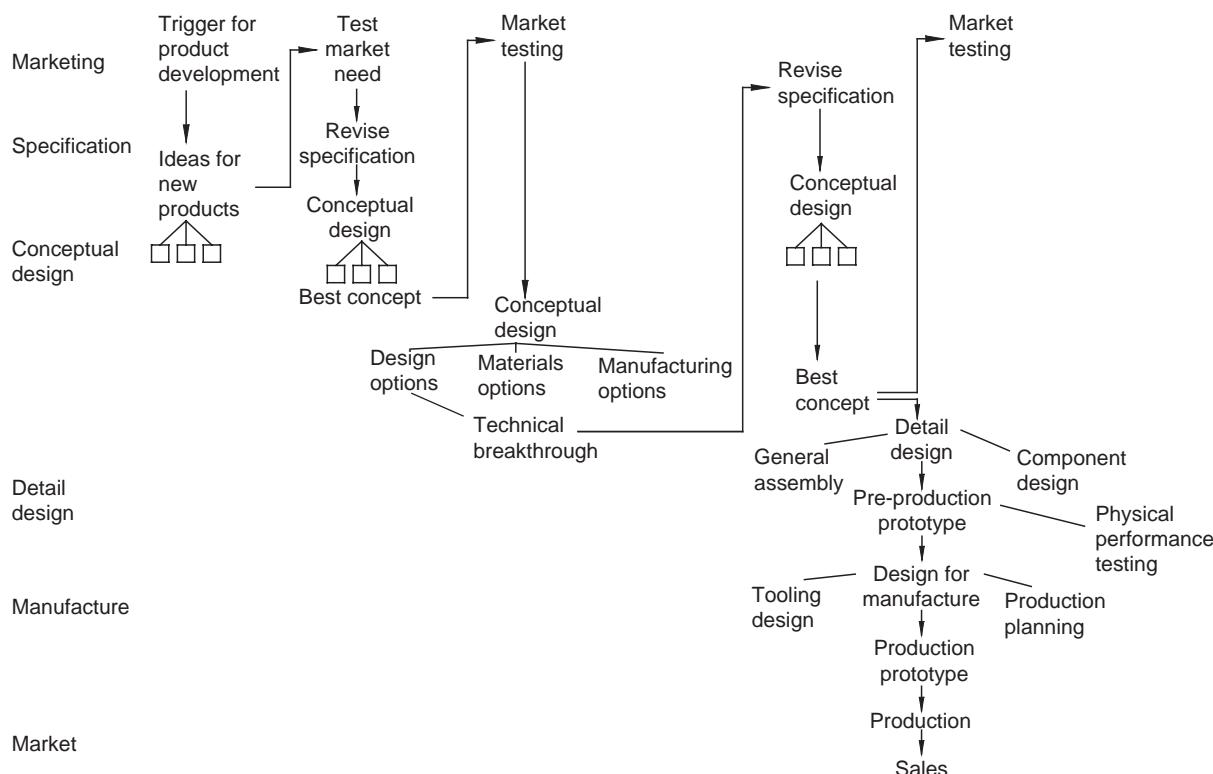


Figure 1.8 Design activities at different stages in product development (adapted from Baxter, 1995).

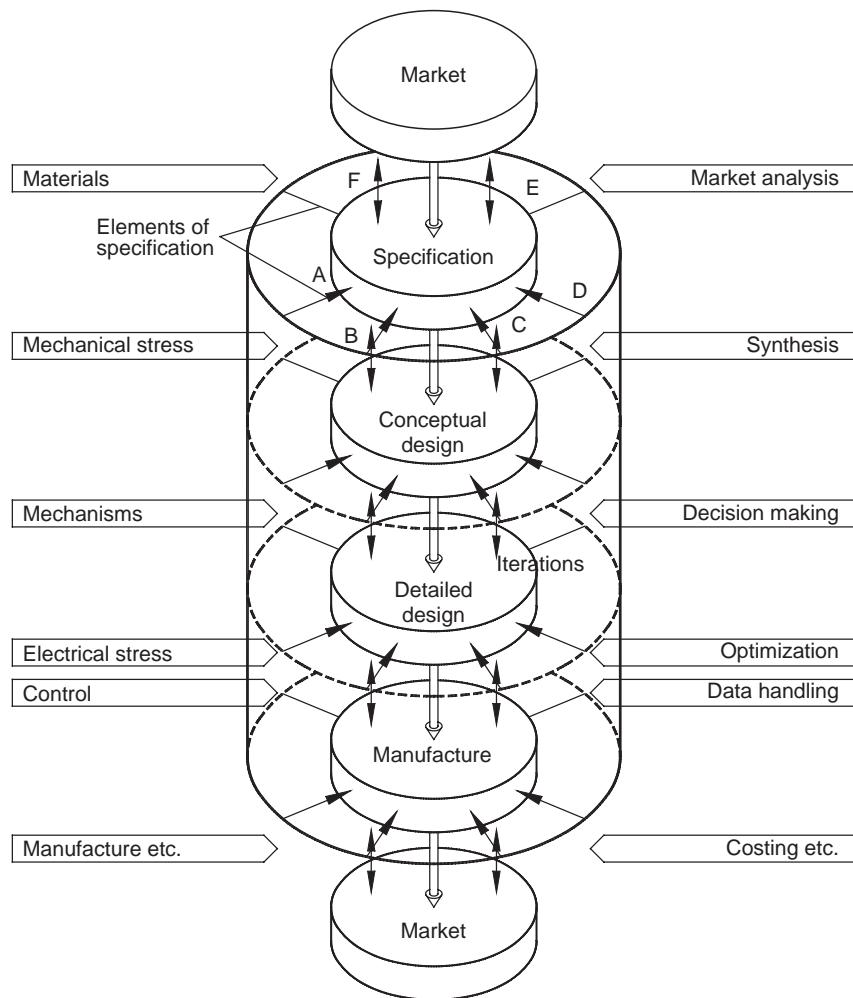


Figure 1.9 The total design process (after Pugh, 1990).

design is not feasible and must be reconsidered. Alternatively conceptual work may yield features, which have potential for additional marketing opportunities. In other words the activity on one level can and does interact dynamically with activities on other levels. Figure 1.8 illustrates the possible flow of this process during the development of a product.

Almost any product such as a vacuum cleaner, kettle, automobile or cordless hand-tool requires input from people of many disciplines, including engineering, legal and marketing and this requires considerable co-ordination. In industrial terms,

the integration comes about as a result of the partial design inputs from each discipline. In Figure 1.9 additional activities, such as market analysis, stressing and optimization, have been added to the design core as inputs. The effective and efficient design of any product invariably requires the use of different techniques and skills. The disciplines indicated are the designer's tool-kit and indicate the multi-disciplinary nature of design. The forklift truck example mentioned in Section 1.2.1 will require engine management and control systems, as well as the design of mechanical components. Although this text concentrates

on mechanical design, this is just one, albeit an important, interesting and necessary aspect of the holistic or total design activity.

A number of circumferential inputs have been shown as arrows in Figures 1.3, 1.5, 1.7 and 1.9. These represent elements of the specification listed in order of importance for each phase of design. The priority order of these specifications may alter for different phases of the design activity. The exact number will depend on the actual case under consideration.

Industry is usually concerned with total design. Total design is the systematic activity necessary from the identification of a market need to the commercialization of the product to satisfy the market need. Total design can be regarded as having a central core of activities consisting of the market potential, product specification, conceptual design, detailed design, manufacture and marketing.

1.4 Product design specification

Specification has different meanings depending on the context and the individuals concerned. Here specification refers to an activity that assists in providing a framework within which design can take place. Work on a design may commence with an identification of a need, which is sometimes summarized in the form of a statement commonly known as the ‘design brief’ or ‘design intent’. A design brief may provide some direction to initial design activity, but rarely sets sufficient bounds on it to allow a design team to know when their efforts are adequate, or not, to fulfil the requirements. In order to overcome this shortcoming and compel the individuals concerned to explore what is actually required of the product or process concerned a ‘product design specification’ (PDS) should be developed.

In developing a product design specification, bounds should be set on a wide range of relevant parameters for the product or process concerned.

The spider chart shown in Figure 1.10 provides a helpful checklist of aspects for consideration. A useful rule associated with specification, which will be stated frequently, is the mantra:

quantify wherever possible.

Having identified a general need or requirement and documented this in the form of a brief, the specification of the product to be developed should be generated. This is called the ‘product design specification’ or PDS. The PDS acts as the controlling mechanism, mantle or envelope for the total design activity. Whatever you are concerned with, the PDS gives you the terms of reference. The starting point for design activity is market research, analysis of the competition, literature and patent searching. Once this has been undertaken the PDS can be developed. Figure 1.11 shows the areas of information and research required to produce the PDS and Table 1.1 shows the information content of a typical PDS.

In a new design it is preferable to consider each aspect identified in Figure 1.10, however for the re-engineering of a current product, engineering expediency dictates that some aspects will be passed over. The PDS thus consists of a document covering a wide range of considerations. A possible format for the PDS is shown in Table 1.2 and explored in Example 1.1, although other types of pro-forma exist and are widely used. Table 1.3 shows just two of the 30+ items listed in Figure 1.10, but gives a sensible format for documentation of the PDS. The PDS is dynamic. If during the design of a product there is a good reason for changing the basic PDS this can be done. It should be noted, however, that the PDS may actually form part of contractual obligations and the legal implications must be addressed.

Undertaking a PDS will typically be an iterative process where initial parameters are identified and limits set, but more information may be required in order to confirm these. The process of developing a PDS will require input from people with a variety of skills and knowledge. It will very

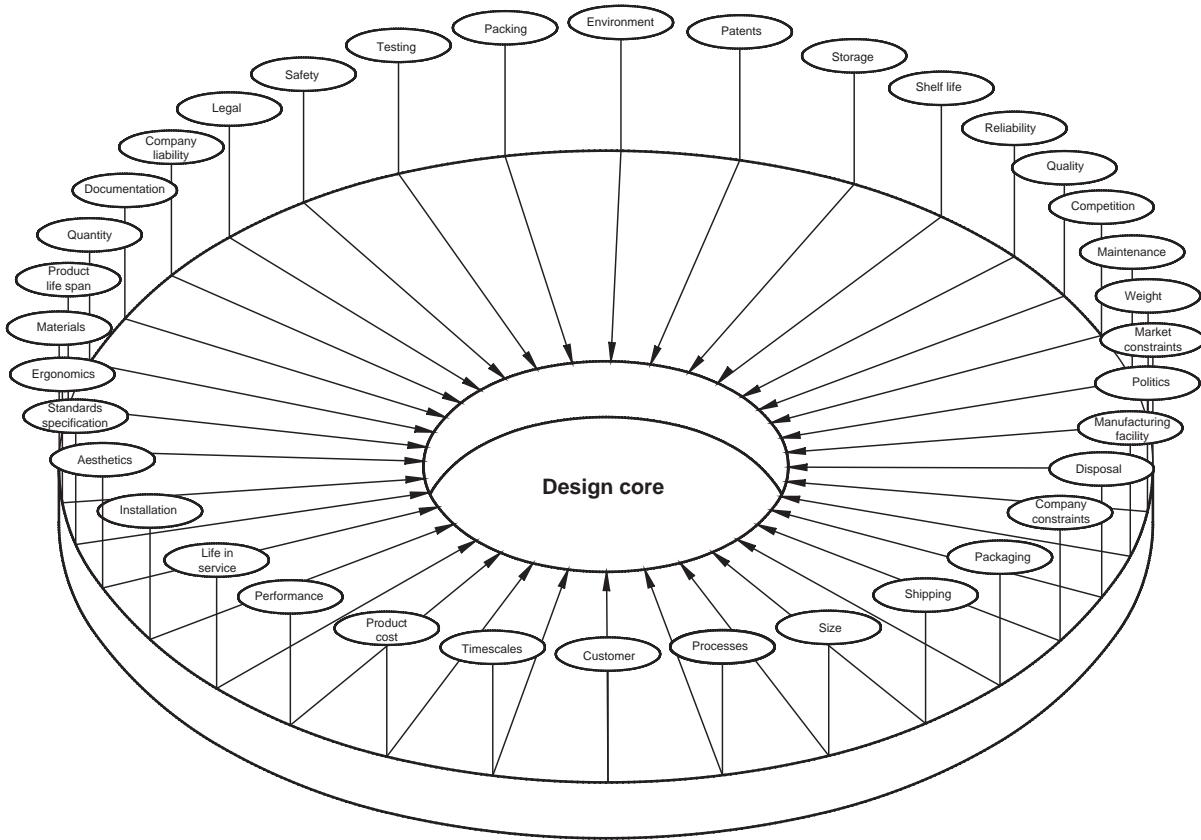


Figure 1.10 Information content of a typical product design specification (after Pugh, 1990).

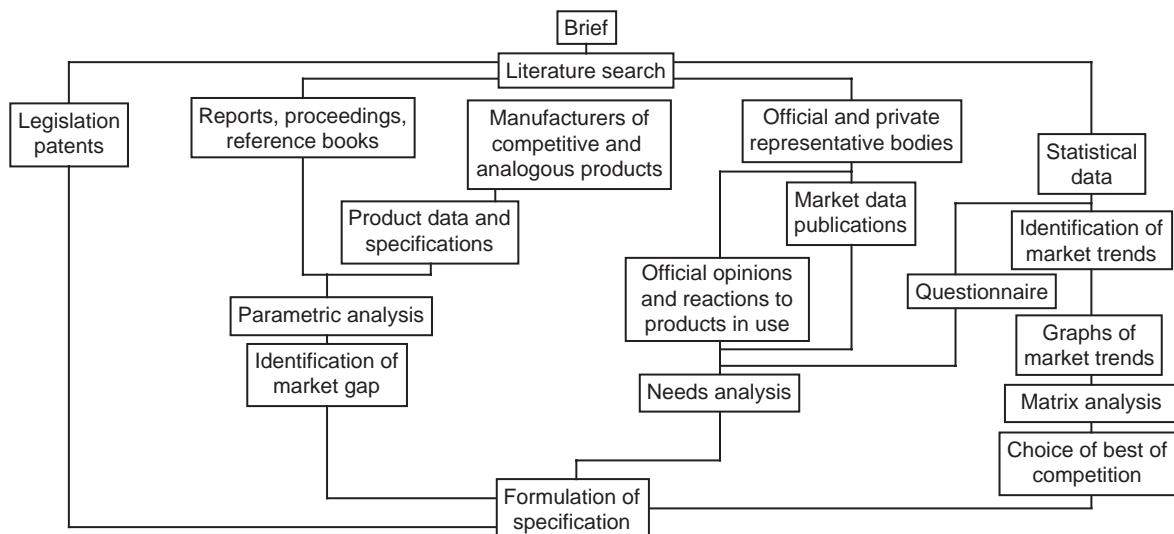


Figure 1.11 Information required for the production of the product design specification (after Pugh, 1990).

Table 1.1 Format for the documentation of the product design specification (after Pugh, 1990)

Date:	Product:	Issue:		
Parameters	Competition on best	Current model (ours)	This design (intent)	World class (target)
Performance description				
Safety description				

Table 1.2 Example table for developing a product design specification for one particular aspect. All relevant aspects identified in Figure 1.10 should be considered in a similar fashion

Aspect	Objective	Criteria	Test conditions
Performance	Low wear of moving parts	Switch will operate 10 000 times without fault Electric motor will operate for 250 h without fault Cutting blade will cut effectively for 100 h without attention	Test to ISO Standard Continuous-run laboratory test until failure

Table 1.3 Performance and weight aspects of a product design specification for a lawn mower

Aspect	Objective	Criteria	Test Conditions
Performance	Low wear of moving parts	Switch will operate 10 000 times without fault Electric motor will operate for 250 h without fault Cutting blade will cut effectively for 1000 h without attention	Test to British Standards Continuous run laboratory test
	Low probability of damage	Body of lawnmower can withstand a side impact without cracking as a result of being dropped from a height of 1 m	Practical test
	Low level of vibration	Static out of balance of cutting blades not to exceed 2 g	To British Standards
Weight	Low weight	Total weight of machine not to exceed 50 kg	Weigh it!

Adapted from Walker *et al.*, 1991.

rarely be simply one day's work and may extend to many man months. Recall that conceptual design, let alone detailed design, cannot sensibly proceed towards any firm status until the specification has been completed.

Example 1.1

The managing director of a company specializing in gardening equipment has called a meeting to explain a marketing opportunity for a power-assisted grass-cutting device. The director explains that the key objectives of

this device are safety and reliability. The director has asked you to develop part of the product design specification for this device considering low wear of moving parts, low probability of damage, low level of vibration and low weight.

Solution

There are no right answers to this activity. The suggestions given in Table 1.3 are a possibility. Note the inclusion of numbers. Think about the values given. Why 250 h operation for the motor?

1.5 Conceptual design

Conceptual design is the generation of solutions to meet specified requirements. Conceptual design can represent the sum of all subsystems and component parts that go on to make up the whole system. Ion and Smith (1996) describe conceptual design as an iterative process comprising of a series of generative and evaluative stages that converge to the preferred solution. At each stage of iteration, the concepts are defined in greater detail allowing more thorough evaluation. It is important to generate as many concepts and ideas as possible or economically expedient. There is a temptation to accept the first promising concept and proceed towards detailed design and the final product. This should be resisted as such results can invariably be bettered. It is worth noting that sooner or later your design will have to compete against those from other manufacturers, so the generation of developed concepts is prudent.

Some of the popular methods for assisting concept generation are described in Sections 1.5.1 to 1.5.3.

1.5.1 Boundary shifting

Boundary shifting involves challenging the constraints defined in the PDS to identify whether they are necessary. For example, the PDS may define that steel should be used for a component. Boundary shifting would challenge this specification to see whether it is appropriate and, if not, other materials could be considered.

Example 1.2

The mayor of Stockholm has outlined funding for a sculpture, based upon an architect's sketches, to span the entrance of Stockholm harbour and has presented you as a leading engineering company with the contract to design and construct this (see Matthews (1998)).

Solution

There are often problems with designing something new. This sculpture is not an exception. Consideration of fatigue due to wind loading is critical for a structure of this sort. In this case the designer challenged the original specification for a sculpture that spanned the harbour and in place proposed a half arch structure with the remainder of the geometry completed by a stream of water, Figure 1.12.

1.5.2 Brain-storming

Brain-storming is a group-based activity for the generation of multiple ideas and concepts to fulfil a given design requirement. The emphasis in brain-storming is on quantity rather than quality. Following a brain-storming session a review or debrief should be undertaken to identify concepts or aspects of the concepts generated that have merit and should be explored further. A typical outcome is that aspects of a few of the concepts generated should be combined to form a solution to the design need that gives a better overall solution than any one of the individual concepts from the original brain-storming session.

There are a number of rules that have been found helpful for brain-storming sessions.

- Brain-storming should be a group activity undertaken by an interdisciplinary team. For design-based applications, this might typically involve individuals with marketing, styling, mechanical and electronic engineering, packaging, materials, software and manufacture skills.
- There should be no more than ten people present in order to preserve an effective group dynamic.
- No criticism or mockery of any idea, statement or individual is allowed in the brain-storming session. Any infringement on this should be dealt with by exclusion of the individual concerned from the session for say a period of 15 min.

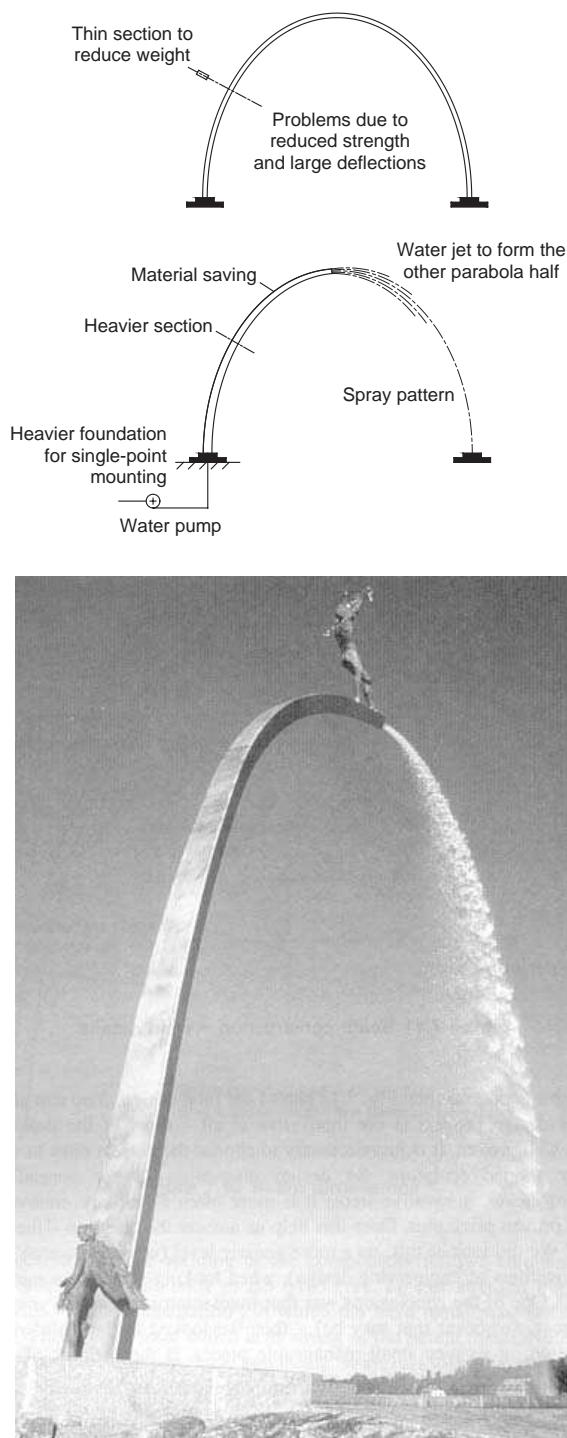


Figure 1.12 Conceptual and final solutions (Matthews, 1998).

- Flip charts, marker pens, pencils and paper should be provided to facilitate communication and recording of ideas.
- The design brief should be clearly stated at the start of the session.
- A set period of time should be allocated to the brain-storming session with the length of time typically being less than 2 h.
- An individual should be identified to introduce the brief, enforce exclusion of any misbehaving individual from the session and to close the session and gather relevant notes and sketches arising from the exercise. The session should, however, not be chaired. Brain-storming activities tend to be most productive when participants are able to freely share their concepts and participate.

The activities outlined here involve an element of role-play and group work. It is envisaged that this will be undertaken in an appropriate environment, such as a design studio or meeting room. It is also envisaged that only a selection of the questions will be attempted as advised by your course leader.

1. Working in groups of five, numbers allowing, develop as many ideas as you can in 10 min to ensure that the driver of a vehicle is able to see adequately during wet weather.
2. Working in a group of seven, numbers allowing, develop as many ideas as you can in 20 min to move 150 kg of gravel piled at floor level in a garden centre to the boot of a car parked at a distance of up to 70 m away.

1.5.3 Morphological analysis

Morphological analysis is a technique that can be used to generate additional ideas for products that would not normally spring to mind. The technique involves considering the function of a generic solution to a problem and breaking it down into a number of systems or subfunctions. The next step is to generate a variety of means to fulfil each of these systems or subfunctions. The subfunctions and potential means of fulfilling

Table 1.4 A general morphological analysis table

Subsystem	Means			
1	Method 1 of fulfilling subsystem 1	Method 2 of fulfilling subsystem 1	Method 3 of fulfilling subsystem 1	Method n of fulfilling subsystem 1
2	Method 1 of fulfilling subsystem 2	Method 2 of fulfilling subsystem 2	Method 3 of fulfilling subsystem 2	Method n of fulfilling subsystem 2
3	Method 1 of fulfilling subsystem 3	Method 2 of fulfilling subsystem 3	Method 3 of fulfilling subsystem 3	Method n of fulfilling subsystem 3
4	Method 1 of fulfilling subsystem 4	Method 2 of fulfilling subsystem 4	Method 3 of fulfilling subsystem 4	Method n of fulfilling subsystem 4
5	Method 1 of fulfilling subsystem 5	Method 2 of fulfilling subsystem 5	Method 3 of fulfilling subsystem 5	Method n of fulfilling subsystem 5

Table 1.5 Morphological chart for a pallet moving device

Feature	Means				
Support	Track	Wheels	Air cushion	Slides	Pedipulators
Propulsion	Driven wheels	Air thrust	Moving cable	Linear induction	
Power	Electric	Diesel	Petrol	Bottled gas	Steam
Transmission	Belts	Chains	Gears and shafts	Hydraulics	Flexible cable
Steering	Turning wheels	Air thrust	Rails	Magnetism	
Stopping	Brakes	Reverse thrust	Ratchet	Magnetism	Anchor
Lifting	Hydraulic ram	Rack and pinion	Screw	Chain or rope hoist	Linkage
Operator	Standing	Walking	Seated at front	Seated at rear	Remote control

Reproduced with modifications from Cross (2000).

each of these subfunctions are arranged in a grid. An overall solution is then formulated by selecting one means for each subfunction and the combination of these forms the overall solution.

A general morphological analysis table is illustrated in Table 1.4. The grid can be filled by text or by sketches depicting the potential means of fulfilling the subsystem requirement.

The use of morphological charts requires the designer to consider the function of components rather than their specific details. The pallet-moving device considered in Section 1.2.1 would probably comprise of support, propulsion, power, transmission, steering, stopping and lifting systems. Within each of these criteria all plausible options should be listed as shown in Table 1.5. The task of the designer is to look at this chart and select a justifiable option for each of the criteria producing the make-up of the pallet-moving device. In this case there would not necessarily be any surprises in the final make-up of the product and

**Figure 1.13** Forklift truck.

selection of wheels, driven wheels, petrol, gears and shafts, turning wheels, brakes, hydraulic ram, and seated at the front would result in the traditional forklift truck illustrated in Figure 1.13. However the use of this method can assist in the

Table 1.6 Morphological chart for a pallet moving device with choices identified

Feature	Means				
Support	Track	Wheels	Air cushion	Slides	Pedipulators
Propulsion	Driven wheels	Air thrust	Moving cable	Linear induction	
Power	Electric	Diesel	Petrol	Bottled gas	Steam
Transmission	Belts	Chains	Gears and shafts	Hydraulics	Flexible cable
Steering	Turning wheels	Air thrust	Rails	Magnetism	
Stopping	Brakes	Reverse thrust	Ratchet	Magnetism	Anchor
Lifting	Hydraulic ram	Rack and pinion	Screw	Chain or rope hoist	Linkage
Operator	Standing	Walking	Seated at front	Seated at rear	Remote control

Reproduced with modifications from Cross (2000).



Figure 1.14 The Teletruk concept by JCB. Teletruk photograph courtesy of J.C. Bamford Excavators Ltd.

production of alternatives as shown in Table 1.6 and Figure 1.14 where the option for a linkage based lifting method has been considered.

Morphological analysis can be made up using text, icons or sketches. Table 1.7 gives an example of a morphological analysis for a pallet-wrapping device.

1.5.4 Function trees

The function or objective tree method (Cross, 2000) involves decomposing the function of a product into different subfunctions. Using the function tree method, conceptual design takes place at the subsystem or component level. For

Table 1.7 A morphological chart for some elements of a machine to stretch wrap a pallet and its load (after Wright, 1998)

Feature	Means
Pallet Support drive	Ram
Rollers	
Pallet rotation	Pulley
Pneumatic roller	
Geared motor	
Film carriage guidance	Guide pillar

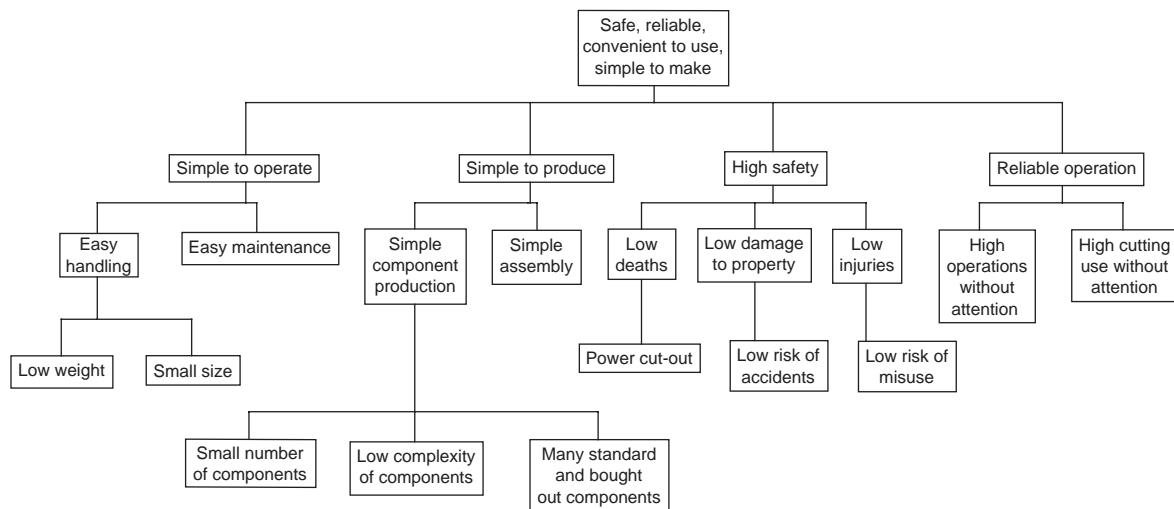


Figure 1.15 An objectives tree for a lawnmower that is to be safe, reliable, convenient to use and simple to make (after Walker *et al.*, 1991).

example, it may not be possible to produce new concepts for the power plant of an automobile, but it may be possible to generate new concepts for the subsystems or for individual components.

For example, in the design of a lawnmower, the market brief may include key objectives such as product safety and reliability. The function tree method helps expand on these objectives by use of the questioning adverbs: why, what, where and how? For instance, what is meant by reliable? As a result of this question this statement can be developed into: high number of operations without attention, high cutting use without attention, low wear of moving parts, low level of vibration, low risk of misuse.

The process of developing a list of functions or objectives will usually involve several different levels. For example the means of achieving an objective will be an objective in itself, albeit a lower level of objective. The process of developing functions in this way results in a branching of functions from the original key objectives. The objectives tree for a lawnmower that is to be safe, reliable, convenient to use and simple to make has been developed in Figure 1.15.

1.6 The technology base

The range of existing and well-tried technology is extensive. Many items are available as standard components and can be purchased from specialist suppliers. In addition standard practice and design methodology have been specified for many others. The designer is able to exploit available technology as it is or by combination and adaptation. The scope of mechanical machine elements available to the designer is outlined in Table 1.8. This list is not exhaustive, but does give an idea of some of the ‘building blocks’ available. Many of these machine elements are considered within this book and are identified by shading in the table.

Developing an overview of the available technology base is useful to the designer as it avoids time spent repeating the design process on a technology that has already been developed. In addition, knowledge of science and engineering enables a designer to judge what is technically and scientifically feasible so that ‘far-fetched’ ideas can be objectively ruled out.

Table 1.8 An overview of the scope of machine elements

Energy conversion	Energy transmission	Energy storage	Locating	Friction reduction	Switching	Sealing	Sensors	Miscellaneous mechanisms
Turbomachinery Gas turbine engines Rotodynamic pumps and compressors fans propellers turbines	Gears spur helical bevel worm conformal Belts flat vee wedge round synchronous	Flywheel Springs helical leaf Belleville rubber spiral garter Fluid accumulator Gas spring Reservoir Pressure vessel Solid mass Chemical Charge Torsion bar	Threaded fasteners bolts nuts and lock nuts grub screws studs screws Washers Nails Pins Cylindrical taper spring Rivets Tolerance rings Expanding bolts Keys Flat, round profiled gib head Woodruff Splines Circclips Snap rings Clamps Retaining rings Shoulders Spacers Grooves Fits clearance transition interference Adhesives Welds	Rolling element bearings deep groove cylindrical roller needle roller taper roller angular contact self aligning thrust recirculating ball Sliding bearings plain rubbing hydrodynamic hydrostatic hydrodynamic slideway Wheels and rollers Brushes	Clutches square jaw multiple serration sprag roller disk drum cone magnetic synchromesh Ratchet and pawl Geneva Mechanisms Valves Latches Triggers Bimetallic strips	Dynamic seals mechanical face lip ring bush labyrinth brush ferrofluidic rim seals O rings packings piston rings Static seals gaskets O rings gaskets sealants	Motion Dimensional Mass Force Torque Power Pressure pitot tubes static tappings manometers piezoelectric Sound Flow laser doppler hot wire ultrasonic Level Humidity Temperature thermocouples resistance thermometers pyrometers Heat flux thermopile Gardon gauges Strain and stress Time Chemical composition	Hinges, pivots Linkages Levers Tools cutters shears drills formers Grips Guides Followers Housings frames casings enclosures Sprayers Shutters Hooks Pulleys Handles Rollers and drums Centrifuges Filters
Internal combustion engines rotary reciprocating								
Boilers and combustors								
Electric motors								
Alternators and generators								
Solenoids								
Pneumatic and hydraulic actuators								
Brakes disk drum and band	Cables and ropes Couplings rigid flexible universal							
Pumps								
Rockets								
Heat exchangers								
Guns								
Dampers and shock absorbers	Power screws and threads							
Fuel cells	Levers and linkages Pipes, hoses and ducts Ball screws							

After private communication, C. McMahon, Bristol University, 1997.

1.7 Conclusions

The total design model illustrated in Figures 1.7 and 1.9 was originally proposed some years ago (1985). Since then the ethos of design has developed to include the design of products for total life recycling. This means that the end of a product should be considered at the design stage. When the product finally fails or is deliberately scheduled for withdrawal, consideration should be given to its disposal or recycling. For instance, are there any environmentally threatening aspects that must be dealt with? Some automotive companies, for example, have developed recycling policies whereby approaching 100 per cent of an old exchanged vehicle is recycled. This has the added benefit of giving the company the opportunity to sell a new product to the customer.

Although the total design model has been adopted for this course, other models have been developed. The reader is recommended to review the models proposed by Pahl and Beitz (1996), March (1976) and Cross (2000). For example, in the total design model the conceptual design phase includes the development of bulk concepts as well as the process of providing form and layout to the design. The model presented by Pahl and Beitz (1996) models this process as distinct conceptual and embodiment phases. In the embodiment phase the layout and form of the design concept are quantified with consideration of the overall objectives of the specification. The subject of design has been practised for thousands of years but understanding of the process of design is in its infancy and all these models will no doubt be developed further in the future.

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Websites

At the time of going to press the World Wide Web contains useful information relating to this chapter at the following sites.

www.bsi.org.uk

www.iso.org

www.hyster.com/products/

www.nissanforklift.com

www.ski-doo.com

www.jcb.co.uk

Nomenclature

BS	British Standard
DFA	Design for assembly
DFM	Design for manufacture
ISO	International Organisation for Standardisation
PDS	product design specification

Worksheet

- 1.1 The managing director of a small tools company has outlined a requirement for an automatic nailing machine to augment an existing range of mid-priced professional tools. List approximately 10 additional items of information that need to be determined in order to develop the specification.
- 1.2 After a demonstration at a national ski exposition the managing director of a recreational equipment company has outlined a requirement for a snow sports activity vehicle. The director has marched into your office and asked you to develop the specification for this during the next 20 min. Although you know that doing this job will take

orders of magnitude more time and resource than this, outline a first attempt at the requested specification.

- 1.3 Following an examination of the marketplace a national brand-name do-it-yourself store company has commissioned your design consultancy to develop a product for managing the length of grass. Spend about 1 h on this, outlining an initial potential product design specification.
- 1.4 Following a demonstration at the local marina the managing director of a recreational equipment company has outlined a requirement for a water sports activity vehicle. The director has marched into your office and asked you to develop the specification for this during the next 20 min. Outline the specification.
- 1.5 A European-based manufacturer of wheelchairs has identified market potential for personal transportation devices for non-infirm people. Your design consultancy has been awarded the contract to develop the PDS and conceptual designs to meet this requirement. The brief is the design of a transportation device for an adult and their shopping with a range in a town environment of up to 6 km. Spending about an hour, develop a PDS for further consideration by the sponsor company and your colleagues.
- 1.6 A clutch is required for a UK-manufactured motorcycle. The engine develops 90 PS at 8000 rpm. Working in groups of three, spend 45 min developing an outline PDS.
- 1.7 A braking system is required for a small city vehicle. The maximum speed of the car is 110 km/h and the 95 percentile laden mass of the vehicle is 1400 kg. Working in groups of three spend 45 min developing an outline PDS.
- 1.8 A worldwide OEM has recognized the need to update its principal C sector vehicle. Competitive products are now regularly outclassing its own in the motoring surveys and although the company had initiated a reskinning team immediately after the launch of the present vehicle it is recognized that this will not be sufficient to produce a product that has the potential to be profitable. Working in groups of seven spend an hour developing an outline PDS.
- 1.9 A Netherlands-based white goods supplier has identified a market gap for a kettle. The market potential is for a product with a selling price of

20 Euros, selling 10 million items annually, for 3 years in 20 key markets in a variety of European, Asian and South American countries. Working in groups of three spend 30 min developing an outline PDS for the kettle.

- 1.10 Following a demonstration at the local marina, the managing director of a recreational equipment company has outlined a requirement for a winch. The director has marched into your office and asked you to develop the specification for this during the next 20 min. Outline the specification.
- 1.11 After a demonstration at the national ski exposition, the managing director of a recreational equipment company has outlined a requirement for a snow sports activity vehicle trailer. The director has marched into your office and asked you to develop the specification for the trailer during the next 20 min. Outline the specification.
- 1.12 Develop a morphological chart to aid the design for a small vehicle to enable the movement of building waste and aggregates around a building site. Identify your preferred configuration on the chart.
- 1.13 Develop a morphological chart to aid the design of a product to facilitate the transportation of a standard size pallet around a factory. Identify your preferred solution on the chart if the specification demands the capability for the pallet to be deposited on warehouse shelving at a height of up to 5 m. Produce a sketch of your preferred solution.
- 1.14 Develop a morphological chart to aid the design of a device for transporting a shepherd, a dog and one sheep a distance of up to 30 km in hilly terrain. Identify your preferred solution on the chart based on the criteria that the shepherd concerned has considerable influence on the purchase recommendation.
- 1.15 Develop a morphological chart to aid the design of device for transporting up to two people and 20 kg of luggage over both mud tracks and snow for a distance of up to 60 km in any one journey. Identify your preferred solution on the chart,

based on a high reliability demanded in the specification.

- 1.16 Develop a morphological chart to aid the design of a fun, sports activity-based product to transport up to two people over water at speeds up to 40 km/h. Identify your preferred solution on the chart based on a first-purchase price of less than 6000 Euros.
- 1.17 Develop a morphological chart to aid the design of a product to transport one adult and their daily shopping a distance of up to 6 km within a city environment. Identify your preferred solution on the chart based on a first-purchase price of less than 5000 Euros. Produce a sketch of your preferred solution.
- 1.18 Develop a morphological chart to aid the design for a personal transportation device in an arctic environment in line with the specification developed in 1.2. Identify your preferred configuration on the chart.
- 1.19 Working in groups of five, numbers allowing, develop as many ideas as you can in 30 min to deliver a discrete quantity of toothpaste on to a toothbrush.
- 1.20 Working in a group of 10, numbers allowing, develop as many ideas as you can in 20 min to atomize a powdered drug for inhalation by a human.
- 1.21 Working in groups of three, numbers allowing, develop as many ideas as you can in 20 min to enable twice as much mass flow of water in one direction of a pipe than in the opposite direction.
- 1.22 Working in a group of 10, numbers allowing, develop as many ideas as you can in 1 h to break the foil seal on a Tetra-Pak™ carton.
- 1.23 Working in groups of five, numbers allowing, develop as many ideas as you can in 30 min to seal and release a liquid fuel in a pressurized cigarette lighter.
- 1.24 Working in groups of five, numbers allowing, develop as many ideas as you can in 20 min to set light to a stream of fuel released from a pressurized liquid reservoir of a cigarette lighter.

Learning objectives achievement

Can you identify the iterative steps associated with the design process?	<input type="checkbox"/>	✓ ↓	X → Section 1.3
Can you develop a product design specification from a market brief?	<input type="checkbox"/>	✓ ↓	X → Section 1.4
Can you participate in brain-storming activities?	<input type="checkbox"/>	✓ ↓	X → Section 1.5.2
Can you develop a morphological design chart to aid the development of conceptual solutions?	<input type="checkbox"/>	✓ ↓	X → Section 1.5.3
Do you realize the scope of technologies available to the designer?	<input type="checkbox"/>	✓ ↓	X → Section 1.6

MECHANICAL ENGINEERING

The aim of this chapter is to introduce mechanical engineering and its interrelationships to mechanical design. Defining mechanical engineering is itself a challenge. Some people hold the opinion that it should not suffer the indignity of a definition, as this would invariably set bounds on the activity. Nevertheless, it is considered here that the subject of definition should be explored as this serves the purpose of introducing some vital concepts and setting the context and the interrelationships with the topics considered in this book.

2.1 Introduction

A brief look at the publicity material for the world's larger bodies representing mechanical engineers and mechanical engineering, such as the Institution of Mechanical Engineers (about 80 000 members) and the American Society of Mechanical Engineers (about 125 000 members), will reveal that mechanical engineering relates to activities including:

- Aerospace
- Automobiles
- Rail
- Shipping
- Power generation
- Mining
- White goods
- Sports
- Agriculture
- Manufacture/production
- Military hardware

The above activities involve a challenge. Take the automobile, for example, which comprises a power plant, fuel storage, a form of support (traditionally wheels and suspension), a skin, furniture and entertainment systems. Which aspects relate to the activity of the mechanical engineer? The engine, a typical example of which is illustrated in Figure 2.1, is invariably controlled by a microprocessor-based engine management system as only this can currently perform the necessary monitoring and control functions to maintain acceptable emissions standards. Where do the mechanical engineer's responsibilities start and end and how should the interaction with software experts and electronic

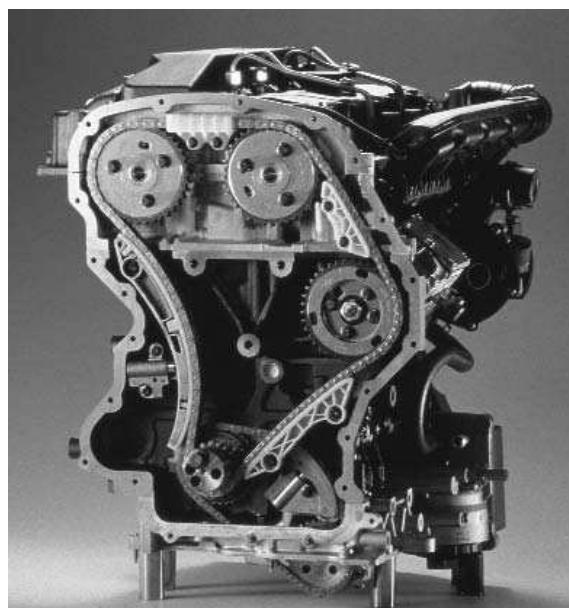


Figure 2.1 An engine from the Mondeo. (Figure courtesy of Ford Motor Co.)

and control engineers occur? Should in fact just one individual undertake all the tasks associated with producing a functioning engine? The answer to this last question lies in the complexity of the task and the timescales involved. It is unlikely that any one individual has all the necessary skills and furthermore, the timescales demanded in industry to fulfil profit objectives and changes in available technology usually preclude an individual working in isolation from being effective.

Taking another example from the aerospace sector using Airbus Industrie's A380 illustrated in Figures 2.2 to 2.4. The design work here was not



Figure 2.2 The Airbus A380. (Figure courtesy of Airbus Industrie.)

only divided across overlapping disciplines, but across countries and cultural divides. Wings were designed and made in the United Kingdom, the fuselage in France, cabin interior and seats in

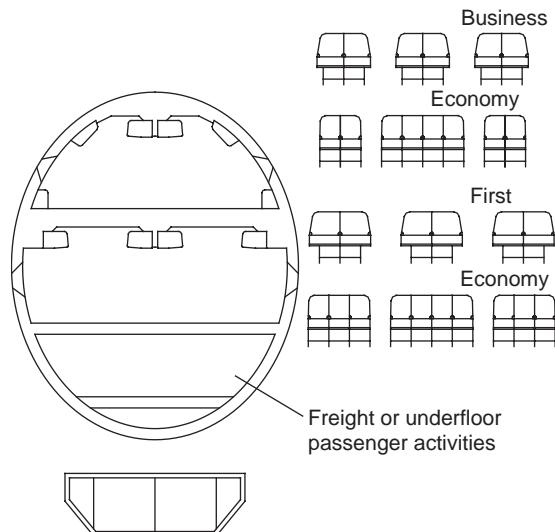


Figure 2.4 Cabin options for the A380. (Figure courtesy of Airbus Industrie.)

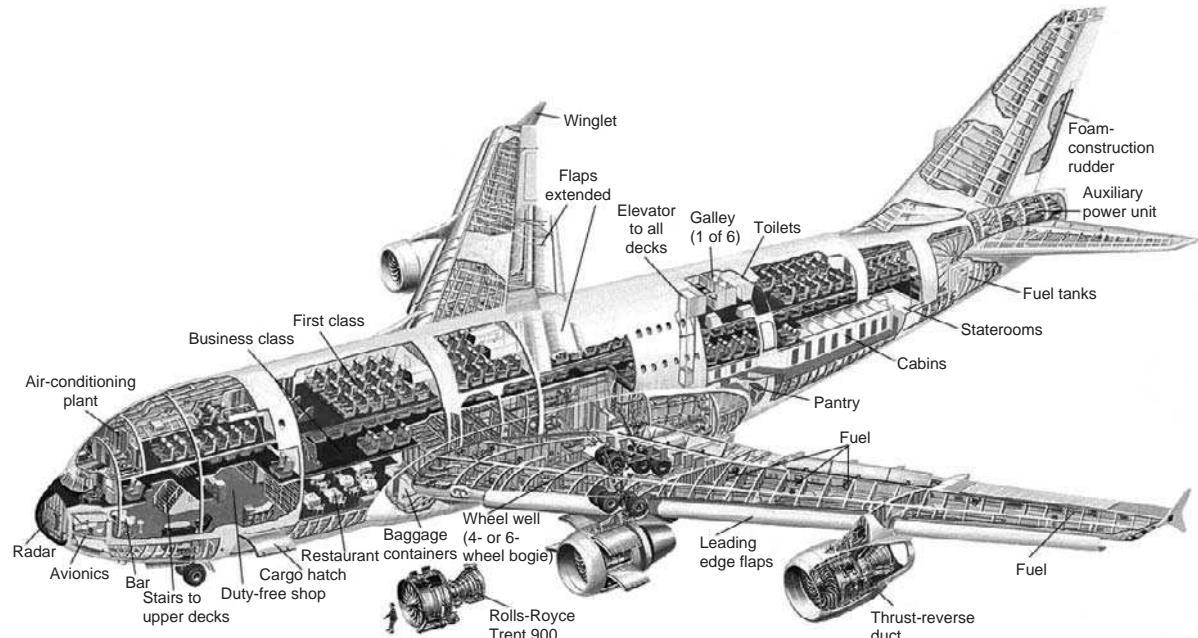


Figure 2.3 Exploded view of the A380. (Figure courtesy of Airbus Industrie.)

Germany and the assembly undertaken in France. To give an idea of the complexity of the task involved, psychologists can be involved in determining the layout of cockpit design in order to advise on human-machine interactions.

Mechanical engineers can choose to limit their activity to items they perceive to be inherently mechanical, such as gearboxes, clutches and internal combustion engines. If this is the case then some form of deception has invariably occurred, as it is next to impossible to isolate a mechanism or component entirely and thereby decouple it from its relationship to other factors and influences. Devices of a mechanical nature can be considered to be those involving relative motion, stressed components and prime movers amongst many others. This limitation, however, is too constraining for today's products where added value is a key consideration as exemplified by the variety in white goods, such as vacuum cleaners and dishwashers available in any electrical hardware store. Instead the mechanical engineer needs to be a master of his own discipline and also have skills and awareness of many related disciplines. This need is reflected in the demands of accrediting bodies for first degrees such as the Institution of Mechanical Engineers. A typical requirement is for an interdisciplinary education followed up by continuing professional development where the engineer covers the fundamentals of his primary profession in depth and breadth accompanied by courses that widen the perspective, such as studies in materials, electronics, software, management, etc.

The foundations of mechanical engineering can be considered to be the sciences of thermodynamics and mechanics. Thermodynamics is the study of energy transformation and mechanics is concerned with the motion and equilibrium of bodies. Design is the integrating activity that marries thermodynamics and mechanics together to enable the production of useful products. The skill set required by the mechanical engineer is not however wholly fulfilled by thermodynamics and mechanics. In addition, skills in languages and communication, mathematics, physics, chemistry

and biology, electronics, control and software engineering, psychology and management are necessary to name but a few.

Professions are not immune to advances in understanding and technology. Stress analysis was once the function of trained engineers with extensive grounding in mechanics. Sophisticated software, enabling the modelling of axisymmetric and three-dimensional geometry, such as ANSYS® and its competitors, is now advanced enough and has sufficient support in the form of on-line tutorials and diagnosis to allow an inexperienced user to identify whether a component is over-stressed. The notion that, for a contour plot of stress, blue is acceptable and that red means danger is now commonplace. Such advances have already been made in stress analysis, albeit with major misgivings amongst some subject experts especially where safety critical failure is concerned. In the area of fluid mechanics, the sophistication of CFD (computational fluid mechanics) is less advanced. But even here, on-line tutorials enable a computer-literate individual to import geometry, apply boundary conditions and produce a solution. The issue is, however, how valid is the output and this is where engineering skills and know-how are important.

Many individuals have produced innovative and groundbreaking products without a foundation in engineering and related disciplines. Examples include the Dyson vacuum development where thousands of prototypes were trialled, see Dyson (1997), indicative of a trial and error evolutionary approach to design. Such an approach is perfectly acceptable and can result in market-dominating products and breakthroughs in science, but it is important to recognize the contribution that a grounding in engineering can provide, which is the prediction of the performance of a product at the design stage. If prediction at the design stage is achieved then the performance of different concepts can be evaluated before expensive effort and extensive resources are committed to developing a particular design. It is for this reason that the traditional disciplines of mechanical engineering

are still relevant and necessary in a wide range of design work. These subjects are introduced briefly in Sections 2.2, 2.3 and 2.4.

2.2 Thermodynamics

Thermodynamics is the science of energy transformations. It is concerned with the relationships between various physical properties and qualities of substances that are affected by energy transformations. Thermodynamics enables the design of cycles for power plants to operate on. An example is the determination of pressure ratios and temperatures in a jet engine. The limitations on these levels are invariably mechanical in nature and stem from the peak temperatures and stresses that materials can operate at for sustained periods of time. Furthermore, in the case of a jet engine, weight is critical and the designer is faced with the challenge of designing components operating under extreme conditions that are low in weight, production cost and in the case of power-by-the-hour^{TM1} agreements, service cost. Energy is very important due to limited oil supplies, pollution concerns and cost factors. Despite engine technology being relatively mature the efficiency levels for the various power plants given in Table 2.1 show that a considerable proportion of energy is being wasted. This can be addressed by improved cycles involving additional equipment or by working with better materials. Either option will involve the mechanical designer in producing low weight, low cost, highly stressed components.

2.2.1 Fluid mechanics

Fluid mechanics deals with modelling fluid behaviour and applications where fluid systems are used. A fluid is defined as a substance that deforms continuously when acted upon by a shear stress. Fluids can be subdivided into gases and liquids. A fixed quantity of liquid has a definite

Table 2.1 Typical thermal efficiencies for a variety of power-producing and energy converters (after Wark, 1983)

Type	Conditions	Efficiency (%)
Automobile internal combustion engine (ICE)	Optimal	25 (35) ^b
	Steady 60 mph	18 (23) ^b
	Steady 30 mph	12 (16) ^b
Truck diesel	Full load	35 (37) ^b
Gas turbine engine without regeneration (>7.5 MW)	Optimum	25 (30) ^b
Gas turbine engine with regeneration (>7.5 MW)	Optimum	34 (37) ^b
Jet engine		40
Steam power plant (>350 MW)	Optimum	41 (55) ^b
Home furnace (chemical to thermal)	Serviced and well-maintained	70
Chemical to electrical dry cell battery		90
Hydrogen fuel cells	100°C	82 ^a
	1000°C	52 ^a

^a Larminie and Dicks (2000).

^b Private communication A.B. Turner, University of Sussex.

volume that varies only slightly with pressure and temperature. For example, it takes 200 bar (200×10^5 Pa, 2900 lb/in²) to increase the density of water by 1 per cent. Conversely a fixed quantity of gas will expand until its volume equals that of the container.

The importance of fluid mechanics is evident when we consider the air we breathe, water we drink and blood that flows in our veins. The atmosphere around us provides a medium to support transportation systems in the form of aircraft and to provide a source of oxygen for internal combustion engines. Cars are driven on tyres that contain air to form part of the suspension system to improve the ride comfort and conformance to the terrain, and gas is used in air bags in an attempt to improve car safety. Water is used in power

¹ Power-by-the-hour is a trade mark of Rolls-Royce plc.

generation cycles to produce electricity to enable our society to operate in a dispersed urban environment. Oil is used in damping systems to cushion shock in suspension systems and in bearings to reduce friction and wear on components. Understanding of fluid mechanics will be used in the design of bearings in Chapter 4, seals in Chapter 9, hydraulics and pneumatics and measurement in Chapter 14 and the cooling of various machine elements.

2.2.2 Heat transfer

Heat transfer is energy in transit due to a temperature difference. Heat transfer plays a vital role in many industrial applications, such as energy production and conversion, space and water heating systems, incinerators, refrigeration and air conditioning. The science of heat transfer allows the modelling of energy in transit and therefore assists us in the design of a wide range of mechanical components and systems. A critical aspect affecting many designs is the operating temperature as this usually dictates the system efficiency and influences choice of material and therefore operating and first cost price.

Heat can be transferred by means of thermal conduction, convection or thermal radiation. In addition, heat transfer by a combination of any of these modes is also possible. If a temperature gradient exists in a solid or stationary fluid, then the term used to describe the mode of heat transfer is 'heat conduction' or just conduction and occurs due to interactions between molecules of different random translational, rotational and internal energies. If a temperature gradient exists in a fluid, then it is possible for heat transfer to occur due to both random molecular motion and bulk macroscopic motion of the fluid and this mode of heat transfer is described as convection. All matter will emit electromagnetic radiation due to its temperature and heat transfer due to radiation between an object and its surroundings can be modelled by Planck's law of radiation, which models heat transfer as proportional to the fourth power of the temperature.

Heat transfer is a substantial discipline in itself, but it is often necessary for the designer to produce a thermal model for a design. An example is illustrated in Figure 2.5 of a thermal-structural model for a turbine assembly of a jet engine. Here, it is important to know the operating temperatures of the components throughout the cycle of use from idle, through take-off, cruise, landing and shut down in order to select appropriate materials, predict the life for components prior to service or replacement and to determine the cooling flows.

The modelling of heat transfer and determination of cooling flows for machine casings is considered in Chapter 13.

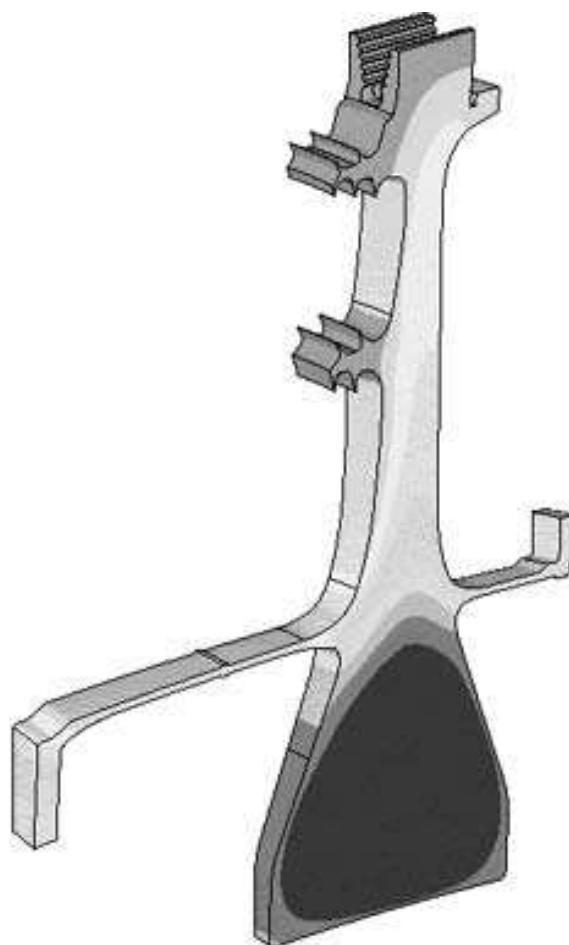


Figure 2.5 Temperature distribution in the high pressure turbine rotor for the Trent 800. (Courtesy of Rolls-Royce plc, Christopher Barnes.)

2.3 Mechanics

Mechanics is concerned with the motion and equilibrium of bodies and is usually subdivided into three categories: statics, kinematics and dynamics. Statics is the science of equilibrium of bodies subjected to the action of forces. Kinematics deals with motion without reference to its cause. It is important in determining the relationship, for example between piston speed and crankshaft speed in an engine. Dynamics considers the influence of forces on motion. It is the science of dynamics that enables the analysis of shafts in gas turbine and reciprocating engines to ensure that vibrations are not catastrophic and the analysis of gyroscopes to enable their use as a control in a wide range of applications such as the Segway human transport system.

2.4 Materials

For physical products, materials often provide significant constraints. Solids can fail, amongst other modes of failure, under compression, in shear or in tension or some combination of these loads.

Furthermore if the load is not constant then the maximum strength used in calculations will likely need to be reduced appropriately to account for the dynamic loading on the molecular bonds in the material. An example is a shaft rotating on its axis as illustrated in Figure 2.6. Here the point P is in tension when it is at the bottom of shaft, but when it has rotated through 180° it is now completely in compression. This fluctuation in loading can lead to failure by fatigue and may occur at stress levels an order of magnitude below the yield strength for the material concerned.

In the selection of materials for a design it can be helpful to realize that there are a number of general types of material:

- metals
- polymers and elastomers
- composites
- ceramics and glasses.

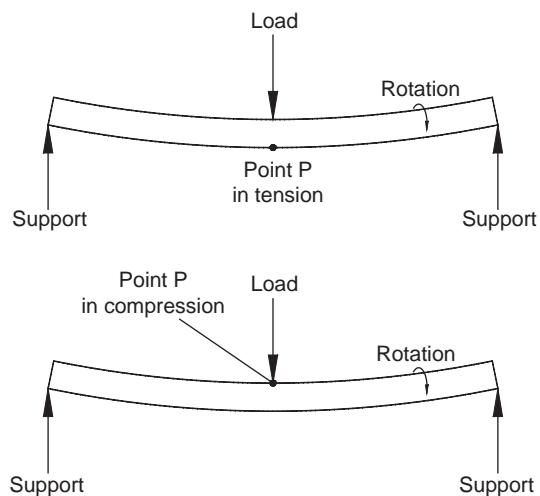


Figure 2.6 Variation in loading for a rotating shaft.

Each type of material has certain characteristics and the task facing the designer becomes that of selecting the appropriate class of material and then within that particular class selecting the appropriate material and grade. It should be noted that the choice of a material for a given application should not be made independently from the choice of processes by which the material is formed, joined, treated and finished and the cost of these processes, as well as the raw material itself.

Each material can be thought of as having a set of attributes or properties such as density, modulus of elasticity, strength, toughness, thermal conductivity, melting point and so on. A list of standard design limiting properties is presented in Table 2.2. The designer seeks for a given application a particular set or profile of properties to match the demands of the application. The task can be formidable when you consider that there are estimated to be between 40 000 and 80 000 engineering materials available (Ashby, 1999).

It is not the intention of this brief introduction to materials to provide a skill set sufficient to select an appropriate material for a particular design need. Instead brief introductions to the four classes of materials are given in Sections 2.4.1 to 2.4.4 to provide an overview of the principal properties and characteristics. Materials for

Table 2.2 Design limiting materials (after Ashby, 1999)

Class	Property	Common symbol	SI units	English units
General	Cost			
Mechanical	Density	ρ	kg/m ³	lb _m /ft ³
	Young's modulus	E	GPa	lb/in ²
	Shear modulus	G	GPa	lb/in ²
	Bulk modulus	K	GPa	lb/in ²
	Yield strength	σ_y	MPa	lb/in ²
	Ultimate strength	σ_{uts}	MPa	lb/in ²
	Toughness	G_C	kJ/m ²	Btu/ft ²
	Fracture toughness	K_{lc}	MPa m ^{0.5}	lb/in ² in ^{0.5}
Thermal	Fatigue endurance limit	σ_e	MPa	lb/in ²
	Thermal conductivity	k	W/m K	Btu/(h ft°F)
	Thermal diffusivity	α	m ² /s	ft ² /s
	Specific heat capacity	c_p	J/kg K	Btu/(lb _m °F)
	Melting point	T_m	K	°F
	Glass temperature	T_g	K	°F
Wear	Coefficient of thermal expansion	α	K ⁻¹	°F ⁻¹
	Archard wear constant	k_A	MPa ⁻¹	in ² /lb
Corrosion	Corrosion rate	K	mm/year	in/year
Oxidation	Parabolic rate constant	k_p	m ² /s	ft ² /s

individual machine elements are discussed in the relevant chapters in this book. For a comprehensive treatment of materials and their selection the reader is referred to Ashby (1999) and the Cambridge Engineering Selector software.

2.4.1 Metals

Metals have relatively high moduli. They can be made to have high strength by alloying and by mechanical and heat treatments. Metals can have low ductilities and although this is often a merit it means that they are prone to failure by fatigue and are the least resistant of the general classes of materials to corrosion.

2.4.2 Polymers and elastomers

Polymers and elastomers have low moduli in comparison with metals, roughly 50 times less, but they can be nearly as strong as metals. The consequence of this is that deflections can be large. They

are subject to creep, even at room temperature, which means that a polymer under load may with time acquire a permanent set. The properties of polymers are highly sensitive to temperature. For example, a polymer that is tough and flexible at 20°C may be brittle in a refrigerator at 4°C and creep rapidly in the dishwasher at 60°C. Provided the limitations of polymers are accounted for they can be very useful in providing design solutions. When combinations of properties, such as strength per unit weight, are important, polymers can be as good or better than metals. They are easy to form into complex shapes and the ability to accommodate large elastic deformations means that they can be designed to form components that snap together making assembly cheap and simple. A further advantage is the production of finished or near-finished components from appropriate-sized moulds by using precoloured polymers. Polymers are resistant to corrosion and have low coefficients of friction. Elastomers are particularly useful in seals and as springs and their applications are considered in Chapters 9 and 11.

2.4.3 Ceramics and glasses

Similar to metals, ceramics and glasses also have relatively high moduli but have the disadvantage that they are relatively brittle. As ceramics have very low ductility they have very little tolerance to stress concentrations, such as holes and cracks, or for high contact stresses, such as clamping. Ductile materials can accommodate stress concentrations by deforming in the region of the concentrations and thus redistributing the load. Ceramics and glasses are not capable of this and are prone to cracking in regions of high stress concentrations. Nevertheless, ceramics and glasses are an important class of material with many applications. The merits of ceramics are that they are stiff, hard, abrasion-resistant and they retain their strength to high temperatures relative to other types of materials, and they resist corrosion well.

2.4.4 Composites

Composites tend to combine the merits of other classes of materials, whilst avoiding some of the disadvantages. They tend to be light, stiff and strong and can be tough. Most of the available composites have a polymer matrix, such as epoxy or polyester, and are reinforced by fibres of glass, carbon or Kevlar. They cannot be used at elevated temperatures because the matrix softens, at about 250°C in the case of polyester. Composite materials tend to be expensive and are relatively difficult to join and form.

2.5 Conclusions

In the words of a past president of the Institution of Mechanical Engineers, ‘Scientists study what is whilst engineers create what never was’. This chapter has explored the importance of a range of scientific disciplines traditionally associated with mechanical engineering, such as thermodynamics, mechanics and materials. The subjects considered

are not exhaustive and the explanations have been deliberately concise. Nevertheless the treatment serves to illustrate the importance of the subjects and to introduce methodologies that are useful in the complex design process.

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- Larminie, J. and Dicks, A. (2000). *Fuel Cell Systems Explained*. John Wiley & Sons Ltd.
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Websites

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www.asme.org
www.granta.co.uk
www.imeche.org.uk
www.matweb.com

Nomenclature

Generally, preferred SI units have been stated.

- c_p Specific heat capacity (J/kg K)
- E Young's modulus (N/m²)
- G Shear modulus (N/m²)
- G_C Toughness (kJ/m²)
- k Thermal conductivity (W/m K)
- k_A Archard wear constant (MPa⁻¹)
- k_p Parabolic rate constant (m²/s)
- K Bulk modulus (N/m²), corrosion rate (mm/year)
- K_{Ic} Fracture toughness (MPa m^{0.5})
- T_g Glass temperature (K)
- T_m Melting point (K)
- α Thermal diffusivity (m²/s), coefficient of thermal expansion (K⁻¹)
- σ_e Fatigue endurance limit (N/m²)
- σ_{uts} Ultimate strength (N/m²)
- σ_y Yield strength (N/m²)

3

MACHINE ELEMENTS

The aim of this chapter is to introduce machine design and the individual machine elements that go together to form an overall machine design. The chapter introduces fundamental design philosophies that will be used in each of the subsequent chapters dedicated to an individual machine element.

3.1 Introduction

The design activity involves endowing form and ensuring functionality of a given product. The function of many products depends on devices that modify force or motion and consist of a number of interrelated units. An example is a gearbox that modifies the torque speed characteristic of a prime mover to match that of the load and comprises gears, shafts, bearings and seals. Such devices that modify force or motion are called machines and the interrelated units are sometimes known as 'machine elements'. The technology base has developed to a mature status in this area providing a number of such devices that can in some cases be used as they are or designed fit-for-purpose.

In the design of a machine and its machine elements it is the designer's task to determine the motion, forces and changes in energy involved so that sizes, shapes and materials for each machine element making up the whole machine can be determined. Although design work may involve concentrating on one component at a given time, it is essential to consider its interrelationship with the whole product following the total design philosophy, taking into account the market

requirement, detailed specifications, the conceptual design, design of other components and manufacture requirements.

The goal in machine design is to determine the size, shape and selection of materials and manufacturing processes for each of the parts of the machine so that the machine will perform its intended function without failure. This requires the designer to model and predict the performance and operation of each component and overall assemblies studying the mode of failure for each and ensuring that conditions expected in service for the product are met. This may involve determining stresses, deflections, temperatures and material degradation for each and every component and overall assembly, as well as its influence within the total design. As stresses are a function of forces, moments and torques, consideration of the dynamics of the system will often be necessary before detailed stress analysis can be undertaken. Furthermore, as the strength of materials is a function of temperature, a thermal analysis may also be necessary in order to determine operating temperatures throughout the performance cycle of the machine.

If a machine has no moving parts then the task for the designer is simpler since only static force analysis and thermal analysis is necessary. However, even structures are subject to loading from external loads, such as weather conditions of wind and rain, earth tremors and traffic. As such a consideration of the environment into which the machine is to be installed should be made and appropriate failure modes mitigated against. Mitigation involves considering what may go wrong and determining

what action to take to either prevent it from going wrong in the first case or if it does go wrong what is going to be done about it.

If the motions involved in a machine are slow then it is likely that a static force analysis will be sufficient. In a static structure such as the floor of a casing designed to support a particular weight, the payload that the structure can support can be increased by adding appropriately distributed weight (dead weight) to its structure. If, however, the components of a machine have significant accelerations, a dynamic force analysis will be necessary. Adding additional mass to a moving component increases the forces involved and it is possible for such components to become victims of their own mass. This is because some of the loading causing stress in a moving component is due to inertial forces modelled by Newton's second law of motion

$$F = ma \quad (3.1)$$

Although addition of mass may increase the strength of a component, the benefit may be reduced by the increases in inertial forces.

To meet a design requirement, the designer needs to conceive a form and express the connectivity of its parts in order to meet the desired goal. A proportion of the mental sculpting involved is geometric in nature because shape often permits the achievement of a given function. The design procedures presented here follow this approach with an initial proposal for the geometric configuration followed by analysis to justify the proposal or provide guidance for its modification. Boden (1990) discusses creativity in terms of the connection of conceptual spaces in the mind. In order to create a design it is often necessary to combine ideas from different fields or disciplines. This may or may not take place as part of a structured design procedure or in a prescribed time period and over-reliance on procedure should in some cases actually be guarded against as this may stymie creativity.

The variety of machine elements and technologies already available to the designer as a

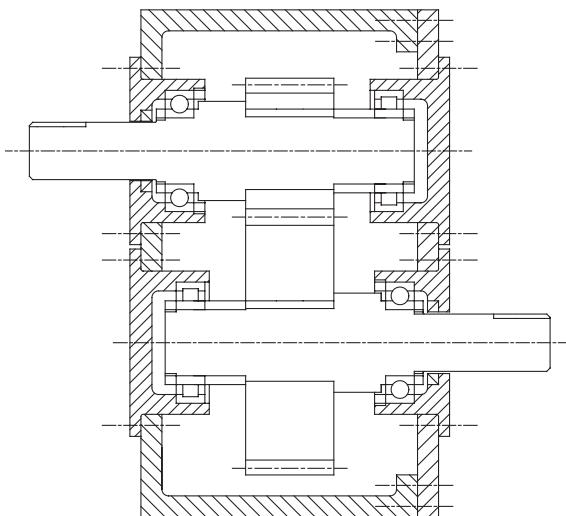


Figure 3.1 Transmission for an engine driven compressor.

result of years of development is extensive as indicated in Table 1.8 in Chapter 1. Machine elements considered in this book are those more commonly found in machines transmitting power by means of moving components, such as motors and engines. The diagram given in Figure 3.1 illustrates the range of machine elements involved in the design of a gear box for a single cylinder air compressor and comprises: shafts, bearings, gears, seals and enclosure. The machine elements considered in this book provide skills in the design of such applications and cover bearings, shafts, gears, belts and chains, seals, clutches and brakes, springs, fasteners and enclosures. These subjects are introduced in Sections 3.3 to 3.9 prior to more thorough consideration in the relevant chapters later in this book. A common feature to many machine elements is the wear of surfaces and this subject is introduced in Section 3.2.

3.2 Tribology

Tribology is the study of friction, lubrication and wear of surfaces in relative motion. Because of

the desire to produce efficient machines where the loss of energy due to friction is minimal, extensive research and effort has been put into studying tribology. As a subject it has considerable interrelationships and crossover with materials and fluid mechanics. The study and reduction of friction has been of concern to humans for as long as machines have been in use and the subject of tribology includes consideration of machine elements such as bearings, gears, belts, chains, clutches, brakes and seals.

3.3 Bearings

The term ‘bearing’, in its general sense, is used to refer to the assembly formed by two surfaces that have the capacity for relative motion. A wide range of bearings has been developed some of which involve rolling motion, some sliding motion and some both. Lubrication of a bearing is frequently used to reduce friction and therefore the wear and power absorbed by the bearing, and to remove heat to ensure operation of the bearing assembly at temperatures compatible with the materials and lubricant used.

Typical examples of bearings are shown in Figures 3.2 and 3.3. Figure 3.2 illustrates rolling bearings. These come in many forms each designed for a given set of performance characteristics, such as speed and load capability. Rolling element bearings usually need to be lubricated by grease or oil with the choice depending on the speed and temperature of operation. Rolling element bearings are typically available as stock items. This is of significant potential advantage to a machine designer, as the task becomes that of bearing selection rather than bearing design. Bearing design is a significant undertaking involving consideration of very high point contact stresses and use of special materials. The technology involved relies heavily on the science of tribology and there are a number of worldwide companies already operating in this market.



Figure 3.2 Examples of rolling element bearings available from existing suppliers.



Figure 3.3 Journal bearing for a turbocharger rotor.

A designer needs to carefully consider the issue of whether time is better spent producing a new design for a bearing that is already available from an existing supplier or whether it is better spent on designing and adding value to the product in hand.

Plain bearings rely on either rubbing contact between the bearing surfaces or pressure to separate the two surfaces. The pressure required can be induced in a lubricant by movement of one component relative to another or by the external supply of the lubricant under the required pressure. Plain bearings are also available from specialist bearing suppliers. In principle they can be significantly cheaper than rolling element bearings as they involve fewer moving components. The selection of an appropriate bearing for a given task,

however, is an involved activity, which needs to take into account, amongst other factors:

- load
- speed
- location
- size
- cost
- starting torque
- noise
- lubrication supply

The design of plain bearings and the selection of rolling element bearings as stock items from bearing supplies is considered more fully in Chapter 4. The related topic of shafts is introduced in Chapter 5. A consequence of bearings being supplied by a select number of specialist manufacturers is that they have been subject to standardization and are usually only available in a limited number of standard sizes. Standardization has the advantage that parts from one original equipment manufacturer (OEM) will be interchangeable in terms of geometry with another. This does not, however, mean that the performance of two similarly sized components from different manufacturers will be the same.

3.4 Gears, belts and chains

Gears, belts and chains can be used to transmit power from one shaft to another. Typical examples are illustrated in Figures 3.4, 3.5 and 3.6.

Many power producing machines, or prime movers, such as internal combustion engines, gas turbines and electric motors produce power in the form of rotary motion. The operating characteristics of prime movers vary according to their type and size. Typical examples are illustrated in Figure 3.7 for an internal combustion engine, an alternating current electric motor and a series wound direct current electric motor. Unfortunately these characteristics may not match those of a given application. For example, the internal combustion

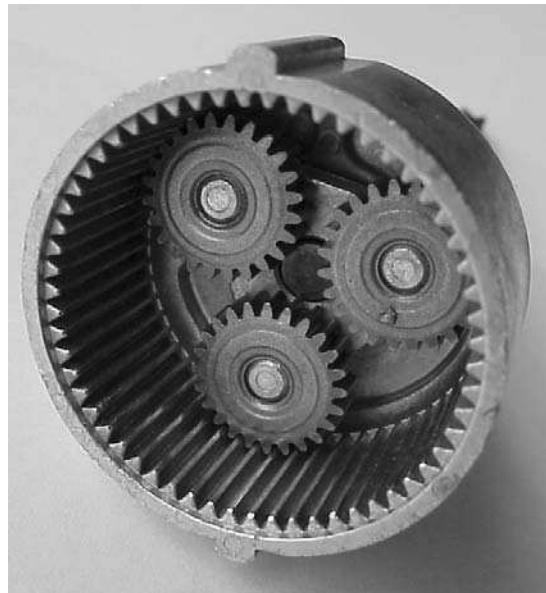


Figure 3.4 Spur gears.

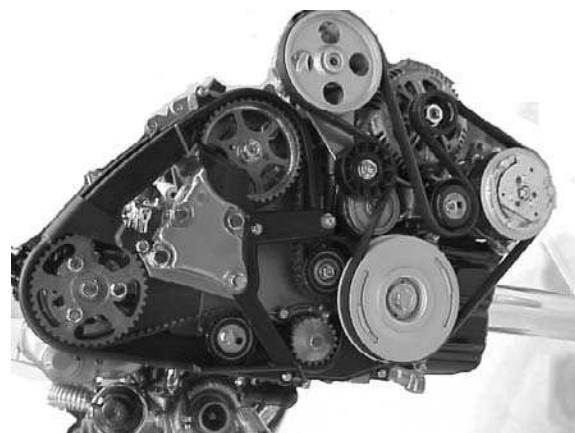


Figure 3.5 Flat belt and synchronous belt drives.

engine gives little torque at the low speeds associated with starting. It is for this reason that a car engine requires a clutch to decouple the engine from the load during starting and a gearbox to transform the characteristic of the prime mover to match that of the load.

A change of speed can be used to transform the torque speed characteristic of a prime mover

to a useful output characteristic. An example is the use of a reduction gearbox to reduce the speed and increase the torque seen by a load as illustrated in Figure 3.8. Recall the relationship between torque, angular velocity and power,

$$\text{torque} = \text{power}/\text{speed}. \quad (3.2)$$

Examination of this equation shows that as the speed reduces the torque rises for a given power. So if increased torque is required then a reduction gearbox, which decreases the speed of the output



Figure 3.6 Roller chain.

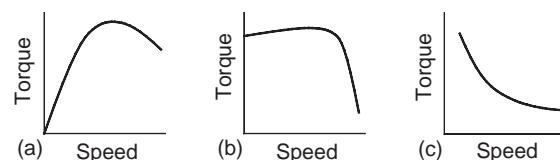


Figure 3.7 Typical torque speed characteristics.
(a) Internal combustion engine. (b) Three phase induction alternating current motor. (c) Series wound direct current motor.

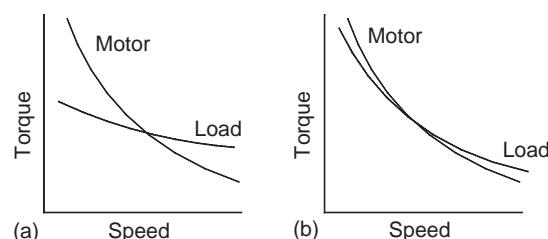


Figure 3.8 Transformation of the torque speed characteristic using a reduction gear ratio. (a) Mismatched system. (b) Transformed system.

shaft relative to the input shaft can be used to increase the torque delivered by the output shaft of the gearbox.

When transmitting power from a source to the required point of application a series of devices is available including belts, pulleys, chains, hydraulic and electrical systems, and gears. Generally, if the distances of power transmission are large, gears are not suitable and chains and belts can be considered. However, when a compact, efficient or high-speed drive is required gear trains offer a competitive and suitable solution.

Gears, belts and chains are all available from specialist suppliers. The manufacture of gears requires specialist machinery and the stresses in the region of contact between gears in mesh can be exceptionally high requiring the use of specialist materials. Similarly the design of roller chain and associated sprockets involves high contact stresses, and in addition the design of an articulated joint and associated bearings. For these reasons, combined with financial expediency, leaving the cost of specialist manufacture infrastructure to other companies, gear, chain and belt manufacture is usually best left to the specialist and the task of the machine designer becomes that of selecting and specifying appropriate components from the specialist stock suppliers and original equipment manufacturers. Gears are considered in Chapters 6 and 7 and belts and chains in Chapter 8.

3.5 Seals

Seals are devices that are used to reduce or eliminate the flow of fluid or particulates between two locations. Seals are an important part of a machine design where fluids under relatively high differential pressures must be contained within a particular region. An example is the fluid within the cylinder of an internal combustion engine. Here the working fluid needs to be contained in the cylinder for part of the cycle and excluded, in as much as possible, from contaminating the lubricating oil for the crankshaft bearings. This is

conventionally achieved by means of piston rings, illustrated in Figure 3.9.

There are many types of seal, but they can generally be classified as either static or dynamic depending on whether relative movement is involved. Many static seals involve use of a gasket or sealing material, such as an elastomer. These are widely available from specialist suppliers and the task facing the designer is the selection of appropriate materials for the application. Some dynamic seals are also commercially available such as radial lip seals and mechanical face seals. However, there is not the level of standardization in shaft design and the range of possible applications is seemingly infinite with differing pressure ratios and fluids being sealed. As a result the range of seal geometries required is too extensive to warrant an original equipment manufacturer designing or stocking a wide number of seal sizes and types. Instead many dynamic seals are designed fit-for-purpose. Typical considerations in the selection and design of seals include the nature of the fluid or particulates to be contained, the differential pressure, the nature of any relative motion between components, the level of sealing required, operating conditions, life expectancy and total costs. Seals are introduced more fully in Chapter 9.



Figure 3.9 Sealing of an internal combustion engine cylinder.

3.6 Clutches and brakes

Clutches and brakes provide frictional, magnetic, hydraulic or mechanical connection between two machine elements, usually shafts. There are significant similarities between clutches and brakes. If both shafts rotate then the machine element will be classed as a clutch and the usual function is to connect or disconnect a driven load from a driving shaft. If one shaft rotates and the other is stationary then the machine element is classed as a brake and the likely function is to decelerate a shaft. In reality, however, the same device can function as a brake or clutch by fixing its output element to a shaft that can rotate or to ground, respectively.

Brakes and clutches are familiar devices from their use in automotive applications as illustrated in Figures 3.10 and 3.11. They are, however, also used extensively in production machinery. Clutches allow a high inertia load to be rotated with a smaller electric motor than would otherwise be required if it was directly connected. Clutches are also used to maintain a constant torque on a shaft for tensioning of webs and filaments. They can be used, in emergencies, to disconnect a driven

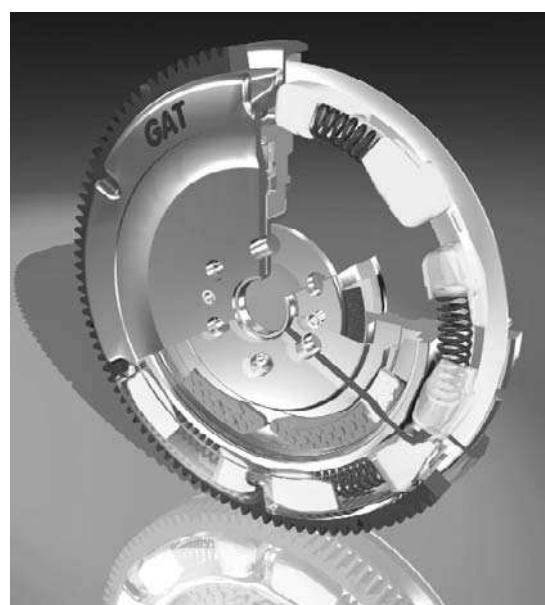


Figure 3.10 An automotive clutch.

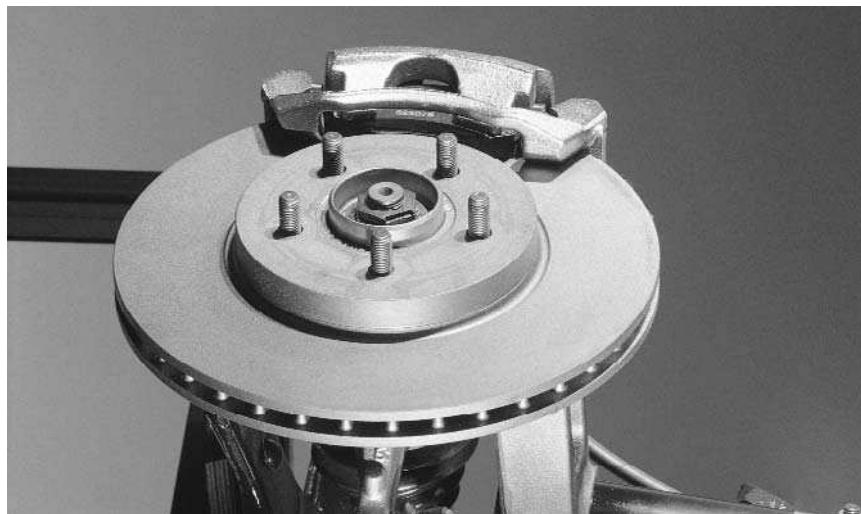


Figure 3.11 An automotive brake.

machine from a motor in the event of jamming of the machinery. In such cases a brake may also be installed in order to bring the machinery to a rapid stop.

Complete clutches and brakes are available from specialist manufacturers. In addition key components such as discs and hydraulic and pneumatic actuators are also available enabling the designer to opt, if appropriate, to design the overall configuration of the clutch or brake and buy in say the frictional lining surface. Clutches and brakes are considered more fully in Chapter 10.

3.7 Springs

Springs are flexible elements used to exert force or torque and store energy. The force exerted by a spring can be a linear push or pull or a radial force similar to that of a rubber band around a roll of paper. Alternatively a spring can be configured to produce a torque with applications including door closers. Springs store energy when they are deformed and return energy when the force that is causing the deflection is removed. A further type of spring is the power springs or spring motor, which once wound up can dissipate energy at a steady pace.

Springs can be made from round or rectangular wire bent into a suitable form, such as a coil, or made from flat stock and loaded as a cantilevered beam. Many standard size spring configurations are available as stock items from specialist manufacturers and suppliers and it is usually expedient to use these where possible. As well as paying attention to the performance characteristics of the spring, attention should also be focussed on the spring ends or terminations. Many options are available such as twist loops or hooks, side loops, extended hooks and so on. Some applications require a custom spring design. Spring design involves consideration of many variables such as length, wire diameter, forces, spring rate, spring index, number of coils, pitch, pitch angle, installation considerations, coil clearance, materials, types of loading and allowable stresses. The selection and design of springs is considered in Chapter 11.

3.8 Fasteners

Fasteners connect or join two or more components. It has been estimated that there are over 2 500 000 fasteners in a Boeing jumbo jet. For more modern airliners, such as the Airbus

Industrie A340, this figure has been dramatically reduced, but the total is still significant. This type of example illustrates the importance of fasteners to the designer.

There are thousands of types of fastener commercially available from specialist suppliers. The most common include threaded fasteners such as bolts, screws, nuts, studs and set screws, rivets, fast operating fasteners requiring say a quarter turn to connect or disconnect and snap joints commonly used in plastic component assembly. In addition, there is the range of permanent fastening techniques by means of welding, brazing, soldering and use of adhesive.

Given the wide range of fastening techniques available and the costs involved in sourcing, stocking and assembly, the correct choice of fastener is very important. Chapter 12 serves to introduce a variety of types of fastener and fastening and their selection based on appropriate analysis and consideration of the intended function.

3.9 Enclosures

Individual machine elements, such as shafts or bearings, must themselves be supported and this is the function of an enclosure or frame. The total design model requires attention to be applied to all other phases of the design process, whilst undertaking any one task. As such the design of any given machine element should not occur in isolation, but instead give due consideration to how it will fit in and interact with the overall assembly. The enclosure for a machine provides not only the support for the machine, but also provides form to a product. Attention to visual attractiveness is not always given the attention it warrants by designers of machine elements. However, we produce our components to be purchased and current thinking indicates that visual perception is the dominant amongst the senses with over half the cerebral cortex dedicated to visual processing. The shape of our products is therefore important and we should design accordingly.

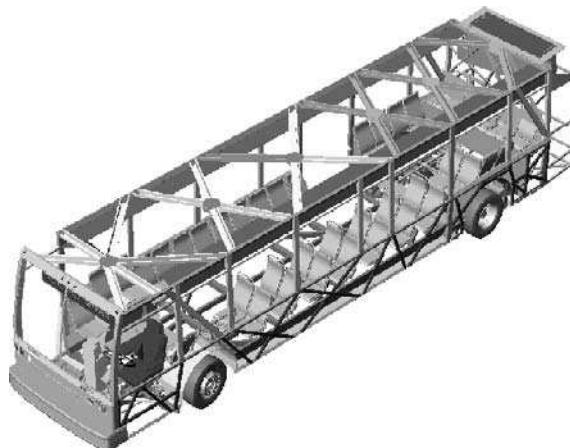


Figure 3.12 Bus frame design (Allin and Simon, 2002).

Enclosures are usually unique to a given class of machine, as each machine tends to be different in terms of the number, size and function of its constituent components. It is not practical, therefore, to have a completely general approach to enclosure and frame design. Common examples of the variation of frames and enclosures are those for vehicles and toys. Toys must be safe, functional and fun, whilst minimizing the material and personnel effort in producing the toy, in order to minimize the production cost and maximize profit.

The design of an enclosure or frame is to some extent an art in that the constituent machine components must be accommodated. There may be constraints on where supports can be placed in order not to interfere with the function and operation of the machine or in order to ensure access to particular components. The frame for the bus shown in Figure 3.12 illustrates this point in that access for doors, windows and occupants must be provided.

Some of the more important design parameters to consider for an enclosure or frame include strength, size, assembly, appearance, corrosion resistance, stiffness, vibration, weight, noise, cost to produce and maintain, and life. A list such as this may be too all-embracing to be of much use at the start of a design process and a more helpful

list of factors to consider as the starting point for a frame or enclosure include:

- forces exerted by the machine components through mounting points, such as bearings, pivots, brackets and feet;
- how the frame itself will be supported;
- allowable deflections of constituent machine elements;
- cooling requirements;
- the environments into which the machine will be transported and installed and the relationship to other machines and infrastructure;
- the quantity required and production facilities available;
- the expertise available for the design task.

The complexity involved when considering more than one machine element is evident and because of the need to keep the bigger picture in mind, models for the design process such as total design are useful. The design of frames and enclosures is considered more fully in Chapter 13.

3.10 Conclusions

The technology base available to us includes a wide range of machine elements, such as bearings, shafts, gears, chains, belts, seals, clutches, brakes and fasteners. This chapter has served to introduce these items, many of which are available as stock

items from specialist manufacturers. This means that the designer's task becomes that of selecting and specifying the appropriate item and relieves him or her of the task of the detailed design of, for instance, a roller chain so the design of the overall machine can be concentrated on.

It is appropriate to take time in overviewing the technologies available to the designer in order to be able to consider and take into account the interrelationships between individual machine elements. This is in keeping with the total design model where each phase takes into account all other phases and aspects of the design process.

References

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Nomenclature

Generally, preferred SI units have been stated.

<i>a</i>	acceleration (m/s^2)
<i>F</i>	force (N)
<i>m</i>	mass (kg)
OEM	original equipment manufacturer

4

BEARINGS

The purpose of a bearing is to support a load, typically applied to a shaft, whilst allowing relative motion between two elements of a machine. The aims of this chapter are to describe the range of bearing technology, to outline the identification of which type of bearing to use for a given application, to introduce journal bearing design and to describe the selection of standard rolling element bearings.

LEARNING OBJECTIVES

At the end of this chapter you should be able to:

- distinguish what sort of bearing to use for a given application;
- specify when to use a boundary lubricated bearing and select an appropriate bearing material to use for given conditions;
- determine the principal geometry for a full-film boundary lubricated bearing;
- determine the life of a rolling element bearing using the life equation;

- select an appropriate rolling element bearing from a manufacturer's catalogue;
- specify the layout for a rolling bearing sealing and lubrication system.

4.1 Introduction

The term 'bearing' typically refers to contacting surfaces through which a load is transmitted. Bearings may roll or slide or do both simultaneously. The range of bearing types available is extensive, although they can be broadly split into two categories: sliding bearings also known as plain surface bearings, where the motion is facilitated by a thin layer or film of lubricant, and rolling element bearings, where the motion is aided by a combination of rolling motion and lubrication. Lubrication is often required in a bearing to reduce friction between surfaces and to remove heat. Figure 4.1 illustrates two of the more commonly known bearings: a deep groove ball bearing and a journal

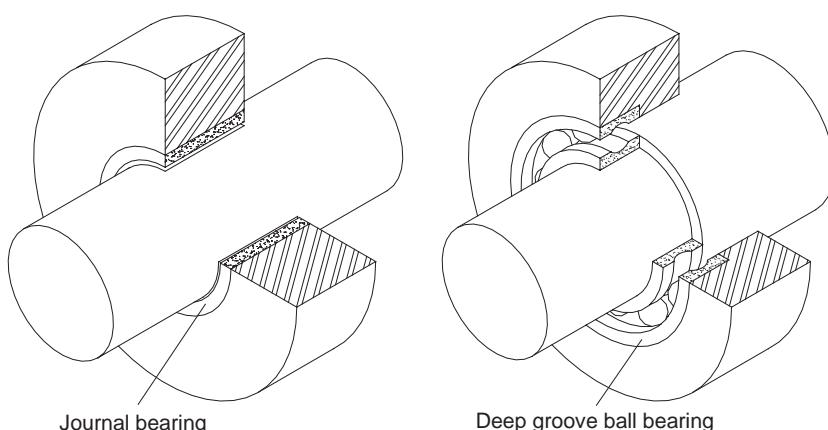


Figure 4.1 A journal bearing and a deep groove ball bearing.

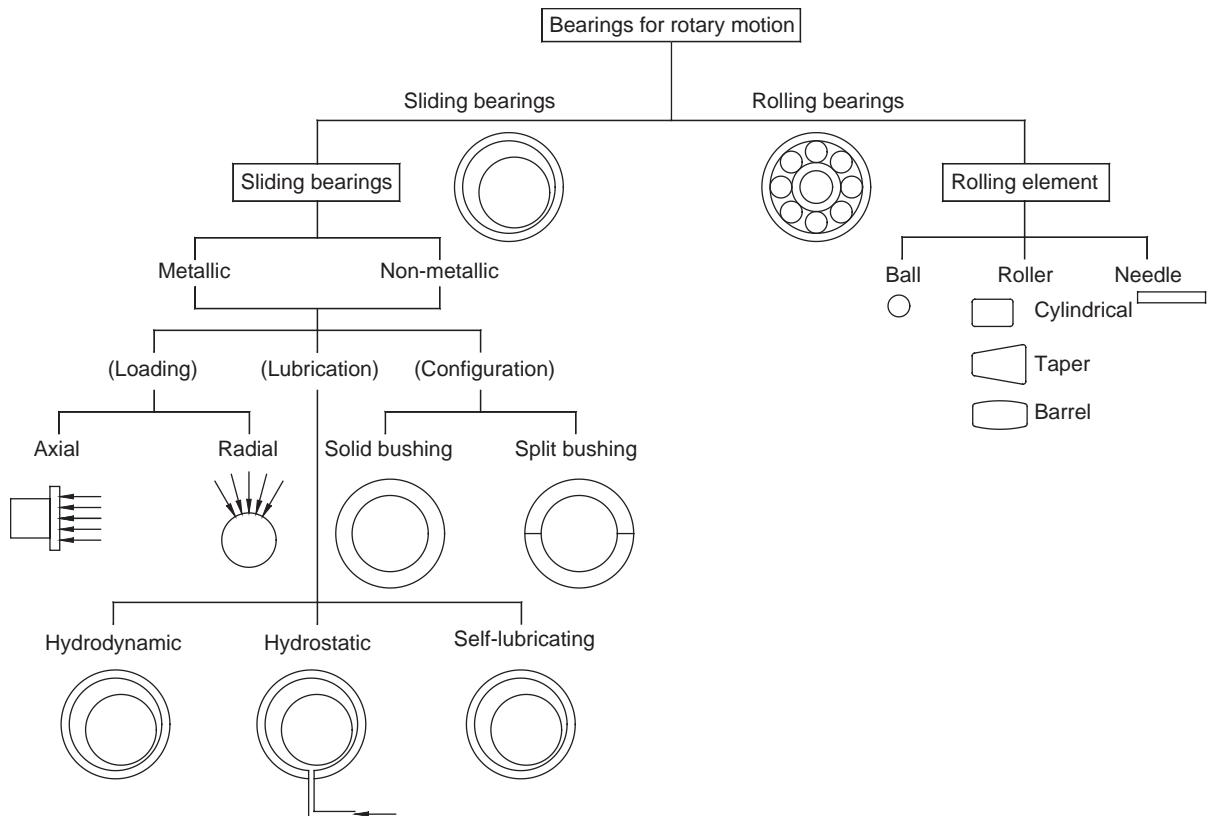


Figure 4.2 Bearing classification. Reproduced with adaptations from Hindhede (1983).

bearing. A general classification scheme for the distinction of bearings is given in Figure 4.2.

As can be seen from Figure 4.2, the scope of choice for a bearing is extensive. For a given application it may be possible to use different bearing types. For example, in a small gas turbine engine rotating at say 50 000 rpm, either rolling bearings or journal bearings could typically be used, although the optimal choice will depend on a number of factors such as life, cost and size. Figure 4.3 can be used to give guidance for which kind of bearing has the maximum load capacity at a given speed and shaft size and Table 4.1 gives an indication of the performance of the various bearing types for some criteria other than load capacity.

The design of lubricant film sliding bearings is introduced in Section 4.2 and the selection and installation of rolling element bearings is described in Section 4.3.

4.2 Sliding bearings

The term ‘sliding bearing’ refers to bearings where two surfaces move relative to each other without the benefit of rolling contact. The two surfaces slide over each other and this motion can be facilitated by means of a lubricant which gets squeezed by the motion of the components and can generate sufficient pressure to separate them, thereby reducing frictional contact and wear.

A typical application of sliding bearings is to allow rotation of a load-carrying shaft. The portion of the shaft at the bearing is referred to as the journal and the stationary part, which supports the load, is called the bearing (see Figure 4.4). For this reason, sliding bearings are often collectively referred to as journal bearings, although this term ignores the existence of sliding bearings that support linear translation of components. Another

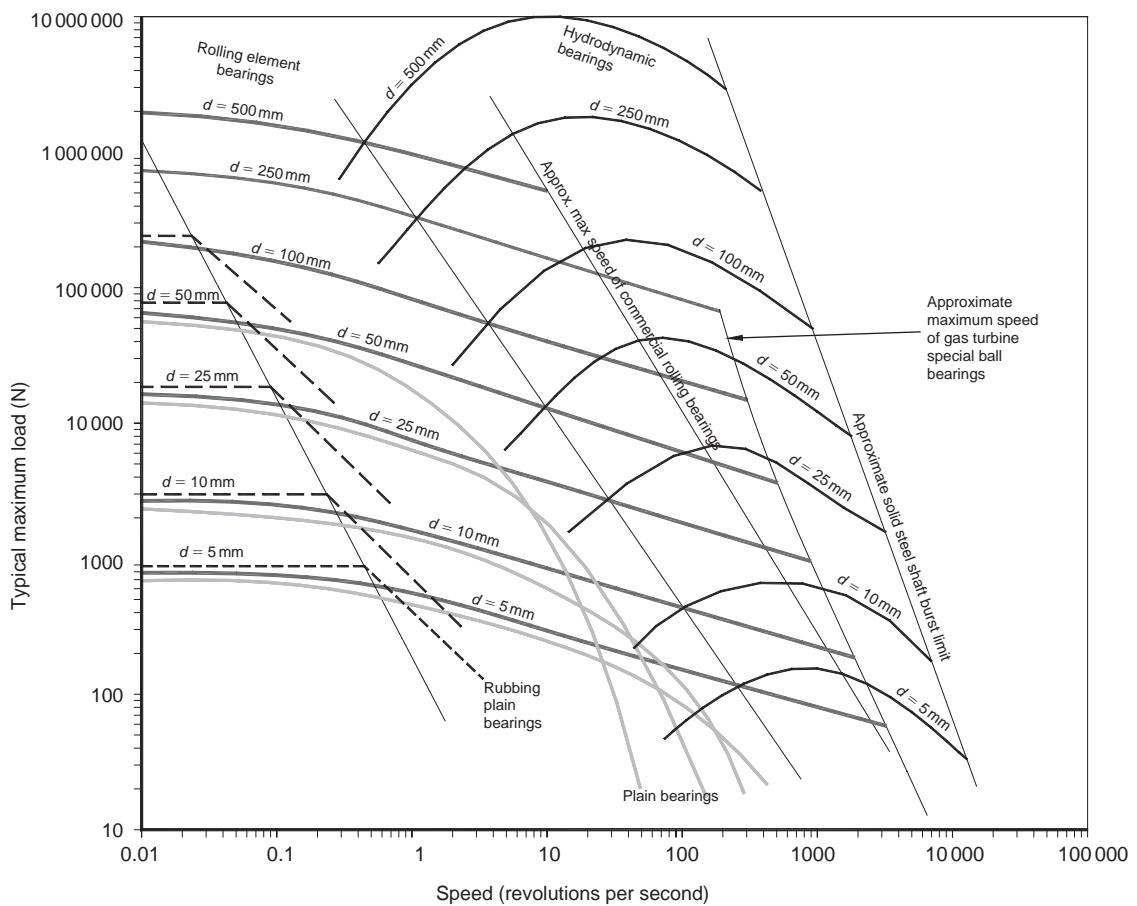


Figure 4.3 Bearing type selection by load capacity and speed. Reproduced courtesy of Neale (1993).

Table 4.1 Comparison of bearing performance for continuous rotation

Bearing type	Accurate radial location	Combined axial and radial load capability	Low starting torque capability	Silent running	Standard parts available	Lubrication simplicity
Rubbing plain bearings (non-metallic)	Poor	Some in most cases	Poor	Fair	Some	Excellent
Porous metal plain bearings oil impregnated	Good	Some	Good	Excellent	Yes	Excellent
Fluid film hydrodynamic bearings	Fair	No, separate thrust bearing needed	Good	Excellent	Some	Usually requires a recirculation system
Hydrostatic bearings	Excellent	No, separate thrust bearing needed	Excellent	Excellent	No	Poor special system needed
Rolling bearings	Good	Yes, in most cases	Very good	Usually satisfactory	Yes	Good when grease lubricated

Reproduced courtesy of Neale (1993).

common term is 'plain surface bearings'. This section is principally concerned with bearings for rotary motion and the terms 'journal' and 'sliding' bearing are used interchangeably.

There are three regimes of lubrication for sliding bearings:

1. boundary lubrication;
2. mixed film lubrication;
3. full film lubrication.

Boundary lubrication typically occurs at low relative velocities between the journal and the bearing surfaces and is characterized by actual physical contact. The surfaces, even if ground to a low value of surface roughness, will still consist of a series of peaks and troughs as illustrated schematically in Figure 4.5. Although some lubricant may be present, the pressures generated within it are not significant

and the relative motion of the surfaces brings the corresponding peaks periodically into contact. Mixed film lubrication occurs when the relative motion between the surfaces is sufficient to generate high enough pressures in the lubricant film, which can partially separate the surfaces for periods of time. There is still contact in places around the circumference between the two components. Full film lubrication occurs at higher relative velocities. Here the motion of the surface generates high pressures in the lubricant, which separate the two components and the journal can 'ride' on a wedge of fluid. All of these types of lubrication can be encountered in a bearing without external pressuring of the bearing. If lubricant under high enough pressure is supplied to the bearing to separate the two surfaces it is called a 'hydrostatic bearing'.

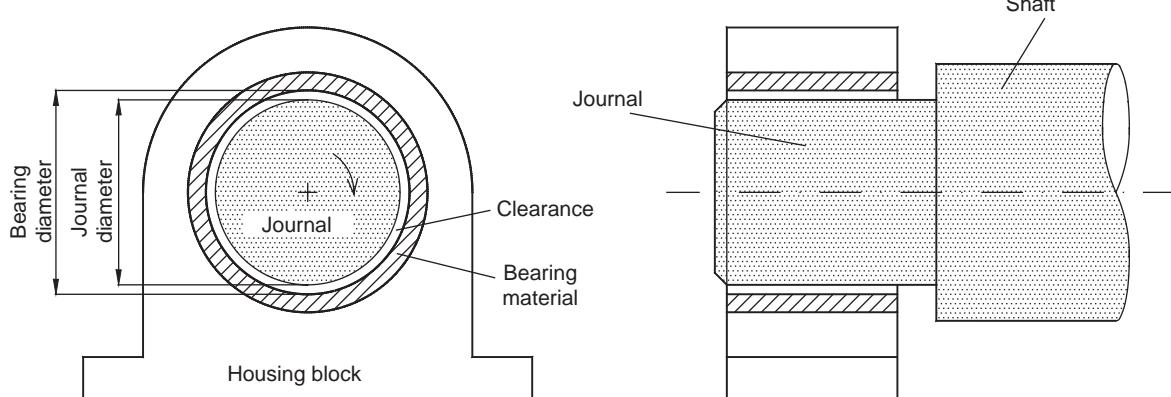


Figure 4.4 A plain surface, sliding or journal bearing.

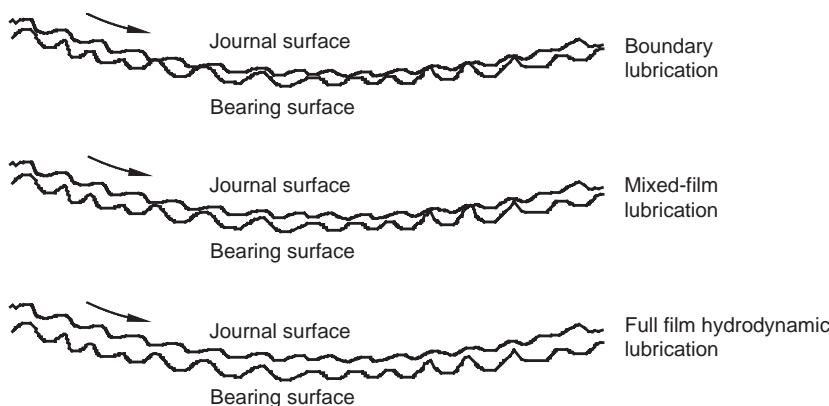


Figure 4.5 A schematic representation of the surface roughness for sliding bearings and the relative position depending on the type of lubrication occurring.

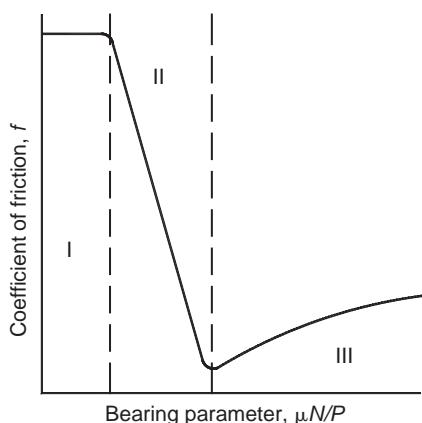


Figure 4.6 Schematic representation of the variation of bearing performance with lubrication. I, boundary; II, mixed film; III, hydrodynamic.

The performance of a sliding bearing differs markedly depending on which type of lubrication is physically occurring. This is illustrated in Figure 4.6, which shows the variation of the coefficient of friction with a group of variables called the ‘bearing parameter’ which is defined by:

$$\frac{\mu N}{P} \quad (4.1)$$

where μ , viscosity of lubricant (Pas); N , speed (for this definition normally in rpm); P , load capacity (N/m^2) given by

$$P = \frac{W}{LD} \quad (4.2)$$

where W , applied load (N); L , bearing length (m); D , journal diameter (m).

The bearing parameter, $\mu N/P$, groups several of the bearing design variables into one number. Normally, of course, a low coefficient of friction is desirable. In general, boundary lubrication is used for slow speed applications where the surface speed is less than approximately 1.5 m/s. Mixed film lubrication is rarely used because it is difficult to quantify the actual value of the coefficient of friction (note the steep gradient in Figure 4.6 for this zone). The design of boundary-lubricated bearings is outlined in Section 4.2.2 and full film hydrodynamic bearings in Section 4.2.3.

4.2.1 Lubricants

As can be seen from Figure 4.6 bearing performance is dependent on the type of lubrication occurring and the viscosity of the lubricant. The viscosity is a measure of a fluid’s resistance to shear. Lubricants can be solid, liquid or gaseous, although the most commonly known are oils and greases. The principal classes of liquid lubricants are mineral oils and synthetic oils. Their viscosity is highly dependent on temperature as illustrated in Figure 4.7. They are typically solid at -35°C , thin as paraffin at 100°C and burn above 240°C . Many additives are used to affect their performance. For example, EP (extreme pressure) additives add fatty acids and other compounds to the oil, which attack the metal surfaces to form ‘contaminant’ layers, which protect the surfaces and reduce friction even when the oil film is squeezed out by high contact loads. Greases are oils mixed with soaps to form a thicker lubricant that can be retained on surfaces.

The viscosity variation with temperature of oils has been standardized and oils are available with a class number, for example SAE 10, SAE 20, SAE 30, SAE 40, SAE 5W, SAE 10W, etc. The origin of this identification system developed by the Society of Automotive Engineers was to class oils for general purpose use and winter use, the ‘W’ signifying the latter. The lower the numerical value, the thinner or less viscous the oil. Multigrade oil, e.g. SAE 10W/40, is formulated to meet the viscosity requirements of two oils giving some of the benefits of the constituent parts. An equivalent identification system is also available from the International Organisation for Standardisation (ISO 3448).

4.2.2 Design of boundary lubricated bearings

As described in Section 4.2, the journal and bearing surfaces in a boundary lubricated bearing are in direct contact in places. These bearings are typically used for very low speed applications such as bushes and linkages where their simplicity and compact nature are advantageous. Examples include

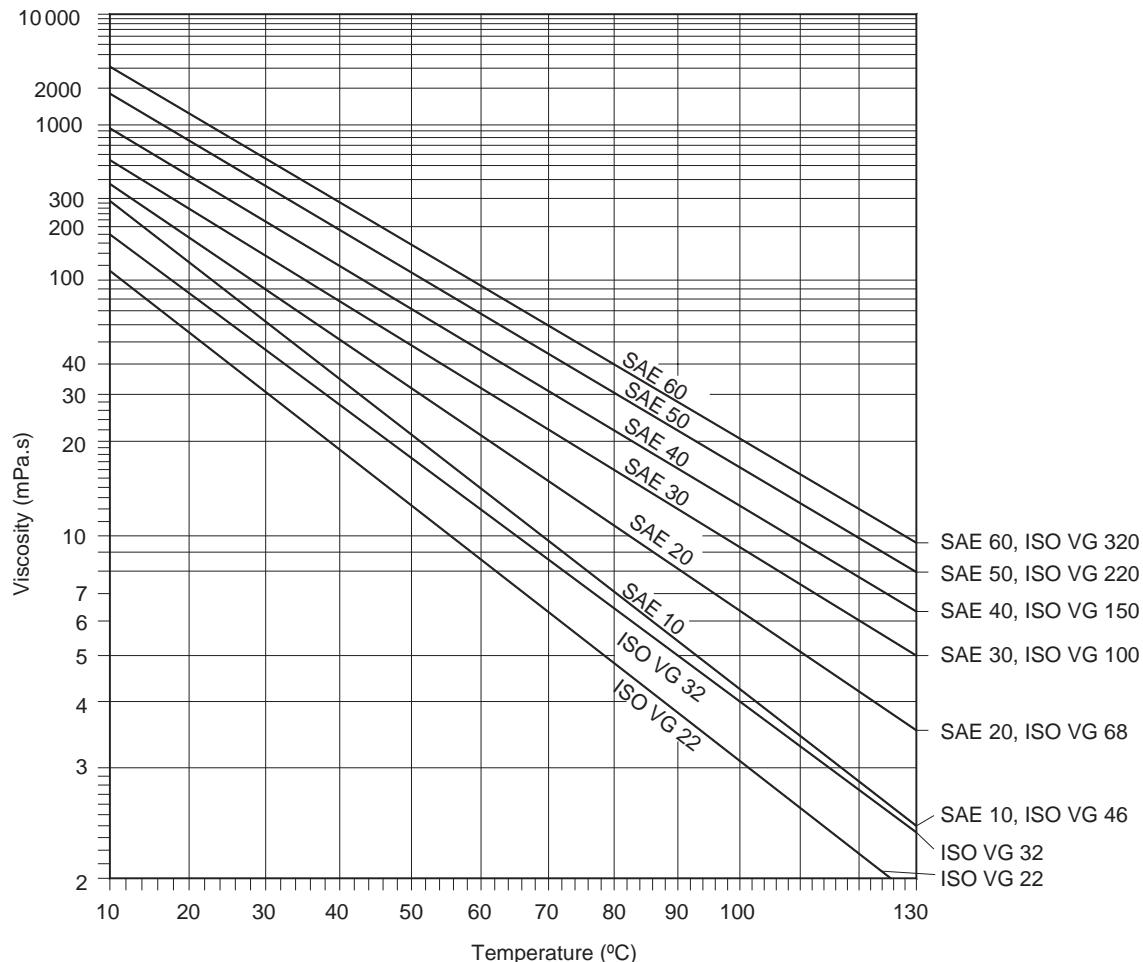


Figure 4.7 Variation of absolute viscosity with temperature for various lubricants.

household lawnmower wheels, garden hand tools, such as shears, ratchet wrenches, and domestic and automotive door hinges incorporating the journal as a solid pin riding inside a cylindrical outer member. A further example, involving linear motion is the slider in an ink jet printer.

General considerations in the design of a boundary lubricated bearing are:

- the coefficient of friction (both static and dynamic);
- the load capacity;
- the relative velocity between the stationary and moving components;
- the operating temperature;

- wear limitations; and
- the production capability.

A useful measure in the design of boundary lubricated bearings is the PV factor (load capacity \times peripheral speed), which indicates the ability of the bearing material to accommodate the frictional energy generated in the bearing. At the limiting PV value the temperature will be unstable and failure will occur rapidly. A practical value for PV is half the limiting PV value. Values for PV for various bearing materials are given in Table 4.2. The preliminary design of a boundary lubricated bearing essentially consists of setting the bearing proportions, its length and its diameter,

Table 4.2 Characteristics of some rubbing materials

Material	Maximum load capacity P (MN/m ²)	Limiting PV value (MN/m s)	Maximum operating temperature (°C)	Coefficient of friction	Coefficient of expansion ($\times 10^{-6}/^{\circ}\text{C}$)	Comments	Typical application
Carbon/graphite	1.4–2	0.11	350–500	0.1–0.25 dry	2.5–5.0	For continuous dry operation	Food and textile machinery
Carbon/graphite with metal	3.4	0.145	130–350	0.1–0.35 dry	4.2–5		
Graphite impregnated metal	70	0.28–0.35	350–600	0.1–0.15 dry	12–13		
Graphite/thermosetting resin	2	0.35	250	0.13–0.5 dry	3.5–5	Suitable for sea water operation	
Reinforced thermosetting plastics	35	0.35	200	0.1–0.4 dry	25–80		Roll-neck bearings
Thermoplastic material without filler	10	0.035	100	0.1–0.45 dry	100		Bushes and thrust washers
Thermoplastic with filler or metal backed	10–14	0.035–0.11	100	0.15–0.4 dry	80–100		Bushes and thrust washers
Thermoplastic material with filler bonded to metal back	140	0.35	105	0.2–0.35 dry	27	With initial lubrication only	For conditions of intermittent operation or boundary lubrication, e.g. ball joints, suspension, steering
Filled PTFE	7	up to 0.35	250	0.05–0.35 dry	60–80	Glass, mica, bronze, graphite	For dry operations where low friction and wear required
PTFE with filler, bonded to steel backing	140	up to 1.75	280	0.05–0.3 dry	20	Sintered bronze bonded to steel backing impregnated with PTFE/lead	Aircraft controls, linkages, gearbox, clutch, conveyors, bridges
Woven PTFE reinforced and bonded to metal backing	420	up to 1.6	250	0.03–0.3	–	Reinforcement may be interwoven glass fibre or rayon	Aircraft and engine controls, linkages, engine mountings, bridge bearings

Reproduced courtesy of Neale (1995).

and selecting the bearing material such that an acceptable PV value is obtained. This approach is set out as a step-by-step procedure below.

1. Determine the speed of rotation of the bearing and the load to be supported.
2. Set the bearing proportions. Common practice is to set the length to diameter ratio between 0.5 and 1.5. If the diameter D is known as an initial trial, set L equal to D .
3. Calculate the load capacity, $P = W/(LD)$.
4. Determine the maximum tangential speed of the journal.
5. Calculate the PV factor.
6. Multiply the PV value obtained by a factor of safety of 2.
7. Interrogate manufacturer's data or Table 4.2 to identify an appropriate bearing material with a value for PV factor greater than that obtained in (6) above.

Example 4.1

A bearing is to be designed to carry a radial load of 700 N for a shaft of diameter 25 mm running at a speed of 75 rpm (see Figure 4.8). Calculate the PV value and by comparison with the available materials listed in Table 4.2 determine a suitable bearing material.

Solution

The primary data are $W = 700 \text{ N}$, $D = 25 \text{ mm}$ and $N = 75 \text{ rpm}$.

Use $L/D = 1$ as an initial suggestion for the length to diameter ratio for the bearing. $L = 25 \text{ mm}$.

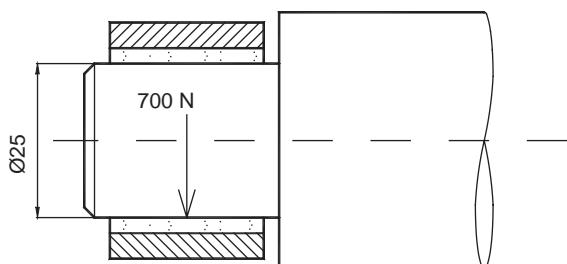


Figure 4.8 Boundary lubricated bearing design example (dimensions in mm).

Calculating the load capacity, P

$$P = \frac{W}{LD} = \frac{700}{0.025 \times 0.025} = 1.12 \text{ MN/m}^2$$

$$V = \omega r = \frac{\downarrow 2\pi}{60} N \frac{D}{2} = 0.1047 \times 75 \frac{0.025}{2} = 0.09816 \text{ m/s}$$

$$PV = 0.11 (\text{MN/m}^2)(\text{m/s})$$

Multiplying this by a factor of 2 gives $PV = 0.22 (\text{MN/m s})$.

A material with PV value greater than this, such as filled PTFE (limiting PV value up to 0.35), or PTFE with filler bonded to a steel backing (limiting PV value up to 1.75), would give acceptable performance.

4.2.3 Design of full film hydrodynamic bearings

In a full film hydrodynamic bearing, the load on the bearing is supported on a continuous film of lubricant, so that no contact between the bearing and the rotating journal occurs. The motion of the journal inside the bearing creates the necessary pressure to support the load, Figure 4.9. Hydrodynamic bearings are commonly found in internal

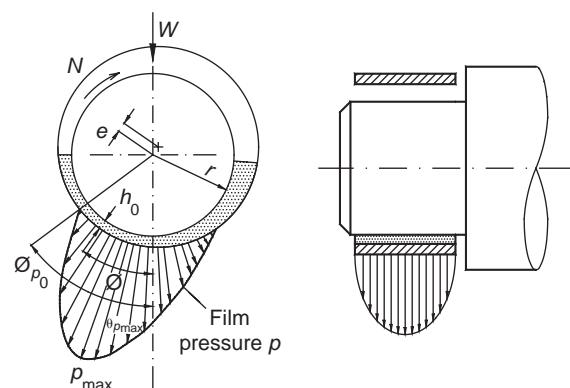


Figure 4.9 Motion of the journal generates pressure in the lubricant separating the two surfaces. Beyond h_0 , the minimum film thickness, the pressure terms go negative and the film is ruptured.

combustion engines for supporting the crankshaft and in turbocharger applications. Hydrodynamic bearings can consist of a full circumferential surface or a partial surface around the journal.

Bearing design can involve so many parameters that it is probably not possible to develop a procedure capable of designing the best possible bearing. One route towards this goal would be to assign attribute points to each aspect of the design and undertake an optimization exercise. This, however, would be time consuming if the software were not already in a developed state and would not necessarily produce an optimum result due to inadequacies in modelling and incorrect assignment of attribute weightings. An alternative approach which is sensible as a starting point and outlined here is to develop one or a number of feasible designs and use judgement to select the best, or combine the best features of the proposed designs.

The design procedure for a journal bearing, recommended here as a starting point, includes the specification of the journal radius r , the radial clearance c , the axial length of the bearing surface L , the type of lubricant and its viscosity μ , the

journal speed N and the load W . Values for the speed, the load and possibly the journal radius are usually specified by the machine requirements and stress and deflection considerations. As such journal bearing design consists of the determination of the radial clearance, the bearing length and the lubricant viscosity. The design process for a journal bearing is usually iterative. Trial values for the clearance, the length and the viscosity are chosen, various performance criteria calculated and the process repeated until a satisfactory or optimized design is achieved. Criteria for optimization may be minimizing of the frictional loss, minimizing the lubricant temperature rise, minimizing the lubricant supply, maximizing the load capability, and minimizing production costs.

The clearance between the journal and the bearing depends on the nominal diameter of the journal, the precision of the machine, surface roughness and thermal expansion considerations. An overall guideline is for the radial clearance, c , to be in the range $0.001 r < c < 0.002 r$, where r is the nominal bearing radius ($0.001 D < 2c < 0.002 D$). Figure 4.10 shows values for the recommended

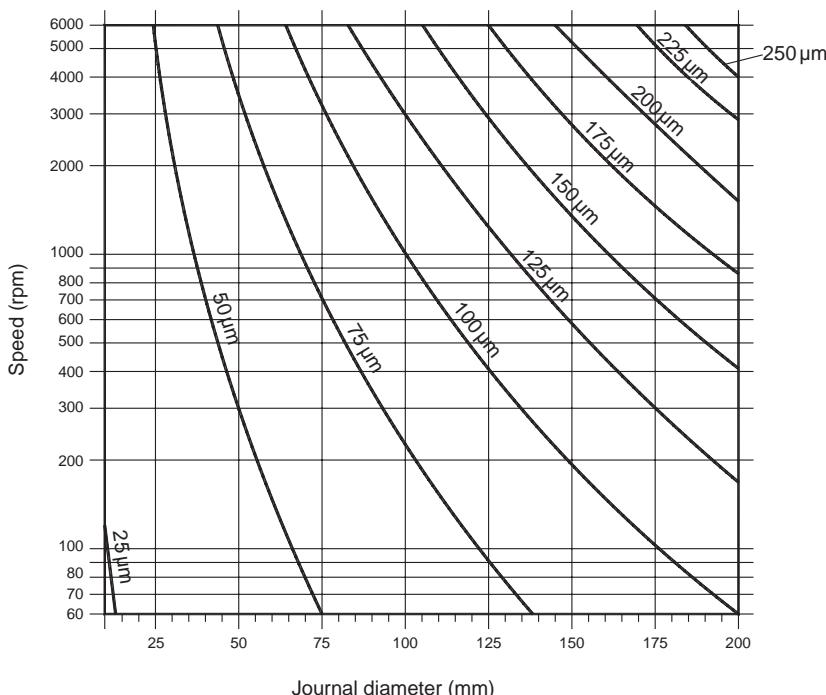


Figure 4.10 Minimum recommended values for the diametral clearance ($2 \times c$) for steadily loaded journal bearings.
Reproduced from Welsh (1983).

diametral clearance ($2 \times c$) as a function of the journal diameter and rotational speed for steadily loaded bearings.

For a given combination of r , c , L , μ , N and W , the performance of a journal bearing can be calculated. This requires determining the pressure distribution in the bearing, the minimum film thickness h_0 , the location of the minimum film thickness θ_{pmax} , the coefficient of friction f , the lubricant flow Q , the maximum film pressure p_{max} and the temperature rise ΔT of the lubricant.

The pressure distribution in a journal bearing (see Figure 4.9) can be determined by solving the relevant form of the Navier–Stokes fluid flow equations, which in the reduced form for journal bearings is called the Reynolds equation. This is given in its steady form as

$$\frac{\downarrow h^3}{x - \mu} \frac{p}{x} + \frac{\downarrow h^3}{z - \mu} \frac{p}{z} = 6V \frac{h}{x} + 6h \frac{V}{x} \quad (4.3)$$

where h , is film thickness (m); μ , dynamic viscosity (Pa s); p , fluid pressure (Pa); V , journal velocity (m/s); and x, y, z are rectangular coordinates (m).

The Reynolds equation can be solved by approximate mathematical methods or numerically. Once the pressure distribution has been established the journal performance can be determined in terms of the bearing load capacity, frictional losses, lubricant flow requirements and the lubricant temperature rise.

If the designer wishes to avoid the direct solution of Eq. 4.3, use can be made of a series of design charts. These were originally produced by Raimondi and Boyd (1958a,b,c) who used an iterative technique to solve the Reynolds equations. These charts give the film thickness, coefficient of friction, lubricant flow, lubricant side flow ratio, minimum film thickness location, maximum pressure ratio, maximum pressure ratio position and film termination angle versus the Sommerfield number (see Figures 4.11 to 4.18).

The Sommerfield number, which is also known as the bearing characteristic number, is defined in Eq. 4.4. It is used as it encapsulates all the parameters usually defined by the designer. Great care needs to be taken with units in the design of journal bearings. Many design charts have been produced using English units (psi, reyn, Btu, etc.). As long as a consistent set of units is maintained, use of the charts will yield sensible results. In particular note the use of revolutions per second in the definition of speed in the Sommerfield number.

$$S = \frac{\downarrow r}{\rightarrow}^2 \frac{\mu N_s}{P} \quad (4.4)$$

where S is the bearing characteristic number; r , the journal radius, (m); c , the radial clearance (m); μ , the absolute viscosity (Pa s); N_s , the journal speed (revolutions per second, rps); $P = W/LD$, the load per unit of projected bearing area (N/m²); W , the load on the bearing (N); D , the journal diameter (m); L , the journal bearing length (m).

Consider the journal shown in Figure 4.19. As the journal rotates it will pump lubricant in a clockwise direction. The lubricant is pumped into a wedge-shaped space and the journal is forced over to the opposite side. The angular position where the lubricant film is at its minimum thickness h_0 is called the attitude angle, ϕ . The centre of the journal is displaced from the centre of the bearing by a distance e , called the eccentricity. The ratio of the eccentricity e to the radial clearance c is called the eccentricity ratio ($\varepsilon = e/c$). The relationship between the film thickness, the clearance, eccentricity and eccentricity ratio is defined in Eqs 4.5 to 4.7.

$$h_0 = c - e \quad (4.5)$$

$$\varepsilon = \frac{e}{c} \quad (4.6)$$

$$\frac{h_0}{c} = 1 - \varepsilon \quad (4.7)$$

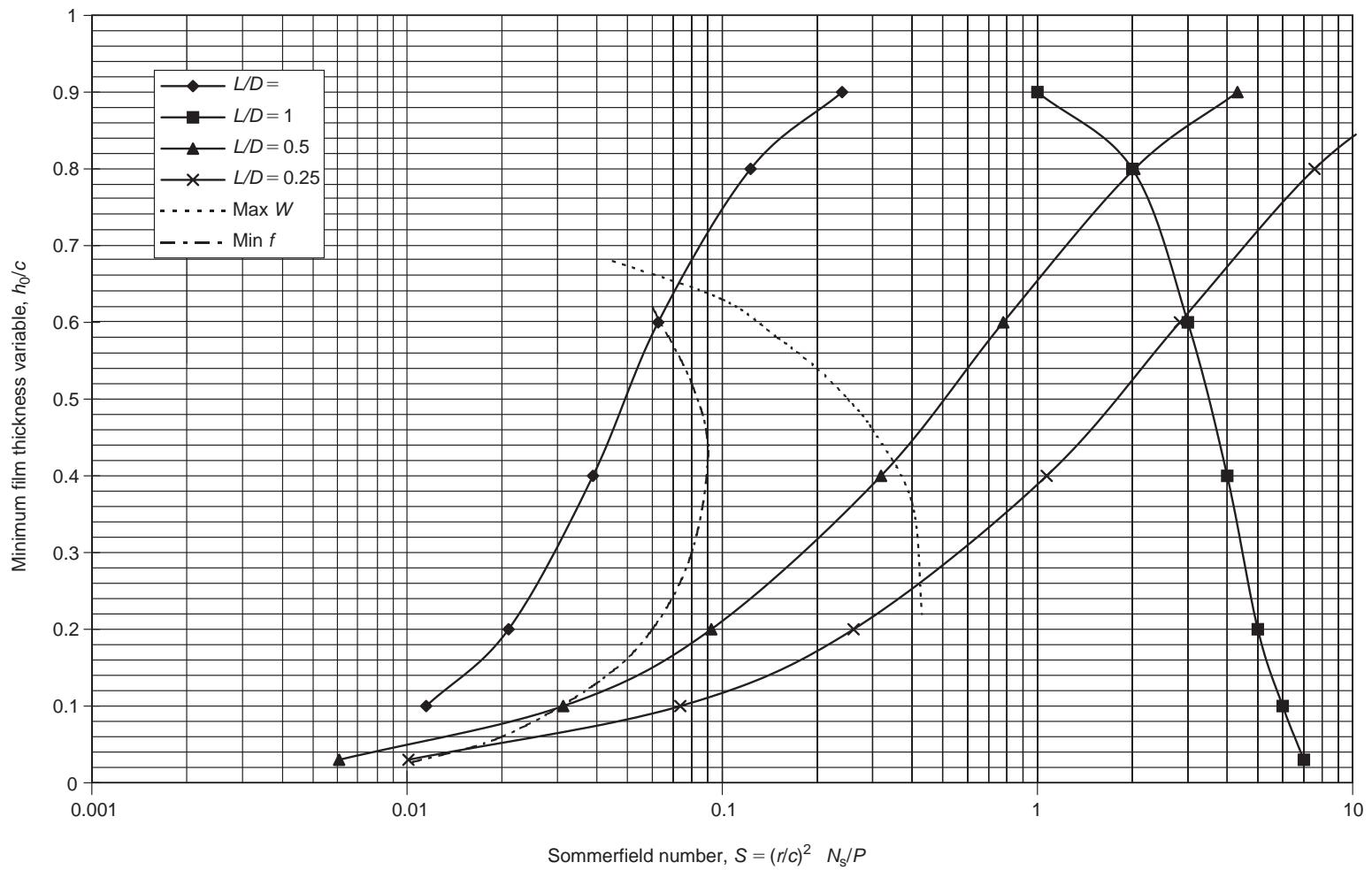


Figure 4.11 Chart for the minimum film thickness variable, (h_0/c) versus the Sommerfield number. Data from Raimondi and Boyd (1958c).

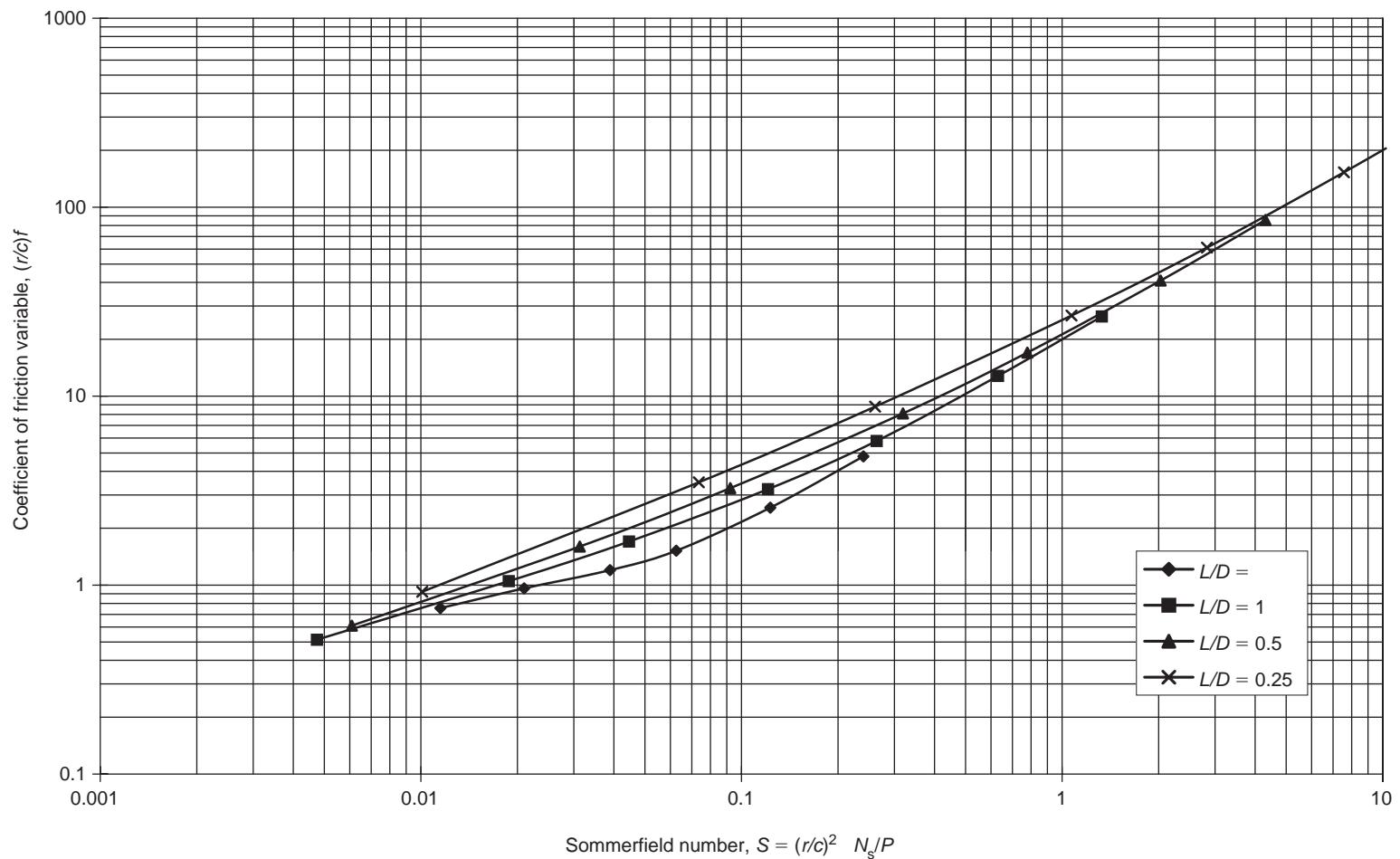


Figure 4.12 Chart for determining the coefficient of friction variable, $(r/c)f$. Data from Raimondi and Boyd (1958c).

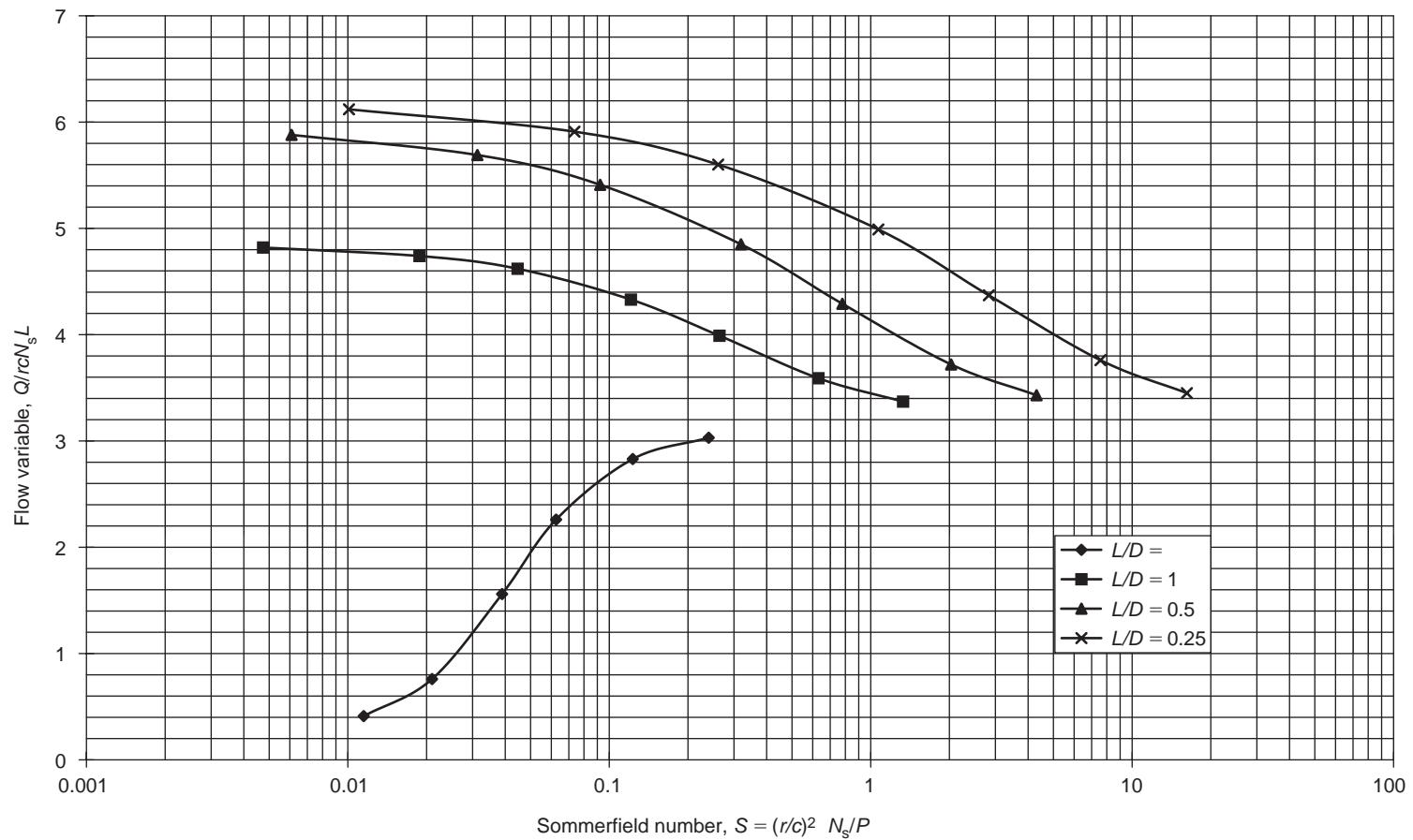


Figure 4.13 Chart for determining the flow variable, $Q/(rcN_s L)$. Data from Raimondi and Boyd (1958c).

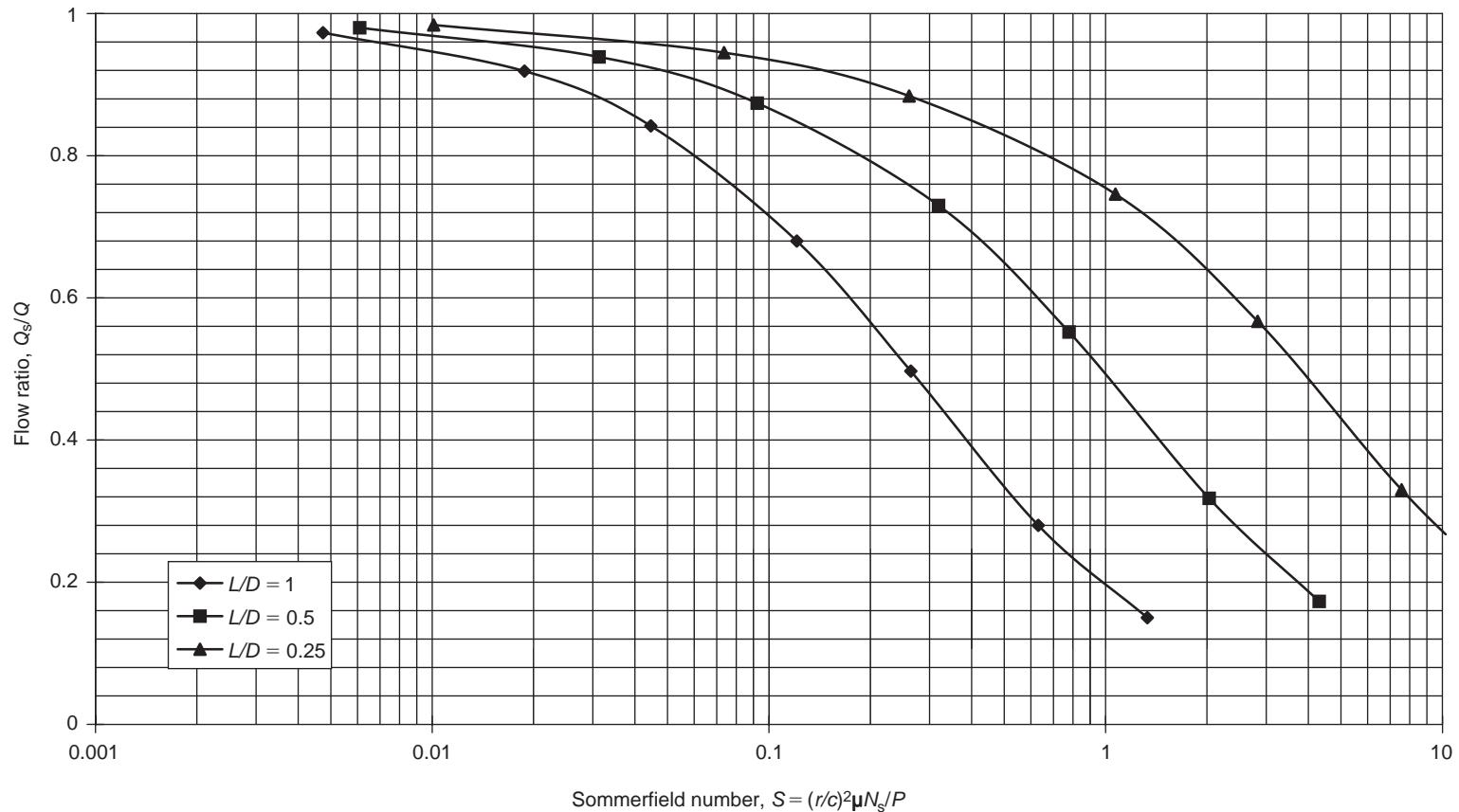


Figure 4.14 Chart for determining the ratio of side flow, Q_s , to total flow, Q . Data from Raimondi and Boyd (1958c).

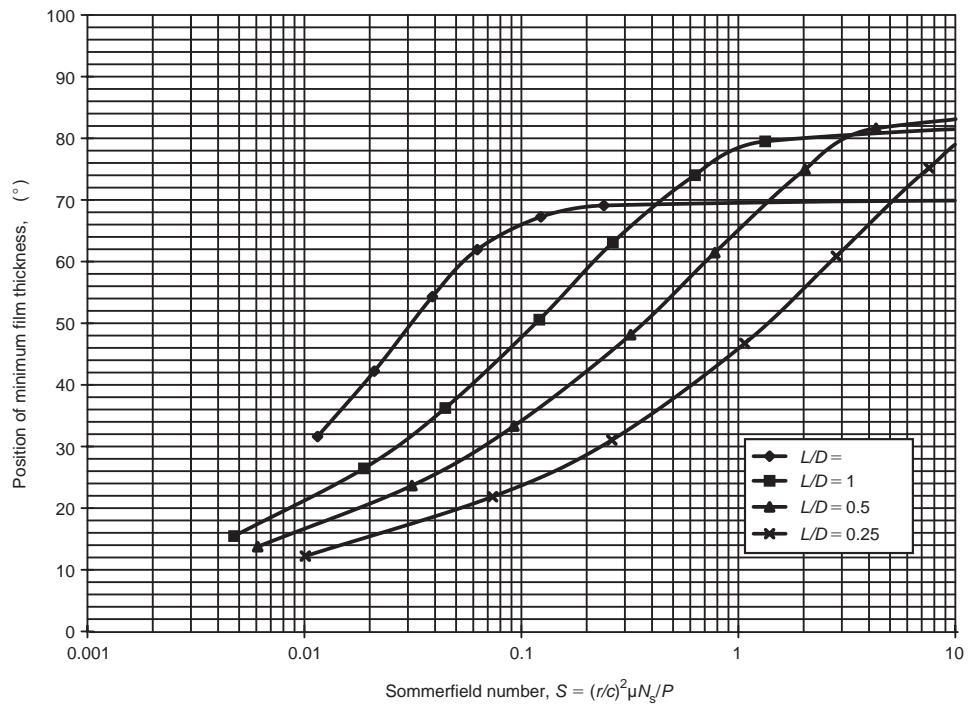


Figure 4.15 Chart for determining the position of the minimum film thickness ϕ . Data from Raimondi and Boyd (1958c).

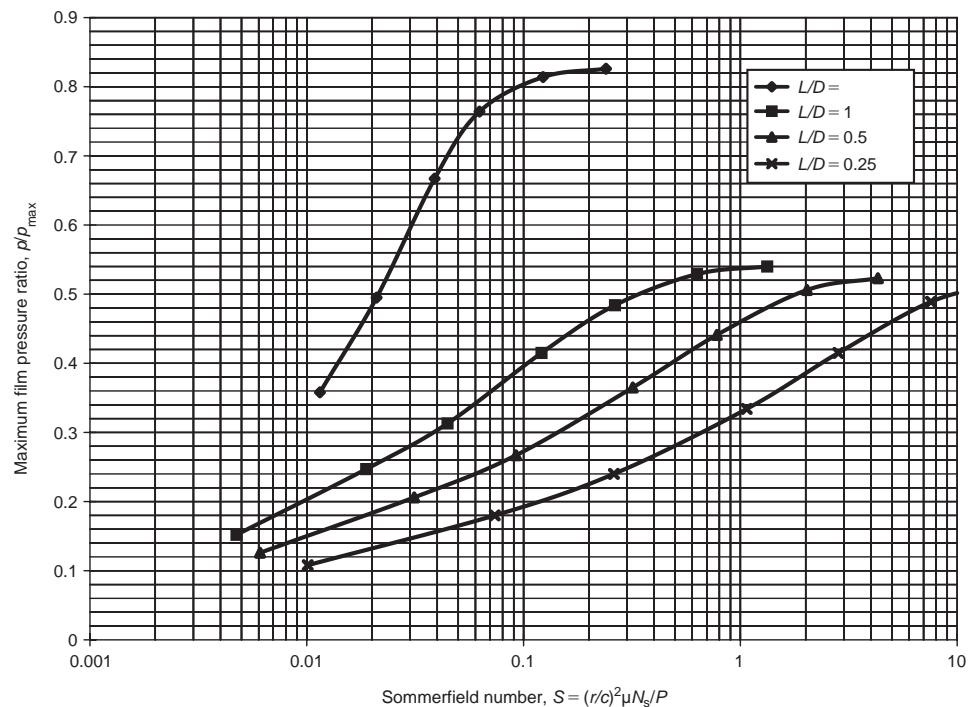


Figure 4.16 Chart for determining the maximum film pressure ratio, p/p_{\max} . Data from Raimondi and Boyd (1958c).

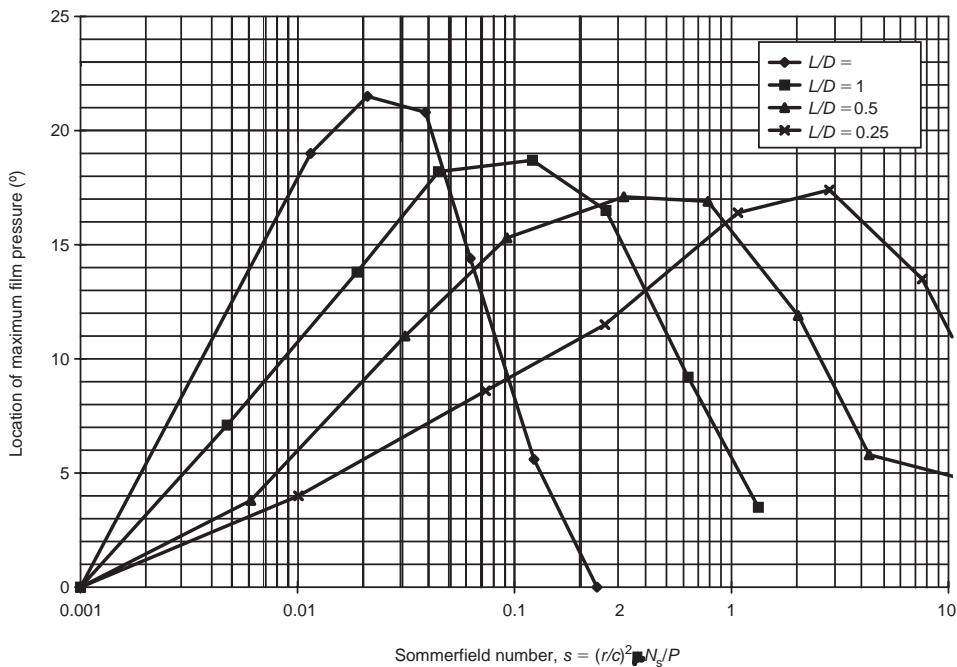


Figure 4.17 Chart for determining the position of maximum pressure, $\theta_{p\max}$. Data from Raimondi and Boyd (1958c).

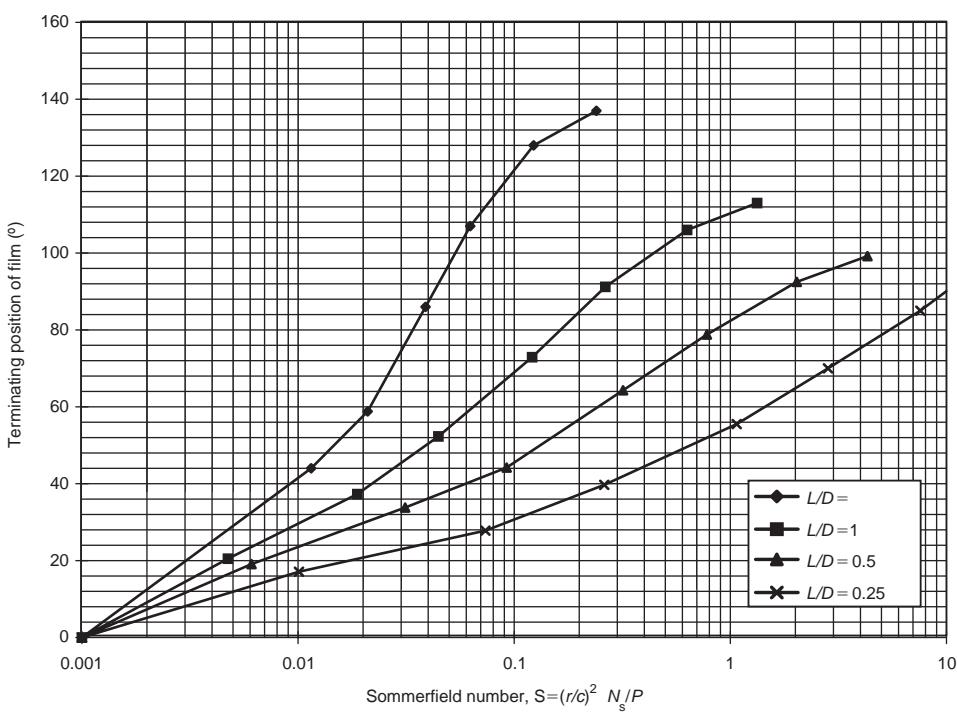


Figure 4.18 Chart for determining the film termination angle, θ_{po} . Data from Raimondi and Boyd (1958c).

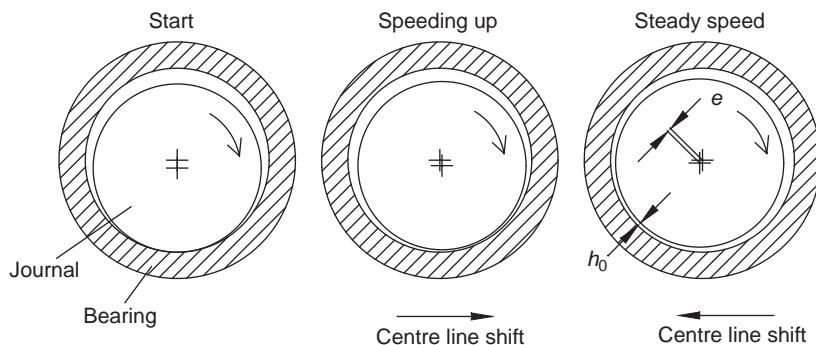


Figure 4.19 Full film hydrodynamic bearing motion from start up.

One of the assumptions made in the analysis of Raimondi and Boyd is that the viscosity of the lubricant is constant as it passes through the bearing. However, work is done on the lubricant in the bearing and the temperature of the lubricant leaving the bearing zone will be higher than the entrance value. Figure 4.7 shows the variation of viscosity with temperature for some of the SAE and ISO-defined lubricants and it can be seen that the value of the viscosity for a particular lubricant is highly dependent on the temperature. Some of the lubricant entrained into the bearing film emerges as side flow which carries away some of the heat. The remainder flows through the load-bearing zone and carries away the remainder of the heat generated. The temperature used for determining the viscosity can be taken as the average of the inlet and exit lubricant temperatures:

$$T_{av} = T_1 + \frac{T_2 - T_1}{2} = T_1 + \frac{\Delta T}{2} \quad (4.8)$$

where T_1 is the temperature of the lubricant supply and T_2 is the temperature of the lubricant leaving the bearing. Generally petroleum lubrication oils should be limited to a maximum temperature of approximately 70°C in order to prevent excessive oxidation.

One of the parameters that needs to be determined is the bearing lubricant exit temperature T_2 . This is a trial and error or iterative process. A value of the temperature rise, ΔT , is guessed, the viscosity for a standard oil, corresponding to this

value, determined and the analysis performed. If the temperature rise calculated by the analysis does not correspond closely to the guessed value the process should be repeated using an updated value for the temperature rise until the two match. If the temperature rise is unacceptable it may be necessary to try a different lubricant or modify the bearing configuration.

Given the length, the journal radius, the radial clearance, the lubricant type and its supply temperature the steps for determining the various bearing operating parameters are listed below.

1. Determine the speed of the journal and the load to be supported.
2. If L and D are not already determined, set the proportions of the bearing so that the load capacity, $P = W/LD$, is somewhere between 0.34 MN/m² for light machinery to 13.4 MN/m² for heavy machinery.
3. Determine a value for the radial clearance of the bearing using the data presented in Figure 4.10.
4. If not already specified select a lubricant. Lubricant oil selection is a function of speed or compatibility with other lubricant requirements. Generally as the design speed rises, oils with a lower viscosity should be selected.
5. Estimate a value for the temperature rise ΔT across the bearing. The value taken for the initial estimate is relatively unimportant. As a guide, a value of $\Delta T = 10^\circ\text{C}$ is generally a good starting guess. This value can be

- increased for high speed bearings and for low bearing clearances.
6. Determine the average lubricant temperature $T_{av} = T_1 + \Delta T/2$ and find the corresponding value for the viscosity for the chosen lubricant.
 7. Calculate the Sommerfield number, $S = (r/c)^2 \mu N_s / P$ and the length to diameter ratio.
 8. Use the charts (Figures 4.12, 4.13 and 4.14) to determine values for the coefficient of friction variable, the total lubricant flow variable, and the ratio of the side flow to the total lubricant flow with the values for the Sommerfield number and the L/D ratio.
 9. Calculate the temperature rise of the lubricant through the bearing using
- $$\Delta T = \frac{8.30 \times 10^{-6} P}{1 - \frac{1}{2}(Q_s/Q)} \times \frac{(r/c)f}{Q/r c N_s L} \quad (4.9)$$
10. If this calculated value does not match the estimated value for ΔT to within say 1°C repeat the procedure from step 6 using the updated value of the temperature rise to determine the average lubricant temperature.
 11. Check that the values for the Sommerfield number and the length to diameter ratio give a design that is in the optimal operating region for minimal friction and maximum load capability on the chart for the minimum film thickness variable (Figure 4.11). If the operating Sommerfield number and L/D ratio combination do not fall within this zone, then it is likely that the bearing design can be improved by altering the values for c , L , D , the lubricant type and the operating temperature as appropriate.
 12. If the value for the temperature rise across the bearing has converged, values for the total lubricant flow rate, Q , the side flow rate Q_s and the coefficient of friction f can be calculated. The journal bearing must be supplied with the value of the total lubricant calculated in order for it to perform as predicted by the charts.
 13. The charts given in Figures 4.11 to 4.18 can be used to determine values for the maximum film thickness, the maximum film pressure ratio, the location of the maximum film pressure and its terminating angular location as required.
 14. The torque required to overcome friction in the bearing can be calculated by
- $$\text{Torque} = fWr \quad (4.10)$$
15. The power lost in the bearing is given by
- $$\text{Power} = \omega \times \text{Torque} = 2\pi N_s \times \text{Torque} \quad (4.11)$$
- where ω is the angular velocity (rad/s).
16. Specify the surface roughness for the journal and bearing surfaces. A ground journal with an arithmetic average surface roughness of 0.4 to $0.8 \mu\text{m}$ is recommended for bearings of good quality. For high precision equipment both surfaces can be lapped or polished to give a surface roughness of 0.2 to $0.4 \mu\text{m}$. The specified roughness should be less than the minimum film thickness.
 17. Design the recirculation and sealing system for the bearing lubricant.
- For values of the length to diameter (L/D) ratio other than those shown in the charts given in Figures 4.11 to 4.18, values for the various parameters can be found by interpolation using Eq. 4.12 (see Raimondi and Boyd, 1958b).
- $$\gamma = \frac{1}{(L/D)^3} - \frac{1}{8} \downarrow_1 - \frac{L}{D} \downarrow_1 - 2 \frac{L}{D} \downarrow_1 - 4 \frac{L}{D} \downarrow_1 - 4 \frac{L}{D} \downarrow_1 + \frac{1}{3} \downarrow_1 - 2 \frac{L}{D} \downarrow_1 - 4 \frac{L}{D} \downarrow_1 - \frac{1}{4} \downarrow_1 - \frac{L}{D} \downarrow_1 - 4 \frac{L}{D} \downarrow_1 - 4 \frac{L}{D} \downarrow_1 + \frac{1}{24} \downarrow_1 - \frac{L}{D} \downarrow_1 - 2 \frac{L}{D} \downarrow_1 \quad (4.12)$$

Here γ is the desired variable and $\gamma_\infty, \gamma_1, \gamma_{1/2}$ and $\gamma_{1/4}$ are the variable at $L/D = \infty, 1, 1/2$ and $1/4$, respectively.

Example 4.2

A full journal bearing has a nominal diameter of 50.0 mm and a bearing length of 25.0 mm (see Figure 4.20). The bearing supports a load of 3000 N, and the journal design speed is 3000 rpm. The radial clearance has been specified as 0.04 mm. An SAE 10 oil has been chosen and the lubricant supply temperature is 50°C.

Find the temperature rise of the lubricant, the lubricant flow rate, the minimum film thickness, the torque required to overcome friction and the heat generated in the bearing.

Solution

The primary data are $D = 50.0$ mm, $L = 25$ mm, $W = 3000$ N, $N = 3000$ rpm, $c = 0.04$ mm, SAE 10, $T_1 = 50^\circ\text{C}$.

Guess a value for the lubricant temperature rise ΔT across the bearing to be say $\Delta T = 20^\circ\text{C}$.

$$T_{av} = T_1 + \frac{\Delta T}{2} = 50 + \frac{20}{2} = 60^\circ\text{C}$$

From Figure 4.7 for SAE 10 at 60°C, $\mu = 0.014$ Pa s.

$$N_s = 3000/60 = 50 \text{ rps},$$

$$L/D = 25/50 = 0.5.$$

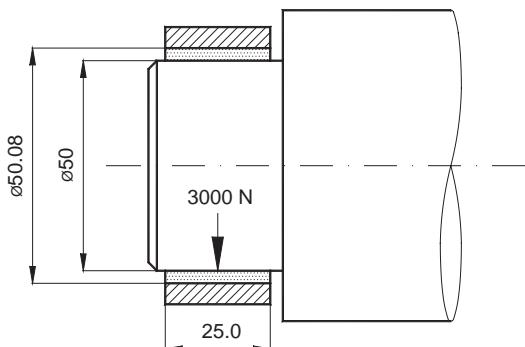


Figure 4.20 Bearing design example.

$$P = \frac{W}{LD} = \frac{3000}{0.025 \times 0.05}$$

$$= 2.4 \times 10^6 \text{ N/m}^2$$

$$S = \frac{\downarrow r^2 \mu N_s}{\rightarrow P}$$

$$= \frac{\downarrow 25 \times 10^{-3}}{-0.04 \times 10^{-3}} \frac{2}{2.4 \times 10^6} \frac{0.014 \times 50}{2.4 \times 10^6}$$

$$= 0.1139.$$

From Figure 4.12 with $S = 0.1139$ and $L/D = 0.5$, $(r/c)f = 3.8$.

From Figure 4.13 with $S = 0.1139$ and $L/D = 0.5$, $Q/(rcN_s L) = 5.34$.

From Figure 4.14 with $S = 0.1139$ and $L/D = 0.5$, $Q_s/Q = 0.852$.

The value of the temperature rise of the lubricant can now be calculated using Eq. 4.9:

$$\begin{aligned} \Delta T &= \frac{8.3 \times 10^{-6} P}{1 - 0.5 (Q_s/Q)} \times \frac{(r/c)f}{Q/(rcN_s L)} \\ &= \frac{8.3 \times 10^{-6} \times 2.4 \times 10^6}{1 - (0.5 \times 0.852)} \times \frac{3.8}{5.34} \\ &= 24.70^\circ\text{C} \end{aligned}$$

As the value calculated for the lubricant temperature rise is significantly different to the estimated value, it is necessary to repeat the above calculation, but using the new improved estimate for determining the average lubricant temperature.

Using $\Delta T = 24.70^\circ\text{C}$ to calculate T_{av} gives:

$$T_{av} = 50 + \frac{24.70}{2} = 62.35^\circ\text{C}$$

Repeating the procedure using the new value for T_{av} gives:

$$\mu = 0.0136 \text{ Pa s}$$

$$S = 0.1107.$$

From Figure 4.12 with $S = 0.1107$ and $L/D = 0.5$, $(r/c)f = 3.7$.

From Figure 4.13 with $S = 0.1107$ and $L/D = 0.5$, $Q/(rcN_s L) = 5.35$.

From Figure 4.14 with $S = 0.1107$ and $L/D = 0.5$, $Q_s/Q = 0.856$.

$$\Delta T = \frac{8.3 \times 10^{-6} \times 2.4 \times 10^6}{1 - (0.5 \times 0.856)} \times \frac{3.7}{5.35} = 24.08^\circ\text{C},$$

$$T_{av} = 50 + \frac{24.08}{2} = 62.04^\circ\text{C}.$$

This value for T_{av} is close to the previous calculated value suggesting that the solution has converged. For $T_{av} = 62.04^\circ\text{C}$, $\mu = 0.0136 \text{ Pa s}$ and $S = 0.1107$.

The other parameters can now be found.

$$\begin{aligned} Q &= nN_s L \times 5.35 \\ &= 25 \times 0.04 \times 50 \times 25 \times 5.35 \\ &= 6688 \text{ mm}^3/\text{s}. \end{aligned}$$

From Figure 4.11, $h_0/c = 0.22$, $h_0 = 0.0088 \text{ mm}$.

$$\begin{aligned} f &= 3.7 \times (c/r) = 3.7 \times (0.04/25) \\ &= 0.00592. \end{aligned}$$

The torque is given by

$$\begin{aligned} \text{Torque} &= fWr \\ &= 0.00592 \times 3000 \times 0.025 \\ &= 0.444 \text{ N m} \end{aligned}$$

The power dissipated in the bearing is given by

$$\text{Power} = 2\pi \times \text{Torque} \times N_s = 139.5 \text{ W}.$$

Example 4.3

A full journal bearing is required for a 30 mm diameter shaft, rotating at 1200 rpm supporting a load of 1500 N (see Figure 4.21). From previous experience, the lubrication system available is capable of delivering the lubricant to the bearing

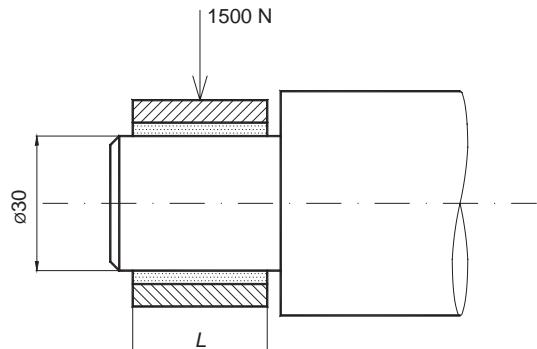


Figure 4.21 Bearing design example.

at 40°C . Select an appropriate radial clearance, length and lubricant for the bearing and determine the temperature rise of the lubricant, the total lubricant flow rate required and the power absorbed in the bearing. Check that your chosen design operates within the optimum zone indicated by the dotted lines on the minimum film thickness ratio chart (Figure 4.11).

Solution

The primary design information is: $D = 30.0 \text{ mm}$, $W = 1500 \text{ N}$, $N = 1200 \text{ rpm}$, SAE 10, $T_1 = 40^\circ\text{C}$.

From Figure 4.10 for a speed of 1200 rpm and a nominal journal diameter of 30 mm, a suitable value for the diametral clearance is 0.05 mm. The radial clearance is therefore 0.025 mm. The next step is to set the length of the bearing. Typical values for L/D ratios are between 0.5 and 1.5. Here a value for the ratio is arbitrarily set as $L/D = 1$, i.e. $L = 30 \text{ mm}$.

Given the speed of the bearing, 1200 rpm, as an initial proposal an SAE 10 lubricant is selected.

The procedure for determining the average lubricant temperature is the same as for the previous example. As a first estimate the temperature rise of the lubricant is taken as 10°C and the results of the iterative procedure are given in Table 4.3.

Table 4.3 Tabular data for Example 4.3

Temperature rise, ΔT ($^{\circ}\text{C}$)	10	15.90	13.76	13.76
Average lubricant temperature, T_{av} ($^{\circ}\text{C}$)	45	47.95	46.88	46.88
Average lubricant viscosity, μ (Pa s)	0.028	0.023	0.024	Converged
Sommerfield number, S	0.1209	0.09934	0.1037	
Coefficient of friction variable, $(r/c)f$	3.3	2.8	2.8	
Flow variable, Q/rcN_sL	4.35	4.4	4.4	
Side flow to total flow ratio, Q_s/Q	0.68	0.72	0.72	

$$\begin{aligned}
 P &= \frac{W}{LD} = \frac{1500}{0.03 \times 0.03} \\
 &= 1.667 \times 10^6 \text{ N/m}^2 \\
 S &= \frac{\frac{\downarrow r}{\rightarrow}^2 \mu N_s}{P} \\
 &= \frac{\downarrow 15 \times 10^{-3}}{-0.025 \times 10^{-3}}^2 \frac{\mu 20}{1.667 \times 10^6} \\
 &= 4.319 \mu \\
 T_{\text{av}} &= T_1 + \frac{\Delta T}{2} \\
 &= 40 + \frac{10}{2} = 45^{\circ}\text{C}
 \end{aligned}$$

The total lubricant flow rate required is given by

$$\begin{aligned}
 Q &= rcN_sL \times 4.4 \\
 &= 15 \times 0.025 \times 20 \times 30 \times 4.4 = 990
 \end{aligned}$$

With $S = 0.1037$ and $L/D = 1$, the design selected is within the optimum operating zone for minimum friction and optimum load capacity indicated in Figure 4.11.

$$\begin{aligned}
 \text{Friction factor, } f &= 2.8 \times (c/r) \\
 &= 2.8 \times (0.025/15) \\
 &= 0.004667
 \end{aligned}$$

$$\begin{aligned}
 \text{Torque} &= fWr \\
 &= 0.004667 \times 1500 \times 0.015 \\
 &= 0.105 \text{ N m}
 \end{aligned}$$

$$\text{Power} = 2\pi N_s \times \text{Torque} = 13.19 \text{ W.}$$

4.2.4 Alternative method for the design of full film hydrodynamic bearings

An alternative method to using the design charts of Raimondi and Boyd is to use an approximate method developed by Reason and Narang (1982). They defined an approximation for the pressure variation in a journal bearing given by:

$$p = \frac{\frac{3\mu V}{rc^2} \frac{L^2}{4} - z^2 \frac{\varepsilon \sin \theta}{(1 + \varepsilon \cos \theta)^3}}{1 + \frac{2 + \varepsilon^2}{2r^2} \frac{L^2}{4} - z^2 \frac{1}{(1 + \varepsilon \cos \theta)(2 + \varepsilon \cos \theta)}} \quad (4.13)$$

where μ , viscosity (Pa s); V , surface velocity (m/s); r , journal radius (m); c , radial clearance (m); L , bearing length (m); z , axial coordinate (m); ε , eccentricity variable; θ , circumferential coordinate.

This equation was integrated using an approximate method and the solution given in terms of two integrals I_s and I_c , the sine and cosine load integrals, respectively. Both I_s and I_c are functions of the L/D ratio and the eccentricity ratio ε . The values of these integrals are given in Table 4.4 along with the corresponding Sommerfeld number calculated using

$$S = \frac{1}{6\pi\sqrt{I_c^2 + I_s^2}} \quad (4.14)$$

Knowing values for the integrals I_s and I_c enables the bearing parameters to be calculated as shown below.

Table 4.4 Values of the sine and cosine integrals, I_s and I_c , and the Sommerfield number S , as functions of the L/D ratio and the eccentricity ratio ε

ε	$L/D = 0.25$	$L/D = 0.5$	$L/D = 0.75$	$L/D = 1.0$	$L/D = 1.5$	$L/D = 2$	$L/D = \infty$
0.10	$I_s = 0.0032$ $I_c = -0.0004$ $S = 16.4506$	0.0120 −0.0014 4.3912	0.0244 −0.0028 2.1601	0.0380 −0.0041 1.3880	0.0636 −0.0063 0.8301	0.0839 −0.0076 0.6297	0.1570 −0.0100 0.3372
0.20	$I_s = 0.0067$ $I_c = -0.0017$ $S = 7.6750$	0.0251 −0.0062 2.0519	0.0505 −0.0118 1.0230	0.0783 −0.0174 0.6614	0.1300 −0.0259 0.4002	0.1705 −0.0312 0.3061	0.3143 −0.0408 0.1674
0.30	$I_s = 0.0109$ $I_c = -0.0043$ $S = 4.5276$	0.0404 −0.0153 1.2280	0.0804 −0.0289 0.6209	0.1236 −0.0419 0.4065	0.2023 −0.0615 0.2509	0.2628 −0.0733 0.1944	0.4727 −0.0946 0.1100
0.40	$I_s = 0.0164$ $I_c = -0.0089$ $S = 2.8432$	0.0597 −0.0312 0.7876	0.1172 −0.0579 0.4058	0.1776 −0.0825 0.2709	0.2847 −0.1183 0.1721	0.3649 −0.1391 0.1359	0.6347 −0.1763 0.0850
0.50	$I_s = 0.0241$ $I_c = -0.0174$ $S = 1.7848$	0.0862 −0.0591 0.5076	0.1656 −0.1065 0.2694	0.2462 −0.1484 0.1845	0.3835 −0.2065 0.1218	0.4831 −0.2391 0.0984	0.8061 −0.2962 0.0618
0.60	$I_s = 0.0363$ $I_c = -0.0338$ $S = 1.0696$	0.1259 −0.1105 0.3167	0.2345 −0.1917 0.1752	0.3396 −0.2590 0.1242	0.5102 −0.3474 0.0859	0.6291 −0.3949 0.0714	0.9983 −0.4766 0.0480
0.70	$I_s = 0.0582$ $I_c = -0.0703$ $S = 0.5813$	0.1927 −0.2161 0.1832	0.3430 −0.3549 0.1075	0.4793 −0.4612 0.0798	0.6878 −0.5916 0.0585	0.8266 −0.6586 0.0502	1.2366 −0.7717 0.0364
0.80	$I_s = 0.1071$ $I_c = -0.1732$ $S = 0.2605$	0.3264 −0.4797 0.0914	0.5425 −0.7283 0.0584	0.7220 −0.8997 0.0460	0.9771 −1.0941 0.0362	1.1380 −1.1891 0.0322	1.5866 −1.3467 0.0255
0.90	$I_s = 0.2761$ $I_c = -0.6644$ $S = 0.0737$	0.7079 −1.4990 0.0320	1.0499 −2.0172 0.0233	1.3002 −2.3269 0.0199	1.6235 −2.6461 0.0171	1.8137 −2.7932 0.0159	2.3083 −3.0339 0.0139
0.95	$I_s = 0.6429$ $I_c = -2.1625$ $S = 0.0235$	1.3712 −3.9787 0.0126	1.8467 −4.8773 0.0102	2.1632 −5.3621 0.0092	2.5455 −5.8315 0.0083	2.7600 −6.0396 0.0080	3.2913 −6.3776 0.0074
0.99	$I_s = 3.3140$ $I_c = -22.0703$ $S = 0.0024$	4.9224 −28.5960 0.0018	5.6905 −30.8608 0.0017	6.1373 −31.9219 0.0016	6.6295 −32.8642 0.0016	6.8881 −33.2602 0.0016	8.7210 −33.5520 0.0015

From Reason and Narang, 1982.

- Given W, N_s, L, D, c , the type of lubricant and T_1 , guess a value for the temperature rise of the lubricant ΔT and calculate T_{av} . Determine the viscosity at T_{av} from Figure 4.7.
- Calculate the bearing loading pressure

$$P = \frac{W}{LD}.$$

- Calculate the Sommerfield number

$$S = \frac{\downarrow r}{\rightarrow} \frac{\mu N_s^2}{P}.$$

- Knowing the length to diameter ratio and the Sommerfield number, search the correct L/D column in Table 4.4 for a matching Sommerfield number. A matching Sommerfield number

will give the eccentricity ratio reading across the table to the left. Interpolation may be necessary for values of S between the values listed in the table. Once the correct eccentricity has been found, determine the values for the sine and cosine integrals I_s and I_c to correspond with the value of the Sommerfeld number.

Recall that for a linear function, $y = f(x)$, the value for y between the bounding points (x_1, y_1) and (x_2, y_2) for a given value of x can be found by linear interpolation:

$$y = \frac{\downarrow x - x_1}{\rightarrow x_2 - x_1} (y_2 - y_1) + y_1 \quad (4.15)$$

- Calculate the minimum film thickness $h_0 = c(1 - \varepsilon)$.
- Calculate the attitude angle $\phi, (\circ)$

$$\phi = \tan^{-1} \frac{\downarrow I_s}{\rightarrow I_c} \quad (4.16)$$

- Calculate the flow variable $Q/(rcN_s L)$, where Q is the lubricant supply.

$$\frac{Q}{rcN_s L} = \pi(1 + \varepsilon) - \pi\varepsilon\eta_1 \frac{\downarrow D}{\rightarrow L}^2$$

$$\frac{1}{\sqrt{\frac{D}{L}^2 \frac{\eta_1}{2} + \frac{1}{4}}} \tan h^{-1} \frac{1}{\rightarrow 2 \sqrt{\frac{D}{L}^2 \frac{\eta_1}{2} + \frac{1}{4}}} \quad (4.17)$$

where

$$\eta_1 = \frac{(1 + \varepsilon)(2 + \varepsilon)}{(2 + \varepsilon^2)} \quad (4.18)$$

- Calculate the flow through the minimum film thickness Q_π .

$$\frac{Q_\pi}{rcN_s L} = \pi(1 - \varepsilon) + \pi\varepsilon\eta_2 \frac{\downarrow D}{\rightarrow L}^2$$

$$\frac{1}{\sqrt{\frac{D}{L}^2 \frac{\eta_2}{2} + \frac{1}{4}}} \tan h^{-1} \frac{1}{\rightarrow 3 \sqrt{\frac{D}{L}^2 \frac{\eta_2}{2} + \frac{1}{4}}} \quad (4.19)$$

where

$$\eta_2 = \frac{(1 - \varepsilon)(2 - \varepsilon)}{(2 + \varepsilon^2)} \quad (4.20)$$

Note that

$$\tan h^{-1} x = \frac{1}{2} \ln \frac{\downarrow 1 + x}{\rightarrow 1 - x} \quad (4.21)$$

- The flow ratio can be calculated using

$$\frac{Q_s}{Q} = 1 - \frac{Q_\pi}{Q} \quad (4.22)$$

- Calculate the friction variable $(r/c)f$

$$\frac{\downarrow r}{\rightarrow} = 6\pi S \frac{\pi}{3\sqrt{1 - \varepsilon^2}} + \frac{\varepsilon}{2} I_s \quad (4.23)$$

- Calculate the temperature rise:

$$\Delta T = \frac{8.30 \times 10^{-6} P}{1 - \frac{1}{2}(Q_s/Q)} \times \frac{(r/c)f}{Q/rcN_s L}$$

Although these equations may appear complex the advantage of the method developed by Reason and Narang is that it is suitable for conventional programming or utilization within a spreadsheet.

Example 4.4

A full journal bearing has a nominal diameter of 20.0 mm and a bearing length of 20.0 mm (see Figure 4.22). The bearing supports a load of 1000 N, and the journal design speed is 6000 rpm. The radial clearance has been specified as 0.02 mm. An SAE 20 oil has been chosen to be compatible with requirements elsewhere

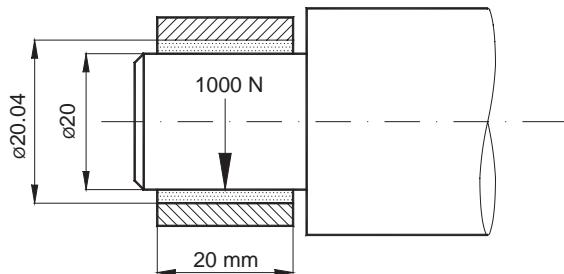


Figure 4.22 Bearing design example.

in the machine and the lubricant supply temperature is 60°C. Using the approximate method of Reason and Narang find the temperature rise of the lubricant, the total lubricant flow rate, the minimum film thickness, the torque required to overcome friction and the heat generated in the bearing.

Solution

The primary design data are $D = 20 \text{ mm}$, $L = 20 \text{ mm}$, $W = 1000 \text{ N}$, $N = 6000 \text{ rpm}$, $c = 0.02 \text{ mm}$, SAE 20, $T_1 = 60^\circ\text{C}$.

Guess $\Delta T = 15^\circ\text{C}$. Hence:

$$T_{av} = T_1 + \frac{\Delta T}{2} = 60 + \frac{15}{2} = 67.5^\circ\text{C}$$

For SAE 20, at 67.5°C , from Figure 4.7, $\mu = 0.016 \text{ Pa s}$.

$$\begin{aligned} P &= \frac{W}{LD} = \frac{1000}{0.02 \times 0.02} = 2.5 \text{ MPa} \\ S &= \frac{\downarrow r^2 \mu N_s}{\rightarrow P} \\ &= \frac{\downarrow 0.01^2 \frac{0.016 \times 100}{2.5 \times 10^6}}{\rightarrow -0.00002} = 0.16. \end{aligned}$$

From Table 4.4 for $L/D = 1$, the nearest bounding values of the Sommerfield number to $S = 0.16$, are for $\varepsilon = 0.5$ ($S = 0.1845$) and $\varepsilon = 0.6$ ($S = 0.1242$).

Interpolating to find the values of ε , I_s and I_c which correspond to $S = 0.16$:

$$\begin{aligned} \varepsilon &= \frac{0.16 - 0.1845}{0.1242 - 0.1845} (0.6 - 0.5) + 0.5 \\ &= 0.5406, \end{aligned}$$

$$\begin{aligned} I_s &= \frac{0.16 - 0.1845}{0.1242 - 0.1845} (0.3396 - 0.2462) \\ &\quad + 0.2462 = 0.2841, \end{aligned}$$

$$\begin{aligned} I_c &= \frac{0.16 - 0.1845}{0.1242 - 0.1845} (-0.2590 - (-0.1484)) \\ &\quad + (-0.1484) = -0.1933. \end{aligned}$$

From Eq. 4.17 with $\eta_1 = 1.708$,

$$\begin{aligned} \frac{Q}{rcN_s L} &= 4.84 - 2.901(1 - 1.708) \\ &\quad \times 0.9517 \tanh^{-1}(0.4759)) = 4.379. \end{aligned}$$

From Eq. 4.19 with $\eta_2 = 0.2925$,

$$\begin{aligned} \frac{Q_\pi}{rcN_s L} &= 1.443 + 0.4968 (1 - 0.2925) \\ &\quad \times 1.589 \tanh^{-1}(0.7943)) = 1.690. \end{aligned}$$

From Eq. 4.22,

$$Q_s/Q = 0.6141.$$

From Eq. 4.23,

$$\begin{aligned} f \frac{\downarrow r}{\rightarrow} &= 3.016(1.245 + 0.07679) = 3.987 \\ \Delta T &= \frac{\downarrow 8.3 \times 10^{-6} P}{\rightarrow -0.5(Q_s/Q)} \frac{(r/c)f}{Q/rcN_s L} \\ &= 27.26^\circ\text{C}. \end{aligned}$$

Repeating the procedure with this value for ΔT gives:

$$\begin{aligned} T_{av} &= 73.6^\circ\text{C}, \text{ hence } \mu = 0.013 \text{ Pa s}, \\ S &= 0.13, \text{ hence } \varepsilon = 0.5904, \\ Q/(rcN_s L) &= 4.492, \\ Q_\pi/(rcN_s L) &= 1.531, \\ (r/c)f &= 3.418. \end{aligned}$$

Using these values gives $\Delta T = 23.56^\circ\text{C}$.

Repeating the procedure with this value for ΔT gives:

$$T_{av} = 71.78^\circ\text{C}, \text{ hence } \mu = 0.014 \text{ Pa s},$$

$$S = 0.14, \text{ hence } \varepsilon = 0.5738,$$

$$Q/(rcN_sL) = 4.454,$$

$$Q_\pi/(rcN_sL) = 1.585,$$

$$(r/c)f = 3.613.$$

Using these values gives $\Delta T = 24.83^\circ\text{C}$. Hence $T_{av} = 72.4^\circ\text{C}$. This is close enough to the previous value, indicating that the correct value for T_{av} has been determined.

The values for the total lubricant supply, minimum film thickness, coefficient of friction, torque and the heat generated in the bearing can now be determined:

$$Q = 1782 \text{ mm}^3/\text{s},$$

$$h_0 = c(1 - \varepsilon) = 0.009 \text{ mm},$$

$$f = 0.007226,$$

$$\text{Torque} = 0.072 \text{ N m},$$

$$\text{Power} = 45.4 \text{ W}.$$

in hard copy (just telephone and ask the manufacturer for a brochure) and via the internet. The comparative ratings shown in Table 4.5 can be justified for say radial load carrying capacity; roller bearings are better than ball bearings because of their shape and the area over which the load is spread. Thrust load capacity varies dramatically with design. The grooves in the races of the deep groove ball bearing permit the transfer of moderate thrust in combination with radial load. The angular contact bearing (Figure 4.23f) is better than the single row ball bearing because the races are higher on one side providing a more favourable load path. Cylindrical and needle bearings should not generally be subjected to any thrust load.

4.3.1 Bearing life and selection

The load on a rolling contact bearing is exerted on a very small area as illustrated in Figure 4.24. The resulting contact stresses are very high and of the order of 2000 MPa. Despite very strong steels (e.g. BS 970 534 A99, AISI 52100) all bearings have a finite life and will eventually fail due to fatigue. For two groups of apparently identical bearings tested under loads P_1 and P_2 , the respective lives L_1 and L_2 are related by:

$$\frac{L_1}{L_2} = \left(\frac{P_2}{P_1}\right)^k \quad (4.24)$$

where L_1 , life at load P_1 (number of revolutions); L_2 , life at load P_2 (number of revolutions); P_1 , load (N); P_2 , load (N); and k , 3 for ball bearings; and k , 3.33 for cylindrical roller bearings.

The dimensions for rolling element bearings have been standardized and can be purchased as stock items from specialist manufacturers and suppliers. The selection of a bearing from a manufacturer's catalogue involves consideration of the bearing load carrying capacity and the bearing geometry. For a given bearing the load carrying capacity is given in terms of the basic dynamic load rating and the basic static load rating. The various commonly used definitions for rolling

4.3 Rolling contact bearings

The term 'rolling contact bearings' encompasses the wide variety of bearings that use spherical balls or some type of roller between the stationary and moving elements as illustrated in Figure 4.23. The most common type of bearing supports a rotating shaft resisting a combination of radial and axial (or thrust) loads. Some bearings are designed to carry only radial or only thrust loads.

Selection of the type of bearing to be used for a given application can be aided by the comparison charts, an example of which is given in Table 4.5. Several bearing manufacturers produce excellent catalogues (e.g. NSK/RHP, SKF, FAG, INA) including design guides. The reader is commended to gain access to this information, which is available

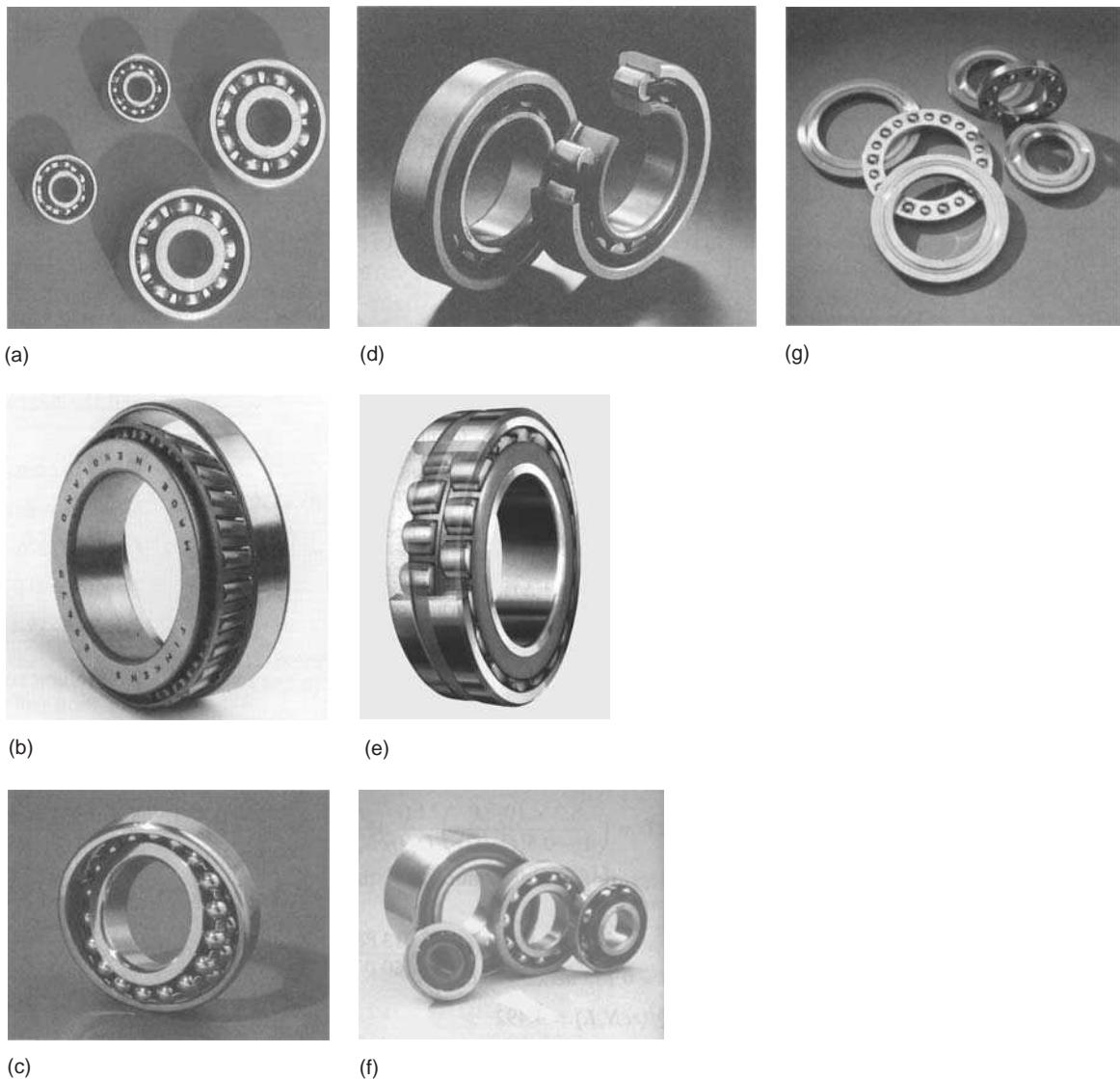


Figure 4.23 A selection of the various types of rolling element bearings. (a) Deep groove ball bearing. (b) Taper roller bearing. (c) Self-aligning ball bearing. (d) Cylindrical roller bearing. (e) Spherical roller bearing. (f) Angular contact ball bearing. (g) Thrust ball bearing. Photographs courtesy of NSK RHP Bearings.

element bearing life specification are outlined and illustrated by the example below.

The basic dynamic load rating, C , is the constant radial load which a bearing can endure for 1×10^6 revolutions without evidence of the development of fatigue in any of the bearing components.

The life of a ball bearing, L , is the number of revolutions (or hours at some constant

speed), which the bearing runs before the development of fatigue in any of the bearing components.

Fatigue occurs over a large number of cycles of loading. For a bearing this would mean a large number of revolutions. Fatigue is a statistical phenomenon with considerable spread of the actual life of a group of bearings of a given design. The rated life is the standard means of

reporting the results of many tests of bearings. It represents the life that 90 per cent of the bearings would achieve successfully at a rated load. The rated life is referred to as the L_{10} life at the rated load. The rated life, L_{10} , of a group of apparently identical bearings is defined as the number of revolutions (or hours at some constant speed) that 90 per cent of the group of bearings will complete before the first evidence of fatigue develops.

If in Eq. 4.24, $P_2 = C$ and the corresponding life $L_2 = 1 \times 10^6$, then the life of a bearing L , with basic dynamic load rating C with a load P is given by:

Table 4.5 Merits of different rolling contact bearings

Bearing type	Radial load capacity	Axial or thrust load capacity	Misalignment capability
Single row	Good	Fair	Fair
Double row deep groove ball	Excellent	Good	Fair
Angular contact	Good	Excellent	Poor
Cylindrical roller	Excellent	Poor	Fair
Needle roller	Excellent	Poor	Poor
Spherical roller	Excellent	Fair/good	Excellent
Tapered roller	Excellent	Excellent	Poor

$$L = \frac{\downarrow C}{\rightarrow P}^k \text{ million revolutions} \quad (4.25)$$

where L , life (millions of revolutions); C , basic dynamic load rating (N); P , load (N).

When selecting a particular bearing from a manufacturer's catalogue, it is useful to know the required basic dynamic load rating C for a given load P and life L , which is given by:

$$C = P \frac{\downarrow L}{\rightarrow 10^6}^{1/k} \quad (4.26)$$

Example 4.5

A straight cylindrical roller bearing operates with a load of 7.5 kN. The required life is 8760 hours at 1000 rpm. What load rating should be used for selection from the catalogue?

Solution

Using Eq. 4.26,

$$\begin{aligned} C &= P_d (L_d / 10^6)^{1/k} \\ &= 7500 \frac{\downarrow 8760 \times 1000 \times 60}{\rightarrow 10^6}^{1/3.33} \\ &= 49.2 \text{ kN} \end{aligned}$$

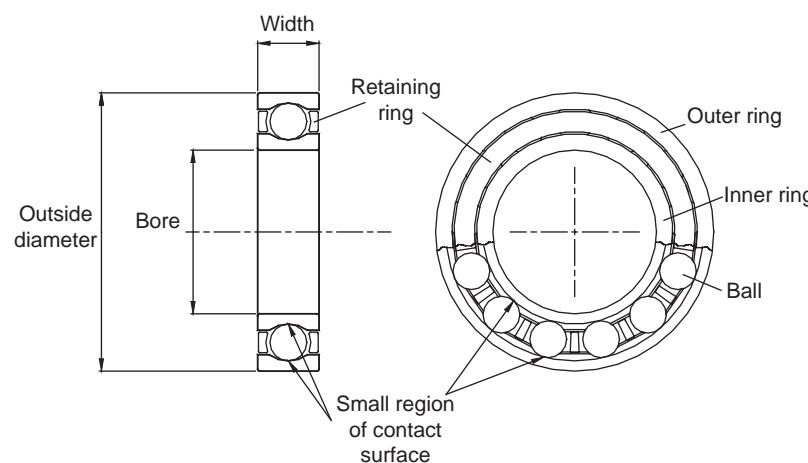


Figure 4.24 Contact area for a ball bearing.

Example 4.6

A catalogue lists the basic dynamic load rating for a ball bearing to be 33 800 N for a rated life of 1 million revolutions. What would be the expected L_{10} life of the bearing if it were subjected to 15 000 N and determine the life in hours that this corresponds to if the speed of rotation is 2000 rpm. Comment on the value obtained and its suitability for a machine.

Solution

$C_{\text{cat}} = 33800 \text{ N}$, $P_d = 15000 \text{ N}$, $L_{\text{cat}} = 10^6$ (L_{10} life at load C), $k = 3$ (ball). Using Eq. 4.25, the life is given by:

$$L = 10^6 \frac{\downarrow 33800}{-45000}^3 \\ = 11.44 \times 10^6 \text{ revolutions} \\ (= L_{10} \text{ life at } 15000 \text{ N}).$$

If the rotational speed is 2000 rpm, $L = 11.44 \times 10^6 / (2000 \times 60) = 95$ hours operation. This is not very long and illustrates the need to use a bearing with a high basic dynamic load rating.

The basic static load rating, C_o , is the load the bearing can withstand without any permanent deformation of any component. If this load is exceeded it is likely the bearing races will be indented by the rolling elements (called Brinelling). Subsequently the operation of the bearing would be noisy and impact loads on the indented area would produce rapid wear and progressive failure of the bearing would ensue. Unintentional damage to bearings can occur for machines during transportation. If a rotor is unsecured it can vibrate and although not rotating the loads are transferred through the small areas of contact between the bearing elements and raceways. If these loads are above the basic static load rating, Brinelling can take place.



Figure 4.25 Thrust bearing from the Rolls-Royce Trent series of engines.

Loads on bearings often vary with time and may not be entirely radial. An example is the thrust bearing illustrated in Figure 4.25 from the Rolls-Royce Trent series of engines, see Figure 4.26. This bearing has to take a substantial thrust load, as well as a radial load. In addition and somewhat unusually for a bearing both inner and outer raceways rotate. This is because Rolls-Royce plc use a three-spool technology for their engines in order to operate the turbine and compressor stages at optimum speeds. The combined effects of radial and thrust loads can be accommodated in the life equation by an equivalent load.

The equivalent load, P , is defined as the constant radial load which if applied to a bearing would give the same life as that which the bearing would attain under the actual conditions of load and rotation. When both radial and thrust loads are exerted on a bearing the equivalent load is the constant radial load that would produce the same rated life for the bearing as the combined loading.

Normally,

$$P = VXR + YT \quad (4.27)$$

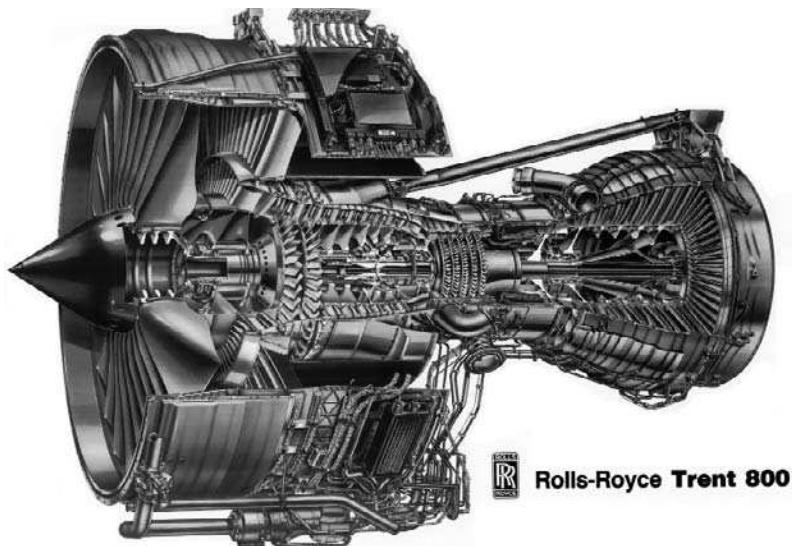


Figure 4.26 The Trent 800 engine illustrating three spools, each rotating at a different speed. Figure courtesy of Rolls-Royce plc.

Table 4.6 Values for the radial and thrust factors for determining the equivalent load for deep groove ball single bearings and bearing pairs arranged in tandem

F_a/C_0	Normal clearance			C3 clearance			C4 clearance		
	e	X	Y	e	X	Y	e	X	Y
0.025	0.22	0.56	2	0.31	0.46	1.75	0.4	0.44	1.42
0.04	0.24	0.56	1.8	0.33	0.46	1.62	0.42	0.44	1.36
0.07	0.27	0.56	1.6	0.36	0.46	1.46	0.44	0.44	1.27
0.13	0.31	0.56	1.4	0.41	0.46	1.3	0.48	0.44	1.16
0.25	0.37	0.56	1.2	0.46	0.46	1.14	0.53	0.44	1.05
0.5	0.44	0.56	1	0.54	0.46	1	0.56	0.44	1

After SKF.

where P , equivalent load (N); V , 1.2 if mounting rotates is recommended, V , 1.0 if shaft rotates; X , radial factor (given in bearing catalogues, see Table 4.6 for example data); R , applied radial load (N); Y , thrust factor (given in bearing catalogues, see Table 4.6 for example data); T , applied thrust load (N).

Tables 4.7 to 4.9 give an overview of the information typically available in bearing manufacturers' catalogues and the example given below illustrates their basic use.

Example 4.7

A bearing is required to carry a radial load of 2.8 kN and provide axial location for a shaft of 30 mm diameter rotating at 1500 rpm. An L_{10} life of 10 000 hours is required. Select and specify an appropriate bearing.

Solution

Axial shaft location is required, so a deep groove ball bearing, which provides axial location capability in both directions, would be suitable.

Table 4.7 Selected example single row deep groove ball bearing ratings

d (mm)	D (mm)	B (mm)	Basic dynamic	Basic static	Speed limit for	Speed limit for oil	Code
			load rating C (N)	load rating C_o (N)	grease lubrication (rpm)	lubrication (rpm)	
15	24	5	1570	800	28 000	34 000	61802
	32	8	5600	2850	22 000	28 000	16002
	32	9	5600	2850	22 000	28 000	6002
	35	11	7850	3750	19 000	24 000	6202
	42	13	11 500	5400	17 000	20 000	6302
17	26	5	1690	930	24 000	30 000	61803
	35	8	6060	3250	19 000	24 000	16003
	35	10	6060	3250	19 000	24 000	6003
	40	12	9550	4750	17 000	20 000	6203
	47	14	13 600	6550	16 000	19 000	6303
	62	17	23 000	10 800	12 000	15 000	6403
20	32	7	2750	1500	19 000	24 000	61804
	42	8	6900	4050	17 000	20 000	16004
	42	12	9400	5000	17 000	20 000	6004
	47	14	12 800	6550	15 000	18 000	6204
	52	15	16 000	7800	13 000	16 000	6304
	72	19	30 800	15 000	10 000	13 000	6404
25	37	7	4400	2600	17 000	20 000	61805
	47	8	7600	4750	14 000	17 000	16005
	47	12	11 300	6550	15 000	18 000	6005
	52	15	14 050	7800	12 000	15 000	6205
	62	17	22 600	11 600	11 000	14 000	6305
	80	21	36 000	19 300	9000	11 000	6405
30	42	7	4500	2900	15 000	18 000	61806
	55	9	11 300	7350	12 000	15 000	16006
	55	13	13 400	8300	12 000	15 000	6006
	62	16	19 600	11 200	10 000	13 000	6206
	72	19	28 200	16 000	9000	11 000	6306
	90	23	43 700	23 600	8500	10 000	6406

The total number of revolutions in life is

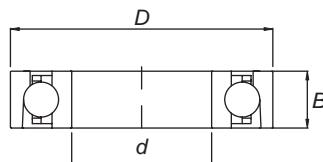
$$10\ 000 \times 1500 \times 60 = 900 \text{ million}$$

so $L = 900$. The load is purely radial, so $P = 2800 \text{ N}$. The required dynamic loading is given by

$$C = PL^{1/3} = 2800 \times 900^{1/3} = 27\ 033 \text{ N.}$$

Reference to the deep groove bearing chart (Table 4.7) shows a suitable bearing could be:

- ISO designation 6306
- bore diameter 30 mm, outer diameter 72 mm
- width 19 mm
- $C = 28\ 200 \text{ N}$, $C_o = 16\ 000 \text{ N}$
- speed limit (using grease) 9000 rpm
- speed limit (using oil) 11 000 rpm.

Table 4.8 Selected example angular contact ball bearing ratings

<i>d</i> (mm)	<i>D</i> (mm)	<i>B</i> (mm)	Basic dynamic load rating <i>C</i> (N)	Basic static load rating <i>C_o</i> (N)	Speed limit for grease lubrication (rpm)	Speed limit for oil lubrication (rpm)	Code
15	35	11	8850	4800	17 000	24 000	7202B
	42	13	13 050	6700	15 000	20 000	7302B
17	40	12	11 200	6100	15 000	20 000	7203B
	47	14	16 000	8300	13 000	18 000	7303B
20	47	14	14 050	8300	12 000	17 000	7204B
	52	15	19 050	10 400	11 000	16 000	7304B
25	52	15	15 500	10 200	10 000	15 000	7205B
	62	17	26 100	15 600	9000	13 000	7305B
30	62	16	23 900	15 600	8500	12 000	7206B
	72	19	34 600	21 200	8000	11 000	7306B
40	80	18	36 500	26 000	7000	9500	7208B
	90	23	49 500	33 500	6700	9000	7308B
50	90	20	39 050	30 500	6000	8000	7210B
	110	27	74 000	51 000	5300	7000	7310B
55	100	21	48 850	38 000	5600	7500	7211B
	120	29	85 300	60 000	4800	6300	7311B
60	110	22	57 250	45 500	5000	6700	7212B
	130	31	95 700	69 500	4500	6000	7312B
65	120	23	66 400	54 000	4500	6000	7213B
	140	33	109 000	80 000	4300	5600	7313B
70	125	24	71 600	60 000	4300	5600	7214B

4.3.2 Bearing installation

The practical use of rolling element bearings requires consideration of their installation as well as correct selection. Bearing installation considerations include the bearing combination, the mounting of the bearings and the provision of lubrication.

A typical application of rolling element bearings is the support of a rotating shaft. If the operating temperature of the machine varies, the shaft length can grow relative to the casing or mounting arrangement. An idea of the magnitude of the axial shaft growth can be estimated by:

$$\Delta L = L\alpha\Delta T \quad (4.28)$$

where ΔL is the change in length (m); L , the original length (m); α , the coefficient of linear thermal expansion ($^{\circ}\text{C}^{-1}$); and ΔT is the temperature rise ($^{\circ}\text{C}$).

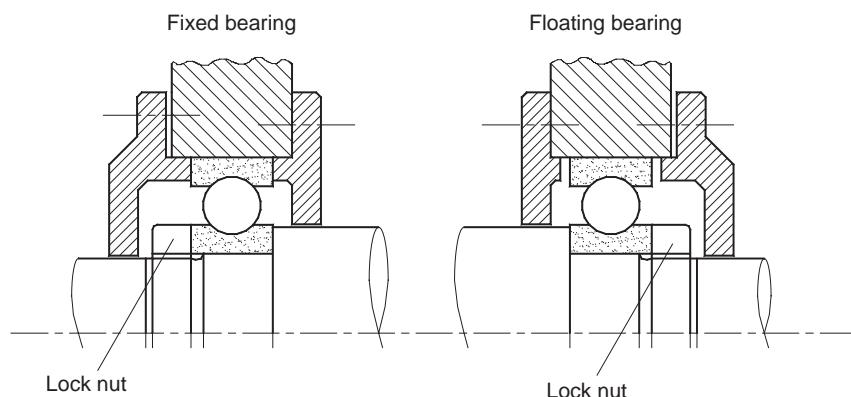
For a gas turbine engine the difference in temperature between the casing and the shaft can be 50°C . If the original length of the steel shaft was 1.0 m the growth of the shaft would be

$$\begin{aligned} \Delta L &= L\alpha\Delta T = 1.0 \times 11 \times 10^{-6} \times 50 \\ &= 5.5 \times 10^{-4} \text{ m} = 0.55 \text{ mm}. \end{aligned}$$

This is a considerable axial movement within a machine and must be allowed for if significant loadings and resultant stresses and possible contact

Table 4.9 Selected example cylindrical roller bearing ratings

d (mm)	D (mm)	B (mm)	Basic dynamic load rating C (N)	Basic static load rating C_o (N)	Speed limit for grease lubrication (rpm)	Speed limit for oil lubrication (rpm)	Code
15	35	11	12 600	10 200	18 000	22 000	NU202E
	42	13	19 500	15 300	16 000	19 000	NU302E
25	52	15	28 700	27 000	11 000	14 000	NU205E
	62	17	40 300	36 500	9 500	12 000	NU305E
30	62	16	38 100	36 500	9 500	12 000	NU206E
	72	19	51 300	48 000	9 000	11 000	NU306E
50	90	20	64 500	69 500	6 300	7 500	NU210E
	110	27	111 000	112 000	5 000	6 000	NU310E
	130	31	131 000	127 000	5 000	6 000	NU410
100	180	34	252 000	305 000	3 200	3 800	NU220E
	250	58	430 000	475 000	2 400	3 000	NU420
200	360	58	766 000	1 060 000	1 500	1 800	NU240E
	420	80	990 000	1 320 000	1 300	1 600	NU340
600	870	118	2 750 000	510 000	600	700	NU10/600

**Figure 4.27** Basic bearing mounting using two deep groove ball bearing for a rotating horizontal shaft for moderate radial and axial loading.

between stationary and rotating components are to be avoided. A typical solution to this kind of situation is to allow for a limited axial movement on one bearing as illustrated in Figure 4.27. Here one bearing has the location of its inner and outer races 'fixed' relative to the shaft and housing by means of shoulders and locking rings. The location of the other bearing is fixed only for the inner race. If the shaft expands, the axial movement can be accommodated by limited

sliding motion of the outer race of the right hand bearing within the housing bore. For this kind of arrangement the bearings are referred to as fixed and floating bearings. Similar movement is also possible for the arrangement shown in Figure 4.28. Here the right hand bearing is a cylindrical roller bearing and the axial location of the roller is not fixed and can move or float axially to the limited extents of the race to take up any axial movement or expansion of the shaft.

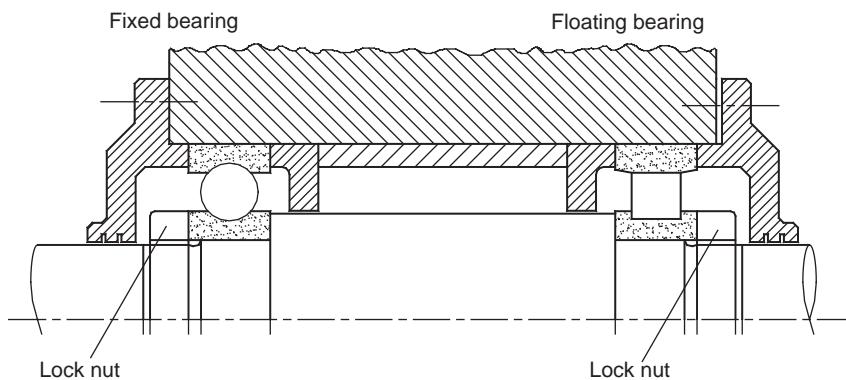


Figure 4.28 Basic bearing mounting using a deep groove and a cylindrical roller bearing for moderate radial loads at the 'locating deep groove bearing' and high radial load capacity at the cylindrical roller bearing.

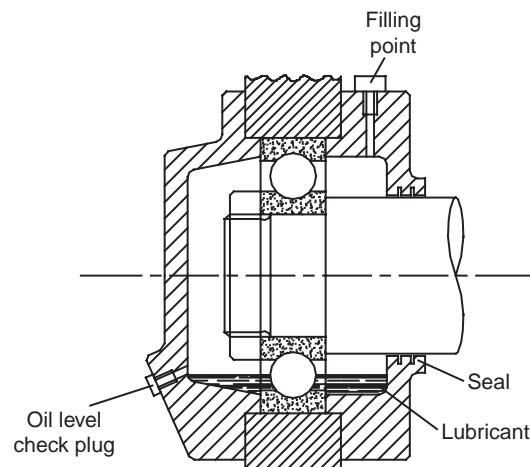


Figure 4.29 Partial submersion lubrication arrangement.

Correct lubrication is essential in order to ensure the calculated life of the bearing is achieved. Too much lubrication can result in increased levels of viscous dissipation of energy in the bearings and resultant overheating of the lubricant and the bearing. Too little lubrication can result in excessive wear and early failure. The form of lubrication depends on that required by the bearing application. Grease lubrication is normally the easiest and requires injection of a specific quantity of grease into the bearing and some method of dirt and dust exclusion. Bearings can be purchased which are grease filled and sealed for life. Oil lubrication can be supplied by means of partial submersion as shown in Figure 4.29, a recirculation circuit as shown in Figure 4.30 or oil spot lubrication. A recirculation system typically consists of a cooler, an oil

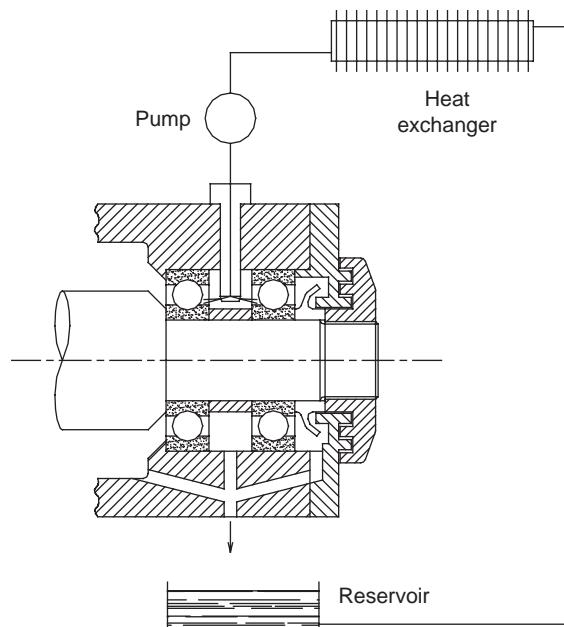


Figure 4.30 Recirculation lubrication system.

pump, a filter, oil jets, scavenge or collector holes and a reservoir. Oil spot lubrication involves the application of very small quantities of lubricant directly to the bearing by means of compressed air. Once again the bearing manufacturers' handbooks are a good source of information for lubricant supply rates and systems.

4.4 Conclusions

Bearings are used to support a load whilst allowing relative motion between two elements of a

machine. Typical applications are shafts where the shaft rotates relative to a housing, but bearings can also be used to facilitate linear relative motion for, say, machine slides. This chapter has introduced the design of journal bearings and the selection and specification of rolling element bearings. Rolling element bearings are produced by specialist manufacturers and many types and sizes are available as stock items. For further information on bearings the reader is recommended to view specific manufacturers' literature and the texts by Harris (2001) and Brandlein *et al.* (1999).

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- www.revolve.com
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Nomenclature

Generally, preferred SI units have been stated.

<i>c</i>	radial clearance (m)
<i>C</i>	basic dynamic load rating (N)
<i>D</i>	journal diameter (m)
<i>E</i>	eccentricity
<i>f</i>	coefficient of friction
<i>h</i>	film thickness (m)
<i>h</i> ₀	minimum film thickness (m)
<i>I</i> _c	cosine load integral
<i>I</i> _s	sine load integral
<i>ISO</i>	International Organisation for Standardisation
<i>k</i>	constant
<i>L</i>	bearing length (m), life (number of revolutions)
<i>L</i> _d	design life (number of revolutions)
<i>L</i> ₁	life at load P ₁ (number of revolutions)
<i>L</i> ₂	life at load P ₂ (number of revolutions)
<i>N</i>	speed (normally in rpm)
<i>N</i> _s	journal speed (revolutions per second)
<i>p</i>	fluid pressure
<i>P</i>	load capacity (N/m ²), load (N)
<i>P</i> _d	design load (N)
<i>P</i> _{max}	maximum film pressure (Pa)
<i>P</i> ₁	load (N)
<i>P</i> ₂	load (N)
<i>Q</i>	lubricant flow rate (m ³ /s)
<i>Q</i> _s	side flow rate (m ³ /s)
<i>Q</i> _π	flow through the minimum film thickness (m ³ /s)
<i>r</i>	radius (m)
<i>S</i>	bearing characteristic number

SAE	Society of Automotive Engineers
T	temperature ($^{\circ}\text{C}$), thrust load (N)
T_{av}	average temperature ($^{\circ}\text{C}$)
T_1	temperature of the lubricant supply ($^{\circ}\text{C}$)
T_2	temperature of the lubricant leaving the bearing ($^{\circ}\text{C}$)
V	velocity (m/s)
W	applied load (N)
x	coordinate
X	radial factor
y	coordinate
Y	thrust factor
z	coordinate
α	coefficient of expansion (K^{-1})
ϕ	position of the minimum film thickness ($^{\circ}$), attitude angle ($^{\circ}$)
ε	eccentricity ratio
μ	viscosity of lubricant (Pa s)
θ	circumferential coordinate
θ_{po}	film termination angle ($^{\circ}$)
θ_{pmax}	location of the minimum film thickness ($^{\circ}$)
ω	angular velocity (rad/s)
ΔL	change in length (m)
ΔT	temperature rise ($^{\circ}\text{C}$)

Worksheet

- 4.1 Identify appropriate bearings for the following applications:
- A single spool (shaft) gas turbine operating at 12 000 rpm with a shaft diameter of 40 mm.
 - A turbocharger spinning at up to 150 000 rpm with a shaft diameter of 10 mm.
 - A photocopier roller operating at 150 rpm with a spindle diameter of 10 mm.
 - A ship's propeller shaft operating at 1500 rpm.
- 4.2 Which of the bearings in Figure 4.31:
- can support axial loads?
 - can support combined axial and radial loads?
 - has the highest load capacity?
 - has the highest speed capacity?
 - is the most common?
- 4.3 A plain surface bearing has been partially specified to support a load of 2000 N at a rotational speed of 20 rpm. The nominal diameter of the journal is 50 mm and the length is 50 mm. Select an appropriate material for the bearing surface.

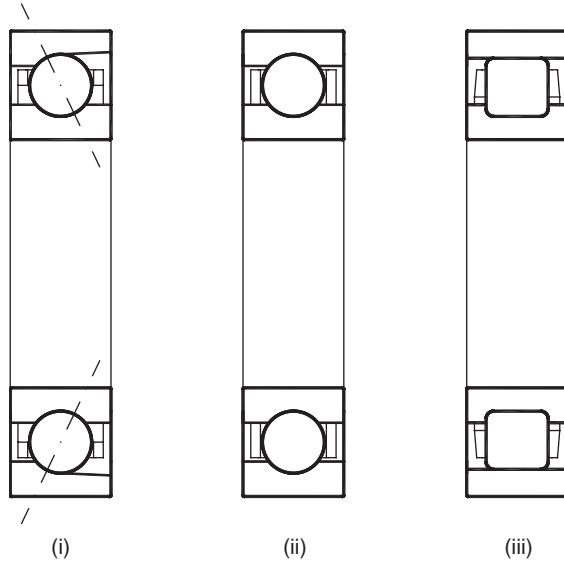


Figure 4.31 Rolling element bearings.

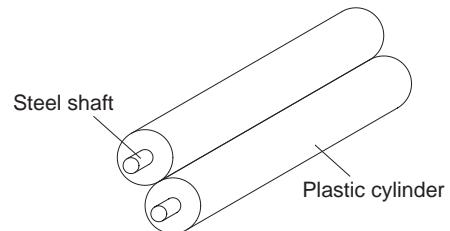


Figure 4.32 Copier rollers. Reproduced from IMechE (1994).

- 4.4 The paper feed for a photocopier is controlled by two rollers which are sprung together with a force of approximately 20 N. The rollers each consist of a 20 mm outer diameter plastic cylinder pressed onto a 10 mm diameter steel shaft (see Figure 4.32). The maximum feed rate for the copier is 30 pages per minute. Select bearings to support the rollers (after IMechE, 1994).
- 4.5 A full journal bearing has a nominal diameter of 50.0 mm, a bearing length of 25.0 mm and supports a load of 2500 N when running at the design speed of 3000 rpm. The radial clearance has been specified as 0.04 mm. An SAE 10 oil has been chosen and the lubricant supply system is capable of delivering the lubricant to the bearing at 45°C. Using the Raimondi and Boyd design charts, find the temperature rise of the lubricant,

- the total lubricant flow rate required, the torque required to overcome friction and the heat generated in the bearing.
- 4.6 A full journal bearing has a nominal diameter of 25.0 mm and bearing length 25.0 mm. The bearing supports a load of 1450 N and the journal design speed is 2500 rpm. Two radial clearances are proposed (i) 0.025 mm, (ii) 0.015 mm. An SAE 10 oil has been chosen and the lubricant supply temperature is 50°C. Using the standard Raimondi and Boyd design charts, find the temperature rise of the lubricant, the lubricant flow rate, the torque required to overcome friction and the heat generated in the bearing for both of the proposed clearances. Determine which clearance, if any, should be selected for the bearing.
- 4.7 A full journal bearing is required for a shaft with a nominal diameter of 40 mm rotating at 1000 rpm. The load on the bearing is 2200 N. The lubricant supply available uses SAE 20 oil at an inlet temperature of 60°C. Select appropriate values for the radial clearance and bearing length and using the Raimondi and Boyd charts determine the overall temperature rise in the bearing. Establish that the selected design lies within the optimum zone for minimum friction and maximum load on the minimum film thickness ratio chart (Figure 4.11).
- 4.8 The proposed design for a full journal bearing, supporting a load of 1600 N and running at 5000 rpm, has a nominal diameter of 40 mm and bearing length of 20 mm. As part of the initial design, two radial clearances of 50 and 70 μm are proposed running with SAE 10 oil with a lubricant supply temperature of 60°C. Using the standard Raimondi and Boyd design charts determine the overall lubricant temperature rise, the total lubricant flow, the friction coefficient and the minimum film thickness for both of the proposed radial clearances and give justification for the selection of one of the clearances for use in the bearing. As a first guess for the lubricant temperature rise, ΔT can be taken as 10°C.
- 4.9 A full film hydrodynamic journal bearing is required for a shaft supporting a radial load of 7500 N, whilst rotating at 800 rpm. The nominal diameter of the shaft is 80 mm. Select a suitable lubricant and appropriate values for the radial clearance and bearing length. Determine the overall temperature rise in the bearing and the flow rate of lubricant required. Establish that the selected design lies within the optimum zone for minimum friction and maximum load on the minimum film thickness ratio chart. The lubricant can be supplied at an inlet temperature of 50°C using an available cooling system.
- 4.10 A full film hydrodynamic journal bearing is required for a shaft supporting a radial load of 2200 N, whilst rotating at 1000 rpm. The nominal diameter of the shaft is 40 mm. The lubricant can be supplied at an inlet temperature of 60°C, using an available cooling system. Select a suitable lubricant and appropriate values for the radial clearance and bearing length. Determine the overall temperature rise in the bearing and the flow rate of lubricant required. Establish that the selected design lies within the optimum zone for minimum friction and maximum load on the minimum film thickness ratio chart.
- 4.11 A full journal bearing has a nominal diameter of 30.0 mm, bearing length 15.0 mm. The bearing supports a load of 1000 N, and the journal design speed is 6000 rpm. The radial clearance has been specified as 0.02 mm. An SAE 10 oil has been chosen and the lubricant supply temperature is 50°C. Using the approximate method of Reason and Narang find the temperature rise of the lubricant, the lubricant flow rate, the minimum film thickness, the torque required to overcome friction and the heat generated in the bearing.
- 4.12 A full journal bearing has a nominal diameter of 80.0 mm, bearing length 60.0 mm. The bearing supports a load of 7500 N, and the journal design speed is 800 rpm. The radial clearance has been specified as 0.05 mm. An SAE 30 oil has been chosen and the lubricant supply temperature is 50°C. Using the approximate method of Reason and Narang for determining the design and performance of full film hydrodynamic bearings, find the temperature rise of the lubricant, the lubricant flow rate, the torque required to overcome friction, the heat generated in the bearing and the minimum film thickness.
- 4.13 A full film hydrodynamic journal bearing is required for a machine tool shaft which supports a radial load of 5800 N and rotates at 1470 rpm. The nominal diameter of the shaft is 50 mm. A lubrication system is available that can supply lubricant

with an inlet temperature of 60°C. Select a suitable lubricant and appropriate values for the radial clearance and bearing length. Determine the overall temperature rise in the bearing and the flow rate of lubricant required. Establish that the selected design lies within the optimum zone for minimum friction and maximum load on the minimum film thickness ratio chart.

- 4.14 A full journal bearing is required for a shaft with a nominal diameter of 40 mm rotating at 1450 rpm. The load on the bearing is 2100 N. The lubricant can be supplied at an inlet temperature of 60°C using an available cooling system. Select a suitable lubricant and appropriate values for the radial clearance and bearing length. Determine the overall temperature rise in the bearing and the flow rate of lubricant required. Establish that the selected design lies within the optimum zone for minimum friction and maximum load on the minimum film thickness ratio chart.
- 4.15 A design proposal for a full film hydrodynamic bearing comprises a bearing of nominal diameter 30 mm, length 15 mm using ISOVG 22 lubricant supplied at 50°C. The bearing load is 1280 N and the rotational speed is 6000 rpm. The radial clearance for the bearing is 0.022 mm. Use the viscosity chart given in Figure 4.7 and the Raimondi and Boyd design charts given in Figures 4.11 to 4.14 to determine the lubricant flow rate, lubricant temperature rise and minimum film thickness. The initial guess for the temperature rise across the bearing can be taken as 30°C. If in place of ISOVG 22 oil, SAE 10 oil was used in the bearing determine the resulting temperature rise of the lubricant. Comment on the suitability of the proposed design and justify the selection of either ISO VG 22 or SAE 10 as a lubricant.
- 4.16 The drive shaft for a combine harvester supports two pulleys: one transmitting power from the engine and the other driving the cutter. The loads on the pulleys both act in the vertical plane as shown in Figure 4.33. If the diameter of the shaft is 50 mm and the maximum rotational speed is 300 rpm, design suitable hydrodynamic bearings for the shaft (after IMechE, 1994).
- 4.17 A bearing is required to support a radial load of 3200 N for a shaft of 50 mm nominal diameter spinning at 700 rpm. The desired life is 10 000 hours. Select and specify an appropriate bearing.
- 4.18 A straight cylindrical roller bearing operates with a load of 14.2 kN. The required life is 3800 hours at 925 rpm. What load rating should be used for selection from a bearing manufacturer's catalogue?
- 4.19 A bearing is required for the floating end of a heavy-duty lathe to carry a radial load of up to 9 kN. The shaft diameter is 50 mm and rotates at 3000 rpm. A life of 7500 hours for the bearings is desired. Select and specify an appropriate bearing.
- 4.20 A bearing is required to support a radial load of 2800 N for a shaft of 30 mm nominal diameter spinning at 750 rpm. The desired life is 10 000 hours. From the limited range available in Tables 4.7 to 4.9, select and specify an appropriate bearing, justifying the choice.
- 4.21 A bearing is required to provide axial location and support a radial load of 940 N for a shaft of 17 mm nominal diameter spinning at 570 rpm. The desired life is 10 years continuous operation. From the limited range available in Tables 4.7 to 4.9, select and specify an appropriate bearing, justifying the choice.
- 4.22 A bearing is required to support an equivalent radial load of 1290 N. The nominal diameter of the shaft is 25 mm and its design speed is 730 rpm. The desired life is 2 years continuous operation. If the bearing should support the load and provide axial location, then from the limited range available in Tables 4.7 to 4.9, select and specify an appropriate bearing, justifying the choice.
- 4.23 A bearing is required to support an equivalent radial load of 1130 N. The nominal diameter of the shaft is 20 mm and its design speed is 7000 rpm. The desired life is 1 year continuous operation. If the bearing should support the load and provide axial location, then from the limited range available in Tables 4.7 to 4.9, select and specify, detailing the bore, width and outer diameter an appropriate bearing, justifying the choice.
- 4.24 The bearing for a power transmission arrangement is required to carry an equivalent radial load of 2.4 kN at 3000 rpm. The nominal shaft diameter at the bearing is 30 mm. A life of 8760 hours is required. Select and specify an appropriate bearing from the limited range available in Tables 4.7 to 4.9.
- 4.25 The angular contact bearings of a dedicated lathe spindle are subjected to a cyclic load during a manufacturing process in an automated plant.

During the cycle a bearing experiences a radial load of 95 N for 30 s at 2500 rpm, 75 N for 45 s at 3500 rpm, and 115 N at 3100 rpm for 15 s. Calculate the mean cubic load which should be used for 4000 hours life.

- 4.26 The design for a 4 km long conveyor system consist of idler stations located 1 m apart. Each idler station comprises three rollers which rotate about stationary axles fixed to supports as illustrated in Figure 4.34. The outer diameter of the rollers is 100 mm and each roller has a mass of 6 kg. The two end rollers are angled at 30° to the horizontal. A belt weighing 40 kg/m runs over the rollers at 3 m/s. The expected maximum loading is 3000 tonnes of aggregate per hour and experience suggests that the central idler supports approximately 60 per cent of the load.

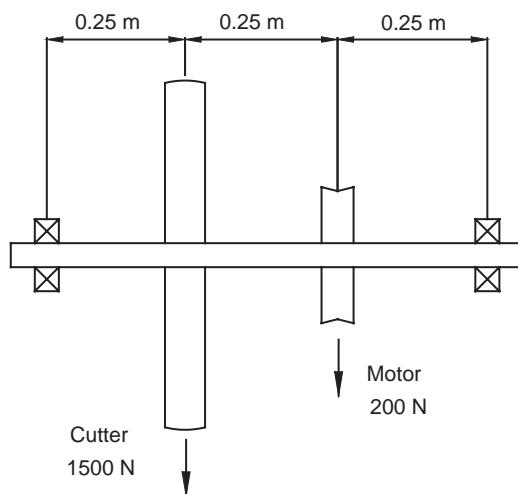


Figure 4.33 Agricultural machinery drive shaft.
Reproduced from IMechE (1994).

Select appropriate bearings for the rollers (after IMechE, 1994).

Answers

- 4.3 $PV = 0.04188 \text{ MN m}^{-1}\text{s}^{-1}$. Carbon graphite.
- 4.4 $PV = 0.0297 \text{ MN m}^{-1}\text{s}^{-1}$. Thermoplastic.
- 4.5 24.5°C , $6525 \text{ mm}^3/\text{s}$, 0.45 N m , 141.1 W .
- 4.6 (i) 15.4°C , $1471 \text{ mm}^3/\text{s}$, 0.08 N m , 21 W .
(ii) 25.9°C , $822 \text{ mm}^3/\text{s}$, 0.085 N m , 22 W .
- 4.7 No unique solution.
- 4.8 (i) 17.1°C , $9200 \text{ mm}^3/\text{s}$, 0.00675 , $0.008 \mu\text{m}$.
(ii) 9.9°C , $13\,160 \text{ mm}^3/\text{s}$, 0.0063 , $0.00798 \mu\text{m}$.
- 4.9 No unique solution.
- 4.10 No unique solution.
- 4.11 39.3°C , $2264 \text{ mm}^3/\text{s}$, $6.9 \mu\text{m}$, 0.13 N m , 83.5 W .
- 4.12 19.2°C , $7544 \text{ mm}^3/\text{s}$, 1.7 N m , 143 W , $20 \mu\text{m}$.
- 4.13 No unique solution.
- 4.14 No unique solution.
- 4.15 $2624 \text{ mm}^3/\text{s}$, 31°C , $5.3 \mu\text{m}$, 38.6°C .
- 4.16 No unique solution.
- 4.17 NU210E, $d = 50 \text{ mm}$, $D = 90 \text{ mm}$, $B = 20 \text{ mm}$. Grease limit 6300 rpm.
- 4.18 70.7 kN.
- 4.19 $C = 78.2 \text{ kN}$. No unique solution.
- 4.20 NU206E, $d = 30 \text{ mm}$, $D = 62 \text{ mm}$, $B = 16 \text{ mm}$, $C = 38\,100 \text{ N}$. Grease limit 9500 rpm.
- 4.21 6403, $d = 17 \text{ mm}$, $D = 62 \text{ mm}$, $B = 17 \text{ mm}$, $C = 23\,000 \text{ N}$. Grease limit 12 000 rpm.
- 4.22 6205, $d = 25 \text{ mm}$, $D = 52 \text{ mm}$, $B = 15 \text{ mm}$, $C = 14\,050 \text{ N}$. Grease limit 12 000 rpm.
- 4.23 6404, $d = 20 \text{ mm}$, $D = 72 \text{ mm}$, $B = 19 \text{ mm}$.
- 4.24 6306, $d = 30 \text{ mm}$, $D = 72 \text{ mm}$, $B = 19 \text{ mm}$. Grease limit 9000 rpm.
- 4.25 89.7 N.
- 4.26 Deep groove ball bearing. No unique answer.

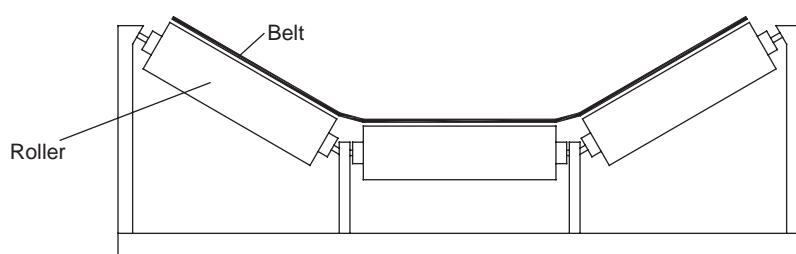


Figure 4.34 Conveyer belt idler station. Reproduced from IMechE (1994).

Learning objectives achievement

Can you distinguish what sort of bearing to use for a given application?	<input type="checkbox"/>	✓	✗	Section 4.1
Can you specify when to use a boundary lubricated bearing and select an appropriate material to use for given conditions?	<input type="checkbox"/>	✓	✗	Section 4.2.2
Can you determine the life of a rolling element bearing using the life equation?	<input type="checkbox"/>	✓	✗	Section 4.3.1
Can you select an appropriate rolling element bearing from a manufacturer's catalogue?	<input type="checkbox"/>	✓	✗	Section 4.3.1
Are you able to specify the layout for a rolling bearing sealing and lubrication system?	<input type="checkbox"/>	✓	✗	Section 4.3.2

5

SHAFTS

The objective of this chapter is to introduce the concepts of shaft design. An overall shaft design procedure is presented, including consideration of bearing and component mounting and shaft dynamics. At the end of the chapter the reader should be able to scheme out a general shaft arrangement, determine deflections and critical speeds, and specify shaft dimensions for strength and fluctuating load integrity.

LEARNING OBJECTIVES

At the end of this chapter you should be able to:

- select appropriate methods for mounting and locating components on shafts;
- produce a scheme for a shaft design to locate and mount standard machine elements;
- determine the deflection of a shaft;
- calculate the first two critical frequencies of a shaft;
- determine the minimum diameter of a shaft to avoid fatigue failure.

5.1 Introduction

The term ‘shaft’ usually refers to a component of circular cross-section that rotates and transmits power from a driving device, such as a motor or engine, through a machine. Shafts can carry gears, pulleys and sprockets to transmit rotary motion and power via mating gears, belts and chains. Alternatively, a shaft may simply connect to another via a coupling. A shaft can be stationary and support a rotating member, such as the short shafts that support the non-driven wheels of automobiles

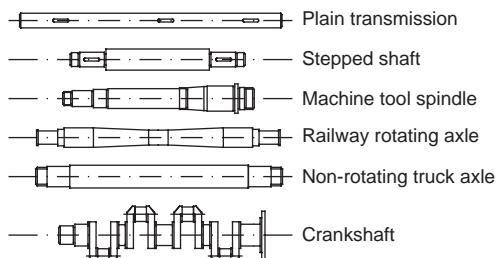


Figure 5.1 Typical shaft arrangements (adapted from Reshetov, 1978).

often referred to as spindles. Some common shaft arrangements are illustrated in Figure 5.1.

Shaft design considerations include:

1. size and spacing of components (as on a general assembly drawing), tolerances
2. material selection, material treatments
3. deflection and rigidity
 - bending deflection
 - torsional deflection
 - slope at bearings
 - shear deflection
4. stress and strength
 - static strength
 - fatigue
 - reliability
5. frequency response
6. manufacturing constraints.

Shafts typically consist of a series of stepped diameters accommodating bearing mounts and providing shoulders for locating devices, such as gears, sprockets and pulleys to butt up against and keys are often used to prevent rotation, relative to the shaft, of these ‘added’ components. A typical

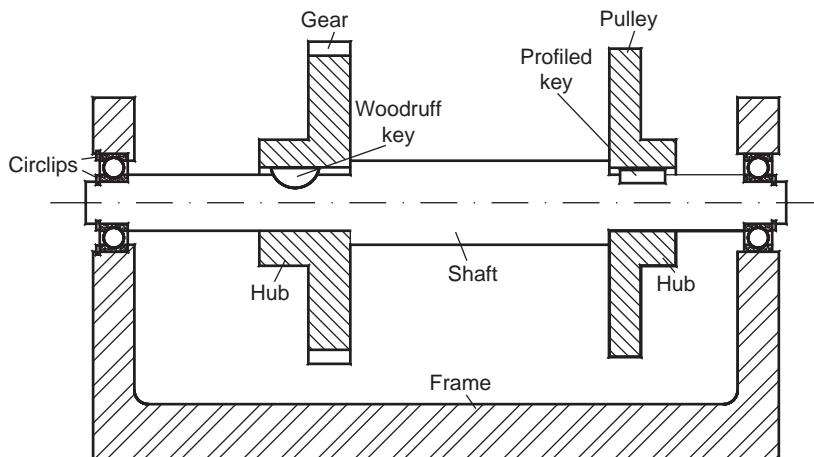


Figure 5.2 Typical shaft arrangement incorporating constant diameter sections and shoulders for locating added components.

arrangement illustrating the use of constant diameter sections and shoulders is illustrated in Figure 5.2 for a transmission shaft supporting a gear and pulley wheel.

Shafts must be designed so that deflections are within acceptable levels. Too much deflection can for example degrade gear performance, and cause noise and vibration. The maximum allowable deflection of a shaft is usually determined by limitations set on the critical speed, minimum deflections required for gear operation and bearing requirements. In general, deflections should not cause mating gear teeth to separate more than about 0.13 mm and the slope of the gear axes should not exceed about 0.03°. The deflection of the journal section of a shaft across a plain bearing should be small in comparison with the oil film thickness. Torsional and lateral deflection both contribute to lower critical speed. In addition, shaft angular deflection at rolling element bearings should not exceed 0.04°, with the exception being self-aligning rolling element bearings.

Shafts can be subjected to a variety of combinations of axial, bending and torsional loads (see Figure 5.3) which may fluctuate or vary with time. Typically a rotating shaft transmitting power is subjected to a constant torque together with a completely reversed bending load, producing a mean torsional stress and an alternating bending stress, respectively.

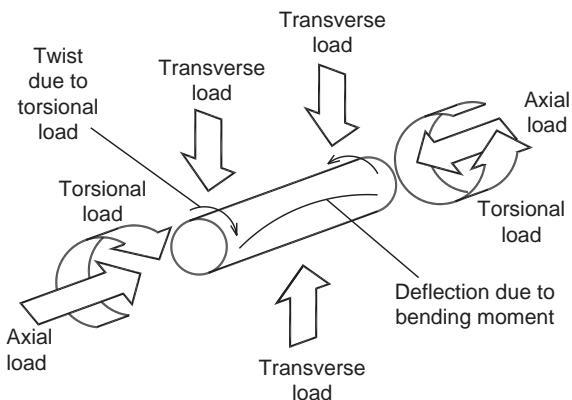


Figure 5.3 Typical shaft loading and deflection (adapted from Beswarick, 1994a).

Shafts should be designed to avoid operation at, or near, critical speeds. This is usually achieved by the provision of sufficient lateral rigidity so that the lowest critical speed is significantly above the range of operation. If torsional fluctuations are present (e.g. engine crankshafts, cam-shafts, compressors) the torsional natural frequencies of the shaft must be significantly different to the torsional input frequency. This can be achieved by providing sufficient torsional stiffness so that the shaft's lowest natural frequency is much higher than the highest torsional input frequency.

Rotating shafts must generally be supported by bearings. For simplicity of manufacture it is

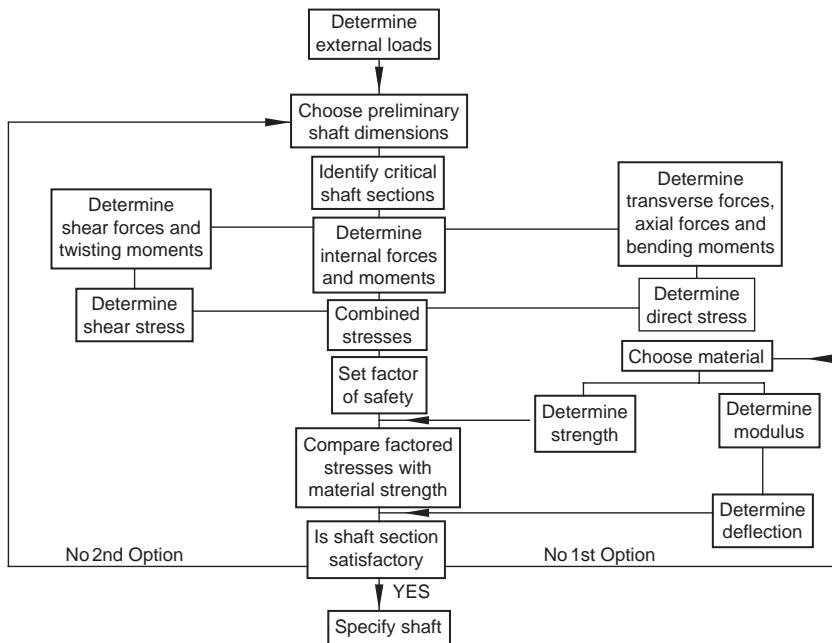


Figure 5.4 Design procedure flow chart for shaft strength and rigidity (adapted from Beswarick, 1994a).

desirable to use just two sets of bearings. If more bearings are required, precise alignment of the bearings is necessary. Provision for thrust load capability and axial location of the shaft is normally supplied by just one thrust bearing, taking thrust in each direction. It is important that the structural members supporting the shaft bearings are sufficiently strong and rigid.

The list below outlines a shaft design procedure for a shaft experiencing constant loading. The flow charts given in Figures 5.4 and 5.5 can be used to guide and facilitate design for shaft strength and rigidity and fluctuating load capability.

1. Determine the shaft rotational speed.
2. Determine the power or torque to be transmitted by the shaft.
3. Determine the dimensions of the power transmitting devices and other components mounted on the shaft and specify locations for each device.
4. Specify the locations of the bearings to support the shaft.
5. Propose a general form or scheme for the shaft geometry considering how each component

will be located axially and how power transmission will take place.

6. Determine the magnitude of the torques throughout the shaft.
7. Determine the forces exerted on the shaft.
8. Produce shearing force and bending moment diagrams, so that the distribution of bending moments in the shaft can be determined.
9. Select a material for the shaft and specify any heat treatments, etc.
10. Determine an appropriate design stress taking into account the type of loading (whether smooth, shock, repeated, reversed).
11. Analyse all the critical points on the shaft and determine the minimum acceptable diameter at each point to ensure safe design.
12. Determine the deflections of the shaft at critical locations and estimate the critical frequencies.
13. Specify the final dimensions of the shaft. This is best achieved using a detailed manufacturing drawing to a recognized standard (see the *Manual of British Standards in Engineering and Drawing Design* (BSI, 1984)) and the drawing should include all the information required

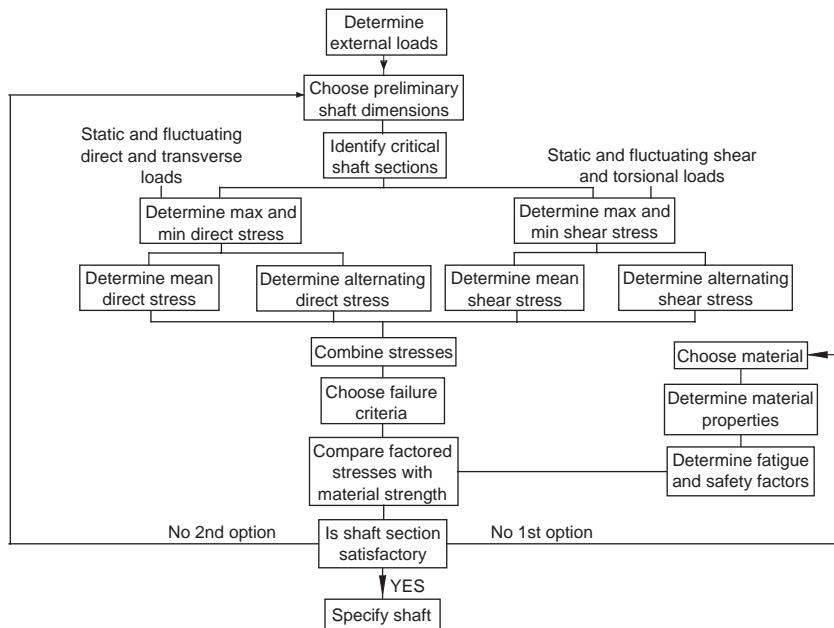


Figure 5.5 Design procedure flow chart for a shaft with fluctuating loading (adapted from Beswarick, 1994b).

to ensure the desired quality. Typically, this will include material specifications, dimensions and tolerances (bilateral, runout, datums, etc. (see Chapter 15)), surface finishes, material treatments and inspection procedures.

The following general principles should be observed in shaft design.

- Keep shafts as short as possible with the bearings close to applied loads. This will reduce shaft deflection and bending moments and increase critical speeds.
- If possible locate stress raisers away from highly stressed regions of the shaft. Use generous fillet radii and smooth surface finishes and consider using local surface strengthening processes, such as shot peening and cold rolling.
- If weight is critical use hollow shafts.

An overview of shaft hub connection methods is given in Section 5.2, shaft to shaft connection methods in Section 5.3 and the determination of critical speeds in Section 5.4. In Section 5.5, the ASME equation for the design of transmission shafts is introduced.

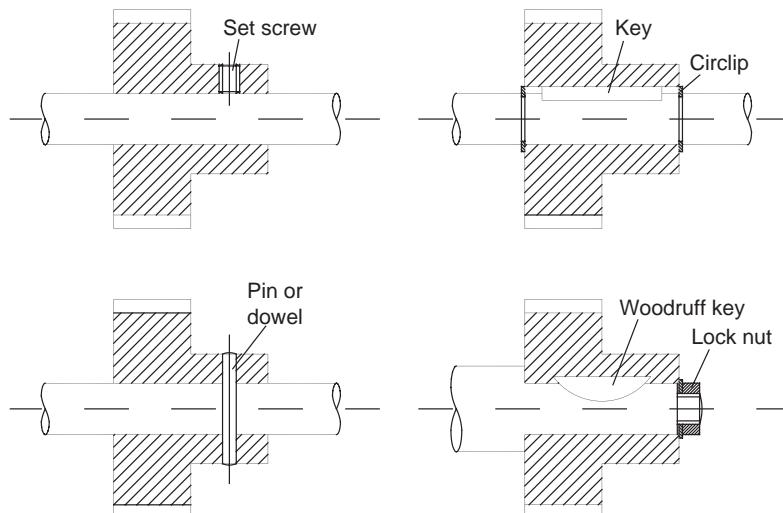
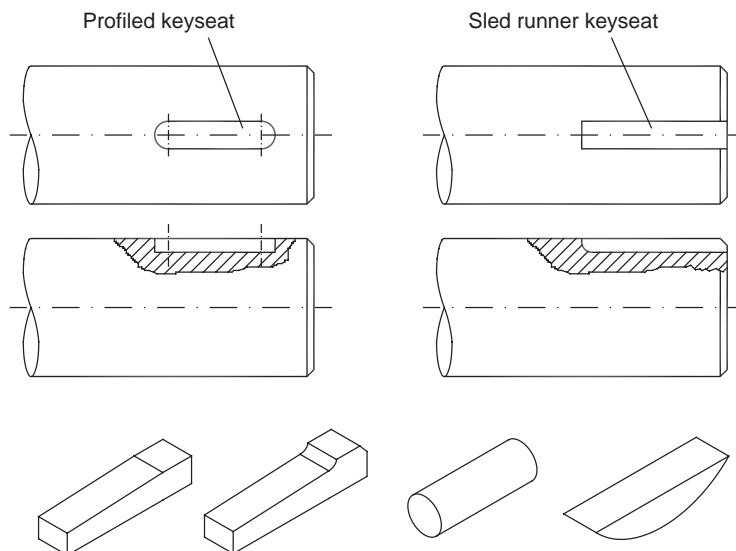
5.2 Shaft-hub connection

Power transmitting components such as gears, pulleys and sprockets need to be mounted on shafts securely and located axially with respect to mating components. In addition, a method of transmitting torque between the shaft and the component must be supplied. The portion of the component in contact with the shaft is called the hub and can be attached to, or driven by, the shaft by keys, pins, setscrews, press and shrink fits, splines and taper bushes. Table 5.1 identifies the merits of various connection methods. Alternatively the component can be formed as an integral part of a shaft as, for example, the cam on an automotive cam-shaft.

Figure 5.6 illustrates the practical implementation of several shaft hub connection methods. Gears, for example, can be gripped axially between a shoulder on a shaft and a spacer with the torque transmitted through a key. Various configurations of keys exist including square, flat and round keys as shown in Figure 5.7. The grooves in the shaft and hub into which the key fits are called keyways or keyseats. A simpler and less expensive method for transmitting light loads

Table 5.1 Merits of various shaft–hub connections (after Hurst, 1994)

	Pin	Grub screw	Clamp	Press fit	Shrink fit	Spline	Key	Taper bush
High torque capacity	✓	✓	✗	✓	✗	✗	✗	✗
Large axial loads	✗	✓	✗	✓	✗	✓	✓	✗
Axially compact	✓	✓	✓	✗	✗	✗	✗	✗
Axial location provision	✗	✗	✗	✗	✗	✓	✓	✗
Easy hub replacement	✓	✗	✗	✓	✓	✗	✗	✗
Fatigue	✓	✓	✗	✗	✗	✓	✓	✗
Accurate angular positioning	✗	✓	✓	✓	✓	✗	✗	✗
Easy position adjustment	✓	✗	✗	✓	✓	✓	✓	✗

**Figure 5.6** Alternative methods of shaft–hub connection.**Figure 5.7** Keys for torque transmission and component location.

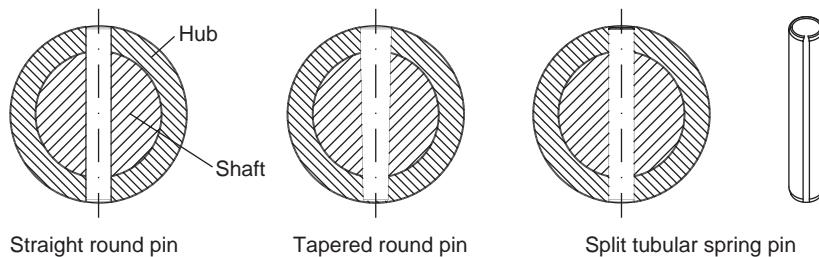


Figure 5.8 Pins for torque transmission and component location.

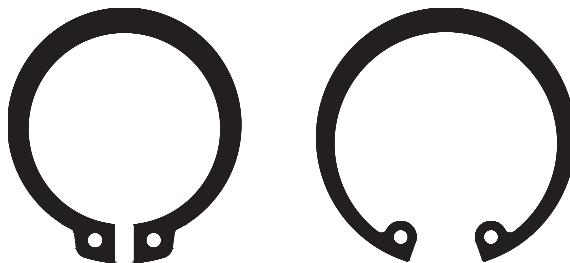


Figure 5.9 Snap rings or circlips.

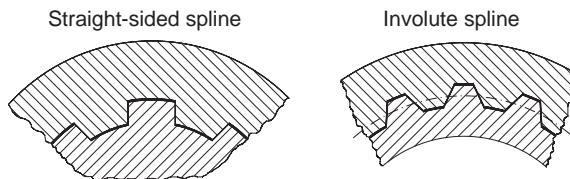


Figure 5.10 Splines.

is to use pins, and various pin types are illustrated in Figure 5.8. An inexpensive method of providing axial location of hubs and bearings on shafts is to use circlips as shown in Figures 5.2 and 5.9. One of the simplest hub–shaft attachments is to use an interference fit, where the hub bore is slightly smaller than the shaft diameter. Assembly is achieved by press fitting, or thermal expansion of the outer ring by heating and thermal contraction of the inner by use of liquid nitrogen. The design of interference fits is covered in greater detail in Section 15.2.2. Mating splines, as shown in Figure 5.10, comprise teeth cut into both the shaft and the hub and provide one of the strongest methods of transmitting torque. Both splines and keys can be designed to allow axial sliding along the shaft.

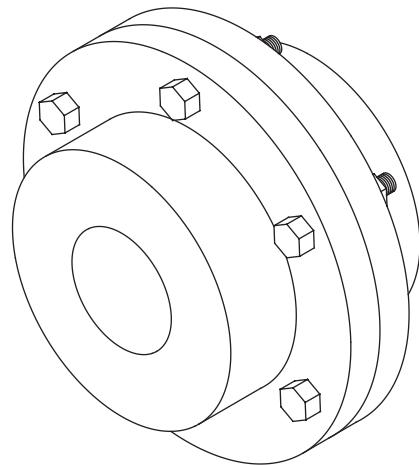


Figure 5.11 Rigid coupling.

5.3 Shaft–shaft connection – couplings

In order to transmit power from one shaft to another, a coupling or clutch can be used (for clutches see Chapter 10). There are two general types of coupling, rigid and flexible. Rigid couplings are designed to connect two shafts together so that no relative motion occurs between them (see Figure 5.11). Rigid couplings are suitable when precise alignment of two shafts is required. If significant radial or axial misalignment occurs high stresses may result which can lead to early failure. Flexible couplings (see Figure 5.12) are designed to transmit torque, whilst permitting some axial, radial and angular misalignment. Many forms of flexible coupling are available (e.g. see manufacturers' catalogues such as Turboflex and Fenner). Each

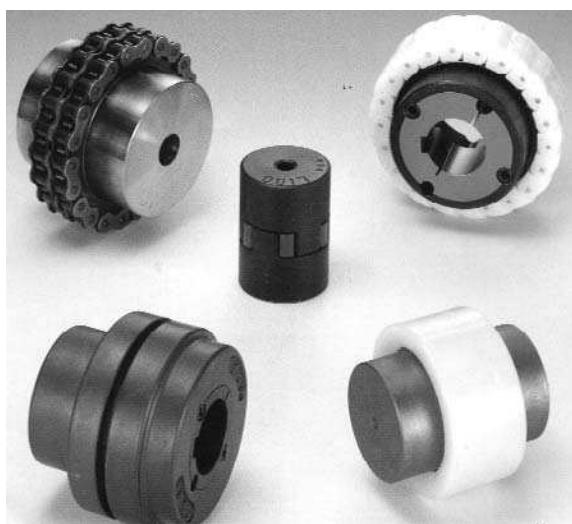


Figure 5.12 Flexible couplings (photograph courtesy of Cross and Morse).



Figure 5.13 Universal joints.

coupling is designed to transmit a given limiting torque. Generally flexible couplings are able to tolerate up to $\pm 3^\circ$ of angular misalignment and up to 0.75 mm parallel misalignment depending on their design. If more misalignment is required a universal joint can be used (see Figure 5.13). Couplings are considered in detail by Neale *et al.* (1998) and Piotrowski (1995).

5.4 Critical speeds and shaft deflection

The centre of mass of a rotating system (for example a mid-mounted disc on a shaft supported by

bearings at each end) will never coincide with the centre of rotation due to manufacturing and operational constraints. As the shaft rotational speed is increased, the centrifugal force acting at the centre of mass tends to bow the shaft. The more the shaft bows, the greater the eccentricity and the greater the centrifugal force. Below the lowest critical speed of rotation the centrifugal and elastic forces balance at a finite value of shaft deflection. At the critical speed, equilibrium theoretically requires infinite deflection of the centre of mass although realistically bearing damping, internal hysteresis and windage causes equilibrium to occur at a finite displacement. This displacement can be large enough to break the shaft, damage bearings and cause destructive machine vibration and therefore deflections need to be determined along the shaft and the consequences evaluated.

The critical speed of rotation is the same as the lateral frequency of vibration, which is induced when rotation is stopped and the centre displaced laterally and suddenly released (i.e. the same as the frequency you would obtain if you struck the stationary shaft with a hammer and monitored the frequency at which it vibrated). For all shafts, except for the single concentrated mass shaft, critical speeds also occur at higher frequencies.

At the first critical speed the shaft will bend into the simplest possible shape, and at the second critical speed it will bend into the next simplest shape. For example, the shapes the shaft will bend into at the first two critical speeds (or modes) for an end-supported shaft with two masses are illustrated in Figure 5.14.

In certain circumstances, the fundamental frequency of a shaft system cannot be made higher than the shaft design speed. If the shaft can be accelerated rapidly through and beyond the first resonant critical frequency, before the vibrations have a chance to build up in amplitude, then the system can be run at speeds higher than the natural frequency. This is the case with steam and gas turbines where the size of the turbomachinery and generators give low natural frequency, but must be run at high speed due to efficiency considerations.

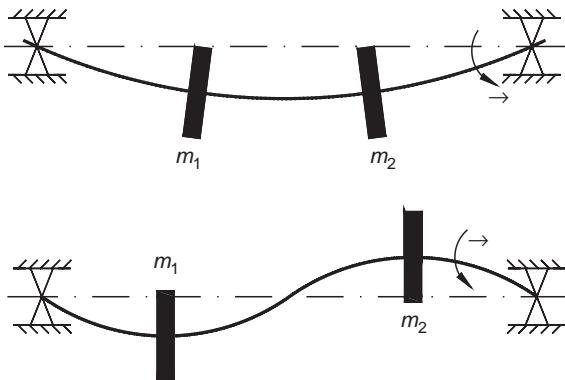


Figure 5.14 Shaft shapes for a simply supported shaft with two masses at the first and second critical speeds. Masses large in comparison with shaft mass.

As a general design principle maintaining the operating speed of a shaft below half the shaft whirl critical frequency is normally acceptable.

A complete analysis of the natural frequencies of a shaft can be performed using a finite element analysis package, such as ANSYS, called a 'nodal analysis'. This can give a large number of natural frequencies in three dimensions from the fundamental upwards. This is the sensible and easiest approach for complex systems, but a quick estimate for a simplified system can be undertaken for design purposes as outlined in this section.

The critical speed of a shaft with a single mass attached can be approximated by:

$$\omega_c = \sqrt{g/\gamma} \quad (5.1)$$

where ω_c is critical angular velocity (rad/s); γ , static deflection at the location of the mass (m); and g , acceleration due to gravity (m/s^2).

The first critical speed of a shaft carrying several concentrated masses is approximated by the Rayleigh–Ritz equation (see Eq. 5.2). The dynamic deflections of a shaft are generally unknown. Rayleigh showed that an estimate of the deflection curve is suitable provided it represents the maximum deflection and the boundary conditions. The static deflection curve due to the shaft's own weight and the weight of any attached components gives a suitable estimate. Note that external

loads are not considered in this analysis only those due to gravitation. The resulting calculation gives a value higher than the actual natural frequency by a few per cent.

$$\omega_c = \sqrt{\frac{g}{\sum_{i=1}^n W_i \gamma_i}} \quad (5.2)$$

where ω_c is critical angular velocity (rad/s); W_i , mass or weight of node i (kg or N); and γ_i , static deflection of W_i (m).

Alternatively the Dunkerley equation can be used to estimate the critical speed:

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \dots \quad (5.3)$$

where ω_1 is the critical speed if only mass number 1 is present, etc.

Both the Rayleigh–Ritz and the Dunkerley equations are approximations to the first natural frequency of vibration, which is assumed nearly equal to the critical speed. The Dunkerley equation tends to underestimate and the Rayleigh–Ritz equation to overestimate the critical frequency.

Often we need to know the second critical speed. For a two mass system, as illustrated in Figure 5.14, approximate values for the first two critical speeds can be found by solving the frequency equation given in Eq. 5.4.

$$\frac{1}{\omega^4} - (a_{11}m_1 + a_{22}m_2)\frac{1}{\omega^2} + (a_{11}a_{22} - a_{12}a_{21})m_1m_2 = 0 \quad (5.4)$$

where a_{11} , a_{12} , etc., are the influence coefficients. These correspond to the deflection of the shaft at the locations of the loads as a result of 1 N loads. The first subscript refers to location of the deflection, the second to the location of the 1 N force. For example, the influence coefficients for a simply supported shaft with two loads, as illustrated in Figure 5.15, are listed below.

- a_{11} is the deflection at the location of mass 1, that would be caused by a 1 N weight at the location of mass 1.

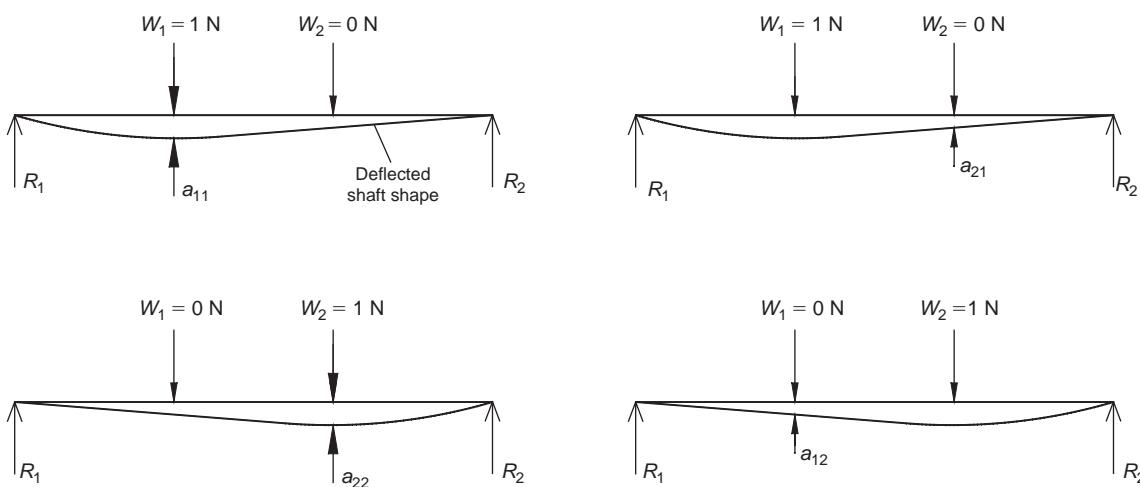


Figure 5.15 Example of influence coefficient definition for a simply supported shaft with two concentrated loads.

- a_{21} is the deflection at the location of mass 2, that would be caused by a 1 N weight at the location of mass 1.
- a_{22} is the deflection at the location of mass 2, that would be caused by a 1 N weight at the location of mass 2.
- a_{12} is the deflection at the location of mass 1, that would be caused by a 1 N weight at the location of mass 2.

Note that values for a_{np} and a_{pn} are equal by the principle of reciprocity (e.g. $a_{12} = a_{21}$).

For a multimass system the frequency equation can be obtained by equating the following determinate to zero:

$$\begin{vmatrix} a_{11}m_1 - \frac{1}{\omega^2} & a_{12}m_2 & a_{13}m_3 & \dots \\ a_{21}m_1 & a_{22}m_2 - \frac{1}{\omega^2} & a_{23}m_3 & \dots \\ a_{31}m_1 & a_{32}m_2 & a_{33}m_3 - \frac{1}{\omega^2} & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix} = 0 \quad (5.5)$$

It should be noted that lateral vibration requires an external source of energy. For example, vibrations can be transferred from another part of a machine and the shaft will vibrate in one or more lateral planes regardless of whether the shaft is rotating.

Shaft whirl is a self-excited vibration caused by the shaft's rotation acting on an eccentric mass.

These analysis techniques for calculating the critical frequency require the determination of the shaft deflection. Section 5.4.1 introduces the Macaulay method which is suitable for calculating the deflection of a constant diameter shaft and Section 5.4.2 introduces the strain energy method which is suitable for more complex shafts with stepped diameters.

5.4.1 Macaulay's method for calculating the deflection of beams

Macaulay's method can be used to determine the deflection of a constant cross-section shaft. The general rules for this method are given below. Having calculated the deflections of a shaft this information can then be used to determine critical frequencies.

1. Take an origin at the left hand side of the beam.
2. Express the bending moment at a suitable section XX in the beam to include the effect of all the loads.
3. Uniformly distributed loads must be made to extend to the right hand end of the beam. Use negative loads to compensate.

4. Put in square brackets all functions of length other than those involving single powers of x .
5. Integrate as a whole any term in square brackets.
6. When evaluating the moment, slope or deflection, neglect the square brackets terms when they become negative.
7. In the moment equation, express concentrated moments in the form $M_1[x - a]^0$, where M_1 is the concentrated moment and $x - a$ is its point of application relative to the section XX.

Macaulay's method for calculating the deflection of beams and shafts is useful in that it is relatively simple to use and easily programmed.

Example 5.1

As part of the preliminary design of a machine shaft, a check is undertaken to determine that the critical speed is significantly higher than the design speed of 7000 rpm. The components can be represented by three point masses as shown in Figure 5.16. Assume the bearings are stiff and act as simple supports. The shaft diameter is 40 mm and the material is steel with a Young's modulus of $200 \times 10^9 \text{ N/m}^2$.

Solution

Macaulay's method is used to determine the shaft deflections: Resolving vertically:

$$R_1 + R_2 = W_1 + W_2 + W_3.$$

Clockwise moments about O:

$$W_1 L_1 + W_2 (L_1 + L_2) - R_2 (L_1 + L_2 + L_3) \\ + W_3 (L_1 + L_2 + L_3 + L_4) = 0$$

Hence

$$R_2 = \frac{W_1 L_1 + W_2 (L_1 + L_2) + W_3 (L_1 + L_2 + L_3 + L_4)}{L_1 + L_2 + L_3}$$

Calculating the moment at XX:

$$M_{xx} = -R_1 x + W_1 [x - L_1] + W_2 [x - (L_1 + L_2)] \\ - R_2 [x - (L_1 + L_2 + L_3)] \quad (5.6)$$

The relationship for the deflection y of a beam subjected to a bending moment M is given by Eq. 5.7.

$$EI \frac{d^2y}{dx^2} = M \quad (5.7)$$

where E is the modulus of elasticity or Young's modulus (N/m^2); I , second moment of area (m^4); Y , deflection (m); x , distance from the end of the beam to the location at which the deflection is to be determined (m); M , moment (N m).

Eq. 5.7 can be integrated once to find the slope dy/dx and twice to find the deflection y .

Integrating Eq. 5.6 to find the slope:

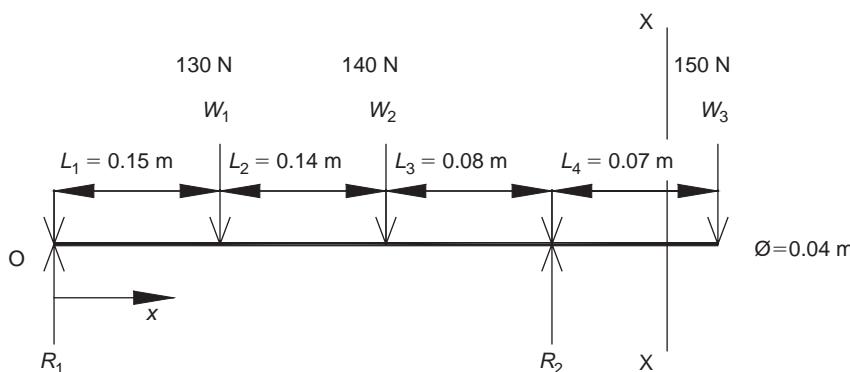


Figure 5.16 Machine shaft example.

$$EI \frac{dy}{dx} = -R_1 \frac{x^2}{2} + \frac{W_1}{2}[x - L_1]^2 + \frac{W_2}{2}[x - (L_1 + L_2)]^2 - \frac{R_2}{2}[x - (L_1 + L_2 + L_3)]^2 + A$$

Integrating again to find the deflection equation gives:

$$EIy = -R_1 \frac{x^3}{6} + \frac{W_1}{6}[x - L_1]^3 + \frac{W_2}{6}[x - (L_1 + L_2)]^3 - \frac{R_2}{6}[x - (L_1 + L_2 + L_3)]^3 + Ax + B.$$

Note that Macaulay's method requires that terms within square brackets be ignored when the sign of the bracket goes negative.

It is now necessary to substitute boundary conditions to find the constants of integration. In the case of a shaft the deflection at the bearings may be known in which case these can be used as boundary conditions.

Assuming that the deflection at the bearing is zero then substituting $y = 0$ at $x = 0$ into the equation for deflection above gives $B = 0$. At $x = L_1 + L_2 + L_3$, $y = 0$.

$$0 = -\frac{R_1}{6}(L_1 + L_2 + L_3)^3 + \frac{W_1}{6}(L_2 + L_3)^3 + \frac{W_2}{6}(L_3)^3 + A(L_1 + L_2 + L_3)$$

Hence

$$A = \frac{1}{L_1 + L_2 + L_3} \left[\frac{R_1}{6}(L_1 + L_2 + L_3)^3 - \frac{W_1}{6}(L_2 + L_3)^3 - \frac{W_2}{6}(L_3)^3 \right]$$

The second moment of area for a solid round shaft is given by

$$I = \frac{\pi d^4}{64}$$

so

$$I = \frac{\pi 0.04^4}{64} = 1.2566 \times 10^{-7} \text{ m}^4$$

With $W_1 = 130 \text{ N}$, $W_2 = 140 \text{ N}$, $W_3 = 150 \text{ N}$, $\phi = 0.04 \text{ m}$, and $E = 200 \times 10^9 \text{ N/m}^2$, substitution of these values into the above equations gives:

$$R_1 = 79.19 \text{ N}$$

$$R_2 = 340.8 \text{ N}$$

$$A = 1.151 \text{ N m}^2$$

At $x = 0.15 \text{ m}$, $y = 5.097 \times 10^{-6} \text{ m}$.

At $x = 0.29 \text{ m}$, $y = 2.839 \times 10^{-6} \text{ m}$.

At $x = 0.44 \text{ m}$, $y = -1.199 \times 10^{-6} \text{ m}$.

Use absolute values for the displacement in the Rayleigh–Ritz equation:

$$\omega_c = \sqrt{\frac{9.81 \cdot 130 \times 5.097 \times 10^{-6} + 140 \times 2.839 \times 10^{-6} + 150 \times |-1.199 \times 10^{-6}|}{130 \times (5.097 \times 10^{-6})^2 + 140 \times (2.839 \times 10^{-6})^2 + 150 \times (-1.199 \times 10^{-6})^2}} = \sqrt{\frac{9.81 \times 1.240 \times 10^{-3}}{4.722 \times 10^{-9}}} = 1605 \text{ rad/s}$$

so the critical speed is $1605 \times 2\pi/60 = 15530 \text{ rpm}$.

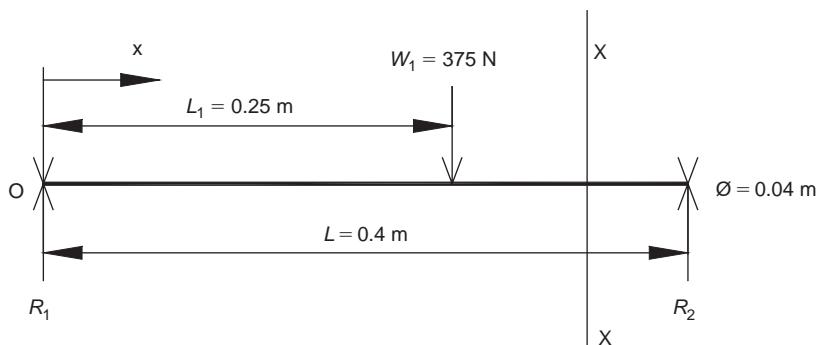


Figure 5.17 Simple shaft with single concentrated mass.

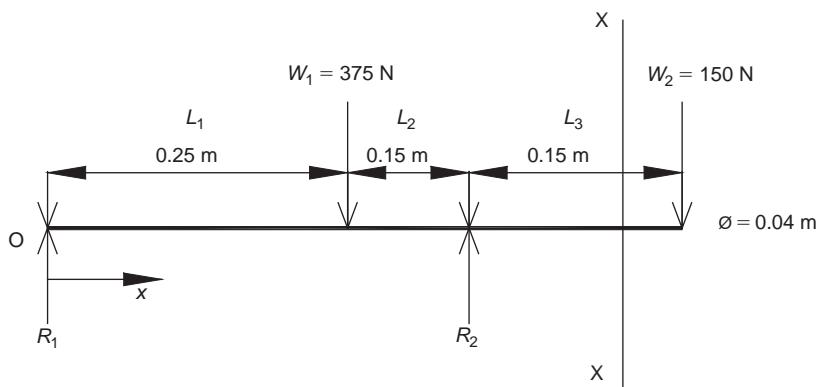


Figure 5.18 Simple shaft with a single concentrated mass between bearing and overhang mass.

Example 5.2

This example explores the influence on the critical frequency of adding an overhang load to a beam.

- Determine the critical frequency for a 0.4 m long steel shaft running on bearings with a single point load as illustrated in Figure 5.17. Assume that the bearings are rigid and act as simple supports. Ignore the self-weight of the shaft. Take Young's modulus of elasticity as $200 \times 10^9 \text{ N/m}^2$.
- Calculate the critical frequency if an overhang load of 150 N is added at a distance of 0.15 m from the right hand bearing as shown in Figure 5.18.

Solution

- Resolving vertically: $R_1 + R_2 = W_1$. Clockwise moments about O : $375 \times 0.25 - R_2 \cdot 0.4 = 0$.

$$\text{Hence } R_2 = 234.375 \text{ N}, R_1 = 140.625 \text{ N.}$$

$$EI \frac{d^2y}{dx^2} = -R_1 x + W_1 [x - L_1]$$

$$EI \frac{dy}{dx} = -R_1 \frac{x^2}{2} + \frac{W_1}{2} [x - L_1]^2 + A$$

$$EIy = -R_1 \frac{x^3}{6} + \frac{W_1}{6} [x - L_1]^3 + Ax + B.$$

Boundary conditions:

Assuming that the deflection of the shaft is zero at the bearings, then substituting $y = 0$ and $x = 0$ into the above equation gives $B = 0$. At $x = L$, $y = 0$.

Hence

$$\begin{aligned} A &= \frac{1}{L} R_1 \frac{L^2}{6} - \frac{W_1}{6} (L - L_1)^3 \\ &= \frac{1}{0.4} (1.5 - 0.2109) = 3.22266. \end{aligned}$$

For a solid circular shaft

$$I = \frac{\pi d^4}{64} = \frac{\pi 0.04^4}{64} \\ = 1.2566 \times 10^{-7} \text{ m}^4$$

$$IE = 200 \times 10^9 \text{ N/m}^2$$

$$EIy = -140.625 \times \frac{0.25^3}{6} \\ + (3.22266 \times 0.25)$$

$$\gamma = 1.7485 \times 10^{-5} \text{ m.}$$

Application of Macaulay's method to the geometry given in Figure 5.17 yields $\gamma_1 = 1.74853 \times 10^{-5} \text{ m}$. So from Eq. 5.1:

$$\omega_c = \sqrt{\frac{g}{\gamma_1}} = 749.03 \text{ rad/s} = 7152 \text{ rpm}$$

$$2. EI \frac{d^2y}{dx^2} = -R_1x + W_1[x - L_1] \\ - R_2[x - (L_1 + L_2)]$$

$$EI \frac{dy}{dx} = -R_1 \frac{x^2}{2} + \frac{W_1}{2}[x - L_1]^2 \\ - \frac{R^2}{2}[x - (L_1 + L_2)]^2 + A$$

$$EIy = -R_1 \frac{x^3}{6} + \frac{W_1}{6}[x - L_1]^3 \\ - \frac{R_2}{6}[x - (L_1 + L_2)]^3 + Ax + B$$

Boundary conditions:

Assuming negligible deflection at the bearings. At $x = 0$, $y = 0$. Hence $B = 0$.

At $x = L_1 + L_2$, $y = 0$. Hence

$$A = \frac{1}{L_1 + L_2} \frac{R_1}{6}(L_1 + L_2)^3 - \frac{W_1}{6}L_2^3 .$$

Resolving vertically: $R_1 + R_2 = W_1 + W_2$.

Moments about O: $W_1L_1 + W_2(L_1 + L_2 + L_3) - R_2(L_1 + L_2) = 0$.

Hence

$$R_2 = \frac{1}{L_1 + L_2} W_1L_1 + W_2(L_1 + L_2 + L_3)$$

$$R_1 = W_1 + W_2 - R_2$$

$$R_1 = 84.375 \text{ N}$$

$$R_2 = 440.625 \text{ N.}$$

Substitution for A gives $A = 1.722656 \text{ N m}^2$.

Evaluating γ at $x = L_1$ gives $\gamma = 8.3929381 \times 10^{-6} \text{ m}$.

Evaluating γ at $x = L_1 + L_2 + L_3$ gives $\gamma = 1.8884145 \times 10^{-6} \text{ m}$.

Hence

$$\omega_c = \sqrt{\frac{9.81 \cdot 3.147 \times 10^{-3} + 2.8326 \times 10^{-4}}{2.64155 \times 10^{-8} + 5.34061 \times 10^{-10}}} \\ = \sqrt{\frac{9.81 \times 3.4306 \times 10^{-3}}{2.695 \times 10^{-8}}} = 1117.5 \text{ rad/s}$$

$$\omega_c = 10671 \text{ rpm.}$$

Comparison of the results from 1 and 2 shows that the addition of an overhang weight can have a significant effect in raising the critical frequency from 7150 to 10670 rpm in this case. This technique of adding overhang masses to increase the critical speed is sometimes used in plant machinery.

Example 5.3

- Calculate the first two critical speeds for the loadings on a steel shaft indicated in Figure 5.19. Assume the mass of the shaft can be ignored for the purpose of this calculation and that the bearings can be considered to be stiff simple supports. The internal and external diameters of the shaft are 0.03 and 0.05 m, respectively. Take Young's modulus as 200 GN/m².
- The design speed for the shaft is nominally 5000 rpm; what would be the implication of the bearing data given in Table 5.2 on the

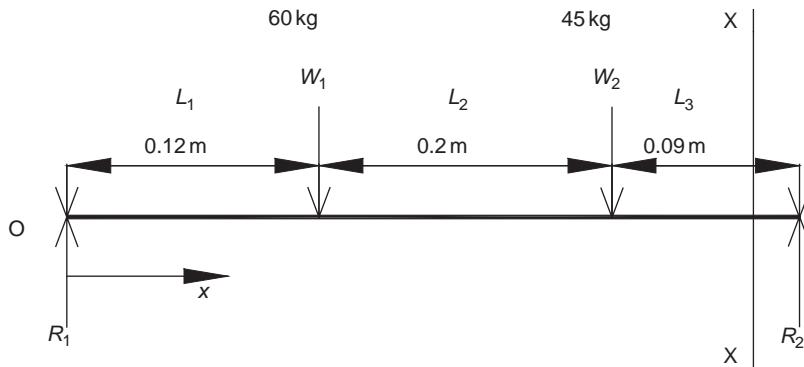


Figure 5.19 Simple shaft.

Table 5.2 Example bearing data for deflection versus load

Load on bearing (N)	Deflection at bearing (mm)
50	0.003
100	0.009
200	0.010
300	0.0115
400	0.0126
500	0.0136
1000	0.0179
2000	0.0245

shaft operation assuming the bearings allow flexibility only in directions perpendicular to the shaft axis?

Solution

- Applying Macaulay's method to find the influence coefficients: first calculate the deflections at L_1 and L_2 with $W_1 = 1\text{ N}$ and $W_2 = 0\text{ N}$ to give a_{11} and a_{21} . Then calculating the deflections at L_1 and L_2 with $W_1 = 0\text{ N}$ and $W_2 = 1\text{ N}$ to give a_{12} and a_{22} .

$$EIy = -R_1 \frac{x^3}{6} + \frac{W_1}{6}[x - L_1]^3 + \frac{W_2}{6}[x - (L_1 + L_2)]^3 + Ax + B.$$

With $W_1 = 1\text{ N}$, $W_2 = 0\text{ N}$, $R_1 = 0.707317\text{ N}$, $R_2 = 0.292683\text{ N}$, $A = 9.90244 \times 10^{-3}\text{ N m}^2$ ($B = 0$).

Hence

$$a_{11} = 1.84355 \times 10^{-8}\text{ m}, \\ a_{21} = 1.19688 \times 10^{-8}\text{ m}.$$

With $W_1 = 0\text{ N}$, $W_2 = 1\text{ N}$, $R_1 = 0.2195122\text{ N}$, $R_2 = 0.7804878\text{ N}$, $A = 5.853659 \times 10^{-3}\text{ N m}^2$.

Hence

$$a_{22} = 1.26264 \times 10^{-8}\text{ m}, \\ a_{12} = 1.19688 \times 10^{-8}\text{ m}.$$

For this example, $a_{11} = 1.84355 \times 10^{-8}\text{ m}$, $a_{21} = 1.19688 \times 10^{-8}\text{ m}$, $a_{12} = 1.19688 \times 10^{-8}\text{ m}$, $a_{22} = 1.26264 \times 10^{-8}\text{ m}$.

Solving

$$\frac{1}{\omega^4} - (a_{11}m_1 + a_{22}m_2)\frac{1}{\omega^2} + (a_{11}a_{22} - a_{12}a_{21})m_1m_2 = 0$$

by multiplying through by ω^4 and solving as a quadratic:

$$\omega^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

with

$$a = (a_{11}a_{22} - a_{12}a_{21})m_1m_2,$$

$$b = -(a_{11}m_1 + a_{22}m_2),$$

$$c = 1,$$

gives $\omega_{c1} = 812.5\text{ rad/s}$ (7759 rpm) and $\omega_{c2} = 2503.4\text{ rad/s}$ (23905 rpm).

2. The deflection at the bearings should also be taken into account in determining the critical frequency of the shaft. The reaction at each of the bearings is approximately 500 N ($R_1 = 513.2$ N and $R_2 = 516.8$ N). The deflection at each bearing can be assumed roughly equal (so from Table 5.2, $y_{\text{bearing}} = 0.0136$ mm). This deflection can be simply added to the static deflection of the shaft due to bending by superposition. If the deflections were not equal, similar triangles could be used to calculate the deflection at the mass locations.

The influence coefficients can also be used to determine the deflection under the actual loads applied. $W_1 = 60 \times 9.81 = 588.6$ N, $W_2 = 45 \times 9.81 = 441.45$ N:

$$y_{1 \text{ Load}} = W_1 a_{11} + W_2 a_{12} = 1.613 \times 10^{-5} \text{ m},$$

$$y_{2 \text{ Load}} = W_1 a_{21} + W_2 a_{22} = 1.26 \times 10^{-5} \text{ m},$$

$$\begin{aligned} y_{1 \text{ Total}} &= 1.61 \times 10^{-5} + 1.36 \times 10^{-5} \\ &= 2.97 \times 10^{-5} \text{ m}, \end{aligned}$$

$$\begin{aligned} y_{2 \text{ Total}} &= 1.26 \times 10^{-5} + 1.36 \times 10^{-5} \\ &= 2.62 \times 10^{-5} \text{ m}. \end{aligned}$$

These deflections give (using the Rayleigh–Ritz equation) a first critical frequency of:

$$\begin{aligned} \omega_c &= \sqrt{\frac{9.81 \times 2.96 \times 10^{-3}}{8.38 \times 10^{-8}}} \\ &= 588.6 \text{ rad/s} = 5620 \text{ rpm}. \end{aligned}$$

This is very close to the design speed. Different bearings or a stiffer shaft would be advisable.

5.4.2 Castigiano's theorem for calculating shaft deflections

The strain energy, U , in a straight beam subjected to a bending moment M is:

$$U = \frac{M^2 dx}{2EI} \quad (5.8)$$

Castigiano's strain energy equation can be applied to problems involving shafts of non-constant section to calculate deflections, and hence critical speeds.

Example 5.4

Determine the deflection of the stepped shaft illustrated in Figure 5.20 under a single concentrated load.

Solution

The total strain energy due to bending is

$$U = \frac{M^2 dx}{2EI} = U_1 + U_2$$

where U_1 is the energy from $x = 0$ to L_1 , and U_2 is the energy from $x = L_1$ to L .

In general, consider the strain energy for each section of a beam, i.e. between loads and between any change of shaft section.

Resolving vertical loads:

$$R_1 + R_2 = W.$$

Taking moments about the left hand bearing:

$$R_2 = \frac{WL_1}{L}.$$

Hence

$$R_1 = W - \frac{L_1}{L}.$$

Splitting the beam between $x = 0$ and L_1 the moment can be expressed as R_1x .

$$\begin{aligned} U_1 &= \int_0^{L_1} \frac{(R_1x)^2}{2EI_1} dx = \frac{(W(1 - (L_1/L)))^2 x^3}{6EI_1} \Big|_0^{L_1} \\ &= \frac{(W(1 - (L_1/L)))^2 L_1^3}{6EI_1}. \end{aligned}$$

Splitting the beam between $x = L_1$ and L the moment can be expressed as $R_2(L - x)$.

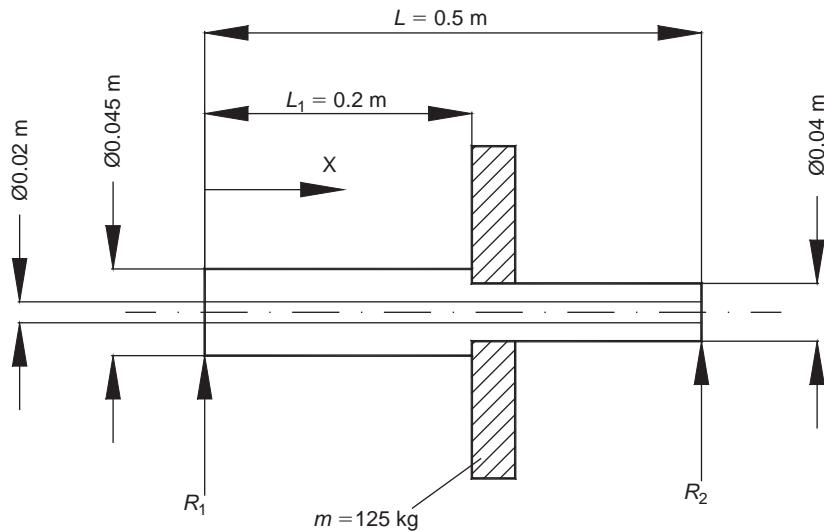


Figure 5.20 Stepped shaft with single load.

$$\begin{aligned} U_2 &= \frac{\int_{L_1}^L (R_2(I - X))^2 dx}{2EI} \\ &= \frac{(WL_1/L)^2(L^2x - Lx^2 + x^3/3)}{2EI_2} \Big|_{L_1}^L \\ &= \frac{W^2L_1^2}{2EI_2L^2} L^3 - L^3 + \frac{L^3}{3} \\ &\quad - L^2L_1 + LL_1^2 - \frac{L_1^3}{3}. \end{aligned}$$

$$\begin{aligned} U &= U_1 + U_2 \\ &= \frac{(W(1 - (L_1/L))^2L_1^3}{6EI_1} \\ &\quad + \frac{W^2L_1^2}{2EI_2L^2} L^3 - L^3 + \frac{L^3}{3} \\ &\quad - L^2L_1 + LL_1^2 - \frac{L_1^3}{3}. \end{aligned}$$

Differentiating the above expression with respect to W gives the deflection at W :

$$\frac{U}{W} = \frac{L_1^3}{6EI_1} 2W - \frac{4WL_1}{L} + \frac{2WL_1^2}{L^2}$$

$$\begin{aligned} &+ \frac{2WL_1^2}{2EI_2L^2} \frac{L^3}{3} - L^2L_1 + LL_1^2 - \frac{L_1^3}{3}. \\ I_1 &= \frac{\pi(d_o^4 - d_i^4)}{64} = \frac{\pi(0.045^4 - 0.02^4)}{64} \\ &= 1.9343 \times 10^{-7} \text{ m}^4. \\ I_2 &= \frac{\pi(0.04^4 - 0.02^4)}{64} = 1.1781 \times 10^{-7} \text{ m}^4. \end{aligned} \quad (5.9)$$

Substitution of $W = 125 \times 9.81 \text{ N}$, $L_1 = 0.2 \text{ m}$, $L = 0.5 \text{ m}$, $E = 200 \times 10^9 \text{ N/m}^2$, I_1 and I_2 into Eq. 5.9 gives a deflection $U/W = 1.05372 \times 10^{-4} \text{ m}$ and hence a first critical frequency using the approximation of Eq. 5.1 of:

$$\sqrt{\frac{9.81}{1.05372 \times 10^{-4}}} = 305.121 \text{ rad/s} \\ = 2913 \text{ rpm.}$$

5.5 ASME design code for transmission shafting

According to Fuchs and Stephens (1980) between 50 and 90 per cent of all mechanical failures are

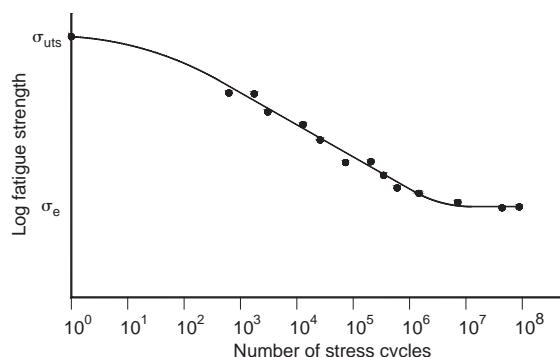


Figure 5.21 A typical strength cycle diagram for various steels.

fatigue failures. This challenges the engineer to consider the possibility of fatigue failure at the design stage. Figure 5.21 shows the characteristic variation of fatigue strength for steel with the number of stress cycles. For low strength steels a ‘levelling off’ occurs in the graph between 10^6 and 10^7 cycles under non-corrosive conditions and, regardless of the number of stress cycles beyond this, the component will not fail due to fatigue. The value of stress corresponding to this levelling off is called the ‘endurance stress’ or the ‘fatigue limit’.

In order to design hollow or solid rotating shafts under combined cyclic bending and steady torsional loading for limited life the ASME (American Society of Mechanical Engineers) design code for the design of transmission shafting can be used. The ASME procedure ensures that the shaft is properly sized to provide adequate service life, but the designer must ensure that the shaft is stiff enough to limit deflections of power transfer elements, such as gears, and to minimize misalignment through seals and bearings. In addition the shaft’s stiffness must be such that it avoids unwanted vibrations through the running range.

The equation for determining the diameter for a solid shaft is given by

$$d = \frac{32n_s}{\pi} \sqrt{\left(\frac{M}{\sigma_e} \right)^2 + \frac{3}{4} \left(\frac{T}{\sigma_y} \right)^2}^{1/3} \quad (5.10)$$

where d , diameter (m); n_s , factor of safety; M , bending moment (N m); σ_e , endurance limit of the item (N/m^2); T , torque (N m); and σ_y , yield strength (N/m^2).

There is usually some uncertainty regarding what level a component will actually be loaded to, how strong a material is and how accurate the modelling methods are. Factors of safety are frequently used to account for these uncertainties. The value of a factor of safety is usually based on experience of what has given acceptable performance in the past. The level is also a function of the consequences of component failure and the cost of providing an increased safety factor. As a guide, typical values for the factor of safety based on strength recommended by Vidosek (1957) are:

- 1.25–1.5 for reliable materials under controlled conditions subjected to loads and stresses known with certainty;
- 1.5–2 for well-known materials under reasonably constant environmental conditions subjected to known loads and stresses;
- 2–2.5 for average materials subjected to known loads and stresses;
- 2.5–3 for less well-known materials under average conditions of load, stress and environment;
- 3–4 for untried materials under average conditions of load, stress and environment;
- 3–4 for well-known materials under uncertain conditions of load, stress and environment.

The endurance limit, σ_e , for a mechanical element can be estimated by Eq. 5.11 (after Marin, 1962). Here a series of modifying factors are applied to the endurance limit of a test specimen for various effects such as size, load and temperature.

$$\sigma_e = k_a k_b k_c k_d k_e k_f k_g \sigma'_e \quad (5.11)$$

where k_a , surface factor; k_b , size factor; k_c , reliability factor; k_d , temperature factor; k_e , duty cycle factor; k_f , fatigue stress concentration factor; k_g , miscellaneous effects factor; σ'_e , endurance limit of test specimen (N/m^2).

Table 5.3 Surface finish factors (Noll and Lipson, 1946)

Surface finish	<i>a</i> (MPa)	<i>b</i>
Ground	1.58	-0.085
M/c or cold-drawn	4.51	-0.265
Hot rolled	57.7	-0.718
Forged	272.0	-0.995

If the stress at the location under consideration is greater than σ_e , then the component will fail eventually due to fatigue (i.e. the component has a limited life).

Mischke (1987) has determined the following approximate relationships between the endurance limit of test specimens and the ultimate tensile strength of the material (for steels only):

$$\sigma'_e = 0.504\sigma_{uts} \quad \text{for } \sigma_{uts} \leq 1400 \text{ MPa} \quad (5.12)$$

$$\sigma'_e = 700 \text{ MPa} \quad \text{for } \sigma_{uts} \geq 1400 \text{ MPa} \quad (5.13)$$

The surface finish factor is given by

$$k_a = a\sigma_{uts}^b \quad (5.14)$$

Values for *a* and *b* can be found in Table 5.3.

The size factor k_b can be calculated from Eq. 5.15 or 5.16 (Kuguel, 1969).

$$k_b = \frac{d}{7.62}^{-0.1133} \quad (\text{for } d < 50 \text{ mm}) \quad (5.15)$$

$$k_b = 1.85d^{-0.19} \quad (\text{for } d > 50 \text{ mm}) \quad (5.16)$$

The reliability factor k_c is given in Table 5.4 as a function of the nominal reliability desired.

For temperatures between -57°C and 204°C , the temperature factor, k_d , can be taken as 1. The ASME standard documents values to use outside this range.

The duty cycle factor k_e is used to account for cycle loading experienced by the shaft, such as stops and starts, transient overloads, shock loading, etc., and requires prototype fatigue testing for

Table 5.4 Reliability factors for use in the ASME transmission shaft equation

Shaft nominal reliability	<i>k_c</i>
0.5	1.0
0.9	0.897
0.99	0.814
0.999	0.753

Source, ANSI/ASME B106.1 M-1985.

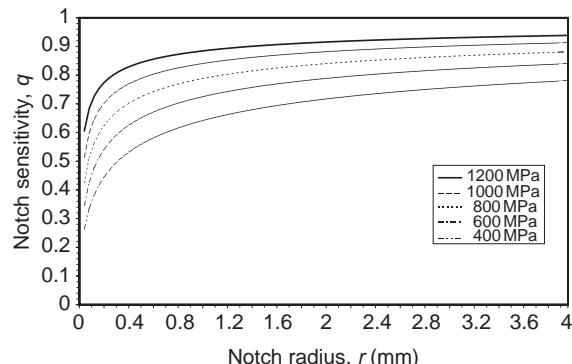


Figure 5.22 Notch sensitivity index versus notch radius for a range of steels subjected to reversed bending or reversed axial loads (data from Kuhn and Handrath, 1952).

its quantification. k_e is taken as 1 in the examples presented here.

The fatigue stress concentration factor k_f is used to account for stress concentration regions, such as notches, holes, keyways and shoulders. It is given by

$$k_f = \frac{1}{K_f} \quad (5.17)$$

where K_f is the component fatigue stress concentration factor which is given by:

$$K_f = 1 + q(k_t - 1) \quad (5.18)$$

where q is notch sensitivity; and k_t is geometric stress concentration factor.

Values for the notch sensitivity and typical geometric stress concentration factors are given in Figures 5.22–5.25 and Table 5.5.

The miscellaneous factor k_g is used to account for residual stresses, heat treatment, corrosion,

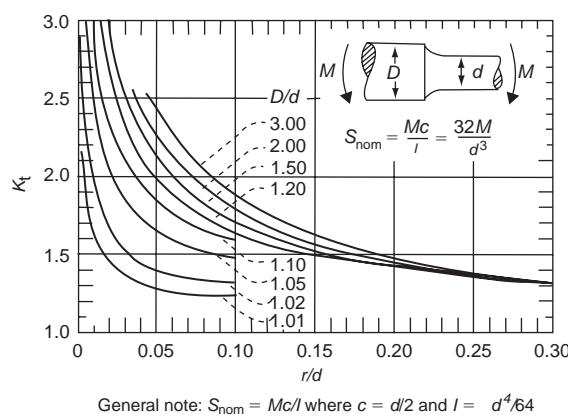


Figure 5.23 Stress concentration factors for a shaft with a fillet subjected to bending (reproduced from Peterson, 1974).

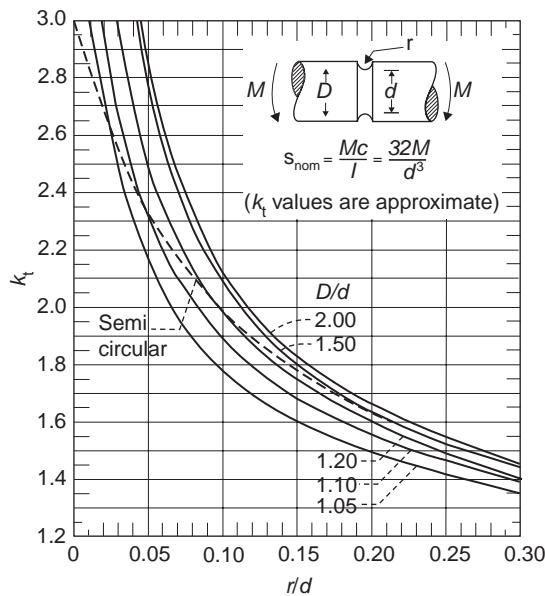


Figure 5.24 Stress concentration factors for a shaft with a groove subjected to bending (reproduced from Peterson, 1974).

surface coatings, vibration, environment and unusual loadings. k_g is taken as 1 here.

In general, the following principles should be used when designing to avoid fatigue failures.

- Calculations should allow an appropriate safety factor, particularly where stress concentrations occur (e.g. keyways, notches, change of section).

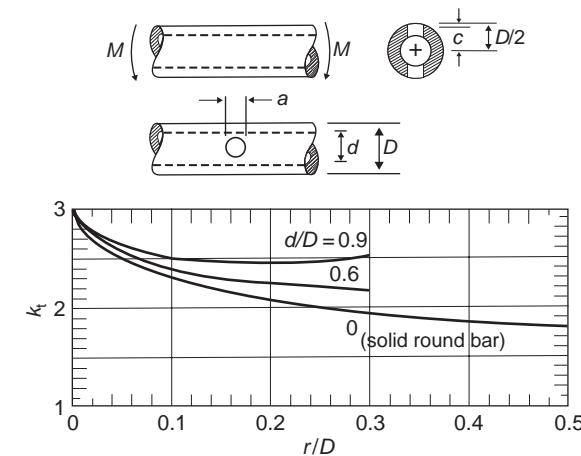


Figure 5.25 Stress concentration factors for a shaft with a transverse hole subjected to bending (reproduced from Peterson, 1974).

Table 5.5 Fatigue stress concentration factors, k_f for keyways in steel shafts (Juvinall, 1967)

Steel	Keyway bending stress	
	Profiled	Sled runner
Annealed < 200 BHN	0.63	0.77
Quenched and drawn	0.5	0.63
> 200 BHN		

- Provide generous radii at changes of section and introduce stress relief grooves, etc.
- Choose materials if possible that have limiting fatigue stresses, e.g. most steels.
- Provide for suitable forms of surface treatment, e.g. shot-peening, work-hardening, nitriding. Avoid treatments which introduce residual tensile stresses, such as electroplating.
- Specify fine surface finishes.
- Avoid corrosive conditions.
- Stress relieving should be used where possible, particularly for welded structures.

Example 5.5

Using the ASME equation for the design of transmission shafting determine a sensible

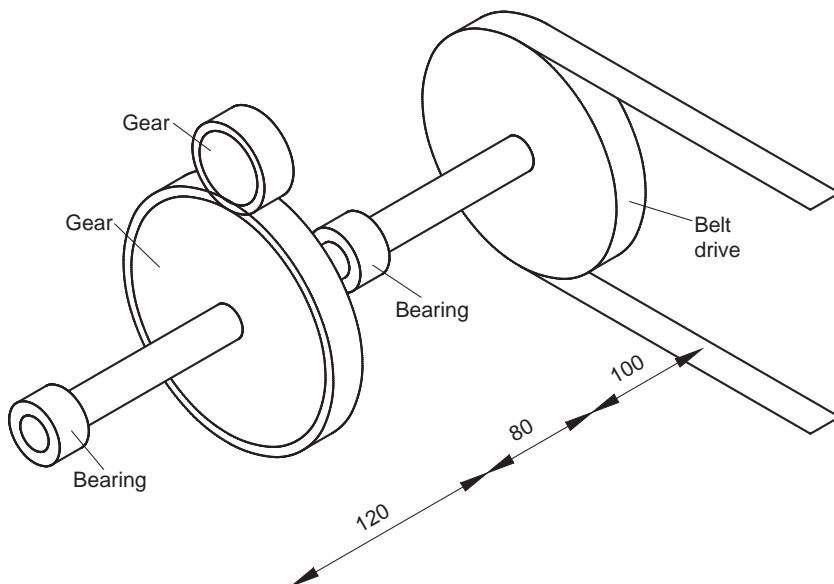


Figure 5.26 Transmission drive shaft.

minimum nominal diameter for the drive shaft illustrated in Figure 5.26 consisting of a mid-mounted spur gear and overhung pulley wheel. The shaft is to be manufactured using 817M40 hot rolled alloy steel with $\sigma_{uts} = 1000$ MPa, $\sigma_y = 770$ MPa and Brinell Hardness approximately 220 BHN. The radius of the fillets at the gear and pulley shoulders is 3 mm. The power to be transmitted is 8 kW at 900 rpm. The pitch circle diameter of the 20° pressure angle spur gear is 192 mm and the pulley diameter is 250 mm. The masses of the gear and pulley are 8 and 10 kg, respectively. The ratio of belt tensions should be taken as 2.5. Profiled keys are used to transmit torque through the gear and pulley. A shaft nominal reliability of 90 per cent is desired. Assume the shaft is of constant diameter for the calculation.

Solution

In order to use the ASME design equation for transmission shafts, the maximum combination of torque and bending moment must be determined. A sensible approach is to determine the overall bending moment diagram for the shaft as this information may be of use in other

design calculations such as calculating the shaft deflection.

The loading on the shaft can be resolved into both horizontal and vertical planes and must be considered in determining the resulting bending moments on the shaft. Figure 5.27 shows the combined loadings on the shaft from the gear forces, the gear's mass, the belt tensions and the pulley's mass. The mass of the shaft itself has been ignored. The tension on a pulley belt is tighter on the 'pulling' side than on the 'slack' side and the relationship of these tensions is normally given as a ratio, which in this case is 2.5:1.

The power transmitted through the shaft is 8000 W.

The torque is given by

$$\text{Torque} = \frac{\text{Power}}{\omega} = \frac{8000}{(2\pi/60) \times 900} \\ = 84.9 \text{ N m.}$$

Belts are introduced in more detail in Chapter 8. Here only the tensions will be considered as necessary for the development of the solution.

The ratio of belt tensions is $T_1/T_2 = 2.5$, so $T_1 = 2.5T_2$.

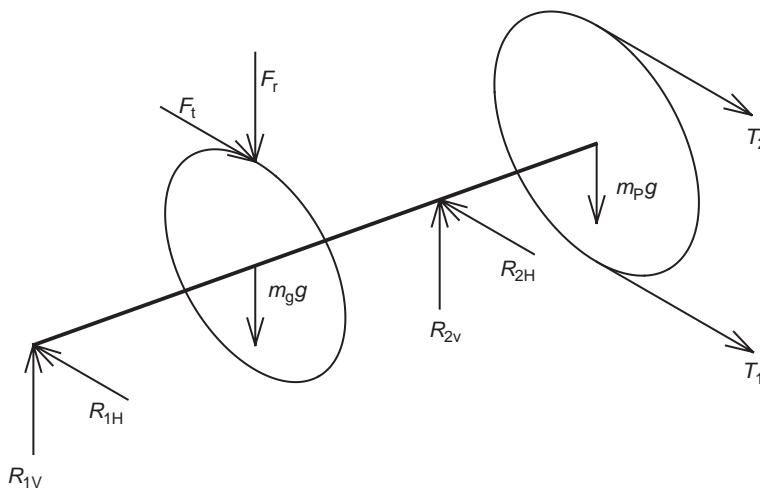


Figure 5.27 Shaft loading diagram.

The torque on the pulley in terms of the belt tensions is given by

$$\begin{aligned}\text{Torque} &= T_1 r_p - T_2 r_p \\ &= 2.5 T_2 r_p - T_2 r_p = 1.5 T_2 r_p.\end{aligned}$$

The belt tensions are

$$\begin{aligned}T_2 &= 84.9 / (1.5 \times 0.125) = 452.8 \text{ N}, \\ T_1 &= 2.5 T_2 = 1131.8 \text{ N}.\end{aligned}$$

Forces on spur gears are considered in detail in Chapter 6. Here the relevant relationships will be stated as necessary and used in the development of the solution. A spur gear experiences both radial and tangential loading as shown in Figure 5.24. The tangential load is given by $F_t = \text{Torque}/\text{pitch circle radius}$.

$$F_t = \frac{\text{Torque}}{r_g} = \frac{84.9}{0.192/2} = 884.4 \text{ N}.$$

The radial load is given by $F_r = F_t \tan \phi$, where ϕ is the pressure angle.

$$F_r = F_t \tan \phi = 884.4 \times \tan 20 = 321.9 \text{ N}.$$

Note that when determining the horizontal and vertical bending moment diagrams both

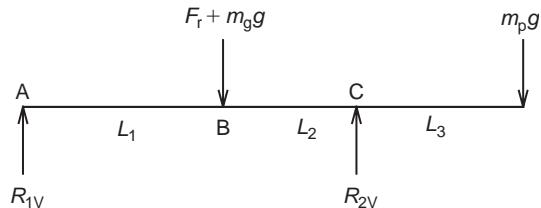


Figure 5.28 Vertical loading diagram.

transmitted and gravitational loads should be included.

The vertical loads on the shaft are illustrated in Figure 5.28.

Moments about A:

$$(F_r + m_g g)L_1 - R_{2V}(L_1 + L_2) + m_p g(L_1 + L_2 + L_3) = 0.$$

$$\begin{aligned}R_{2V} &= \frac{1}{L_1 + L_2} (F_r + m_g g)L_1 + m_p g(L_1 + L_2 + L_3) \\ &= \frac{1}{0.12 + 0.08} (321.9 + (8 \times 9.81))0.12 \\ &\quad + (10 \times 9.81)0.3 \\ &= 387.4 \text{ N}.\end{aligned}$$

Resolving vertical forces

$$R_{1V} + R_{2V} = F_r + m_g g + m_p g.$$

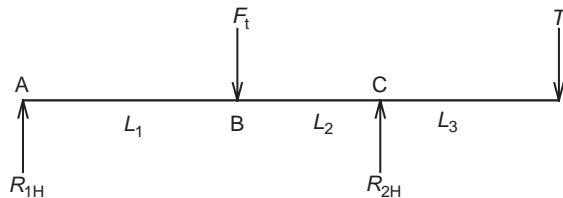


Figure 5.29 Horizontal loading diagram.

Hence

$$R_{1V} = 321.9 + 78.5 + 98.1 - 387.4 = 111.1 \text{ N.}$$

The horizontal loads on the shaft are illustrated in Figure 5.29. The total tension T is

$$T_1 + T_2 = 1131.8 + 452.8 = 1585 \text{ N.}$$

Moments about A

$$F_t L_1 - R_{2H}(L_1 + L_2) + T(L_1 + L_2 + L_3) = 0.$$

Hence

$$\begin{aligned} R_{2H} &= \frac{F_t L_1 + T(L_1 + L_2 + L_3)}{L_1 + L_2} \\ &= \frac{(884.4 \times 0.12) + (1585 \times 0.3)}{0.2} = 2908 \text{ N.} \end{aligned}$$

Resolving horizontal forces

$$R_{1H} + R_{2H} = F_t + T.$$

Hence

$$R_{1H} = 884.4 + 1585 - 2908 = -438.5 \text{ N.}$$

The bending moment diagrams can now be determined. You may wish to recall that the bending moment is the algebraic sum of the moments of the external forces to one side of the moments of the external forces to one side of the section about an axis through the section.

The vertical bending moments at B and C (Figure 5.30) are given by:

$$\begin{aligned} M_{BV} &= R_{1V} L_1 = 111.1 \times 0.12 = 13.3 \text{ Nm,} \\ M_{CV} &= R_{1V}(L_1 + L_2) - ((F_t + m_g g)L_2) \\ &= 111.1(0.12 + 0.08) \\ &\quad - (321.9 + 8 \times 9.81)0.08 = -9.810 \text{ Nm.} \end{aligned}$$

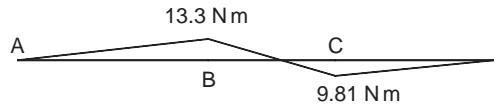


Figure 5.30 Vertical bending moment diagram.

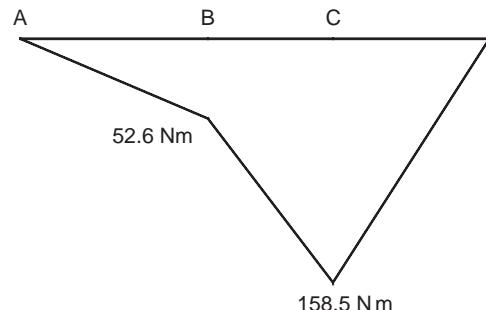


Figure 5.31 Horizontal bending moment diagram.

The horizontal bending moments at B and C (Figure 5.31) are given by:

$$\begin{aligned} M_{BH} &= R_{1H} L_1 = -438.5 \times 0.12 = -52.62 \text{ Nm,} \\ M_{CH} &= R_{1H}(L_1 + L_2) - (F_t L_2) \\ &= -438.5 \times 0.2 - 884.4 \times 0.08 \\ &= -158.5 \text{ N m.} \end{aligned}$$

The resultant bending moment diagram can be determined by calculating the resultant bending moments at each point.

$$|M_B| = \sqrt{(13.3)^2 + (-52.62)^2} = 54.27 \text{ Nm.}$$

$$|M_C| = \sqrt{(-9.81)^2 + (-158.5)^2} = 158.8 \text{ Nm.}$$

From Figure 5.32, it can be seen that the maximum bending moment occurs at the right hand bearing. The value of the resultant bending moment here should be used in the ASME design equation.

The next task is to determine the endurance limit of the shaft and the modifying factors. The shaft material is hot rolled steel and the endurance limit of the test specimen if unknown can be estimated using $\sigma'_e = 0.504\sigma_{uts}$. The ultimate tensile strength for 817M40 is

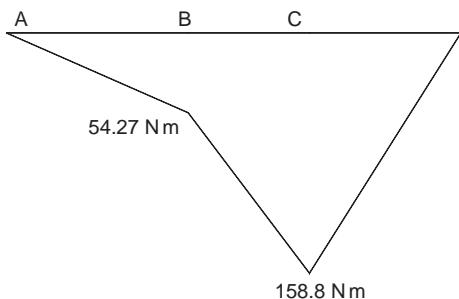


Figure 5.32 Resultant bending moment diagram.

1000 MPa and σ_y is 770 MPa. So $\sigma'_e = 0.504 \times 1000 = 504$ MPa.

The material is hot rolled, so from Eq. 5.14 and Table 5.3, $k_a = a\sigma_{uts}^b = 57.7(1000)^{-0.718} = 0.405$.

Assuming that the diameter will be about 30 mm, the size factor can be estimated using $k_b = (d/7.62)^{-0.1133} = (30/7.62)^{-0.1133} = 0.856$. If the shaft is significantly different in size to 30 mm, then the calculation should be repeated until convergence between the assumed and the final calculated value for the shaft diameter is achieved.

The desired nominal reliability is 90 per cent, so $k_c = 0.897$.

The operating temperature is not stated, so a value of temperature factor of $k_d = 1$ is assumed. The duty cycle factor is assumed to be $k_e = 1$.

The fillet radius at the shoulders is 3 mm. The ratio of diameters $D/d = (3 + 3 + 30)/30 = 36/30 = 1.2$. $r/d = 3/30 = 0.1$. So from Figure 5.23 the geometric stress concentration factor is $k_t = 1.65$. From Figure 5.22 the notch sensitivity index for a 1000 MPa strength material with notch radius of 3 mm is $q = 0.9$.

$$K_f = 1 + q(k_t - 1) = 1 + 0.9(1.65 - 1) = 1.59$$

$$k_f = 1/K_f = 0.629.$$

The miscellaneous factor is taken as $k_g = 1$.

The endurance limit can now be calculated from

$$\sigma_e = k_a k_b k_c k_d k_e k_f k_g \sigma'_e.$$

$$\sigma_e = 0.405 \times 0.856 \times 0.897 \times 1 \times 1 \times 0.629 \times 1 \times 504 = 98.6 \text{ MPa.}$$

As a well-known material has been selected subject to known loads the factor of safety can be taken as $n_s = 2$. The diameter can now be calculated from the ASME equation.

$$d = \frac{32n_s}{\pi} \sqrt{\left(\frac{M}{\sigma_e}\right)^2 + \frac{3}{4} \left(\frac{T}{\sigma_y}\right)^2}^{1/3}$$

$$= \frac{32 \times 2}{\pi} \sqrt{\frac{158.8}{98.6 \times 10^6}^2 + 0.75 \frac{84.9}{770 \times 10^6}^2}^{1/3}$$

$$= 0.032 \text{ m}$$

As this value is close to the assumed value used to evaluate the size and fatigue stress factors further iteration is not necessary. For manufacturing convenience, it may be necessary to modify this diameter to the nearest standard size as used within the company or taking into account materials readily available from suppliers, in this case 35 mm. Standard sizes can be found in texts such as the *Machinery's Handbook* (Schubert, 1982). An advantage of using standard sizes is that standard stock bearings can be selected to fit.

5.6 Conclusions

Shaft design involves consideration of the layout of features and components to be mounted on the shaft, specific dimensions and allowable tolerances, materials, deflection, frequency response, life and manufacturing constraints. This chapter has introduced the use of steps and shoulders and miscellaneous devices to locate components, methods to calculate the deflection of a shaft and its critical speeds, and a method to determine the minimum safe diameter for a shaft experiencing torque and bending for a given life.

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- www.ameridrives.com
- www.couplingcorp.com
- www.heli-cal.com
- www.magnaloy.com
- www.mayrcorp.com
- www.peterstubs.com
- www.rw-america.com
- www.servometer.com
- www.zero-max.com

Nomenclature

Generally, preferred SI units have been stated.

- a influence coefficient (m)
- d diameter (m)
- E Young's modulus (N/m^2)
- F force (N)
- g acceleration due to gravity (m^2/s)
- I second moment of area (m^4)
- k_a surface factor
- k_b size factor
- k_c reliability factor
- k_d temperature factor
- k_e duty cycle factor
- k_f fatigue stress concentration factor
- K_f component fatigue stress concentration factor

k_g	miscellaneous effects factor
k_t	geometric stress concentration factor
L	length (m)
m	mass (kg)
M	moment (N m)
n_s	factor of safety
q	notch sensitivity
R	reaction (N)
T	torque (N m), tension (N)
U	strain energy (J)
W_i	mass (kg) or weight of node i (N)
x	coordinate (m)
y	static deflection (m)
γ_i	static deflection of W_i (m)
σ_e	endurance limit of the item (N/m^2)
σ'_e	endurance limit of test specimen (N/m^2)
σ_{uts}	ultimate tensile strength (N/m^2)
σ_y	yield strength (N/m^2)
ω	frequency
ω_c	critical speed (rad/s)

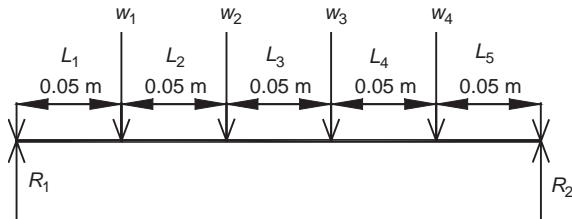


Figure 5.33 Air compressor shaft.

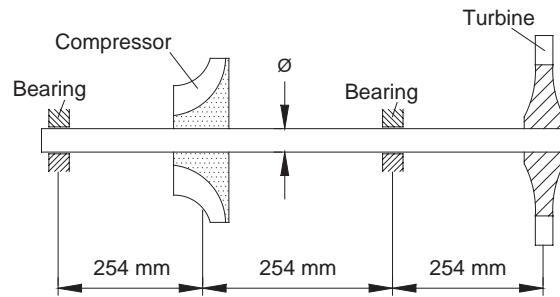


Figure 5.34 Turbine shaft.

Worksheet

5.1 An air compressor consists of four compressor discs mounted on a steel shaft 50 mm apart as shown in Figure 5.33. The shaft is simply supported at either end. Each compressor wheel mass is 18 kg. Calculate the deflection at each wheel using Macaulay's method and calculate, using the Rayleigh–Ritz equation, the first critical frequency of the shaft. The shaft outer diameter is 0.05 m, the inner diameter is 0.03 m.

5.2 A 22.63 kg compressor impeller wheel is driven by a 13.56 kg turbine mounted on a common shaft (see Figure 5.34) manufactured from steel with Young's modulus $E = 207 \text{ GN/m}^2$. The design speed is 10 000 rpm. Determine the shaft diameter so that the first critical speed is 12 000 rpm giving a safety margin of 2000 rpm. Use Rayleigh's equation and assume rigid bearings and a massless shaft.

5.3 A research turbocharger shaft is simply supported by bearings as shown in Figure 5.35. Assuming there is no deflection at the bearings and that the shaft mass is negligible, calculate the first two critical frequencies of the shaft. The compressor mass is 3 kg, the turbine mass 2.5 kg. The steel shaft diameter is 0.02 m. Take Young's modulus for steel as 200 GN/m^2 .

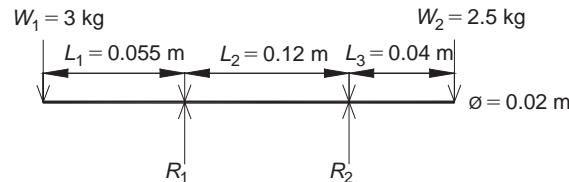


Figure 5.35 Turbocharger.

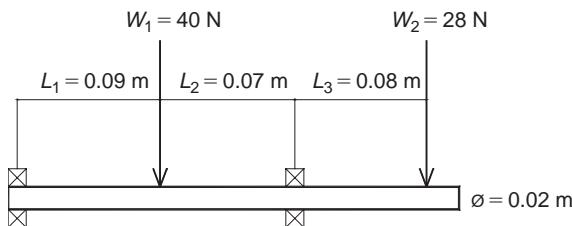


Figure 5.36 Turbocharger shaft.

5.4 As part of a preliminary design for a turbocharger determine the first two critical frequencies, stating any assumptions made, for the shaft arrangement shown in Figure 5.36. To assist with your calculation, a_{22} may be taken as $3.2594926 \times 10^{-7} \text{ m}$. Young's modulus should be taken as 200 GN/m^2 .

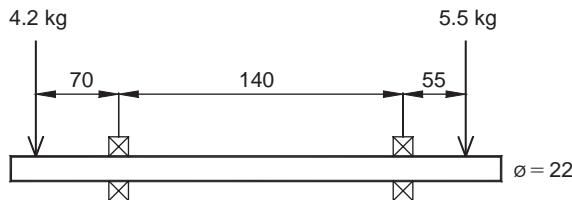


Figure 5.37 Turbocharger shaft (dimensions in mm).

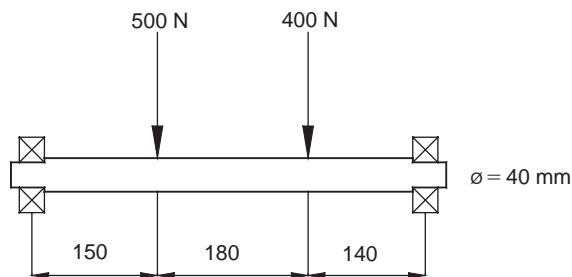


Figure 5.38 Shaft loading diagram.

- 5.5 As an initial step in the design of an experimental turbocharger, calculate the first two critical frequencies, stating any assumptions made, for the steel shaft shown in Figure 5.37. Young's modulus can be taken as 200 GN/m^2 .
- 5.6 Determine the first two critical speeds for the shaft shown in Figure 5.38 stating and discussing the practical implications for any assumptions made. Young's modulus for steel should be taken as 200 GN/m^2 .
- 5.7 Using Castiglione's theorem for calculating deflections, determine the critical frequency of the steel shaft shown in Figure 5.39. Neglect the shaft mass.
- 5.8 Determine the diameter of the drive shaft for a chain conveyor, which has the loading parameters illustrated in Figure 5.40. A roller chain sprocket of 500 mm pitch diameter, weighing 90 kg will be mid-mounted between two bearings. A 400 mm, 125 kg roller chain sprocket will be mounted overhung. The drive shaft is to be manufactured using cold drawn 070M20 steel. Operating temperatures are not expected to exceed 65°C and the operating environment is non-corrosive. The shaft is to be designed for non-limited life of greater than 10^8 cycles with a 90 per cent survival rate. The shaft will carry a steady driving torque of 1600 N m and rotate at 36 rpm. A sled runner keyway will be used for the overhung pulley and a

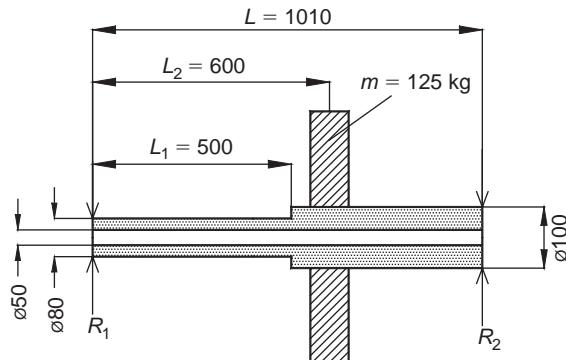


Figure 5.39 Simple stepped shaft and load (dimensions in mm).

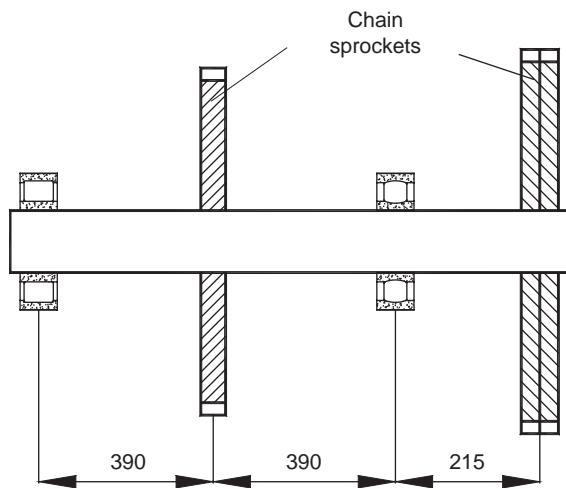


Figure 5.40 Chain conveyor drive shaft (dimensions in mm).

profile keyway for the mid-mounted pulley (example adapted from ASME B106.1 M, 1985).

- 5.9 The geometry and loading for the drive shaft of a snow-track mobile is given in Figure 5.41. Discuss and outline, with sketches as appropriate, the design decisions that should be taken in scheming out a general arrangement for the shaft (after Juvinall and Marshek, 1991).
- 5.10 A shaft is required for the chain drive illustrated in Figure 5.42. The power transmitted is 12 kW at 100 rpm. The masses of the sprockets are 5 and 16.3 kg, with respective diameters of 254.6 and 460.8 mm. The material selected for the shaft is quenched and drawn 817M40 with $\sigma_{uts} = 850 \text{ MPa}$ and $\sigma_y = 700 \text{ MPa}$. The torque is

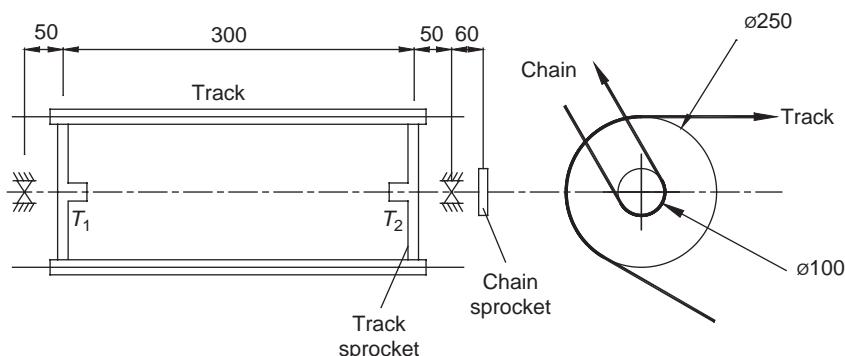


Figure 5.41 Schematic representation of the geometry and loading for a snow-track mobile drive shaft (dimensions in mm) (reproduced from Juvinall and Marshek, 1991).

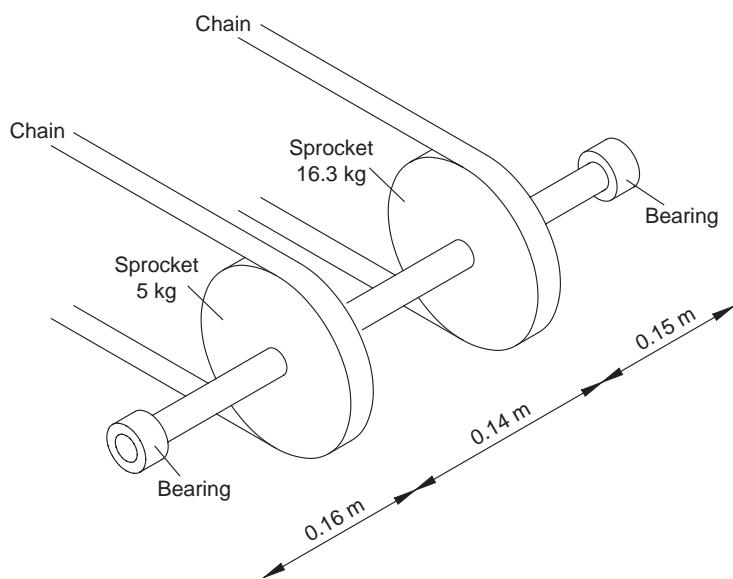


Figure 5.42 Chain drive.

transmitted between the shaft and sprockets by means of keys in profiled keyways. Determine the minimum diameter for the shaft based on the ASME equation for the design of transmission shafting if the desired reliability is 90 per cent. Use an initial estimate for the shaft diameter of 60 mm.

Answers

- 5.1 $\gamma_{w1} = 1.929 \times 10^{-6} \text{ m}$, $\gamma_{w2} = 3.100 \times 10^{-6} \text{ m}$,
 $\gamma_{w3} = 3.100 \times 10^{-6} \text{ m}$, $\gamma_{w4} = 1.929 \times 10^{-6} \text{ m}$,
 $\omega = 1923.8 \text{ rad/s} = 18371 \text{ rpm}$.

- 5.2 $d = 67 \text{ mm}$.
- 5.3 $\omega_1 = 1660 \text{ rad/s}$, $\omega_2 = 3017 \text{ rad/s}$.
- 5.4 $\omega_1 = 985.3 \text{ rad/s}$, $\omega_2 = 2949 \text{ rad/s}$.
- 5.5 $\omega_1 = 1153 \text{ rad/s}$, $\omega_2 = 1703 \text{ rad/s}$.
- 5.6 $\omega_1 = 1521.6 \text{ rad/s}$, $\omega_2 = 432.8 \text{ rad/s}$.
- 5.7 $\omega_c = 482.3 \text{ rad/s}$.
- 5.8 74 mm.
- 5.9 No unique solution.
- 5.10 58 mm.

Learning objectives achievement

Can you identify different methods for mounting and locating components on shafts?	<input type="checkbox"/>	✓ Section 5.2
Can you select an appropriate method for mounting and locating given components on a shaft?	<input type="checkbox"/>	✓ Section 5.2
Can you produce a scheme of a shaft design to locate and mount standard machine elements?	<input type="checkbox"/>	✓ Section 5.1
Can you determine the deflection of a shaft?	<input type="checkbox"/>	✓ Sections 5.4.1, 5.4.2
Can you calculate the first two critical frequencies of a shaft?	<input type="checkbox"/>	✓ Section 5.4
Can you determine the diameter for a shaft using the ASME equation for the design of transmission shafting?	<input type="checkbox"/>	✓ Section 5.5

6

GEARS

Gears are toothed cylindrical wheels used for transmitting mechanical power from one rotating shaft to another. Several types of gears are in common use. This chapter introduces various types of gears and details the design, specification and selection of spur gears in particular.

LEARNING OBJECTIVES

At the end of this section you should be:

- familiar with gear nomenclature;
- able to select a suitable gear type for different applications;
- able to determine gear train ratios;
- able to determine the bending stress for a spur gear using the Lewis formula;
- able to select appropriate gears for a compound gearbox using spur gears;
- able to select appropriate gears for an epicyclic gearbox using spur gears.

6.1 Introduction

Many power producing machines, or prime movers, such as internal combustion (IC) engines, industrial gas turbine engines and electric motors produce power in the form of rotary motion. The operating characteristics of prime movers vary according to their type and size as discussed in Chapter 3, Section 3.4, and a change of speed is often necessary to transform the torque speed characteristic of a prime mover to a useful output characteristic. When transmitting power from a source to the required point of application, a series of devices is available including gears, belts, pulleys,

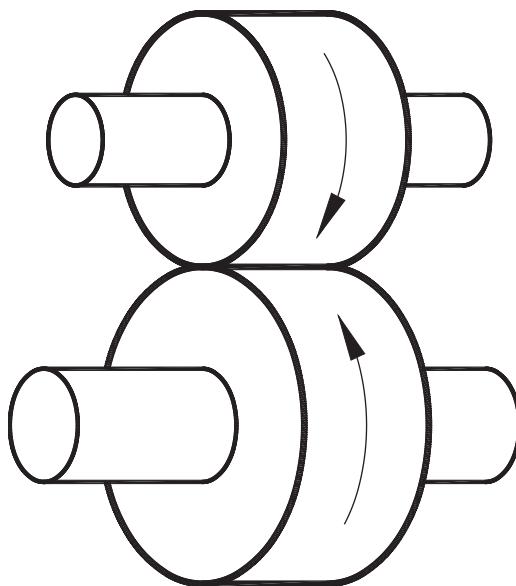


Figure 6.1 Disc or roller drives.

chains, hydraulic and electrical systems. Generally if the distances of power transmission are large, gears are not suitable and chains and belts can be considered which are introduced in Chapter 8. However when a compact, efficient or high-speed drive is required gear trains offer a competitive and suitable solution. Additional benefits of gear drives include reversibility, configuration at almost any angle between input and output and their suitability to operate in arduous conditions.

Simplistically, a speed change can be achieved by running discs of different diameter together, Figure 6.1, or alternatively cones for turning corners as well, Figure 6.2. However, the torque capacity of disc or cone drives is limited by the frictional properties of the surfaces. The addition

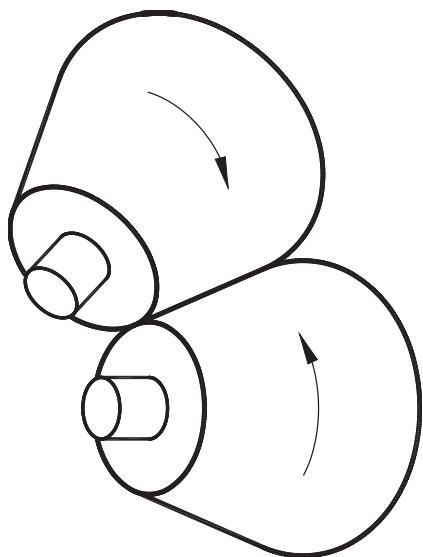


Figure 6.2 Cone drives.

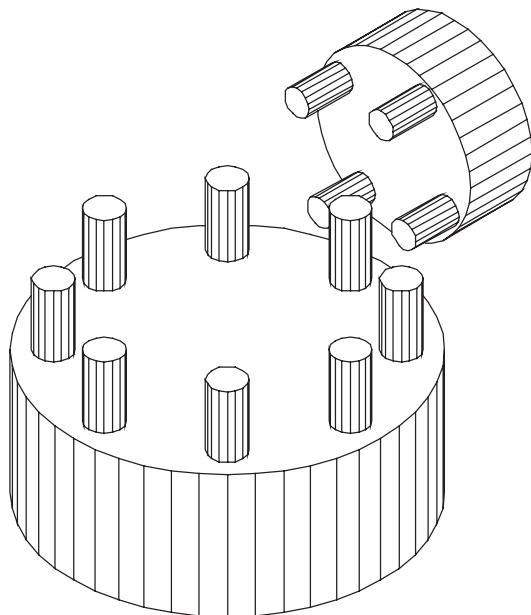


Figure 6.3 Primitive teeth gears.

of teeth to the surfaces of the discs or cones makes the drive positive, ensuring synchronization, and substantially increases the torque capacity.

Primitive gear trains consisted of teeth or pegs located on discs as illustrated in Figures 6.3 and 6.4. The disadvantage of these simple teeth is that

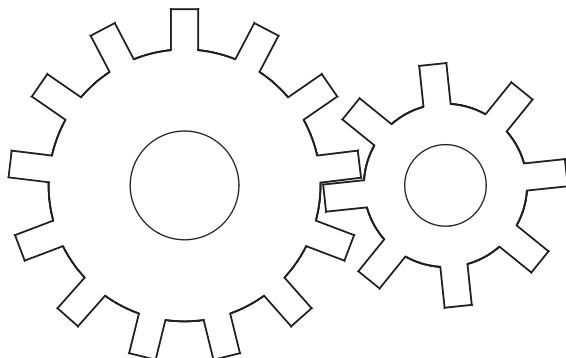


Figure 6.4 Primitive gears.

the velocity ratio is not constant and changes as the teeth go through the meshing cycle causing noise and vibration problems at elevated speeds. The solution to this problem can be achieved by using a profile on the gear teeth, which gives a constant velocity ratio throughout the meshing cycle. Several different geometrical forms can be used, but the full depth involute form is primarily used in current professional engineering practice.

Gears can be divided into several broad classifications.

1. Parallel axis gears:
 - (a) spur gears (see Figure 6.5),
 - (b) helical gears (see Figures 6.6 and 6.7),
 - (c) internal gears.
2. Non-parallel, coplanar gears (intersecting axes):
 - (a) bevel gears (see Figure 6.8),
 - (b) face gears,
 - (c) conical involute gearing.
3. Non-parallel, noncoplanar gears (nonintersecting axes):
 - (a) crossed axis helicals (see Figure 6.9),
 - (b) cylindrical worm gearing (see Figure 6.10),
 - (c) single enveloping worm gearing,
 - (d) double enveloping worm gearing,
 - (e) hypoid gears,
 - (f) spiroid and helicon gearing,
 - (g) face gears (off centre).
4. Special gear types:
 - (a) square and rectangular gears,
 - (b) elliptical gears,



Figure 6.5 Spur gears. Photograph courtesy of Hinchliffe Precision Components Ltd.



Figure 6.6 Helical gears. Photograph courtesy of Hinchliffe Precision Components Ltd.

- (c) scroll gears,
- (d) multiple sector gears.

Spur gears (Figure 6.5) are the cheapest of all types for parallel shaft applications. Their straight teeth allow running engagement or disengagement using sliding shaft and clutch mechanisms. Typical applications of spur gears include manual and automatic



Figure 6.7 Double helical gears.



Figure 6.8 Bevel gears.

motor vehicle gearboxes, machine tool drives, conveyor systems, electric motor gearboxes, timing mechanisms, and power tool drives. The majority of power gears are manufactured from hardened and case hardened steel. Other materials used



Figure 6.9 Crossed axis helical gears. Photograph courtesy of Hinchliffe Precision Components Ltd.

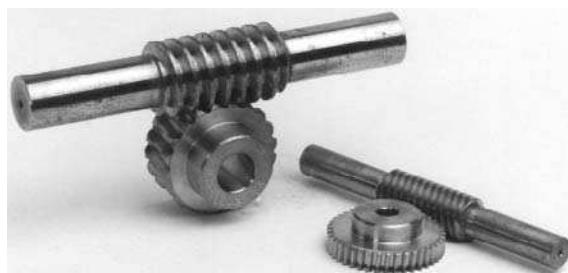


Figure 6.10 Worm gears. Photograph courtesy of Hinchliffe Precision Components Ltd.

Table 6.1 Typical material matches for gears and pinions

Gear material	Pinion material
Cast iron	Cast iron
Cast iron	Carbon steel
Carbon steel	Alloy steel
Alloy steel	Alloy steel
Alloy steel	Case hardened steel

include iron, brass and bronze, and polymers such as polyamide (e.g. nylon), polyacetal (e.g. Delrin) and SR BF (e.g. Tufnol). Typical material matches are listed in Table 6.1. A harder material may sometimes be used for a pinion, than that for the mating gear wheel, as the pinion teeth will experience more use than those on the gear wheel. Steel gears are often supplied in unhardened form to allow for grub screws, keyways or splines to be added before hardening and finishing.

A helical gear is a cylindrical gear whose tooth traces are helixes as shown in Figure 6.6. Common helix angles are 15° to 30° . Helical gears are typically used for heavy-duty high speed (>3500 rpm) power transmission, turbine drives, locomotive gearboxes and machine tool drives. Helical gears are generally more expensive than spur gears. Noise levels are lower than for spur gears because teeth in mesh make point contact rather than line contact. The forces arising from meshing of helical gears can be resolved into three components loads, radial, tangential and axial (axial loads are often called thrust loads). Bearings used to support the gear on a shaft must be able to withstand the axial or thrust force component. A solution to this problem is the use of double helical gears, as illustrated in Figure 6.7. These eliminate the need for thrust bearings because the axial force components cancel each other. Whilst spur gears are generally cheaper, a comparable helical gear will be smaller.

Bevel gears have teeth cut on conical blanks (see Figure 6.8) and a gear pair can connect non-parallel intersecting shafts. Bevel gears are used for motor transmission differential drives, valve control and mechanical instruments.

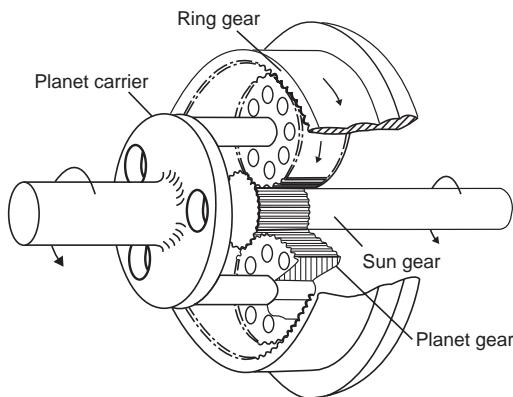


Figure 6.11 Epicyclic gears (reproduced from Townsend, 1992).

A worm gear is a cylindrical helical gear with one or more threads (see Figure 6.10). A worm wheel is a cylindrical gear with flanks cut in such a way as to ensure contact with the flanks of the worm gear. Worm gears are used for steering gear, winch blocks, low speed gearboxes, rotary tables and remote valve control. Worm gear sets are capable of high-speed reduction and high load applications, where nonparallel, noninteracting shafts are used. The 90° configuration is most common. Frictional heat generation is high in worm gears, so continuous lubrication is required and provision for heat dissipation must be made.

Some gear axes can be allowed to rotate about others. In such cases the gear trains are called planetary or epicyclic (see Figure 6.11). Planetary trains always include a sun gear, a planet carrier or arm, and one or more planet gears.

All gear mechanisms and gear trains demand continuous lubrication, which must be pressure fed for high-speed gears in order to counteract centrifugal effects on the oil. Plastic gears, made from for example nylon, can be used in certain applications and have the advantage that there is no need for lubrication, but are only suitable for low speed applications. Plastic gears can reduce noise levels significantly.

Generally the pinion of a pair of gears should have the largest number of teeth consistent with adequate resistance to failure by bending stress in

Table 6.2 Useful range of gear ratios

Gear	Ratio range	Pitch line velocity (m/s)	Efficiency
Spur	1:1 to 6:1	25	98–99%
Helical	1:1 to 10:1	50	98–99%
Double helical	1:1 to 15:1	150	98–99%
Bevel	1:1 to 4:1	20	
Worm	5:1 to 75:1	30	20–98%
Crossed helical	1:1 to 6:1	30	

the teeth. For a given diameter the larger the number of teeth the finer the pitch and consequently the weaker they are and the greater the liability to fracture. Table 6.2 lists the range of gear ratios and performance characteristics typically achievable.

When the number of teeth selected for a gear wheel is equal to the product of an integer and the number of teeth in the mating pinion, then the same tooth on the gear wheel will touch the same tooth on the pinion in each revolution of the gear wheel. If there is no common factor between the numbers of teeth on the gear wheel and pinion, then each tooth on the gear wheel will touch each tooth on the pinion in regular succession with the frequency of contact between a particular pair of teeth, being the speed of the pinion divided by the number of teeth on the gear wheel. The avoidance of an integer ratio between the number of teeth on the gear wheel and the pinion can be achieved by the addition of an extra tooth to the gear wheel, provided that there is no operational need for an exact velocity ratio. This extra tooth is called a hunting tooth and can have advantages for wear equalization (see Tuplin (1962) for a fuller discussion).

Various definitions used for describing gear geometry are illustrated in Figure 6.12 and listed below. For a pair of meshing gears, the smaller gear is called the ‘pinion’, the larger is called the ‘gear wheel’ or simply the ‘gear’.

- **Pitch circle.** This is a theoretical circle on which calculations are based. Its diameter is called the pitch diameter.

$$d = mN \quad (6.1)$$

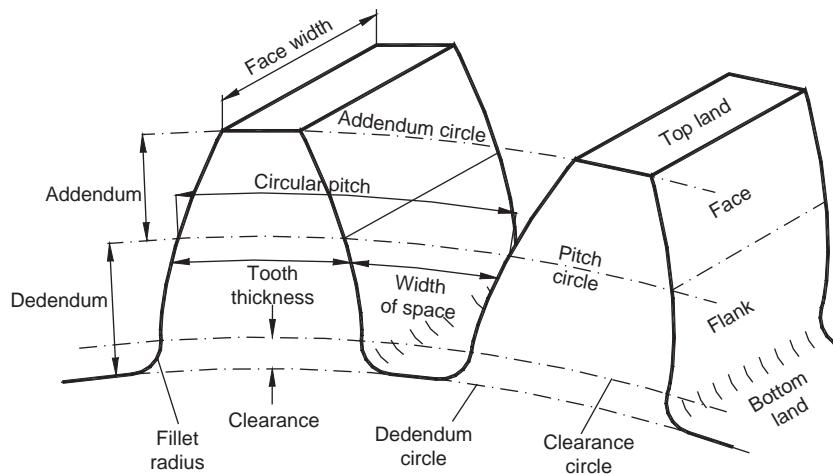


Figure 6.12 Spur gear schematic showing principle terminology.

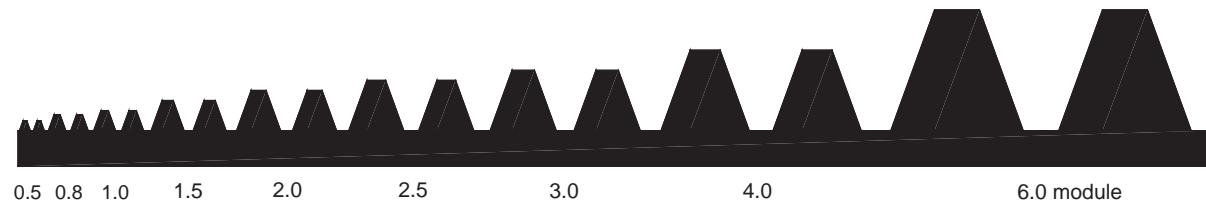


Figure 6.13 Gear tooth size as a function of the module.

where d is the pitch diameter (mm); m is the module (mm); and N is the number of teeth. Care must be taken to distinguish the module from the unit symbol for a metre.

- **Circular pitch.** This is the distance from a point on one tooth to the corresponding point on the adjacent tooth measured along the pitch circle.

$$p = \pi m = \frac{\pi d}{N} \quad (6.2)$$

where p is the circular pitch (mm); m the module; d the pitch diameter (mm); and N the number of teeth.

- **Module.** This is the ratio of the pitch diameter to the number of teeth. The unit of the module should be millimetres (mm). The module is defined by the ratio of pitch diameter and number of teeth. Typically the height

of a tooth is about 2.25 times the module. Various modules are illustrated in Figure 6.13.

$$m = \frac{d}{N} \quad (6.3)$$

- **Addendum, a.** This is the radial distance from the pitch circle to the outside of the tooth.
- **Dedendum, b.** This is the radial distance from the pitch circle to the bottom land.
- **Backlash.** The amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circle.

Prior to use of the metric module, the term ‘diametral pitch’ was commonly used. The diametral pitch is the ratio of the number of teeth in the gear to the pitch diameter, $P_d = N/d$ (usually in US/English units only, i.e. teeth per inch (tpi)). To convert from diametral pitch P_d (tpi) to the module m (in mm) use $m = 25.4/P_d$.

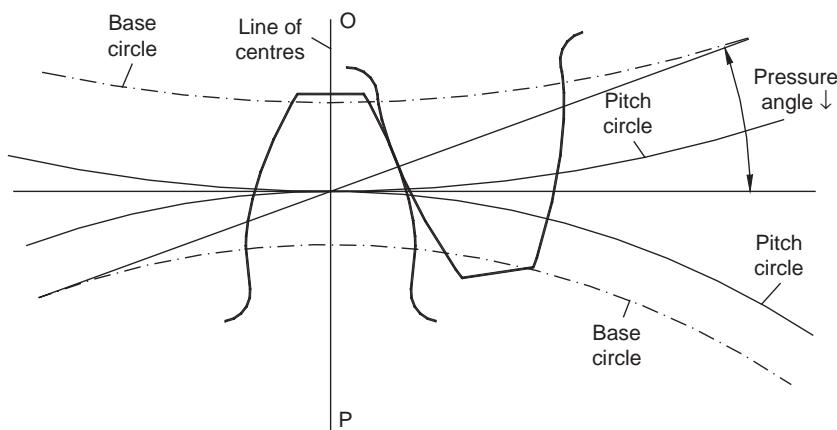


Figure 6.14 Schematic showing the pressure line and pressure angle.

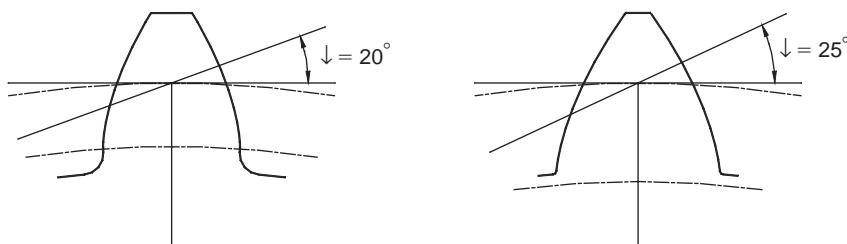


Figure 6.15 Full depth involute form for varying pressure angles.

Figure 6.14 shows the line of centres OP connecting the rotation axes of a pair of meshing gears. The angle ϕ is called the pressure angle. The pressure line (also called the generating line or line of action) is defined by the pressure angle. The resultant force vector between a pair of operating meshing gears acts along this line. The actual shape or form of the gear teeth depends on the pressure angle chosen. Figure 6.15 shows two gear teeth of differing pressure angle for a gear with identical number of teeth, pitch and pitch diameter. Current standard pressure angles are 20° and 25° , with the 20° form most widely available.

6.2 Construction of gear tooth profiles

The most widely used tooth form for spur gears is the full depth involute form as illustrated in Figure 6.16. An involute is one of a class of curves called ‘conjugate curves’. When two involute-form gear teeth are in mesh, there is a constant velocity ratio between them. From the moment

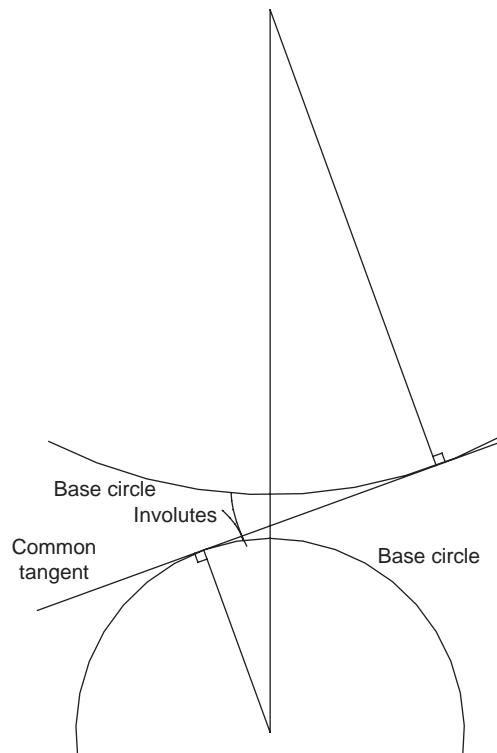


Figure 6.16 Schematic of the involute form.

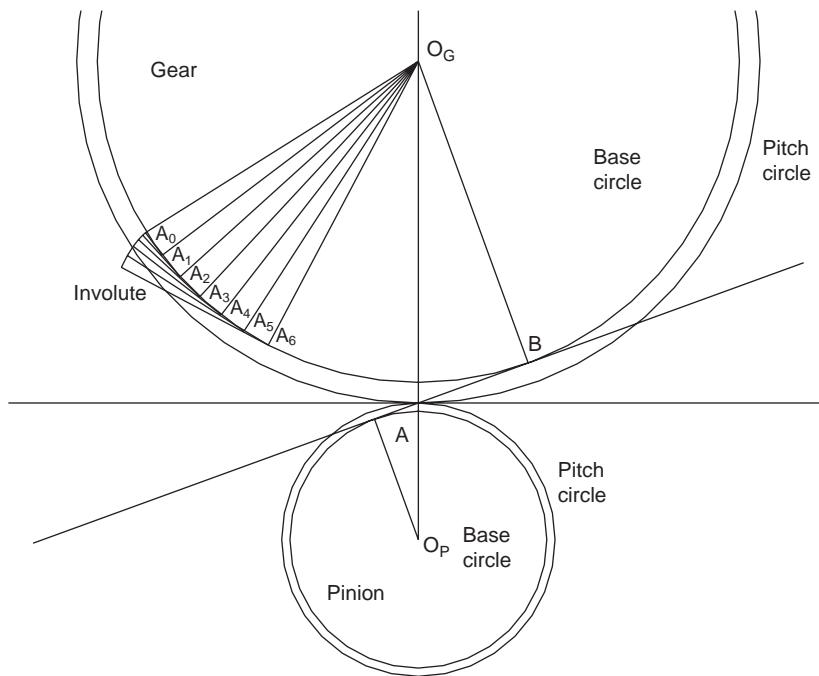


Figure 6.17 Construction of gear geometry.

of initial contact to the moment of disengagement, the speed of the gear is in constant proportion to the speed of the pinion. The resulting action of the gears is very smooth. If the velocity ratio were not constant, there would be accelerations and decelerations during the engagement and disengagement, causing vibration noise and potentially damaging torsional oscillations.

The layout and geometry for a pair of meshing spur gears can be determined by the procedure set out below. This procedure assumes access to a computer aided design (CAD) drawing package. It should be noted that gears are commonly available as standard items from specialist manufacturers and suppliers and need not necessarily be designed from scratch.

1. Calculate the pitch diameter and draw pitch circles tangential to each other (see Figure 6.17).

$$d = mN$$

$$d_p = mN_p$$

$$d_g = mN_g$$

In the example shown, the module has been selected as $m = 2.5$, the number of teeth in the pinion 20, and in the gear 50. So $d_p = 2.5 \times 20 = 50 \text{ mm}$ and $d_g = 2.5 \times 50 = 125 \text{ mm}$.

2. Draw a line perpendicular to the line of centres through the pitch point (this is the point of tangency of the pitch circles). Draw the pressure line at an angle equal to the pressure angle from the perpendicular. It is called the pressure line because the resultant tooth force is along this line during meshing. Here the pressure angle is 20° .
3. Construct perpendiculars $O_p A$ and $O_g B$ to the pressure line through the centres of each gear. The radial distances of each of these lines are the radii of the base circles of the pinion and gear, respectively. Draw the base circles.
4. Draw an involute curve on each base circle. This is illustrated on the gear. First divide the base circle in equal parts $A_0, A_1, A_2, A_3, A_4, A_5, \dots, A_n$. Construct radial lines $O_g A_0, O_g A_1, O_g A_2, O_g A_3, \dots, O_g A_n$. Construct perpendiculars to these radial lines. The involute begins at A_0 . The second point is obtained by measuring

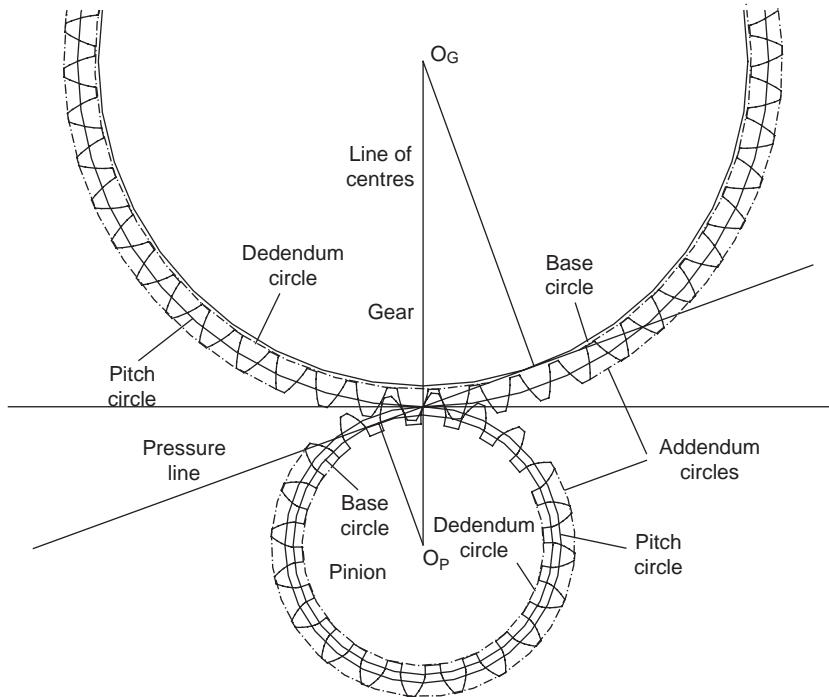


Figure 6.18 Layout of a pair of meshing spur gears.

off the distance A_0A_1 on the perpendicular through A_1 . The next point is found by measuring off twice the distance A_0A_1 on the perpendicular through A_2 and so on. The curve constructed through these points is the involute for the gear. The involute for the pinion is constructed in the same way on the base circle of the pinion.

5. Calculate the circular pitch

$$p = \pi m$$

The width of the teeth and the width of the spaces are equal to half the circular pitch. Mark these distances off on the pitch circles (here $p = \pi \times 2.5$).

6. Draw the addendum and dedendum circles for the pinion and gear (see Figure 6.18). Here a tooth system (see Section 6.4 and Table 6.3) has been selected with

$$a = m$$

$$b = 1.25 m$$

Table 6.3 Tooth dimension formulas for $\phi = 20^\circ$ and 25°

Addendum	$a = m$
Dedendum	$b = 1.25 m$
Working depth	$h_k = 2 m$
Whole depth (min)	$h_t = 2.25 m$
Tooth thickness	$t = \pi m/2$
Fillet radius of basic rack	$r_f = 0.3 m$
Clearance (min)	$c = 0.25 m$
Clearance for shaved or ground teeth	$c = 0.35 m$
Minimum number of pinion teeth $\phi = 20^\circ$	$N_p = 18$
Minimum number of pinion teeth $\phi = 25^\circ$	$N_p = 12$
Minimum number of teeth per pair $\phi = 20^\circ$	$N_p + N_G = 36$
Minimum number of teeth per pair $\phi = 25^\circ$	$N_p + N_G = 24$
Width of top land (min)	$t_o = 0.25 m$

N.B. In this table 'm' stands for module.

7. Mirror the involute profile about a line constructed using a distance half the tooth width along the pitch circle and the gear centre. Using a polar array generate all of the teeth for the gear. Construct the root fillets as appropriate. Construct the tooth top and bottom lands.

Gears can be purchased as standard items from specialist manufacturers and the exact geometry

for the blade teeth is not necessary for engineering drawings and general design purposes (unless you are designer gear cutters/forming tools). However, the tooth geometry is necessary for detailed stress/reliability and dynamic analysis.

6.3 Gear trains

A gear train is one or more pairs of gears operating together to transmit power. Figure 6.19 shows examples of a simple gear train, a reverted gear train and a non-reverted gear train.

When two gears are in mesh, their pitch circles roll on each other without slippage. If r_1 is pitch radius of gear 1; r_2 is pitch radius of gear 2; ω_1 is angular velocity of gear 1; and ω_2 is angular velocity of gear 2 then the pitch line velocity is given by

$$V = |r_1\omega_1| = |r_2\omega_2| \quad (6.4)$$

The velocity ratio is

$$\left| \frac{\omega_1}{\omega_2} \right| = \frac{r_2}{r_1} \quad (6.5)$$

and can be defined in any of the following ways

$$\frac{\omega_p}{\omega_G} = \frac{n_p}{n_G} = -\frac{N_G}{N_p} = -\frac{d_G}{d_p} \quad (6.6)$$

where ω_p and ω_G are the angular velocities of the pinion and gear, respectively (rad/s); n_p and n_G are the rotational speeds of the pinion and gear, respectively (rpm); N_p and N_G are the number of teeth in the pinion and gear, respectively; and d_p and d_G are the pitch diameter of the pinion and gear respectively (mm).

Consider a pinion 1, driving a gear 2. The speed of the driven gear is:

$$n_2 = \left| \frac{N_1}{N_2} n_1 \right| = \left| \frac{d_1}{d_2} n_1 \right| \quad (6.7)$$

where n is revolutions or rpm; N is number of teeth; and d is pitch diameter.

Equation 6.7 applies to any gear set (spur, helical, bevel or worm). For spur and parallel helical gears, the convention for direction is positive for anti-clockwise rotation, i.e.

$$n_2 = -\frac{N_1}{N_2} n_1 \quad (6.8)$$

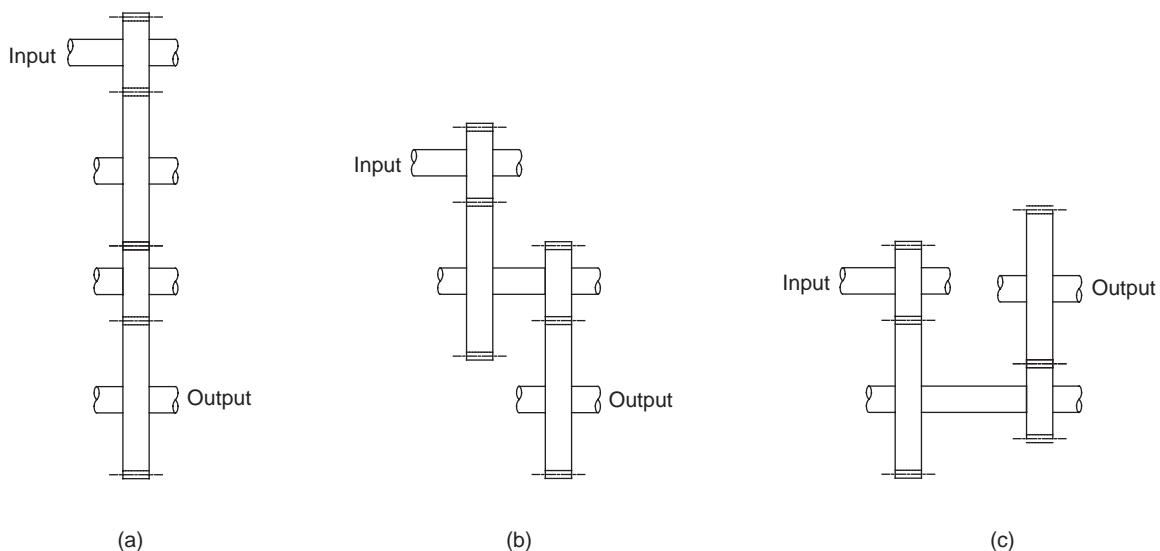


Figure 6.19 Gear trains. (a) Simple gear train. (b) Reverted compound gear train. (c) Non-reverted compound gear train.

Example 6.1

Consider the gear train shown in Figure 6.20. Calculate the speed of gear five.

Solution

$$n_2 = -\frac{N_1}{N_2} n_1$$

$$n_3 = -\frac{N_2}{N_3} n_2$$

$n_4 = n_3$ (on the same shaft)

$$n_5 = -\frac{N_4}{N_5} n_4$$

$$n_5 = -\frac{N_4 N_2 N_1}{N_5 N_3 N_2} n_1$$

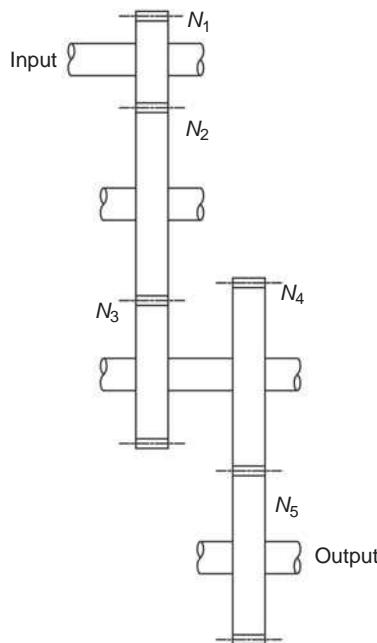
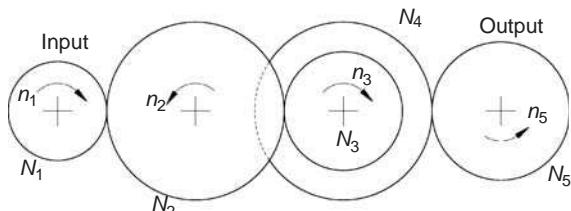


Figure 6.20 Example gear train.

Example 6.2

For the double reduction gear train shown in Figure 6.21, if the input speed is 1750 rpm in a clockwise direction what is the output speed?

Solution

$$n_2 = -\frac{N_1}{N_2} n_1$$

$n_3 = n_2$ (on the same shaft)

$$n_4 = -\frac{N_3}{N_4} n_3$$

$$n_4 = \frac{N_3}{N_4} \frac{N_1}{N_2} n_1 = \frac{-48}{54} \frac{-20}{70} (-1750)$$

$$= -166.7 \text{ rpm}$$

Example 6.3

For the double reduction gear train with an idler shown in Figure 6.22, if the input speed is 1750 rpm in a clockwise direction what is the output speed?

Solution

$$n_5 = -\frac{N_4}{N_5} n_4$$

$$n_4 = -\frac{N_3}{N_4} n_3$$

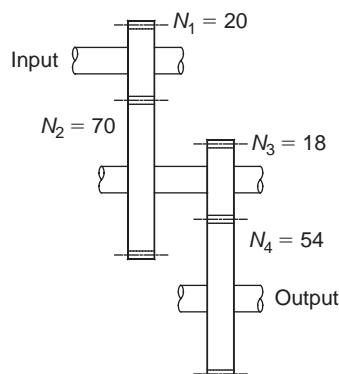


Figure 6.21 Double reduction gear train.

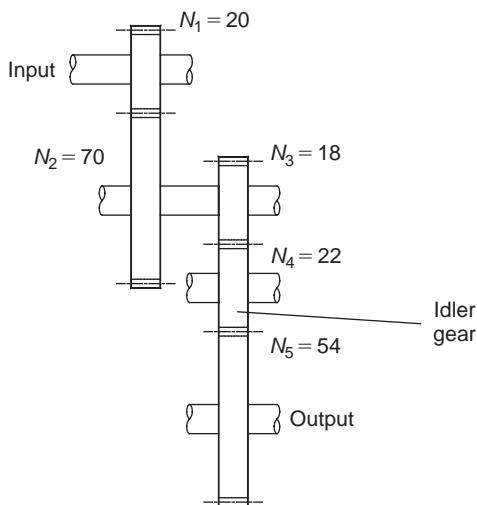


Figure 6.22 Double reduction gear with idler.

$$\begin{aligned}
 n_3 &= n_2 \\
 n_2 &= -\frac{N_1}{N_2}n_1 \\
 n_5 &= -\frac{N_4}{N_5} \frac{N_3}{N_4} \frac{N_1}{N_2} n_1 \\
 &= -\frac{22}{54} \frac{-48}{22} \frac{-20}{70} (-1750) \\
 &= 166.7 \text{ rpm}
 \end{aligned}$$

Notice that the presence of the idler gear has caused the gear train output to reverse direction, but has not altered the gear ratio in comparison to the previous example.

6.3.1 Manually shifted automotive transmissions

The torque speed characteristic of a petrol internal combustion engine delivers low torque at starting and at low speeds. This does not match the load, which, to enable a car to pull away from stationary, demands a high torque. The torque from an internal combustion engine can be increased by means of a gearing. Similarly at other load conditions such as high acceleration, hills and cruising

the torque speed characteristics of the engine may not be compatible with the load. A multiratio gearbox is the traditional solution to this problem.

A typical manually shifted gearbox for a passenger car is illustrated in Figure 6.23. This has five forward speeds and reverse. The basic elements of a manually shifted transmission are a single or multiplate clutch for engaging and dis-engaging the power from the load, a variable ratio transmission unit with permanent mesh gears and a gear shift mechanism and lever. Clutches are considered more fully in Chapter 10 and their function here is to interrupt the power flow. The demands of the automobile product mean that harshness in gear changes is unacceptable to the majority of customers. A method of overcoming shocks from gears crashing into each other as they are engaged is to hold the gears in permanent mesh, but not necessarily transmitting any power. This is illustrated for gears 2, 3 and 4 which, although in mesh in Figure 6.23a, do not transmit torque because one of each of the pairs of gears concerned is free to rotate relative to its shaft. In order to transmit torque, a spline is moved on the shaft causing the gear to rotate at the same speed as the shaft and therefore it is able to transmit power. This is the principle of the synchromesh illustrated in Figure 6.24. Depending on the gear train desired, the gear lever shift mechanism moves the synchromesh splines to engage the appropriate gears.

6.3.2 Epicyclic gear trains

Figure 6.11 illustrates an example of an epicyclic gear train. Epicyclic gear trains include a sun gear, a planet carrier and one or more planetary gears. The planetary gears may also mesh with an internal ring gear. Epicyclic gear trains have two degrees of freedom and in order to determine their motion two inputs must be defined, such as the speed of both the sun and the ring gear. For the example shown in Figure 6.11, the rotational speed of the sun gear is given by

$$n_{\text{sun}} = n_{\text{arm}} + n_{\text{sun/arm}} \quad (6.9)$$

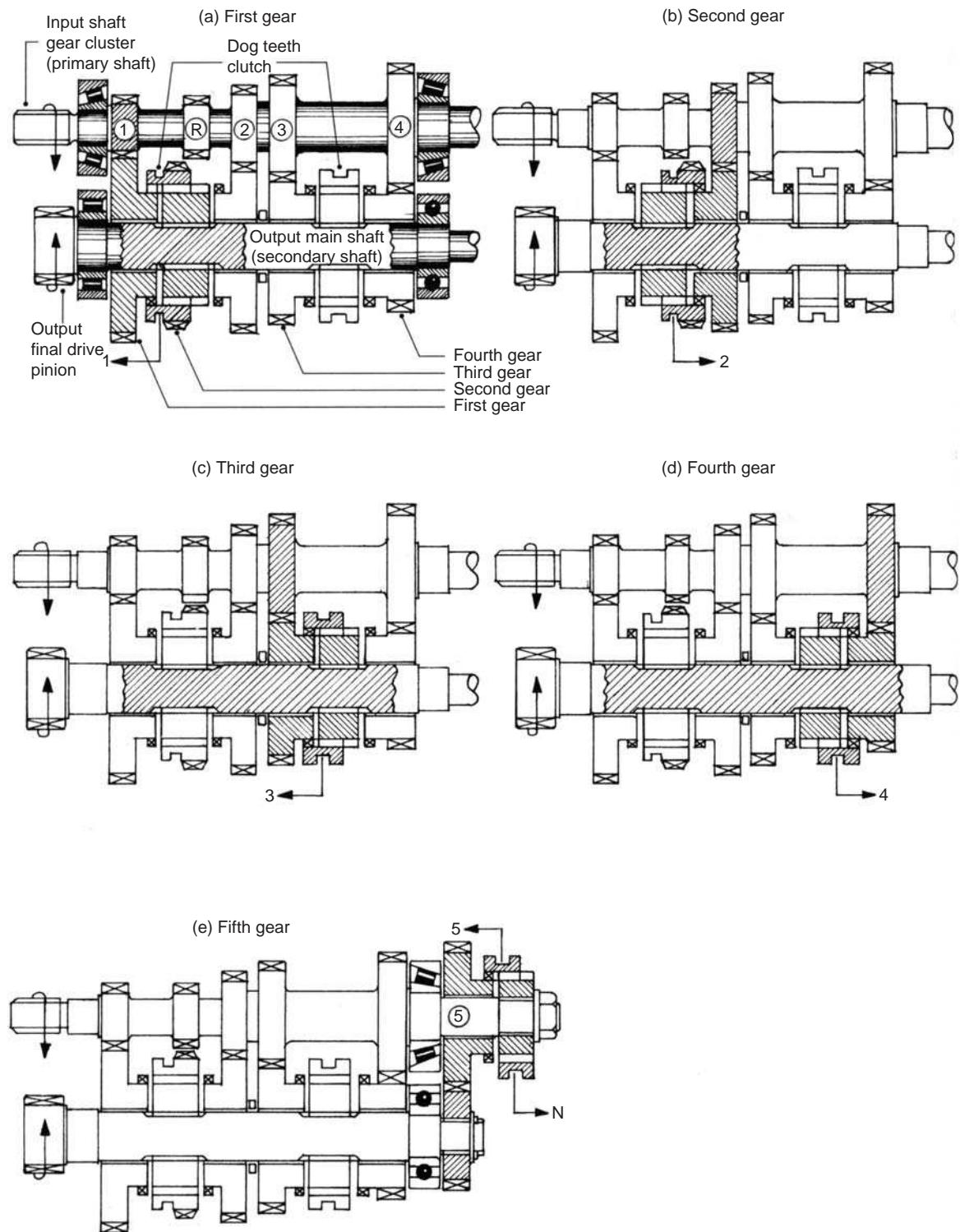
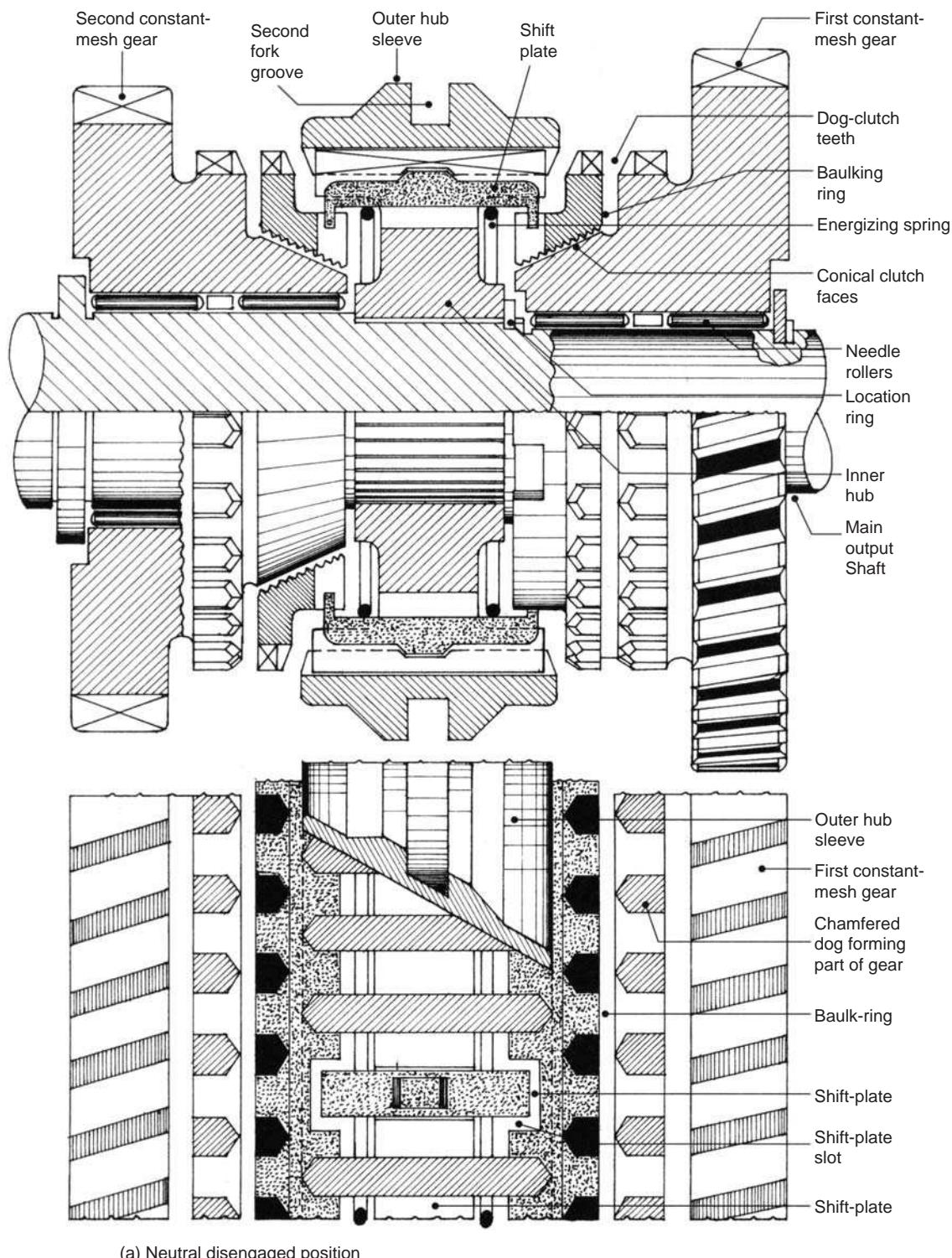
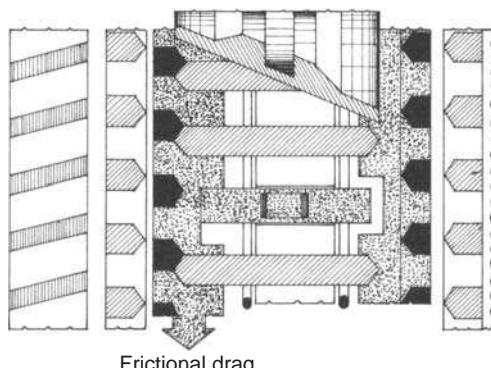
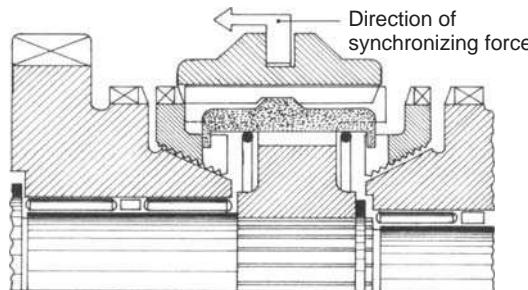


Figure 6.23 A five speed and reverse single-stage constant mesh transmission (reproduced from Heisler, 1999).

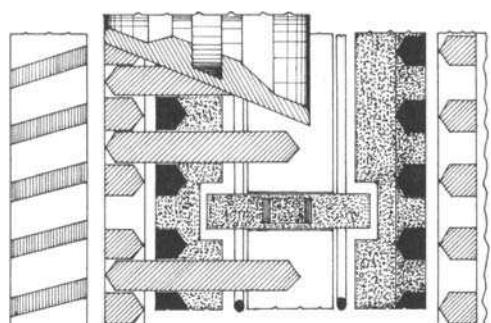
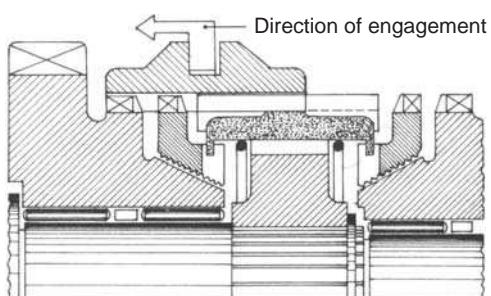


(a) Neutral disengaged position

Figure 6.24 Positive baulk-ring synchromesh (reproduced from Heisler, 1999).



b) Synchronization position



c) Engaged position

Figure 6.24 (Continued).

If n_F represents the speed of the first gear in the train and n_L the speed of the last gear in the train

$$n_{F/arm} = n_F - n_{arm} \quad (6.10)$$

$$n_{L/arm} = n_L - n_{arm} \quad (6.11)$$

Dividing Eq. 6.11 by 6.10 gives

$$\begin{aligned} \frac{n_{L/arm}}{n_{F/arm}} &= \frac{n_L - n_{arm}}{n_F - n_{arm}} = e \\ &= \pm \frac{\text{Product of driving teeth numbers}}{\text{Product of driven teeth numbers}} \\ &= \frac{N_S}{N_P} \frac{N_P}{N_R} \end{aligned} \quad (6.12)$$

where e is the train value. For spur and helical gears, the train value is positive if the last gear rotates in the same direction as the first.

Equation 6.12 can be rearranged to give the reduction ratio defined as n_{sun}/n_{arm} which for an epicyclic gearbox, with the ring fixed stationary, $n_L = 0$, $n_F = n_{sun}$, is given by

$$\frac{n_{sun}}{n_{arm}} = \frac{N_R}{N_S} + 1 \quad (6.13)$$

For calculations of stress, it may also be necessary to determine the speed of the planetary gears. Velocity difference equations can be used for this. For the gear train illustrated

$$n_{planet/arm} = n_{planet} - n_{arm} \quad (6.14)$$

$$n_{sun/arm} = n_{sun} - n_{arm} \quad (6.15)$$

From knowledge of n_{arm} , N_p and N_s these equations can be solved to determine n_p .

Note that if equi-spacing of the planetary gears is desired, the ratio given in Eq. 6.16 must be an integer. Equi-spacing of planets is desirable to produce balanced forces on the pinion and ring gear.

$$\frac{N_S + N_R}{\text{Number of planets}} = \text{integer} \quad (6.16)$$

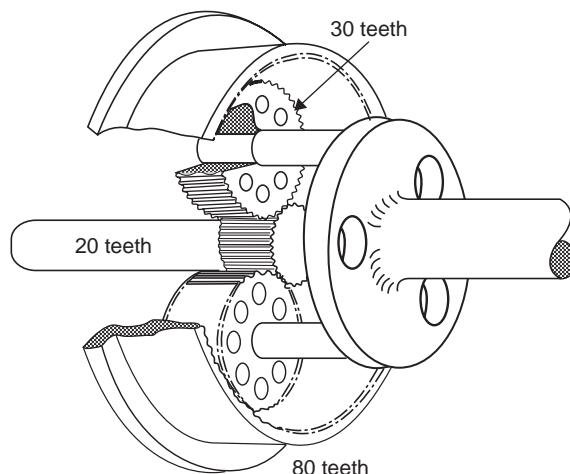


Figure 6.25 Epicyclic gear train.

Example 6.4

For the epicyclic gearbox illustrated in Figure 6.25, determine the speed and direction of the final drive and also the speed and direction of the planetary gears. The teeth numbers of the sun, planets and ring gear are 20, 30 and 80, respectively. The speed and direction of the sun gear is 1000 rpm clockwise and the ring gear is held stationary.

Solution

From Eq. 6.13,

$$n_{\text{arm}} = \frac{n_{\text{sun}}}{(80/20) + 1} = \frac{-1000}{5} = -200 \text{ rpm}$$

The speed of the final drive is 200 rpm clockwise. The reduction ratio for the gearbox is given by $n_{\text{sun}}/n_{\text{arm}} = 1000/200 = 5$. To determine the speed of the planets use Eqs 6.14 and 6.15. The planets and sun are in mesh, so

$$\frac{n_{\text{planet}}/n_{\text{arm}}}{n_{\text{sun}}/n_{\text{arm}}} = -\frac{N_S}{N_P}$$

$$\frac{n_{\text{planet}} - n_{\text{arm}}}{n_{\text{sun}} - n_{\text{arm}}} = -\frac{N_S}{N_P}$$

Table 6.4 Preferred values for the module m

m	0.5	0.8	1	1.25	1.5	2	2.5	3	4	5
6	8	10	12	16	20	25	32	40	50	

Table 6.5 Preferred standard gear teeth numbers

12	13	14	15	16	18	20	22	24	25	28	30
32	34	38	40	45	50	54	60	64	70	72	75
80	84	90	96	100	120	140	150	180	200	220	250

$$\frac{n_{\text{planet}} - (-200)}{-1000 - (-200)} = -\frac{20}{30}$$

$$n_{\text{planet}} = -\frac{20}{30} \times (-800) - 200 = 333 \text{ rpm}$$

The speed of rotation of the planetary gears is 333 rpm counter-clockwise.

6.4 Tooth systems

Tooth systems are standards that define the geometric proportions of gear teeth. Table 6.3 lists the basic tooth dimensions for full depth teeth with pressure angles of 20° and 25° .

Table 6.4 lists preferred values for the module, m , which are used to minimize gear cutting tool requirements, and Table 6.5 lists the preferred standard gear teeth numbers.

The failure of gears can principally be attributed to tooth breakage, surface failure and scuffing and scoring of the teeth surfaces. Section 6.5 outlines the forces involved on spur gear teeth and Section 6.6 defines methods for estimating gear stresses and the preliminary selection of standard stock gears.

6.5 Force analysis

Figure 6.26 shows the forces involved for two spur gears in mesh. The force acting at the pressure angle ϕ can be subdivided into two components: the

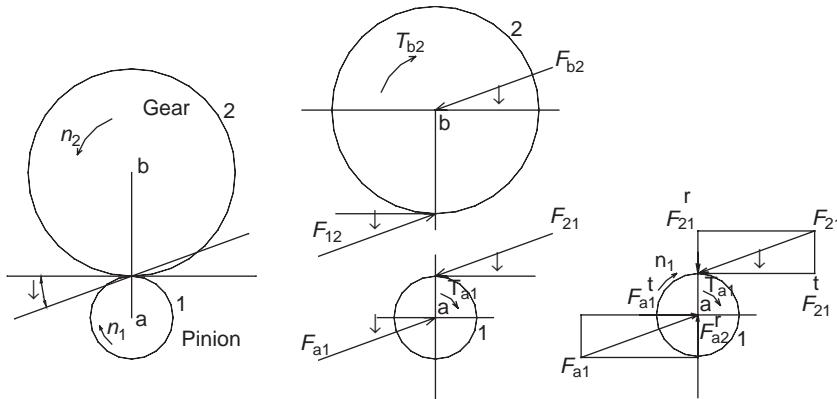


Figure 6.26 Forces for two spur gears in mesh.

tangential component F_t and the radial component F_r . The radial component serves no useful purpose. The tangential component F_t transmits the load from one gear to the other. If W_t is defined as the transmitted load, $W_t = F_t$. The transmitted load is related to the power transmitted through the gears by the equation:

$$W_t = \frac{P}{V} \quad (6.17)$$

where W_t is transmitted load (N); P is power (W); and V is pitch line velocity (m/s).

Alternatively the transmitted load can be defined by

$$W_t = \frac{60 \times 10^3 H}{\pi d n} \quad (6.18)$$

where W_t is transmitted load (kN); H is power (kW); d is pitch diameter (mm); and n is speed (rpm).

6.5.1 Introduction to gear stresses

Gears experience two principle types of stress, bending stress at the root of the teeth due to the transmitted load and contact stresses on the flank of the teeth due to repeated impact, or sustained contact, of one tooth surface against another. A simple method of calculating bending stresses is presented in Section 6.5.2 and this is utilized

within a gear selection procedure given in Section 6.6. However, this methodology, whilst suitable for determining the geometry for a gear train at the conceptual phase is inadequate for designing a gear train for highly stressed applications or where the life of the components is important. For the determination of life-critical parts, national and international standards for gears are available such as the AGMA, BS and ISO standards for gears. These are considered in Chapter 7 and the AGMA equations recommended for bending and contact stress for spur gears are introduced.

6.5.2 Bending stresses

The calculation of bending stress in gear teeth can be based on the Lewis formula

$$\sigma = \frac{W_t}{F m Y} \quad (6.19)$$

where W_t is transmitted load (N); F is face width (m or mm); m is module (m or mm); and Y is the Lewis form factor and can be found from Table 6.6.

When teeth mesh, the load is delivered to the teeth with some degree of impact. The velocity factor is used to account for this and is given, in the case of cut or milled profile gears, by the Barth equation, 6.20.

$$K_v = \frac{6.1}{6.1 + V} \quad (6.20)$$

Table 6.6 Values for the Lewis form factor Y defined for two different tooth standards (Mitchener and Mabie, 1982)

N , number of teeth	$Y(\phi = 20^\circ; a = 0.8\text{ m}; b = \text{m})$	$Y(\phi = 20^\circ; a = \text{m}; b = 1.25\text{ m})$
12	0.33512	0.22960
13	0.34827	0.24317
14	0.35985	0.25530
15	0.37013	0.26622
16	0.37931	0.27610
17	0.38757	0.28508
18	0.39502	0.29327
19	0.40179	0.30078
20	0.40797	0.30769
21	0.41363	0.31406
22	0.41883	0.31997
24	0.42806	0.33056
26	0.43601	0.33979
28	0.44294	0.34790
30	0.44902	0.35510
34	0.45920	0.36731
38	0.46740	0.37727
45	0.47846	0.39093
50	0.48458	0.39860
60	0.49391	0.41047
75	0.50345	0.42283
100	0.51321	0.43574
150	0.52321	0.44930
300	0.53348	0.46364
Rack	0.54406	0.47897

a , addendum; b , dedendum; ϕ , pressure angle; m , module.

where V is the pitch line velocity which is given by

$$V = \frac{d}{2} \times 10^{-3} n \frac{2\pi}{60} \quad (6.21)$$

where d is in mm and n in rpm.

Introducing the velocity factor into the Lewis equation gives:

$$\sigma = \frac{W_t}{K_v F_m Y} \quad (6.22)$$

Equation 6.22 forms the basis of a simple approach to the calculation of bending stresses in gears.

Example 6.5

A 20° full depth spur pinion is to transmit 1.25 kW at 850 rpm. The pinion has 18 teeth.

Determine the Lewis bending stress if the module is 2 and the face width is 25 mm.

Solution

Calculating the pinion pitch diameter

$$d_p = mN_p = 2 \times 18 = 36 \text{ mm}$$

Calculating the pitch line velocity

$$V = \frac{d_p}{2} \times 10^{-3} \times n \frac{2\pi}{60} \\ = \frac{0.036}{2} \times 850 \times 0.1047 = 1.602 \text{ m/s}$$

Calculating the velocity factor

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 1.602} = 0.7920$$

Calculating the transmitted load

$$W_t = \frac{\text{Power}}{V} = \frac{1250}{1.602} = 780.2 \text{ N}$$

From Table 6.6, for $N_p = 18$, the Lewis form factor $Y = 0.29327$.

The Lewis Equation for bending stress gives:

$$\sigma = \frac{W_t}{K_v F_m Y} \\ = \frac{780.2}{0.792 \times 0.025 \times 0.002 \times 0.29327} \\ = 67.18 \times 10^6 \text{ N/m}^2 = 67.18 \text{ MPa.}$$

6.6 Simple gear selection procedure

The Lewis formula in the form given by Eq. 6.22 ($\sigma = W_t/(K_v F_m Y)$) can be used in a provisional spur gear selection procedure for a given transmission power, input and output speeds. The procedure is outlined below.

1. Select the number of teeth for the pinion and the gear to give the required gear ratio (observe the guidelines presented in Table 6.2 for

Table 6.7 Spur gears: 1.0 module, heavy duty steel 817M40, 655M13, face width 15 mm

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)	Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG1-9	9	10.00	12.00	12	6	SG1-46	46	46.00	48.00	30	8
SG1-10	10	11.00	13.00	13	6	SG1-47	47	47.00	49.00	30	8
SG1-11	11	12.00	14.00	14	6	SG1-48	48	48.00	50.00	30	8
SG1-12	12	12.00	14.00	14	6	SG1-49	49	49.00	51.00	30	8
SG1-13	13	13.00	15.00	15	6	SG1-50	50	50.00	52.00	35	10
SG1-14	14	14.00	16.00	16	6	SG1-51	51	51.00	53.00	35	10
SG1-15	15	15.00	17.00	17	6	SG1-52	52	52.00	54.00	35	10
SG1-16	16	16.00	18.00	18	8	SG1-53	53	53.00	55.00	35	10
SG1-17	17	17.00	19.00	18	8	SG1-54	54	54.00	56.00	35	10
SG1-18	18	18.00	20.00	18	8	SG1-55	55	55.00	57.00	35	10
SG1-19	19	19.00	21.00	18	8	SG1-56	56	56.00	58.00	35	10
SG1-20	20	20.00	22.00	20	8	SG1-57	57	57.00	59.00	35	10
SG1-21	21	21.00	23.00	20	8	SG1-58	58	58.00	60.00	35	10
SG1-22	22	22.00	24.00	20	8	SG1-59	59	59.00	61.00	35	10
SG1-23	23	23.00	25.00	20	8	SG1-60	60	60.00	62.00	35	10
SG1-24	24	24.00	26.00	20	8	SG1-61	61	61.00	63.00	35	10
SG1-25	25	25.00	27.00	20	8	SG1-62	62	62.00	64.00	35	10
SG1-26	26	26.00	28.00	25	8	SG1-63	63	63.00	65.00	35	10
SG1-27	27	27.00	29.00	25	8	SG1-64	64	64.00	66.00	35	10
SG1-28	28	28.00	30.00	25	8	SG1-65	65	65.00	67.00	35	10
SG1-29	29	29.00	31.00	25	8	SG1-66	66	66.00	68.00	35	10
SG1-30	30	30.00	32.00	25	8	SG1-68	68	68.00	70.00	35	10
SG1-31	31	31.00	33.00	25	8	SG1-70	70	70.00	72.00	35	10
SG1-32	32	32.00	34.00	25	8	SG1-72	72	72.00	74.00	35	10
SG1-33	33	33.00	35.00	25	8	SG1-74	74	74.00	76.00	45	10
SG1-34	34	34.00	36.00	25	8	SG1-76	76	76.00	78.00	45	10
SG1-35	35	35.00	37.00	25	8	SG1-78	78	78.00	80.00	45	10
SG1-36	36	36.00	38.00	30	8	SG1-80	80	80.00	82.00	45	10
SG1-37	37	37.00	39.00	30	8	SG1-84	84	84.00	86.00	45	10
SG1-38	38	38.00	40.00	30	8	SG1-88	88	88.00	90.00	45	10
SG1-39	39	39.00	41.00	30	8	SG1-90	90	90.00	92.00	45	10
SG1-40	40	40.00	42.00	30	8	SG1-96	96	96.00	98.00	45	10
SG1-41	41	41.00	43.00	30	8	SG1-100	100	100.00	102.00	45	10
SG1-42	42	42.00	44.00	30	8	SG1-112	112	112.00	114.00	45	10
SG1-43	43	43.00	45.00	30	8	SG1-120	120	120.00	122.00	45	10
SG1-44	44	44.00	46.00	30	8	SG1-130	130	130.00	132.00	45	10
SG1-45	45	45.00	47.00	30	8	SG1-150	150	150.00	152.00	45	10

- maximum gear ratios). Note that the minimum number of teeth permissible when using a pressure angle of 20° is 18 (Table 6.3). Use either the standard teeth numbers as listed in Table 6.5 or as listed in a stock gear catalogue.
- Select a material. This will be limited to those listed in the stock gear catalogues.
 - Select a module, m from Table 6.4 or as listed in a stock gear catalogue (see Tables 6.7 to 6.10 which give examples of a selection of stock gears available).

- Calculate the pitch diameter, $d = mN$.
- Calculate the pitch line velocity, $V = (d/2) \times n \times (2\pi/60)$. Ensure this does not exceed the guidelines given in Table 6.2.
- Calculate the dynamic factor, $K_v = 6/(6 + V)$.
- Calculate the transmitted load, $W_t = \text{Power}/V$.
- Calculate an acceptable face width using the Lewis formula in the form,

$$F = \frac{W_t}{K_v m Y \sigma_p} \quad (6.23)$$

Table 6.8 Spur gears: 1.5 module, heavy duty steel 817M40, 655M13, face width 20 mm

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)	Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG1.5-9	9	15.00	18.00	18	8	SG1.5-47	47	70.50	73.50	50	15
SG1.5-10	10	16.50	19.50	19.5	8	SG1.5-48	48	72.00	75.00	50	15
SG1.5-11	11	18.00	21.00	21	8	SG1.5-49	49	73.50	76.50	50	15
SG1.5-12	12	18.00	21.00	21	8	SG1.5-50	50	75.00	78.00	50	15
SG1.5-13	13	19.50	22.50	20	8	SG1.5-51	51	76.50	79.50	50	15
SG1.5-14	14	21.00	24.00	20	8	SG1.5-52	52	78.00	81.00	50	15
SG1.5-15	15	22.50	25.50	20	8	SG1.5-53	53	79.50	82.50	60	15
SG1.5-16	16	24.00	27.00	25	10	SG1.5-54	54	81.00	84.00	60	15
SG1.5-17	17	25.50	28.50	25	10	SG1.5-55	55	82.50	85.50	60	15
SG1.5-18	18	27.00	30.00	25	10	SG1.5-56	56	84.00	87.00	60	15
SG1.5-19	19	28.50	31.50	25	10	SG1.5-57	57	85.50	88.50	60	15
SG1.5-20	20	30.00	33.00	25	10	SG1.5-58	58	87.00	90.00	60	15
SG1.5-21	21	31.50	34.50	25	10	SG1.5-59	59	88.50	91.50	60	15
SG1.5-22	22	33.00	36.00	30	10	SG1.5-60	60	90.00	93.00	60	15
SG1.5-23	23	34.50	37.50	30	10	SG1.5-62	62	93.00	96.00	60	15
SG1.5-24	24	36.00	39.00	30	10	SG1.5-64	64	96.00	99.00	60	15
SG1.5-25	25	37.50	40.50	30	10	SG1.5-65	65	97.50	100.50	60	15
SG1.5-26	26	39.00	42.00	30	10	SG1.5-66	66	99.00	102.00	60	15
SG1.5-27	27	40.50	43.50	30	10	SG1.5-68	68	102.00	105.00	60	15
SG1.5-28	28	42.00	45.00	30	10	SG1.5-70	70	105.00	108.00	60	15
SG1.5-29	29	43.50	46.50	30	10	SG1.5-71	71	106.50	109.50	60	15
SG1.5-30	30	45.00	48.00	30	10	SG1.5-72	72	108.00	111.00	60	15
SG1.5-31	31	46.50	49.50	30	10	SG1.5-73	73	109.50	112.50	60	15
SG1.5-32	32	48.00	51.00	30	10	SG1.5-74	74	111.00	114.00	60	15
SG1.5-33	33	49.50	52.50	30	10	SG1.5-75	75	112.50	115.50	60	15
SG1.5-34	34	51.00	54.00	30	10	SG1.5-76	76	114.00	117.00	60	15
SG1.5-35	35	52.50	55.50	50	15	SG1.5-78	78	117.00	120.00	75	15
SG1.5-36	36	54.00	57.00	50	15	SG1.5-80	80	120.00	123.00	75	15
SG1.5-37	37	55.50	58.50	50	15	SG1.5-86	86	129.00	132.00	75	15
SG1.5-38	38	57.00	60.00	50	15	SG1.5-90	90	135.00	138.00	75	15
SG1.5-39	39	58.50	61.50	50	15	SG1.5-96	96	144.00	147.00	75	15
SG1.5-40	40	60.00	63.00	50	15	SG1.5-98	98	147.00	150.00	75	15
SG1.5-41	41	61.50	64.50	50	15	SG1.5-100	100	150.00	153.00	75	15
SG1.5-42	42	63.00	66.00	50	15	SG1.5-105	105	157.50	160.50	75	15
SG1.5-43	43	64.50	67.50	50	15	SG1.5-110	110	165.00	168.00	75	15
SG1.5-44	44	66.00	69.00	50	15	SG1.5-115	115	175.50	178.50	75	15
SG1.5-45	45	67.50	70.50	50	15	SG1.5-120	120	180.00	183.00	75	15
SG1.5-46	46	69.00	72.00	50	15						

The Lewis form factor, Y , can be obtained from Table 6.6.

The permissible bending stress, σ_p , can be taken as $\sigma_{uts}/\text{factor of safety}$, where the factor of safety is set by experience, but may range from 2 to 5. Alternatively use values of σ_p as listed for the appropriate material in a stock gear catalogue. Certain plastics are suitable for use as gear materials in application where low weight, low friction,

high corrosion resistance, low wear and quiet operation are beneficial. The strength of plastic is usually significantly lower than that of metals. Plastics are often formed using a filler to improve strength, wear, impact resistance, temperature performance, as well as other properties and it is therefore difficult to regulate or standardize properties for plastics and these need to be obtained instead from the manufacturer or a

Table 6.9 Spur gears: 2.0 module, heavy duty steel 817M40, 655M13, face width 25 mm

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)	Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG2-9	9	20.00	24.00	24	12	SG2-47	47	94.00	98.00	60	20
SG2-10	10	22.00	26.00	26	12	SG2-48	48	96.00	100.00	60	20
SG2-11	11	24.00	28.00	28	12	SG2-49	49	98.00	102.00	60	20
SG2-12	12	24.00	28.00	28	12	SG2-50	50	100.00	104.00	60	20
SG2-13	13	26.00	30.00	30	12	SG2-51	51	102.00	106.00	60	20
SG2-14	14	28.00	32.00	30	12	SG2-52	52	104.00	108.00	60	20
SG2-15	15	30.00	34.00	30	12	SG2-53	53	106.00	110.00	60	20
SG2-16	16	32.00	36.00	30	12	SG2-54	54	108.00	112.00	60	20
SG2-17	17	34.00	38.00	35	15	SG2-55	55	110.00	114.00	60	20
SG2-18	18	36.00	40.00	35	15	SG2-56	56	112.00	116.00	60	20
SG2-19	19	38.00	42.00	35	15	SG2-57	57	114.00	118.00	60	20
SG2-20	20	40.00	44.00	35	15	SG2-58	58	116.00	120.00	60	20
SG2-21	21	42.00	46.00	35	15	SG2-59	59	118.00	122.00	60	20
SG2-22	22	44.00	48.00	35	15	SG2-60	60	120.00	124.00	60	20
SG2-23	23	46.00	50.00	35	15	SG2-62	62	124.00	128.00	75	20
SG2-24	24	48.00	52.00	35	15	SG2-64	64	128.00	132.00	75	20
SG2-25	25	50.00	54.00	35	15	SG2-65	65	130.00	134.00	75	20
SG2-26	26	52.00	56.00	35	15	SG2-66	66	132.00	136.00	75	20
SG2-27	27	54.00	58.00	50	20	SG2-68	68	136.00	140.00	75	20
SG2-28	28	56.00	60.00	50	20	SG2-70	70	140.00	144.00	75	20
SG2-29	29	58.00	62.00	50	20	SG2-71	71	142.00	146.00	75	20
SG2-30	30	60.00	64.00	50	20	SG2-72	72	144.00	148.00	75	20
SG2-31	31	62.00	66.00	50	20	SG2-73	73	146.00	150.00	75	20
SG2-32	32	64.00	68.00	50	20	SG2-74	74	148.00	152.00	75	20
SG2-33	33	66.00	70.00	50	20	SG2-75	75	150.00	154.00	75	20
SG2-34	34	68.00	72.00	50	20	SG2-76	76	152.00	156.00	75	20
SG2-35	35	70.00	74.00	50	20	SG2-78	78	156.00	160.00	100	20
SG2-36	36	72.00	76.00	50	20	SG2-80	80	160.00	164.00	100	20
SG2-37	37	74.00	78.00	50	20	SG2-86	86	172.00	176.00	100	20
SG2-38	38	76.00	80.00	50	20	SG2-90	90	180.00	184.00	100	20
SG2-39	39	78.00	82.00	50	20	SG2-96	96	192.00	196.00	100	20
SG2-40	40	80.00	84.00	50	20	SG2-98	98	196.00	200.00	100	20
SG2-41	41	82.00	86.00	50	20	SG2-100	100	200.00	204.00	100	20
SG2-42	42	84.00	88.00	50	20	SG2-105	105	210.00	214.00	100	20
SG2-43	43	86.00	90.00	60	20	SG2-110	110	220.00	224.00	100	20
SG2-44	44	88.00	92.00	60	20	SG2-115	115	230.00	234.00	100	20
SG2-45	45	90.00	94.00	60	20	SG2-120	120	240.00	244.00	100	20
SG2-46	46	92.00	96.00	60	20						

traceable testing laboratory. Plastics used for gears include acrylonitrile-butadiene-styrene (ABS), acetal, nylon, polycarbonate, polyester, polyurethane and styrene-acrylonitrile (SAN). Values of permissible bending stress for a few gear materials are listed in Table 6.11.

The design procedure consists of proposing teeth numbers for the gear and pinion, selecting a suitable material, selecting a module, calculating

the various parameters as listed resulting in a value for the face width. If the face width is greater than those available in the stock gear catalogue or if the pitch line velocity is too high, repeat the process for a different module. If this does not provide a sensible solution try a different material, etc. The process can be optimized taking cost and other performance criteria into account if necessary and can be programmed.

Table 6.10 Spur gears: 3.0 module, heavy duty steel 817M40, 655M13, face width 35 mm

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)	Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SG3-9	9	30.00	36.00	36	15	SG3-45	45	135.00	141.00	100	25
SG3-10	10	33.00	39.00	39	15	SG3-46	46	138.00	144.00	100	25
SG3-11	11	36.00	42.00	42	15	SG3-47	47	141.00	147.00	100	25
SG3-12	12	36.00	42.00	42	20	SG3-48	48	144.00	150.00	100	25
SG3-13	13	39.00	45.00	45	20	SG3-49	49	147.00	153.00	100	25
SG3-14	14	42.00	48.00	45	20	SG3-50	50	150.00	156.00	100	25
SG3-15	15	45.00	51.00	45	20	SG3-51	51	153.00	159.00	100	25
SG3-16	16	48.00	54.00	45	20	SG3-52	52	156.00	162.00	100	25
SG3-17	17	51.00	57.00	45	20	SG3-53	53	159.00	165.00	100	25
SG3-18	18	54.00	60.00	45	20	SG3-54	54	162.00	168.00	100	25
SG3-19	19	57.00	63.00	45	20	SG3-55	55	165.00	171.00	100	25
SG3-20	20	60.00	66.00	60	25	SG3-56	56	168.00	174.00	100	25
SG3-21	21	63.00	69.00	60	25	SG3-57	57	171.00	177.00	127	25
SG3-22	22	66.00	72.00	60	25	SG3-58	58	174.00	180.00	127	25
SG3-23	23	69.00	75.00	60	25	SG3-59	59	177.00	183.00	127	25
SG3-24	24	72.00	78.00	60	25	SG3-60	60	180.00	186.00	127	25
SG3-25	25	75.00	81.00	60	25	SG3-62	62	186.00	192.00	127	25
SG3-26	26	78.00	84.00	60	25	SG3-63	63	189.00	195.00	127	25
SG3-27	27	81.00	87.00	60	25	SG3-64	64	192.00	198.00	127	25
SG3-28	28	84.00	90.00	60	25	SG3-65	65	195.00	201.00	127	25
SG3-29	29	87.00	93.00	60	25	SG3-68	68	204.00	210.00	127	25
SG3-30	30	90.00	96.00	60	25	SG3-70	70	210.00	216.00	150	30
SG3-31	31	93.00	99.00	60	25	SG3-72	72	216.00	222.00	150	30
SG3-32	32	96.00	102.00	60	25	SG3-75	75	225.00	231.00	150	30
SG3-33	33	99.00	105.00	60	25	SG3-76	76	228.00	234.00	150	30
SG3-34	34	102.00	108.00	75	25	SG3-78	78	234.00	240.00	150	30
SG3-35	35	105.00	111.00	75	25	SG3-80	80	240.00	246.00	150	30
SG3-36	36	108.00	114.00	75	25	SG3-82	82	246.00	252.00	150	30
SG3-37	37	111.00	117.00	75	25	SG3-84	84	252.00	258.00	150	30
SG3-38	38	114.00	120.00	75	25	SG3-86	86	258.00	264.00	150	30
SG3-39	39	117.00	123.00	75	25	SG3-90	90	270.00	276.00	150	30
SG3-40	40	120.00	126.00	75	25	SG3-92	92	276.00	282.00	150	30
SG3-41	41	123.00	129.00	75	25	SG3-94	94	282.00	288.00	150	30
SG3-42	42	126.00	132.00	75	25	SG3-95	95	285.00	291.00	150	30
SG3-43	43	129.00	135.00	75	25	SG3-96	96	288.00	294.00	150	30
SG3-44	44	132.00	138.00	100	25						

Data adapted from HPC Gears Ltd.

Example 6.6

A gearbox is required to transmit 18 kW from a shaft rotating at 2650 rpm. The desired output speed is approximately 12 000 rpm. For space limitation and standardization reasons a double step-up gearbox is requested with equal ratios. Using the limited selection of gears presented in Tables 6.7 to 6.10, select suitable gears for the gear wheels and pinions.

Solution

$$\text{Overall ratio} = 12\ 000/2650 = 4.528.$$

$$\text{First stage ratio} = \sqrt{4.528} = 2.128.$$

This could be achieved using a gear with 38 teeth and pinion with 18 teeth (ratio = 38/18 = 2.11).

The gear materials listed in Tables 6.7 to 6.10 are 817M40 and 655M13 steels. From

Table 6.11 Permissible bending stresses for various commonly used gear materials

Material	Treatment	σ_{uts} (MPa)	Permissible bending stress σ_p (MPa)
Nylon		65 (20°C)	27
Tufnol		110	31
080M40		540	131
080M40	Induction hardened	540	117
817M40		772	221
817M40	Induction hardened	772	183
045M10		494	117
045M10	Case hardened	494	276
655M13	Case hardened	849	345

Table 6.11, the 655M13 is the stronger steel and this is selected for this example prior to a more detailed consideration. For 655M13 case hardened steel gears, the permissible stress $\sigma_p = 345$ MPa.

Calculations for gear 1: $Y_{38} = 0.37727$, $n = 2650$ rpm.

$$\begin{array}{lll} m & 1.5 & 2.0 \\ d = mN \text{ (mm)} & 57 & 76 \end{array}$$

$$V = \frac{d}{2} \times 10^{-3} \times n \frac{2\pi}{60} \text{ (m/s)} \quad 7.9 \quad 10.5$$

$$W_t = \frac{\text{Power}}{V} \text{ (N)} \quad 2276 \quad 1707$$

$$K_v = \frac{6.1}{6.1 + V} \quad 0.4357 \quad 0.3675$$

$$F = \frac{W_t}{K_v m Y \sigma_p} \text{ (m)} \quad 0.027 \quad 0.018$$

$m = 1.5$ gives a face width value greater than the catalogue value of 20 mm, so try $m = 2$. $m = 2$ gives a face width value less than the catalogue value of 25 mm, so design is OK.

Calculations for pinion 1: $Y_{18} = 0.29327$, $n = 5594$ rpm. (No need to do calculations for $m = 1.5$, because it has now been rejected.)

$$\begin{array}{lll} m & 2.0 \\ d \text{ (mm)} & 36 \end{array}$$

V (m/s)	10.5
W_t (N)	1707
K_v	0.3676
F (m)	0.023

$m = 1.5$ gives a face width value greater than the catalogue value of 20 mm, so try $m = 2$. $m = 2$ gives a face width value less than the catalogue value of 25 mm, so design is OK.

Calculations for gear 2: $Y_{38} = 0.37727$, $n = 5594$ rpm.

m	2.0
d (mm)	76
V (m/s)	22.26
W_t (N)	808.6
K_v	0.215
F (m)	0.0144

$m = 2$ gives a face width value lower than catalogue specification, so the design is OK.

Calculations for pinion 2: $Y_{18} = 0.29327$, $n = 11810$ rpm.

m	2.0
d (mm)	36
V (m/s)	22.26
W_t (N)	808.6
K_v	0.215
F (m)	0.0186

$m = 2$ gives a face width value lower than catalogue specification, so the design is OK. A general arrangement for the solution is shown in Figure 6.27.

Example 6.7

A gearbox for a cordless hand tool, Figure 6.28, is required to transmit 5 W from an electric motor running at 3000 rpm to an output drive running at approximately 75 rpm. The specification for the gearbox is that it should be reliable, efficient, maintenance free for the life of the device, light and fit within an enclosure of 50 mm diameter. The length is not critical.

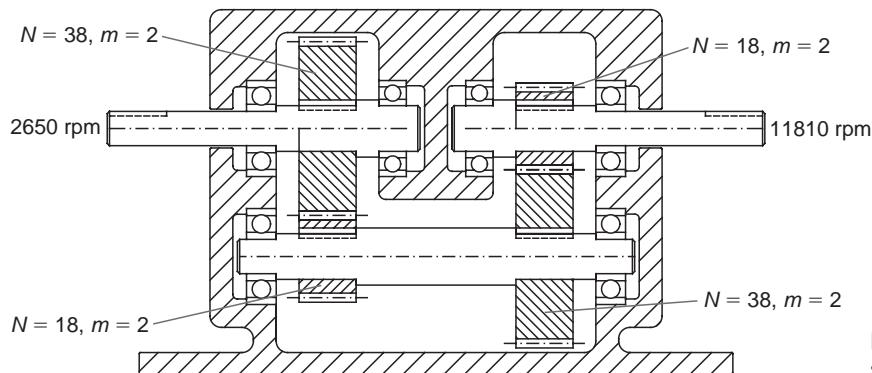


Figure 6.27 Step up gearbox solution.

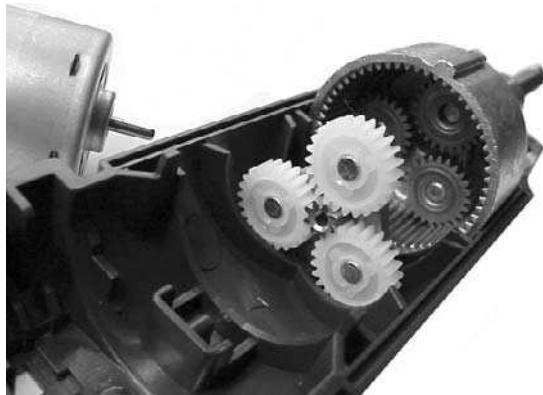


Figure 6.28 Cordless hand tool moulding, electric motor and gearbox.

The tolerance on the output speed is acceptable at ± 10 per cent. Specify an appropriate gear train including values for the module, teeth numbers and gear materials.

Solution

The reduction ratio $3000/75$ is quite large and could be achieved by a compound gear train, a worm and wheel or a two stage reduction epicyclic. The restriction on the maximum diameter precludes the use of compound gear trains, especially if the drive input is axial as would be expected from an electric motor. The high efficiency demand rules out the use of a worm and wheel.

We want an overall ratio of 40 ($=3000/75$). This could be achieved in a two-stage

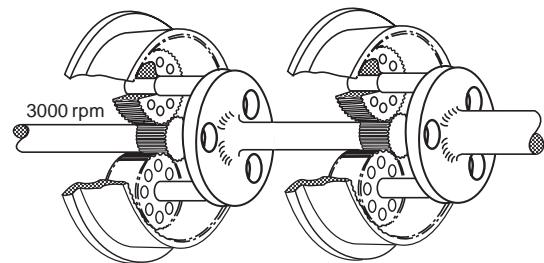


Figure 6.29 Two stage reduction epicyclic gear train (after Townsend 1992).

reduction epicyclic gearbox as illustrated in Figure 6.29. For identical stages the gear ratio of each stage would be $\sqrt{40} = 6.3$ ($6.3 \times 6.3 \approx 39.7$).

Try $N_S = 18, N_R = 72$. This gives a reduction ratio per stage of $N_R/N_S + 1 = 72/18 + 1 = 5$. Overall reduction ratio of the two stage gear train $= 5 \times 5 = 25$. No good.

Try $N_S = 12, N_R = 72$. This gives a reduction ratio of 7. Overall reduction ratio $7 \times 7 = 49$. No good.

Try $N_S = 12, N_R = 63$. This gives a reduction ratio of 6.25. Overall reduction ratio $6.25 \times 6.25 = 39.1$. OK.

Can this fit in the space?

Propose a module. Say $m = 0.5$.

With $N_S = 12, d_{\text{sun}} = mN = 0.5 \times 12 = 6 \text{ mm}$.

With $N_R = 63$, $d_{ring} = mN = 0.5 \times 63 = 31.5$ mm.

Room available for planet gear is $31.5/2 - 6/2 = 12.75$ mm. With a module of 0.5 there are no gears with this diameter. Impossible solution.

Try $N_S = 12$, $N_R = 66$. This gives a reduction ratio of $6.5 \times 6.5 = 42.3$.

This would give an output speed of 71 rpm. This is within the tolerance allowable on the output speed. OK.

Can this fit in the space?

Propose a module. Say $m = 0.5$

With $N_S = 12$, $d_{sun} = mN = 0.5 \times 12 = 6$ mm.

With $N_R = 66$, $d_{ring} = mN = 0.5 \times 66 = 33$ mm.

Room available for planet gear is $33/2 - 6/2 = 13.5$ mm.

With a module of 0.5, the planetary gears would have $13.5/0.5 = 27$ teeth. $N_p = 27$.

Proposed design: $N_S = 12$, $N_p = 27$, $N_R = 66$, module 0.5.

What face width should the gears have? For a module of 0.5 HPC Gears Ltd offers standard 0.5 module gears with a face width of 5 mm available in Tufmol, delrin, brass and 045M10 steel.

The Lewis formula can be used as an approximate check to determine whether the gears with a face width of 5 mm manufactured from a particular material are strong enough.

Analysing the input sun gear

$$V_S = \frac{6 \times 10^{-3}}{2} \times 3000 \times \frac{2\pi}{60} = 0.942 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.942} = 0.866$$

$$W_t = \frac{\text{power}}{V} = \frac{5}{0.942} = 5.3 \text{ N}$$

From Table 6.6

$$Y_{12} = 0.2296$$

Try Tufmol

$$\sigma_p = 31 \text{ MPa}$$

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

$$= \frac{5.3}{0.866 \times 0.0005 \times 0.2296 \times 31 \times 10^6}$$

$$= 1.72 \times 10^{-3} \text{ m}$$

This face width of 1.7 mm is less than the 5 mm available, indicating that the gear will be more than strong enough. If this figure had been greater than 5 mm, then another material such as steel or brass could be considered.

Analysing the planets

$$n_{arm} = 3000/6.5 = 461.5 \text{ rpm}$$

To determine the speed of the planets use Eqs 6.14 and 6.15

$$\frac{n_p - n_{arm}}{n_S - n_{arm}} = -\frac{N_S}{N_p}$$

$$\frac{n_p - (-461.5)}{-3000 - (-461.5)} = -\frac{12}{27}$$

$$n_p = -\frac{12}{27} \times (-2538) - 461.5 = 666.7 \text{ rpm}$$

$$V_p = \frac{13.5 \times 10^{-3}}{2} \times 666.7 \times \frac{2\pi}{60} = 0.47 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.47} = 0.928$$

The power is divided between three planets, so:

$$W_t = \frac{\text{power}}{V} = \frac{5/3}{0.47} = 3.55 \text{ N}$$

From Table 6.6, $Y_{27} = 0.34385$. Try Tufnol

$$\begin{aligned}\sigma_p &= 31 \text{ MPa} \\ F &= \frac{W_t}{K_v m Y \sigma_p} \\ &= \frac{3.55}{0.928 \times 0.0005 \times 0.34385 \times 31 \times 10^6} \\ &= 0.72 \text{ mm}\end{aligned}$$

This face width of 0.72 mm is less than the 5 mm available, indicating that the gear will be more than strong enough.

Analysing the ring gear, the stress on the ring gear will be less than on the planets so does not necessarily need to be analysed at this stage.

Analysing the stage 2 sun gear

$$\begin{aligned}(n_{\text{sun}} &= n_{\text{planet carrier}}) \\ V_s &= \frac{6 \times 10^{-3}}{2} \times 461.5 \times \frac{2\pi}{60} = 0.145 \text{ m/s} \\ K_v &= \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.145} = 0.98 \\ W_t &= \frac{\text{power}}{V} = \frac{5}{0.145} = 34.5 \text{ N}\end{aligned}$$

From Table 6.6, $Y_{12} = 0.2296$

Try Tufnol

$$\begin{aligned}\sigma_p &= 31 \text{ MPa} \\ F &= \frac{W_t}{K_v m Y \sigma_p} \\ &= \frac{34.5}{0.98 \times 0.0005 \times 0.2296 \times 31 \times 10^6} \\ &= 0.0099 \text{ m}\end{aligned}$$

This is larger than the available standard size (5 mm), so try a stronger material.

Using 045M10 steel, $\sigma_p = 117 \text{ MPa}$

$$\begin{aligned}F &= \frac{W_t}{K_v m Y \sigma_p} \\ &= \frac{34.5}{0.98 \times 0.0005 \times 0.2296 \times 117 \times 10^6} \\ &= 2.62 \times 10^{-3} \text{ m}\end{aligned}$$

This face width of 2.62 mm is less than the 5 mm available, indicating that the gear will be more than strong enough. So the sun gear must be manufactured from 045M10 steel. Manufacturing expediency might dictate that the planet carrier should also be made from the same steel.

Analysing the planets

$$\begin{aligned}\frac{n_p - (-71)}{461.5 - (-71)} &= -\frac{12}{27} \\ n_p &= -\frac{12}{27} \times (-390.5) - 71 = 102.6 \text{ rpm} \\ V_p &= \frac{13.5 \times 10^{-3}}{2} \times 102.6 \times \frac{2\pi}{60} \\ &= 0.0725 \text{ m/s} \\ K_v &= \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.0725} = 0.99\end{aligned}$$

The power is divided between three planets, so

$$\begin{aligned}W_t &= \frac{\text{power}}{V} = \frac{5 / 3}{0.0725} = 23.0 \text{ N} \\ Y_{27} &= 0.34385\end{aligned}$$

Try Tufnol

$$\begin{aligned}\sigma_p &= 31 \text{ MPa} \\ F &= \frac{W_t}{K_v m Y \sigma_p} \\ &= \frac{23.0}{0.99 \times 0.0005 \times 0.34385 \times 31 \times 10^6} \\ &= 4.4 \times 10^{-3} \text{ m}\end{aligned}$$

This face width of 4.4 mm is less than the 5 mm available, indicating that the gear will be more than strong enough.

Design summary:

Stage 1: module 0.5, $F = 5$ mm, $N_S = 12$ (Tufnol), $N_P = 27$ (three planets, Tufnol), $N_R = 66$ (Tufnol).

Stage 2: module 0.5, $F = 5$ mm, $N_S = 12$ (steel), $N_P = 27$ (three planets, Tufnol), $N_R = 66$ (Tufnol).

6.7 Conclusions

For transmissions where compact size, high efficiency or high speed are required, gears offer a competitive solution in comparison to other types of drive, such as belts and chains. This chapter has introduced a wide range of types of gears and calculations enabling simple gear trains to be analysed. A preliminary selection procedure for spur gears based on the Lewis formula for bending stresses has been presented. This serves as the starting point in proposing the details for a gear train, but more sophisticated analysis is necessary if a robust gearbox is required. Analysis for determining bending and contact stresses and associated safety factors based on national standards for spurs gears is outlined in Chapter 7.

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Websites

At the time of going to press, the internet contains useful information relating to this chapter at the following sites.

- moorecustomgear.com
- www.bga.org.uk
- www.cross-morse.co.uk
- www.davall.co.uk
- www.drgears.com
- www.gearing-hk.com
- www.geartechology.com
- www.hpcgears.com
- www.omnigearandmachine.com
- www.penfold.co.nz
- www.powertransmission.com
- www.precipart.com
- www.qtcgears.com
- www.rapidgear.com
- www.reliance.co.uk
- www.shanthigears.com
- www.textronpt.com
- www.textronpt.com
- www.ugaco.com
- www.wgear.com

Nomenclature

Generally, preferred SI units have been stated.

<i>a</i>	addendum (mm)
<i>b</i>	dedendum (mm)
<i>c</i>	clearance (mm)
<i>d</i>	pitch diameter (mm)
<i>d_G</i>	pitch diameter of the gear (mm)
<i>d_P</i>	pitch diameter of the pinion (mm)
<i>d_{ring}</i>	pitch diameter of the ring gear (mm)
<i>d_{sun}</i>	pitch diameter of the sun gear (mm)
<i>e</i>	train value
<i>F</i>	face width (mm)
<i>F_r</i>	radial force component (N)
<i>F_t</i>	tangential force component (N)
<i>h_k</i>	working depth (mm)
<i>h_t</i>	whole depth (mm)
<i>H</i>	power (kW)
<i>K_v</i>	velocity factor
<i>m</i>	module (mm)
<i>n</i>	rotational speed (rpm)
<i>n_P</i>	rotational speeds of the pinion (rpm)
<i>n_G</i>	rotational speeds of the gear (rpm)
<i>N</i>	number of teeth
<i>N_G</i>	number of teeth in the gear
<i>N_P</i>	number of teeth in the pinion or planet gear
<i>N_R</i>	number of teeth in annulus gear
<i>N_s</i>	number of teeth in sun gear
O/D	outside diameter (mm)
<i>p</i>	circular pitch (mm)
PCD	pitch circle diameter (mm)

<i>P_d</i>	diametral pitch (teeth per inch)
<i>r</i>	pitch radius (mm)
<i>r_f</i>	fillet radius of basic rack (mm)
<i>t</i>	tooth thickness (mm)
<i>t_o</i>	width of top land (mm)
<i>V</i>	velocity (m/s)
<i>W_t</i>	transmitted load (N)
<i>Y</i>	Lewis form factor
ϕ	pressure angle (°)
σ	bending stress (N/m ²)
σp	permissible stress (MPa)
σ_{uts}	ultimate tensile strength (MPa)
ω	angular velocity (rad/s)
ωG	angular velocities of the gear (rad/s)
ωP	angular velocities of the pinion (rad/s)

Worksheet

- 6.1 For the gear train illustrated in Figure 6.30, determine the output speed and direction of rotation if the input shaft rotates at 1490 rpm clockwise. Gears A to D have a module of 1.5 and gears E to H a module of 2.
- 6.2 What type of gear could be selected for the following applications.
 - (a) To transmit power between two shafts with intersecting axes at 90° to each other.
 - (b) To transmit power between two shafts with non-intersecting axes at 90° to each other.

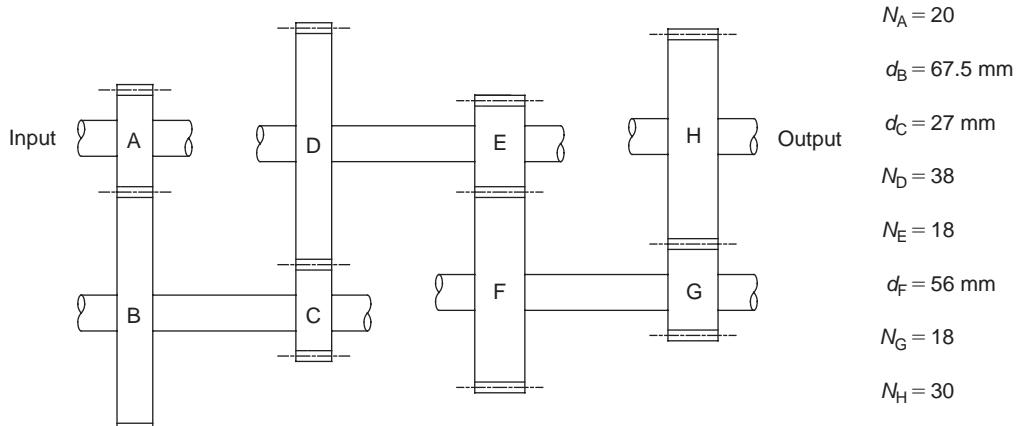


Figure 6.30 Gear train.

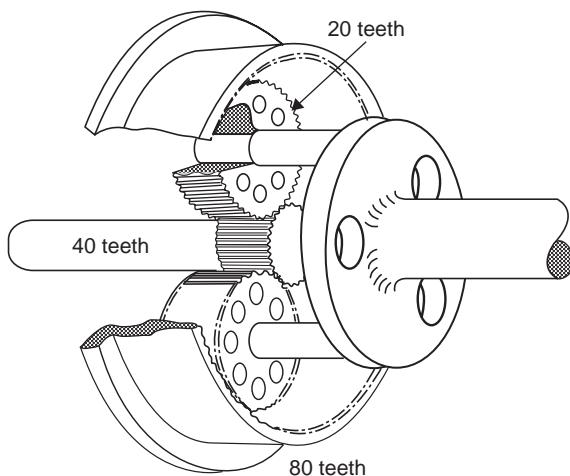


Figure 6.31 Epicyclic gears (reproduced from Townsend, 1992).

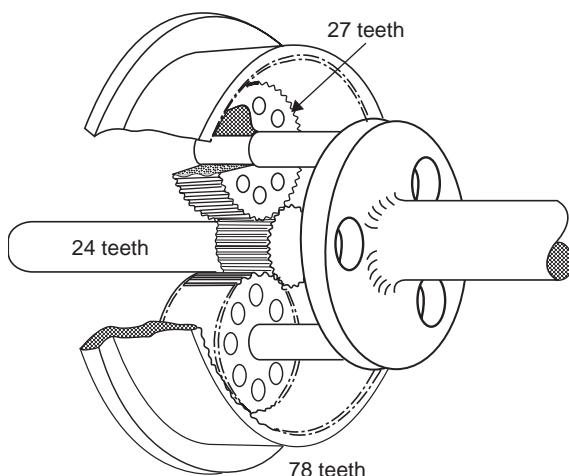


Figure 6.32 Epicyclic gears (reproduced from Townsend, 1992).

- (c) To transmit 30kW between parallel axis shafts. What material might be specified for these gears?
 - (d) To transmit a low power load in an epicyclic gearbox. What material might be specified for this application?
- 6.3 Determine the ratio, defined as the sun/arm speed ratio, and the speed and direction of rotation of the planet carrier for the epicyclic gearbox illustrated in Figure 6.31. The sun, planets and annulus gears have 40, 20 and 80 teeth, respectively. The sun gear is the input and rotates at 3000 rpm anticlockwise. The planet carrier is the output. The annulus gear is held stationary.
- 6.4 Determine the ratio, defined as the sun/arm speed ratio, and the speed and direction of the planet carrier for the epicyclic gear box illustrated in Figure 6.32. The sun, planets and annulus gears have 24, 27 and 78 teeth, respectively. The sun gear is rotating at 5000 rpm clockwise. The sun gear is the input and the planet carrier the output. The annulus gear is held stationary.
- 6.5 A 20° full depth spur pinion is to transmit 1.75 kW at 1200 rpm. If the pinion has 18 teeth with a module of 2, determine a suitable value for the face width, based on the

Lewis formula, if the bending stress should not exceed 75 MPa.

- 6.6 A 20° full depth spur pinion is required to transmit 1.8 kW at a speed of 1100 rpm. If the pinion has 18 teeth and is manufactured from heavy duty 817M40 steel select a suitable gear from the limited choice given in Tables 6.7–6.10, specifying the module and face width based on the Lewis formula.
- 6.7 A gearbox is required to transmit 6 kW from a shaft rotating at 2200 rpm. The desired output speed is approximately 300 rpm. Using the Lewis formula to determine gear strength based upon bending stresses select suitable gears from the limited selection presented in Tables 6.7–6.10.
- 6.8 A 20° full depth spur pinion is required to transmit 1.5 kW at a speed of 1230 rpm. If the pinion has 21 teeth and is manufactured from heavy duty 817M40 steel select a suitable gear from the limited choice given in Tables 6.7–6.10, specifying the module and face width based on the Lewis formula.
- 6.9 A gearbox is required to transmit 5.5 kW from a shaft rotating at 2000 rpm. The desired output speed is approximately 350 rpm. The output must rotate in the same direction as the input and along the same axis. Select suitable gears.

- 6.10 A double reduction gearbox is required to transmit 15 kW from a shaft rotating at 1500 rpm. The desired output speed is approximately 200 rpm. Using the Lewis formula to determine gear strength based upon bending stresses select suitable gears from the limited selection presented in Tables 6.7–6.10.
- 6.11 A gearbox is required to transmit 8 kW from a shaft rotating at 1470 rpm. The desired output speed is approximately 10 000 rpm. The output must rotate in the same direction as the input and along the same axis. Select and specify appropriate gears for the gearbox.
- 6.12 An epicyclic gearbox is required to transmit 12 kW from a shaft rotating at 3000 rpm. The desired output speed is approximately 500 rpm. Using a module 3 hardened steel gear, determine the number of teeth in each gear for a single stage reduction gearbox.

Answers

- 6.1 121 rpm, clockwise.
- 6.2 (a) Bevel; (b) crossed axis helical gears; (c) helical gears, case hardened steel; (d) spur gears, Tufnol
- 6.3 3, 1000 rpm, anti-clockwise
- 6.4 4.25, 1176 rpm, clockwise
- 6.5 25 mm
- 6.6 $m = 2, F = 25 \text{ mm}$ would be acceptable.
- 6.7 No unique solution. $m = 2, F = 25 \text{ mm}, N_1 = 18, N_2 = 48, N_3 = 18, N_4 = 48$, case hardened 655M13 steel would be acceptable.
- 6.8 No unique solution. $m = 2, N = 21, F = 25 \text{ mm}$ would be acceptable.
- 6.9 No unique solution.
- 6.10 No unique solution.
- 6.11 No unique solution.
- 6.12 No unique solution. $N_S = 18, N_P = 36, N_R = 90$ would be acceptable.

Learning objectives achievement

Can you name the various features on a gear?

✓ X Section 6.1

Can you select a suitable gear type for different applications?

✓ X Section 6.1

Can you determine a gearbox ratio?

✓ X Section 6.3

Can you determine the bending stress for a spur gear using the Lewis formula?

✓ X Section 6.5.2

Are you able to select appropriate gears for a compound gearbox using spur gears?

✓ X Section 6.6

Are you able to select appropriate gears for an epicyclic gearbox using spur gears?

✓ X Section 6.3.2, 6.6

7

DETAILED GEAR STRESSING

Although suitable for initial selection of a gear, the Lewis equation is inadequate for detailed design calculations where life of the machine is critical or where the gears are highly loaded. National and international standards are available detailing the calculation of bending and contact stresses for gears. This chapter outlines the AGMA standards for the calculation of bending and contact stresses in spur gears.

LEARNING OBJECTIVES

At the end of this chapter you should be able to:

- determine the AGMA bending stress for a spur gear;
- determine the AGMA contact stress for a spur gear;
- determine the allowable bending and contact stresses for a spur gear;
- determine a factor of safety for a spur gear;
- propose a design strategy for spur gears.

7.1 Introduction

The Lewis equation introduced in Chapter 6 provides a useful starting point allowing the configuration of a gear set that is to be developed. The Lewis equation takes account of the bending stress on a gear. Gears, however, can fail due to excessive bending stress or wear. Wear occurs as the teeth move in and out of contact with each other with accompanying local deformation of the gear teeth surfaces in the region of contact. The stresses resulting from the surface deformation are known as contact stresses. If the stresses are too high then

material failure can take the form of loss of material from the surfaces, which is also known as pitting. Pitting is a surface fatigue failure due to too many repetitions of high contact stresses. Wear can also occur due to scoring, due to failure of lubrication, or abrasion, due to the presence of a foreign material, such as grit or powder.

The modelling of gear stresses has been the subject of extensive research. The analysis is complex because of the dynamic nature of the loads and the very high stresses involved for some gear designs. Whilst techniques such as finite element analysis allow detailed modelling of a specific gear geometry, the results are labour-intensive in terms of setting up the model and uncertainty often remains concerning the validity of boundary conditions and material properties. An alternative to finite element analysis is the use of procedural approaches such as those outlined in standards developed by the American Gear Manufacturers Association (AGMA), the British Standards Institution (BSI) and the International Organization for Standardization (ISO).

This chapter concentrates on analysing whether failure due to bending and contact stresses in a gear set is likely. In Section 7.2 a relatively simple relationship for contact stresses is outlined prior to the introduction of the current AGMA equations for bending and contact stresses in Section 7.3. The AGMA equations rely on the evaluation of a significant number of factors and geometrical parameters, which depend on the specific geometry and materials concerned. In this chapter, the basic approach is outlined for 20° pressure angle, full depth teeth spur gears only. Many of the detailed

charts and equations given in the source standards have been omitted and the reader is encouraged to view the most recent standards as necessary. A general strategy for gear design is outlined in Section 7.4.

7.2 Wear failure

As well as failure due to bending stresses in gears, the failure due to wear on the surface of gear teeth should be considered. Possible surface failures are pitting, which is a surface fatigue failure due to many repetitions of high contact stresses, scoring due to failure of lubrication and abrasion due to the presence of foreign particles.

The surface compressive, Hertzian or contact stress for a gear can be modelled by:

$$\sigma_c = -C_p \frac{K'_v W_t}{F \cos \phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)^{0.5} \quad (7.1)$$

where C_p is an elastic coefficient; r_1 and r_2 are radii of curvature; K'_v is the velocity factor; W_t is the transmitted load (N); F is the face width (m); and ϕ is the pressure angle ($^\circ$).

If the units of C_p are in $\sqrt{\text{MPa}}$, W_t in N, r_1, r_2 in meters, then Eq. 7.1 gives the contact stress σ_c in kPa.

The radii of curvature are given by

$$r_1 = \frac{d_p \sin \phi}{2} \quad (7.2)$$

Table 7.1 Values of the elastic coefficient C_p ($\sqrt{\text{MPa}}$) (AGMA 218.01)

Pinion material	E_{pinion} (GPa)	Gear material				
		Steel	Malleable iron	Nodular iron	Cast iron	Aluminium bronze
Steel	200	191	181	179	174	162
Malleable iron	170	181	174	172	168	158
Nodular iron	170	179	172	170	166	156
Cast iron	150	174	168	166	163	154
Aluminium bronze	120	162	158	156	154	145
Tin bronze	110	158	154	152	149	141

$$r_2 = \frac{d_G \sin \phi}{2} \quad (7.3)$$

where d_p and d_G are the pitch diameters of the pinion and gear, respectively.

The velocity factor K'_v for cut or milled profile gears is given by

$$K'_v = \frac{6.1 + V}{6.1} \quad (7.4)$$

where V is the pitch line velocity (m/s).

Note that this definition for the velocity factor, now used in the latest AGMA standards is the reciprocal of that previously defined. For this reason, following the notation suggested by Mischke in Shigley and Mischke (2001), a superscript ' has been added to allow it to be distinguished from previous definitions.

The elastic coefficient C_p can be calculated from Eq. 7.5 or obtained from Table 7.1.

$$C_p = \frac{1}{\pi \frac{\downarrow 1 - v_p^2}{\rightarrow E_p} + \frac{1 - v_g^2}{E_g}}^{0.5} \quad (7.5)$$

where v_p is Poisson's ratio for the pinion; v_g is Poisson's ratio for the gear; E_p is Young's modulus for the pinion; and E_g is Young's modulus for the gear.

Example 7.1

A speed reducer has a 22 tooth spur pinion made of steel, driving a 60 tooth gear made of cast iron. The transmitted power is 10 kW. The pinion speed is 1200 rpm, module 4 and face width 50 mm. Determine the contact stress.

Solution

$N_p = 22$; steel pinion, $N_g = 60$; cast iron gear, $n = 1200$ rpm, $m = 4$, $F = 50$ mm, $H = 10$ kW.

$$d_p = mN_p = 4 \times 22 = 88 \text{ mm},$$

$$d_G = 4 \times 60 = 240 \text{ mm}$$

$$V = \frac{0.088}{2} \times 1200 \times \frac{2\pi}{60} = 5.529 \text{ m/s}$$

$$K'_v = \frac{6.1 + 5.529}{6.1} = 1.906$$

$$W_t = \frac{10000}{5.529} = 1809 \text{ N}$$

From Table 7.1 for a steel pinion and cast iron gear, $C_p = 174 \sqrt{\text{MPa}}$

From Eqs 7.2 and 7.3

$$r_1 = \frac{88 \times 10^{-3} \sin 20}{2} = 0.01505 \text{ m}$$

$$r_2 = \frac{240 \times 10^{-3} \sin 20}{2} = 0.04104 \text{ m}$$

From Eq. 7.1

$$\sigma_c = -174 \frac{1.906 \times 1809}{0.05 \cos 20} (90.81)^{0.5}$$

$$= -174 \times 2581 \text{ kPa} = -449200 \text{ kPa}$$

$$-449 \text{ MPa}$$

7.3 AGMA equations for bending and contact stress

The calculation of bending and contact stresses in spur and helical gears can be determined using standardized methods presented by the British Standards Institution, the International Organization for Standardization (ISO), the Deutsches Institut für Normung (DIN), and the American Gear Manufacturers Association (AGMA). The AGMA standards have recently been revised (ANSI/AGMA 2001-C95) and are widely used and have therefore been selected for presentation here. The procedures make extensive use of a series of geometry and design factors, which can be determined from design charts and tables.

The AGMA formula for bending stress is given in SI units by

$$\sigma = W_t K_O K'_v K_s \frac{1}{Fm} \frac{K_H K_B}{Y_J} \quad (7.6)$$

or US customary units by:

$$\sigma = W_t K_O K'_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \quad (7.7)$$

where σ is the bending stress (N/m^2); W_t , the transmitted tangential load (N); K_O , the overload factor; K'_v , the dynamic factor; K_s , the size factor; P_d , the transverse diametral pitch (in); F , the face width (m or in); m , the module (with the unit of metres); K_H or K_m , the load distribution factor; K_B , the rim thickness factor; and Y_J or J , the geometry factor for bending strength.

The AGMA equation for pitting resistance is given in SI units by

$$\sigma_c = Z_e \frac{\downarrow}{\rightarrow} W_t K_O K'_v K_s \frac{K_H}{Fd} \frac{Z_R}{Z_I}^{0.5} \quad (7.8)$$

or in US customary units by:

$$\sigma_c = C_p \frac{\downarrow}{\rightarrow} W_t K_O K'_v K_s \frac{K_m}{Fd} \frac{C_f}{I}^{0.5} \quad (7.9)$$

where σ_c is the absolute value of contact stress (kPa); Z_e , an elastic coefficient (MPa)^{0.5}; W_t , the transmitted tangential load (N); K_O , the overload factor; K'_v , the velocity factor; K_s , the size factor; K_H or K_m , the load distribution factor; F , the face width; d , the pitch diameter of the pinion; Z_R or C_F , the surface condition factor for pitting resistance; and Z_I or I , the geometry factor for pitting resistance.

For Eq. 7.8, if W_t is in N, F in m, d in m, C_P in $\sqrt{\text{MPa}}$, then the units of the contact stress σ_c are in kPa.

Based on safe working practices, the American Gear Manufacturers Association has defined allowable stress equations for gears for the allowable bending stress and for the allowable contact stress. The AGMA equation for determining a safe value for the allowable bending stress in SI units is given by

$$\sigma_{\text{all}} = \frac{\sigma_{\text{FP}} Y_N}{S_F Y_\theta Y_Z} \quad (7.10)$$

or US customary units by

$$\sigma_{\text{all}} = \frac{S_t Y_N}{S_F K_T K_R} \quad (7.11)$$

where S_t or σ_{FP} is the allowable bending stress (lb/in² or MPa); Y_N , the stress cycle factor for bending stress; S_F , the AGMA factor of safety; Y_θ or K_T , the temperature factor; and Y_Z or K_R , the reliability factor.

The AGMA equation for determining a safe value for the allowable contact stress in SI units is given by

$$\sigma_{c,\text{all}} = \frac{\sigma_{\text{HP}} Z_N Z_W}{S_H Y_\theta Y_Z} \quad (7.12)$$

or in US customary units by

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R} \quad (7.13)$$

where S_c or σ_{HP} is the allowable contact stress (lb/in² or MPa); Z_N , the stress cycle life factor; Z_W or C_H , a hardness ratio factor; Y_θ or K_T , the

temperature factor; Y_Z or K_R , the reliability factor; and S_H , the AGMA factor of safety.

The purpose of the overload factor, K_O , is to account for externally applied loads in excess of the nominal tangential load W_t . A typical example is the variation in torque from an internal combustion engine. Values for K_O tend to be derived from extensive experience of a particular application. Values for the overload factor can be estimated from Table 7.2. Within this text the value for the overload factor will generally be assumed to be unity.

The dynamic factor, K'_v , is used to account for the effect of tooth spacing and profile errors, the magnitude of the pitch line velocity, the inertia and stiffness of the rotating components and the transmitted load per unit face width. The American Gear Manufacturers Association has defined a set of quality control numbers which can be taken as equal to the transmission accuracy level number, Q_V , to quantify these parameters (AGMA 390.01). Classes $3 \leq Q_V \leq 7$ include most commercial quality gears and are generally suitable for applications, such as agricultural and plant machinery, presses and conveyors, classes of $8 \leq Q_V \leq 12$ are of precision quality and are generally suitable for power tools, washing machines and automotive transmissions. Generally the higher the peripheral speed and the smaller the specific tooth load, the more accurate the gear manufacture must be and a gear with a higher quality control number should be used. The dynamic factor K'_v can be determined from

$$K'_v = \frac{\downarrow A + \sqrt{200V}}{\rightarrow A}^B \quad (7.14)$$

where

$$B = \frac{12 - Q_V}{4}^{2/3} \quad (7.15)$$

and

$$A = 50 + 56(1 - B) \quad (7.16)$$

The size factor K_s is usually taken as unity. Higher values can be used if there is any non-uniformity in the material properties due to size.

Table 7.2 Estimate for overload factors (after Mott, 1999)

Driven machine				
Driving machine	Uniform. For example continuous duty generator sets	Light shock. For example fans, low speed centrifugal pumps, variable duty generators, uniformly loaded conveyors, positive displacement pumps	Moderate shock. For example high speed centrifugal pumps, reciprocating pumps and compressors, heavy duty conveyors, machine tools drives and saws	Heavy shock. For example punch presses and crushers
Uniform. For example electric motors or constant speed gas turbines	1	1.25	1.5	1.75
Light shock. For example, variable speed drives	1.2	1.4	1.75	2.25
Moderate shock. For example multi-cylinder engines	1.3	1.7	2	2.75

The load distribution factor K_H is used to account for non-uniform distribution of load across the line of contact. Ideally gears should be sited at a location where there is zero slope on the shaft under the load. This is normally at mid-span.

For $F/d \leq 2$, gears mounted between bearings, $F \leq 1.016$ m, contact across the full width of the narrowest member,

$$K_H = K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \quad (7.17)$$

For uncrowned teeth $C_{mc} = 1$ and for crowned teeth $C_{mc} = 0.8$

If $F \leq 0.0254$ m then

$$C_{pf} = \frac{F}{10d} - 0.025 \quad (7.18)$$

If $0.0254 \text{ m} < F \leq 0.4318 \text{ m}$ then

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.4921F \quad (7.19)$$

If $0.4318 \text{ m} < F \leq 1.016 \text{ m}$ then

$$C_{pf} = \frac{F}{10d} + 0.815F - 0.3534F^2 \quad (7.20)$$

For values of $F/10d < 0.05$, then set $F/10d = 0.05$ in Eqs 7.18 to 7.20.

For a straddle-mounted pinion with $S_1/S < 0.175$ use $C_{pm} = 1$.

For a straddle-mounted pinion with $S_1/S \geq 0.175$ use $C_{pm} = 1.1$.

The definition for S_1 and S is given in Figure 7.1.

$$C_{ma} = A + BF + CF^2 \quad (7.21)$$

where the coefficients A , B and C can be found from Table 7.3.

For gearing adjusted at assembly or if compatibility is improved by lapping or both, $C_e = 0.8$, for all other conditions take $C_e = 1$.

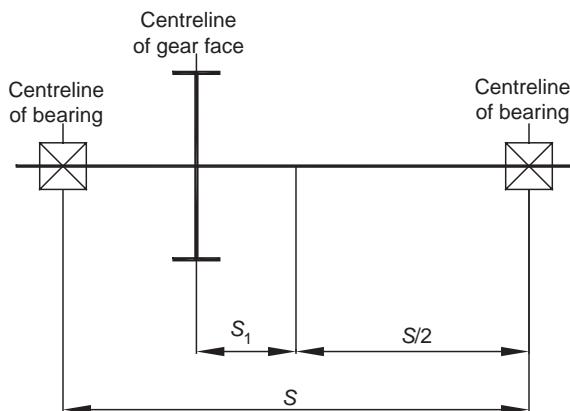


Figure 7.1 Definition of distances S_1 and S used for evaluating C_{pm} (ANSI/AGMA 2001/C95).

If the rim thickness is insufficient to provide full support for the bearing then this can be accounted for by the rim thickness factor K_B . This is necessary when the ratio $t_R/h_t < 1.2$. In the examples given here it will be assumed that $t_R/h_t \geq 1.2$ in which case $K_B = 1$.

The bending strength geometry factor, Y_J , for 20° pressure angle full depth teeth spur gears can be obtained from Figure 7.2.

The surface condition factor Z_R is used to account for surface finish, residual stress and work hardening. A value for Z_R greater than unity is suggested when a detrimental surface finish is known to exist.

Table 7.3 Coefficients used in determining C_{ma} , Eq. 7.21

Condition	A	B	C
Open gearing	0.247	0.657	-0.119
Commercial enclosed gear units	0.127	0.622	-0.0144
Precision enclosed gear units	0.0675	0.504	-0.144
Extra-precision enclosed gear units	0.00360	0.402	-0.127

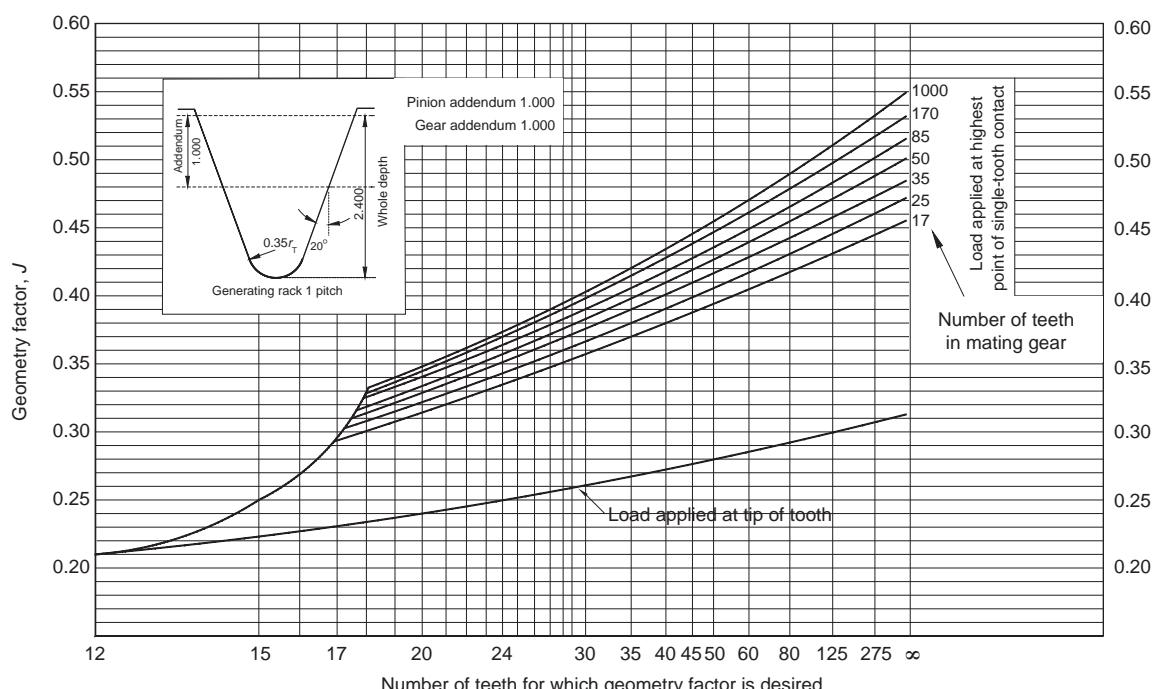


Figure 7.2 Bending strength geometry factor Y_J or J for 20° pressure angle full depth teeth spur gears.

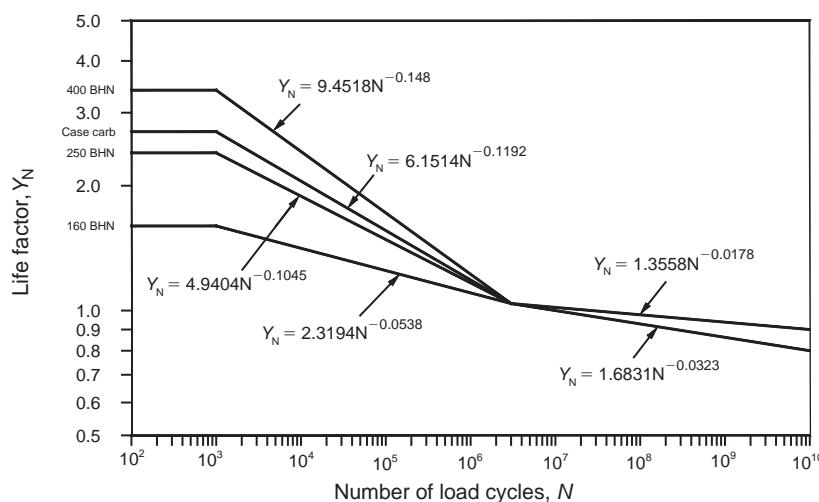


Figure 7.3 Bending strength life factor Y_N . Reproduced from ANSI/AGMA 2001-C95.

For spur and helical gears the surface strength geometry factor, I , can be determined from Eqs 7.22 or 7.23.

For external gears

$$Z_I = I = \frac{\cos \phi_t \sin \phi_t}{2m_N} \frac{m_G}{m_G + 1} \quad (7.22)$$

For internal gears

$$Z_I = I = \frac{\cos \phi_t \sin \phi_t}{2m_N} \frac{m_G}{m_G - 1} \quad (7.23)$$

where $m_N = 1$ for spur gears. (ϕ_t is the transverse pressure angle used for helical gears. Replace ϕ_t by ϕ for spur gears.) For helical gears $m_N = p_N/0.95Z$ where p_N is the base pitch and Z is the length of the line of action in the transverse plane.

The speed ratio m_G is

$$m_G = \frac{N_G}{N_P} \quad (7.24)$$

The elastic coefficient Z_e or C_p can be found from Table 7.1 or Eq. 7.5.

The life factors Y_N and Z_N are used to modify the AGMA strengths for lives other than 10^7 cycles. The life required for a design is a critical decision. Consideration should be given to the maximum number of cycles that a machine in service will see. Obviously different customers may use machines

in different ways. A common approach is to consider 95 per cent of the machines in use. This allows for exclusion of extreme cases. Take for instance a washing machine. Market analysis would be required to identify what the maximum use of the machine would be. The 95 percentile excludes the tail of extreme users who might only rarely use their machines or use them 24 hours a day. Such high use may well be excluded from warranty claims in any case. An estimate of use can also be made. It is conceivable that a washing machine in a family household might be used twice a day, every day of the year. Assuming that the washing process involves operation at 30 rpm for 1 h and 800 rpm for 15 min, then the total number of cycles in a year is given by $365 \times 2 \times ((30 \times 60) + (800 \times 15)) = 10.074 \times 10^6$ or about 10^7 cycles. If a 1-year warranty is to be considered then the number of cycles to be used in calculations should be 10^7 . The life factor Y_N can be determined from Figure 7.3 and Z_N from Figure 7.4. Note that if the pinion and gear have different teeth numbers then they will experience different numbers of cycles.

For oil or gear temperatures up to 120°C the temperature factor Y_θ can be taken as unity. Operation at temperatures above this value are not recommended due to degradation of the lubricant and use of a heat exchanger to reduce

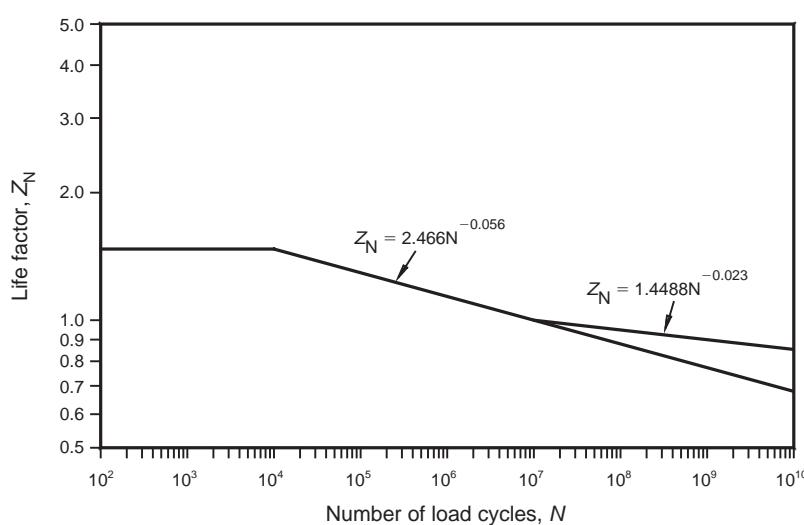


Figure 7.4 Pitting resistance life factor Z_N . Reproduced from ANSI/AGMA 2001-C95.

Table 7.4 AGMA reliability factor Y_Z or K_R (after ANSI/AGMA 2001-C95)

Reliability	Y_Z, K_R
0.5	0.70
0.90	0.85
0.99	1.00
0.999	1.25
0.9999	1.50

the lubricant temperature is advised in such circumstances. If the temperature exceeds 120°C the temperature factor Y_θ needs to be increased above unity.

The reliability factor, K_R or Y_Z , is used to account for the statistical distributions of failure of material by fatigue. Depending on the level of reliability desired the factor Y_Z should be adjusted accordingly as identified in Table 7.4. For values other than those listed in Table 7.4 use logarithmic interpolation.

The allowable bending stress for through hardened steels for grade 1 components is given by

$$\sigma_{FP} = 0.533H_B + 88.3 \text{ MPa} \quad (7.25)$$

and for grade 2

$$\sigma_{FP} = 0.703H_B + 113 \text{ MPa} \quad (7.26)$$

The allowable bending stress for nitrided through hardened steels for grade 1 components is given by

$$\sigma_{FP} = 0.568H_B + 83.8 \text{ MPa} \quad (7.27)$$

and for grade 2

$$\sigma_{FP} = 0.749H_B + 110 \text{ MPa} \quad (7.28)$$

Values for the allowable bending stress can be found from Eqs 7.25 to 7.28 for through hardened and nitrided through hardened steels and from Table 7.5 for iron and bronze gears. Alternatively an estimate for σ_{FP} can be obtained from Eq. 7.29.

$$\sigma_{FP} = 20.55 + 1.176H_B - 9.584 \times 10^{-4}H_B^2 \quad (7.29)$$

where σ_{FP} is the allowable bending stress in MPa.

The allowable contact stress, σ_{HP} , assuming unidirectional loading, 10^7 cycles and 99 per cent reliability for grade 1 through hardened steel gears is given by

$$\sigma_{HP} = 2.22H_B + 200 \quad (7.30)$$

and for grade 2 through hardened steel gears by

$$\sigma_{HP} = 2.41H_B + 237 \quad (7.31)$$

Values for the allowable contact stress can be found from Eqs 7.30 and 7.31 for grade 1 and 2

Table 7.5 Repeatedly applied allowable bending stress and allowable contact stress for a selection of iron and bronze gear materials at 10^7 cycles and 99 per cent reliability (ANSI/AGMA 2001/C95)

Material	Designation	Heat treatment	Typical minimum surface hardness	Allowable bending stress (MPa)	Allowable contact stress (MPa)
ASTM A48 gray cast iron	Class 20	As cast	—	34	345 to 414
	Class 30	As cast	174 H _B	59	448 to 517
	Class 40	As cast	201 H _B	90	517 to 586
ASTM A536 ductile (nodular) iron	Grade 60-40-18	Annealed	140 H _B	152 to 228	530 to 634
	Grade 80-55-06	Quenched and tempered	179 H _B	152 to 228	530 to 634
	Grade 100-70-03	Quenched and tempered	229 H _B	186 to 276	634 to 772
	Grade 120-90-02	Quenched and tempered	269 H _B	214 to 303	710 to 869
Bronze	ASTM B-148 alloy 954	Sand cast Heat treated	$\sigma_{uts \min} = 275 \text{ MPa}$ $\sigma_{uts \min} = 620 \text{ MPa}$	39 163	207 448

through hardened steels and from Table 7.5 for iron and bronze gears. Alternatively, an estimate for σ_{HP} can be obtained from Eq. 7.32.

$$\sigma_{HP} = 182.7 + 2.382H_B \quad (7.32)$$

where σ_{HP} is the allowable contact stress in MPa.

The hardness ratio factor is used only for the gear. Its purpose is to adjust the surface strengths to account for different hardnesses of the gear and pinion. The hardness ratio factor for a gear is given by:

$$Z_W = C_H = 1 + A'(m_G - 1) \quad (7.33)$$

where

$$A' = 8.98 \times 10^{-3} \frac{\downarrow H_{BP}}{-H_{BG}} - 8.29 \times 10^{-3} \quad (7.34)$$

for

$$1.2 \leq \frac{H_{BP}}{H_{BG}} \leq 1.7 \quad (7.35)$$

where H_{BP} and H_{BG} are the Brinell hardness of the pinion and gear, respectively.

$$m_G = \frac{N_G}{N_P}$$

Example 7.2

A gear drive proposal consists of a 20° pressure angle spur pinion with 18 teeth driving a 50 tooth gear. The module of the gears is 2.5 and the face width is 30 mm. The pinion runs at 1425 rpm. The gear set is to transmit 3 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 6 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.

Solution

The pinion diameter, $d_P = 18 \times 2.5 = 45 \text{ mm}$.

The gear diameter, $d_G = 50 \times 2.5 = 125 \text{ mm}$.

The speed ratio, Eq. 7.24, is given by

$$m_G = \frac{N_G}{N_P} = \frac{50}{18} = 2.778$$

Assume the overload factor, $K_O = 1$.

$$V = \omega r = 1425 \times \frac{2\pi}{60} \times \frac{0.045}{2} = 3.358 \text{ m/s}$$

$$W_t = \frac{\text{Power}}{V} = \frac{3000}{3.358} = 893.5 \text{ N}$$

From Eqs 7.15 and 7.16

$$B = \frac{(12 - Q_V)^{2/3}}{4} = \frac{(12 - 6)^{2/3}}{4} = 0.8255$$

$$\begin{aligned} A &= 50 + 56(1 - B) \\ &= 50 + 56(1 - 0.8255) = 59.77 \end{aligned}$$

The dynamic factor, Eq. 7.14, is given by

$$\begin{aligned} K'_v &= \frac{\downarrow A + \sqrt{200V}}{\rightarrow A}^B \\ &= \frac{\downarrow 59.77 + \sqrt{200 \times 3.358}}{\rightarrow 59.77}^{0.8255} \\ &= 1.346 \end{aligned}$$

Assume the size factor $K_s = 1$.

The teeth are uncrowned, so $C_{mc} = 1$.

$F = 30 \text{ mm}$, so from Eq. 7.19

$$\begin{aligned} C_{pf} &= \frac{F}{10d} - 0.0375 + 0.4921F \\ &= \frac{0.03}{10 \times 0.045} - 0.0375 + 0.4921 \times 0.03 \\ &= 0.04392 \end{aligned}$$

$$C_{pm} = 1$$

$$C_e = 1$$

For commercially enclosed units, from Eq. 7.21

$$\begin{aligned} C_{ma} &= 0.127 + 0.622F - 0.01442F^2 \\ &= 0.0127 + 0.622 \times 0.03 - 0.01442 \\ &\quad \times 0.03^2 \\ &= 0.1456 \end{aligned}$$

The load distribution factor K_H , Eq. 7.17, is given by

$$\begin{aligned} K_H &= 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \\ &= 1 + 1(0.04392 \times 1 + 0.1456 \times 1) \\ &= 1.190 \end{aligned}$$

The repeatedly applied bending strength stress cycle factor for the pinion, Y_{NP} , from Figure 7.3, taking the worst case line for 10^8 cycles, is given by

$$Y_{NP} = 1.6831(1 \times 10^8)^{-0.0323} = 0.9284$$

The gear does comparatively fewer cycles and the repeatedly applied bending strength stress cycle factor for the gear, Y_{NG} , is given by

$$\begin{aligned} Y_{NG} &= 1.6831 \frac{\downarrow 1 \times 10^8}{\rightarrow m_G}^{-0.0323} \\ &= 1.6831 \frac{\downarrow 1 \times 10^8}{\rightarrow 2.778}^{-0.0323} \\ &= 0.9284 \end{aligned}$$

The pitting resistance stress cycle factor for the pinion, Z_{NP} , from Figure 7.4, taking the worst case line for 10^8 cycles, is given by

$$\begin{aligned} Z_{NP} &= 2.466N^{-0.056} \\ &= 2.466(1 \times 10^8)^{-0.056} = 0.879 \end{aligned}$$

The gear does comparatively fewer cycles and the pitting resistance stress cycle factor for the gear, Z_{NG} , is given by

$$\begin{aligned} Z_{NG} &= 2.466 \frac{\downarrow 1 \times 10^8}{\rightarrow m_G}^{-0.056} \\ &= 2.466 \frac{\downarrow 1 \times 10^8}{\rightarrow 2.778}^{-0.056} = 0.9308 \end{aligned}$$

From Table 7.4, the reliability factor $Y_Z = 0.85$ for a reliability of 90 per cent.

Assume the temperature factor $Y_\theta = 1$.

Assume the surface condition factor $Z_R = 1$.

For a steel pinion in mesh with a steel gear the elastic coefficient, $Z_E = 191$ (MPa)^{0.5} from Table 7.1.

The allowable bending stress for through hardened grade 1 steel, in the case of the pinion from Eq. 7.25 is given by

$$\begin{aligned}\sigma_{FP} &= 0.533H_B + 88.3 \\ &= 0.533 \times 240 + 88.3 = 216 \text{ MPa}\end{aligned}$$

For the gear

$$\sigma_{FG} = 0.533 \times 200 + 88.3 = 195 \text{ MPa}$$

The allowable contact stress for through hardened steel gears, in the case of the pinion, Eq. 7.30, is given by

$$\begin{aligned}\sigma_{HP} &= 2.22H_B + 200 \\ &= 2.22 \times 240 + 200 \\ &= 733 \text{ MPa}\end{aligned}$$

For the gear

$$\sigma_{HG} = 2.22 \times 200 + 200 = 644 \text{ MPa}$$

From Eq. 7.34

$$\begin{aligned}A' &= 8.98 \times 10^{-3} \frac{\downarrow H_{BP}}{-H_{BG}} - 8.29 \times 10^{-3} \\ &= 8.98 \times 10^{-3} \frac{\downarrow 240}{-200} - 8.29 \times 10^{-3} \\ &= 2.486 \times 10^{-3}\end{aligned}$$

The hardness ratio factor, Z_W , Eq. 7.33, is given by

$$\begin{aligned}Z_W &= 1 + A'(m_G - 1) \\ &= 1 + 2.486 \times 10^{-3}(2.778 - 1) \\ &= 1.004\end{aligned}$$

From Figure 7.2, the bending strength geometry factors $Y_{JP} = 0.316$ and $Y_{JG} = 0.396$.

The geometry factor Z_I , Eq. 7.22, is given by

$$\begin{aligned}Z_I &= \frac{\cos \phi_t \sin \phi_t}{2m_N} \frac{m_G}{m_G + 1} \\ &= \frac{\cos 20 \sin 20}{2} \frac{2.778}{2.778 + 1} = 0.1182\end{aligned}$$

The AGMA bending stress, Eq. 7.6, is given by

$$\sigma = W_t K_O K'_v K_s \frac{1}{Fm} \frac{K_H K_B}{Y_J}$$

Substitution for the factors and geometrical variables gives, for the pinion,

$$\begin{aligned}\sigma_P &= 893.5 \times 1 \times 1.346 \times 1 \\ &\quad \times \frac{1}{0.03 \times 0.0025} \times \frac{1.190 \times 1}{0.316} \\ &= 60.39 \times 10^6 \text{ Pa}\end{aligned}$$

For the gear

$$\begin{aligned}\sigma_G &= 893.5 \times 1 \times 1.346 \times 1 \\ &\quad \times \frac{1}{0.03 \times 0.0025} \times \frac{1.190 \times 1}{0.396} \\ &= 48.18 \times 10^6 \text{ Pa}\end{aligned}$$

The allowable bending stress, Eq. 7.10, is given by

$$\sigma_{all} = \frac{\sigma_{FP} Y_N}{S_F Y_\theta Y_Z}$$

Setting $S_F = 1$, and determining the allowable bending stress for the pinion gives

$$\sigma_{all} = \frac{216 \times 0.9284}{1 \times 1 \times 0.85} = 236 \text{ MPa}$$

The factor of safety for the pinion, S_{FP} , is given by

$$S_{FP} = \frac{236}{60.4} = 3.91$$

Setting $S_F = 1$, and determining the allowable bending stress for the gear gives

$$\sigma_{\text{all}} = \frac{195 \times 0.9583}{1 \times 1 \times 0.85} = 220 \text{ MPa}$$

The factor of safety for the gear, S_{FG} , is given by

$$S_{FG} = \frac{220}{48.2} = 4.56$$

The values for S_{FP} and S_{FG} are greater than 1, indicating that the design seems acceptable at this stage.

The contact stress, Eq. 7.8, is given by

$$\sigma_c = Z_e \frac{\downarrow}{\rightarrow} W_t K_O K'_v K_s \frac{K_H}{F_d} \frac{Z_R}{Z_I}^{0.5}$$

For the pinion

$$\begin{aligned} \sigma_c &= 191 \sqrt{\frac{893.5 \times 1 \times 1.346 \times 1}{\times \frac{1.190}{0.045 \times 0.03} \times \frac{1}{0.1182}}} \text{ kPa} \\ &= 191 \times 2994 \text{ kPa} = 572 \text{ MPa} \end{aligned}$$

For the gear

$$\begin{aligned} \sigma_c &= 191 \sqrt{\frac{893.5 \times 1 \times 1.346 \times 1}{\times \frac{1.190}{0.125 \times 0.03} \times \frac{1}{0.1182}}} \text{ kPa} \\ &= 191 \times 1796 \text{ kPa} = 342 \text{ MPa} \end{aligned}$$

The allowable contact stress, Eq. 7.12, is given by

$$\sigma_{c,\text{all}} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_0 Y_Z}$$

For the pinion, omitting Z_W and setting $S_H = 1$ gives

$$\begin{aligned} \sigma_{c,\text{all}} &= \frac{\sigma_{HP}}{S_H} \frac{Z_N}{Y_0 Y_Z} = \frac{732.8}{1} \times \frac{0.879}{1 \times 0.85} \\ &= 758 \text{ MPa} \end{aligned}$$

$$S_{HP} = \frac{758}{572} = 1.33$$

For the gear

$$\begin{aligned} \sigma_{c,\text{all}} &= \frac{644}{1} \times \frac{0.9308 \times 1.004}{1 \times 0.85} \\ &= 708 \text{ MPa} \end{aligned}$$

$$S_{HG} = \frac{708}{342} = 2.06$$

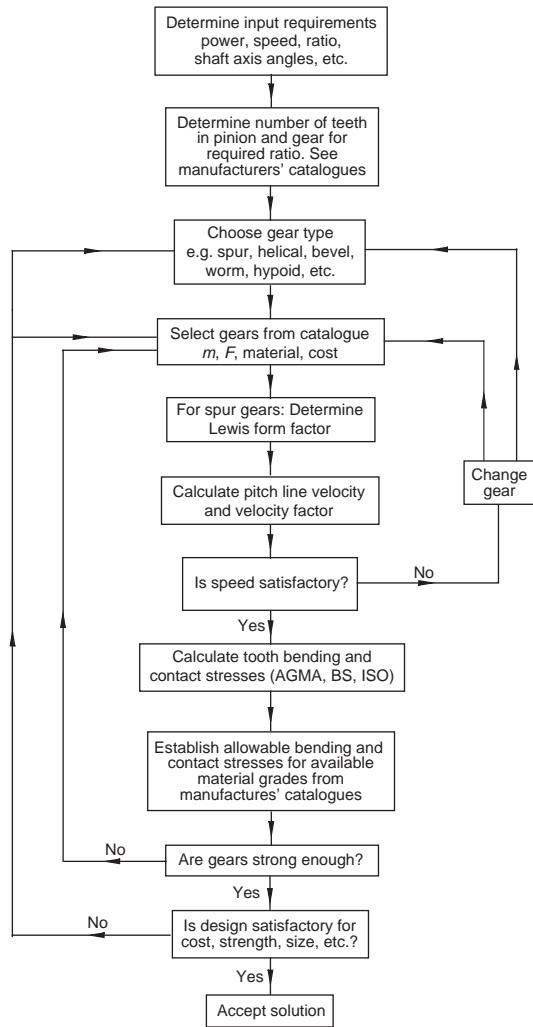
The values for S_{HP} and S_{HG} are greater than 1 indicating that the design seems acceptable at this stage.

Care needs to be taken when comparing values for the factors of safety for bending and contact stress. S_F should be compared with S_H^2 if the teeth are uncrowned or S_F with S_H^3 if the teeth are crowned. The exponent linearizes the safety factor for contact stress with respect to the transmitted load.

In Example 7.2, $S_{FP} = 3.91$ and $S_{HP} = 1.33$. Comparing $S_{FP} = 3.91$ with $S_{HP}^2 = 1.77$ shows that the risk of failure for the pinion is more likely due to wear. Similarly for the gear, comparing $S_{FP} = 4.56$ with $S_{HG}^2 = 4.24$ shows that the risk of failure is evenly balanced between bending and wear. In conclusion, failure due to wear is most likely to cause retirement of the gear set.

7.4 Gear selection procedure

Figure 7.5 illustrates a general selection procedure for gears. Normally the required rotational speeds for the gear and pinion are known from the design

**Figure 7.5** General selection procedure for gears.

requirements, as is the power the drive must transmit. A designer has to decide on the type of gears to be used, their arrangements on the shafts, materials, heat treatments, gear geometry: number of teeth, module, tooth form, face width, quality. There is no one solution to a gear design requirement. Several 'good' or optimum designs are possible. The designer's judgement and creativity are important aspects. While Figure 7.5 shows the general selection procedure, the steps given below outline a general calculation procedure for a pinion and gear.

Select the number of teeth for the pinion and the gear to give the required gear ratio.

Propose a material for the pinion and gear.

Select a module, m . Figure 6.13 can be used as a visual guide to the size of gear teeth for a given module.

Calculate the pitch diameters

$$d_p = mN_p \quad d_G = mN_G$$

Calculate the pitch line velocity

$$V = \omega r$$

Calculate the transmitted load

$$W_t = \frac{\text{Power}}{V}$$

Specify the face width or calculate using the procedure outlined in Section 6.6 from

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

As a general rule spur gears should be designed with a face width between three and five times the circular pitch.

Check the proposed gears for bending and contact stress. Determine the AGMA factors K_O , K'_v , K_H , Y_N , Z_N , Z_W , Z_I , Y_J .

Calculate the AGMA bending stress (Eq. 7.6)

$$\sigma = W_t K_O K'_v K_s \frac{1}{Fm} \frac{K_H K_B}{Y_J}$$

Calculate the AGMA contact stress (Eq. 7.8)

$$\sigma_c = Z_e \downarrow W_t K_O K'_v K_s \frac{K_H}{Fd} \frac{Z_R}{Z_I} 0.5$$

Calculate the allowable bending stress (Eq. 7.10)

$$\sigma_{\text{all}} = \frac{\sigma_{\text{FP}} Y_N}{S_F Y_\theta Y_Z}$$

Calculate the allowable contact stress (Eq. 7.12)

$$\sigma_{c,all} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z}$$

If values for the factors of safety S_F and S_H are unsatisfactory, either change initial geometry or change materials.

This procedure can be re-ordered say if a particular gear material is to be used and stress is to be limited to a certain value, or if a particular factor of safety is required etc.

7.5 Conclusions

Gears can fail due to excessive bending stresses or by wear. Wear can take the form of pitting of the surface due to excessive contact stresses, scoring due to failure of lubrication or abrasion due to the presence of foreign particulates. This chapter has introduced the AGMA equations for modelling bending and contact stresses in spur gears. The calculations involved in evaluating the AGMA bending and contact stresses are lengthy and rely on a large number of factors and geometrical parameters. These can be readily embedded in a program or spreadsheet calculation procedure if required.

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Websites

At the time of going to press, the internet contains useful information relating to this chapter at the following sites.

- www.agma.org
- www.bsi.org.uk
- www.esdu.com
- www.iso.org

Nomenclature

Generally, preferred SI units have been stated.	
C_e	used in evaluating the load distribution factor K_H
C_f	surface condition factor for pitting resistance
C_H	hardness ratio factor
C_{ma}	used in evaluating the load distribution factor K_H
C_{mc}	used in evaluating the load distribution factor K_H
C_p	elastic coefficient $((\text{MPa})^{0.5})$
C_{pf}	used in evaluating the load distribution factor K_H
C_{pm}	used in evaluating the load distribution factor K_H
d	pitch diameter (m)
E_G	Young's modulus for the gear (N/m^2)
E_p	Young's modulus for the pinion (N/m^2)
F	face width (m)
h_t	gear tooth whole depth (m)
H	power
H_B	Brinell hardness
H_{BG}	Brinell hardness of the gear
H_{BP}	Brinell hardness of the pinion
I	geometry factor for pitting resistance
J	geometry factor for bending strength
K_B	rim thickness factor
K_H	load distribution factor
K_m	load distribution factor
K_O	overload factor
K_R	reliability factor
K_s	size factor
K_T	temperature factor
K'_v	velocity factor
m	module
m_G	speed ratio
n	speed (rpm)
N	number of teeth
N_G	number of teeth in the gear
N_p	number of teeth in the pinion
P_d	transverse diametral pitch
Q_v	quality control numbers, transmission accuracy level number
r_1	radius of curvature (m)
r_2	radius of curvature (m)
S	distance (m)
S_c	allowable contact stress (lb/in^2)
S_F	AGMA factor of safety

S_H	AGMA factor of safety
S_t	allowable bending stress (lb/in^2)
S_1	distance (m)
t_R	rim thickness below the tooth (m)
V	velocity (m/s)
W_t	transmitted load (N)
Y	Lewis form factor
Y_J	geometry factor for bending strength
Y_N	stress cycle factor for bending stress
Y_Z	reliability factor
Y_θ	temperature factor
Z_e	elastic coefficient $(\text{MPa})^{0.5}$
Z_I	geometry factor for pitting resistance
Z_N	stress cycle life factor
Z_R	surface condition factor for pitting resistance
Z_W	hardness ratio factor
ϕ	pressure angle ($^\circ$)
ϕ_t	transverse pressure angle ($^\circ$)
ν_G	Poisson's ratio for the gear
ν_p	Poisson's ratio for the pinion
σ	bending stress (N/m^2)
σ_c	contact stress (N/m^2)
σ_{FP}	allowable bending stress (MPa)
σ_{HP}	allowable contact stress (MPa)
σ_p	permissible stress (N/m^2)

Worksheet

- 7.1 A gear drive proposal consists of a 20° pressure angle spur pinion with 18 teeth driving a 48 tooth gear. The module of the gears is 4 and the face width is 50 mm. The pinion runs at 350 rpm. The gear set is to transmit 5 kW and the load and drive can be considered smooth. The pinion material proposed is grade 1 steel and the Brinell hardness is 240. Cast iron is proposed for the gear with a Brinell hardness of 175. The gears are to be manufactured to number 7 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.
- 7.2 A gear drive proposal consists of a 20° pressure angle spur pinion with 22 teeth driving a 32 tooth gear. The module of the gears is 2.5 and the face width is 30 mm. The pinion runs at 450 rpm.

The gear set is to transmit 1 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 6 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.

- 7.3 A gear drive proposal consists of a 20° pressure angle spur pinion with 20 teeth driving a 36 tooth gear. The module proposed for the gears is either 1.5 with a face width of 20 mm or a module of 2 with a face width of 25 mm. The pinion runs at 100 rpm. The gear set is to transmit 0.15 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 6 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear and identify which size of gear should be selected.
- 7.4 A gear drive proposal for a sawing machine consists of a 20° pressure angle spur pinion with 19 teeth driving a 84 tooth gear. The module of the gears is 3 and the face width is 40 mm. The pinion runs at 750 rpm. The gear set is to transmit 3.75 kW and in order to accommodate variations in the load, an overload factor of two is considered appropriate. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 6 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.
- 7.5 A gear drive proposal consists of a 20° pressure angle spur pinion with 20 teeth driving a 38 tooth gear. The module of the gears is 5 and the face width is 45 mm. The pinion runs at 1450 rpm. The gear set is to transmit 12 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 7 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 99.9 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.
- 7.6 A gear drive proposal consists of a 20° pressure angle spur pinion with 19 teeth driving a 77 tooth gear. The module of the gears is 6 and the face width is 75 mm. The pinion runs at 375 rpm. The gear set is to transmit 14.5 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 300 and 200, respectively. The gears are to be manufactured to number 6 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.
- 7.7 A gear drive proposal consists of a 20° pressure angle spur pinion with 17 teeth driving a 52 tooth gear. The module of the gears is 2.5 and the face width is 40 mm. The pinion runs at 1800 rpm. The gear set is to transmit 3 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 6 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.
- 7.8 A gear drive proposal for an electric motor driven pump consists of a 20° pressure angle spur pinion with 18 teeth driving a 76 tooth gear. The module of the gears is 3 and the face width is

35 mm. The pinion runs at 2970 rpm. The gear set is to transmit 5 kW and the load and drive can be considered smooth. The gear material proposed is grade 1 steel and the Brinell hardness of the pinion and gear is 240 and 200, respectively. The gears are to be manufactured to number 7 AGMA quality standard and the teeth are uncrowned. The gears are straddle mounted within an enclosed unit. Assume a pinion life of 10^8 cycles and a reliability of 90 per cent. Calculate the AGMA bending and contact stresses and the corresponding factors of safety for both the pinion and the gear.

Answers

- 7.1 Pinion: $\sigma = 51.6 \text{ MPa}$, $\sigma_c = 484 \text{ MPa}$, $S_F = 4.6$, $S_H = 1.6$. Gear: $\sigma = 41.1 \text{ MPa}$, $\sigma_c = 296 \text{ MPa}$, $S_F = 5$, $S_H = 1.7$.
- 7.2 Pinion: $\sigma = 43.9 \text{ MPa}$, $\sigma_c = 507 \text{ MPa}$, $S_F = 5.4$, $S_H = 1.5$. Gear: $\sigma = 40.4 \text{ MPa}$, $\sigma_c = 420 \text{ MPa}$, $S_F = 5.3$, $S_H = 1.6$.

- 7.3 $m = 1.5$: Pinion: $\sigma = 124 \text{ MPa}$, $\sigma_c = 844 \text{ MPa}$, $S_F = 1.9$, $S_H = 0.9$. Gear: $\sigma = 108 \text{ MPa}$, $\sigma_c = 629 \text{ MPa}$, $S_F = 1.9$, $S_H = 0.76$. $m = 2$: Pinion: $\sigma = 56.2 \text{ MPa}$, $\sigma_c = 570 \text{ MPa}$, $S_F = 4.2$, $S_H = 1.33$. Gear: $\sigma = 49.1 \text{ MPa}$, $\sigma_c = 425 \text{ MPa}$, $S_F = 4.4$, $S_H = 1.6$.
- 7.4 Pinion: $\sigma = 131 \text{ MPa}$, $\sigma_c = 795 \text{ MPa}$, $S_F = 1.8$, $S_H = 0.95$. Gear: $\sigma = 102 \text{ MPa}$, $\sigma_c = 328 \text{ MPa}$, $S_F = 2.2$, $S_H = 1.9$.
- 7.5 Pinion: $\sigma = 35.8 \text{ MPa}$, $\sigma_c = 451 \text{ MPa}$, $S_F = 4.5$, $S_H = 1.1$. Gear: $\sigma = 30.9 \text{ MPa}$, $\sigma_c = 327 \text{ MPa}$, $S_F = 4.8$, $S_H = 1.4$.
- 7.6 Pinion: $\sigma = 70.5 \text{ MPa}$, $\sigma_c = 584 \text{ MPa}$, $S_F = 3.8$, $S_H = 1.6$. Gear: $\sigma = 54.5 \text{ MPa}$, $\sigma_c = 290 \text{ MPa}$, $S_F = 4.1$, $S_H = 2.5$.
- 7.7 Pinion: $\sigma = 44.3 \text{ MPa}$, $\sigma_c = 485 \text{ MPa}$, $S_F = 5.3$, $S_H = 1.6$. Gear: $\sigma = 33.2 \text{ MPa}$, $\sigma_c = 278 \text{ MPa}$, $S_F = 6.6$, $S_H = 2.6$.
- 7.8 Pinion: $\sigma = 29.8 \text{ MPa}$, $\sigma_c = 388 \text{ MPa}$, $S_F = 6.7$, $S_H = 1.7$. Gear: $\sigma = 23.2 \text{ MPa}$, $\sigma_c = 189 \text{ MPa}$, $S_F = 8.2$, $S_H = 3.3$.

Learning objectives achievement

- Can you determine the AGMA bending stress for a spur gear?
Can you determine the AGMA contact stress for a spur gear?



Section 7.3
Section 7.3

BELT AND CHAIN DRIVES

Belt and chain drives are used to transmit power from one rotational drive to another. A belt is a flexible power transmission element that runs tightly on a set of pulleys. A chain drive consists of a series of pin-connected links that run on a set of sprockets. The aims of this chapter are to introduce various types of belt and chain drives and to develop selection procedures for wedge and flat belts and for roller chains.

LEARNING OBJECTIVES

At the end of this chapter you should:

- be able to differentiate whether to use a chain, belt or gear drive for a given application;
- be able to select a particular wedge belt for a given speed and power requirement;
- be able to determine the width of flat belt for a given speed and power requirement;
- be able to determine an appropriate chain ratio using standard components;
- be able to select an appropriate chain type and pitch for a given application;
- be able to specify an appropriate chain drive for a given speed and power requirement.

8.1 Introduction

Belt and chain drives consist of flexible elements running on either pulleys or sprockets as illustrated in Figure 8.1. The purpose of a belt or chain drive is to transmit power from one rotating shaft to another. The speed ratio between the driving and

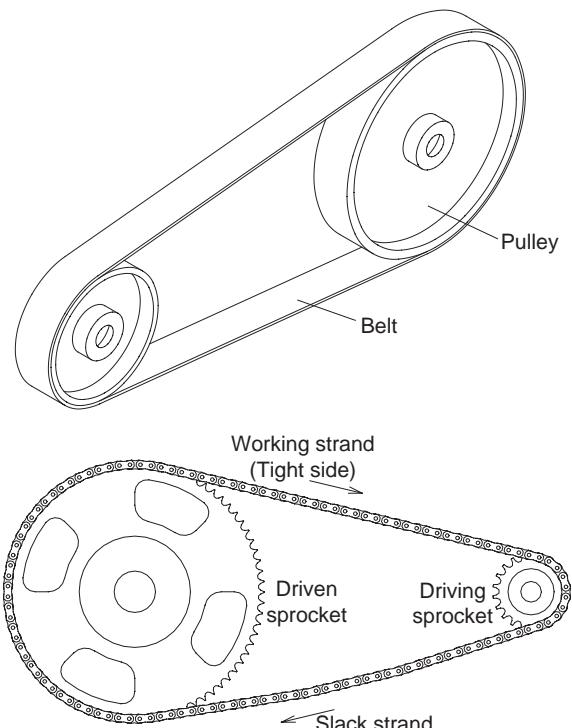


Figure 8.1 Belt drive and chain drive.

driven shaft is dependent on the ratio of the pulley or sprocket diameters as is given by:

$$V_{\text{pitchline}} = \omega_1 R_1 = \omega_2 R_2 \quad (8.1)$$

$$\text{Angular velocity ratio} = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} \quad (8.2)$$

where $V_{\text{pitchline}}$ is pitchline velocity (m/s); ω_1 , angular velocity of driving pulley or sprocket (rad/s); ω_2 , angular velocity of driven pulley or sprocket

(rad/s); R_1 , radius of driving pulley or sprocket (m); and R_2 , radius of driven pulley or sprocket (m).

A belt drive transmits power between shafts by using a belt to connect pulleys on the shafts by means of frictional contact or mechanical interference. A chain consists of a series of links connected by pins. The chain is designed to mesh with corresponding teeth on sprockets located on both the driving and the driven shafts.

Power transmission between shafts can be achieved by a variety of means including belt, chain and gear drives and their use should be compared for suitability and optimization for any given application. In addition to power transmission, constant speed ratio or synchronization of the angular position of the driving and driven shaft may be critical to operation. This can be achieved by means of gears, chains or special toothed belts, called synchronous or timing belts.

Both belt and chain drives can transmit power between shafts that are widely separated giving designers greater scope for control over machine layout. In comparison to gears they do not require such precision in the location of centre distances. Both belt and chain drives are often cheaper than the equivalent gear drive. Belts and chains are generally complementary covering a range of operational requirements.

In general, belt drives are used when rotational speeds are of the order of 10 to 60 m/s. At lower speeds the tension in the belt becomes too high for typical belt sections. At higher speeds centrifugal forces throw the belts off the pulleys reducing the torque capacity and dynamic phenomena reduce the effectiveness and life of the belt drive. Chain drives are typically used at lower speeds and consequently higher torques than belts. Recall that for a rotating machine torque is proportional to power/ ω . As the angular velocity reduces, for a given power, the torque increases.

Belt drives have numerous advantages over gear and chain drives including easy installation, low maintenance, high reliability, adaptability to non-parallel drive and high transmission speeds. The principle disadvantages of belt drives are

their limited power transmission capacity and limited speed ratio capability. Belt drives are less compact than either gear or chain drives and are susceptible to changes in environmental conditions, such as contamination with lubricants. In addition, vibration and shock loading can damage belts.

Chains are usually more compact than belt drives for a given speed ratio and power capacity. Chain drives are generally more economical than the equivalent gear drive and are usually competitive with belt drives. Chains are inherently stronger than belt drives due to the use of steels in their manufacture and can therefore support higher tension and transmit greater power. The disadvantages of chain drives are limited speed ratios and power transmission capability and also safety issues. Chain drives can break and get thrown off the sprockets with large forces and high speeds. Guards should be provided for chain drives and some belt drives to prevent damage caused by a broken chain or belt and also to prevent careless access to the chain or belt drive.

8.2 Belt drives

As mentioned previously, a belt drive consists of a flexible element that runs on a set of pulleys mounted on the shafts. Belt drives can be used to simply transmit power between one and another with the speed of the driving and driven shaft equal. In this case the pulley diameters would be equal. Alternatively the driven shaft velocity can be decreased by using a bigger diameter pulley on the driven shaft than on the driving shaft. It is also possible to use belt drives to step up or increase the speed of a driven shaft, but this is a less common application.

There are various types of belt drive configurations including flat, round, V, wedge and synchronous belt drives, each with their individual merits. The cross-sections of various belts are illustrated in Figure 8.2. Most belts are manufactured from rubber or polymer-based materials.

Power is transmitted by means of friction in the case of flat, round, wedge and V belts and by a combination of friction and positive mechanical interference in the case of synchronous belt drives. When the driven shaft rotates, friction between the pulley and the belt causes the belt to grip the pulley increasing the tension in the side near to the point of first rotational contact. The tensile force exerts a tangential force, and associated torque, on the driven pulley. The opposite side of the belt is also under tension, but to a lesser extent and is referred to as the slack side.

A frequent application of belt drives is to reduce the speed output from electric motors which typically run at specific synchronous speeds which are high in comparison to the desired application drive speed. Because of their good 'twistability', belt drives are well suited to applications where the rotating shafts are in different planes. Some of the

standard layouts are shown in Figure 8.3. Belts are installed by moving the shafts closer together, slipping the belt over the pulleys and then moving the shafts back into their operating locations.

Flat belts have high strength, can be used for large speed ratios ($>8:1$), have a low pulley cost, give low noise levels and are good at absorbing torsional vibration. The belts are typically made from multiple plies with each layer serving a special purpose. A typical three-ply belt consists of a friction ply made from synthetic rubber, polyurethane or chrome leather, a tension ply made from polyamide strips or polyester cord and an outer skin made from polyamide fabric, chrome leather or an elastomer. The corresponding pulleys are made from cast iron or polymer materials and are relatively smooth to limit wear. The driving force is limited by the friction between the belt and the pulley. Applications include manufacturing tools, saw mills,

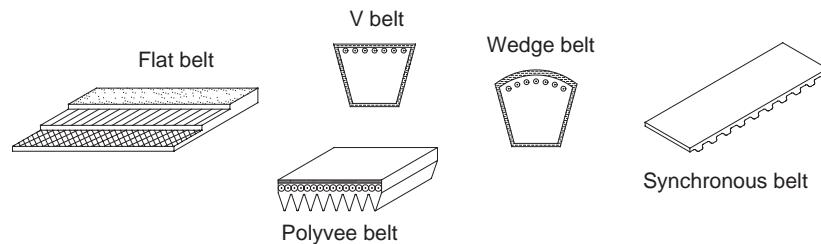


Figure 8.2 Various belt cross-sections.

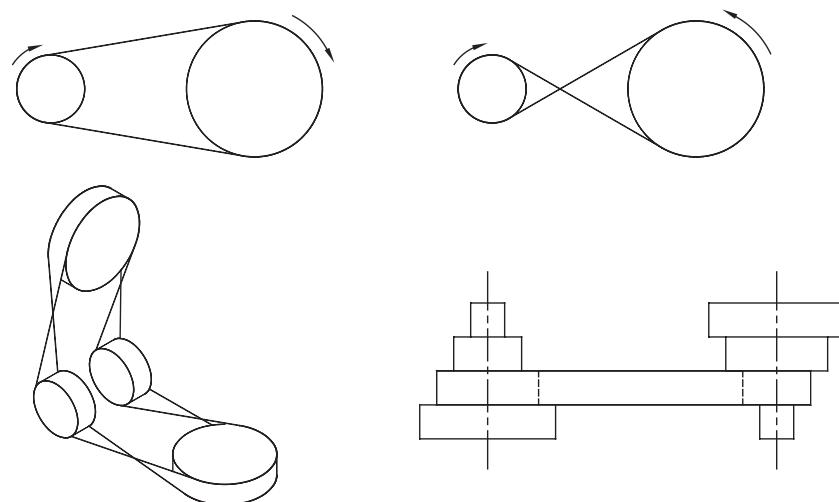


Figure 8.3 Pulley configurations.

textile machinery, food processing machines, multiple spindle drives, pumps and compressors.

The most widely used type of belt in industrial and automotive applications is the V belt or wedge belt. It is familiar from its automotive application where it is used to connect the crankshaft to accessory drives such as the alternator, water pump and cooling fan. It is also used for general engineering purposes from domestic appliances to heavy duty rolling machines. The V- or wedge-shape causes the belt to wedge into the corresponding groove in the pulley increasing friction and torque capacity. Multiple belts are commonly used so that a cheaper small cross-sectional area belt can be used to transmit more power. Note that long centre distances are not recommended for V or wedge belts.

Synchronous belts, also called timing belts, have teeth which mesh with corresponding teeth on the

pulleys. This mechanical interference or positive contact between the pulleys and the belt provides angular synchronization between the driving and the driven shafts and ensures a constant speed ratio. Synchronous belts combine the advantages of normal friction belt drives with the capability of synchronous drive. The meshing of the belt and pulleys is critical to their effective operation. The teeth of the advancing belt must mesh correctly with the corresponding grooves on the pulley wheels and remain in mesh throughout the arc of contact. To achieve this, the pitch of the belts and the pulleys must correspond exactly. A disadvantage of synchronous belts can be the noise generated by compression of air between the teeth especially at high speeds.

Table 8.1 lists the comparative merits of various belt drives. Figure 8.4 provides a guide for the selection of belt type.

Table 8.1 Comparison of belt performance

Parameter	Flat belt drive	V belt drive	Wedge belt drive	Synchronous belt drive
Optimum efficiency	98%	80%	86%	98%
Maximum speed (m/s)	70	30	40	50
Minimum pulley diameter (mm)	40	67	60	16
Maximum speed ratio	20	7	8	9
Optimum tension ratio	2.5	5	5	–

Reproduced from Hamilton, 1994.

8.2.1 Belt selection

Two approaches are presented here for the selection of a belt drive. Use can be made of the design procedures and accompanying charts provided by most belt drive manufacturers. The use of these charts is illustrated by considering a wedge belt drive. Alternatively use can be made of fundamental relationships for the belt tensions and torque transmission and a belt selected based on its maximum permissible tensile stress. These equations are illustrated by an example using a flat belt drive.

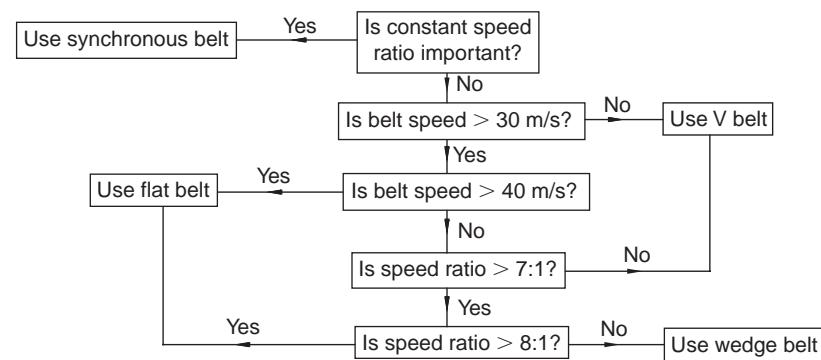


Figure 8.4 Procedure for the selection of belt type (after Hamilton, 1994).

Table 8.2 Service factors

Types of driven machine	Type of prime mover			Duty (hours per day)		
	<10	10–16	>16	<10	10–16	>16
Light duty, e.g. agitators (uniform density), blowers, exhausters and fans (up to 7.5 kW), centrifugal compressors, rotodynamic pumps, uniformly loaded belt conveyors	1.0	1.1	1.2	1.1	1.2	1.3
Medium duty, e.g. agitators (variable density), blowers exhausters and fans (over 7.5 kW), rotary compressors and pumps (other than centrifugal), nonuniformly loaded conveyors, generators, machine tools, printing machinery, sawmill machinery	1.1	1.2	1.3	1.2	1.3	1.4
Heavy duty, e.g. brick machinery, bucket elevators, reciprocating compressors and pumps, heavy duty conveyors, hoists, pulverizers, punches, presses, quarry plant, textile machinery	1.2	1.3	1.4	1.4	1.5	1.6
Extra heavy duty, e.g. crushers	1.3	1.4	1.5	1.5	1.6	1.8

After Fenner Power Transmission UK.

The typical design and selection procedure for use in conjunction with power-speed rating charts supplied by commercial belt companies is outlined below.

1. Define the operating conditions: These include the nominal power to be transmitted, the rotational speeds of the shafts, and any space, layout or other constraints such as environmental conditions.
2. Determine the service factor: Service factors are used to down-rate the power transmission capability listed by belt suppliers to account for the differences between practical applications and test conditions. Table 8.2 lists typical values for service factors.
3. Calculate the design power: This is the product of the nominal power and the service factor.

4. Select the belt type: To assist in the selection of which type of belt drive to use, the procedure given in Figure 8.4 can be used as a guideline.
5. Select a belt: Using a manufacturer's rating chart, select a specific belt for the design power and speed.
6. Select the pulley diameters: Pulleys are normally available in standard sizes. Choose the smallest sizes available, such that the speed ratio is acceptable.
7. Set the centre distance: This is dependent on the application. As a general guideline the centre distance should be greater than the diameter of the larger pulley.
8. Determine the belt length: Note that belts are usually manufactured in standard lengths. So some iteration around the design parameters

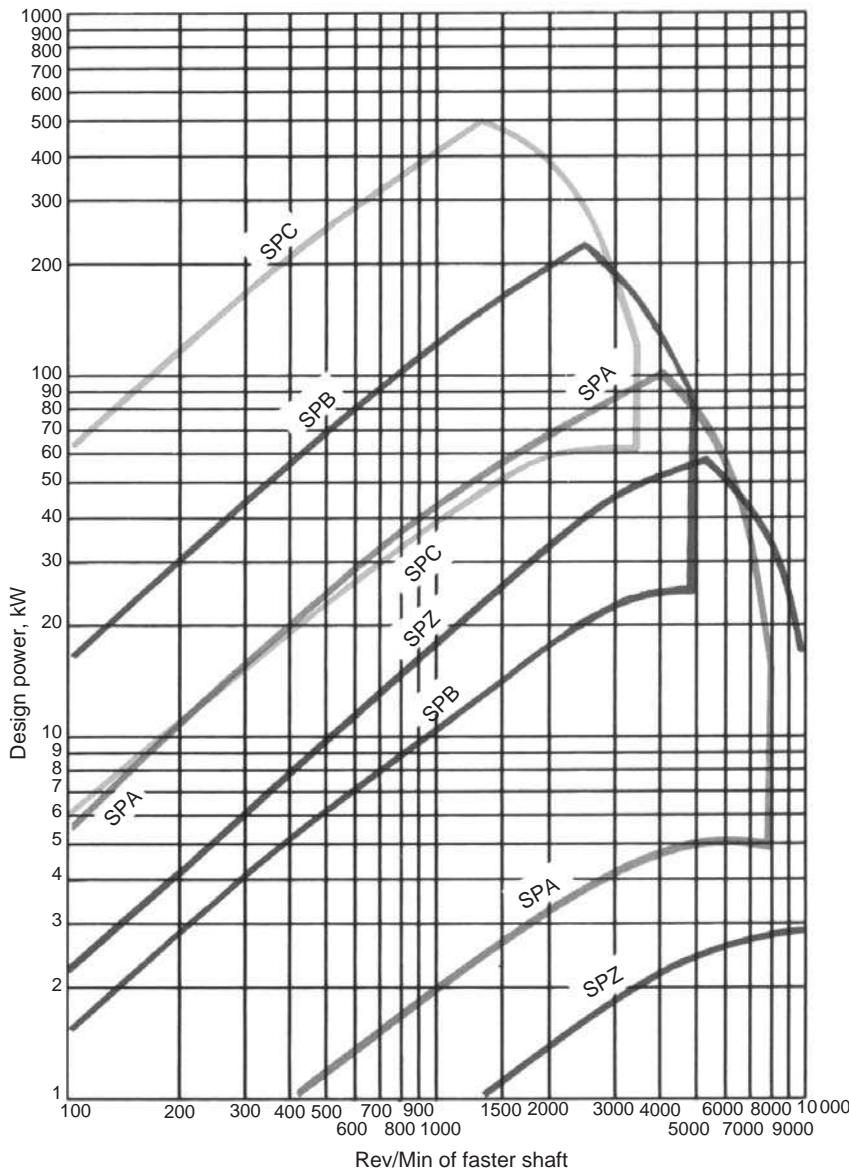


Figure 8.5 Selection chart for wedge belts (courtesy of Fenner Drives UK).

- may be necessary to arrive at a satisfactory compromise.
9. Apply power correction factors: These are used to compensate for speed ratio and the belt geometry and are provided within the belt manufacturer's design guides.
 10. Determine the allowable power per belt (or per belt width for flat belts): This is a function of the belt dimensions and is available from manufacturer's design guides.

11. Determine the number of belts: The number of belts is given by dividing the design power by the allowable power per belt and rounding up to the nearest integer.

Figure 8.5 shows a design chart for power rating versus speed and Tables 8.3 to 8.6 list values for the minimum pulley diameter, centre distance and power ratings for wedge belts. The use of these in conjunction with the procedure given is illustrated by the following example.

Table 8.3 Minimum recommended pulley diameters for wedge belt drives

Speed of faster shaft (rpm)	Minimum pulley diameters (mm)																			
	<1	3.0	4.0	5.0	7.5	10.0	15.0	20.0	25	30	40	50	60	75	90	110	130	150	200	250
500	56	90	100	112	125	140	180	200	212	236	250	280	280	315	375	400	450	475	500	560
600	56	85	90	100	112	125	140	180	200	212	224	250	265	280	300	335	375	400	475	500
720	56	80	85	90	100	106	132	150	160	170	200	236	250	265	280	300	335	375	450	500
960	56	75	80	85	95	100	112	132	150	180	180	200	224	250	280	280	300	335	400	450
1200	56	71	80	80	95	95	106	118	132	150	160	180	200	236	236	250	265	300	335	355
1440	56	63	75	80	85	85	100	112	125	140	160	170	190	212	236	236	250	280	315	335
1800	56	63	71	75	80	85	95	106	112	125	150	160	170	190	212	224	236	265	300	335
2880	56	60	67	67	80	80	85	90	100	112	125	140	160	170	180	212	224	236	—	—

Courtesy of Fenner Power Transmission UK.

Example 8.1

Select a wedge belt and determine the pulley diameters for a reciprocating compressor driven by a 28 kW two-cylinder diesel engine. The engine speed is 1500 rpm and the compressor speed is 950 rpm. The proposed distance between the engine and compressor shaft centres is approximately 1.5 m. The system is expected to be used for less than 10 h per day.

Solution

The speed ratio is

$$\frac{1500}{950} = 1.58.$$

From Table 8.2 for ‘heavy start’, ‘heavy duty’ and less than 10 h operation per day, the service factor is 1.4.

$$\text{Design power} = 28 \times 1.4 = 39.2 \text{ kW.}$$

From Figure 8.5, the combination of power equal to 39.2 kW and the speed equal to 1500 rpm is found to be within the range suitable for SPB belt drives.

From Table 8.3, for a design power of 39.2 kW and driving shaft speed of 1500 rpm an approximate minimum pulley diameter of 160 mm is suitable.

From Table 8.4, the actual pulley diameters can be selected by tracing down the left-hand side to the speed ratio and reading across for the pitch diameters of the driving and driven pulleys. The minimum diameter from Table 8.3 can be used as a guideline when there is a choice.

From Table 8.4, for a speed ratio of 1.58 suitable pitch diameters are $D_1 = 224 \text{ mm}$ and $D_2 = 355 \text{ mm}$. The nearest centre distance listed to the 1.5 m desired is 1.544 m and tracing the column upwards defines the belt length as 4000 mm and by tracing the shading upwards the arc-length correction factor is 1.05.

From Table 8.5, the rated power per belt for $n_1 = 1500 \text{ rpm}$ and $D_1 = 224 \text{ mm}$ is 15.97 kW/belt.

From Table 8.6, the additional power per belt accounting for the speed ratio is 1.11 kW.

The corrected value for the power per belt is $(15.97 + 1.11) \times 1.05 = 17.93 \text{ kW/belt}$.

The total number of belts is therefore

$$\frac{39.2}{17.93} = 2.19.$$

Table 8.4 Centre distances for selected SPB wedge belts

Combined arc and belt length correction factor			0.85		0.90		0.95		1.00		1.05		1.10		1.15					
Speed ratio	Pitch diameter of pulleys		Power per belt (kW)		Belt length for SPB belt type															
	Driver	Driven	1440 rpm	960 rpm	1250	1400	1800	2000	2240	2500	2800	3150	3550	4000	4500	5000	5600	6300	7100	8000
1.27	315	400	24.56	17.91	—	—	—	436	557	687	837	1013	1213	1438	1688	1938	2238	2588	2988	3438
1.27	118	150	6.37	4.58	414	489	689	789	—	—	—	—	—	—	—	—	—	—	—	—
1.28	125	160	7.24	5.18	401	476	676	776	—	—	—	—	—	—	—	—	—	—	—	—
1.29	140	180	7.69	5.54	373	448	648	748	869	998	1148	1342	1524	1749	1999	2249	2549	2899	3299	3749
1.29	132	170	8.10	5.78	387	462	663	763	—	—	—	—	—	—	—	—	—	—	—	—
1.53	118	180	6.51	4.67	390	465	665	765	—	—	—	—	—	—	—	—	—	—	—	—
1.56	180	280	12.03	8.61	259	335	536	637	757	887	1038	1213	1413	1638	1888	2138	2438	2788	3188	3638
1.56	160	250	9.95	7.13	300	375	576	676	797	927	1077	1252	1452	1677	1927	2178	2478	2828	3228	3678
1.57	150	236	8.89	6.37	319	394	595	696	816	946	1096	1271	1471	1696	1946	2196	2496	2847	3247	3697
1.57	200	315	14.05	10.05	—	290	492	593	713	844	994	1169	1369	1594	1845	2095	2395	2745	3145	3595
1.58	224	355	16.54	11.86	—	—	440	541	662	793	943	1118	1319	1544	1794	2044	2344	2694	3095	3545
1.59	315	500	24.81	18.09	—	—	—	—	471	603	754	930	1131	1357	1607	1858	2158	2508	2908	3359
1.60	125	200	7.49	5.35	368	443	644	744	—	—	—	—	—	—	—	—	—	—	—	—
1.60	140	224	7.95	5.71	336	412	613	713	833	963	1113	1288	1489	1714	1964	2214	2514	2864	3264	3714
1.60	250	400	19.02	13.68	—	—	382	484	605	736	886	1062	1262	1488	1738	1988	2288	2368	3039	3489
1.87	190	355	13.17	9.42	—	—	465	566	687	818	968	1144	1344	1570	1820	2070	2371	2721	3121	3571
1.89	212	400	15.37	11.00	—	—	409	511	633	764	915	1090	1291	1516	1767	2017	2317	2668	3068	3518
1.89	125	236	7.49	5.35	337	413	614	714	—	—	—	—	—	—	—	—	—	—	—	—
1.89	112	212	5.88	4.23	367	443	644	744	—	—	—	—	—	—	—	—	—	—	—	—
1.89	132	250	8.36	5.95	320	396	597	697	—	—	—	—	—	—	—	—	—	—	—	—
2.09	170	355	11.21	8.02	—	—	479	580	702	833	983	1159	1360	1585	1835	2086	2386	2736	3136	3586
2.10	150	315	9.11	6.53	—	324	528	629	750	881	1031	1207	1407	1633	1883	2133	2433	2784	3184	3634
2.11	190	400	13.26	9.48	—	—	424	526	648	780	931	1107	1307	1533	1784	2034	2334	2685	3085	3535
2.11	112	236	5.97	4.30	346	422	624	724	—	—	—	—	—	—	—	—	—	—	—	—
2.12	118	250	6.72	4.81	429	406	607	708	—	—	—	—	—	—	—	—	—	—	—	—

(continued)

Table 8.4 Continued

Combined arc and belt length correction factor				0.85		0.90		0.95		1.00		1.05		1.10		1.15				
Speed ratio	Pitch diameter of pulleys		Power per belt (kW)	Belt length for SPB belt type																
	Driver	Driven		1440 rpm	960 rpm	1250	1400	1800	2000	2240	2500	2800	3150	3550	4000	4500	5000	5600	6300	7100
2.23	224	500	16.64	11.92	—	—	—	408	534	667	820	997	1198	1425	1676	1926	2227	2578	2978	3429
2.24	125	280	7.59	5.42	297	374	577	677	—	—	—	—	—	—	—	—	—	—	—	—
2.25	140	315	8.04	5.78	252	331	535	637	758	888	1039	1214	1415	1640	1891	2141	2441	2791	3191	3642
2.25	280	630	21.87	15.81	—	—	—	—	364	505	662	842	1046	1273	1525	1777	2078	2429	2830	3281
2.35	170	400	11.21	8.02	—	—	437	540	663	794	945	1121	1322	1548	1799	2049	2350	2700	3100	3550
2.54	315	800	24.81	18.15	—	—	—	—	—	—	654	865	1097	1353	1606	1909	2261	2663	3115	
2.63	190	500	13.26	9.48	—	—	—	430	557	691	844	1021	1223	1450	1701	1952	2253	2603	3004	3455
2.67	150	400	9.11	6.53	—	—	451	554	677	808	960	1136	1337	1563	1814	2064	2365	2715	3116	3566
2.67	118	315	6.72	4.81	267	346	551	652	—	—	—	—	—	—	—	—	—	—	—	
2.67	236	630	17.79	12.77	—	—	—	—	390	533	692	873	1077	1305	1557	1809	2111	2462	2863	3314
3.39	236	800	17.86	12.82	—	—	—	—	—	—	705	918	1152	1408	1662	1966	2319	2722	3174	
3.50	180	630	12.31	8.80	—	—	—	425	569	729	911	1116	1345	1598	1850	2152	2504	2905	3356	
3.57	112	400	6.04	4.34	—	—	476	580	—	—	—	—	—	—	—	—	—	—	—	
3.57	140	500	8.11	5.82	—	—	351	462	590	725	879	1057	1259	1486	1738	1989	2290	2641	3042	3493
3.57	224	800	16.71	11.97	—	—	—	—	—	—	713	926	1160	1416	1671	1975	2328	2731	3183	
3.57	280	1000	21.94	15.85	—	—	—	—	—	—	673	925	1190	1450	1758	2114	2519	2973		
3.71	170	630	11.28	8.06	—	—	—	431	576	736	918	1123	1352	1605	1857	2159	2511	2913	3364	
3.77	212	800	15.53	11.11	—	—	—	—	—	—	720	934	1168	1425	1679	1983	2337	2739	3192	
3.79	132	500	8.52	6.06	—	—	356	467	—	—	—	—	—	—	—	—	—	—		
3.94	160	630	10.23	7.32	—	—	—	—	437	582	742	925	1130	1359	1612	1865	2167	2519	2920	3371
4.72	212	1000	15.53	11.11	—	—	—	—	—	—	—	714	968	1235	1496	1805	2162	2568	3022	
5.00	160	800	10.23	7.32	—	—	—	—	—	—	554	753	968	1203	1461	1716	2021	2374	2778	3230
5.00	200	1000	14.34	10.25	—	—	—	—	—	—	—	722	976	1243	1504	1813	2171	2576	3031	
5.26	190	1000	13.33	9.53	—	—	—	—	—	—	—	728	982	1250	1511	1820	2178	2584	3038	
5.33	150	800	9.18	6.57	—	—	—	—	—	—	759	975	1210	1468	1723	2028	2382	2785	3238	

NB This is only a partial selection from a typical catalogue relevant to the worked examples and worksheet questions.

Courtesy of Fenner Power Transmission UK.

Table 8.5 Power ratings for SPB wedge belts

Speed of faster shaft (rpm)	Rated power (kW) per belt for small pulley pitch diameter (mm)												
	140	150	160	170	180	190	200	212	224	236	250	280	315
100	0.73	0.82	0.92	1.01	1.10	1.20	1.29	1.40	1.51	1.62	1.74	2.01	2.33
200	1.33	1.51	1.69	1.87	2.05	2.22	2.40	2.61	2.82	3.02	3.26	3.78	4.37
300	1.89	2.15	2.41	2.67	2.93	3.18	3.44	3.74	4.04	4.35	4.70	5.44	6.30
400	2.42	2.76	3.09	3.43	3.77	4.10	4.43	4.83	5.22	5.61	6.07	7.04	8.15
500	2.92	3.33	3.75	4.16	4.57	4.98	5.39	5.87	6.36	6.84	7.39	8.58	9.94
600	3.40	3.89	4.38	4.87	5.35	5.83	6.31	6.89	7.45	8.02	8.67	10.06	11.66
700	3.86	4.43	4.99	5.55	6.11	6.66	7.21	7.87	8.52	9.17	9.92	11.50	13.32
720	3.95	4.53	5.11	5.69	6.26	6.82	7.39	8.06	8.73	9.39	10.16	11.79	13.65
800	4.31	4.95	5.59	6.22	6.84	7.47	8.08	8.82	9.55	10.28	11.12	12.90	14.93
900	4.75	5.46	6.16	6.86	7.56	8.25	8.93	9.75	10.56	11.36	12.29	14.25	16.47
960	5.00	5.75	6.50	7.24	7.98	8.71	9.43	10.29	11.15	11.99	12.97	15.03	17.37
1000	5.17	5.95	6.72	7.49	8.25	9.01	9.76	10.65	11.53	12.41	13.42	15.55	17.96
1100	5.58	6.42	7.27	8.10	8.93	9.75	10.56	11.52	12.48	13.43	14.52	16.80	19.39
1200	5.97	6.89	7.79	8.69	9.58	10.46	11.34	12.37	13.40	14.41	15.57	18.01	20.75
1300	6.36	7.34	8.31	9.27	10.22	11.16	12.09	13.19	14.28	15.36	16.59	19.17	22.05
1400	6.73	7.77	8.81	9.83	10.84	11.84	12.82	13.99	15.14	16.27	17.57	20.28	23.28
1440	6.88	7.95	9.00	10.05	11.08	12.10	13.11	14.30	15.47	16.63	17.96	20.70	23.75
1500	7.09	8.20	9.29	10.37	11.44	12.49	13.53	14.76	15.97	17.15	18.51	21.33	24.43
1600	7.44	8.61	9.76	10.90	12.02	13.12	14.21	15.50	16.76	18.00	19.41	22.33	25.51
1700	7.78	9.01	10.21	11.40	12.58	13.73	14.87	16.21	17.52	18.81	20.27	23.27	26.51
1800	8.11	9.39	10.65	11.90	13.12	14.32	15.50	16.89	18.25	19.58	21.08	24.15	27.43
1900	8.43	9.76	11.08	12.37	13.64	14.88	16.11	17.54	18.94	20.31	21.85	24.97	28.27
2000	8.73	10.12	11.48	12.82	14.14	15.43	16.69	18.16	19.60	20.99	22.57	25.72	29.01
2100	9.02	10.46	11.88	13.26	14.62	15.94	17.24	18.75	20.22	21.64	23.23	26.41	29.67
2200	9.31	10.79	12.25	13.68	15.07	16.44	17.76	19.31	20.80	22.24	23.85	27.03	30.22
2300	9.57	11.11	12.61	14.08	15.51	16.90	18.26	19.83	21.35	22.80	24.42	27.57	30.68
2400	9.83	11.41	12.95	14.46	15.92	17.34	18.72	20.32	21.85	23.31	24.93	28.05	31.04
2500	10.08	11.70	13.28	14.82	16.31	17.76	19.16	20.77	22.31	23.78	25.38	28.44	—
2600	10.31	11.97	13.59	15.16	16.68	18.14	19.56	21.19	22.73	24.19	25.78	28.76	—
2700	10.53	12.23	13.88	15.47	17.02	18.50	19.93	21.56	23.11	24.56	26.12	28.99	—
2800	10.73	12.47	14.15	15.77	17.33	18.83	20.27	21.90	23.44	24.87	26.40	—	—
2880	10.89	12.65	14.35	15.99	17.57	19.07	20.51	22.14	23.67	25.08	26.57	—	—
2900	10.93	12.69	14.40	16.04	17.62	19.13	20.57	22.20	23.72	25.12	26.61	—	—
3000	11.10	12.90	14.63	16.30	17.89	19.40	20.84	22.46	23.96	25.33	26.76	—	—

Courtesy of Fenner Power Transmission UK.

As a partial belt cannot be used, round this number up to the nearest integer. Thus three SPB4000 belts should be used running on sprockets of 224 mm and 315 mm pitch diameter with a centre distance of 1.544 m.

For the simple belt drive configuration shown in Figure 8.6, the angles of contact between the belt and the pulleys are given by

$$\theta_d = \pi - 2\sin^{-1} \frac{D-d}{2C} \quad (8.3)$$

$$\theta_d = \pi + 2\sin^{-1} \frac{D-d}{2C} \quad (8.4)$$

where d is the diameter of the small pulley (m); D , the diameter of the large pulley (m); C , the distance between the pulley centres (m); θ_d , the angle of contact between the belt and the small pulley (rad); and θ_D , the angle of contact between the belt and the large pulley (rad).

Table 8.6 Additional power increment per SPB belt

Speed of faster shaft (rpm)	Additional power (kW) per belt for speed ratio									
	1.00 to 1.01	1.02 to 1.05	1.06 to 1.11	1.12 to 1.18	1.19 to 1.26	1.27 to 1.38	1.39 to 1.57	1.58 to 1.94	1.95 to 3.38	3.39 and over
100	0.00	0.01	0.02	0.04	0.04	0.06	0.07	0.07	0.08	0.08
200	0.00	0.01	0.04	0.07	0.09	0.11	0.13	0.15	0.16	0.17
300	0.00	0.02	0.06	0.10	0.14	0.17	0.20	0.22	0.24	0.25
400	0.00	0.03	0.07	0.13	0.19	0.22	0.26	0.29	0.32	0.34
500	0.00	0.04	0.09	0.17	0.23	0.28	0.33	0.37	0.40	0.43
600	0.00	0.04	0.12	0.20	0.28	0.34	0.40	0.45	0.48	0.51
700	0.00	0.05	0.13	0.24	0.33	0.39	0.46	0.52	0.57	0.59
720	0.00	0.05	0.14	0.25	0.33	0.41	0.48	0.54	0.59	0.62
800	0.00	0.06	0.16	0.28	0.37	0.45	0.53	0.60	0.65	0.69
900	0.00	0.07	0.18	0.31	0.42	0.51	0.60	0.66	0.72	0.77
960	0.00	0.07	0.19	0.32	0.44	0.54	0.62	0.70	0.77	0.81
1000	0.00	0.07	0.19	0.34	0.46	0.56	0.66	0.74	0.81	0.86
1100	0.00	0.08	0.22	0.37	0.51	0.62	0.72	0.81	0.89	0.94
1200	0.00	0.09	0.23	0.41	0.56	0.68	0.79	0.89	0.97	1.03
1300	0.00	0.09	0.25	0.44	0.60	0.73	0.86	0.96	1.05	1.11
1400	0.00	0.10	0.28	0.48	0.65	0.79	0.93	1.04	1.13	1.20
1440	0.00	0.10	0.28	0.48	0.66	0.79	0.94	1.06	1.15	1.21
1500	0.00	0.10	0.29	0.51	0.69	0.84	0.99	1.11	1.21	1.28
1600	0.00	0.11	0.31	0.54	0.75	0.90	1.05	1.19	1.29	1.37
1700	0.00	0.12	0.34	0.58	0.79	0.95	1.12	1.26	1.37	1.45
1800	0.00	0.13	0.35	0.61	0.84	1.01	1.19	1.34	1.45	1.54
1900	0.00	0.13	0.37	0.65	0.88	1.07	1.25	1.41	1.54	1.63
2000	0.00	0.14	0.39	0.68	0.93	1.13	1.32	1.48	1.62	1.71
2100	0.00	0.15	0.41	0.72	0.98	1.18	1.39	1.56	1.69	1.79
2200	0.00	0.16	0.43	0.75	1.02	1.24	1.45	1.63	1.78	1.88
2300	0.00	0.16	0.45	0.78	1.07	1.29	1.51	1.71	1.86	1.97
2400	0.00	0.17	0.47	0.82	1.11	1.35	1.58	1.78	1.94	2.05
2500	0.00	0.18	0.49	0.85	1.16	1.41	1.65	1.86	2.02	2.14
2600	0.00	0.19	0.51	0.89	1.21	1.46	1.72	1.92	2.10	2.22
2700	0.00	0.19	0.53	0.92	1.25	1.52	1.78	1.99	2.18	2.31
2800	0.00	0.20	0.54	0.95	1.29	1.57	1.84	2.07	2.26	2.39
2880	0.00	0.20	0.56	0.97	1.32	1.60	1.88	2.11	2.31	2.44
2900	0.00	0.21	0.57	0.99	1.34	1.63	1.91	2.15	2.34	2.48
3000	0.00	0.22	0.59	1.02	1.39	1.69	1.98	2.23	2.42	2.57

Courtesy of Fenner Power Transmission UK.

The length of the belt can be obtained by summing the arc lengths of contact and the spanned distances and is given by

$$L = \sqrt{4C^2 - (D - d)^2} + \frac{1}{2} (D\theta_D + d\theta_d) \quad (8.5)$$

The power transmitted by a belt drive is given by

$$\text{Power} = (F_1 - F_2)V \quad (8.6)$$

where F_1 is belt tension in the tight side (N); F_2 , belt tension in the slack side (N); and V , belt speed (ms^{-1}).

The torque is given by

$$\text{Torque} = (F_1 - F_2)r \quad (8.7)$$

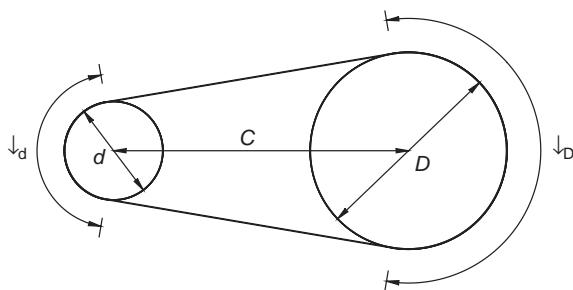


Figure 8.6 Belt drive geometry definition.

Assuming that the friction is uniform throughout the arc of contact and ignoring centrifugal effects the ratio of the tensions in the belts can be modelled by Eytlewein's formula

$$\frac{F_1}{F_2} = e^{\mu\theta} \quad (8.8)$$

where μ is the coefficient of friction; θ , the angle of contact (rad), usually taken as the angle for the smaller pulley.

The centrifugal forces acting on the belt along the arcs of contact reduce the surface pressure. The centrifugal force is given by

$$F_c = \rho V^2 A = mV^2 \quad (8.9)$$

where ρ is the density of the belt material (kg/m^3); A , cross-sectional area of the belt (m^2); and m , the mass per unit length of the belt (kg/m).

The centrifugal force acts on both the tight and the slack sides of the belt and Eytlewein's formula can be modified to model the effect:

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta} \quad (8.10)$$

The maximum allowable tension, $F_{1,\max}$, in the tight side of a belt depends on the allowable stress of the belt material, σ_{\max} . A selection of values for certain materials is given in Table 8.7.

$$F_{1,\max} = \sigma_{\max} A \quad (8.11)$$

Table 8.7 Guideline values for the maximum permissible stress for high performance flat belts

Multiply structure	Friction surface coating	Core	Top surface	Maximum permissible stress (MN/m^2)
Elastomer	Polyamide sheet	Polyamide fabric	Elastomer	8.3–19.3
Elastomer	Polyamide sheet	Polyamide sheet	Elastomer	6.6–13.7
Chrome leather	Polyamide sheet	None	Polyamide fabric	6.3–11.4
Chrome leather	Polyamide sheet	Polyamide sheet	Chrome leather	5.7–14.7
Chrome leather	Polyamide sheet	Polyester cord	Elastomer	4–8
Elastomer	Polyester cord	Polyester cord	Polyamide fabric	≤ 21.8
Chrome leather	Polyester cord	Polyester cord	Chrome leather	5.2–12
Chrome leather	Polyester cord	Polyester cord	Chrome leather	3.1–8

The required cross-sectional area for a belt drive can be found from

$$A = \frac{F_1 - F_2}{\sigma_1 - \sigma_2} \quad (8.12)$$

Example 8.2

A fan is belt driven by an electric motor running at 1500 rpm. The pulley diameters for the fan and motor are 500 and 355 mm, respectively. A flat belt has been selected with a width of 100 mm, thickness of 3.5 mm, coefficient of friction of 0.8, density of 1100 kg/m^3 and permissible stress of 11 MN/m^2 . The centre distance is 1500 mm. Determine the power capacity of the belt.

Solution

The arcs of contact for the driving and driven pulleys are:

$$\theta_d = \pi - 2 \sin^{-1} \frac{D - d}{2C}$$

$$\begin{aligned}
 &= \pi - 2\sin^{-1} \frac{500 - 355}{2 \times 1500} \\
 &= 3.045 \text{ rad}, \\
 \theta_D &= \pi + 2\sin^{-1} \frac{D - d}{2C} \\
 &= \pi + 2\sin^{-1} \frac{500 - 355}{2 \times 1500} \\
 &= 3.238 \text{ rad}.
 \end{aligned}$$

The maximum tension in the tight side is given as a function of the maximum permissible stress in the belt by:

$$\begin{aligned}
 F_1 &= \sigma_{\max} A = 11 \times 10^6 (3.5 \times 10^{-3} \\
 &\quad \times 100 \times 10^{-3}) = 3850 \text{ N}.
 \end{aligned}$$

The belt velocity is

$$V = 1500 \times \frac{2\pi}{60} \times \frac{0.355}{2} = 27.88 \text{ m/s.}$$

The mass per unit length is

$$m = \rho A = 1100 \times 0.1 \times 0.0035 = 0.385 \text{ kg/m.}$$

The centrifugal load is given by

$$F_c = 0.385(27.88)^2 = 299.3 \text{ N}$$

Using Eq. 8.10

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta} = \frac{3850 - 299.3}{F_2 - 299.3} = e^{0.8 \times 3.045}$$

which gives

$$F_2 = 610.0 \text{ N.}$$

The power capacity is given by $(F_1 - F_2) V = (3850 - 610)27.88 = 90.3 \text{ kW.}$

8.3 Chain drives

A chain is a power transmission device consisting of a series of pin-connected links as illustrated in Figure 8.7. The chain transmits power between two rotating shafts by meshing with toothed sprockets as shown in Figure 8.8.

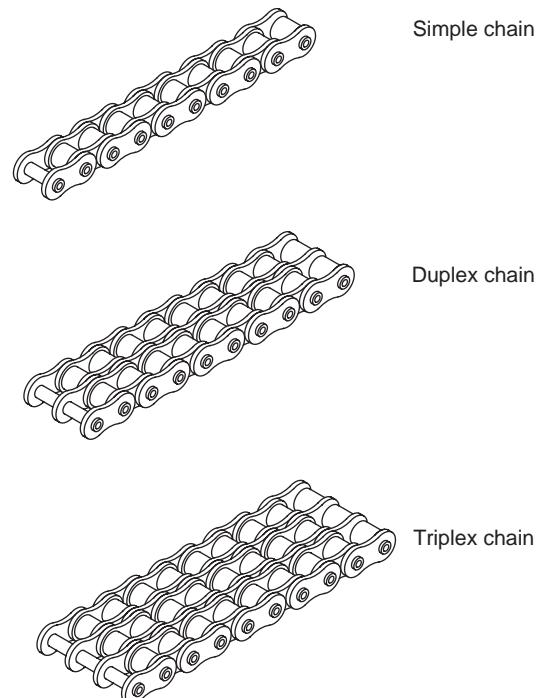


Figure 8.7 Roller chain.

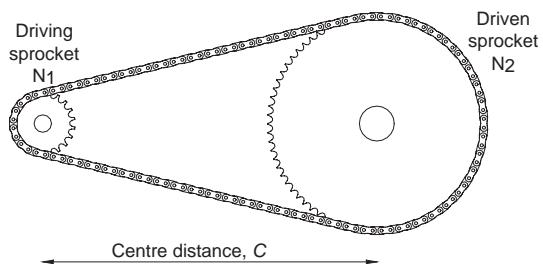


Figure 8.8 Simple chain drive.

Chain drives are usually manufactured using high strength steel and for this reason are capable of transmitting high torque. Chain drives are complementary and competitive with belt drives serving the function of transmitting a wide range of powers for shaft speeds up to about 6000 rpm. At higher speeds the cyclic impact between the chain links and the sprocket teeth, high noise and difficulties in providing lubrication, limit the application of chain drives. Table 8.8 shows a comparison of chain, belt and gear attributes. Chain drives are principally used for power transmission, conveyors

and for supporting or lifting loads. Applications range from motorcycle and bicycle transmissions, automotive camshaft drives, machine tools and aerospace drives, such as the thruster nozzles for the Harrier, to conveyors and packaging machinery. Efficiencies of up to 98.9 per cent, where necessary ratios of up to 9:1 and power transmission of several hundred kilowatts can be achieved. Chain drives are typically used with reduction ratios of up to three to one giving high efficiency and ratios of up to five giving reasonable efficiency.

The range of chain drives is extensive as illustrated in Figure 8.9. The most common type is roller chain, which is used for high power transmission and conveyor applications. The selection and specification of this type of chain is introduced in Section 8.3.1. Roller chain is made up of a series of links. Each chain consists of side plates, pins,

bushes and rollers as shown in Figure 8.10. The chain runs on toothed sprockets and as the teeth of the sprockets engage with the rollers, rolling motion occurs and the chain articulates on to the sprocket. Roller chain is classified by its pitch. This is the distance between the corresponding points of adjacent links. Roller chain is available in stainless steel and nylon (for hygienic applications), as well as a series of other materials for specialist applications.

Conveyor chain is specially designed for use in materials handling and conveyor equipment and is characterized by its long pitch, large roller diameters and high tensile strength. The applications of conveyor chain demand that it is capable of pulling significant loads usually along a straight line at relatively low speeds. The sides of conveyor chains often incorporate special features to aid connection to conveyor components as illustrated in Figure 8.11.

Leaf chain consists of a series of pin connected side plates as illustrated in Figure 8.12. It is generally used for load balancing applications and is essentially a special form of transmission chain.

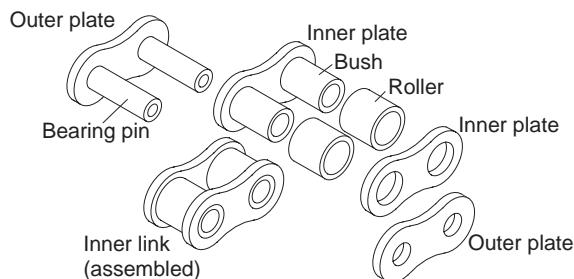


Figure 8.10 Roller chain components.

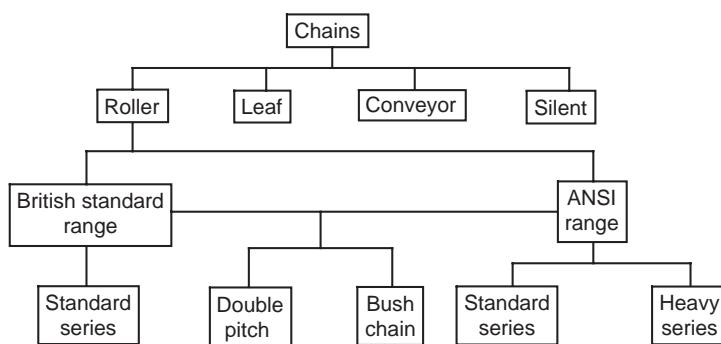


Figure 8.9 Chain types.

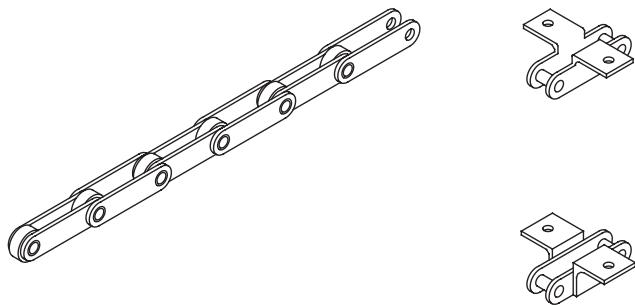


Figure 8.11 Conveyor chain.

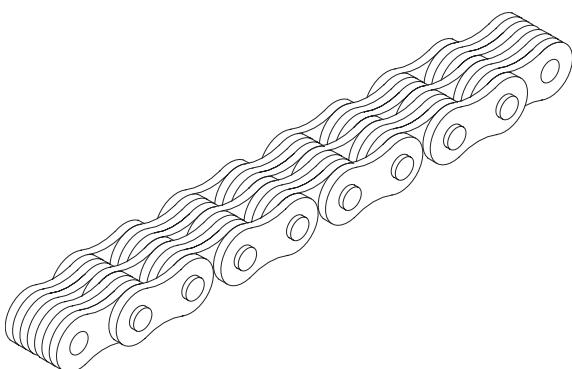


Figure 8.12 Leaf chain.

Silent chain (also called inverted tooth chain) has the teeth formed in the link plates as shown in Figure 8.13. The chain consists of alternately mounted links so that the chain can articulate on to the mating sprocket teeth. Silent chains can operate at higher speeds than comparatively sized roller chain and, as the name suggests, more quietly.

The direction of rotation relative to the chain layout is important to the effective operation of a chain drive. Figure 8.14 shows recommended and acceptable practice for chain drive layouts. The idler sprockets shown are used to engage the chain in the slack region for chain drives with large centre distances or when the drive is vertical.

Chain design is based on ensuring that the power transmission capacity is within limits for three modes of failure: fatigue, impact and galling. Chains are designed so that the maximum tensile stress is below the fatigue endurance limit for finite life of the material. Failure would nevertheless eventually

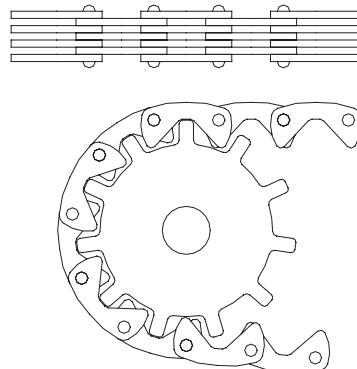


Figure 8.13 Silent chain.

occur, but it would be due to wear, not fatigue. In service, failures due to wear can be eliminated by inspection and replacement intervals. When the chain rollers mesh with the sprocket teeth an impact occurs and a Hertz contact stress occurs, similar to that found for gear meshing. The power rating charts for chain drives limit the selection of the drive so that these modes of failure should not occur assuming proper installation, operation and lubrication.

Details of the selection procedure for standard roller chains are outlined in Section 8.3.1. For details of chain geometry the reader is referred to the standards listed in the references and for details of selection and design procedures for other chain types to manufacturers' catalogues.

8.3.1 Roller chain selection

Once the use of a chain drive has been shown to be preferable to other forms of drive the type of

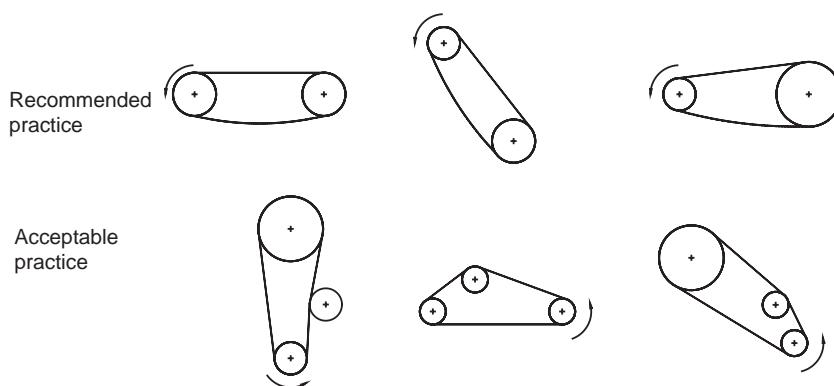


Figure 8.14 Typical chain drive layouts.

chain to be used can be selected from the range available as illustrated in Figure 8.9. The next step is the design of the chain drive layout and selection of the standard components available from chain manufacturers. The method outlined here is for roller chains. Procedures for the selection of other chain types can be found in manufacturers' catalogues.

The method is based upon the use of power rating charts for the chain drive, which ensure 15 000 hours operation assuming proper installation, operation and lubrication. The steps for the method are itemized below.

1. Determine the power to be transmitted.
2. Determine the speeds of the driving and driven shafts.
3. Determine the characteristics of the driving and driven shaft, e.g. type of running, whether smooth or shock loadings, etc.
4. Set the approximate centre distance. This should normally be in the range of 30 to 50 times the chain pitch.
5. Select the speed ratio. This is dependent on the standard sizes available as listed in Table 8.9. Ideally sprockets should have a minimum of 19 teeth. For high speed drives subjected to transient loads the minimum number of teeth rises to 25. Note that the maximum number of teeth should not exceed 114.
6. Establish the application and tooth factors: Table 8.10 gives values for the application factor f_1 . The tooth factor is given by $f_2 = 19/N_1$

Table 8.9 Chain reduction ratios as a function of the standard sprockets available

Number of teeth in the driven sprocket N_2	Number of teeth in the drive sprocket N_1					
	15	17	19	21	23	25
25	—	—	—	—	—	1.00
38	2.53	2.23	2.00	1.80	1.65	1.52
57	3.80	3.35	3.00	2.71	2.48	2.28
76	5.07	4.47	4.00	3.62	3.30	3.04
95	6.33	5.59	5.00	4.52	4.13	3.80
114	7.60	6.70	6.00	5.43	4.96	4.56

Reproduced from Renold, 1996.

assuming that the selection rating charts are based on a 19 tooth sprocket.

7. Calculate the selection power. Selection power = power $\times f_1 \times f_2$.
8. Select the chain drive pitch. Use power speed rating charts as supplied by chain manufacturers (see Figure 8.15). The smallest pitch of a simple chain should be used as this normally gives the most economical drive. If the power requirement at a given speed is beyond the capacity of a single strand of chain then the use of a multi-strand chain, such as duplex (two strands), triplex (three strands) and up to decuplex (10 strands) for the ANSI range, permits higher power to be transmitted and can be considered.
9. Calculate the chain length. Eq. 8.13 gives the chain length as a function of the number of pitches. Note that the value for the length should be rounded up to the nearest even integer.

Table 8.10 Application factor

Driven machine characteristics	Driver characteristics		
	Smooth running, e.g. electric motors, IC engines with hydraulic coupling	Slight shocks, e.g. IC engines with more than six cylinders, electric motors with frequent starts	Heavy shocks, e.g. IC engines with less than six cylinders
Smooth running, e.g. fans, pumps, compressors, printing machines, uniformly loaded conveyors	1	1.1	1.3
Moderate shocks, e.g. concrete mixing machines, non-uniformly loaded conveyors, mixers	1.4	1.5	1.7
Heavy shocks, e.g. planars, presses, drilling rigs	1.8	1.9	2.1

Reproduced from Renold, 1996.

10. Calculate the exact centre distance. This can be calculated using Eq. 8.14.
11. Specify the lubrication method.

The chain length, in pitches, is given by

$$L = \frac{N_1 + N_2}{2} + \frac{2C}{p} + \frac{(N_2 - N_1)^2}{2\pi} \frac{p}{C} \quad (8.13)$$

where L is number of pitches; N_1 , number of teeth in the driving sprocket; N_2 , number of teeth in the driven sprocket; C , centre distance (m); and p , chain pitch (m).

The exact centre distance is given by

$$C = \frac{p}{8} \overrightarrow{2L} - N_2 - N_1 + \sqrt{(2L - N_2 - N_1)^2 - \frac{\pi}{3.88} (N_2 - N_1)^2} \quad (8.14)$$

Chain drives should be protected against dirt and moisture (tell this to a mountain biker!). Lubrication should be provided using a non-detergent

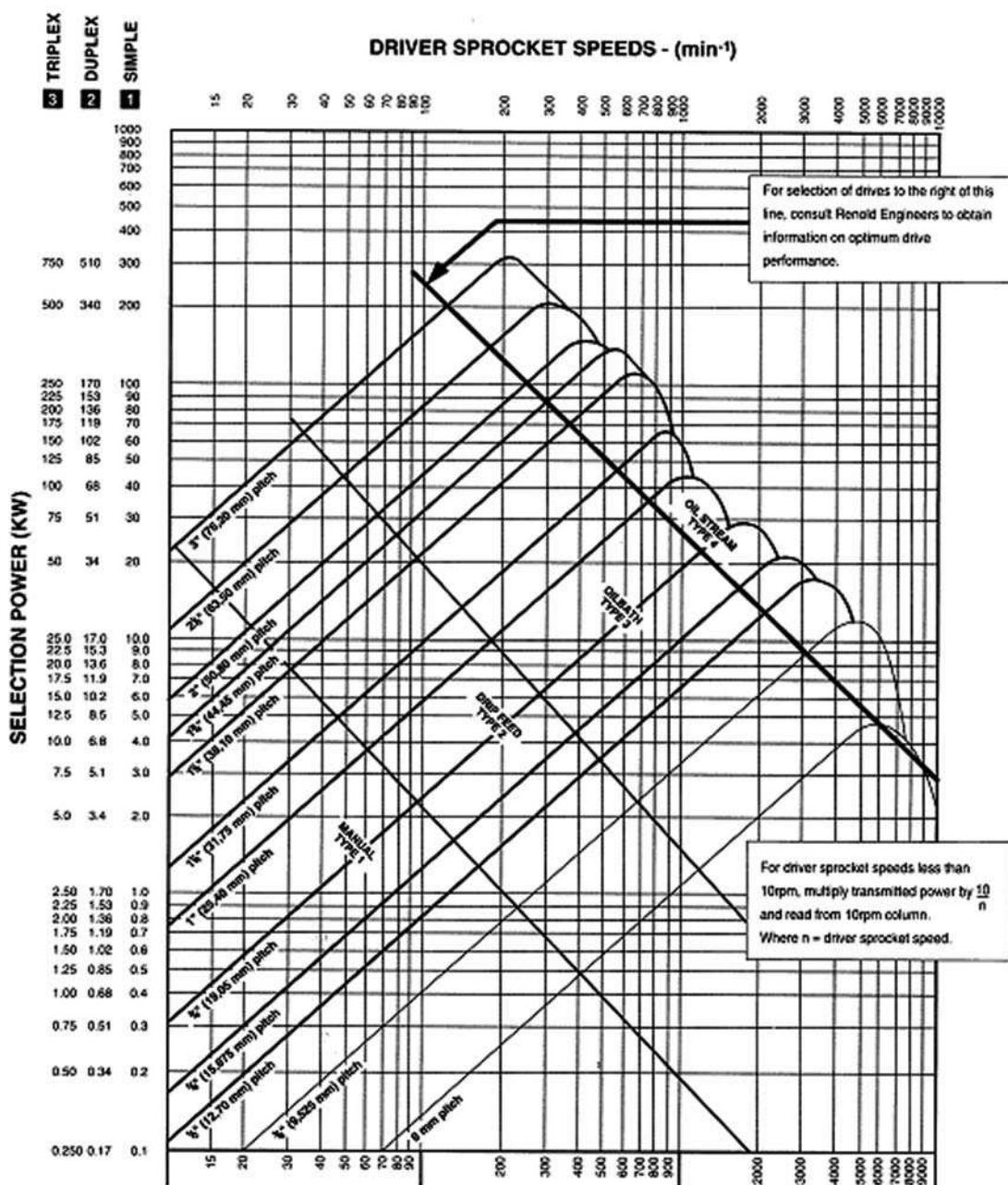
mineral-based oil. For the majority of applications multigrade SAE20/50 is suitable. There are five principle types of lubrication: manual application, drip feed, bath, stream (see Figure 8.16) and dry lubrication. Grease lubrication is not recommended, but can be used for chain speeds of less than 4 m s^{-1} . In order to ensure that the grease penetrates the working parts of the chain, it should be heated until liquid and the chain dipped into the grease until the air has been displaced. This process should be repeated at regular service intervals. For dry lubrication solid lubricant is contained in a volatile carrier fluid. When applied to the chain carrier, it transports the lubricant into the chain and then evaporates leaving the chain lubricated, but dry to touch. Applications for dry lubrication include food processing, dusty environments and fabric handling.

The pitch diameters for the driving and driven sprockets are given by

$$D_1 = \frac{N_1 p}{\pi} \quad D_2 = \frac{N_2 p}{\pi} \quad (8.15)$$

and the angle of contact (in radians) between the chain and the sprockets by

$$\theta_1 = \pi - 2 \sin^{-1} \frac{p(N_2 - N_1)}{2\pi C} \quad (8.16a)$$



1 Kilowatt = 1.34 hp.

Figure 8.15 British Standard chain drives rating chart using 19 tooth drive sprocket (courtesy of Renold Chain).

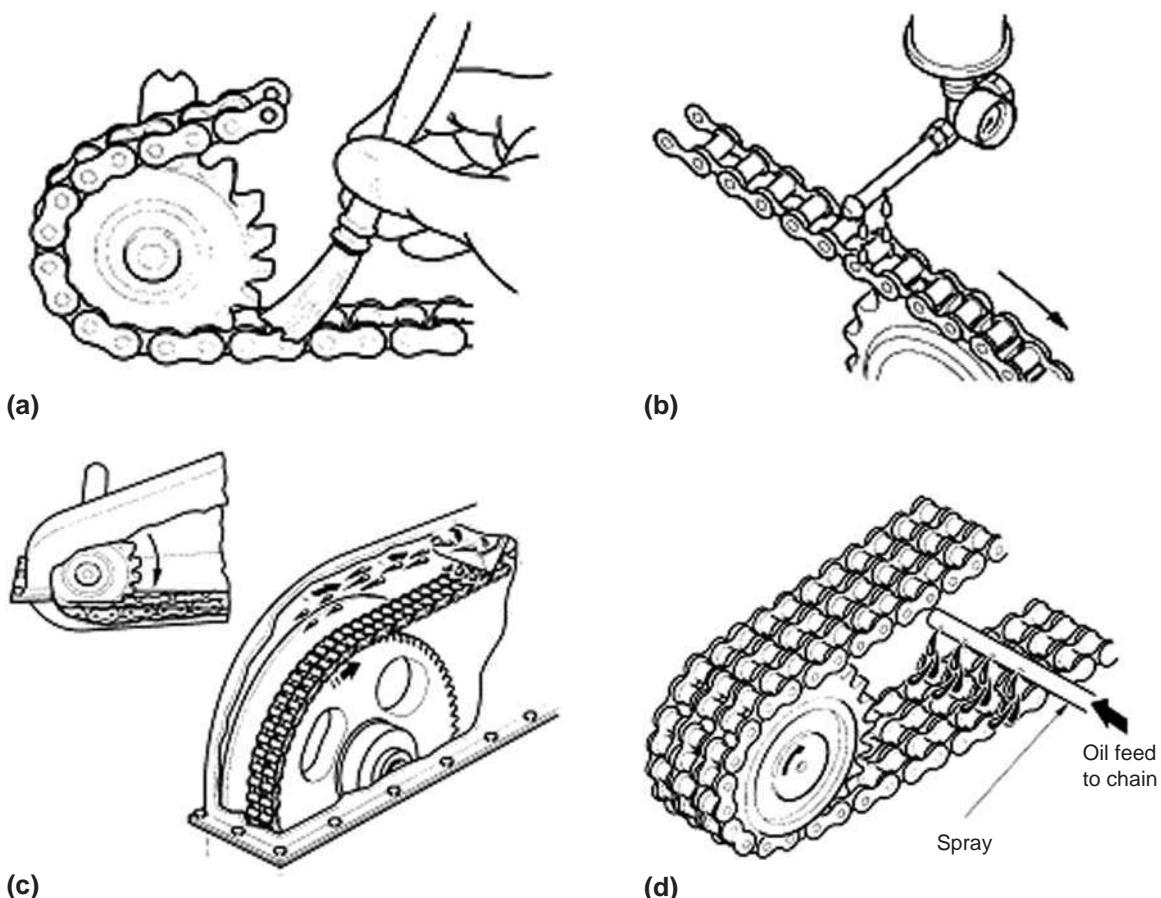


Figure 8.16 Chain lubrication methods (courtesy of Renold Chain).

$$\theta_2 = \pi + 2\sin^{-1} \frac{p(N_2 - N_1)}{2\pi C} \quad (8.16b)$$

Note that the minimum angle of wrap recommended for the small sprocket is 120° .

The chain tension is given by

$$\text{Chain tension} = \frac{\text{Power}}{N_1 \omega_1 p / 2\pi} \quad (8.17)$$

Standard sprockets can be purchased. The choice of sprocket material depends on the number of teeth and the operating conditions as shown in Table 8.11.

Example 8.3

A chain drive is required for a gear pump operating at 400 rpm driven by a 5.5 kW electric motor running at 1440 rpm. The centre distance between the motor and pump shafts is approximately 470 mm.

Solution

The desired reduction ratio is

$$\frac{1440}{400} = 3.6.$$

The nearest ratio available (see Table 8.9) using standard sized sprockets is 3.62. This requires

a driving sprocket of 21 teeth and a driven sprocket of 76 teeth.

The application factor from Table 8.10 is $f_1 = 1.0$.

The tooth factor is

$$f_2 = \frac{19}{N_1} = \frac{19}{21} = 0.905.$$

The selection power is

$$5.5 \times 1.0 \times 0.905 = 4.98 \text{ kW.}$$

Using the BS/ISO selection chart, Figure 8.15, a 12.7 mm pitch simple BS chain drive is suitable.

The chain length is given by

$$\begin{aligned} L &= \frac{N_1 + N_2}{2} + \frac{2C}{p} + \frac{(N_2 - N_1)^2}{2\pi} \frac{p}{C} \\ &= \frac{21 + 76}{2} + \frac{2 \times 470}{12.7} \\ &\quad + \frac{(76 - 21)^2}{2\pi} \frac{12.7}{470} = 124.6 \text{ pitches.} \end{aligned}$$

Rounding up to the nearest even integer gives $L = 126$ pitches.

The exact centre distance can now be calculated using

$$\begin{aligned} C &= \frac{p}{8} \vec{2L} - N_2 - N_1 \\ &\quad + \sqrt{(2L - N_2 - N_1)^2 - \frac{\pi}{3.88} (N_2 - N_1)^2}. \end{aligned}$$

$$\begin{aligned} C &= \frac{12.7}{8} \vec{2} \times 126 - 76 - 21 \\ &\quad + \sqrt{(2 \times 126 - 76 - 21)^2 - \frac{\pi}{3.88} (76 - 21)^2} \\ &= 479.2 \text{ mm.} \end{aligned}$$

From the rating chart, Figure 8.15, the required lubrication type is oil bath. Use of a SAE20/50 multigrade lubricant would likely suffice in the absence of more detailed knowledge concerning the operating conditions.

Table 8.11 Selection of sprocket materials

Sprocket	Smooth running	Moderate shocks	Heavy shocks
Up to 29 teeth	080M40 or 070M55	080M40 or 070M55 hardened and tempered or case hardened mild steel	080M40 or 070M55 hardened and tempered or case hardened mild steel
Over 30 teeth	Cast iron	Mild steel	080M40 or 070M55 hardened and tempered or case hardened mild steel

Adapted from Renold, 1996.

8.4 Conclusions

Belt and chain drives can be used for transmission of mechanical power between two rotating shafts. Belt drives are often cheaper than the equivalent gears and useful for transmitting power between shafts that are widely separated or nonparallel drives. Chain drives are usually more compact than the equivalent belt drive and can be used in oily environments where the equivalent belt would be prone to slipping. There is a wide range of belt and chain drives and this chapter has served to introduce the technology and the selection and specification of wedge and flat belt, and roller chain drives.

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International Organisation for Standardisation. (1991). ISO 22, *Belt drives – Flat transmission belts and corresponding pulleys – Dimensions and tolerances.*

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International Organisation for Standardisation. (1995). ISO 5292, *Belt drives – V belts and V ribbed belts – Calculation of power ratings.*

Websites

At the time of going to press, the internet contains useful information relating to this chapter at the following sites.

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- www.adesco.com.tw
- www.beijingthc.com
- www.beltindustries.com
- www.chiorino.co.uk
- www.chjc-chain.com
- www.daidocorp.com
- www.fennerdrives.com
- www.firstbelt.com.tw
- www.gates.com
- www.goodyearindustrialproducts.com
- www.habasitiakoka.com
- www.hmcross.com
- www.hutchinsontransmission.com
- www.imochain.com
- www.jeffreychain.com
- www.jonsonrubber.com
- www.ketten-wulf.de
- www.kilangrantai.com
- www.kkrubber.com
- www.martinspocket.com

www.ngfeurope.com
www.omfarubbers.com
www.peerbearing.com
www.powertransmission.com
www.renold.com
www.samplabelting-sa.com
www.siegling.com
www.sprocketco.com
www.visusa.com
www.vmchain.com
www.volatapower.com
www.websterchain.com

Nomenclature

Generally, preferred SI units have been stated.

C	centre distance (m)
D	pitch diameter (m)
f_1	application factor
f_2	tooth factor
L	number of pitches
N_1	number of teeth in the driving sprocket
N_2	number of teeth in the driven sprocket
p	chain pitch (m)
R_1	radius of driving pulley or sprocket (m)
R_2	radius of driven pulley or sprocket (m)
$V_{\text{pitchline}}$	pitchline velocity (m/s)
θ	angle of contact (rad)
ω_1	angular velocity of driving pulley or sprocket (rad/s)
ω_2	angular velocity of driven pulley or sprocket (rad/s)

Worksheet

- 8.1 List five merits each of gears, chains and belts.
- 8.2 State and briefly justify an appropriate drive for the following:
- (a) A drive for powering a conveyor belt
 - (b) A drive for a grinding machine
 - (c) A drive for a packaging machine in an oily environment.
- 8.3 A four-cylinder diesel engine running at 2000 rpm developing 45 kW is being used to drive a medium duty agricultural machine running at 890 rpm. The centre distance between the pulley centres is approximately 80 cm. The expected use is less than 10 h per day. Using a

wedge belt drive select suitable pulley diameters and determine the type and number of belts required.

- 8.4 A wedge belt drive is required to transmit 18.5 kW from an electric motor running at 1455 rpm to a uniformly loaded conveyor running at 400 rpm. The desired centre distance is 1.4 m and expected use is 15 h a day. Select a suitable belt, or belts, and determine the pulley diameters.
- 8.5 A flat belt is required to transmit 22 kW from a 250 mm diameter pulley running at 1450 rpm to a 355 mm diameter pulley. The coefficient of friction can be taken as 0.7, the density of the belt is 1100 kg/m^3 and the maximum permissible stress is 7 MPa. The distance between the shaft centres is 1.8 m. The proposed belt is 3.5 mm thick. Calculate the width required.
- 8.6 The application of an existing machine has changed, such that the desired transmission power has increased to 120 kW. The existing design consists of a driving pulley running at 3000 rpm with a pulley diameter of 200 mm and the driven pulley of 250 mm diameter. The belt is 150 mm wide, 4.2 mm thick and has a maximum permissible stress of 6.6 MPa. The belt density and coefficient of friction are 1100 kg/m^3 and 0.75, respectively. The centre distance is 1.4 m. Is the belt drive suitable?
- 8.7 A chain drive is required for a gear pump operating at 470 rpm driven by a 3 kW electric motor running at 2820 rpm. The centre distance between the motor and the pump is approximately 300 mm.
- (a) Calculate the reduction ratio
 - (b) Identify the nearest available ratio for standard sprockets
 - (c) Determine the application factor
 - (d) Calculate the tooth factor
 - (e) Determine the selection power
 - (f) Identify a suitable drive from the ISO selection chart
 - (g) Calculate the approximate chain length
 - (h) Determine the number of pitches in the chain
 - (i) Calculate the exact centre distance
- 8.8 Specify a suitable chain drive for a gear pump operating at 400 rpm driven by an 18.5 kW electric motor running at 725 rpm. The centre distance between the motor and pump shafts is approximately 470 mm.

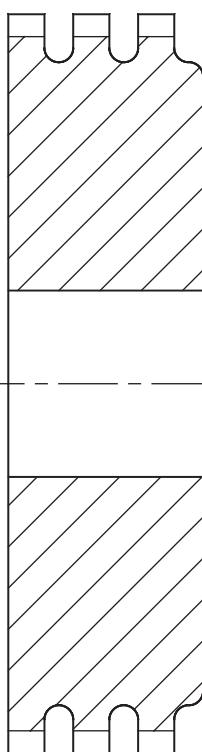


Figure 8.17 Sprocket.

- 8.9 Sketch a suitable method for locating the sprocket illustrated in Figure 8.17 on to a shaft.

- 8.10 Specify a suitable chain drive for a packaging machine operating at 75 rpm driven by a 2.2 kW electric motor running at 710 rpm. The maximum permissible centre distance between the motor and packaging machine shafts is approximately 1 m.
- 8.11 Specify a suitable drive for a gear pump operating at 400 rpm driven by a 30 kW electric motor running at 728 rpm. The centre distance between the motor and pump shafts is approximately 1 m.

Answers

- 8.3 $D_1 = 280$ mm, $D_2 = 630$ mm, $C = 662$ mm, SPB2800, 3 belts.
- 8.4 $D_1 = 224$ mm, $D_2 = 800$ mm, $C = 1.416$ m, SPB4500, 2 belts.
- 8.5 $w = 57$ mm.
- 8.6 Power capacity = 98.5 kW. No.
- 8.7 6, 6, 1.0, 1, 3 kW, 9.525 mm pitch simple or 8 mm duplex, 136.7 pitches, 138 pitches, 306.7 mm.
- 8.8 Simple chain, 25.4 mm pitch, 68 pitches, $C = 484.1$ mm.
- 8.9 No unique solution.
- 8.11 Simple, 25.4 mm pitch, 110 pitches, $C = 1020$ mm.

Learning objectives achievement

Can you differentiate whether to use a chain, belt or gear drive for a given application?

Sections 8.1, 8.2, 8.3

Can you select an appropriate wedge belt for a given application?

Section 8.2.1

Can you select an appropriate flat belt for a given application?

Section 8.2.1

Can you determine an appropriate chain ratio using standard components?

Section 8.3

Can you select an appropriate chain type and pitch for a given application?

Section 8.3.1

Can you specify an appropriate chain drive for a given speed and power requirement?

Section 8.3.1

9

SEALS

Seals are devices used to prevent or limit leakage of fluids or particulates. The aims of this chapter are to introduce the variety of seal configurations, give guidelines for the selection of seals and introduce calculation methods for the quantification of some seal leakage rates.

LEARNING OBJECTIVES

At the end of this section you should:

- be able to identify a number of the different types of sealing devices;
- be able to select a seal type for rotating, reciprocating or static conditions;
- be able to determine the groove dimensions for a standard O ring;
- be able to estimate the leakage flow through a labyrinth seal;
- be able to calculate the leakage flow through a bush seal.

9.1 Introduction

The purpose of a seal is to prevent or limit flow between components. Seals are an important aspect of machine design where pressurized fluids must be contained within an area of a machine such as a hydraulic cylinder, contaminants excluded or lubricants retained. Seals fall into two general categories.

- 1 Static seals, where sealing takes place between two surfaces that do not move relative to each other.

- 2 Dynamic seals, where sealing takes place between two surfaces that move relative to each other by, for example, rotary or reciprocating motion.

Any clearance between the two components will permit the passage of fluid molecules in either direction, the direction depending on the pressures and momentum associated with the fluid. The basic sealing problem is illustrated in Figure 9.1 where either boundary shown may be stationary or moving. Fluid can move between different regions of space by means of diffusion, free convection or forced convection. The size of a typical gas or vapour molecule is of the order of 10^{-9} m in diameter. They can therefore diffuse through very small gaps, such as pores in a machine casing or seal component. Convection involves the mixing of one portion of fluid with another by means of gross movements of the mass of fluid. The fluid motion may be caused by external mechanical means, such as by a fan or pump in which case the process is called forced convection. Alternatively if the fluid motion is caused by density differences, due to for instance temperature differences, the process is called natural or free convection.

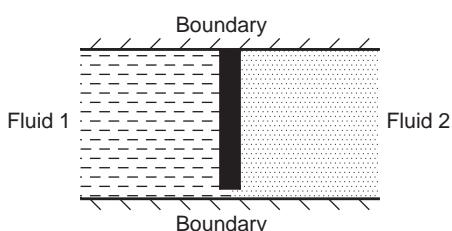


Figure 9.1 The basic sealing geometry.

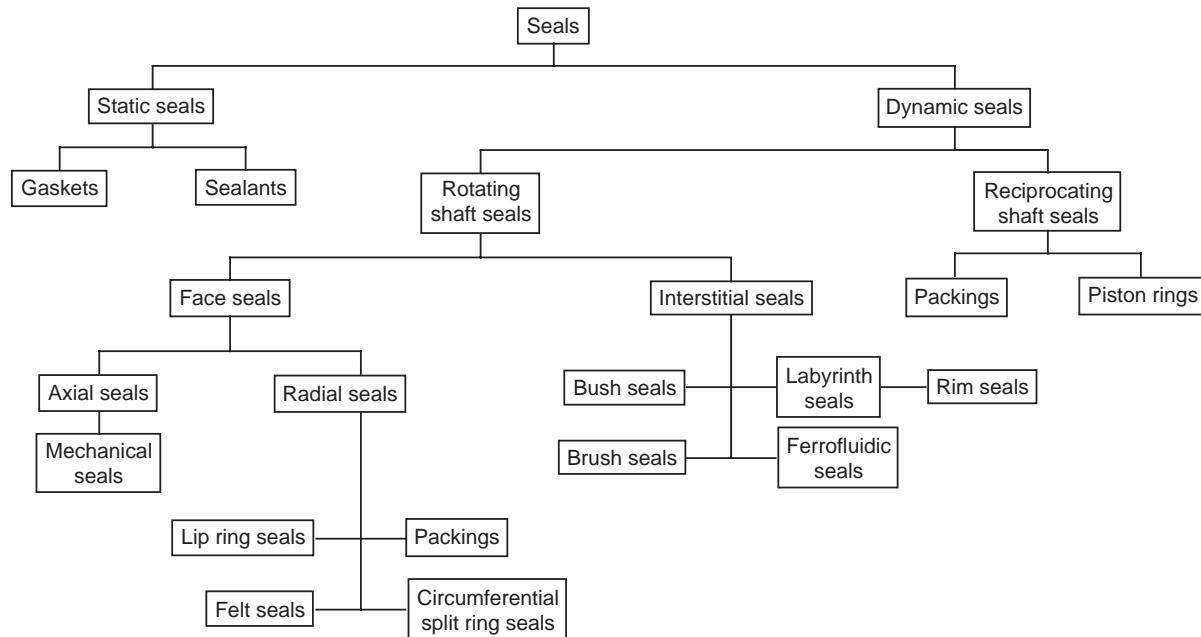


Figure 9.2 General seal classification chart (reproduced with alterations from Buchter, 1979).

Some of the considerations in selecting the type of seal include:

- 1 the nature of the fluid to be contained or excluded;
- 2 pressure levels either side of the seal;
- 3 the nature of any relative motion between the seal and mating components;
- 4 the level of sealing required;
- 5 operating temperatures;
- 6 life expectancy, serviceability;
- 7 total cost, considering components and lifetime service and replacement costs.

The variety of seals and sealing systems is extensive. Figure 9.2 shows a general classification diagram for seals detailing the principal types. Several of these will be introduced in Sections 9.2 and 9.3. It should be noted, however, that once a general sealing requirement and possible solution have been identified, the best source of specific information is usually the seal manufacturers, or for sealing bearings, the bearing manufacturers'. Figure 9.3 provides a general guide to the selection of seal type.

9.2 Static seals

Static seals aim at providing a complete physical barrier to leakage flow. To achieve this the seal material must be resilient enough to flow into and fill any irregularities in the surfaces being sealed and at the same time remain rigid enough to resist extrusion into clearances. Elastomeric seals and gaskets fulfil these criteria and are described in Sections 9.2.1 and 9.2.2, respectively. The particular demands of sealing of foodstuffs are introduced in Section 9.2.3.

9.2.1 Elastomeric seal rings

The 'O' ring, Figure 9.4, is a simple and versatile type of seal with a wide range of applications for both static and dynamic sealing. An 'O' ring seal is a moulded elastomeric ring 'nipped' in a cavity in which the seal is located. The principle of operation for an 'O' ring sealing against a fluid at various pressures is illustrated in Figure 9.5.

Elastomeric seal rings require the seal material to have an interference fit with one of the mating

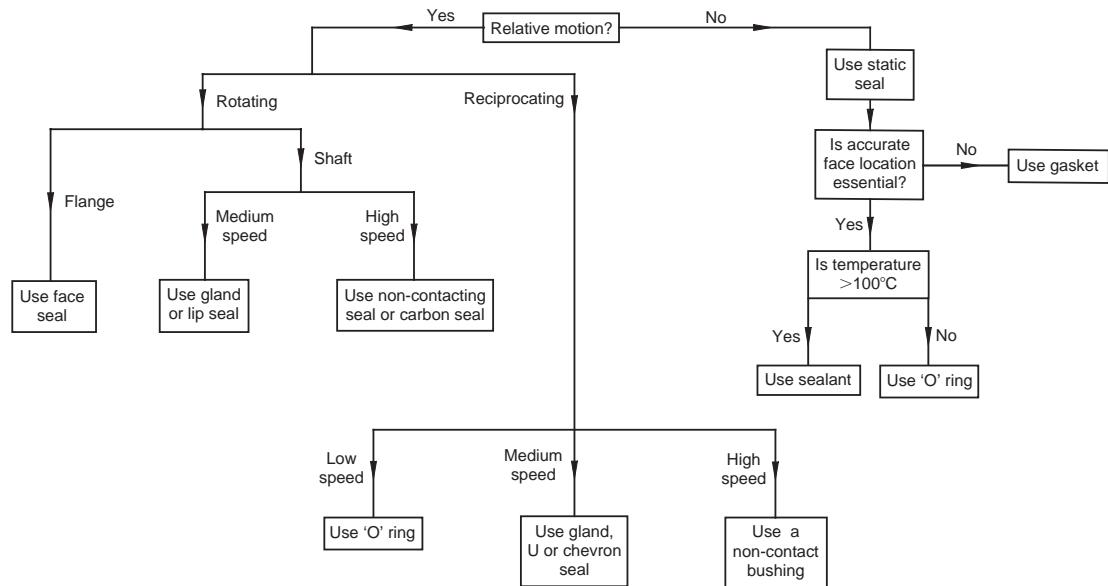


Figure 9.3 Seal selection procedure (reproduced from Hamilton, 1994).

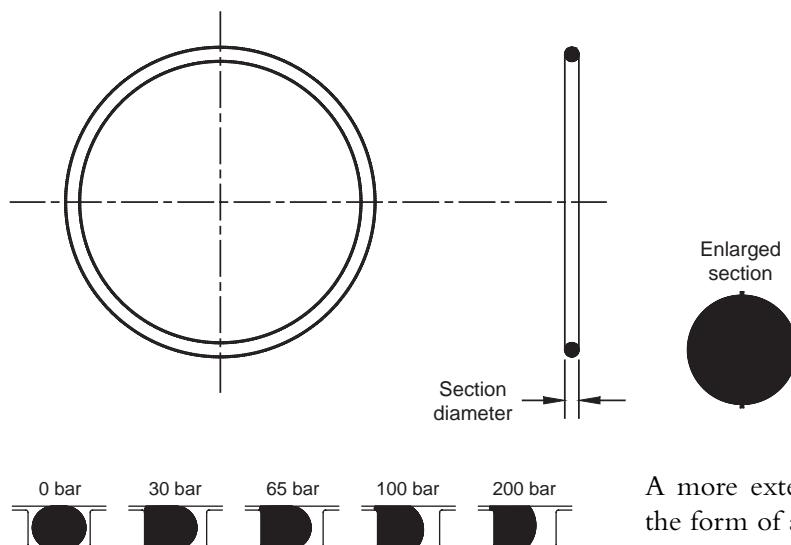


Figure 9.5 Principle of operation of 'O' rings sealing a fluid against pressure.

parts of the assembly. 'O' rings are available in a wide range of sizes with internal diameters from 3.1 to 249.1 mm and section diameters of 1.6, 2.4, 3.0, 4.1, 5.7 and 8.4 mm as defined in British Standard BS 4518. Table 9.1 shows the dimensions for a small number of the seal sizes available.

A more extensive table is normally available in the form of a sales catalogue from manufacturers or in BS 4518. Figure 9.6 shows the groove dimensions which must be specified to house the 'O' ring seal and ensure the seal is nipped or compressed sufficiently to enable effective sealing.

Example 9.1

Specify suitable groove dimensions for an 0195-30 'O' ring to seal against a solid cylinder.

Table 9.1 'O' ring seal dimensions (mm) (limited range tabulated only for illustration)

Reference number	Internal diameter	Section diameter	d_{nominal} (Fig. 9.6a)	D_{nominal} (Fig. 9.6a)	D_{nominal} (Fig. 9.6b)	d_{nominal} (Fig. 9.6b)	B	R
0031-16	3.1	1.6	3.5	5.8	6	3.7	2.3	0.5
0041-16	4.1	1.6	4.5	6.8	7	4.7	2.3	0.5
0051-16	5.1	1.6	5.5	7.8	8	5.7	2.3	0.5
0061-16	6.1	1.6	6.5	8.8	9	6.7	2.3	0.5
0071-16	7.1	1.6	7.5	9.8	10	7.7	2.3	0.5
0081-16	8.1	1.6	8.5	10.8	11	8.7	2.3	0.5
0091-16	9.1	1.6	9.5	11.8	12	9.7	2.3	0.5
0101-16	10.1	1.6	10.5	12.8	13	10.7	2.3	0.5
0111-16	11.1	1.6	11.5	13.8	14	11.7	2.3	0.5
0036-24	3.6	2.4	4	7.7	8	4.3	3.1	0.5
0046-24	4.6	2.4	5	8.7	9	5.3	3.1	0.5
0195-30	19.5	3.0	20	24.8	25	20.2	3.7	1.0
0443-57	44.3	5.7	45	54.7	55	45.3	6.4	1.0
1441-84	144.1	8.4	145	160	160	145	9.0	1.0
2491-84	249.1	8.4	250	265	265	250	9.0	1.0

Source: British Standard BS4518.

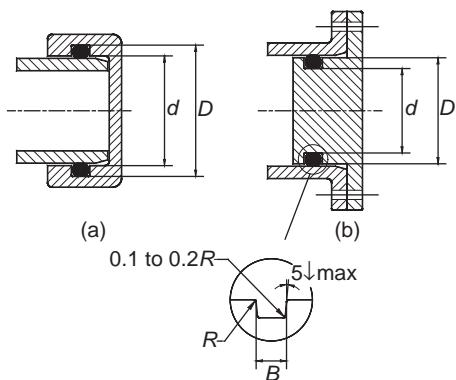


Figure 9.6 'O' ring groove dimensions.

Solution

From Table 9.1 and with reference to Figure 9.6, $B = 3.7 \text{ mm}$, $R = 1 \text{ mm}$, groove fillet radius = 0.2 mm .

A particular problem associated with 'O' rings is the ability to cope with small movements of the housings and sealing faces. A wide range of solutions has been developed to produce seals that are resistant to, for instance, rotation within the seal



Figure 9.7 X ring and rectangular seal.

groove. An X ring, also known as a quadring, and a rectangular seal are illustrated in Figure 9.7.

Aperture seals used, for example, for doors, windows and cabriolet bodies are typically made from elastomeric extrusions as production costs are low relative to fabricated mechanical seals and as their assembly can be automated. In the case of automobiles, the requirements are demanding with the need to seal against differential pressure, exclude dust, air, water and noise. In addition the components need to have a life compatible with that of the vehicle, function over a wide range of operating temperatures, the cost of production and installation must be low, the function of the seal must not deteriorate significantly with time and the seal must not significantly influence the drag of the vehicle. A typical seal for an automotive application is illustrated in Figure 9.8.

9.2.2 Gaskets

A gasket is a material or composite of materials clamped between two components with the purpose of preventing fluid flow. Gaskets are typically made up of spacer rings, a sealing element, internal reinforcement, a compliant surface layer and possibly some form of surface antistick treatment as shown in Figure 9.9.

Figure 9.10 shows a typical application for a gasket seal. When first closed a gasket seal is subject

to compressive stresses produced by the assembly. Under working conditions, however, the compressive load may be relieved by the pressures generated within the assembly or machine. This must be accounted for in the detailed design or by use of a factor to allow for the relaxation of gasket compression. Typical gasket designs are illustrated in Table 9.2. The choice of material depends on the temperature of operation, the type of fluid being contained and the leakage rate that can be tolerated. The order of magnitude of leakage for a range of materials is illustrated in Figure 9.11. Gasket selection is considered in further detail by Winter (1998).

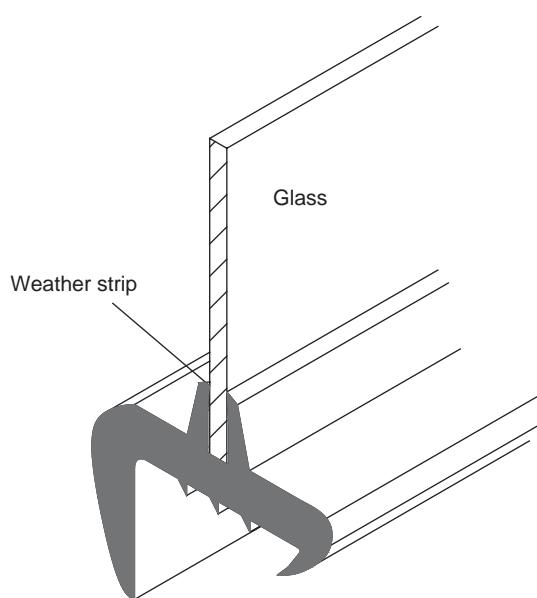


Figure 9.8 An example of an aperture seal for an automotive application.

9.2.3 Foodstuffs containers

The sealing of foodstuffs amounts for a significant proportion of seal designs. Sealing of foodstuffs containers involves consideration of leakage of the

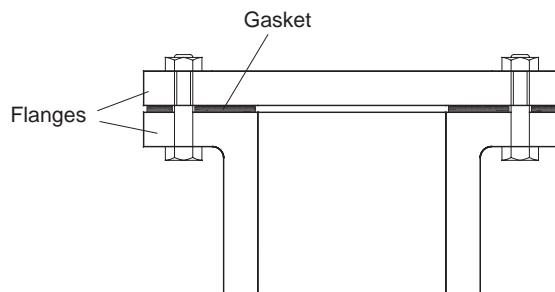


Figure 9.10 Typical gasket application.

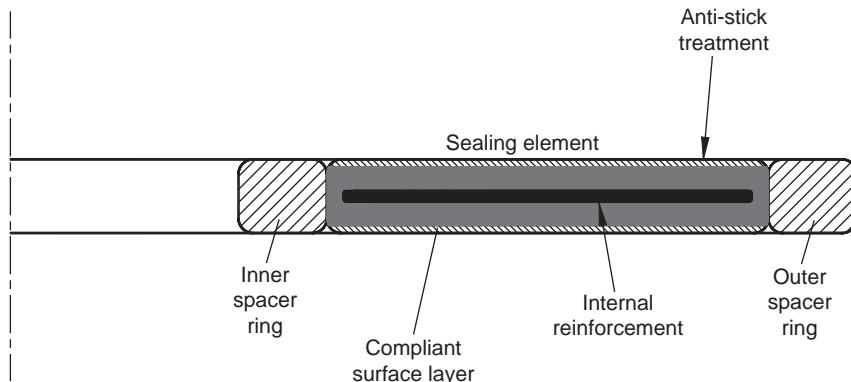
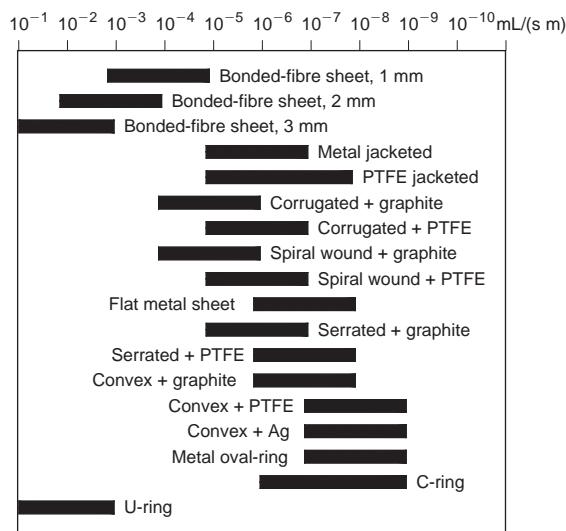
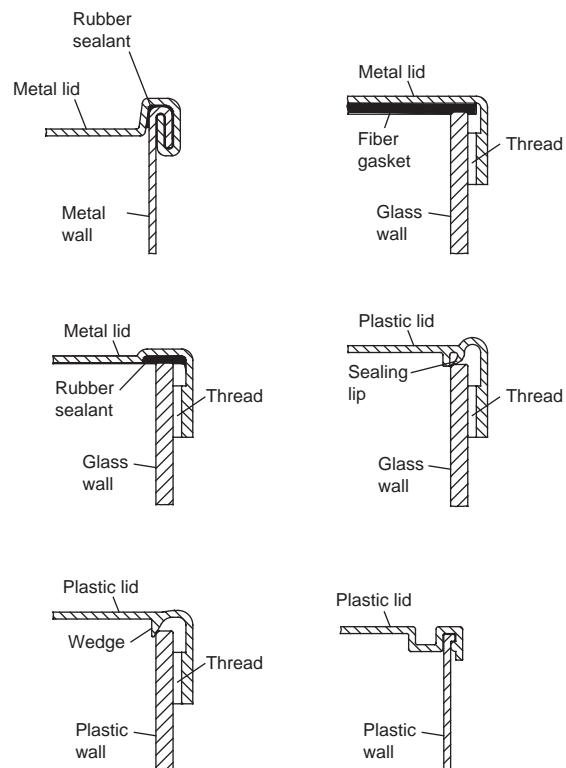


Figure 9.9 The constituent parts of a typical gasket.

Table 9.2 Typical gasket designs

Type	Cross-section	Comment
Flat		Available in a wide variety of materials. Easily formed into other shapes
Reinforced		Fabric or metal reinforced. Improves torque retention and blowout resistance in comparison with flat types
Flat with rubber beads		Rubber beads located on flat or reinforced material. Gives high unit sealing pressure
Flat with metal grommit		The metal grommit gives protection to the base material
Plain metal jacket		The metal jacket gives protection to the filler on one edge and across the surface
Corrugated or embossed		Corrugations provide increased sealing pressure capability
Profile Spiral wound		Multiple sealing surfaces Interleaving pattern of metal and filler

Reproduced with alterations from Czemik, 1996.

**Figure 9.11** Typical order of magnitude of gasket leakage rate per unit diameter for a range of gasket materials (reproduced from Muller and Nau, 1998).**Figure 9.12** Seals for food containers (reproduced from Muller and Nau, 1998).

contents, sealing against biological contamination by, for instance, bacteria and chemical odours. The typical diameter of bacteria is of the order of $1 \mu\text{m}$ and the challenge in designing foodstuffs containers is to exclude bacteria for the shelf life of the product, which may be a number of years. For many designs, the joint must be reusable as with carbonated drink and sauce bottles. A variety of typical joints for food containers is illustrated in Figure 9.12.

9.3 Dynamic seals

The term ‘dynamic seal’ is used to designate a device used to limit the flow of fluid between surfaces that move relative to each other. The range of dynamic seals is extensive with devices for both rotary and reciprocating motion. The requirements

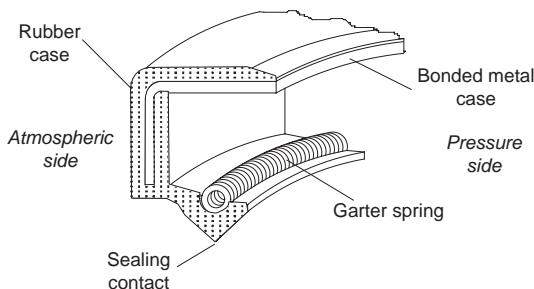


Figure 9.13 Radial lip seal.

of dynamic seals are often conflicting and require compromise. Effective sealing may require high contact pressure between a stationary component and a rotating component, but minimal wear is also desired for long seal life.

9.3.1 Seals for rotating machinery

The functions of seals on rotating shafts include retaining working fluids, retaining lubricants and excluding contaminants, such as dirt and dust. The selection of seal type depends on the shaft speed, working pressure and desired sealing effectiveness. Seals for rotary motion include 'O' rings, lip seals, face seals, sealing rings, compression packings and non-contacting seals such as bush and labyrinth seals.

'O' rings were covered in Section 9.2.1. Their application to rotating shafts is generally limited to use when the shaft speed is below 3.8 m/s and seal pressures below 14 bar.

The typical geometry for a radial lip seal, commonly known as an oil ring, is shown in Figure 9.13. These seals are used to retain lubricants and exclude dirt and are well suited to moderate speed and low pressure applications. The outer case should be retained in the housing by an interference fit. The purpose of the garter spring is to maintain a uniform radial force on the shaft ensuring contact between the elastomeric sealing ring and the rotating shaft. The design of radial lip seals is detailed by Horve (1996) and in the SAE fluid sealing handbook.

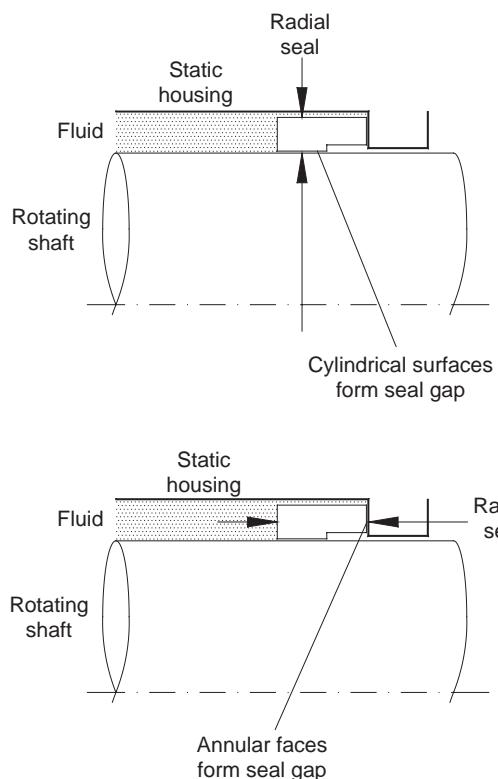


Figure 9.14 Operating principle of mechanical face seals.

For applications where low leakage, high reliability and low wear are important, mechanical face seals may be appropriate. A mechanical face seal consists of two sealing rings, one attached to the rotating member and one attached to the stationary component to form a sealing surface, usually perpendicular to the shaft axis as illustrated in Figure 9.14. During rotation the primary sealing ring attached to the shaft rubs with its seal face against the counter-seal face of the stationary ring. The two interface contact areas function like bearings and are subject to frictional wear. Any leakage flow must pass across this interface.

The overall assembly for a typical mechanical face seal is illustrated in Figure 9.15. Frictional rubbing contact between the sealing faces is maintained by forces acting axially caused by hydraulics or mechanically using, for example, a spring as shown in Figure 9.15. The rubbing action between the surfaces produces heat and wear. In order to

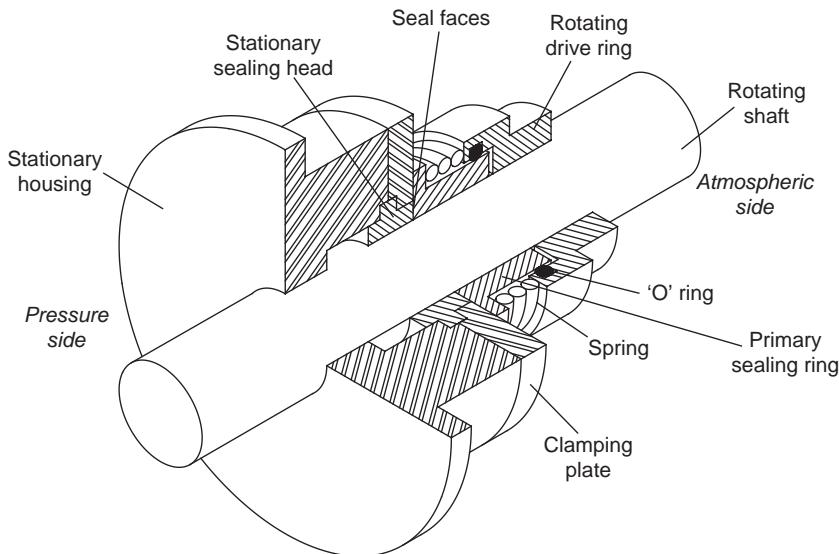


Figure 9.15 A typical mechanical face seal.

minimize this, lubrication is used, which as well as limiting wear and heat build up also serves to generate a fluid film which assists in producing a tight seal. The range of face seal designs is extensive and they are widely used in pumps, compressors, gearboxes and domestic washing machines. The design of face seals is reviewed by Dolan (1992), Summers-Smith (1992), Muller and Nau (1998) and Lebeck (1991).

The term 'interstitial seal' is used for seals that allow unrestricted relative motion between the stationary and moving components (i.e. no seal to shaft contact). Types include labyrinth, brush and bush seals.

A labyrinth seal in its simplest form consists of a series of radial fins forming a restriction to an annular flow of fluid as shown in Figure 9.16. In order for the fluid to pass through the annular restriction, it must accelerate. Just after the restriction the fluid will expand and decelerate with the formation of separation eddies in the cavity downstream of the fin as illustrated schematically in Figure 9.17. These turbulent eddies dissipate some of the energy of the flow reducing the pressure. This process will be repeated in subsequent cavities until the pressure reaches downstream conditions. Labyrinth seals are essentially a controlled clearance seal without rubbing contact. As there is no surface-to-surface

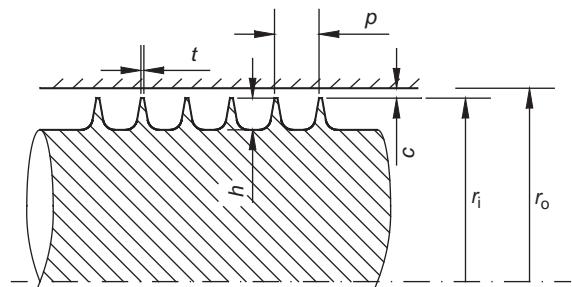


Figure 9.16 Labyrinth seal geometry.

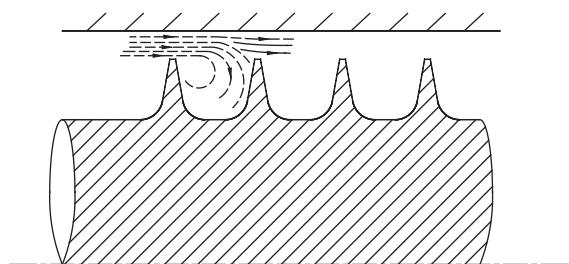


Figure 9.17 Labyrinth seal flow.

contact, very high relative speeds are possible and the geometry can be arranged to limit leakage to tolerable levels. Leakage is, however, inevitable and can be quantified using the method outlined below. Care must be taken in the design of labyrinth seals. If the cavity following the radial fin is too small

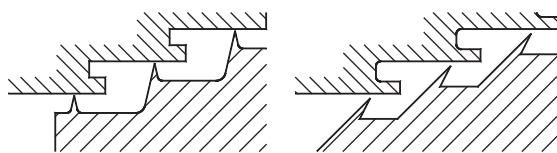


Figure 9.18 Labyrinth seal designs to limit carryover flow (after Stocker, 1978).

the flow will pass straight through without expanding and without the subsequent pressure drop. Despite careful labyrinth design some flow will inevitably be carried straight over from one fin to another. In order to reduce this effect, steps can be incorporated into labyrinth seal design and a wide range of labyrinth variants are possible as illustrated in Figure 9.18.

Flow through a labyrinth can be estimated using Eq. 9.1 (Egli, 1935):

$$\dot{m} = A\alpha\gamma\varphi\sqrt{\rho_o p_o} \quad (9.1)$$

where \dot{m} is the mass flow rate (kg/s); $A = \pi(r_o^2 - r_i^2)$, area of the annular gap between the fin tips and the casing (m^2); α , flow coefficient; γ , carry over correction factor; φ , expansion ratio; p_o , upstream pressure (Pa); and ρ_o , density at the upstream conditions (kg/m^3).

The flow coefficient, α , is a function of the clearance to tip width ratio, but an average value of 0.71 can be used for

$$1.3 < c/t < 2.3 \quad (9.2)$$

where c , radial clearance (m); t , thickness of fin (m).

The carry over correction factor, γ , varies as a function of the clearance to pitch ratio. As a crude approximation, γ can be taken as varying linearly for c/p values of 0 to 0.11 as listed in Table 9.3.

The expansion ratio, φ , is given by:

$$\varphi = \sqrt{\frac{1 - (p_n/p_o)}{n + \ln(p_o/p_n)}} \quad (9.3)$$

where p_n is downstream pressure following the n th labyrinth (Pa); and n is the number of fins.

Although this method was developed some time ago, the results are reasonably reliable. Some

Table 9.3 Relationships for the carry over correction factor as a function of fin number

Carry over correction factor	Number of fins
$\gamma = 1 + 3.27(c/p)$	2
$\gamma = 1 + 5(c/p)$	3
$\gamma = 1 + 6.73(c/p)$	4
$\gamma = 1 + 8.82(c/p)$	6
$\gamma = 1 + 10.2(c/p)$	8
$\gamma = 1 + 11.2(c/p)$	12

c , radial clearance (m); p , pitch (m)

Table 9.4 Guidelines for the selection of fin thickness, pitch and height

t (mm)	p (mm)	h (mm)
0.3–0.4	6–8	4–5
0.28–0.32	4–5	3–3.5
0.18–0.22	1.8–2.2	1.8–2.2

companies have their own proprietary equations and methods for determining leakage rates through both straight and stepped labyrinth seals and as an alternative CFD (computational fluid dynamics) can also be used to model the seal flow. In general, fins are normally included on the rotor, although they can if necessary be incorporated on the stationary component if, for example, significant axial movements are anticipated. Six fins are normally found to be adequate and beyond six or seven fins there is little improvement. Typical values for the selection of values for the fin thickness, pitch and fin height are listed in Table 9.4 as a guideline. The flow rate can then be calculated using Eq. 9.1 for a given labyrinth radius and clearance.

Example 9.2

Determine the mass flow rate through a labyrinth seal on a 100 mm diameter shaft. The labyrinth consists of six fins, height 3.2 mm, pitch 4.5 mm, radial clearance 0.4 mm and tip width 0.3 mm. The pressure is being dropped from 4 bar absolute, 353 K, to atmospheric conditions (1.01 bar). Take the gas constant R as 287 J/(kg K).

Solution

The outer radius of the annular gap is $(100/2) + 3.2 + 0.4 = 53.6$ mm. The inner radius of the annular gap is $(100/2) + 3.2 = 53.2$ mm. The annulus gap area is

$$A = \pi(r_o^2 - r_i^2) = \pi((53.6 \times 10^{-3})^2 - (53.2 \times 10^{-3})^2) = 1.342 \times 10^{-4} \text{ m}^2.$$

$\alpha = 0.71$; $\gamma = 1 + 8.82(c/p)$ for $n = 6$; $c = 0.4$ mm; $p = 4.5$ mm; and $\gamma = 1 + 8.82 \times (10.4/4.5) = 1.784$.

$$\begin{aligned}\phi &= \sqrt{\frac{1 - (p_n/p_o)}{n + \ln(p_o/p_n)}} \\ &= \sqrt{\frac{1 - (1.01 \times 10^5 / 4 \times 10^5)}{6 + \ln(4 \times 10^5 / 1.01 \times 10^5)}} \\ &= 0.3183.\end{aligned}$$

$p = \rho RT$, so the upstream density is given by,

$$\rho_o = p_o/RT_o = 4 \times 10^5 / (287 \times 353) = 3.948 \text{ kg/m}^3.$$

$$\begin{aligned}\dot{m} &= 1.342 \times 10^{-4} \times 0.71 \times 1.784 \\ &\quad \times 0.3183 \sqrt{3.948 \times 4 \times 10^5} \\ &= 0.0680 \text{ kg/s}\end{aligned}$$

Simple axial and radial bush seals are illustrated in Figures 9.19 and 9.20 and can be used for sealing both liquids and gases. Leakage through a concentric axial bush seal can be estimated using Eq. 9.4 for incompressible flow (flow where the density can be considered constant) and Eq. 9.5 for compressible flow. The leakage flow through a radial bush seal can be estimated by Eq. 9.6 for incompressible flow and Eq. 9.7 for compressible flow.

$$Q = \frac{\pi\phi c^3(p_o - p_a)}{12\mu L} \quad (9.4)$$

$$Q = \frac{\pi\phi c^3(p_o^2 - p_a^2)}{24\mu L p_a} \quad (9.5)$$

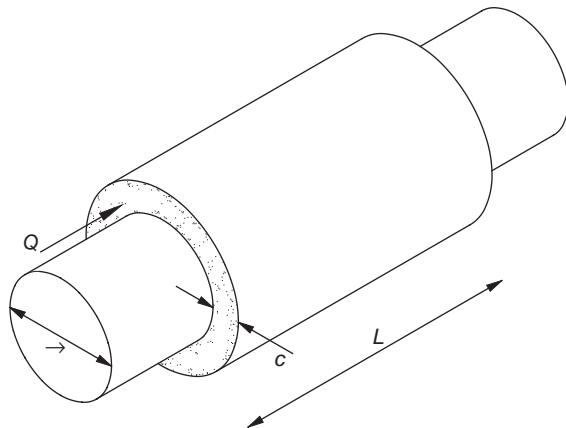


Figure 9.19 Axial bush seal.

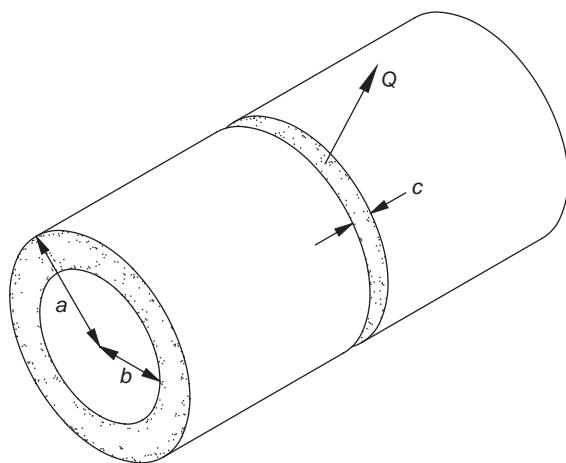


Figure 9.20 Radial bush seal.

where Q is volumetric flow rate (m^3/s); ϕ , diameter of the shaft (m); c , radial clearance (m); p_o , upstream pressure (Pa); p_a , downstream pressure (Pa); μ , absolute viscosity ($\text{Pa} \cdot \text{s}$); and L , length of the axial bush seal (m).

$$Q = \frac{\pi c^3(p_o - p_a)}{6\mu \ln(a/b)} \quad (9.6)$$

$$Q = \frac{\pi c^3(p_o^2 - p_a^2)}{12\mu p_a} \quad (9.7)$$

where c is axial clearance (m); a , outer radius (m); and b , inner radius (m).

Example 9.3

An axial bush seal consists of an annular gap with inner and outer radii of 50 and 50.5 mm, respectively. The length of the seal is 40 mm. Determine the flow rate of oil through the seal if the pressures upstream and downstream of the seal are 7 and 5.5 bar, respectively. The viscosity of the oil can be taken as 0.025 Pa s.

Solution

The radial clearance is $0.0505 - 0.05 = 0.0005$ m; $\phi = 0.1$ m; $L = 0.04$ m.

The volumetric flow rate is given by:

$$Q = \frac{\pi \times 0.1 \times (0.5 \times 10^{-3})^3 (7 \times 10^5 - 5.5 \times 10^5)}{12 \times 0.025 \times 0.04} \\ = 4.909 \times 10^{-4} \text{ m}^3/\text{s.}$$

9.3.2 Seals for reciprocating components

The seals principally used for reciprocating motion are packings and piston rings.

Packing seals are illustrated in Figure 9.21. The seal essentially consists of a cup, V, U or X section of leather, solid rubber or fabric reinforced rubber. The sealing principle is by direct contact with the reciprocating component. The contact pressure can be increased in the case of V packings by axial compression of the seals, although this obviously increases the friction on the shaft and wear rate. The principal uses of cup packings are as piston seals in hydraulic and pneumatic applications, U packings for piston rods and V packings for sealing piston rods or reciprocating shafts.

Piston rings are used to seal cylinders where the operating temperature is above the limit of elastomeric, fabric or polymeric materials. Piston rings are used in automotive cylinders for three purposes:

- 1 to seal the combustion chamber/cylinder head;
- 2 to transfer heat from the piston to the cylinder walls;
- 3 to control the flow of oil.

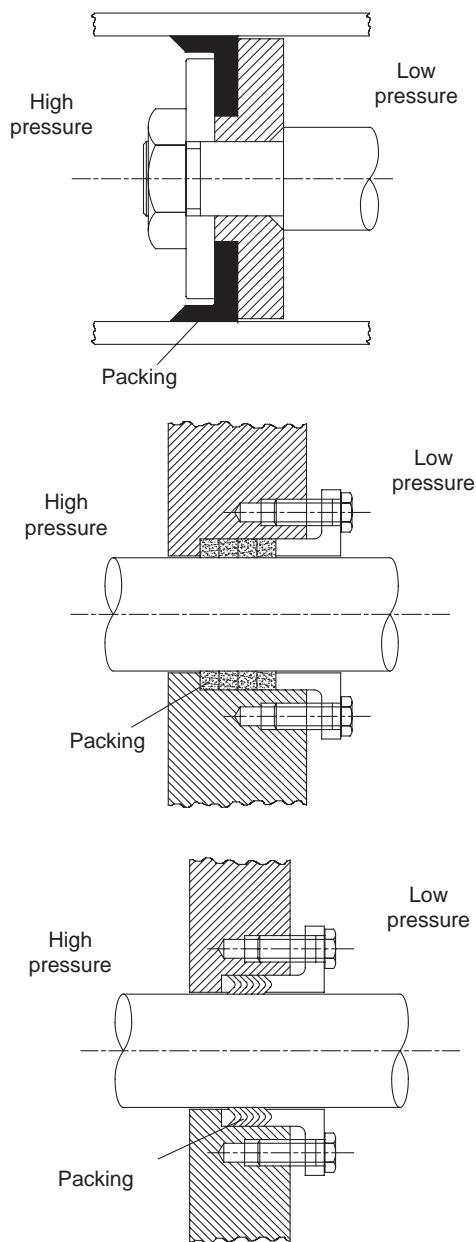
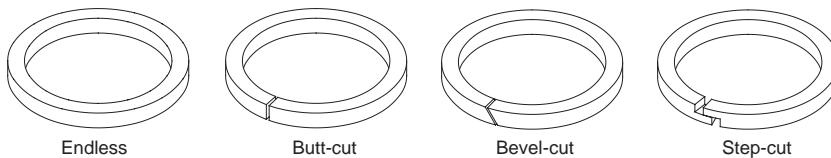
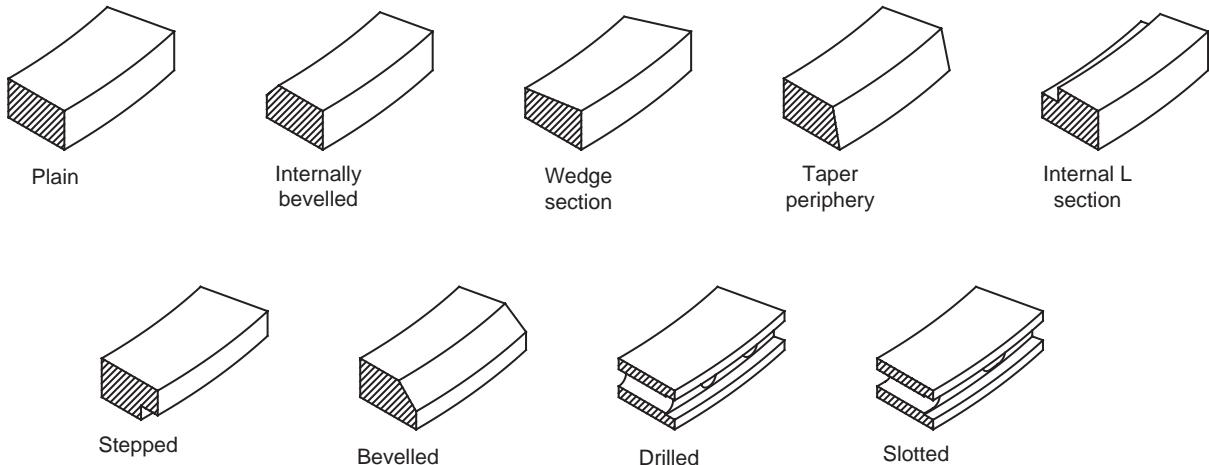


Figure 9.21 Packing seals.

Piston rings are usually machined from a fine grain alloy cast iron and must be split to allow for assembly over the piston. Conventional practice is to use three piston rings with two compression rings sealing the high pressure and one to control the flow of oil. The range of piston ring available

**Figure 9.22** Piston rings.**Figure 9.23** Piston ring sections.**Table 9.5** Number of piston rings required to seal a given pressure

p_o (bar)	Number of rings
<20	2
$20 < p_o < 60$	3
$60 < p_o < 100$	4
$100 < p_o < 200$	5
>200	6+

is extensive as illustrated in Figures 9.22 and 9.23. For other applications the number of piston rings required can be determined using Table 9.5 assuming normal piston temperatures and running speeds.

9.4 Conclusions

It is frequently necessary in machine design to provide some means of containing or limiting the flow of fluid from one region to another. Because of the very small nature of fluid molecules this is a

challenging task. This chapter has reviewed a range of static and dynamic seals. Because of the wide range of applications, seals tend not to be available as stock items and instead must be designed fit for purpose.

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- www.chesterton.com
- www.china-dongya.com
- www.claron.co.uk
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- www.jenp.com.tw
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- www.pspglobal.com
- www.samsunsegman.com
- www.sealsales.com
- www.ses-seal.com.tw
- www.superseal.com
- www.totalseal.com

www.trisunltd.com
 www.uniquerubber.com
 www.usseal.com/
 www.ussealmfg.com
 www.vercoseals.com

Nomenclature

Generally, preferred SI units have been stated.

a	outer radius (m)
A	area (m^2)
b	inner radius (m)
c	clearance (m)
h	height (m)
L	length (m)
\dot{m}	mass flow rate (kg/s)
n	number of fins
p	pitch (m), pressure (Pa)
p_n	downstream pressure following the n th labyrinth (Pa)
p_o	upstream pressure (Pa)
Q	volumetric flow rate (m^3/s)
R	characteristic gas constant ($\text{J}/(\text{kg K})$)
t	thickness of fin (m)
α	flow coefficient
γ	carryover correction factor
ϕ	diameter of the shaft (m)
φ	expansion ratio
μ	absolute viscosity (Pa s)
ρ_o	density at the upstream conditions (kg/m^3)

Worksheet

- 9.1 Determine an appropriate seal type for the following applications:
- A high speed, 100 mm diameter, rotating shaft for a gas turbine engine to limit the escape of air between the shaft and stationary housing.
 - To completely seal the back face of a rotodynamic pump rotating at 2970 rpm.
 - To seal the faces of two mating flanges.
- 9.2 Determine suitable groove dimensions for an 0101-16 'O' ring to seal against a solid cylinder.
- 9.3 Determine the flow rate of air through a labyrinth seal on a 150 mm diameter shaft. The labyrinth consists of 8 fins, height 4.5 mm, pitch 7 mm, radial clearance 0.5 mm and tip width 0.35 mm. The absolute pressures upstream and downstream of the seal are 2 and 1.01 bar, respectively. The temperature

of the air upstream of the seal is approximately 323 K. Take the gas constant R as $287 \text{ J}/(\text{kg K})$.

- 9.4 The shaft for a small high speed rotating machine has a diameter of 60 mm. The absolute pressure within the machine is $1.2 \times 10^5 \text{ Pa}$ and the operating temperature is approximately 318 K. Design a suitable labyrinth seal to limit the flow of air escaping from the machine to atmospheric conditions to 0.01 kg/s .
- 9.5 The shaft for a high speed rotating machine has a diameter of 150 mm. The absolute pressure within the machine is $2.5 \times 10^5 \text{ Pa}$ and the operating temperature is approximately 443 K. Design a suitable labyrinth seal to limit the flow of air escaping from the machine to a chamber at an absolute pressure of 2.2 bar to 0.03 kg/s .
- 9.6 An axial bush seal consists of an annular gap with inner and outer radii of 0.025 and 0.0253 mm, respectively. The length of the seal is 30 mm. Determine the flow rate of oil through the seal if the pressures upstream and downstream of the seal are 6 and 5.5 bar, respectively. The viscosity of the oil can be taken as 0.02 Pas .
- 9.7 Determine the flow rate of air through an axial bush seal. The outer diameter of the inner cylinder is 100 mm and the radial clearance is 0.1 mm. The length of the seal is 50 mm. The pressures upstream and downstream of the seal are 1.4 and 1.01 bar, respectively. The viscosity of air can be taken as $1.85 \times 10^{-5} \text{ Pas}$.
- 9.8 A radial bush seal consists of a cylinder with an inner radius of 20 mm and an outer radius of 40 mm. The axial gap is 0.2 mm. Determine the flow rate of oil through the clearance if the pressure inside the cylinder is 6 bar and the pressure outside the cylinder is 5.2 bar. Take the viscosity of oil as 0.022 Pas .
- Answers**
- 9.1 (a) Labyrinth seal; (b) Mechanical face seal; (c) Gasket.
- 9.2 $d = 10.5 \text{ mm}; D = 12.8 \text{ mm}; B = 2.3 \text{ mm}; R = 0.5 \text{ mm}$.
- 9.3 0.048 kg/s .
- 9.4 No unique solution.
- 9.5 No unique solution.
- 9.6 $2.945 \times 10^{-5} \text{ m}^3/\text{s}$.
- 9.7 $1.317 \times 10^{-3} \text{ m}^3/\text{s}$.
- 9.8 $2.2 \times 10^{-5} \text{ m}^3/\text{s}$.

Learning objectives achievement

Can you identify the different types of sealing devices?



Section 9.1

Can you select a seal type for rotating, reciprocating or static conditions?



Section 9.1

Can you determine the groove dimensions for a standard 'O' ring?



Section 9.2.1

Can you calculate the leakage flow through a labyrinth seal?



Section 9.3.1

Do you calculate the leakage flow through a bush seal?



Section 9.3.1

10

CLUTCHES AND BRAKES

A clutch is a device that permits the smooth, gradual connection of two shafts rotating at different speeds. A brake enables the controlled dissipation of energy to slow down, stop or control the speed of a system. This chapter describes the basic principles of frictional clutches and brakes and outlines design and selection procedures for disc clutches, disc and drum brakes.

LEARNING OBJECTIVES

At the end of this chapter you should be able to determine:

- the primary dimensions for a single disc clutch;
- the principal dimensions and number of discs for a multiple disc clutch for a given speed and power requirement;
- torque capacity for short or long, internal or external brakes;
- the configuration of a brake to be self-energizing.

10.1 Introduction

When a rotating machine is started, it must be accelerated from rest to the desired speed. A clutch is a device used to connect or disconnect a driven component from a prime mover such as an engine or motor. A familiar application is the use of a clutch between an engine's crankshaft and the gearbox in automotive applications. The need for the clutch arises from the relatively high torque

requirement to get a vehicle moving and the low torque output from an internal combustion engine at low levels of rotational speed. The disconnection of the engine from the drive enables the engine to speed up unloaded to about 1000 rpm where it is generating sufficient torque to drive the transmission. The clutch can then be engaged, allowing power to be transmitted to the gearbox, transmission shafts and wheels. A brake is a device used to reduce or control the speed of a system or bring it to rest. Typical applications of a clutch and brake are illustrated in Figure 10.1. Clutches and brakes are similar devices providing frictional, magnetic or mechanical connection between two components. If one component rotates and the other is fixed to a non-rotating plane of reference the device will function as a brake and if both rotate then as a clutch.

Whenever the speed or direction of motion of a body is changed there is force exerted on the body. If the body is rotating, a torque must be applied to the system to speed it up or slow it down. If the speed changes, so does the energy, either by addition or absorption.

The acceleration, α , of a rotating machine is given by

$$\alpha = \frac{T}{I} \quad (10.1)$$

where T is the torque (N m); and I is the mass moment of inertia (kg m^2).

The mass moment of inertia can often be approximated by considering an assembly to be made up of a series of cylinders and discs and

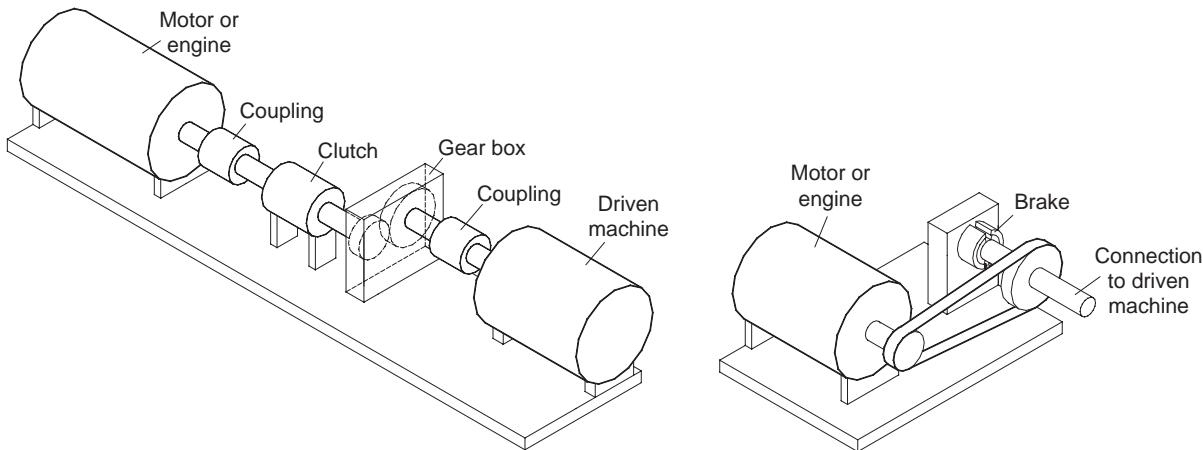


Figure 10.1 Typical applications of clutches and brakes.

summing the individual values for the disc and cylinder mass moments of inertia. The mass moments of inertia for a cylinder and a disc are given by Eqs 10.2 and 10.3, respectively.

$$I_{\text{cylinder}} = \frac{1}{2} \rho \pi L r_o^4 - r_i^4 \rightarrow \quad (10.2)$$

$$I_{\text{disc}} = \frac{1}{2} \rho \pi L r_o^4 \quad (10.3)$$

The desired level of acceleration will depend on the application and once the acceleration has been set and I determined, the approximate value of torque required to accelerate or brake the load can be estimated. This can be used as the principal starting point for the design or selection of the clutch or brake geometry.

Torque is equal to the ratio of power and angular velocity. In other words, torque is inversely proportional to angular velocity. This implies that it is usually advisable to locate the clutch or brake on the highest speed shaft in the system so that the required torque is a minimum. Size, cost and response time are lower when the torque is lower. The disadvantage is that the speed differential between the components can result in increased

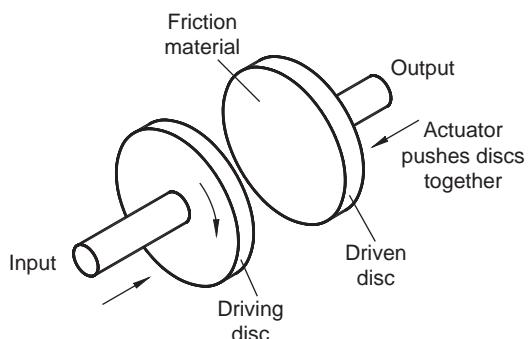


Figure 10.2 Idealized friction disc clutch or brake.

slipping and associated frictional heating potentially causing overheating problems.

Friction type clutches and brakes are the most common. Two or more surfaces are pushed together with a normal force to generate a friction torque (see Figure 10.2). Normally, at least one of the surfaces is metal and the other a high friction material referred to as the lining. The frictional contact can occur radially, as for a cylindrical arrangement, or axially as in a disc arrangement. The function of a frictional clutch or brake surface material is to develop a substantial friction force when a normal force is applied. Ideally a material with a high coefficient of friction, constant

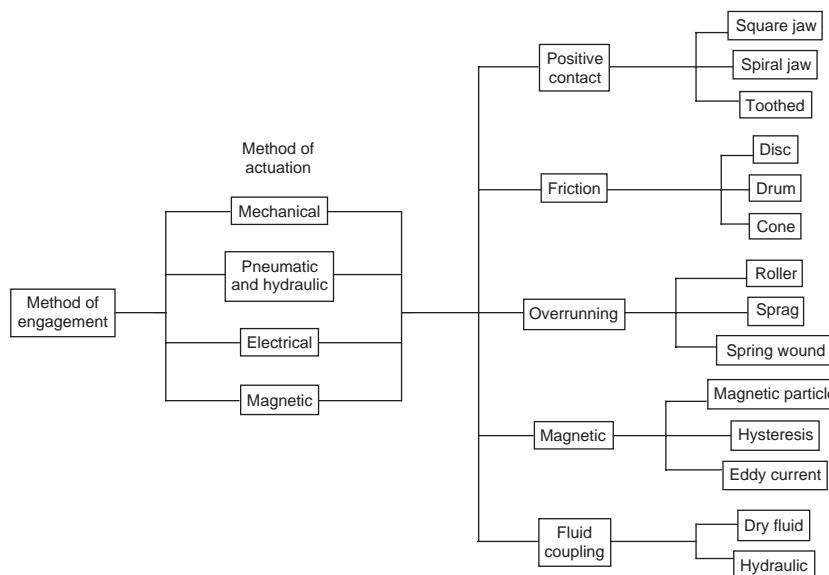


Figure 10.3 Clutch classification (reproduced with adaptations from Hindhede *et al.*, 1983).

properties, good resistance to wear and chemical compatibility is required. Clutches and brakes transfer or dissipate significant quantities of energy and their design must enable the absorption and transfer of this heat without damage to the component parts of the surroundings.

With the exception of high volume automotive clutches and brakes, engineers rarely need to design a clutch or a brake from scratch. Clutch and brake assemblies can be purchased from specialist suppliers and the engineer's task is to specify the torque and speed requirements, the loading characteristics and the system inertias and to select an appropriately sized clutch or brake and the lining materials.

10.2 Clutches

The function of a clutch is to permit the connection and disconnection of two shafts, either when both are stationary or when there is a difference in the relative rotational speeds of the shafts. Clutch connection can be achieved by a number of techniques from direct mechanical friction, electromagnetic coupling, hydraulic or pneumatic means or by some combination. There are various types

of clutches as outlined in Figure 10.3. The devices considered here are of the friction type.

Clutches must be designed principally to satisfy four requirements:

1. The necessary actuation force should not be excessive.
2. The coefficient of friction should be constant.
3. The energy converted to heat must be dissipated.
4. Wear must be limited to provide reasonable clutch life.

Alternatively the objective in clutch design can be stated as maximization of a maintainable friction coefficient and minimization of wear. Correct clutch design and selection are critical because a clutch that is too small for an application will slip and overheat and a clutch that is too large will have a high inertia and may overload the drive.

Positive contact clutches have teeth or serrations, which provide mechanical interference between mating components. Figure 10.4 shows a square jaw and Figure 10.5 shows a multiple serration positive contact clutch. This principle can be combined with frictional surfaces as in an automotive

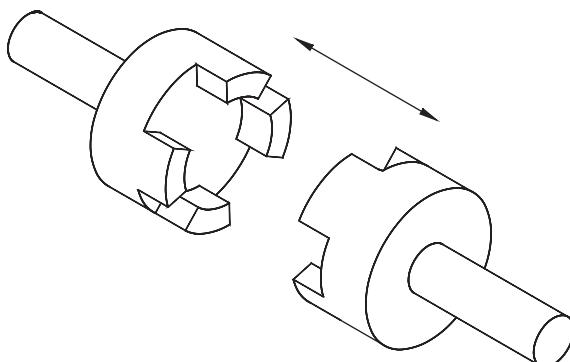


Figure 10.4 Square jaw clutch.

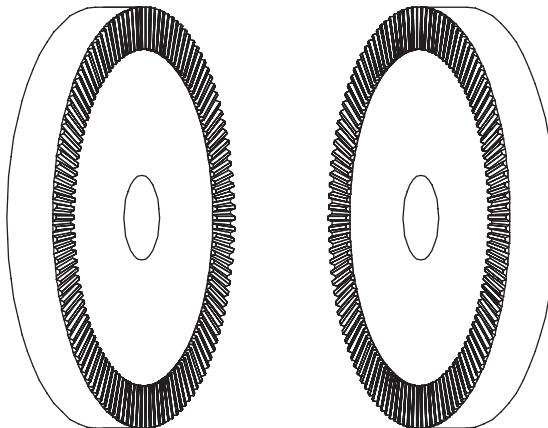


Figure 10.5 Multiple serration clutch.

synchromesh clutch. As helical gears cannot be shifted in and out of mesh easily, the pragmatic approach is to keep the gears engaged in mesh and allow the gear to rotate freely on the shaft when no power is required but provide positive location of the gear on the shaft when necessary (see also Chapter 6, Section 6.3.1).

Friction clutches consist of two surfaces, or two sets of surfaces, which can be forced into frictional contact. A frictional clutch allows gradual engagement between two shafts. The frictional surfaces can be forced together by springs, hydraulic pistons or magnetically. Various forms exist such as disc, cone and radial clutches. The design of the primary geometry for disc clutches is described in Section 10.2.1.

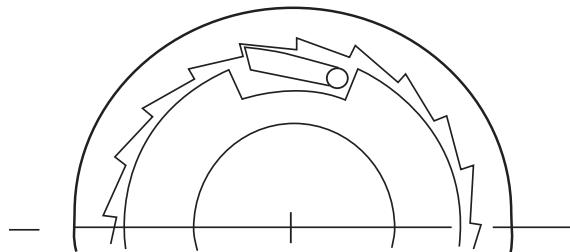


Figure 10.6 Ratchet and pawl (reproduced from Neale, 1995).

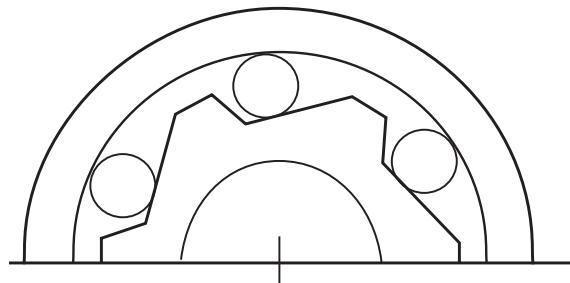


Figure 10.7 Roller clutch (reproduced from Neale, 1995).

Overrunning clutches operate automatically based on the relative velocity of the mating components. They allow relative motion in one direction only. If the rotation attempts to reverse, the constituent components of the clutch grab the shaft and lock up. Applications include backstops, indexing and freewheeling, as on a bicycle when the wheel speed is greater than the drive sprocket. The range of overrunning clutches includes the simple ratchet and pawl (Figure 10.6) roller, sprag and spring wound clutches.

Roller clutches consist of balls or cylindrical rollers located in wedge-shaped chambers between inner and outer races (Figure 10.7). Rotation is possible in one direction but when this is reversed the rollers wedge themselves between the races jamming the inner and outer races together. Figure 10.8 illustrates the principal components of a sprag clutch which consists of an inner and outer race, like a bearing, but instead of balls the space between the races contains specially shaped elements called ‘sprags’ which permit rotation in one direction only.

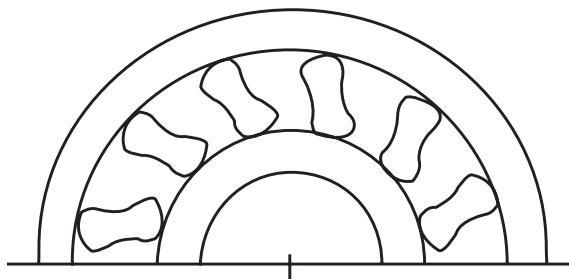


Figure 10.8 Sprag clutch (reproduced from Neale, 1995).

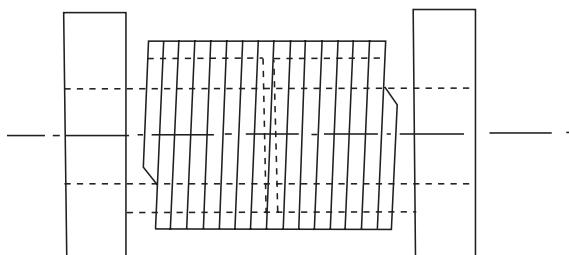


Figure 10.9 Spring wound clutch (reproduced from Neale, 1995).

If the clutch motion is reversed, the sprags jam and lock the clutch. A spring wound clutch is shown in Figure 10.9. Friction between hubs and the coil spring causes the spring to tighten on to hubs and drive in one direction. In the opposite direction the spring unwraps and slips easily on the hubs.

Magnetic clutches use a magnetic field to couple the rotating components together. Magnetic clutches are smooth, quiet and have a long life, as there is no direct mechanical contact and hence no wear, except at the bearings.

Fluid couplings transmit torque through a fluid such as oil. A fluid coupling always transmits some torque because of the swirling nature of the fluid contained within the device. For this reason it is necessary to maintain some braking force in automatic transmission cars that utilize fluid coupling torque converters in the automatic gearbox.

Centrifugal clutches engage automatically when the shaft speed exceeds some critical value. Friction elements are forced radially outwards and engage against the inner radius of a mating cylindrical drum (Figure 10.10). Common applications of

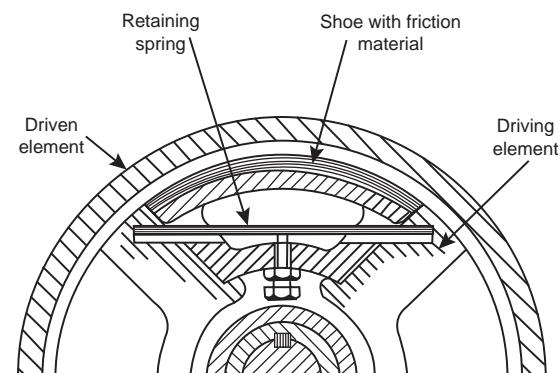


Figure 10.10 Centrifugal clutch (reproduced from Neale, 1995).

centrifugal clutches include chainsaws, overload-releases and go-karts.

The selection of clutch type and configuration depends on the application. Table 10.1 can be used as the first step in determining the type of clutch to be used. Clutches are rarely designed from scratch. Either an existing design is available and is being modified for a new application or a clutch can be bought in from a specialist manufacturer. In the latter case the type, size and the materials for the clutch lining must be specified. This requires determination of the system characteristics such as speed, torque, loading characteristic (e.g. shock loads) and operating temperatures. Many of these factors have been lumped into a multiplier called a service factor. A lining material is typically tested under steady conditions using an electric motor drive. The torque capacity obtained from this test is then derated by the service factor according to the particular application to take account of vibrations and loading conditions. Table 10.2 gives an indication of the typical values for service factors.

10.2.1 Design of disc clutches

Disc clutches can consist of single or multiple discs as illustrated in Figures 10.11, 10.12 and 10.13. Generally multiple disc clutches enable greater torque capacity but are harder to cool. Frictional clutches can be run dry or wet using oil. Typical

Table 10.1 Clutch selection criteria

Type of clutch	Characteristics	Typical applications
Sprag	One way clutch. Profiled elements jam against the outer edge to provide drive. High torque capacity	One way operation, e.g. backstop for hoists
Roller	One way clutch. Rollers ride up ramps and drive by wedging into place	One way operation
Cone clutch	Embodyes the mechanical principle of the wedge which reduces the axial force required to transmit a given torque	Contractor's plant. Feed drives for machine tools
Single disc clutch	Used when diameter is not restricted. Simple construction	Automobile drives
Multiple disc clutch	The power transmitted can be increased by using more plates allowing a reduction in diameter	Machine tool head stocks. Motorcycles
Centrifugal clutch	Automatic engagement at a critical speed	Electric motor drives. Industrial diesel drives
Magnetic	Compact. Low wear	Machine tool gearboxes. Numerical control machine tools

Reproduced with adaptations from Neale, 1994.

Table 10.2 Service factors

Description of general system	Typical driven system	Type of driver			
		Small electric motors, turbine	IC engines (4 to 6 cylinders) Medium to large electric motors	IC engines (2 or 3 cylinders)	Single cylinder engine
Steady power source, steady load, no shock or overload	Belt drive, small generators, centrifugal pumps, fans, machine tools	1.5	1.7	1.9	2.2
Steady power source with some irregularity of load up to 1.5 times nominal power	Light machinery for wood, metal and textiles, conveyor belts	1.8	2.0	2.4	2.7
	Larger conveyor belts, larger machines, reciprocating pumps	2.0	2.2	2.4	2.7
Frequent start-stops, overloads, cycling, high inertia starts, high power, pulsating power source	Presses, punches, piston pumps, cranes, hoists	2.5	2.7	2.9	3.2
	Stone crushers, roll mills, heavy mixers, single cylinder compressors	3.0	3.2	3.4	3.7

Source: Vedamuttu, 1994.

coefficients of friction are 0.07 for a wet clutch and 0.45 for a dry clutch. While running a clutch wet in oil reduces the coefficient of friction it enhances heat transfer and the potential for cool-

ing of the components. The expedient solution to the reduction of the friction coefficient is to use more discs and hence the use of multiple disc clutches.

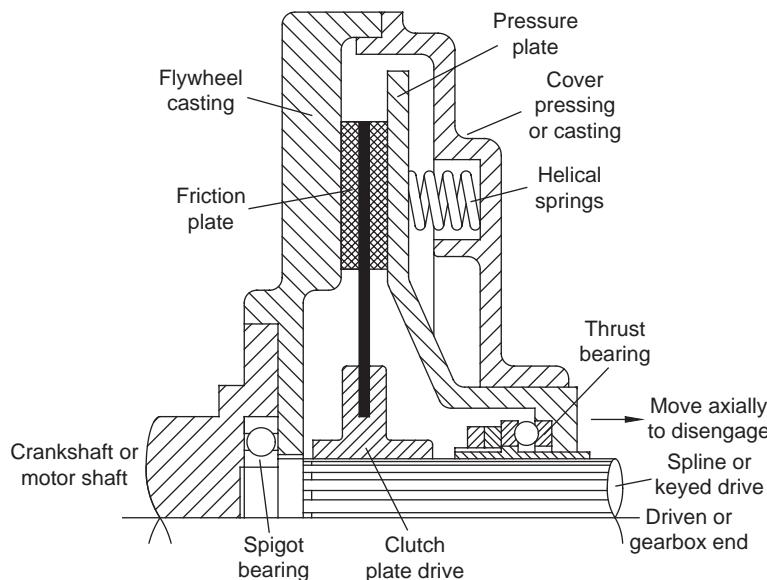


Figure 10.11 Disc clutch (reproduced with adaptations from Neale, 1994).

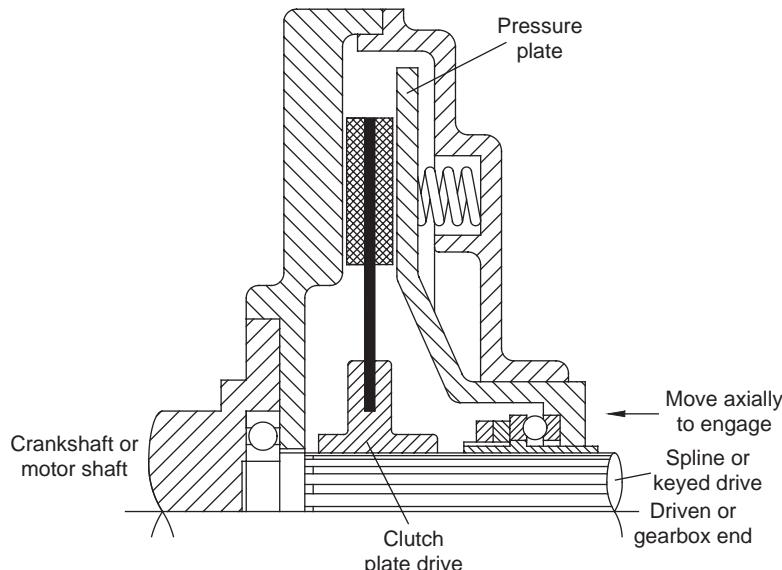


Figure 10.12 Disc clutch shown disengaged. Little or no torque is transferred from the crankshaft to the driven load (reproduced with adaptations from Neale, 1994).

Two basic assumptions are used in the development of procedures for disc clutch design based upon a uniform rate of wear at the mating surfaces or a uniform pressure distribution between the mating surfaces. The equations for both of these methods are outlined in this section.

The assumption of a uniform pressure distribution at the interface between mating surfaces is valid for an unworn accurately manufactured clutch with rigid outer discs.

The area of an elemental annular ring on a disc clutch, Figure 10.14, is $\delta A = 2\pi\delta r$. Now $F = pA$,

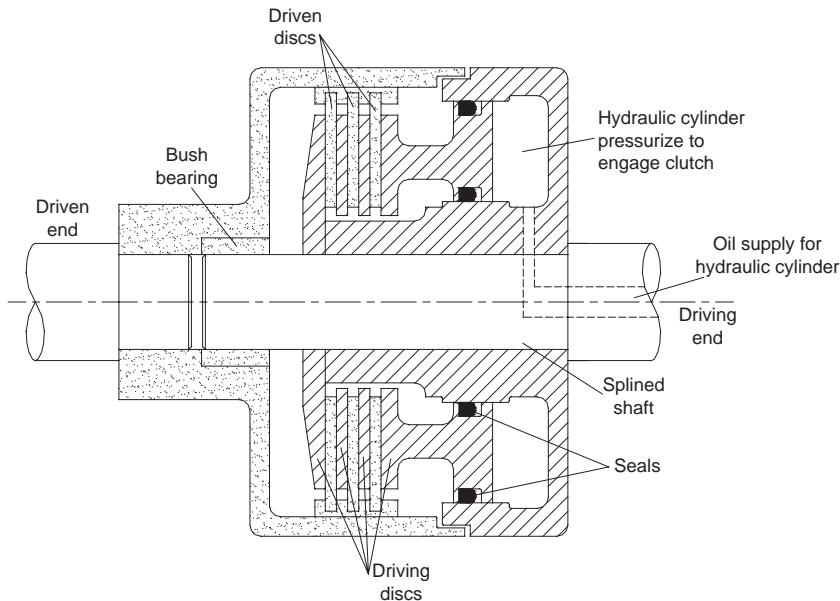


Figure 10.13 Multiple disc clutch (reproduced with adaptations from Juvinal and Marshek, 1991).

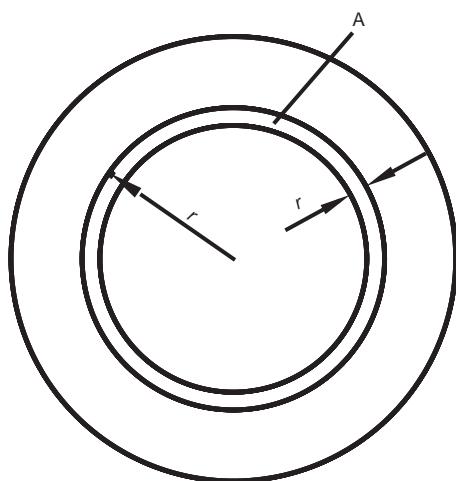


Figure 10.14 Area of an elemental ring on a disc clutch.

where p is the assumed uniform interface pressure, so $\delta F = 2\pi r p \delta r$. For the disc the normal force acting on the entire face is:

$$F = \int_{r_i}^{r_o} 2\pi r p dr = 2\pi p \frac{r^2}{2} \Big|_{r_i}^{r_o} \quad (10.4)$$

$$F = \pi p \left(r_o^2 - r_i^2 \right) \rightarrow \quad (10.5)$$

Note that F is also the necessary force required to clamp the clutch discs together.

The friction torque δT that can be developed on an elemental ring is the product of the elemental normal force, given by $\mu \delta F$ and the radius:

$$\delta T = \mu \delta F = 2\mu \pi r^2 p \delta r \quad (10.6)$$

where μ is the coefficient of friction which models the less than ideal frictional contact which occurs between two surfaces.

The total torque is given by integration of Eq. 10.6 between the limits of the annular ring, r_i and r_o :

$$\begin{aligned} T &= \int_{r_i}^{r_o} 2\mu \pi r^2 p dr = 2\pi p \mu \frac{r^3}{3} \Big|_{r_i}^{r_o} \\ &= \frac{2}{3} \pi p \mu \left(r_o^3 - r_i^3 \right) \rightarrow \end{aligned} \quad (10.7)$$

This equation represents the torque capacity of a clutch with a single frictional interface. In practice clutches use an even number of frictional surfaces as illustrated in Figures 10.11 and 10.13.

For a clutch with N faces the torque capacity is given by:

$$T = \frac{2}{3} \pi p \mu N \downarrow r_o^3 - r_i^3 \rightarrow \quad (10.8)$$

Using Eqs 10.5 and 10.8 and substituting for the pressure, p , gives an equation for the torque capacity as a function of the axial clamping force.

$$T = \frac{2}{3} \mu FN \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \quad (10.9)$$

The equations assuming uniform wear are developed below. The wear rate is assumed to be proportional to the product of the pressure and velocity. So

$$pr\omega = \text{constant} \quad (10.10)$$

where ω is the angular velocity (rad/s).

For a constant angular velocity the maximum pressure will occur at the smallest radius.

$$p_{\max} r_i \omega = \text{constant} \quad (10.11)$$

Eliminating the angular velocity and constant from the above two equations gives a relationship for the pressure as a function of the radius:

$$p = p_{\max} \frac{r_i}{r} \quad (10.12)$$

The value for the maximum permissible pressure is dependent on the clutch lining material and typical values are listed in Table 10.3.

The elemental axial force on an elemental annular ring is given by

$$\delta F = 2\pi pr \delta r \quad (10.13)$$

Integrating to give the total axial force:

$$\begin{aligned} F &= \int_{r_i}^{r_o} 2\pi pr dr = \int_{r_i}^{r_o} 2\pi p_{\max} \frac{r_i}{r} r dr \\ &= 2\pi p_{\max} r_i \downarrow r_o - r_i \rightarrow \end{aligned} \quad (10.14)$$

The elemental torque is given by

$$\delta T = \mu r \delta F \quad (10.15)$$

So

$$T = \int_{r_i}^{r_o} 2\mu \pi p_{\max} r_i r dr = p_{\max} \mu \pi r_i \downarrow r_o^2 - r_i^2 \rightarrow \quad (10.16)$$

Rearranging Eq. 10.16 gives

$$p_{\max} = \frac{F}{2\pi r_i \downarrow r_o - r_i} \rightarrow \quad (10.17)$$

Substituting Eq. 10.17 into Eq. 10.16 gives

$$T = \frac{\mu F}{2} \frac{r_o^2 - r_i^2}{r_o - r_i} = \frac{\mu F}{2} \downarrow r_o + r_i \rightarrow \quad (10.18)$$

Table 10.3 Typical values for dynamic friction coefficients, permissible contact pressures and temperature limits

Material	μ_{dry}	μ_{oil}	p_{\max} (MN/m)	T (°C)
Moulded compounds	0.25–0.45	0.06–0.10	1.035–2.07	200–260
Woven materials	0.25–0.45	0.08–0.10	0.345–0.69	200–260
Sintered metal	0.15–0.45	0.05–0.08	1.035–2.07	230–680
Cork	0.30–0.50	0.15–0.25	0.055–0.1	80
Wood	0.20–0.45	0.12–0.16	0.345–0.62	90
Cast iron	0.15–0.25	0.03–0.06	0.69–1.725	260
Paper based	–	0.10–0.17		
Graphite/resin	–	0.10–0.14		

For N frictional surfaces

$$T = \int_{r_i}^{r_o} 2\mu\pi N p_{\max} r_i dr = p_{\max}\mu\pi N r_i \sqrt{r_o^2 - r_i^2} \rightarrow \quad (10.19)$$

gives

$$T = \frac{\mu NF}{2} \sqrt{r_o^2 + r_i^2} \rightarrow \quad (10.20)$$

By differentiating Eq. 10.16 with respect to r_i and equating the result to zero, the maximum torque for any outer radius r_o is found to occur when

$$r_i = \sqrt{\frac{1}{3}} r_o \quad (10.21)$$

This useful formula can be used to set the inner radius if the outer radius is constrained to a particular value.

Note that Eq. 10.20 indicates a lower torque capacity than the uniform pressure assumption. This is because the higher initial wear at the outer diameter shifts the centre of pressure towards the inner radius.

Clutches are usually designed based on uniform wear. The uniform wear assumption gives a lower torque capacity clutch than the uniform pressure assumption. The preliminary design procedure for disc clutch design requires the determination of the torque and speed, specification of space limitations, selection of materials, i.e. the coefficient of friction and the maximum permissible pressure, and the selection of principal radii, r_o and r_i . Common practice is to set the value of r_i between $0.45r_o$ and $0.8r_o$. This procedure for determining the initial geometry is itemized below.

1. Determine the service factor.
2. Determine the required torque capacity,
 $T = \text{power}/\omega$.
3. Determine the coefficient of friction μ .
4. Determine the outer radius r_o .
5. Find the inner radius r_i .
6. Find the axial actuation force required.

The material used for clutch plates is typically grey cast iron or steel. The friction surface will

consist of a lined material which may be moulded, woven, sintered or solid. Moulded linings consist of a polymeric resin used to bind powdered fibrous material and brass and zinc chips. Table 10.3 lists typical values for the performance of friction linings.

Example 10.1

A clutch is required for transmission of power between a four-cylinder internal combustion engine and a small machine. Determine the radial dimensions for a single face dry disc clutch with a moulded lining which should transmit 5 kW at 1800 rpm (Figure 10.15). Base the design on the uniform wear assumption.

Solution

From Table 10.2, a service factor of two should be used. The design will therefore be undertaken using a power of $2 \times 5 \text{ kW} = 10 \text{ kW}$.

The torque is given by

$$T = \frac{\text{Power}}{\omega} = \frac{10000}{1800 \times (2\pi/60)} = 53 \text{ N.m.}$$

From Table 10.3 taking midrange values for the coefficient of friction and the maximum

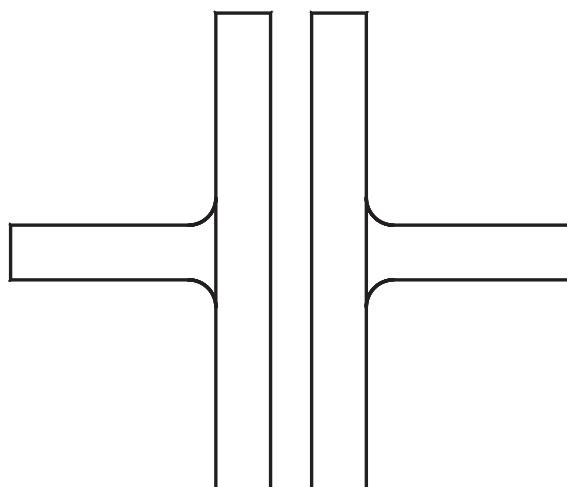


Figure 10.15 Single face clutch.

permissible pressure for moulded linings gives $\mu = 0.35$ and $p_{\max} = 1.55 \text{ MN/m}^2$.

Taking $r_i = \sqrt{1/3} r_o$, and substituting for r_i in Eq. 10.16 gives:

$$T = \pi \mu \sqrt{1/3} r_o p_{\max} r_o^2 - \sqrt{1/3} r_o^2$$

$$= \pi \mu \sqrt{1/3} p_{\max} r_o^3 - \frac{1}{3} r_o^3$$

$$= \pi \mu \sqrt{4/27} p_{\max} r_o^3,$$

$$r_o = \frac{T}{\pi \mu p_{\max} \sqrt{4/27}}^{1/3}$$

$$= \frac{53.05}{\pi 0.35 \times 1.55 \times 10^6 \sqrt{4/27}}^{1/3}$$

$$= 0.04324 \text{ m},$$

$$r_i = \sqrt{1/3} r_o = 0.02497 \text{ m},$$

$$F = 2\pi r_i p_{\max} (r_o - r_i)$$

$$= 2\pi \times 0.02497 \times 1.55 \times 10^6$$

$$\times (0.04324 - 0.02497) = 4443 \text{ N.}$$

So the clutch consists of a disc of inner and outer radius 25 mm and 43 mm respectively, with a moulded lining having a coefficient of friction value of 0.35 and a maximum permissible contact pressure of 1.55 MPa and an actuating force of 4.4 kN.

Example 10.2

A disc clutch, running in oil, is required for a vehicle with a four-cylinder engine. The design power for initial estimation of the clutch specification is 90 kW at 4500 rpm. Determine the radial dimensions and actuating force required. Base the design on the uniform wear assumption.

Solution

From Table 10.2 a service factor of 2.7 should be used to account for starts and stops and the four-cylinder engine. The design will therefore be undertaken using a power of $2.7 \times 90 = 243 \text{ kW}$.

The torque is given by

$$T = \frac{\text{Power}}{\omega} = \frac{243000}{4500 \times (2\pi/60)}$$

$$= 515.7 \text{ N m.}$$

From Table 10.3 taking midrange values for the coefficient of friction and the maximum permissible pressure for moulded linings gives $\mu = 0.35$ and $p_{\max} = 1.55 \text{ MN/m}^2$.

Taking $r_i = \sqrt{1/3} r_o$ and substituting for r_i in Eq. 10.19 gives:

$$r_o = \frac{T}{\pi \mu N p_{\max} \sqrt{4/27}}^{1/3}$$

$$= \frac{515.7}{\pi 0.35 \times 2 \times 1.55 \times 10^6 \sqrt{4/27}}^{1/3}$$

$$= 0.07325 \text{ m},$$

$$r_i = \sqrt{1/3} r_o = 0.04229 \text{ m},$$

$$F = 2\pi r_i p_{\max} (r_o - r_i)$$

$$= 2\pi \times 2 \times 0.04229 \times 1.55 \times 10^6$$

$$\times (0.07325 - 0.04229) = 25500 \text{ N.}$$

So the clutch consists of a disc of inner and outer radius 42.3 and 73.3 mm, respectively, with a moulded lining having a coefficient of friction value of 0.35 and a maximum permissible contact pressure of 1.55 MPa and an actuating force of 25.5 kN.

Example 10.3

A multiple disc clutch, running in oil, is required for a motorcycle with a three-cylinder engine.

The power demand is 75 kW at 8500 rpm. The preliminary design layout indicates that the maximum diameter of the clutch discs should not exceed 100 mm. In addition, previous designs have indicated that a moulded lining with a coefficient of friction of 0.068 in oil and a maximum permissible pressure of 1.2 MPa is reliable. Within these specifications determine the radii for the discs, the number of discs required and the clamping force.

Solution

The torque is given by

$$\begin{aligned} T &= \frac{\text{Service factor} \times \text{Power}}{\omega} \\ &= \frac{3.4 \times 75000}{8500 \times (2\pi/60)} = 286.5 \text{ Nm} \end{aligned}$$

Select the outer radius to be the largest possible, i.e. $r_o = 50$ mm. Using $r_i = \sqrt{1/3} r_o$, $r_i = 28.87$ mm. From Eq. 10.19, the number of frictional surfaces, N , can be determined.

$$\begin{aligned} N &= \frac{T}{\pi p_{\max} r_i \mu \sqrt{r_o^2 - r_i^2}} \\ &= \frac{286.5}{\pi 1.2 \times 10^6 \times 0.02887} \\ &\quad \times 0.068 (0.05^2 - 0.02887^2) \\ &= 23.23 \end{aligned}$$

This must be an even number, so the number of frictional surfaces is taken as $N = 24$. This requires 13 driving discs and 12 driven discs to implement.

Using Eq. 10.14 the clamping force can be calculated:

$$\begin{aligned} F &= \frac{2T}{\mu N(r_o + r_i)} \\ &= \frac{2 \times 286.5}{0.068 \times 24(0.05 + 0.02887)} \\ &= 4452 \text{ N.} \end{aligned}$$

As well as the disc or discs, detailed design of a disc clutch such as the automotive variant illustrated in Figure 10.16, requires specification and consideration of all the associated components such as the shaft, splined hub, damping springs in the disc, Figure 10.17, actuating springs, cover plate, spline, bearings, flywheel, crankshaft and gearbox connections and release mechanism. The choice of whether to use coil springs or a diaphragm spring (Figure 10.18) depends on the clamping force required and cost considerations. Diaphragm springs have been developed from Belleville springs and are made from a steel disc. The inner portion of the disc has a number of radial slots to form actuating fingers. Stress relieving holes are included at the outer end of the fingers to prevent cracking. Multi-coil spring units are used in heavy commercial vehicles where it is difficult to provide the required clamping force from a single diaphragm.

10.3 Brakes

The basic function of a brake is to absorb kinetic energy and dissipate it in the form of heat. An idea of the magnitude of energy that must be dissipated can be obtained from considering the familiar example of a car undergoing an emergency stop in 7 s from 60 mph (96 km/h). If the car's mass is 1400 kg and assuming that 65 per cent of the car's weight is loaded on to the front axles during rapid braking then the load on the front axle is

$$1400 \times 9.81 \times 0.65 = 8927 \text{ N}$$

This will be shared between two brakes so the energy that must be absorbed by one brake is

$$E = \frac{1}{2} m \sqrt{V_i^2 - V_f^2} \rightarrow \quad (10.22)$$

where m is mass (kg); V_i , initial velocity (m/s); V_f , final velocity (m/s).

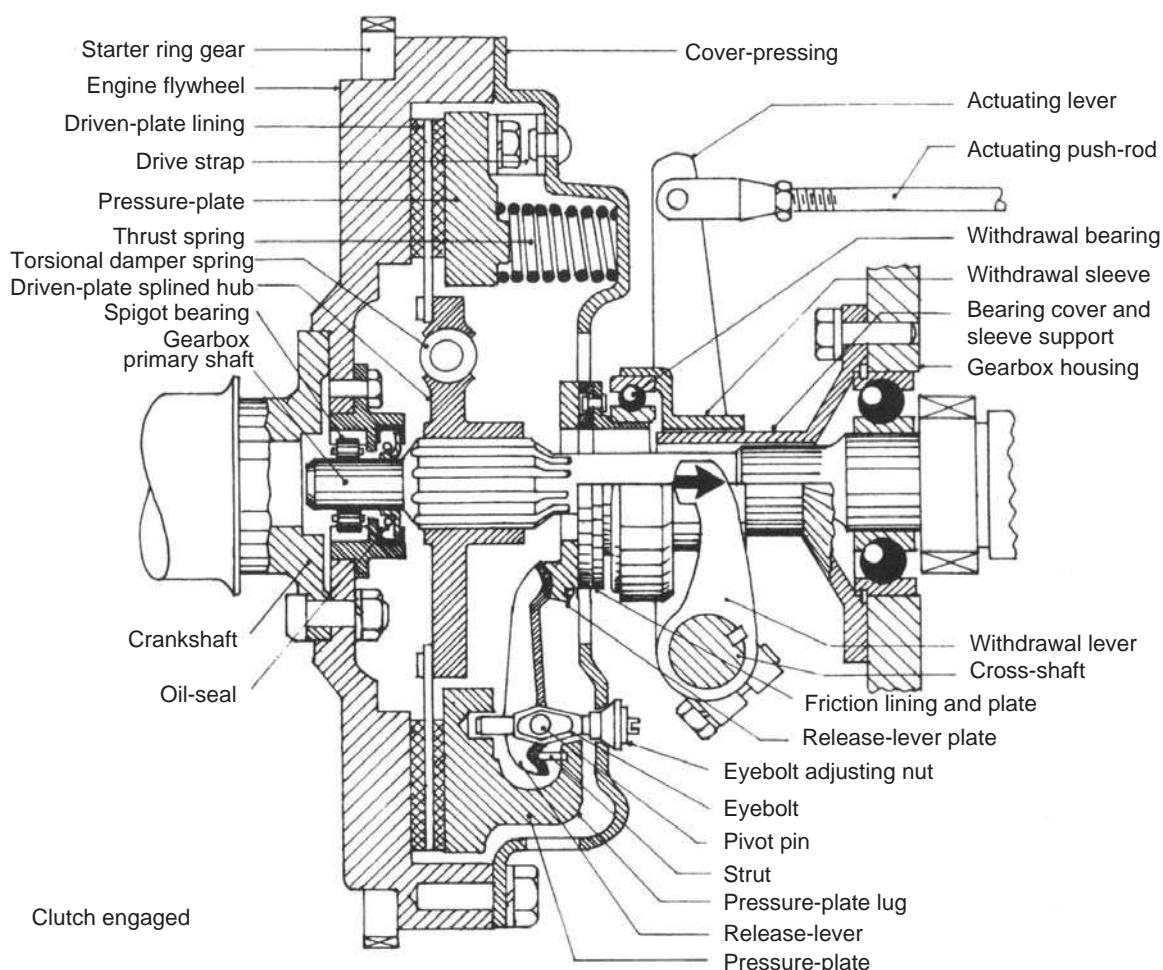


Figure 10.16 Automotive clutch (reproduced from Heisler, 1999).

$$\begin{aligned}
 E = \frac{1}{2}m(V_i^2 - V_f^2) &= \frac{1}{2} \frac{8927}{9.81} \times 0.5 \\
 \times \frac{96 \times 10^3}{3600}^2 - 0^2 &= 161.8 \text{ kJ}
 \end{aligned}$$

If the car brakes uniformly in 7 s, then the heat that must be dissipated is $161.8 \times 10^3 / 7 = 23.1 \text{ kW}$. From your experience of heat transfer from say 1 kW domestic heaters you will recognize that this is a significant quantity of heat to transfer away from the relatively compact components that make up brake assemblies.

Convective heat transfer can be modelled by Fourier's equation:

$$Q = hA\Delta T = hA(T_s - T_f) \quad (10.23)$$

where Q is heat transfer rate (W); h, the heat transfer coefficient ($\text{W/m}^2 \text{ K}$); T_s , temperature of the surface (K or $^\circ\text{C}$); T_f , temperature of the surrounding fluid (K or $^\circ\text{C}$); and A, surface area (m^2).

This equation indicates that the ability of a brake to dissipate the heat generated increases as the surface area increases or as the heat transfer coefficient rises. For air, the heat transfer coefficient is usually dependent on the local flow velocity and

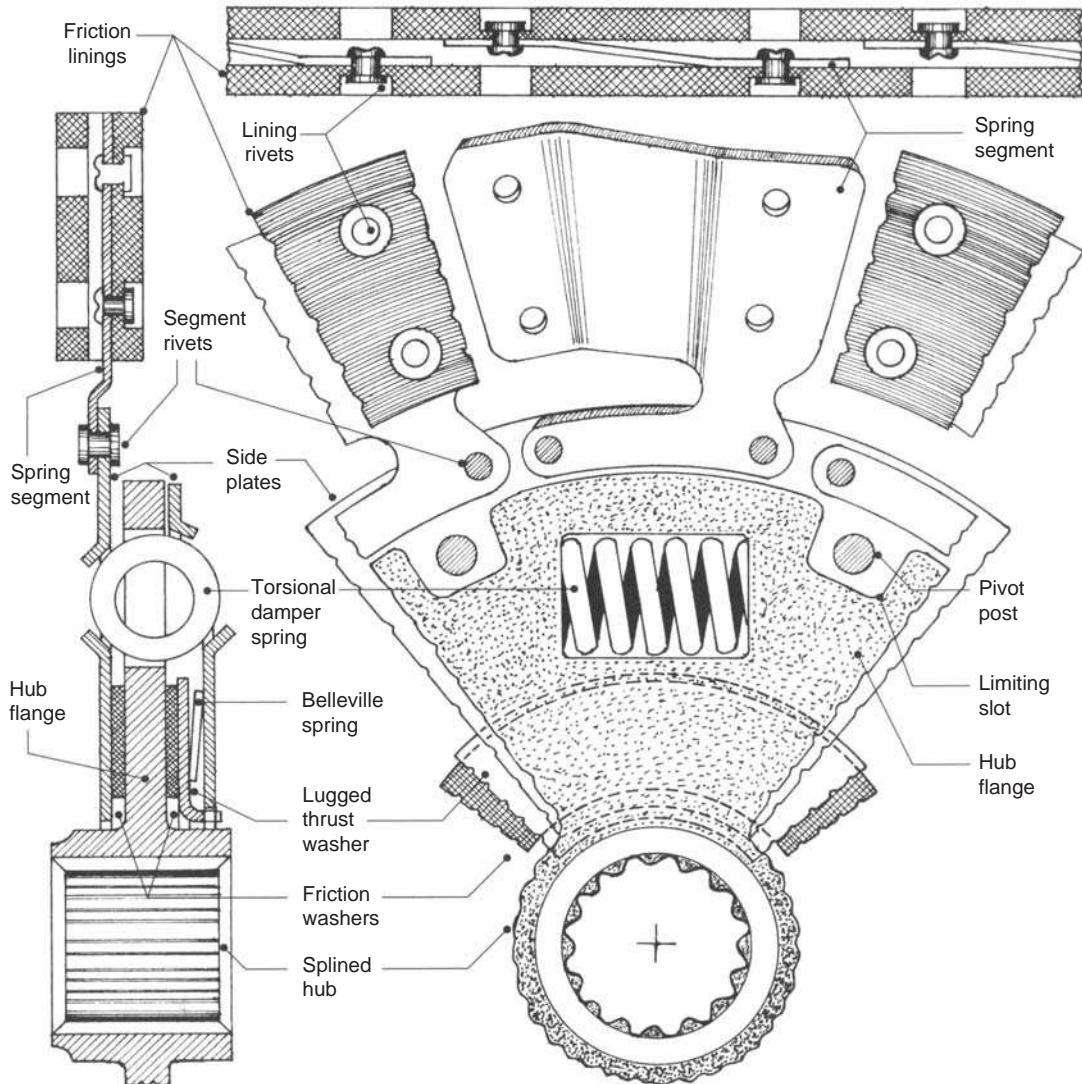


Figure 10.17 Automotive disc illustrating the use of torsional damping springs (reproduced from Heisler, 1999).

on the geometry. A method often used for disc brakes to increase both the surface area and the local flow is to machine multiple axial or radial holes in the disc (this also reduces the mass and inertia).

There are numerous brake types as shown in Figure 10.19. The selection and configuration of a brake depends on the requirements. Table 10.4 gives an indication of brake operation against various criteria. The brake factor listed in Table 10.4

is the ratio of frictional braking force generated to the actuating force applied.

Brakes can be designed so that, once engaged the actuating force applied is assisted by the braking torque. This kind of brake is called a self-energizing brake and is useful for braking large loads. Great care must be exercised in brake design. It is possible and sometimes desirable to design a brake, which once engaged, will grab and lock up (called self-locking action).

A critical aspect of all brakes is the material used for frictional contact. Normally one component will comprise a steel or cast iron disc or drum and this is brought into frictional contact against a geometrically similar component with a brake lining made up of one of the materials listed in Table 10.3.

Section 10.3.1 gives details about the configuration design of disc brakes and Section 10.3.2 introduces the design of drum brakes.

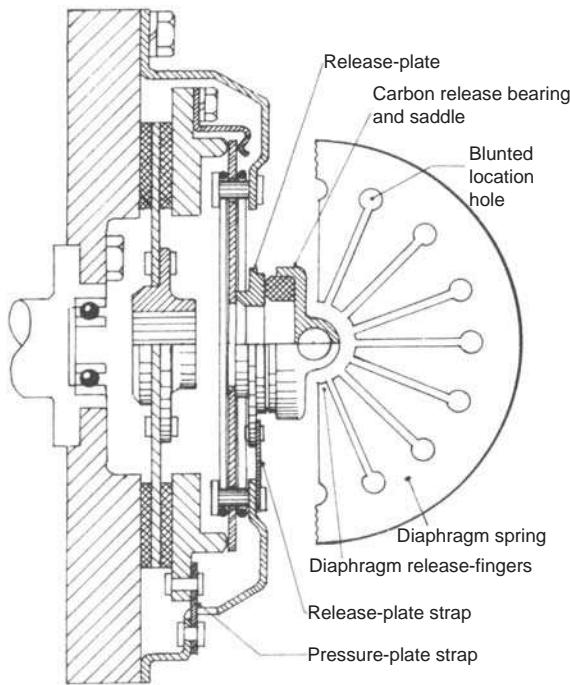


Figure 10.18 Diaphragm spring (reproduced from Heisler, 1999).

10.3.1 Disc brakes

Disc brakes are familiar from automotive applications where they are used extensively for car and motorcycle wheels. These typically consist of a cast iron disc, bolted to the wheel hub. This is sandwiched between two pads actuated by pistons supported in a calliper mounted on the stub shaft (see Figure 10.20). When the brake pedal is pressed, hydraulically pressurized fluid is forced into the cylinders pushing the opposing pistons and brake pads into frictional contact with the disc. The advantages of this form of braking are steady braking, easy ventilation, balancing thrust loads and design simplicity. There is no self-energizing action so the braking action is proportional to the applied force. The use of a discrete pad allows the disc to cool as it rotates, enabling heat transfer between the cooler disc and the hot brake pad. As the pads either side of the disc are pushed on to the disc with equal forces the net thrust load on the disc cancels.

With reference to Figure 10.21, the torque capacity per pad is given by

$$T = \mu F r_e \quad (10.24)$$

where r_e is an effective radius.

The actuating force assuming constant pressure is given by

$$F = p_{av} \theta \frac{r_o^2 - r_i^2}{2} \quad (10.25)$$

or assuming uniform wear by

$$F = p_{max} \theta r_i (r_o - r_i) \quad (10.26)$$

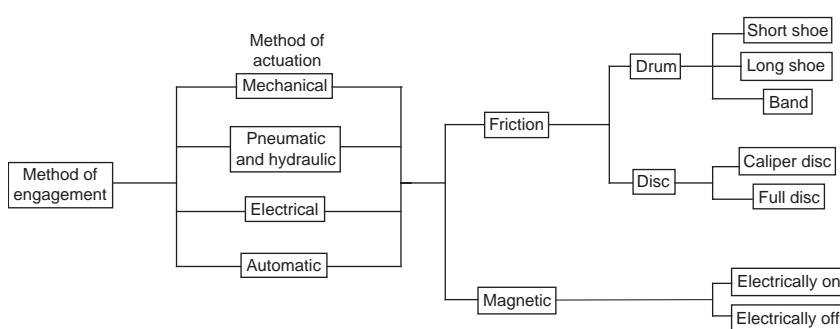


Figure 10.19 Brake classification.

where θ (in radians) is the included angle of the pad, r_i is the inner radius of the pad and r_o is the outer radius of the pad.

The relationship between the average and the maximum pressure for the uniform wear assumption is given by

$$\frac{p_{av}}{p_{max}} = \frac{2r_i/r_o}{1 + r_i/r_o} \quad (10.27)$$

$$r_e = \frac{2\sqrt[3]{r_o^3 - r_i^3}}{3\sqrt[3]{r_o^2 - r_i^2}} \rightarrow \quad (10.28)$$

$$r_e = \frac{r_i + r_o}{2} \quad (10.29)$$

Table 10.4 Comparative table of brake performance

Type of brake	Maximum operating temperature	Brake factor	Stability	Dryness	Dust and dirt	Typical applications
Differential band brake	Low	High	Low	Unstable but still effective	Good	Winches, hoist, excavators, tractors
External drum brake (leading trailing edge)	Low	Medium	Medium	Unstable if humid, poor if wet	Good	Mills, elevators, winders
Internal drum brake (leading trailing edge)	Higher than external brake	Medium	Medium	Unstable if humid, ineffective if wet	Good if sealed	Vehicles (rear axles on passenger cars)
Internal drum brake (two leading shoes)	Higher than external brake	High	Low	Unstable if humid, ineffective if wet	Good if sealed	Vehicles (rear axles on passenger cars)
Internal drum brake (duo-servo)	Low	High	Low	Unstable if humid, ineffective if wet	Good if sealed	Vehicles (rear axles on passenger cars)
Caliper disc brake	High	Low	High	Good	Poor	Vehicles and industrial machinery
Full disc brake	High	Low	High	Good	Poor	Machine tools and other industrial machinery

Reproduced from Neale, 1994.

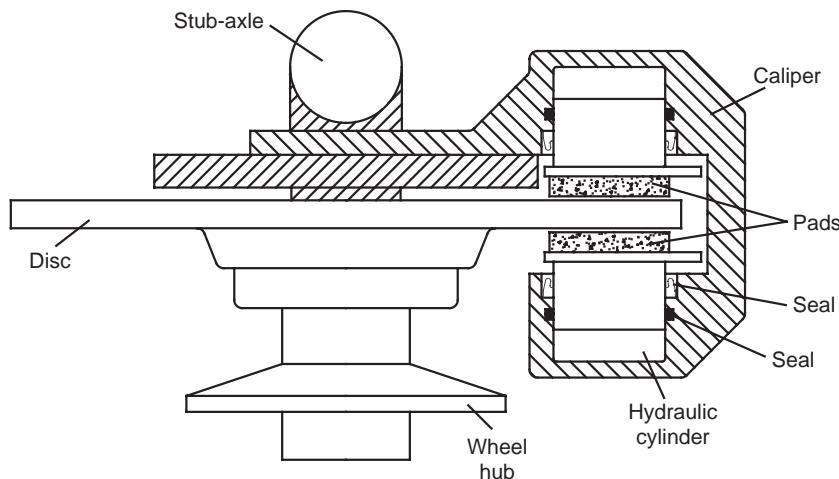


Figure 10.20 Automotive disc brake.

For an annular disc brake the effective radius is given by Eq. 10.28 assuming constant pressure and Eq. 10.29 assuming uniform wear.

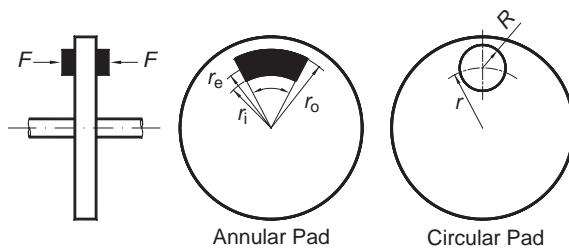


Figure 10.21 Caliper disc brake.

Table 10.5 Circular pad disc brake design values

R/r	$\delta = r_e/r$	p_{\max}/p_{av}
0	1.000	1.000
0.1	0.983	1.093
0.2	0.969	1.212
0.3	0.957	1.367
0.4	0.947	1.578
0.5	0.938	1.875

Source: Fazekas, 1972.

For circular pads the effective radius is given by $r_e = r\delta$, where values for δ are given in Table 10.5 as a function of the ratio of the pad radius and the radial location, R/r . The actuating force for circular pads can be calculated using:

$$F = \pi R^2 p_{av} \quad (10.30)$$

Example 10.4

A caliper brake is required for the front wheels of a sports car with a braking capacity of 820 N m for each brake. Preliminary design estimates have set the brake geometry as $r_i = 100$ mm, $r_o = 160$ mm and $\theta = 45^\circ$. A pad with a coefficient of friction of 0.35 has been selected. Determine the required actuating force and the average and maximum contact pressures.

Solution

The torque capacity per pad = $820/2 = 410$ N m.

The effective radius is

$$r_e = \frac{0.1 + 0.16}{2} = 0.13 \text{ m.}$$

The actuating force is given by

$$F = \frac{T}{\mu r_e} = \frac{410}{0.35 \times 0.13} = 9.011 \text{ kN.}$$

The maximum contact pressure is given by

$$\begin{aligned} p_{\max} &= \frac{F}{\theta r_i (r_o - r_i)} \\ &= \frac{9.011 \times 10^3}{45 \times (2\pi/360) \times 0.1 \times (0.16 - 0.1)} \\ &= 1.912 \text{ MN/m}^2. \end{aligned}$$

The average pressure is given by

$$p_{av} = p_{\max} \frac{2r_i/r_o}{1 + r_i/r_o} = 1.471 \text{ MN/m}^2.$$

Full disc brakes, consisting of a complete annular ring pad, are principally used for industrial machinery. The disc clutch equations developed in Section 10.2.1 are applicable to their design. The disc configuration can be designed to function as either a clutch or a brake (a clutch-brake combination) to transmit a load or control its speed.

10.3.2 Drum brakes

Drum brakes apply friction to the external or internal circumference of a cylinder. A drum brake consists of the brake shoe, which has the friction material bonded to it, and the brake drum. For braking, the shoe is forced against the drum developing the friction torque. Drum brakes can be divided into two groups depending on whether the brake shoe is external or internal to the drum. A further classification can be made in terms of the length of the brake shoe: short, long or complete band.

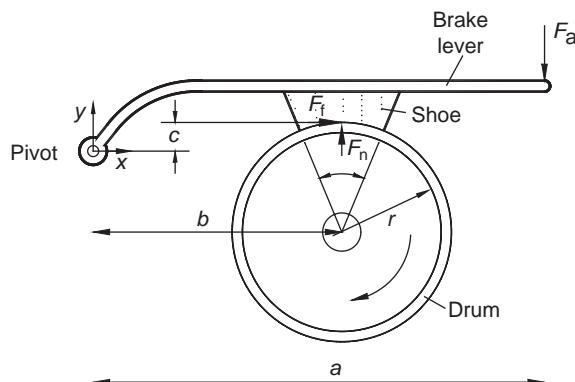


Figure 10.22 Short shoe external drum brake.

Short shoe internal brakes are used for centrifugal brakes that engage at a particular critical speed. Long shoe internal drum brakes are used principally in automotive applications. Drum brakes (or clutches) can be designed to be self-energizing. Once engaged the friction force increases the normal force nonlinearly, increasing the friction torque as in a positive feedback loop. This can be advantageous in braking large loads, but makes control much more difficult. One problem associated with some drum brakes is stability. If the brake has been designed so that the braking torque is not sensitive to small changes in the coefficient of friction, which would occur if the brake is worn or wet, then the brake is said to be stable. If a small change in the coefficient of friction causes a significant change to the braking torque the brake is unstable and will tend to grab if the friction coefficient rises or the braking torque will drop noticeably if the friction coefficient reduces.

10.3.3 Short shoe external drum brakes

The schematic of a short shoe external drum brake is given in Figure 10.22. If the included angle of contact between the brake shoe and the brake drum is less than 45° , the force between the shoe and the drum is relatively uniform and can be modelled by a single concentrated load F_n at the centre of the contact area. If the maximum

permissible pressure is p_{\max} the force F_n can be estimated by

$$F_n = p_{\max} \theta w \quad (10.31)$$

where w is the width of the brake shoe (m); and θ is the angle of contact between the brake shoe and the lining (rad).

The frictional force, F_f , is given by

$$F_f = \mu F_n \quad (10.32)$$

where μ is the coefficient of friction.

The torque on the brake drum is

$$T = F_f r = \mu F_n r \quad (10.33)$$

Summing moments, for the shoe arm, about the pivot gives:

$$\begin{aligned} M_{\text{pivot}} &= aF_a - bF_n + cF_f = 0 \\ F_a &= \frac{bF_n - cF_f}{a} = F_n \frac{b - \mu c}{a} \end{aligned} \quad (10.34)$$

Resolving forces gives the reactions at the pivot:

$$R_x = -F_f \quad (10.35)$$

$$R_y = F_a - F_n \quad (10.36)$$

Note that for the configuration and direction of rotation shown in Figure 10.22, the friction moment $\mu F_n c$ adds or combines with the actuating moment, aF_a . Once the actuating force is applied the friction generated at the shoe acts to increase the braking torque. This kind of braking action is called self-energizing. If the brake direction is reversed the friction moment term $\mu F_n c$ becomes negative and the applied load F_a must be maintained to generate braking torque. This combination is called self de-energizing. From Eq. 10.34, note that if the brake is self-energizing and if $\mu c > b$ then the force required to actuate the brake is zero or negative and the brake action is called self-locking. If the shoe touches the drum it will grab and lock. This is usually undesirable with exceptions being hoist stops or over-running clutch type applications.

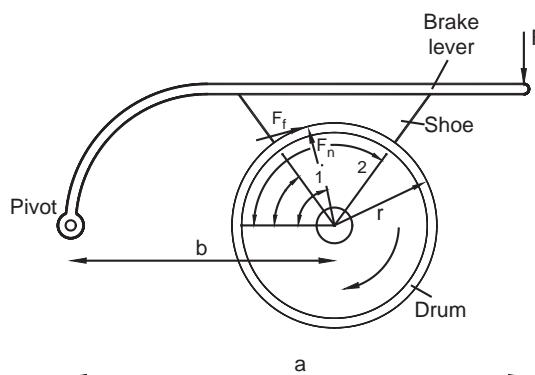


Figure 10.23 Long shoe external drum brake.

10.3.4 Long shoe external drum brakes

If the included angle of contact between the brake shoe and the drum is greater than 45° then the pressure between the shoe and the brake lining cannot be regarded as uniform and the approximations made for the short shoe brake analysis are inadequate. Most drum brakes use contact angles greater than 90° . Brake shoes are not rigid and the local deflection of the shoe affects the pressure distribution. Detailed shoe analysis is possible using finite element software. For initial synthesis/specification of brake geometry, a simpler analysis suffices.

For a single block brake (see Figure 10.23) the force exerted on the drum by the brake shoe must be supported by the bearings. To balance this load and provide a compact braking arrangement two opposing brake shoes are usually used in a caliper arrangement as shown in Figure 10.24.

The following equations can be used (with reference to Figure 10.23), to determine the performance of a long shoe brake.

The braking torque T is given by

$$T = \mu w r^2 \frac{p_{\max}}{(\sin \theta)_{\max}} (\cos \theta_1 - \cos \theta_2) \quad (10.37)$$

where μ is the coefficient of friction; w , width of the brake shoe (m); r , radius of drum (m); p_{\max} , maximum allowable pressure for the lining material (N/m^2); θ , angular location (rad); $(\sin \theta)_{\max}$,

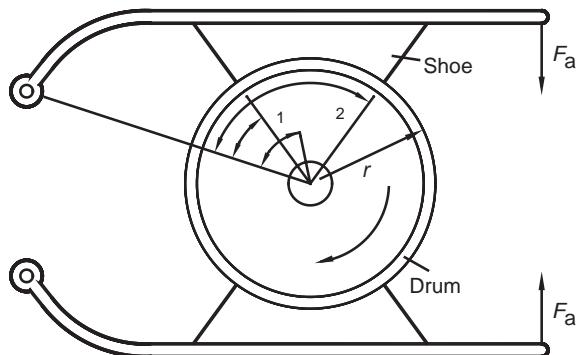


Figure 10.24 Double long shoe external drum brake.

maximum value of $\sin \theta$; θ_1 , centre angle from the shoe pivot to the heel of the lining (rad); and θ_2 , centre angle from the shoe pivot to the toe of the lining (rad).

This is based on the assumption that the local pressure p at an angular location θ is related to the maximum pressure, p_{\max} , by

$$p = \frac{p_{\max} \sin \theta}{(\sin \theta)_{\max}} \quad (10.38)$$

The local pressure p will be a maximum when $\theta = 90^\circ$. If $\theta_2 < 90^\circ$ then the pressure will be a maximum at the toe, θ_2 . The relationship given in Eq. 10.38 assumes that there is no deflection at the shoe or the drum, no wear on the drum and that the shoe wear is proportional to the frictional work and hence the local pressure. Note that if $\theta = 0$ the pressure is zero. This indicates that frictional material near the pivot or heel of the brake does not contribute significantly to the braking action. For this reason, common practice is to leave out the frictional material near the heel and start it at an angle typically between $\theta_1 = 10^\circ$ and $\theta_1 = 30^\circ$.

With the direction of rotation shown in Figure 10.23 (i.e. the brake is self-energizing), the magnitude of the actuation force is given by

$$F_a = \frac{M_n - M_f}{a} \quad (10.39)$$

where M_n is the moment of the normal forces (N m); M_f , the moment due to the frictional forces (N m); and a , the orthogonal distance between the brake pivot and the line of action of the applied force (m).

The normal and frictional moments can be determined using Eqs 10.40 and 10.41, respectively. If the geometry and materials are selected such that $M_f = M_n$, then the actuation force becomes zero. Such a brake would be self-locking. The slightest contact between the shoe and drum would bring the two surfaces into contact and the brake would snatch giving rapid braking. Alternatively values for the brake geometry and materials can be selected to give different levels of self-energization depending on the relative magnitudes of M_n and M_f .

$$M_n = \frac{wrbp_{\max}}{(\sin \theta)_{\max}} \quad \begin{aligned} & \frac{1}{2}(\theta_2 - \theta_1) \\ & - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1) \end{aligned} \quad (10.40)$$

$$M_f = \frac{\mu wrp_{\max}}{(\sin \theta)_{\max}} \quad \begin{aligned} & r(\cos \theta_1 - \cos \theta_2) \\ & + \frac{b}{4}(\cos 2\theta_2 - \cos 2\theta_1) \end{aligned} \quad (10.41)$$

If the direction of rotation for the drum shown in Figure 10.22 is reversed, the brake becomes self de-energizing and the actuation force is given by

$$F_a = \frac{M_n + M_f}{a} \quad (10.42)$$

The pivot reactions can be determined by resolving the horizontal and vertical forces. For a self-energizing brake they are given by

$$R_x = \frac{wrp_{\max}}{(\sin \theta)_{\max}} \quad \begin{aligned} & \sqrt{0.5(\sin^2 \theta_2 - \sin^2 \theta_1)} \rightarrow \\ & - \mu \frac{\theta_2}{2} - \frac{\theta_1}{2} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1) \quad - F_x \end{aligned} \quad (10.43)$$

$$R_y = \frac{wrp_{\max}}{(\sin \theta)_{\max}} \quad \begin{aligned} & \sqrt{0.5(\sin^2 \theta_2 - \sin^2 \theta_1)} \rightarrow \\ & + \mu \frac{\theta_2}{2} - \frac{\theta_1}{2} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1) \quad + F_y \end{aligned} \quad (10.44)$$

and for a self de-energizing brake by

$$R_x = \frac{wrp_{\max}}{(\sin \theta)_{\max}} \quad \begin{aligned} & \sqrt{0.5(\sin^2 \theta_2 - \sin^2 \theta_1)} \rightarrow \\ & + \mu \frac{\theta_2}{2} - \frac{\theta_1}{2} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1) \quad - F_x \end{aligned} \quad (10.45)$$

$$R_y = \frac{wrp_{\max}}{(\sin \theta)_{\max}} \quad \begin{aligned} & \sqrt{0.5(\sin^2 \theta_2 - \sin^2 \theta_1)} \rightarrow \\ & - \mu \frac{\theta_2}{2} - \frac{\theta_1}{2} - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1) \quad + F_y \end{aligned} \quad (10.46)$$

Example 10.5

Design a long shoe drum brake to produce a friction torque of 75 N m to stop a drum rotating at 140 rpm. Initial design calculations have indicated that a shoe lining with $\mu = 0.25$ and using a value of $p_{\max} = 0.5 \times 10^6 \text{ N/m}^2$ in the design will give suitable life.

Solution

First propose trial values for the brake geometry, say $r = 0.1 \text{ m}$, $b = 0.2 \text{ m}$, $a = 0.3 \text{ m}$, $\theta_1 = 30^\circ$, $\theta_2 = 150^\circ$. Using Eq. 10.37 and solving for the width of the shoe,

$$\begin{aligned} w &= \frac{T(\sin \theta)_{\max}}{\mu r^2 p_{\max} (\cos \theta_1 - \cos \theta_2)} \\ &= \frac{75 \sin 90}{0.25 \times 0.1^2 \times 0.5} \\ &\quad \times 10^6 (\cos 30 - \cos 150) \\ &= 0.0346 \text{ m} \end{aligned}$$

Select the width to be 35 mm as this is a standard size. The actual maximum pressure experienced, will be:

$$p_{\max} = 0.5 \times 10^6 \frac{0.0346}{0.035} = 494900 \text{ N/m}^2.$$

From Eq. 10.40, the moment of the normal force with respect to the shoe pivot is:

$$\begin{aligned} M_n &= \frac{0.035 \times 0.1 \times 0.2 \times 0.4949 \times 10^6}{\sin 90} \\ &\times \frac{1}{2} 120 \times \frac{2\pi}{360} \\ &- \frac{1}{4} (\sin 300 - \sin 60) \\ &= 512.8 \text{ N m.} \end{aligned}$$

From Eq. 10.41, the moment of the frictional forces with respect to the shoe pivot is:

$$\begin{aligned} M_f &= \frac{0.25 \times 0.035 \times 0.1 \times 0.4949 \times 10^6}{\sin 90} \\ &\times 0.1(\cos 30 - \cos 150) \\ &+ \frac{0.2}{4} (\cos 300 - \cos 60) \\ &= 75 \text{ N m.} \end{aligned}$$

From Eq. 10.39, the actuation force is:

$$F_a = \frac{M_n - M_f}{a} = \frac{512.8 - 75}{0.3} = 1459 \text{ N.}$$

For the double long shoe external drum brake illustrated in Figure 10.24, the left-hand shoe is self-energizing and the frictional moment reduces the actuation load. The right-hand shoe, however, is self de-energizing and its frictional

moment acts to reduce the maximum pressure which occurs on the right-hand brake shoe. The normal and frictional moments for a self-energizing and self de-energizing brake are related by:

$$M'_n = \frac{M_n p'_{\max}}{p_{\max}} \quad (10.47)$$

$$M'_f = \frac{M_f p'_{\max}}{p_{\max}} \quad (10.48)$$

where M'_n is the moment of the normal forces for the self de-energizing brake (N m); M'_f the moment due to the frictional forces for the self de-energizing brake (N m); and p'_{\max} , the maximum pressure on the self de-energizing brake (N/m^2).

Example 10.6

For the double long shoe external drum brake illustrated in Figure 10.25 determine the limiting force on the lever such that the maximum pressure on the brake lining does not exceed 1.4 MPa and determine the torque capacity of the brake. The face width of the shoes is 30 mm and the coefficient of friction between the shoes and the drum can be taken as 0.28.

Solution

First it is necessary to calculate values for θ_1 and θ_2 as these are not indicated directly on the diagram.

$$\theta_1 = 20^\circ - \tan^{-1} \frac{20}{120} = 10.54^\circ$$

$$\theta_2 = 20^\circ + 130^\circ - \tan^{-1} \frac{20}{120} = 140.5^\circ$$

The maximum value of $\sin \theta$ would be $\sin 90 = 1$.

The distance between the pivot and the drum centre is

$$b = \sqrt{0.02^2 + 0.12^2} = 0.217 \text{ m}$$

The normal moment is given by

$$\begin{aligned} M_n &= \frac{\mu wrp_{\max}}{(\sin \theta)_{\max}} \\ &\times \frac{1}{2}(\theta_2 - \theta_1) - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1) \\ &= \frac{0.03 \times 0.1 \times 0.1217 \times 1.4 \times 10^6}{\sin 90} \\ &\times \frac{1}{2} \downarrow 40.5 - 10.54 \times \frac{2\pi}{360} \\ &- \frac{1}{4}(\sin 281 - \sin 21.08) = 751.1 \text{ N m} \\ M_f &= \frac{\mu wrp_{\max}}{(\sin \theta)_{\max}} r(\cos \theta_1 - \cos \theta_2) \\ &+ \frac{b}{4}(\cos 2\theta_2 - \cos 2\theta_1) \end{aligned}$$

$$= \frac{0.28 \times 0.03 \times 0.1 \times 1.4 \times 10^6}{\sin 90}$$

$$\times 0.1(\cos 10.54 - \cos 140.5)$$

$$+ \frac{0.1217}{4}(\cos 281 - \cos 21.08)$$

$$= 179.8 \text{ N m}$$

The orthogonal distance between the actuation force and the pivot is $a = 0.12 + 0.115 + 0.05 = 0.285 \text{ m}$.

The actuation load on the left-hand shoe is given by

$$\begin{aligned} F_{a \text{ left shoe}} &= \frac{M_n - M_f}{a} \\ &= \frac{751.1 - 179.8}{0.285} \\ &= 2004 \text{ N.} \end{aligned}$$

The torque contribution from the left-hand shoe is given by

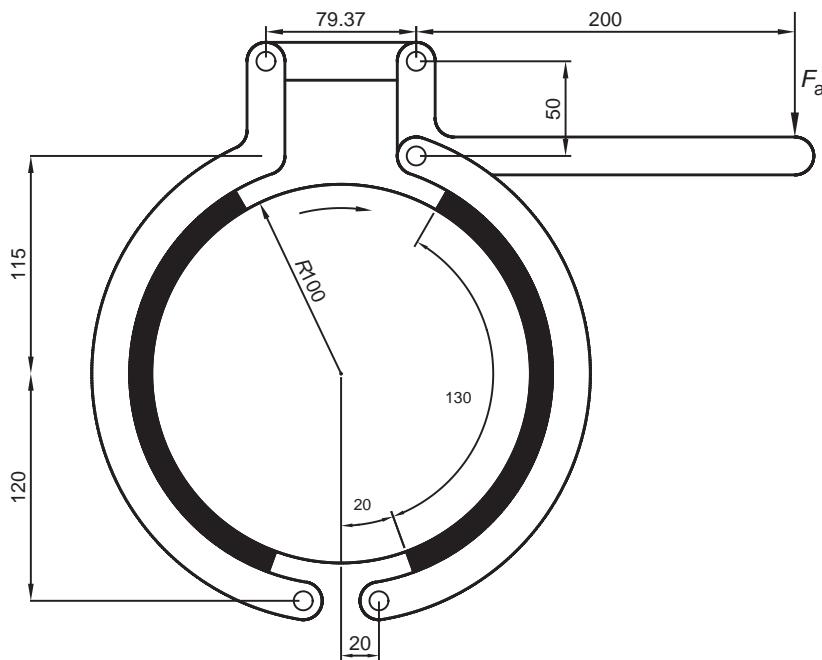


Figure 10.25 Double long shoe external drum brake.

$$\begin{aligned}
 T_{\text{left shoe}} &= \mu wr^2 \frac{p_{\max}}{(\sin \theta)_{\max}} (\cos \theta_1 - \cos \theta_2) \\
 &= 0.28 \times 0.03 \times 0.1^2 \times 1.4 \\
 &\quad \times 10^6 (\cos 10.54 - \cos 140.5) \\
 &= 206.4 \text{ N m.}
 \end{aligned}$$

The actuation force on the right-hand shoe can be determined by considering each member of the lever mechanism as a free body (see Figure 10.26).

$$F - A_V + B_V = 0.$$

$$A_H = B_H.$$

$$B_H = C_H.$$

$$A_H = C_H.$$

$$0.2F = 0.05B_H, F = B_H/4;$$

$$B_H = 2004 \text{ N}; F = 2004/4 = 501 \text{ N.}$$

So the limiting lever force is $F = 501 \text{ N.}$

$$C_V = 0, B_V = 0.$$

The actuating force for the right-hand lever is the resultant of F and B_H . The resultant angle is given by $\tan^{-1}(0.05/0.2) = 14.04^\circ$.

$$F_{\text{a right shoe}} = \frac{2004}{\cos 14.04} = 2065 \text{ N}$$

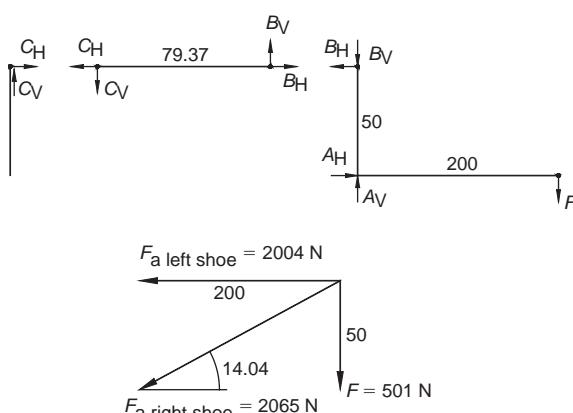


Figure 10.26 Free body diagrams.

The orthogonal distance between the actuation force vector and the pivot (Figure 10.27) is given by

$$\begin{aligned}
 a &= (0.235 - 0.01969 \tan 14.04) \\
 &\quad \times \cos 14.04 = 0.2232 \text{ m}
 \end{aligned}$$

The normal and frictional moments for the right-hand shoe can be determined using Eqs 10.47 and 10.48.

$$\begin{aligned}
 M'_n &= \frac{M_n p'_{\max}}{p_{\max}} = \frac{751.1 p'_{\max}}{1.4 \times 10^6} \\
 M'_f &= \frac{M_f p'_{\max}}{p_{\max}} = \frac{179.8 p'_{\max}}{1.4 \times 10^6}
 \end{aligned}$$

For the right-hand shoe, the maximum pressure can be determined from

$$\begin{aligned}
 F_{\text{a right shoe}} &= \frac{M'_n + M'_f}{a} = 2065 \\
 &= \frac{751.1 p'_{\max} - 179.8 p'_{\max}}{1.4 \times 10^6 \times 0.2232} \\
 p'_{\max} &= 1.130 \times 10^6 \text{ N/m}^2
 \end{aligned}$$

The torque contribution from the right-hand shoe is

$$\begin{aligned}
 T_{\text{right shoe}} &= \mu wr^2 \frac{p'_{\max}}{(\sin \theta)_{\max}} (\cos \theta_1 - \cos \theta_2) \\
 &= 0.28 \times 0.03 \times 0.1^2 \times 1.13 \times 10^6 \\
 &\quad \times (\cos 10.54 - \cos 140.5) \\
 &= 166.6 \text{ N m.}
 \end{aligned}$$

The total torque is given by

$$\begin{aligned}
 T_{\text{total}} &= T_{\text{left shoe}} + T_{\text{right shoe}} \\
 &= 206.4 + 166.6 = 373 \text{ N m.}
 \end{aligned}$$

10.3.5 Long shoe internal drum brakes

Most drum brakes use internal shoes that expand against the inner radius of the drum. Long shoe internal drum brakes are principally used in

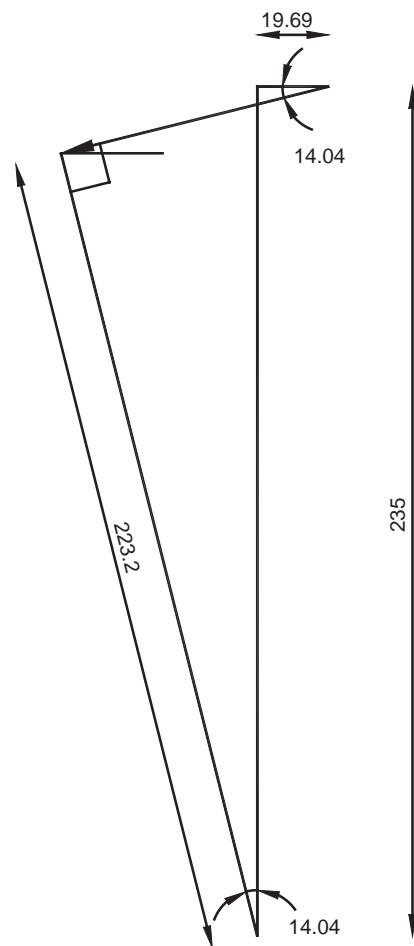


Figure 10.27 Orthogonal distance.

automotive applications. An automotive drum brake typically comprises two brake shoes and linings supported on a back plate bolted to the axle casing. The shoes are pivoted at one end on anchor pins or abutments fixed on to the back plate (see Figure 10.28). The brake can be actuated by a double hydraulic piston expander, which forces the free ends of the brake apart so that the non-rotating shoes come into frictional contact with the rotating brake drum. A leading and trailing shoe layout consists of a pair of shoes pivoted at a common anchor point as shown in Figure 10.28. The leading shoe is identified as the shoe whose expander piston moves in the direction of rotation of the drum. The frictional drag between the shoe and the

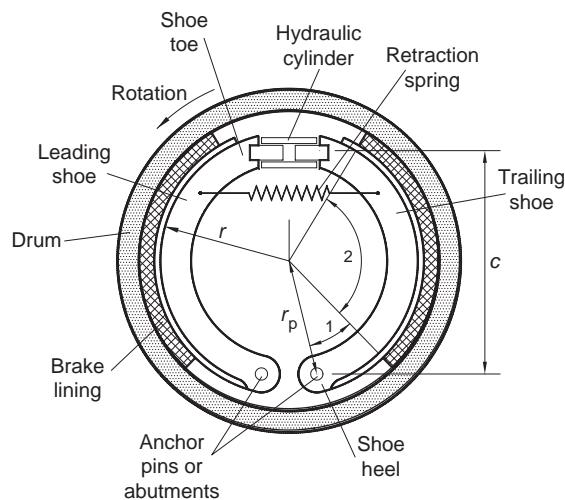


Figure 10.28 Double long shoe internal drum brake.

drum will tend to assist the expander piston in forcing the shoe against the drum and this action is referred to as self-energizing or the self-servo action of the shoe. The trailing shoe is the one whose expander piston moves in the direction opposite to the rotation of the drum. The frictional force opposes the expander and hence a trailing brake shoe provides less braking torque than an equivalent leading shoe actuated by the same force. The equations developed for external

Example 10.7

Determine the actuating force and the braking capacity for the double internal long shoe brake illustrated in Figure 10.29. The lining is sintered metal with a coefficient of friction of 0.32 and the maximum lining pressure is 1.2 MPa. The drum radius is 68 mm and the shoe width is 25 mm.

Solution

$$b = \sqrt{0.015^2 + 0.055^2} = 0.05701 \text{ m}$$

As the brake lining angles relative to the pivot, brake axis line, are not explicitly shown on the diagram, they must be calculated.

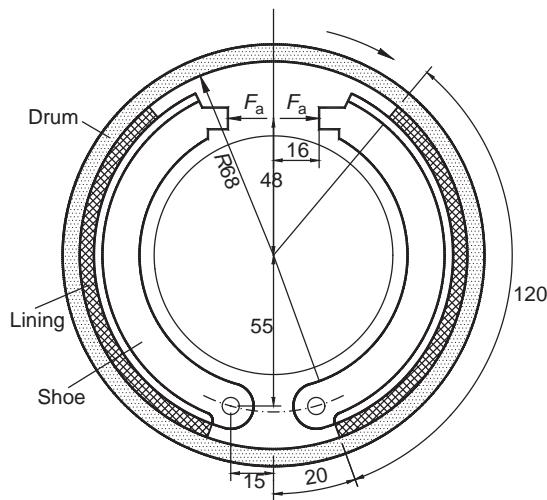


Figure 10.29 Double long shoe internal drum brake.

$$\theta_1 = 4.745^\circ, \theta_2 = 124.7^\circ$$

As $\theta_2 > 90^\circ$, the maximum value of $\sin \theta$ is $\sin 90 = 1 = (\sin \theta)_{\max}$.

For this brake with the direction of rotation as shown, the right-hand shoe is self-energizing.

For the right-hand shoe:

$$\begin{aligned} M_n &= \frac{0.025 \times 0.068 \times 0.05701 \times 1.2 \times 10^6}{1} \\ &\times \frac{1}{2} (124.7 - 4.745) \times \frac{2\pi}{360} \\ &- \frac{1}{4} \sqrt{\sin 249.4 - \sin 9.49} \rightarrow \\ &= 153.8 \text{ N m.} \end{aligned}$$

$$\begin{aligned} M_f &= \frac{0.32 \times 0.025 \times 0.068 \times 1.2 \times 10^6}{1} \\ &\times 0.068 (\cos 4.745 - \cos 124.7) \\ &+ \frac{0.05701}{4} (\cos 249.4 - \cos 9.49) \\ &= 57.1 \text{ m.} \end{aligned}$$

$$a = 0.055 + 0.048 = 0.103 \text{ m.}$$

$$F_a = \frac{M_n - M_f}{a} = \frac{153.8 - 57.1}{0.103} = 938.9 \text{ N.}$$

The actuating force is 938.9 N.

The torque applied by the right-hand shoe is given by

$$\begin{aligned} T_{\text{right shoe}} &= \frac{\mu \nu r^2 p'_{\max} (\cos \theta_1 - \cos \theta_2)}{(\sin \theta)_{\max}} \\ &= \frac{0.32 \times 0.025 \times 0.068^2 \times 1.2 \times 10^6}{1} \\ &\times (\cos 4.745 - \cos 124.7) \\ &= 69.54 \text{ N m.} \end{aligned}$$

The torque applied by the left-hand shoe cannot be determined until the maximum operating pressure p'_{\max} for the left-hand shoe has been calculated.

As the left-hand shoe is self de-energizing the normal and frictional moments can be determined using Eqs 10.47 and 10.48.

$$M'_n = \frac{M_n p'_{\max}}{p_{\max}} = \frac{153.8 p'_{\max}}{1.2 \times 10^6}$$

$$M'_f = \frac{M_f p'_{\max}}{p_{\max}} = \frac{57.1 p'_{\max}}{1.2 \times 10^6}$$

The left-hand shoe is self de-energizing, so,

$$F_a = \frac{M_n + M_f}{a}$$

$F_a = 938.9 \text{ N}$ as calculated earlier.

$$\begin{aligned} 938.9 &= \frac{153.8 p'_{\max} + 57.1 p'_{\max}}{1.2 \times 10^6 \times 0.103} \\ p'_{\max} &= 0.5502 \times 10^6 \text{ N/m}^2. \end{aligned}$$

The torque applied by the left-hand shoe is given by

$$T_{\text{left shoe}} = \frac{\mu \nu r^2 p'_{\max}}{(\sin \theta)_{\max}} (\cos \theta_1 - \cos \theta_2)$$

$$\begin{aligned}
 & 0.32 \times 0.025 \times 0.068^2 \\
 & = \frac{\times 0.5502 \times 10^6}{1} \\
 & \times \downarrow \cos 4.745 - \cos 124.7 \rightarrow \\
 & = 31.89 \text{ N m.}
 \end{aligned}$$

The total torque applied by both shoes is:

$$\begin{aligned}
 T_{\text{total}} &= T_{\text{right shoe}} + T_{\text{left shoe}} \\
 &= 69.54 + 31.89 = 101.4 \text{ N m.}
 \end{aligned}$$

From this example, the advantage in torque capacity of using self-energizing brakes is apparent. Both the left-hand and the right-hand shoes could be made self-energizing by inverting the left-hand shoe, having the pivot at the top. This would be advantageous if rotation occurred in just one direction. If, however, drum rotation is possible in either direction, it may be more suitable to have one brake self-energizing for forward motion and one self-energizing for reverse motion.

long shoe drum brakes are also valid for internal long shoe drum brakes.

10.3.6 Band brakes

One of the simplest types of braking device is the band brake. This consists of a flexible metal band lined with a frictional material wrapped partly around a drum. The brake is actuated by pulling the band against the drum as illustrated in Figure 10.30. For the clockwise rotation shown in Figure 10.30, the friction forces increase F_1 relative to F_2 . The relationship between the tight and slack sides of the band is given by

$$\frac{F_1}{F_2} = e^{\mu\theta} \quad (10.49)$$

where F_1 is the tension in the tight side of the band (N); F_2 , tension in the slack side of the

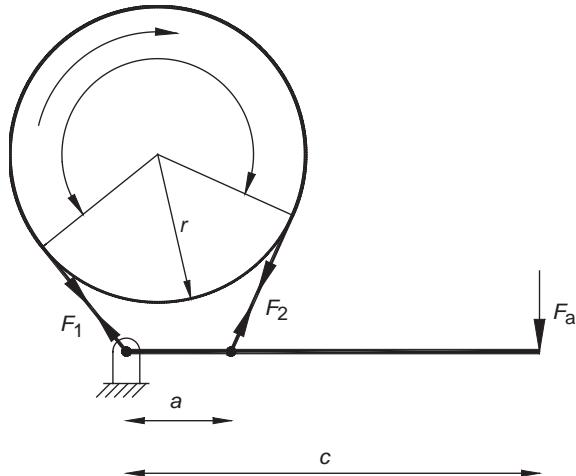


Figure 10.30 Band brake.

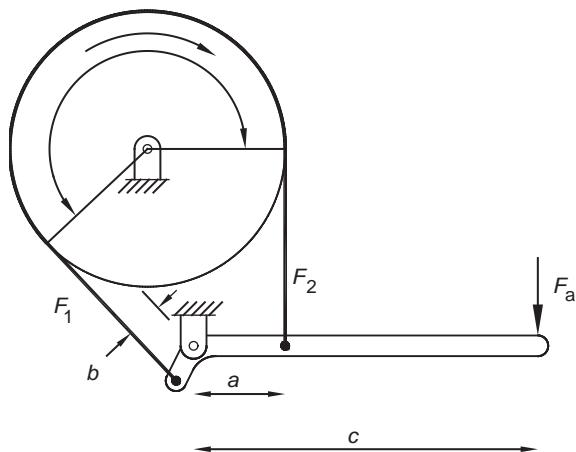


Figure 10.31 Self-energizing band brake.

band (N); μ , coefficient of friction; and θ , angle of wrap (rad).

The point of maximum contact pressure for the friction material occurs at the tight end and is given by:

$$p_{\max} = \frac{F_1}{rw} \quad (10.50)$$

where w is the width of the band (m).

The torque braking capacity is given by

$$T = (F_1 - F_2)r \quad (10.51)$$

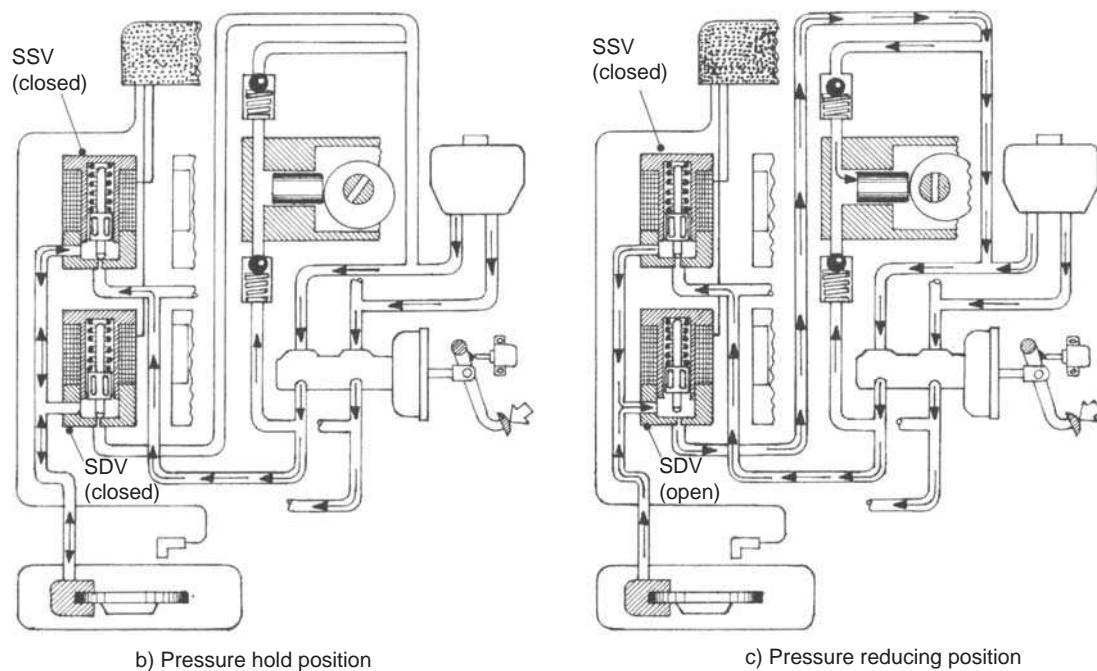
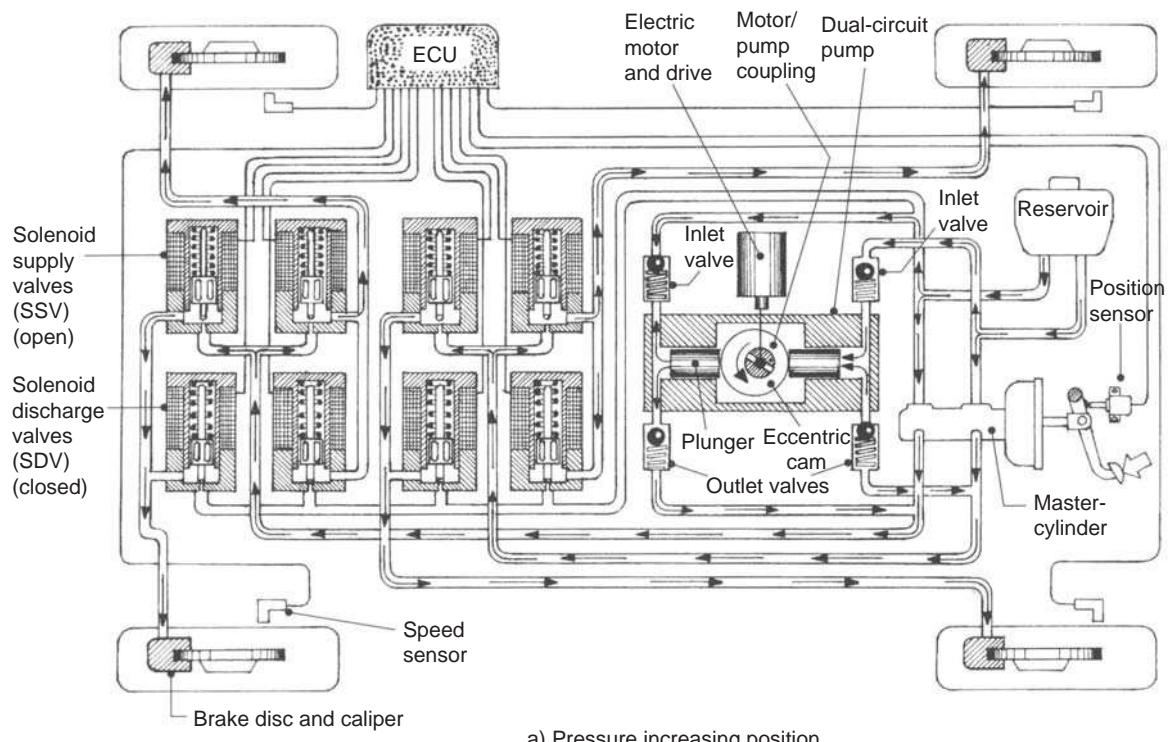


Figure 10.32 Control loop for an antilock braking system (ABS) (reproduced from Heisler, 1999).

The relationship, for the simple band brake shown in Figure 10.30, between the applied lever force F_a and F_2 can be found by taking moments about the pivot point.

$$\begin{aligned} F_a c - F_2 a &= 0 \\ F_a &= F_2 \frac{a}{c} \end{aligned} \quad (10.52)$$

The brake configuration shown in Figure 10.30 is self-energizing for clockwise rotation. The level of self-energization can be enhanced by using the differential band brake configuration shown in Figure 10.31. Summation of the moments about the pivot gives:

$$F_a c - F_2 a + F_1 b = 0 \quad (10.53)$$

So the relationship between the applied load F_a and the band brake tensions is given by:

$$F_a = \frac{F_2 a - F_1 b}{c} \quad (10.54)$$

Note that the value of b must be less than a , so that applying the lever tightens F_2 more than it loosens F_1 . Substituting for F_1 in Eq. 10.54 gives

$$F_a = \frac{F_2(a - be^{\mu\theta})}{c} \quad (10.55)$$

The brake can be made self-locking if $a < be^{\mu\theta}$ and the slightest touch on the lever would cause

Example 10.8

Design a band brake to exert a braking torque of 85 N m. Assume the coefficient of friction for the lining material is 0.25 and the maximum permissible pressure is 0.345 MPa.

Solution

Propose a trial geometry, say $r = 150$ mm, $\theta = 225^\circ$ and $w = 50$ mm.

$$\begin{aligned} F_1 &= p_{\max} rw = 0.345 \times 10^6 \\ &\times 0.15 \times 0.05 = 2587 \text{ N.} \end{aligned}$$

$$F_2 = \frac{F_1}{e^{\mu\theta}} = \frac{2587.5}{e^{0.25(225 \times 2\pi/360)}} = 969 \text{ N.}$$

$$\begin{aligned} T &= \sqrt{F_1 - F_2} \\ &= \sqrt{2587.5 - 969} = 242.7 \text{ N m.} \end{aligned}$$

This torque is much greater than the 80 N m desired, so try a different combination of r , θ and w until a satisfactory design is achieved.

Try $r = 0.1$ m, $\theta = 225^\circ$ and $w = 50$ mm.

$$\begin{aligned} F_1 &= p_{\max} rw = 0.345 \times 10^6 \\ &\times 0.1 \times 0.05 = 1725 \text{ N.} \end{aligned}$$

$$\begin{aligned} F_2 &= \frac{F_1}{e^{\mu\theta}} = \frac{1725}{e^{0.25(225 \times 2\pi/360)}} = 646.3 \text{ N.} \\ T &= \sqrt{F_1 - F_2} \\ &= \sqrt{1725 - 646.3} = 107.9 \text{ N m.} \end{aligned}$$

Try $r = 0.09$ m, $\theta = 225^\circ$ and $w = 50$ mm.

$$\begin{aligned} F_1 &= p_{\max} rw = 0.345 \times 10^6 \\ &\times 0.09 \times 0.05 = 1552.5 \text{ N.} \end{aligned}$$

$$\begin{aligned} F_2 &= \frac{F_1}{e^{\mu\theta}} = \frac{1552.5}{e^{0.25(225 \times 2\pi/360)}} = 581.7 \text{ N.} \\ T &= \sqrt{F_1 - F_2} \\ &= \sqrt{1552.5 - 581.7} = 87.4 \text{ N m.} \end{aligned}$$

The actuating force is given by $F_a = F_2 a/c$. If $a = 0.08$ m and $c = 0.15$ m then,

$$F_a = 581.7 \times \frac{0.08}{0.15} = 310.2 \text{ N.}$$

the brake to grab or lock abruptly. This principle can be used to permit rotation in one direction only as in hoist and conveyor applications.

Practical braking systems comprise an energy supplying device, a control device, a transmission device for controlling the braking force and the brakes themselves. For example, in the case of

a braking system for passenger cars the braking system could control the brake pedal, a vacuum booster, master hydraulic fluid cylinder, brake fluid reservoir, a device to warn the driver of a brake circuit failure, a sensor to warn of low brake fluid level, valves, hydraulic cylinders, springs, pads and discs. For heavy vehicles, say two to three tonnes and over, the force required for braking can become greater than a person can exert. In such cases some assistance needs to be given to the driver. This can be achieved by using a servo mechanism that adds to the driver's effort, for example a vacuum-assisted brake servo unit, or by using power operation in which case the driver's effort is simply for control purposes and is not transmitted directly to the brakes.

The performance of braking systems in, say, automotive applications has been significantly enhanced over the purely mechanical variants described so far by the use of sensors, actuators and sophisticated control systems. An example is antilock braking systems (ABS), which are closed-loop control devices within the braking system. The objective of ABS is to prevent wheel lock-up during braking and as a result retain greater steering control and vehicle stability. The principal components of an ABS system are a hydraulic modulator, wheel-based sensors and the engine control unit (ECU). The ECU processes the signals and controls and triggers the hydraulics. The control loop for an antilock braking system is shown in Figure 10.32. ABS systems are considered in more detail in the *Bosch Automotive Handbook* (2000) and Garrett *et al.* (2001).

It should be noted that the design of braking systems in the case of automotive applications is subject to extensive regulation and requirements are detailed in documents such as StVZO, EC Directive 71320 and ECE Directive 13.

10.4 Conclusions

Clutches are designed to permit the smooth, gradual engagement or disengagement of a prime mover from a driven load. Brakes are designed to

decelerate a system. Clutches and brakes are similar devices providing frictional, magnetic or direct positive connection between two components. In principle one device could function as either a clutch or a brake. If one component rotates and the other is fixed to a non-rotating plane of reference the device will function as a brake and if both rotate then as a clutch.

This chapter has concentrated on rotating clutches and brakes and specifically on the design of friction-based devices. In the case of brakes, control systems enable significant advantages over purely mechanical devices enabling the prevention of wheel lock-up during braking and as a result retain greater driveability. The detailed design of a clutch or braking system involves integration of a wide range of skills, such as bearings, shafts, splines, teeth, flywheels, casings, frictional surfaces, hydraulics, sensors and control algorithms. Both brakes and clutches can be purchased from specialist suppliers or alternatively key components such as brake pads or clutch discs can be specified and bought in from specialist suppliers and integrated into a fit-for-purpose machine design.

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- www.clutchfacing.com
- www.cross-morse.co.uk
- www.dba.com.au
- www.frendisa.com
- www.iatcoinc.com
- www.jbrakes.com
- www.kfabsc.com
- www.luk.de
- www.mondengineering.com/mondProductsMain.htm
- www.nascoaircraft.com
- www.onassisauto.com
- www.trwauto.com
- www.warnernet.com
- www.winnard.co.uk
- www.xyd-autoparts.com

Nomenclature

Generally, preferred SI units have been stated.

- a distance (m)
- b distance (m)
- c distance (m)

A	area (m^2)
ABS	antilock braking system
ECU	engine control unit
F	force (N)
F_f	frictional force (N)
F_n	normal force (N)
F_1	tension in the tight side of the band (N)
F_2	tension in the slack side of the band (N)
h	heat transfer coefficient ($\text{W}/(\text{m}^2 \text{ K})$)
I	mass moment of inertia (kg m^2)
IC	internal combustion
L	length or thickness (m)
m	mass (kg)
M	moment (N m)
M_f	frictional moment (N m)
M'_f	moment due to the frictional forces for a self de-energizing brake (N m)
M_n	normal moment (N m)
M'_n	moment of the normal forces for a self de-energizing brake (N m)
N	number of disc faces
p	pressure (N/m^2)
p_{av}	average pressure (N/m^2)
p_{\max}	maximum permissible pressure (N/m^2)
p'_{\max}	maximum pressure on the self de-energizing brake (N/m^2)
Q	heat transfer rate (W)
r	radius (m)
r_e	effective radius (m)
r_i	inner radius (m)
r_o	outer radius (m)
R	pad radius (m), reaction (N)
T	torque (N m), temperature ($^\circ\text{C}$)
T_f	temperature of the surrounding fluid (K or $^\circ\text{C}$)
T_s	temperature of the surface (K or $^\circ\text{C}$)
V_i	initial velocity (m/s)
V_f	final velocity (m/s)
w	width of the brake shoe (m)
α	acceleration (m/s^2)
δ	ratio of effective radius to radius
δF	elemental axial force (N)
δr	elemental radial distance (m)
δT	elemental torque (N m)
μ	coefficient of friction
θ	angle (rad)
ρ	density (kg/m^3)
ω	angular velocity (rad/s)
ΔT	temperature difference ($^\circ\text{C}$)

Worksheet

- 10.1 Calculate the torque a clutch must transmit to accelerate a pulley with a moment of inertia of 0.25 kg m^2 to
 (a) 500 rpm in 2.5 s
 (b) 1000 rpm in 2 s.
- 10.2 Calculate the energy that must be absorbed in stopping a 100 tonne airbus airliner travelling at 250 km/h in an aborted take-off, stopping in 40 s.
- 10.3 A disc clutch has a single pair of mating surfaces of 300 mm outside diameter and 200 mm inner diameter. If the coefficient of friction is 0.3 and the actuating force is 4000 N determine the torque capacity assuming: (a) the uniform wear assumption, (b) the uniform pressure assumption.
- 10.4 A multiple disc clutch running in oil is required for a touring motorcycle. The power demand is 75 kW at 9000 rpm. Space limitations restrict the maximum outer diameter of the clutch plates to 200 mm. Select an appropriate lining material and determine the radii for the discs, the number of discs required and the clamping force.
- 10.5 A multiple disc clutch is required for a high-performance motorcycle. The power demand is 100 kW at 12 000 rpm. Select an appropriate lining material and determine the radii for the discs, the number of discs required and the clamping force.
- 10.6 A multiple disc clutch is required for a small motorcycle. The power demand is 9 kW at 8500 rpm. Select an appropriate lining material and determine the radii for the discs, the number of discs required and the clamping force.
- 10.7 A caliper brake is required for the front wheels of a passenger car with a braking capacity of 320 N m for each brake. Preliminary design estimates have set the brake geometry as $r_i = 100 \text{ mm}$, $r_o = 140 \text{ mm}$ and $\theta = 40^\circ$. Pads with a coefficient of friction of 0.35 have been selected. Each pad is actuated by means of a hydraulic cylinder of nominal diameter 25.4 mm. Determine the required actuating force, the average and the maximum contact pressures and the required hydraulic pressure for brake actuation.

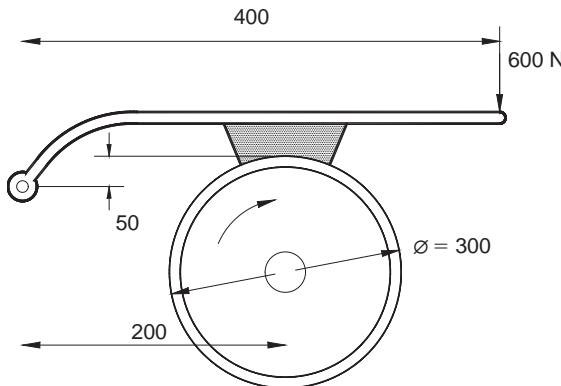


Figure 10.33 Single short shoe brake.

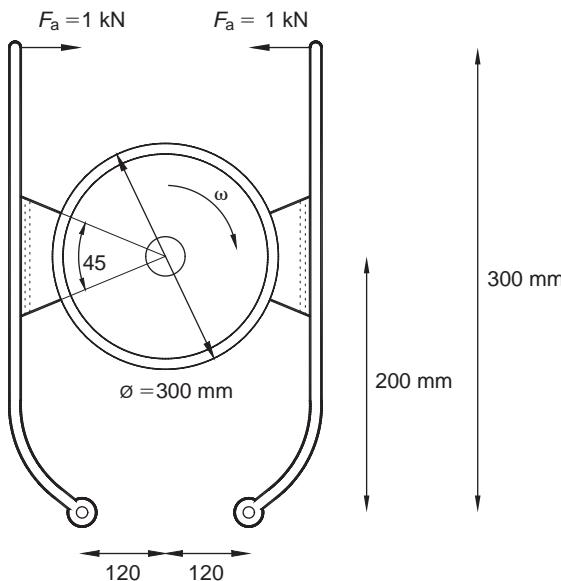


Figure 10.34 Double short shoe external drum brake.

- 10.8 Determine the torque capacity for the single short shoe brake shown in Figure 10.33. The coefficient of friction is 0.35.
- 10.9 The maximum actuating force on the double short shoe external drum brake illustrated in Figure 10.34 is 1 kN. If the coefficient of friction for the shoe lining is 0.3, determine the torque capacity of the brake for clockwise rotation.
- 10.10 A double short shoe external brake is illustrated in Figure 10.35. The actuating force required to limit the drum rotation to 100 rpm is 2.4 kN. The coefficient of

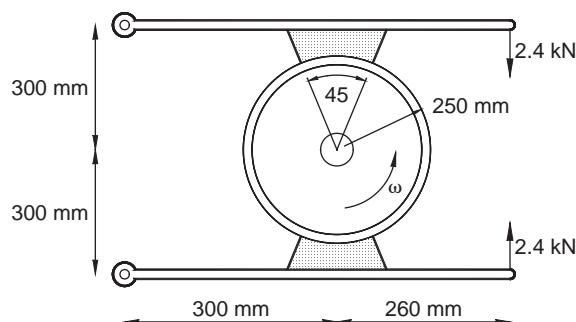


Figure 10.35 Double short shoe external brake.

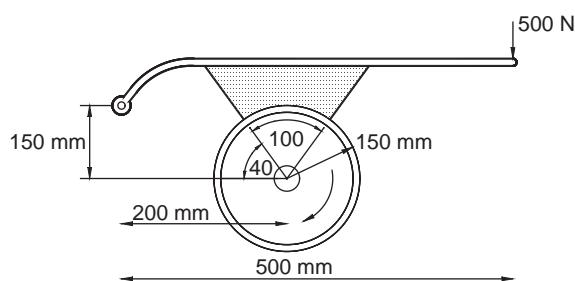


Figure 10.36 Single long shoe external brake.

- friction for the brake lining is 0.35. Determine the braking torque and the rate of heat generation.
- 10.11 The single long shoe external brake illustrated in Figure 10.36 operates on a drum of diameter 300 mm. The coefficient of friction for the brake lining is 0.3 and the face width of the shoes is 40 mm. If the actuating force is 500 N determine the maximum shoe pressure and the braking torque.
- 10.12 A double external long shoe drum brake is illustrated in Figure 10.37. The face width of the shoes is 50 mm and the maximum permissible lining pressure is 1 MPa. If the coefficient of friction is 0.32 determine the limiting actuating force and the torque capacity.

- 10.13 The double internal long shoe drum brake illustrated in Figure 10.38 has a shoe outer diameter of 280 mm and shoe width of 40 mm. Each shoe is actuated by a force of 2 kN. The coefficient of friction for the brake lining is 0.25. Find the magnitude of the maximum pressure and the braking torque.

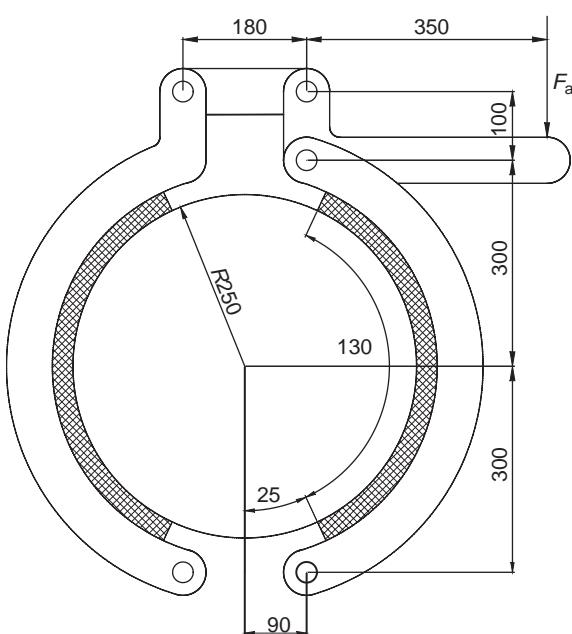


Figure 10.37 Double external long shoe drum brake.

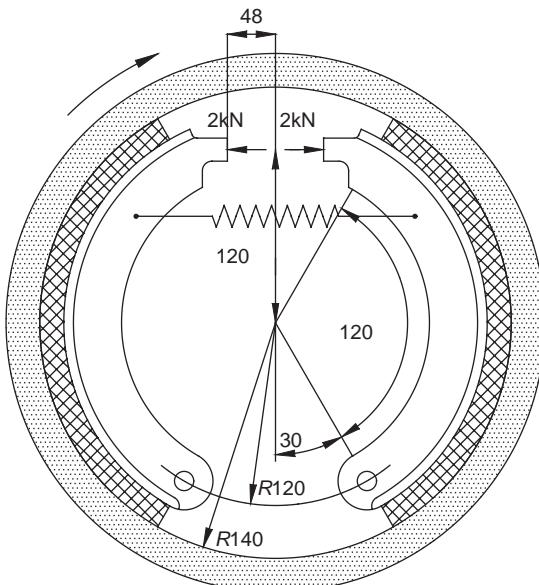


Figure 10.38 Double internal long shoe drum brake.

- 10.14 A double long internal shoe brake as illustrated in Figure 10.39 has a diameter of 200 mm and a shoe face width of 30 mm. The coefficient of friction for the brake lining is 0.4 and the maximum permissible stress is 1.2 MPa. Each brake shoe is actuated

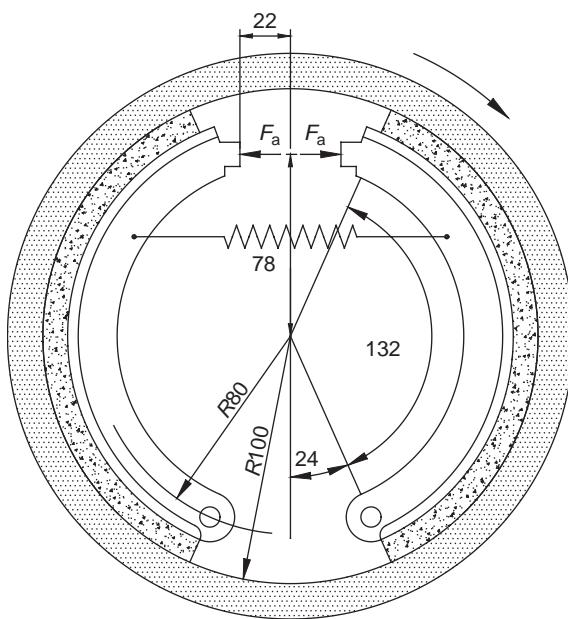


Figure 10.39 Double long internal shoe brake.

- by equal forces. Determine what the value is for the actuating force on each shoe and determine the torque capacity for the brake.
- 10.15 A simple band brake is shown in Figure 10.40. The maximum permissible pressure for the brake lining is 0.6 MPa. The brake band is 100 mm wide and has a coefficient of friction of 0.3. The angle of contact between the band and the drum is 270° . Determine the tensions in the brake band and the torque capacity if the drum diameter is 0.36 m.

Answers

- 10.1 5.2 N m, 13.1 N m.
10.2 241.1 MJ, 6.028 MW.

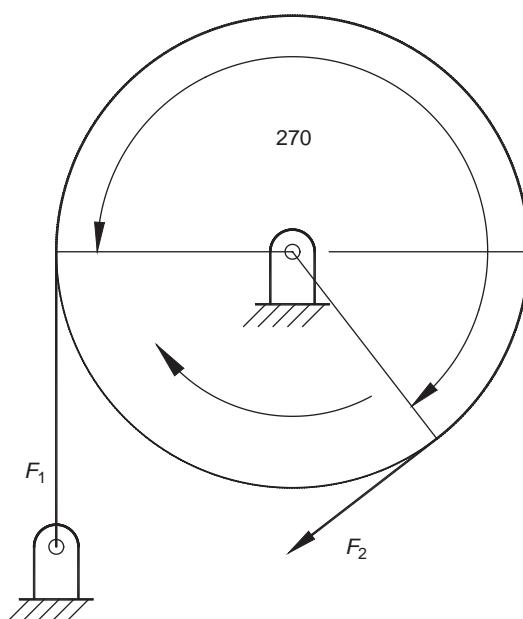


Figure 10.40 Band brake.

- 10.3 300 N m, 304 N m.
10.4 No unique solution.
10.5 No unique solution.
10.6 No unique solution.
10.7 3810 N , $1.137 \times 10^6 \text{ N/m}^2$, $1.364 \times 10^6 \text{ N/m}^2$, 75 bar.
10.8 69.04 N m.
10.9 135.4 N m.
10.10 393.4 N m, 4.1 kW.
10.11 0.179 MN/m^2 , 59.2 N m.
10.12 1.7 kN, 3.2 kN m.
10.13 0.7244 MN/m^2 , 334.9 N m.
10.14 1289 N, 317 N m.
10.15 1.47 kN m.

Learning objectives achievement

- | | | | | |
|--|--------------------------|-------------------------------------|-------------------------------------|----------------|
| Can you determine the primary dimensions for a single disc clutch? | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | Section 10.2.1 |
| Can you determine the principal dimensions and number of discs for a multiple disc clutch for a given speed and power requirement? | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | Section 10.2.1 |
| Can you determine the torque capacity for short or long, internal or external brakes? | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | Section 10.3.2 |
| Can you determine the configuration of a brake to be self-energizing? | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | Section 10.3.2 |

11

SPRINGS

Springs are used to store elastic energy and when required release it. There is a wide range of types of spring that are readily available from specialist suppliers or that can be designed and manufactured fit-for-purpose. The aim of this chapter is to introduce spring technology and to outline the principal steps in the design of helical compression, extension and torsion springs, leaf springs and Belleville washer springs.

LEARNING OBJECTIVES

At the end of this chapter you should be:

- familiar with the range of spring technologies available to the designer;
- able to specify the principal parameters for a helical compression spring;
- able to specify the principal parameters for a helical extension spring;
- able to determine the principal dimensions of a Belleville spring washer for a given duty.

11.1 Introduction

Springs are flexible devices used to exert force or torque and store energy. The force produced by a spring can be compressive or tensile and linear or radial as in the case of a helical torsion spring clip used to hold a tube on the end of a pipe. Alternatively springs can be configured to produce a torque with applications including door closers. The process of deflecting a spring involves the transfer of energy into stored spring energy. When

the force causing the spring deflection is removed the stored spring energy will be returned.

Springs can be classified by the direction and nature of the force exerted when they are deflected. Several types of spring are listed in Table 11.1 according to the nature of force or torque exerted. The principal characteristics of the various classes of springs are summarized in Table 11.2.

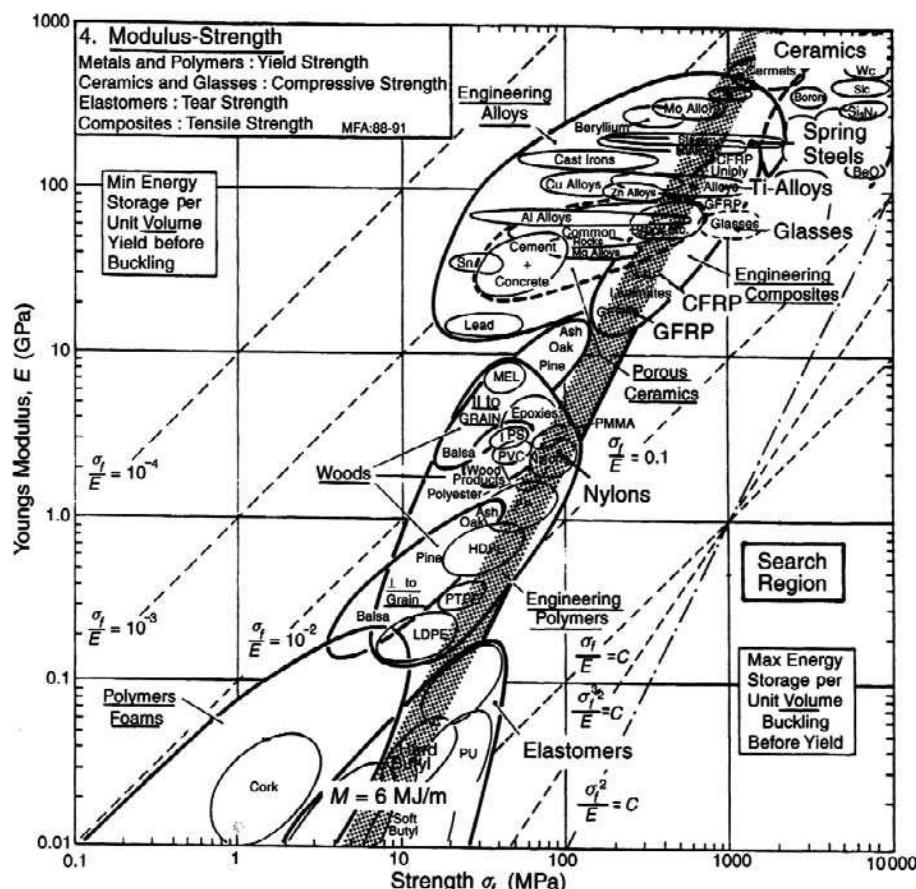
Virtually any material can be used to make springs. However, the ideal material would have a high ultimate strength and yield point and a low modulus of elasticity in order to provide maximum energy storage. Ashby (1999) develops two selection charts for springs, one based on physical size and the other on weight. Both of these charts are reproduced here as Figures 11.1 and 11.2. The highlighted line in Figure 11.1 illustrates the best choices for springs of minimum volume. These include high strength steels, glass-fibre reinforced plastic (GFRP), which is used for certain truck

Table 11.1 Classification of springs according to the nature of force or torque exerted

Actuation	Type of spring
Compressive	Helical compression springs
	Belleville springs
Tensile	Flat springs, e.g. cantilever or leaf springs
	Helical extension springs
	Flat springs, e.g. cantilever or leaf springs
Radial	Drawbar springs
	Constant force springs
	Garter springs
Torque	Elastomeric bands
	Spring clamps
	Torsion springs
	Power springs

Table 11.2 Principal characteristics of a variety of types of spring

Type of spring	Principal characteristics
Helical compression springs	These are usually made from round wire wrapped into a straight cylindrical form with a constant pitch between adjacent coils
Helical extension springs	These are usually made from round wire wrapped into a straight cylindrical form but with the coils closely spaced in the no-load condition. As an axial load is applied the spring will extend but resisting the motion
Drawbar springs	A helical spring is incorporated into an assembly with two loops of wire. As a load is applied the spring is compressed in the assembly resisting the motion
Torsion springs	These exert a torque as the spring is deflected by rotation about their axes. A common example of the application of a torsion spring is the clothes peg
Leaf springs	Leaf springs are made from flat strips of material and loaded as cantilever beams. They can produce a tensile or compressive force depending on the mode of loading applied
Belleville springs	These comprise shallow conical discs with a central hole
Garter springs	These consist of coiled wire formed into a continuous ring so that they can exert a radial inward force when stretched
Volute springs	These consist of a spiral wound strip, that functions in compression. They are subject to significant friction and hysteresis

**Figure 11.1** Materials for springs of minimum volume (reproduced from Ashby, 1999).

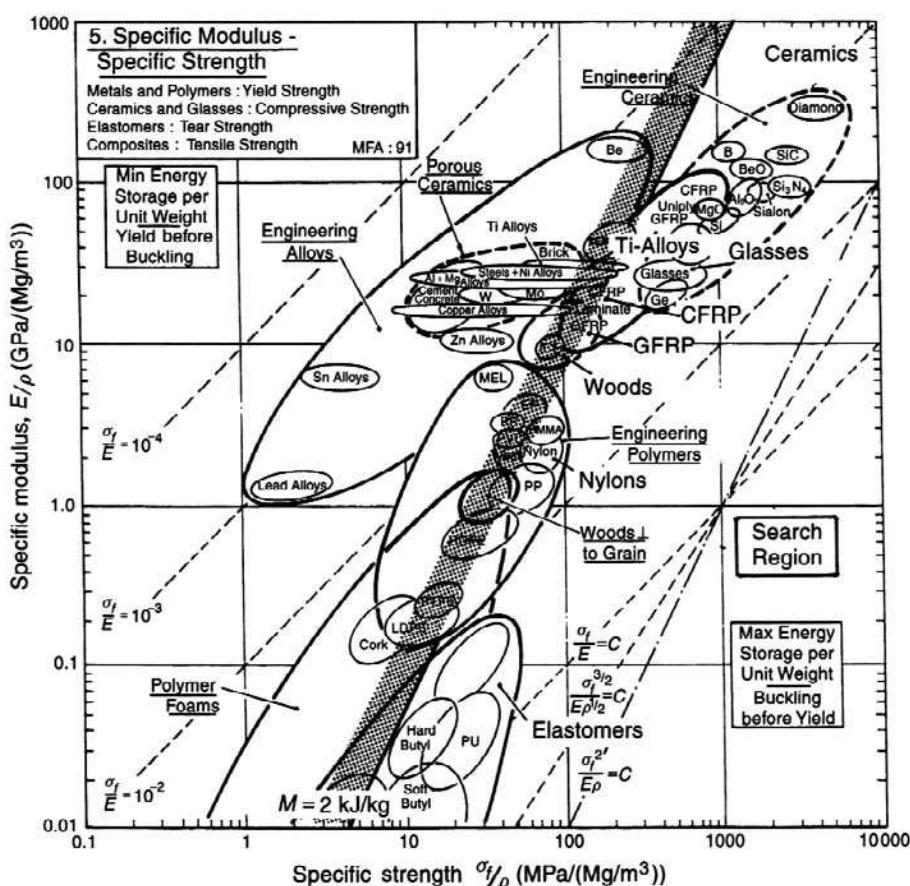


Figure 11.2 Materials for springs of minimum weight (reproduced from Ashby, 1999).

springs, titanium alloys, glass, which is used in galvanometers and nylon, which is used in some toys. The choice for most applications tends to be limited to plain carbon steels, alloy steels, stainless steels, high-nickel steels and copper-based alloys. Most spring materials are manufactured to ASTM, BS or DIN specifications. Typical properties of common spring materials are listed in Table 11.3. Round wire is the most common choice for spring material and preferred diameters are listed in Table 11.4, although any diameter can be produced if necessary.

Important aspects in spring design include determination of the spring material and dimensions in order to ensure that it will not fail due to either static or fluctuating loads for the lifetime required for the application, that it will not buckle or deform beyond allowable limits, that the natural frequencies

of vibration are sufficiently in excess of the frequency of motion that they control and that cost and aesthetic aspirations are met.

All materials deform with load. For a spring, the parameter modelling this is called the spring rate and is defined as the slope of the force deflection curve of a spring. If the slope is constant then

$$k \frac{F}{\delta} \quad (11.1)$$

where k is spring rate (N/m); F , is applied load (N); and δ , deflection (m).

The spring rate is also known as the spring constant or spring scale. Spring rates can be linear or nonlinear.

Springs can be combined in parallel, (Figure 11.3), series (Figure 11.4) or some combination

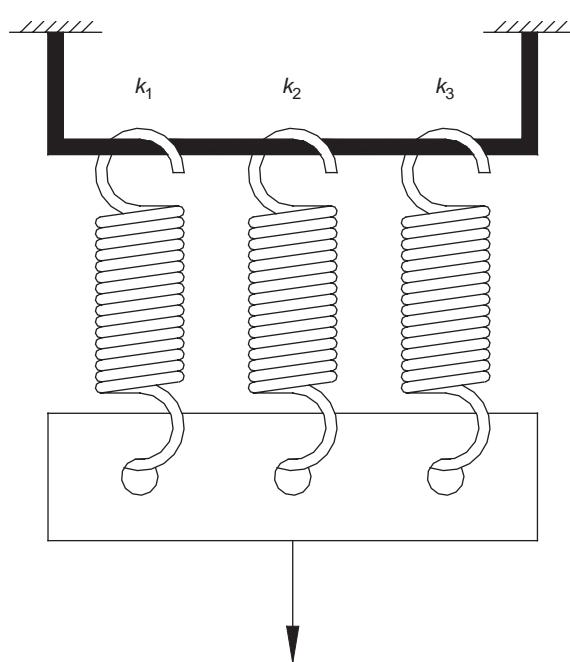
Table 11.3 Typical properties of common spring materials

Material	Young's modulus (GPa)	Modulus of rigidity (GPa)	Density (kg/m ³)	Maximum service temperature (°C)
Music wire	207	79.3	7860	120
Hard drawn wire	207	79.3	7860	150
Oil tempered	207	79.3	7860	150
Valve spring	207	79.3	7860	150
Chrome vanadium alloy steel wire	207	79.3	7860	220
Chrome silicon alloy steel wire	207	79.3	7860	245
302 stainless steel	193	69	7920	260
17-7 PH stainless steel	203	75.8	7810	315
Phosphor bronze (A)	103	43.4	8860	95
Silicon bronze (A)	103	38.6	8530	95
Silicon bronze (B)	117	44.1	8750	95
Beryllium copper	128	48.3	8260	205
Inconel 600	214	75.8	8430	320
Inconel X750	214	79.3	8250	595
AISI 1050	207	79.3	7860	95
AISI 1065	207	79.3	7860	95
AISI 1074	207	79.3	7860	120
AISI 1095	207	79.3	7860	120

Selected data reproduced from Joernes, 1996.

Table 11.4 Preferred wire diameters

Diameter (in)	Diameter (mm)	Diameter (in)	Diameter (mm)
0.004	0.10	0.081	2.00
0.005	0.12	0.085	2.20
0.006	0.16	0.092	
0.008	0.20	0.098	2.50
0.010	0.25	0.105	
0.012	0.30	0.112	2.80
0.014	0.35	0.125	3.00
0.016	0.40	0.135	3.50
0.018	0.45	0.148	
0.020	0.50	0.162	4.00
0.022	0.55	0.177	4.50
0.024	0.60	0.192	5.00
0.026	0.65	0.207	5.50
0.028	0.70	0.225	6.00
0.030	0.80	0.250	6.50
0.035	0.90	0.281	7.00
0.038	1.00	0.312	8.00
0.042	1.10	0.343	9.00
0.045		0.362	
0.048	1.20	0.375	
0.051		0.406	10.0
0.055	1.40	0.437	11.0
0.059		0.469	12.0
0.063	1.60	0.500	13.0
0.067		0.531	14.0
0.072	1.80	0.562	15.0
0.076		0.625	16.0

**Figure 11.3** Springs in parallel.

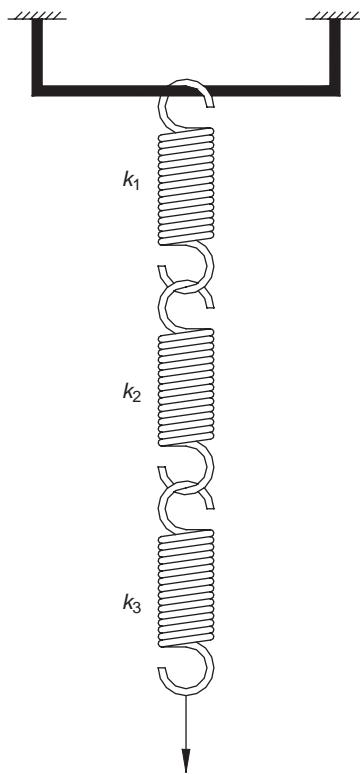


Figure 11.4 Springs in series.

of these. For springs in parallel the spring rates add directly

$$k_{\text{total}} = \downarrow \sum_{i=1}^n k_i = k_1 + k_2 + k_3 + \dots + k_n \quad (11.2)$$

For springs in series the spring rates combine reciprocally:

$$\frac{1}{k_{\text{total}}} = \downarrow \sum_{i=1}^n \frac{1}{k_i} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_n} \quad (11.3)$$

11.2 Helical compression springs

The most familiar type of spring is the helical compression spring. In its most common form it is made from constant diameter round wire with a

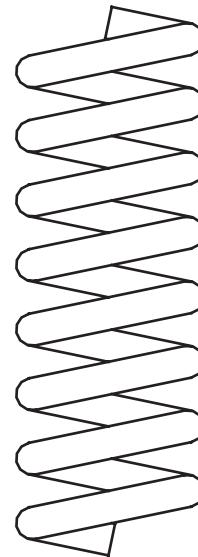
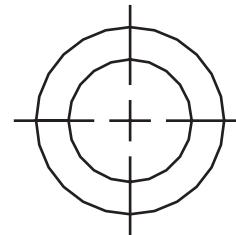


Figure 11.5 A constant pitch helical compression spring.

constant pitch as shown in Figure 11.5. Other forms are possible such as the variable pitch, barrel, hour glass and conical helical compression springs shown in Figure 11.6. In addition to variations on pitch of the coil and diameter the formation of the end is important. A variety of common end treatments is illustrated in Figure 11.7. Plain ends result from cutting the spring stock and leaving the spring with a constant pitch. Treatment of the end by some form of machining or pressing can facilitate alignment and this is the purpose of options (b) to (d) illustrated in Figure 11.7, each of which add to the cost of production of the spring and influence the performance. The end of a spring can also be formed to improve connection to mating components by the incorporation of, for example, hooks and rings.

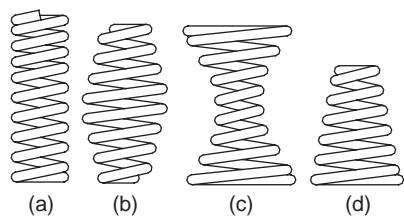


Figure 11.6 Some additional helical spring configurations.
(a) Variable pitch. (b) Barrel. (c) Hour glass. (d) Conical.

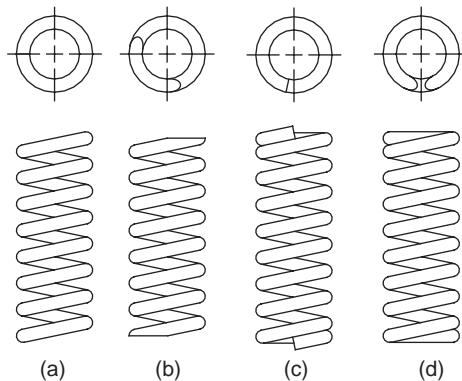


Figure 11.7 Common styles of end treatments for helical compression springs. (a) Plain ends. (b) Plain ground ends. (c) Squared ends. (d) Squared ground ends.

The principal dimensions for a constant pitch helical compression spring are illustrated in Figure 11.8. The wire diameter d , mean diameter D , free length L_f , and either the number of coils N or the pitch p are used to define a helical spring's geometry and in associated analysis. The inside and outer diameter are useful in designing the mating and locating components. The minimum diametral clearance between the outer diameter and a hole or between the inner diameter and a pin recommended, according to Associated Spring (1987), is given by $0.10D$ for $D < 13\text{ mm}$ or $0.05D$ for $D > 13\text{ mm}$.

As well as the geometrical parameters identified in Figure 11.8 for an unloaded spring, there are a number of useful lengths defined for a spring in use as illustrated in Figure 11.9. The installed length is the length after installation with initial deflection δ_{initial} . The operating length is the shortest

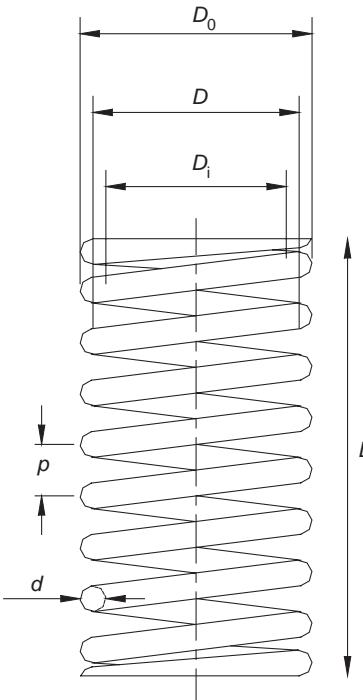


Figure 11.8 Dimensional parameters for helical compression springs.

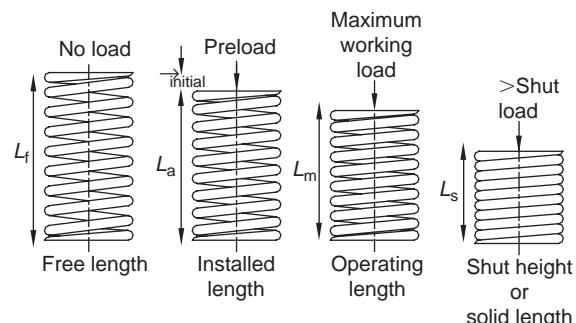


Figure 11.9 Various lengths associated with a spring in use.

dimension to which the spring is compressed in use. The shut height or solid length is the length of the spring when the spring is loaded such that the coils are actually touching. This is the shortest possible length for the spring without crushing it beyond all recognition.

Springs are subject to failure by yielding due to a too high stress in the case of static loading or by fatigue in the case of dynamic loading. In order to

determine the geometry of a spring to avoid such failure, or to determine when failure will occur, it is necessary to consider the stresses experienced by a spring under loading.

The free-body diagram for a helical spring loaded with force F is illustrated in Figure 11.10. There are two components of stress on any cross-section of coil, a torsional shear stress due to the torque and a direct shear stress due to the force. The stresses add and the maximum shear stress will occur at the inner fibre of the wire's cross-section.

$$\begin{aligned}\tau_{\max} &= \frac{Tr}{J} + \frac{F}{A} = \frac{F(D/2)(d/2)}{\pi d^4/32} \\ &+ \frac{F}{\pi d^2/4} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2} \quad (11.4)\end{aligned}$$

where τ_{\max} is maximum shear stress (N/m^2); T , torque (N/m); r , radius (m); J , polar second moment

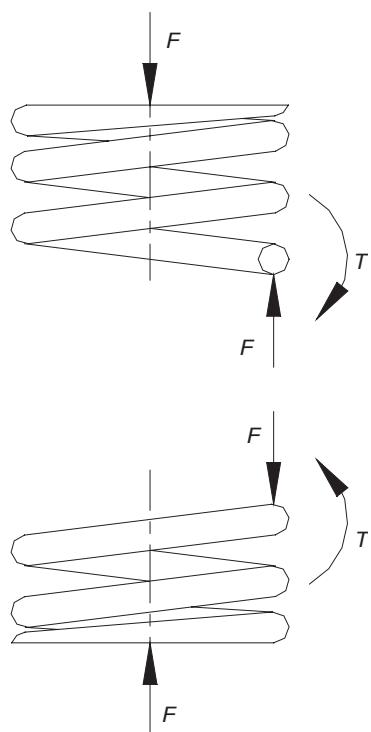


Figure 11.10 Free body diagram for a helical compression spring loaded with force F .

of area (m^4) = $\pi d^4/32$ for a solid circular cylinder; F , load (N); A , area (m^2); d , wire diameter (m); D , mean coil diameter (m).

The spring index is defined by

$$C = \frac{D}{d} \quad (11.5)$$

Preferred values for the spring index are between 4 and 12. For values of the spring index below four, springs are difficult to manufacture and for values greater than 12 they are prone to buckling. Buckling can be prevented by providing either internal or external support, by, for instance, placing the spring in a hole or on a rod. However, frictional contact between the spring and the support may reduce the force that can be delivered at the spring end. The tendency of a helical spring to buckle is proportional to the slenderness factor, which is the ratio of the free length to the mean coil diameter, L_f/D , the ratio of the spring's deflection to its free length, δ/L_f , and the type of end support used. Figure 11.11 provides guidance for determining whether a particular combination of these factors results in a stable spring or a spring prone to buckling.

Substituting for the spring index in Equation 11.4 gives

$$\tau_{\max} = \frac{8FD}{\pi d^3} \left(1 + \frac{1}{2C}\right) = K_s \frac{8FD}{\pi d^3} \quad (11.6)$$

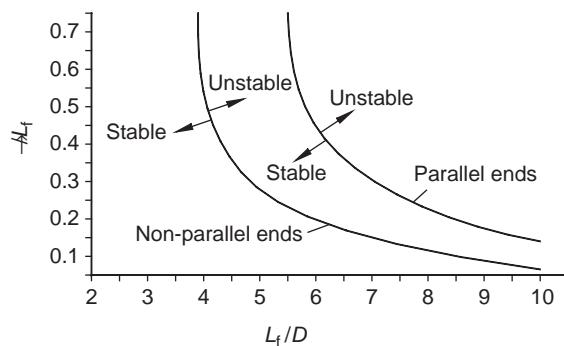


Figure 11.11 Critical buckling curves for two types of end condition (adapted from Associated Spring, Barnes Group, 1987).

where K_s is called the direct shear factor and is given by

$$K_s = 1 + \frac{1}{2C} \quad (11.7)$$

Curved beams have a stress concentration on the inner surface of curvature. The Wahl factor, K_w , includes both direct shear stress effects and the stress concentration factor due to curvature and is given by

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \quad (11.8)$$

$$\tau_{\max} = K_w \frac{8FD}{\pi d^3} = K_w \frac{8FC}{\pi d^2} \quad (11.9)$$

It is commonly recommended that Eq. 11.9 be used for modelling fatigue and Eq. 11.6 for modelling a spring under static loading only.

For springs with end treatments it is necessary to account for whether some of the coils are inactive and can be neglected in calculations for stress and deflection. For a spring with squared and ground ends or simply squared ends, each end coil is inactive and the number of active coils is given by

$$N_a = N - 2 \quad (11.10)$$

where N is the total number of coils in the spring.

For plain coils with ground ends

$$N_a = N - 1 \quad (11.11)$$

The deflection of a helical spring under load F is given by

$$\delta = \frac{8FD^3N_a}{d^4G} = \frac{8FC^3N_a}{dG} \quad (11.12)$$

where N_a is the number of active coils in the spring.

Thus the spring rate for a helical spring is given by

$$k = \frac{F}{\delta} = \frac{d^4G}{8D^3N_a} = \frac{dG}{8C^3N_a} \quad (11.13)$$

It should be noted that it is unlikely that a load on a spring will be directly on its geometric axis. Eccentric loading of a spring causes the stresses on one side of the spring to be higher than indicated in Eqs 11.6 and 11.9.

Springs can vibrate both laterally and longitudinally when excited near their natural frequencies. If a helical spring, fixed at one end, is given a sufficiently rapid compression at the other, the end coil will be pushed against its neighbour before the remaining coils have time to respond to the displacement. This compression then propagates down the spring with first coils one and two in contact, then coils two and three in contact and so on until the compression wave reaches the other end where the disturbance will be reflected back. This process repeats itself until the motion is damped out. This phenomenon is known as spring surge and causes very high stresses in the spring, which are approximately equal to those when the spring is compressed to its solid length. The natural frequency, f_n , of spring surge depends on the boundary conditions. For the fixed-fixed case:

$$f_n = \frac{1}{2} \sqrt{\frac{k}{m}} \quad (11.14)$$

where f_n is natural frequency (Hz); k , spring rate (N/m); and m , mass (kg).

The mass of a helical spring is equal to the product of density and volume, so, for N coils in a spring this is given by

$$m = \rho V = \rho N(\pi D)(\pi d^2/4) = \rho N \pi^2 d^2 D/4 \quad (11.15)$$

Substitution for the mass and spring rate in Eq. 11.14 gives

$$f_n = \frac{d}{2\pi D^2 N} \sqrt{\frac{G}{2\rho}} \quad (11.16)$$

For steel springs with a modulus of rigidity of 79.3 GPa and density 7860 kg/m³,

$$f_n = \frac{357d}{D^2 N} \quad (11.17)$$

where d is the wire diameter (m); D , coil diameter (m); N , number of coils.

In order to avoid surge, the spring should not be cycled at a frequency close to its natural frequency. The natural frequency of spring surge should usually be made higher than the highest significant harmonic of the motion involved, which is typically about the thirteenth. So the natural frequency should be at least 13 times the forcing frequency of the load on the spring in order to avoid resonance. Designing springs with high natural frequencies typically involves operating at the highest possible stress levels as this minimizes the mass of the spring and thereby maximizes f_n , which is proportional to $m^{-0.5}$.

Springs are subject to creep under load. This is sometimes evident in old cars where the sustained

weight on the suspension springs over the years has caused a permanent shortening of the spring's overall length and the car body's ground clearance is reduced. This shortening by creep is known as set. Set is directly related to yield stress. Unfortunately data for yield stress are not as readily available as those for the ultimate tensile strength. Values for the ultimate tensile strength of wire is found to vary with diameter and are illustrated in Figure 11.12 for a range of materials. Juvinall and Marshek (1991) recommend the approximations given in Table 11.5 for shear stress in order to limit set in compression coil springs to 2 per cent.

Prestressing, also known as presetting, of a spring can be used to improve a spring's ability to withstand stress, increasing its load-carrying capability and fatigue resistance. For example compression

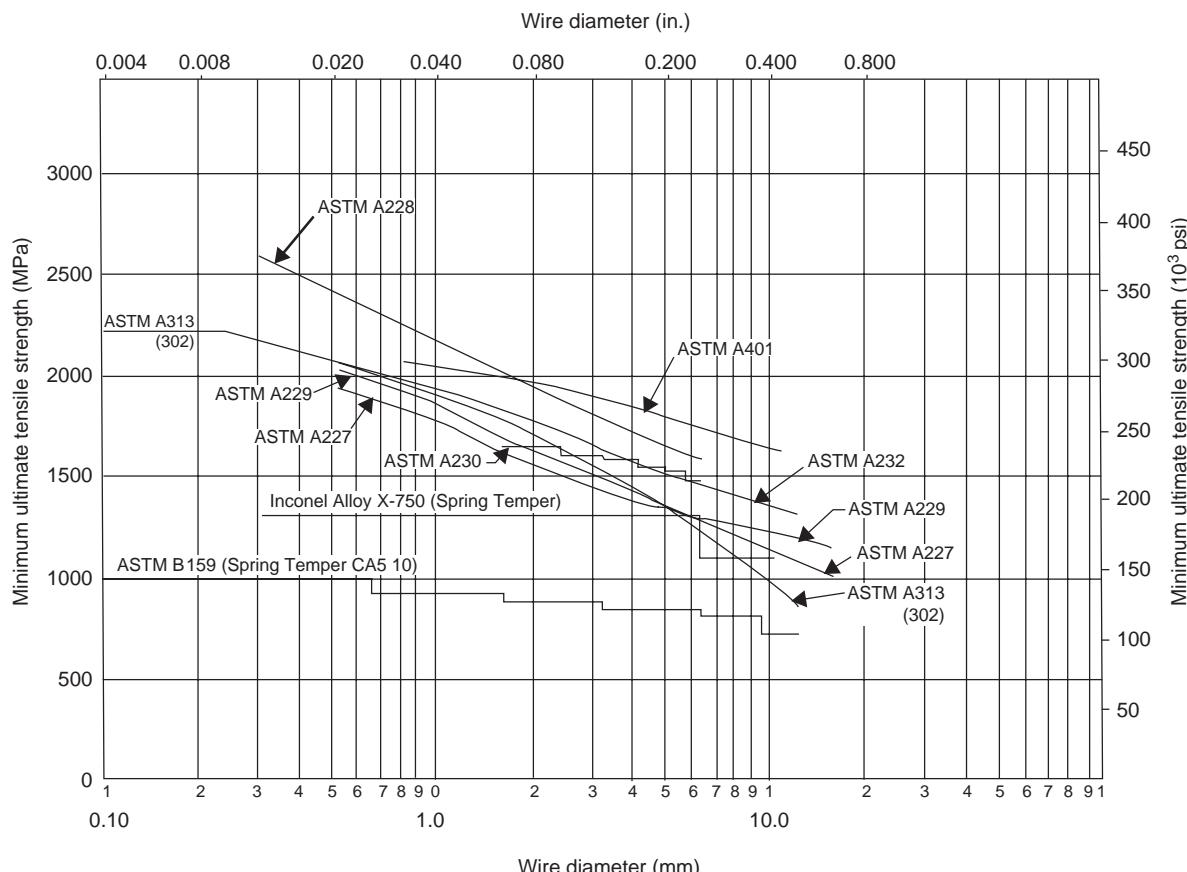


Figure 11.12 Minimum tensile strengths of spring wire (Associated Spring, Barnes Group Inc).

Table 11.5 Solid shear stress to limit long term set in compression coil springs to less than 2%

τ_{solid}	Material condition
$0.45\sigma_{\text{uts}}$	Ferrous without presetting
$0.35\sigma_{\text{uts}}$	Non-ferrous and austenitic stainless without presetting
$0.65\sigma_{\text{uts}}$	Ferrous with presetting
$0.55\sigma_{\text{uts}}$	Non-ferrous and austenitic stainless with presetting

Source: Juvinall and Marshek, 1991.

springs manufactured from BS EN10270-1 cold drawn carbon steel without prestressing can be loaded to 49 per cent of the material's ultimate tensile strength. With prestressing it can be loaded to 70 per cent of its ultimate tensile strength. Prestressing takes place after the spring has been coiled, stress relieved and ground. It involves compressing the spring to its solid length or a fixed position that is greater than its maximum working length. This process is repeated a number of times typically no less than three. During prestressing the spring's dimensions will alter. For a particular desired final length, the spring's dimensional changes must be accounted for by the manufacturer.

There are a number of strategies that can be followed in designing helical compression springs (see Mott, 1999). One approach, knowing the force and length of the spring, is to specify a material, guess a trial diameter for the spring considering the space available, check the values calculated for spring rate and free length and if necessary try a new wire diameter. This approach is outlined below and in the following example. The design procedure requires access to tables of data for material properties and wire diameters.

1. Select a material and identify its shear modulus of elasticity G .
2. Identify the operating force, F_o , operating length, L_o , the installed force, F_i , and the installed length, L_i .
3. Determine the spring rate, $k = (F_o - F_i)/(L_i - L_o)$.
4. Calculate the free length, $L_f = L_i + (F_i/k)$.
5. Specify an initial estimate for the mean diameter.

6. Specify an initial design stress. An estimate for the initial design stress can be made using Table 11.5 and Figure 11.12.
7. Calculate a trial wire diameter, d , by rearranging Eq. 11.6 or 11.9 and assuming a value for K_s or K_w , which is unknown at this stage. $K_w = 1.2$ is generally a suitable estimate at this stage.
8. Based on the value determined in step 7, select a standard wire diameter from a wire manufacturer's catalogue or using Table 11.4 as a guide.
9. Calculate the spring index, C , and the Wahl factor, K_w .
10. Determine the expected stress due to the operating force and compare with the design stress using Eq. 11.6 or 11.9.
11. Determine the number of active coils required to give the desired deflection characteristics for the spring, $N_a = Gd/(8kC^3)$.
12. Calculate the solid length, the force on the spring at the solid length and the shear stress in the spring at the solid length. Compare this value with the allowable shear stress and see whether it is safe. If the value for the shear stress is too high, alter the design parameters set above, such as the wire diameter or material and re-analyse the results.
13. Check whether the spring is likely to buckle.
14. Specify the spring dimensions.

Example 1.1

A helical compression spring is required to exert a force of 35 N when compressed to a length of 60 mm. At a length of 48 mm the force must be 50 N. The spring is to be installed in a hole with a diameter of 24 mm. The application involves slow cycling and a total life of 250 000 cycles is required. The maximum temperature of operation is 80°C.

Solution

A standard material such as ASTM A232 chromium vanadium steel wire is suggested as

an initial starting point for this application. The maximum service temperature for this material is approximately 120°C, which is greater than the operating temperature for the application. The application involves slow cyclic variation of the load so Eq. 11.9 can be used for determining the spring's stress level.

From Table 11.3 for chrome vanadium steel, $G = 79.3 \text{ GPa}$.

The maximum operating force, F_o , is 50 N with an operating length, L_o , of 48 mm. The installed force, F_i , is 35 N at an installed length, L_i , of 60 mm.

The spring rate k is given by

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{50 - 35}{0.06 - 0.048} = 1250 \text{ N/m}$$

The free length L_f is given by

$$L_f = L_i + \frac{F_i}{k} = 0.06 + \frac{35}{1250} = 0.088 \text{ m}$$

A mean diameter of 18 mm is suggested, as this should locate in the 24 mm diameter hole available allowing room for the diameter of the wire itself, which is currently undetermined.

A design shear stress of $0.45 \sigma_{uts}$ is suggested. σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 2 mm is approximately 1700 MPa (see Figure 11.12). The design shear stress will therefore be $0.45 \times 1700 = 765 \text{ MPa}$. If the wire diameter calculated in the design procedure is significantly different then the estimate for the design shear stress will need to be modified.

The equation for shear stress requires a value for K_w . K_w is a function of $C = d/D$, but the wire diameter d is unknown at this stage. A value of $K_w = 1.2$ can be used as an initial estimate here, although this will be checked and modified later in the procedure.

$$d = \frac{8FDK_w}{\pi\tau_{max}}^{1/3} = \frac{8 \times 50 \times 0.018 \times 1.2}{\pi 765 \times 10^6}^{1/3} = 1.532 \times 10^{-3} \text{ m}$$

The estimated wire diameter is therefore 1.53 mm. Examination of Table 11.3 or a stock spring supplier's catalogue will identify a suitably close wire diameter to select. In this case from Table 11.3, the nearest larger wire diameter is 1.6 mm.

The spring index can now be calculated.

$$C = \frac{D}{d} = \frac{0.018}{0.0016} = 11.25$$

The Wahl factor is

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.25 - 1}{4 \times 11.25 - 4} + \frac{0.615}{11.25} = 1.128$$

The maximum shear stress is given by

$$\tau_{max} = K_w \frac{8FD}{\pi d^3} = 1.128 \frac{8 \times 50 \times 0.018}{\pi (0.0016)^3} = 631 \times 10^6 \text{ Pa}$$

This is significantly lower than the allowable maximum shear stress so the design seems acceptable at this stage.

The number of active coils can be determined from

$$N_a = \frac{Gd}{8kC^3} = \frac{79.3 \times 10^9 \times 0.0016}{8 \times 1250 \times (11.25)^3} = 8.91 \text{ coils}$$

The solid length of a spring occurs when all the coils are touching. If the spring is squared and ground then there will be two inactive coils.

$$L_s = d(N + 2) = 0.0016(8.91 + 2) = 0.0175 \text{ m}$$

The force at the solid length would be

$$F_s = k(L_f - L_s) = 1250(0.088 - 0.0175) = 88.2 \text{ N}$$

As stress in a spring is directly proportional to force, the solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max} \frac{F_s}{F_o} = 631 \times 10^6 \frac{88.2}{50} \\ = 1113 \times 10^6 \text{ Pa}$$

This value is greater than the maximum allowable stress of 765 MPa indicating that failure is likely. It is necessary to alter one or more of the design parameters in order to develop a more suitable design. One approach is to try a different wire diameter.

If $d = 0.0018 \text{ m}$ is tried then

$$C = \frac{D}{d} = \frac{0.018}{0.0018} = 10$$

$$K_w = \frac{4 \times 10 - 1}{4 \times 10 - 4} + \frac{0.615}{10} = 1.145$$

$$\tau_{\max} = 1.145 \frac{8 \times 50 \times 0.018}{\pi(0.0018)^3} = 450 \times 10^6 \text{ Pa}$$

This is significantly lower than the allowable maximum shear stress so the design seems acceptable at this stage.

$$N_a = \frac{79.3 \times 10^9 \times 0.0018}{8 \times 1250 \times (10)^3} = 14.27 \text{ coils}$$

$$L_s = 0.0018(14.27 + 2) = 0.0293 \text{ m}$$

$$F_s = 1250(0.088 - 0.0293) = 73.4 \text{ N}$$

$$\tau_{\text{solid}} = 450 \times 10^6 \frac{73.4}{50} = 660 \times 10^6 \text{ Pa}$$

This is below the maximum allowable stress indicating an acceptable design.

The ratio of free length to the mean coil diameter, $L_f/D = 0.088/0.018 = 4.89$. The deflection of the spring under the operating force is 40 mm. Thus the ratio of deflection to free length, $\delta/L_f = 0.04/0.088 = 0.455$. Examination of Figure 11.11 for squared and ground ends shows that this design is stable and unlikely to buckle.

The outer diameter of the spring can be determined from

$$D_o = d + D = 0.0018 + 0.018 = 0.0198 \text{ m}$$

The inner diameter of the spring will be

$$D_i = D - d = 0.018 - 0.0018 = 0.0162 \text{ m}$$

Nominal specification:

ASTM A232 chromium vanadium steel wire helical compression spring,

$$k = 1250 \text{ N/m}$$

$$L_f = 0.088 \text{ m}$$

$$L_s = 0.0293 \text{ m}$$

Squared and ground ends, 16.27 coils

$$d = 1.8 \text{ mm}$$

$$D = 18 \text{ mm}, D_i = 16.2 \text{ mm},$$

$$D_o = 19.8 \text{ mm.}$$

If there are no constraints on the spring diameter then the procedure can be modified as outlined below.

1. Select a material and identify its shear modulus of elasticity G and an estimate of the design stress.
2. Calculate a value for the wire diameter based on the spring material properties and assuming approximate values for C and K_w . Typical estimates for C and K_w of 7 and 1.2, respectively, are generally suitable.

$$d = \sqrt{\frac{8K_w F_o C}{\pi \tau_{\max}}} \quad (11.18)$$

3. Select a standard wire diameter.
4. Determine the maximum number of active coils possible for the spring. The rational here is that the solid length must be less than the operating length. The relationship between the solid length and number of coils is a function of the end treatment. For squared and ground ends,

$$L_s = d(N_a + 2) \quad (11.19)$$

Substituting $L_s = L_o$

$$N_{\max} = \frac{L_o}{d} - 2 \quad (11.20)$$

5. The designer can now select any number of active coils less than the maximum calculated value. Choosing a small value will result in larger clearances between adjacent coils and will use less wire per spring, but will entail higher stresses for a given load. One approach is to try progressively fewer coils until the maximum permissible design stress is approached.
6. Calculate the spring index using

$$C = \frac{Gd}{8kN_a}^{1/3} \quad (11.21)$$

7. The mean diameter can now be calculated from the definition of the spring index, $D = Cd$.
8. Calculate the Wahl factor, K_w .
9. Determine the expected stress due to the operating force and compare with the design stress.
10. Calculate the solid length, the force on the spring at the solid length and the shear stress in the spring at the solid length. Compare this value with the allowable shear stress and see whether it is safe.
11. Check whether the spring is likely to buckle.
12. Specify the spring dimensions.

Example 11.2

A helical spring is required to exert a force of 160 N at a length of 170 mm and 200 N at a length of 150 mm. Specify a suitable spring for this application. The maximum operating temperature is 50°C and the load varies slowly with time and a total life of 200 000 cycles is required.

Solution

A standard material such as ASTM A232 chromium vanadium steel wire is suggested as an initial starting point for this application. The maximum service temperature for this material is approximately 120°C, which is greater than the operating temperature for the application. The application involves slow cyclic variation of the load so Eq. 11.9 should be used for determining the spring's stress level.

From Table 11.3 for chromium vanadium steel, $G = 79.3$ GPa.

The maximum operating force, F_o , is 200 N with an operating length, L_o , of 150 mm. The installed force, F_i , is 160 N at an installed length, L_i , of 170 mm.

The spring rate k is given by

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{200 - 160}{0.17 - 0.15} = 2000 \text{ N/m}$$

The free length L_f is given by

$$L_f = L_i + \frac{F_i}{k} = 0.17 + \frac{160}{2000} = 0.25 \text{ m}$$

A design shear stress of $0.45 \sigma_{uts}$ is suggested. σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 3 mm is approximately 1600 MPa (see Figure 11.12). The design shear stress will therefore be $0.45 \times 1700 = 720$ MPa

$$d = \sqrt{\frac{8K_w F_o C}{\pi \tau_{\max}}} = \sqrt{\frac{8 \times 1.2 \times 200 \times 7}{\pi 7.2 \times 10^8}} = 2.438 \times 10^{-3} \text{ m}$$

Choosing $d = 3$ mm,

$$N_{\max} = \frac{L_o}{d} - 2 = \frac{0.15}{0.003} - 2 = 48$$

Trying $N = 35$ gives

$$C = \frac{Gd}{8kN_a}^{1/3} = \frac{79.3 \times 10^9 \times 0.003}{8 \times 2000 \times 35}^{1/3}$$

$$= 7.517$$

$$D = Cd = 0.02255 \text{ m}$$

$$K_w = 1.197$$

$$\tau_{\max} = K_w \frac{8FD}{\pi d^3} = 1.197 \frac{8 \times 200 \times 0.02255}{\pi(0.003)^3}$$

$$= 509 \times 10^6 \text{ Pa}$$

This is significantly lower than the allowable maximum shear stress so the design seems acceptable at this stage.

$$L_s = d(N_a + 2) = 0.003(35 + 2) = 0.111 \text{ m}$$

The force at the solid length would be

$$F_s = k(L_f - L_s) = 2000(0.25 - 0.111) = 278 \text{ N}$$

The solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max} \frac{F_s}{F_o} = 509 \times 10^6 \frac{278}{200}$$

$$= 708 \times 10^6 \text{ Pa}$$

This value is just below the maximum allowable stress of 720 MPa indicating that failure is unlikely.

The ratio of free length to the mean coil diameter, $L_f/D = 0.25/0.02255 = 11.09$. The deflection of the spring under the operating force is 100 mm. Thus the ratio of deflection to free length, $\delta/L_f = 0.1/0.25 = 0.4$. Examination of Figure 11.11 for squared and ground ends shows that this design is unstable and likely to buckle.

Therefore the design parameters need to be changed in order to ensure the spring does not buckle. Decreasing the number of coils will increase the stress, which is already approaching the limit at the solid length. A different wire diameter is proposed.

If $d = 0.004 \text{ m}$

$$N_{\max} = \frac{L_o}{d} - 2 = \frac{0.15}{0.004} - 2 = 35.5$$

Trying a low value for the number of active coils, say $N_a = 16$ gives

$$C = \frac{Gd}{8kN_a}^{1/3} = \frac{79.3 \times 10^9 \times 0.004}{8 \times 2000 \times 16}^{1/3}$$

$$= 10.74$$

$$D = Cd = 0.04296 \text{ m}$$

$$K_w = 1.134$$

σ_{uts} for ASTM A232 chrome vanadium steel with an assumed wire diameter of 4 mm is approximately 1550 MPa (see Figure 11.12). The design shear stress will therefore be $0.45 \times 1550 = 698 \text{ MPa}$

$$\tau_{\max} = K_w \frac{8FD}{\pi d^3} = 1.134 \frac{8 \times 200 \times 0.04296}{\pi(0.004)^3}$$

$$= 388 \times 10^6 \text{ Pa}$$

This is significantly lower than the allowable maximum shear stress so the design seems acceptable at this stage.

$$L_s = d(N_a + 2) = 0.004(16 + 2) = 0.072 \text{ m}$$

The force at the solid length would be

$$F_s = k(L_f - L_s) = 2000(0.25 - 0.072) = 356 \text{ N}$$

The solid length shear stress can be determined by

$$\tau_{\text{solid}} = \tau_{\max} \frac{F_s}{F_o} = 388 \times 10^6 \frac{356}{200}$$

$$= 691 \times 10^6 \text{ Pa}$$

This value is below the maximum allowable stress of 720 MPa indicating that failure is unlikely.

The ratio of free length to the mean coil diameter, $L_f/D = 0.25/0.04296 = 5.819$.

The deflection of the spring under the operating force is 100 mm. Thus the ratio of deflection to free length, $\delta/L_f = 0.1/0.25 = 0.4$. Examination of Figure 11.11 for squared and ground ends shows that this design is stable and unlikely to buckle.

The outer diameter of the spring can be determined from

$$D_o = d + D = 0.004 + 0.04296 = 0.04696 \text{ m}$$

The inner diameter of the spring will be

$$D_i = D - d = 0.04296 - 0.004 = 0.03896 \text{ m}$$

Nominal specification:

ASTM A232 chromium vanadium steel wire helical compression spring.

$$k = 2000 \text{ N/m}$$

$$L_f = 0.25 \text{ m}$$

$$L_s = 0.072 \text{ m}$$

Squared and ground ends, 18 coils

$$d = 4 \text{ mm}$$

$$D = 42.96 \text{ mm}, D_i = 38.96 \text{ mm}, D_o = 46.96 \text{ mm}.$$

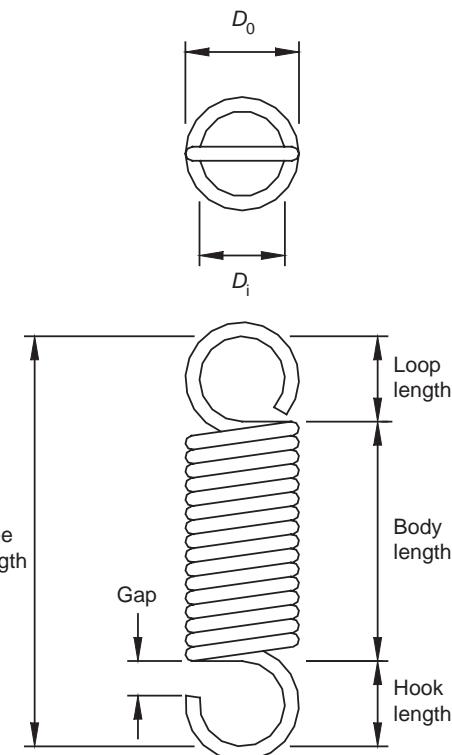


Figure 11.13 Principal dimensions for a helical extension spring with hook and loop end configurations.

overcome in order to separate them. The spring rate for an extension coil is linear after the initial coil separation section and can be modelled by

$$k = \frac{F_o - F_i}{\delta} = \frac{Gd^4}{8N_a D^3} \quad (11.22)$$

The spring index is given by $C = D/d$ and should be kept in the range of 4 to 12.

The deflection of an extension spring can be determined by modifying Eq. 11.10 to account for the preload

$$\delta = \frac{8(F_o - F_i)D^3 N_a}{d^4 G} \quad (11.23)$$

As well as accounting for stresses in the spring coils, it is also necessary to consider the stresses in the hooks and loops used in an extension spring.

11.3 Helical extension springs

Helical extension springs are loaded in tension and exert a pulling force, as well as storing energy. They are similar in appearance to helical compression springs, but tend to be manufactured so that adjacent coils touch when at their free length. Hooks or loops are incorporated as part of the spring to facilitate attachment. The principal dimensions of an extension spring are illustrated in Figure 11.13.

The coils of extension springs are tightly wound together and the wire is twisted as it is wound creating a preload in the coils that must be

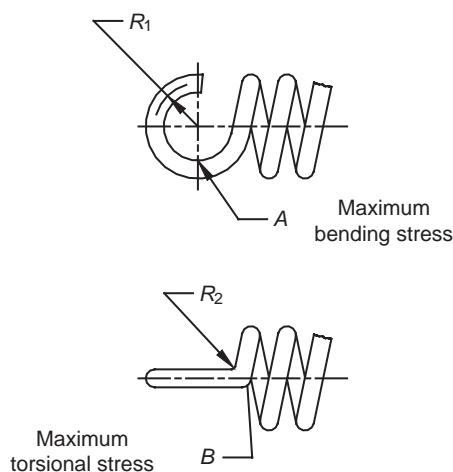


Figure 11.14 Locations of maximum stress in the loop of an extension spring.

The standard loop end illustrated in Figure 11.14 has a high bending stress at point A and a high torsional stress at point B.

The bending stress at point A can be determined using

$$\sigma_A = K_b \frac{16DF}{\pi d^3} + \frac{4F}{\pi d^2} \quad (11.24)$$

where

$$K_b = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \quad (11.25)$$

$$C_1 = \frac{2R_1}{d} \quad (11.26)$$

The torsional stress at B can be found (Wahl, 1963) from:

$$\tau_B = K_{w2} \frac{8DF}{\pi d^3} \quad (11.27)$$

where

$$K_{w2} = \frac{4C_2 - 1}{4C_2 - 4} \quad (11.28)$$

$$C_2 = \frac{2R_2}{d} \quad (11.29)$$

C_2 should be greater than four to avoid high stresses.

In a helical extension spring, all the coils are considered active and in addition a fictitious coil is added to the number of coils in order to calculate the required body length.

Design procedures for extension springs can be developed in a similar fashion to the ones outlined for compression springs. The procedure given here relies upon the designer having access to data for the maximum allowable torsional shear stress. If at any step the values arrived at are deemed unacceptable, then parameters previously determined need to be reconsidered.

1. Select a suitable material for the spring and determine the relevant material parameters, G, τ .
2. Assume a trial mean diameter for the spring.
3. Calculate a diameter for the spring wire, using Eq. 11.6 or 11.9, assuming a value for K of 1.2.
4. Select the nearest larger diameter wire from Table 11.4 or a spring supplier.
5. Determine the design stress for the wire diameter selected.
6. Determine the outer, mean and inner diameters for the spring.
7. Determine the spring index and the Wahl factor.
8. Calculate the maximum shear stress in the spring under the operating load.
9. Determine the number of coils necessary to produce the desired deflection characteristics.
10. Calculate the body length of the spring, propose the end configurations and determine the free length.
11. Calculate the deflection from the free length.
12. Calculate the initial force at which the spring coils just begin to separate. The initial tension in an extension spring is typically 10 to 25 per cent of the maximum design force.
13. Determine the stress in the spring under the initial tension and compare it with the recommended values. Recommended values for the initial shear stress are in the range $0.4\sigma_{uts}/C \leq \tau_{initial} \leq 0.8\sigma_{uts}/C$.

Example 11.3

A helical extension spring is required to exert a force of 30 N when the length between attachment locations is 70 mm and a force of 20 N at a length of 60 mm. The spring will be cycled through its load 100 times a day. ASTM A228 music wire steel has been proposed for the spring. The diameter of the spring should not exceed 20 mm. Determine suitable dimensions for the spring wire diameter, number of coils and mean diameter.

Solution

$$k = \frac{30 - 20}{70 - 60} = 1000 \text{ N/m}$$

Assuming a nominal wire diameter of 1.8 mm, the approximate ultimate tensile strength for A228 music wire is $\sigma_{uts} = 1900 \text{ MPa}$.

$$\tau_{max} = 0.45 \times 1900 = 855 \text{ MPa}$$

A mean coil diameter of 15 mm is proposed.

$$d = \frac{8 \times 30 \times 0.015 \times 1.2}{\pi 855 \times 10^6}^{0.33} \\ = 1.172 \times 10^{-3} \text{ m}$$

Try $d = 1.4 \text{ mm}$

Design stress is $\tau_{max} = 0.45 \times 2100 = 945 \text{ MPa. OK}$

$$D_i = 0.015 - 0.0014 = 0.0136 \text{ m}$$

$$D_o = 0.015 + 0.0014 = 0.0164 \text{ m}$$

$$C = D/d = 0.015/0.0014 = 10.71$$

$$K_w = 1.135$$

$$\tau = \frac{8K_w F_o D}{\pi d^3} = \frac{8 \times 1.135 \times 30 \times 0.015}{\pi 0.0014^3} \\ = 474 \times 10^6 \text{ Pa}$$

$$N_a = \frac{Gd}{8C^3 k} = \frac{79.3 \times 10^9 \times 0.0014}{8 \times 10.71^3 \times 1000} = 11.3$$

$$\text{Body length} = d(N_a + 1) = 0.0172 \text{ m.}$$

An additional fictitious coil has been added to obtain the body length.

If we use a full loop of inside diameter 0.0136 m at each end, $L_f = 0.0172 + 2 \times 0.0136 = 0.04442 \text{ m}$

$$\delta = L_o - L_f = 0.07 - 0.04442 = 0.02558 \text{ m}$$

$$F_{initial} = F_o - k\delta = 30 - 1000 \times 0.02558 \\ = 4.42 \text{ N}$$

This value is between 10 and 25 per cent of F_o so is acceptable.

$$\tau_{initial} = \tau \frac{F_{initial}}{F_o} = 474 \frac{14.42}{30} = 69.8 \text{ MPa}$$

This is less than $0.4 \sigma_{uts}/C$ so is acceptable.

11.4 Helical torsion springs

Many applications require a spring that exerts a torsional moment. A helical coil spring can be loaded in torsion to fulfil this requirement. Various configurations of torsional springs are shown in Figure 11.15. The ends of the coil tend to be extended to provide levers to apply the torque.

The basic equation governing the angular deflection of a torsional spring is

$$\theta = \frac{ML}{EI} \quad (11.30)$$

where M is the applied moment (N m); L , length of wire in the spring (m); E , Young's modulus (N/m²); I , second moment of area (m⁴); and θ , angular deflection (rad)

Substituting for the second moment of area and the length and converting the angular deflection to revolutions gives a more convenient relationship:

$$\theta' = \frac{10.2MDN_a}{d^4 E} \quad (11.31)$$

where θ' is angular deflection (revolutions).

The spring rate for a torsional spring is defined as the moment per unit of angular deflection and is given by

$$k = \frac{M}{\theta'} = \frac{d^4 E}{10.2 D N} \quad (11.32)$$

When a coil is loaded, the coil diameter decreases and its length increases as the coil is wound up. The minimum inner diameter of the coil at full deflection is given by

$$D_{i\min} = \frac{DN_b}{N_b + \theta'} - d \quad (11.33)$$

where $D_{i\min}$ is the minimum inner diameter (m); N_b , number of coils in the body.

The maximum coil body length when the spring is fully wound up is given by

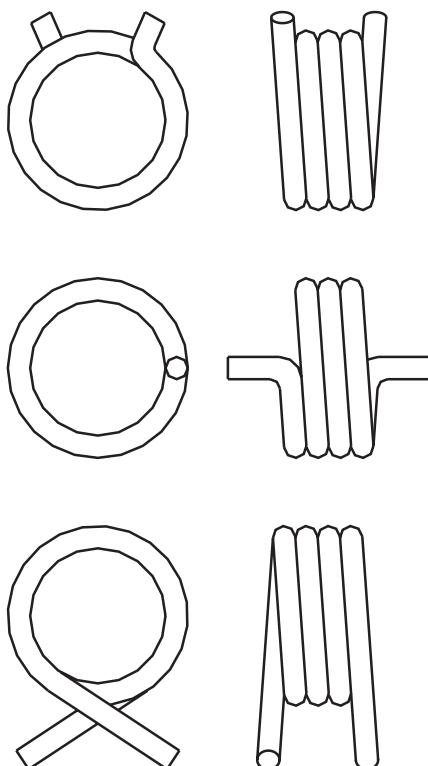


Figure 11.15 Various configurations for torsional springs.

$$L_{\max} = d(N_b + 1 + \theta) \quad (11.34)$$

where L_{\max} is maximum coil body length (m).

The number of active coils in a torsion spring is equal to the number of body turns, N_b plus some contribution from the ends, which also bend. For straight ends this contribution can be expressed by the number of equivalent coils, N_e .

$$N_e = \frac{L_1 + L_2}{3\pi D} \quad (11.35)$$

where L_1 and L_2 are the lengths of the ends. The number of active coils is given by

$$N_a = N_b + N_e \quad (11.36)$$

11.5 Leaf springs

Leaf springs consist of one or more flat strips of material loaded as cantilevers or simple beams as illustrated in Figure 11.16. They can be designed to provide a compressive or tensile force as they are deflected from their free condition. Leaf springs are capable of exerting large forces within comparably small spaces.

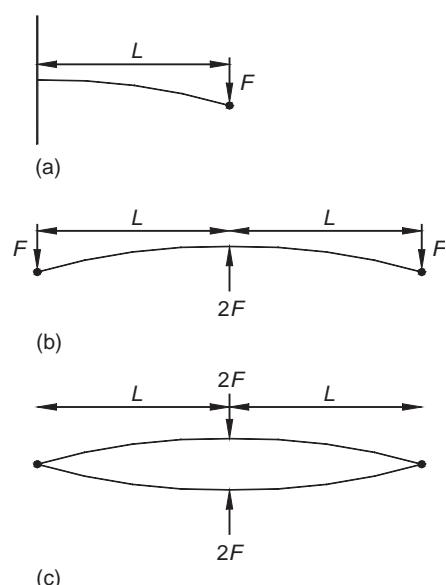


Figure 11.16 Basic types of leaf spring. (a) Simple cantilever. (b) Semi-elliptic. (c) Full-elliptic.

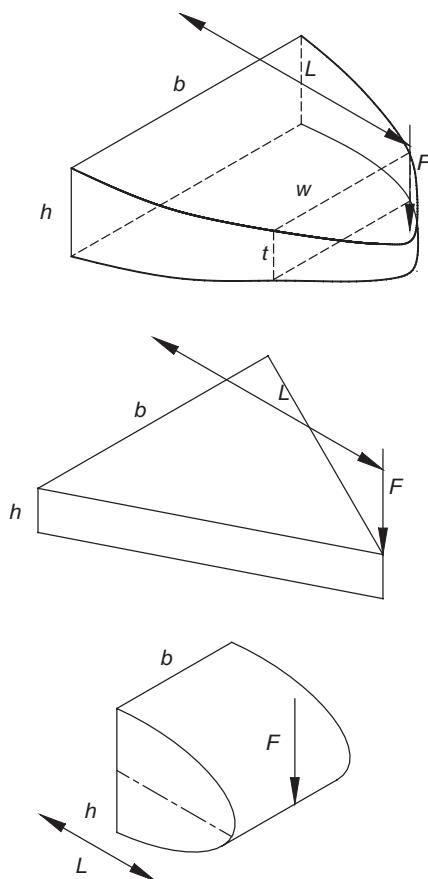


Figure 11.17 Cantilever beams. (a) General form. (b) Uniform bending stress. (c) Uniform stress cantilever of constant width.

Particular force-deflection characteristics can be achieved for a spring by careful dimensioning of the strips and nesting of a number of components. Various forms for cantilever beams are illustrated in Figure 11.17. Each type shown has special characteristics. Figure 11.17a illustrates a generalized beam. If it is desired to maintain uniform bending stresses over the length of the beam, then the width of the cantilever needs to vary linearly with location as illustrated in Figure 11.17b. This triangular form for the cantilever is the basic building block for leaf springs in common use. An alternative requirement might be to maintain uniform stress in a cantilever of constant width. In this case the profile of the beam would need to

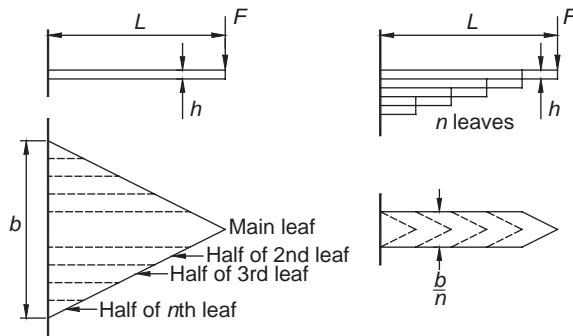


Figure 11.18 Triangular plan view cantilever and equivalent multi-leaf spring.

be parabolic as illustrated in Figure 11.17c. This is the basis used for analysing the bending strength of teeth in spur gears.

The concept used in producing compact cantilever springs of uniform bending stress is to chop the triangular form illustrated in Figure 11.17b into a number of strips and recombine them as illustrated in Figure 11.18. The multi-leaf spring shown and the single triangular section beam both have the same stress and deflection characteristics with the exceptions that the multi-leaf spring is subject to additional damping due to friction between the leaves and that the multi-leaf spring can carry a full load in only one direction due to the tendency for the leaves to separate. Leaf separation can be partially overcome by the provision of clips around the leaves.

The deflection of a triangular leaf spring is given by

$$\delta = \frac{FL^3}{2EI} \quad (11.37)$$

where F is force (N); L , length (m); E , Young's modulus (N/m^2); and I , second moment of area (m^4).

For a rectangular cross-section,

$$I = \frac{bh^3}{12} \quad (11.38)$$

where b is width (m); and h is thickness (m).

The spring rate is given by

$$k = \frac{F}{\delta} = \frac{Ebh^3}{6L^3} \quad (11.39)$$

The corresponding bending stress is given by

$$\sigma = \frac{6FL}{bh^2} \quad (11.40)$$

For a semi-elliptic beam the maximum deflection at the centre is given by

$$\delta = \frac{6FL^3}{Ebh^3} \quad (11.41)$$

The corresponding bending stress for a semi-elliptic beam is given by Eq. 11.40.

For a full-elliptic beam, the maximum deflection at the centre is given by

$$\delta = \frac{12FL^3}{Ebh^3} \quad (11.42)$$

The corresponding bending stress for a full-elliptic beam is given in Eq. 11.40.

11.6 Belleville spring washers

Belleville spring washers comprise a conical shaped disc with a hole through the centre as illustrated in Figure 11.19. They have a nonlinear force deflection characteristic, which makes them useful in

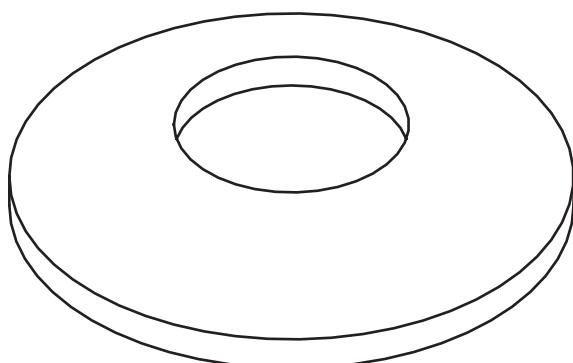


Figure 11.19 A Belleville spring washer.

certain applications. Belleville springs are compact and are capable of large compressive forces, but their deflections are limited. Examples of their use include gun recoil mechanisms, pipe flanges, small machine tools and applications where differential expansion could cause relaxation of a bolt.

The force-deflection characteristics for a Belleville washer depend on material properties and the dimensions illustrated in Figure 11.20. The force-deflection relationship is nonlinear, so it is not stated in the form of a spring rate and is given (Norton, 1996) by:

$$F = \frac{4E\delta}{K_1 D_o^2 (1 - \mu^2)} \quad h - \delta \quad h - \frac{\delta}{2} t + t^3 \quad (11.43)$$

where

$$K_1 = \frac{6}{\pi \ln R_d} \frac{(R_d - 1)^2}{R_d^2} \quad (11.44)$$

$$R_d = \frac{D_o}{D_i} \quad (11.45)$$

The load at the flat position is given by

$$F_{\text{flat}} = \frac{4Eh^3}{K_1 D_o^2 (1 - \mu^2)} \quad (11.46)$$

Eq. 11.43 has been used to illustrate the variation of force with deflection in Figure 11.21 for a variety of h/t ratios. The curves have been normalized to the spring's condition when flat for a

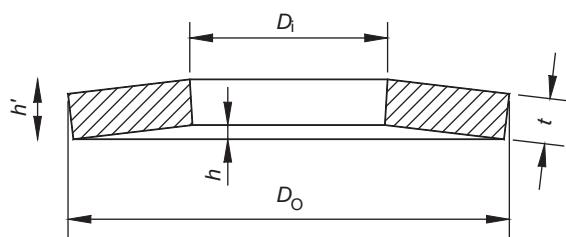


Figure 11.20 Principal Belleville washer dimensions.

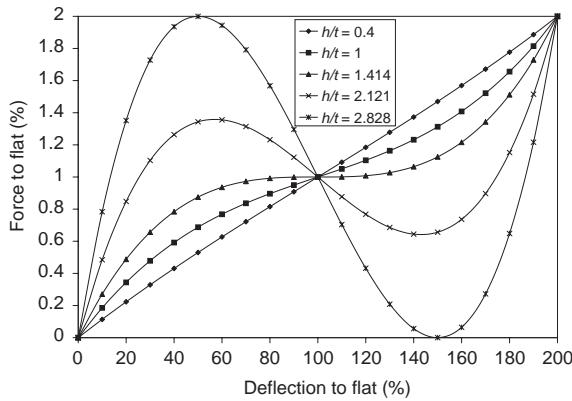


Figure 11.21 Normalized force deflection characteristics for Belleville washer springs for various h/t ratios. Calculations performed with $E = 207 \text{ GPa}$ and $\mu = 0.3$.

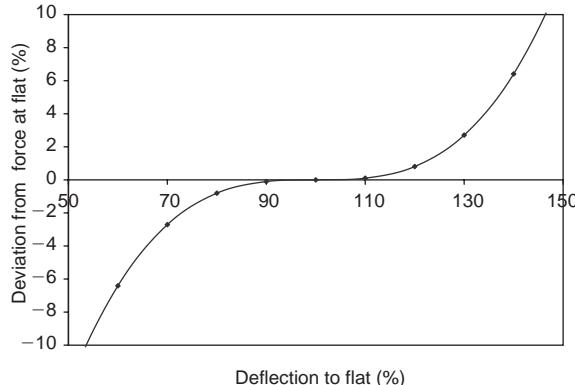


Figure 11.22 Percentage error in a constant force Belleville spring around its flat position. $R_d = 2$, $h/t = 1.414$.

steel Belleville spring. For a ratio of h/t of 0.4, the ratio of force to deflection is close to linear. As the h/t ratio is increased it becomes increasingly nonlinear. For an h/t ratio of 1.414 there is a nearly constant region for the force. For values of h/t greater than 1.414 the curves become bimodal where a given force could cause more than one possible deflection. Once installed and in the flat position a trivial force would be required to trip it in one direction or the other. The deviation of force from the force to flat value with deflection is shown in Figure 11.22 for a spring with $R_d = 2$ and $h/t = 1.414$. The force produced by an $h/t = 1.414$ ratio Belleville spring is within ± 10 per cent

of the force to flat value for deflections of 55 to 145 per cent of the deflection to flat. This is a useful characteristic if a nearly constant force is required over this range of deflection.

The stresses in a Belleville spring are concentrated at the edges of the inside and outside diameters. The largest stress is compressive and occurs at the inside radius on the convex side. The edges on the concave side have tensile stresses given by Eqs 11.48 and 11.49. These equations are quoted from Norton (1996).

$$\sigma_c = \frac{4E\delta}{K_1 D_o^2 (1 - \mu^2)} K_2 h - \frac{\delta}{2} + K_3 t \quad (11.47)$$

$$\sigma_{ti} = \frac{4E\delta}{K_1 D_o^2 (1 - \mu^2)} - K_2 h - \frac{\delta}{2} + K_3 t \quad (11.48)$$

$$\sigma_{to} = \frac{4E\delta}{K_1 D_o^2 (1 - \mu^2)} K_4 h - \frac{\delta}{2} + K_5 t \quad (11.49)$$

where

$$K_2 = \frac{6}{\pi \ln R_d} \frac{R_d - 1}{\ln R_d} - 1 \quad (11.50)$$

$$K_3 = \frac{6}{\pi \ln R_d} \frac{R_d - 1}{2} \quad (11.51)$$

$$K_4 = \frac{R_d \ln R_d - (R_d - 1)}{\ln R_d} \frac{R_d}{(R_d - 1)^2} \quad (11.52)$$

$$K_5 = \frac{R_d}{2(R_d - 1)} \quad (11.53)$$

The maximum deflection of a single Belleville spring tends to be small. They can, however, be stacked in combinations as illustrated in Figure 11.23. If stacked in series the total force will be the

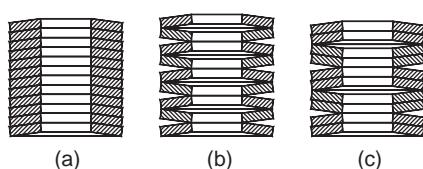


Figure 11.23 Stacking combinations for Belleville springs. (a) Parallel. (b) Series. (c) Series-parallel.

same as for a single Belleville spring, but the deflections will add. If they are stacked in parallel the total deflection will be the same as for a single spring, but the forces will add. Series and parallel combinations are also possible. It should be noted that Belleville spring towers need some form of support, either by inserting them into a hole or over a rod. This, however, will reduce the available load due to friction.

The design or selection of a Belleville spring for a given application requires consideration of the diameter and h/t ratio, as well as the type of material to be used in order to give the desired

force-deflection characteristics. This process invariably involves some iteration. A material is selected, values for the diameter ratio are proposed, often based on given constraints, and values of the h/t ratio proposed either for a single spring or some combination.

In designing Belleville springs it can be useful to estimate the thickness required to give a particular force in the flat position which can be found, substituting $E = 207 \text{ GPa}$, $\mu = 0.3$ and $K_1 = 0.69$ in Eq. 11.46, from:

$$t = \frac{1}{1072} \frac{D_o^2 F_{\text{flat}}}{h/t}^{0.25} \quad (11.54)$$

where t is thickness (m); h , deflection to flat (m); D_o , outer diameter (m); and F_{flat} , load at flat position (N).

Dimensions for a selection of Belleville springs manufactured to DIN 6796 are given in Table 11.6.

Table 11.6 Dimensions for a selection of Belleville washer springs manufactured to DIN 6796 from DIN 17222 spring steel

Notation	D_i (mm)	D_o (mm)	$h' \text{ max}$ (mm)	$h' \text{ min}$ (mm)	t (mm)	Force (N) ^a	Test force (N) ^b	Mass kg/1000	Core diameter (mm)
2	2.2	5	0.6	0.5	0.4	628	700	0.05	2
2.5	2.7	6	0.72	0.61	0.5	946	1100	0.09	2.5
3	3.2	7	0.85	0.72	0.6	1320	1500	0.14	3
3.5	3.7	8	1.06	0.92	0.8	2410	2700	0.25	3.5
4	4.3	9	1.3	1.12	1	3770	4000	0.38	4
5	5.3	11	1.55	1.35	1.2	5480	6550	0.69	5
6	6.4	14	2	1.7	1.5	8590	9250	1.43	6
7	7.4	17	2.3	2	1.75	11 300	13 600	2.53	7
8	8.4	18	2.6	2.24	2	14 900	17 000	3.13	8
10	10.5	23	3.2	2.8	2.5	22 100	27 100	6.45	10
12	13	29	3.95	3.43	3	34 100	39 500	12.4	12
14	15	35	4.65	4.04	3.5	46 000	54 000	21.6	14
16	17	39	5.25	4.58	4	59 700	75 000	30.4	16
18	19	42	5.8	5.08	4.5	74 400	90 500	38.9	18
20	21	45	6.4	5.6	5	93 200	117 000	48.8	20
22	23	49	7.05	6.15	5.5	113 700	145 000	63.5	22
24	25	56	7.75	6.77	6	131 000	169 000	92.9	24
27	28	60	8.35	7.3	6.5	154 000	221 000	113	27
30	31	70	9.2	8	7	172 000	269 000	170	30

^a Force applies to the pressed fault condition and corresponds to twice the calculated value at a deflection $h' - t$.

^b Test force applies for loading tests to DIN 6796.

Example 11.4

A Belleville spring is required to give a constant force of $200 \pm 10\text{ N}$ over a deflection of $\pm 0.3\text{ mm}$. The spring must fit within a 62 mm diameter hole. A carbon spring steel with $\sigma_{uts} = 1700\text{ MPa}$ has been proposed.

Solution

Assume a 60 mm outer diameter to allow some clearance in the hole.

In order to provide a constant force, an h/t ratio of 1.414 is selected.

The variation of force of ± 5 per cent can be met by choosing an appropriate deflection range to operate in from Figure 11.22. If the deflection is limited to between 65 and 135 per cent of the flat deflection, then the tolerance on force can be achieved. The nominal force of 200 N will occur in the flat position and the spring will provide a similar force, between 210 and 190 N, operating on both sides of its centre.

From Eq. 11.54,

$$t = \frac{1}{1072} \left(\frac{D_o^2 F_{flat}}{h/t} \right)^{0.25} = \frac{1}{1072} \left(\frac{0.06^2 \times 200}{1.414} \right)^{0.25} = 7.88 \times 10^{-4} \text{ m}$$

$$h = 1.414; t = 1.414 \times 0.788 = 1.114 \text{ mm}$$

The minimum and maximum deflections are

$$\delta_{min} = 0.65h = 0.65 \times 1.114 = 0.724 \text{ mm}$$

$$\delta_{max} = 1.35h = 1.35 \times 1.114 = 1.504 \text{ mm}$$

$\delta_{max} - \delta_{min}$ is greater than the required deflection range of 0.6 mm, so the force tolerance can be met.

From Eqs 11.44 and 11.50–11.53, K_1 is 0.689; K_2 , 1.220; K_3 , 1.378; K_4 , 1.115; and K_5 , 1.

From Eq. 11.47, $\sigma_c = -840\text{ MPa}$.

From Eq. 11.48, $\sigma_{ti} = 355\text{ MPa}$.

From Eq. 11.49, $\sigma_{to} = 658\text{ MPa}$.

These stresses are well within the capability of a 1700 MPa uts material.

11.7 Conclusions

This chapter has introduced the function and characteristics of a range of spring technologies. Design methods have been outlined for helical compression, extension and torsion springs, leaf springs and Belleville spring washers. There is no single design procedure that is suitable for all types of spring and the procedures outlined here have not considered all of the important design parameters, which also include cost, appearance and environmental considerations. Spring design invariably becomes a process of optimization where various trade-offs need to be balanced in order to provide the best possible total solution.

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www2.din.de

Nomenclature

Generally, preferred SI units have been stated.

A	area (m^2)	N_e	number of equivalent coils
b	width (m)	N_{\max}	maximum number of coils
C	spring index	p	pitch (m)
d	wire diameter (m)	PH	precipitated hardened
D	mean coil diameter (m)	r	radius (m)
D_i	inner diameter (m)	R_d	diameter ratio
$D_{i\min}$	minimum inner diameter (m)	R_1	mean loop radius (m)
D_o	outer diameter (m)	R_2	side bend radius (m)
E	Young's modulus (N/m^2)	t	thickness (m)
f_n	natural frequency (Hz)	T	torque (N m)
F	load (N)	δ	deflection (m)
F_{flat}	load at flat position (m)	δ_{initial}	initial deflection (m)
F_i	installed force (N)	μ	Poisson's ratio
F_o	operating force (N)	θ	angular deflection (rad)
G	modulus of rigidity (N/m^2)	θ'	angular deflection (revolutions)
h	thickness (m), deflection to flat (m)	ρ	density (kg/m^3)
h'	height (m)	τ	shear stress (N/m^2)
I	second moment of area (m^4)	τ_{initial}	shear stress under initial tension (N/m^2)
J	polar second moment of area (m^4/m)	τ_{\max}	maximum shear stress (N/m^2)
k	spring rate (N/m)	τ_{solid}	shear stress at solid (N/m^2)
k_{total}	total spring rate (N/m)	σ	bending stress (N/m^2)
K_b	stress concentration factor	σ_{uts}	ultimate tensile strength (N/m^2)
K_s	direct shear factor		
K_w	Wahl factor		
L	length (m)		
L_a	assembled length (m)		
L_i	installed length (m)		
L_f	free length (m)		
L_m	minimum working length (m)		
L_{\max}	maximum coil body length (m)		
L_o	operating length (m)		
L_s	shut length (m)		
m	mass (kg)		
M	applied moment (N m)		
N	number of coils		
N_a	number of active coils		
N_b	number of coils in the body		

Worksheet

- 11.1 A helical compression spring is required to exert a force of 400 N when compressed to a length of 140 mm. At a length of 170 mm the force must be 160 N. A nominal spring mean diameter of 35 mm has been proposed. The application involves slow cycling and a total life of 150 000 cycles is required. The maximum temperature of operation is 60°C. Determine a suitable wire diameter for this application if ASTM A232 is to be used.
- 11.2 A helical compression spring is required to exert a force of 3800 N when compressed to a length of 230 mm. At a length of 270 mm the force must be 2600 N. A nominal spring mean diameter of 65 mm and material ASTM A232 have been proposed. The application involves slow cycling and a total life of 450 000 cycles is required. The maximum temperature of operation is 50°C. Determine a suitable material and wire diameter for this application.
- 11.3 A helical spring is required to exert a force of 360 N at a length of 210 mm and 240 N at a

- length of 230 mm. Specify a suitable spring for this application.
- 11.4 A helical spring is required to exert a force of 2000 N at a length of 200 mm and 1500 N at a length of 250 mm. Specify a suitable spring for this application.
- 11.5 A Belleville spring is required to give a constant force of 50 ± 5 N over a deflection of ± 0.2 mm. The spring must fit within a 40 mm diameter hole. A carbon spring steel with $\sigma_{uts} = 1700$ MPa has been proposed.
- 11.6 A Belleville spring is required to give a constant force of 10 ± 1 N over a deflection of ± 0.15 mm. The spring must fit within a 16 mm diameter hole. A carbon spring steel with $\sigma_{uts} = 1700$ MPa has been proposed.
- 11.7 A helical spring is required for a pogo stick to exert a force of 975 N at a length of 140 mm and 300 N at a length of 220 mm. Specify a suitable spring for this application if the spring material is ASTM A232.

Learning objectives achievement

Can you specify the principal parameters for a helical compression spring?

✓ ✗ Section 11.2

Can you specify the principal parameters for a helical extension spring?

✓ ✗ Section 11.3

Can you determine the principal dimensions of a Belleville spring washer for a given duty?

✓ ✗ Section 11.6

12

FASTENING AND POWER SCREWS

A fastener is a device used to connect or join two or more components. Traditional forms of fastening include nuts, bolts, screws and rivets. In addition, welding and adhesives can be used to form permanent joins between components. The aim of this chapter is to introduce a wide selection of fastening techniques. Whilst considering threaded fasteners the subject of power screws, which are used for converting rotary motion to linear motion, is also considered here.

LEARNING OBJECTIVES

At the end of this chapter you should be able to determine the:

- preload and tightening torque for a threaded fastener;
- lifting and lowering torques for a power screw;
- safe load for a variety of riveted joints;
- thickness or overlap for simple adhesive lap joint.

12.1 Introduction to permanent and non-permanent fastening

The joining of components is a frequent necessity in the design of products. For example, the Boeing 747 has over 2.5 million fasteners. Fastening techniques can also be a major feature in design. Current styling for automotive vehicles dictates an absence of the means of fastening components together under a cursory inspection. The range of

fastening techniques is extensive including adhesives, welding, brazing, soldering, threaded and unthreaded fasteners, special purpose fasteners and friction joints. Some of these techniques are permanent in nature and some allow the joint to be dismantled. A variety of basic types of join is illustrated in Figure 12.1.

Design considerations include:

- whether the joint should be permanent or non-permanent
- cost
- loads in the fastener or power screw and the associated components
- life
- tooling
- assembly
- tolerances
- aesthetics
- size
- corrosion

Threaded fasteners are introduced in Section 12.2, power screws in Section 12.3, rivets in Section 12.4, adhesives in Section 12.5 and welding and the related subjects of soldering and brazing in Section 12.6.

12.2 Threaded fasteners

There is a large variety of fasteners available using a threaded form to produce connection of components. The common element of screw fasteners is a helical thread that causes the screw to advance into a component or nut when rotated. Screw threads can be either left-handed or right-handed

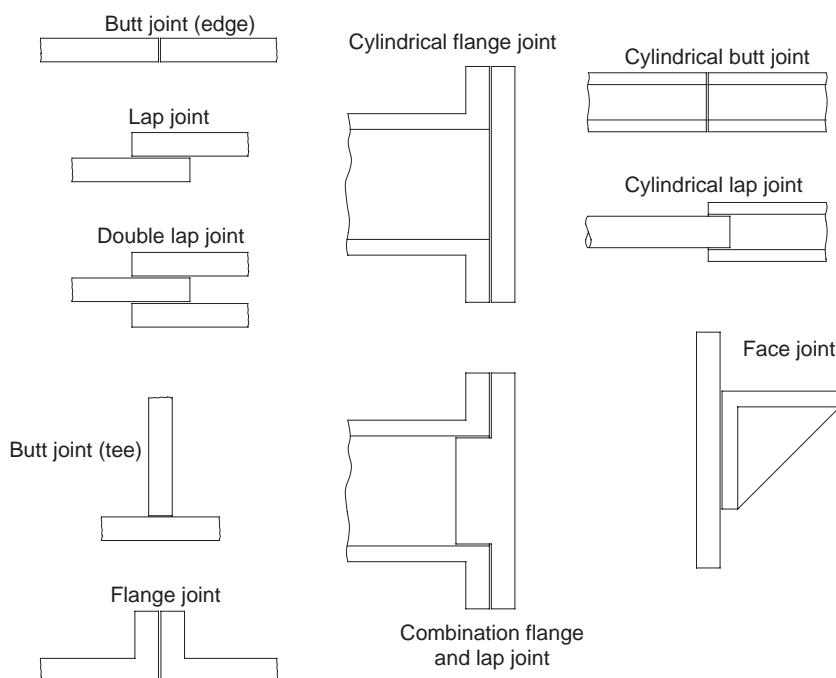
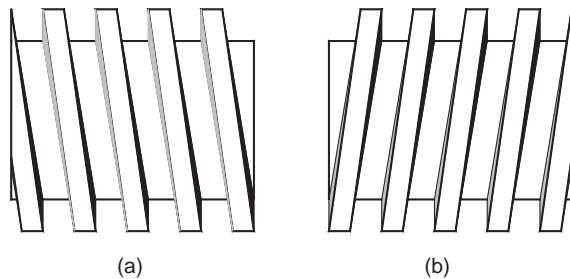


Figure 12.1 A variety of types of join.



depending on the direction of rotation desired for advancing the thread as illustrated in Figure 12.2. Generally, right-hand threads are normally used with left-handed threads being reserved for specialist applications. The detailed aspects of a thread and the specialist terminology used is illustrated in Figure 12.3 and defined in Table 12.1.

Specific thread forms, angle of helix, etc., vary according to specific standards. Common standards developed include UNS (unified national standard series threads) and ISO threads. Both of these use a 60° included angle, but are not interchangeable.

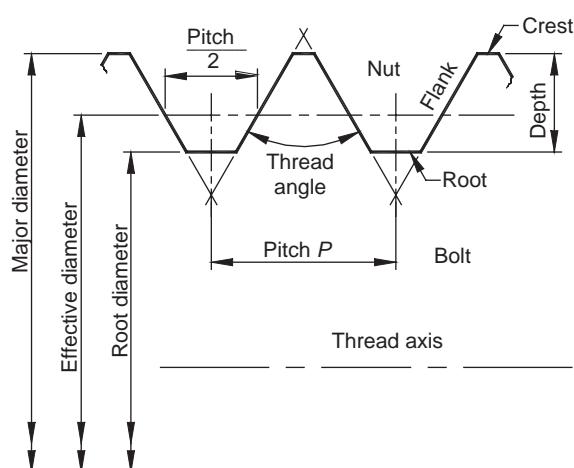


Figure 12.3 Specialist terminology used for describing threads.

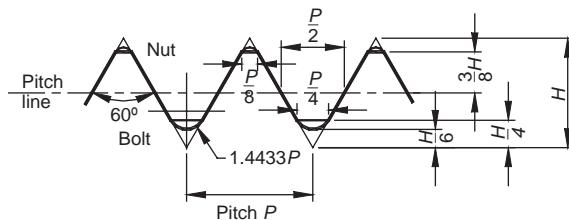
The form for an ISO metric thread for a nut is illustrated in Figure 12.4. In practice, the root of the nut and the crest of the mating bolt are rounded. Both male and female ISO threads are subject to manufacturing tolerances, which are detailed in BS 3643. A coarse series thread and fine series threads

Table 12.1 Thread terminology

Term	Description
Pitch	The thread pitch is the distance between corresponding points on adjacent threads. Measurements must be made parallel to the thread axis
Outside diameter	The outside or major diameter is the diameter over the crests of the thread measured at right angles to the thread axis
Crest	The crest is the most prominent part of thread, either external or internal
Root	The root lies at the bottom of the groove between two adjacent threads
Flank	The flank of a thread is the straight side of the thread between the root and the crest
Root diameter	The root, minor or core diameter is the smallest diameter of the thread measured at right angles to the thread axis
Effective diameter	The effective diameter is the diameter on which the width of the spaces is equal to the width of the threads. It is measured at right angles to the thread axis
Lead	The lead of a thread is the axial movement of the screw in one revolution

are defined in the ISO standard, but fine series threads tend to be more expensive and may not be readily available from all stockists. Table 12.2 gives the standard sizes for a selection of ISO coarse series hexagon bolts, screws and nuts. ISO metric threads are designated by the letter M, followed by the nominal diameter and the pitch required, for example, M6 × 1.5.

The unified system of screw threads was originally introduced in the United Kingdom, Canada and the United States in order to provide a common standard for use by the three countries. Types of unified threads in common use include the unified coarse pitch thread series (UNC) and the unified fine pitch thread series (UNF). Pertinent dimensions for selected UNC and UNF threads are given in Tables 12.3 and 12.4. Unified threads are specified by notation, in the case of a 1/2 bolt, in the form '1/2 in, 13UNC' or '1/2 in, 20UNF' depending on whether a coarse or a fine thread is being used.

**Figure 12.4** ISO metric thread.

The range of threaded fasteners available is extensive including nuts and bolts, machine screws, set screws and sheet metal screws. A variety of machine screws is illustrated in Figure 12.5.

With such an array of types of fastener, the task of selection of the appropriate type for a given application can be time-consuming.

Washers can be used either under the bolt head or the nut or both in order to distribute the clamping load over a wide area and to provide a bearing surface for rotation of the nut. The most basic form of a washer is a simple disc with a hole through which the bolt or screw passes. There are, however, many additional types with particular attributes such as lock washers, which have projections that deform when compressed, producing additional forces on the assembly, decreasing the possibility that the fastener assembly, will loosen in service. Various forms of washer are illustrated in Figure 12.6.

Selection of a particular fastener will depend upon many different criteria such as:

- strength at the operating temperatures concerned
- weight
- cost
- corrosion resistance
- magnetic properties
- life expectancy
- assembly considerations.

Threaded fasteners tend to be used such that they are predominantly loaded in tension. An example is the bolt shown in Figure 12.7 used to fasten a flanged joint. As the fastener is tightened the tension on the bolt increases. It might be

Table 12.2 Selected dimensions for a selection of British Standard ISO Metric Precision Hexagon Bolts. BS 3692:1967

Nominal size and thread diameter	Pitch of thread (coarse pitch series)	Width across flats		Height of head		Tapping drill	Clearance drill
		Max	Min	Max	Min		
M1.6	0.35	3.2	3.08	1.225	0.975	1.25	1.65
M2	0.4	4.0	3.88	1.525	1.275	1.60	2.05
M2.5	0.45	5.0	4.88	1.825	1.575	2.05	2.60
M3	0.5	5.5	5.38	2.125	1.875	2.50	3.10
M4	0.7	7.0	6.85	2.925	2.675	3.30	4.10
M5	0.8	8.0	7.85	3.650	3.35	4.20	5.10
M6	1	10.0	9.78	4.15	3.85	5.00	6.10
M8	1.25	13.0	12.73	5.65	5.35	6.80	8.20
M10	1.5	17.0	16.73	7.18	6.82	8.50	10.20
M12	1.75	19.0	18.67	8.18	7.82	10.20	12.20
M14	2	22.0	21.67	9.18	8.82	12.00	14.25
M16	2	24.0	23.67	10.18	9.82	14.00	16.25
M18	2.5	27.0	26.67	12.215	11.795	15.50	18.25
M20	2.5	30.0	29.67	13.215	12.785	17.50	20.25
M22	2.5	32.0	31.61	14.215	13.785	19.50	22.25
M24	3	36.0	35.38	15.215	14.785	21.00	24.25
M27	3	41.0	40.38	17.215	16.785	24.00	27.25
M30	3.5	46.0	45.38	19.26	18.74	26.50	30.50
M33	3.5	50.0	49.38	21.26	20.74	29.50	33.50
M36	4	55.0	54.26	23.26	22.74	32.00	36.50
M39	4	60.0	59.26	25.26	24.74	35.00	39.50
M42	4.5	65.0	64.26	26.26	25.74	37.50	42.50
M45	4.5	70.0	69.26	28.26	27.74	40.50	45.50
M48	5	75.0	74.26	30.26	29.74	43.00	48.75
M52	5	80.0	79.26	33.31	32.69	47.00	52.75
M56	5.5	85.0	84.13	35.31	34.69	50.50	56.75
M60	5.5	90.0	89.13	38.31	37.69	54.50	60.75
M64	6	95.0	94.13	40.31	39.69	58.00	64.75
M68	6	100.0	99.13	43.31	42.96	62.00	68.75

All dimensions in mm.

envisioned that the strength of a threaded fastener would be limited by the area of its minor diameter. Testing, however, shows that the tensile strength is better defined using an area based on an average of the minor and pitch diameters.

$$A_t = \frac{\pi}{16} (d_p + d_r)^2 \quad (12.1)$$

For UNS threads,

$$d_p = d - \frac{0.649519}{N} \quad \text{and} \quad d_r = d - \frac{1.299038}{N} \quad (12.2)$$

For ISO threads

$$d_p = d - 0.649519p \quad (12.3)$$

and

$$d_r = d - 1.226869p \quad (12.4)$$

The stress in a threaded rod due to a tensile load is

$$\sigma_t = \frac{F}{A_t} \quad (12.5)$$

Theoretically one might think that when a nut engages a thread, all the threads in engagement would share the load. However, inaccuracies in thread spacing cause virtually all the load to be taken by the first pair of threads.

Table 12.3 American Standard thread dimensions for UNC screw threads

Size designation	Nominal major diameter (in)	Threads per inch	Tensile stress area (in ²)
0	0.0600		
1	0.0730	64	0.00263
2	0.0860	56	0.00370
3	0.0990	48	0.0487
4	0.1120	40	0.00604
5	0.1250	40	0.00796
6	0.1380	32	0.00909
8	0.1640	32	0.0140
10	0.1900	24	0.0175
12	0.2160	24	0.0242
Fractional sizes			
1/4	0.2500	20	0.0318
5/16	0.3125	18	0.0524
3/8	0.3750	16	0.0775
7/16	0.4375	14	0.1063
1/2	0.5000	13	0.1419
9/16	0.5625	12	0.182
5/8	0.6250	11	0.226
3/4	0.7500	10	0.334
7/8	0.8750	9	0.462
1	1.000	8	0.606
1 1/8	1.125	7	0.763
1 1/4	1.250	7	0.969
1 3/8	1.375	6	1.155
1 1/2	1.500	6	1.405
1 5/8	1.750	5	1.90
2	2.000	4.5	2.50

Bolts are normally tightened by applying torque to the head or nut, which causes the bolt to stretch. The stretching results in bolt tension, known as preload, which is the force that holds a joint together. Torque is relatively easy to measure using a torque meter during assembly so this is the most frequently used indicator of bolt tension. High preload tension helps to keep bolts tight, increases the strength of a joint, generates friction between parts to resist shear and improves the fatigue resistance of bolted connections. The recommended preload for reusable connections can be determined by

$$F_i = 0.75A_t\sigma_p \quad (12.6)$$

and for permanent joints by

$$F_i = 0.9A_t\sigma_p \quad (12.7)$$

Table 12.4 American Standard thread dimensions for UNF screw threads

Size designation	Nominal major diameter (in)	Threads per inch	Tensile stress area (in ²)
0	0.0600	80	0.00180
1	0.0730	72	0.00278
2	0.0860	64	0.00394
3	0.0990	56	0.00523
4	0.1120	48	0.00661
5	0.1250	44	0.00830
6	0.1380	40	0.01015
8	0.1640	36	0.01474
10	0.1900	32	0.0200
12	0.2160	28	0.0258
Fractional sizes			
1/4	0.2500	28	0.0364
5/16	0.3125	24	0.0580
3/8	0.3750	24	0.0878
7/16	0.4375	20	0.1187
1/2	0.5000	20	0.1599
9/16	0.5625	18	0.203
5/8	0.6250	18	0.256
3/4	0.7500	16	0.373
7/8	0.8750	14	0.509
1	1.000	12	0.663
1 1/8	1.125	12	0.856
1 1/4	1.250	12	1.073
1 3/8	1.375	12	1.315
1 1/2	1.500	12	1.581

where A_t is the tensile stress area of the bolt (m²); σ_p , proof strength of the bolt (N/m²).

Material properties for steel bolts are given in SAE standard J1199 and by bolt manufacturers.

If detailed information concerning the proof strength is unavailable then it can be approximated by

$$\sigma_p = 0.85\sigma_y \quad (12.8)$$

Once the preload has been determined the torque required to tighten the bolt can be estimated from

$$T = KF_i d \quad (12.9)$$

where T is wrench torque (N m); K , constant; F_i , preload (N); d , nominal bolt diameter (m).

The value of K depends on the bolt material and size. In the absence of data from manufacturers or detailed analysis, values for K are given in Table 12.5 for a variety of materials and bolt sizes.

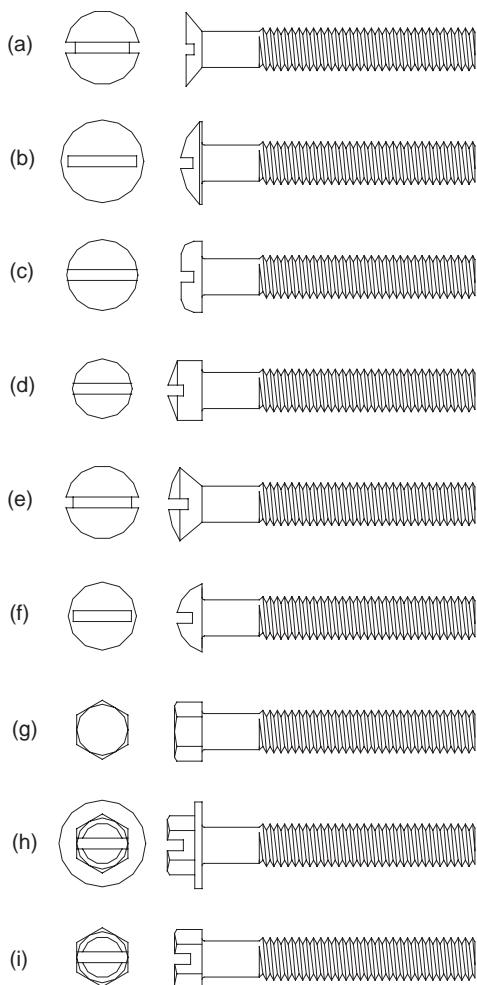


Figure 12.5 Various machine screw styles. (a) Flat countersunk head. (b) Slotted truss head. (c) Slotted pan head. (d) Slotted fillister head. (e) Slotted oval countersunk. (f) Round head. (g) Hex. (h) Hex washer. (i) Slotted hexagon head.

Example 12.1

An M10 bolt has been selected for a re-useable application. The proof stress of the low carbon steel bolt material is 310 MPa. Determine the recommended preload on the bolt and the torque setting.

Solution

From Table 12.2, the pitch for a coarse series M10 bolt is 1.5 mm.

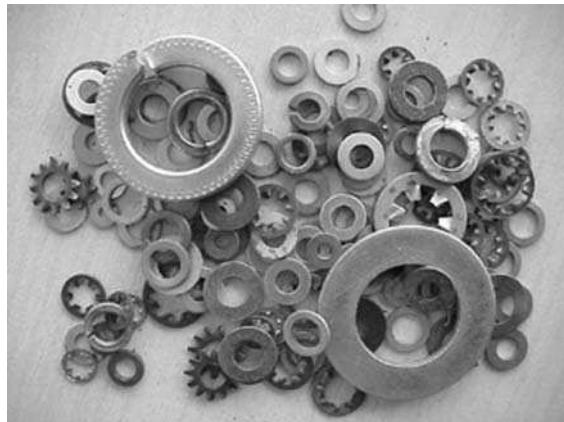


Figure 12.6 Washers.

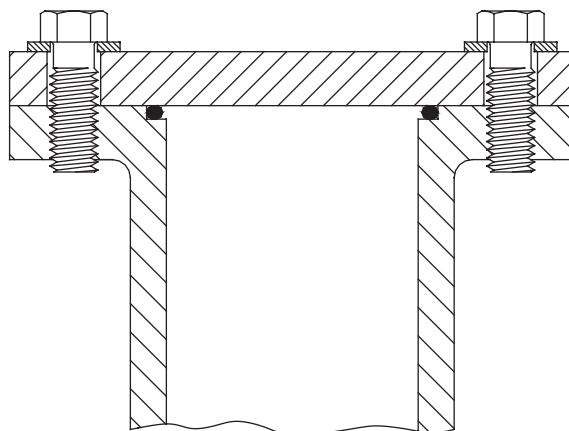


Figure 12.7 Flanged joint.

$$d_p = 10 - 0.649519 \times 1.5 = 9.026 \text{ mm}$$

$$d_r = 10 - 1.226869 \times 1.5 = 8.160 \text{ mm}$$

$$A_t = \frac{\pi}{16} (9.026 + 8.16)^2 = 57.99 \text{ mm}^2$$

For a reusable connection, the recommended preload is

$$F_i = 0.75 A_t \sigma_p = 13.48 \text{ kN}$$

From Table 12.5, $K = 0.2$. The torque required to tighten the bolt is given by

$$\begin{aligned} T &= K F_i d = 0.2 \times 13.48 \times 10^3 \times 0.01 \\ &= 26.96 \text{ Nm} \end{aligned}$$

Table 12.5 Values for the constant K for determining the torque required to tighten a bolt

Conditions	K
½ to 1 inch mild steel bolts	0.2
Non-plated black finish steel bolts	0.3
Zinc plated steel bolts	0.2
Lubricated steel bolts	0.18
Cadmium plated steel bolts	0.16

Source: Oberg et al., 1996.

The principal applications of threaded fasteners, such as bolts and nuts is clamping components together. In such situations the bolt is predominantly in tension. Both the bolt and the clamped components will behave as elastic members, provided material limits are not exceeded. If a load is applied to a bolted joint that is above the clamping load, then the behaviour of the joint itself needs to be considered. As the bolt stretches, the compressive load on the joint will decrease, alleviating some of the load on the bolt. If a very stiff bolt is used to clamp a flexible member, such as a soft gasket, most of the additional force, above the clamping load, is taken by the bolt and the bolt should be designed to take the clamping force and any additional force. Such a joint can be classified as a soft joint. If, however, the bolt is relatively flexible compared to the joint, then nearly all the externally applied load will initially go towards decreasing the clamping force until the components separate. The bolt will then carry all the external load. This kind of joint is classified as a hard joint.

Practical joints normally fall between the two extremes of hard and soft joints. The clamped components of a typical hard joint have a stiffness of approximately three times that of the bolt. An externally applied load will be shared by the bolt and the clamped components according to the relative stiffnesses, which can be modelled by

$$F_b = F_i + \frac{k_b}{k_b + k_c} F_e \quad (12.10)$$

$$F_c = F_i - \frac{k_c}{k_b + k_c} F_e \quad (12.11)$$

where F_b is final force in the bolt (N); F_i , initial clamping load (N); k_b , stiffness of the bolt (N/m); k_c , stiffness of the clamped components (N/m); F_e , externally applied load (N); F_c , final force on the clamped components (N).

Example 12.2

A set of six M8 bolts is used to provide a clamping force of 20 kN between two components in a machine. If the joint is subjected to an additional load of 18 kN after the initial preload of 8.5 kN per bolt has been applied, determine the stress in the bolts. The stiffness of the clamped components can be assumed to be three times that of the bolt material. The proof stress of the low carbon steel bolt material is 310 MPa.

Solution

Taking $k_c = 3k_b$,

$$\begin{aligned} F_b &= F_i + \frac{k_b}{k_b + k_c} F_e = F_i + \frac{k_b}{k_b + 3k_b} F_e \\ &= F_i + \frac{1}{4} F_e = 8500 + \frac{18000/6}{4} \\ &= 9250 \text{ N} \end{aligned}$$

$$\begin{aligned} F_c &= F_i - \frac{k_c}{k_b + k_c} F_e = F_i - \frac{3k_b}{k_b + 3k_b} F_e \\ &= F_i - \frac{3}{4} F_e = 8500 - \frac{3(18000/6)}{4} \\ &= 6250 \text{ N} \end{aligned}$$

As F_c is greater than zero, the joint remains tight. The tensile stress area for the M8 bolt can be determined from

$$d_p = 8 - 0.649519 \times 1.25 = 7.188 \text{ mm}$$

$$d_r = 8 - 1.226869 \times 1.25 = 6.466 \text{ mm}$$

$$A_t = \frac{\pi}{16} (7.188 + 6.466)^2 = 36.61 \text{ mm}^2$$

The stress in each bolt is given by

$$\sigma = \frac{F_b}{A_t} = \frac{9250}{36.61 \times 10^{-6}} = 252.7 \text{ MPa}$$

This is 82 per cent of the proof stress. The bolts are therefore safe.

12.3 Power screws

Power screws, which are also known as lead screws, are used to convert rotary motion into linear movement. With suitably sized threads they are capable of large mechanical advantage and can lift or move large loads. Applications include screw jacks and traverses in production machines. Although suited to fasteners thread forms, such as the ISO metric standard screw threads, UNC and UNF series described in Section 12.2 may not be strong enough for power screw applications. Instead square, Acme and buttress thread forms have been developed and standardized; see Figures 12.8 to 12.10, respectively. Some of the principal dimensions for standard Acme threads are given in Table 12.6.

Self-locking in power screws refers to the condition in which a screw cannot be turned by the application of axial force on the nut. This is very useful in that a power screw that is self-locking will

hold its position and load unless a torque is applied. As an example most screw jacks for cars are self-locking and do not run down when the handle is let go. The opposite condition to self-locking is a screw that can be back-driven. This means that an axial force applied to the nut will cause the screw to turn. A product application of this is the Yankee screwdriver, which has a high-lead thread on its barrel that is attached to the screwdriver bit. The handle acts as the nut and when pushed the barrel will turn driving the bit round. Back-driveable power screws are a useful form of turning linear motion into rotary motion.

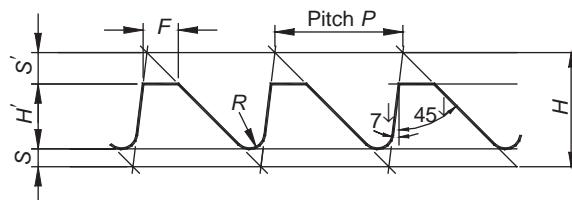


Figure 12.10 Buttress thread.

Table 12.6 Principal dimensions for ACME threads

Major diameter (in)	Threads per inch	Thread pitch (in)	Pitch diameter (in)	Minor diameter (in)	Tensile stress area (in^2)
0.25	16	0.063	0.219	0.188	0.032
0.313	14	0.071	0.277	0.241	0.053
0.375	12	0.083	0.333	0.292	0.077
0.438	12	0.083	0.396	0.354	0.110
0.500	10	0.100	0.450	0.400	0.142
0.625	8	0.125	0.563	0.500	0.222
0.750	6	0.167	0.667	0.583	0.307
0.875	6	0.167	0.792	0.708	0.442
1.000	5	0.200	0.900	0.800	0.568
1.125	5	0.200	1.025	0.925	0.747
1.250	5	0.200	1.150	1.050	0.950
1.375	4	0.250	1.250	1.125	1.108
1.500	4	0.250	1.375	1.250	1.353
1.750	4	0.250	1.625	1.500	1.918
2.000	4	0.250	1.875	1.750	2.580
2.250	3	0.333	2.083	1.917	3.142
2.500	3	0.333	2.333	2.167	3.976
2.750	3	0.333	2.583	2.417	4.909
3.000	2	0.500	2.750	2.500	5.412
3.500	2	0.500	3.250	3.000	7.670
4.000	2	0.500	3.750	3.500	10.321
4.500	2	0.500	4.250	4.000	13.364
5.000	2	0.500	4.750	4.500	16.800

Figure 12.8 Square thread.

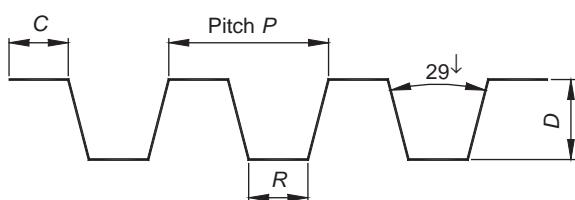


Figure 12.9 Acme thread.

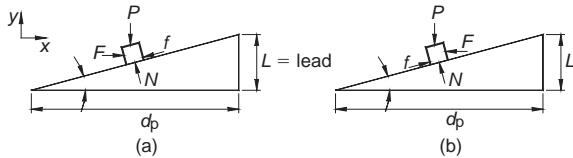


Figure 12.11 Force analysis at the interface of a lead screw and nut (a) lifting a load; (b) lowering a load.

A screw thread can be considered to be an inclined plane wrapped around a cylinder to form a helix. The forces on a single thread of a power screw are illustrated for the case of lifting a load and lowering a load in Figure 12.11.

The inclination of the plane is called the lead angle λ .

$$\tan \lambda = \frac{L}{\pi d_p} \quad (12.12)$$

In the case of lifting a load, summing the forces gives

$$\begin{aligned} \rightarrow F_x &= 0 = F - f \cos \lambda - N \sin \lambda \\ &= F - \mu N \cos \lambda - N \sin \lambda \end{aligned} \quad (12.13)$$

$$F = N(\mu \cos \lambda + \sin \lambda) \quad (12.14)$$

So

$$\begin{aligned} \rightarrow F_y &= 0 = N \cos \lambda - f \sin \lambda - P \\ &= N \cos \lambda - \mu N \sin \lambda - P \end{aligned} \quad (12.15)$$

So

$$N = \frac{P}{\cos \lambda - \mu \sin \lambda} \quad (12.16)$$

where μ is the coefficient of friction between the screw and the nut. The coefficient of friction can typically be taken as 0.15.

Solving Eqs 12.14 and 12.16 to give an expression for F gives

$$F = \frac{P(\mu \cos \lambda + \sin \lambda)}{\cos \lambda - \mu \sin \lambda} \quad (12.17)$$

The screw torque required to lift the load is given by

$$T_s = F \frac{d_p}{2} = \frac{P d_p (\mu \cos \lambda + \sin \lambda)}{2(\cos \lambda - \mu \sin \lambda)} \quad (12.18)$$

It is convenient to rewrite this equation in terms of the lead L , so substituting with Eq. 12.12 and rearranging gives:

$$T_s = \frac{P d_p (\mu \pi d_p + L)}{2(\pi d_p - \mu L)} \quad (12.19)$$

The thrust collar also contributes a friction torque that must be accounted for. The torque required to turn the thrust collar is given by

$$T_c = \mu_c P \frac{d_c}{2} \quad (12.20)$$

where d_c is diameter of the thrust collar; μ_c , coefficient of friction in the thrust bearing.

The total torque to lift the load for a square thread is

$$T_u = \frac{P d_p (\mu \pi d_p + L)}{2(\pi d_p - \mu L)} + \mu_c P \frac{d_c}{2} \quad (12.21)$$

Similar analysis can be performed for lowering a load, in which case:

$$T_d = \frac{P d_p (\mu \pi d_p - L)}{2(\pi d_p + \mu L)} + \mu_c P \frac{d_c}{2} \quad (12.22)$$

For Acme threads the equivalent torque relationship for lifting a load is:

$$T_u = \frac{P d_p (\mu \pi d_p + L \cos \alpha)}{2(\pi d_p \cos \alpha - \mu L)} + \mu_c P \frac{d_c}{2} \quad (12.23)$$

and for lowering:

$$T_d = \frac{P d_p (\mu \pi d_p - L \cos \alpha)}{2(\pi d_p \cos \alpha + \mu L)} + \mu_c P \frac{d_c}{2} \quad (12.24)$$

The work done on a power screw is the product of the torque and angular displacement. For one revolution of a screw,

$$W_{in} = 2\pi T \quad (12.25)$$

The work delivered for one revolution is

$$W_{\text{out}} = PL \quad (12.26)$$

The efficiency of the system defined as work out/work in is

$$\eta = \frac{PL}{2\pi T} \quad (12.27)$$

Substituting for the torque for an Acme thread using Eq. 12.23, neglecting collar friction, gives

$$\eta = \frac{L(\pi d_p \cos \alpha - \mu L)}{\pi d_p (\pi \mu d_p + L \cos \alpha)} \quad (12.28)$$

Simplifying using Eq. 12.12 gives

$$\eta = \frac{\cos \alpha - \mu \tan \lambda}{\cos \alpha + \mu \cot \lambda} \quad (12.29)$$

For a square thread, $\alpha = 0$, so $\cos \alpha = 1$, giving

$$\eta = \frac{1 - \mu \tan \lambda}{1 + \mu \cot \lambda} \quad (12.30)$$

Standard Acme screws have lead angles between about 2° and 5° . Assuming a coefficient of friction of 0.15 gives the efficiency between 18 and 36 per cent. This shows the disadvantage of power screws. Higher efficiencies can, however, be attained by reducing the friction and one of the ways of doing this has been found to be the use of ball screws although these potentially add to the cost of production.

A screw will self-lock if

$$\mu \geq \frac{L}{\pi d_p} \cos \alpha \quad (12.31)$$

or

$$\mu \geq \tan \lambda \cos \alpha \quad (12.32)$$

For a square thread, where $\cos \alpha = 1$,

$$\mu \geq \frac{L}{\pi d_p} \quad (12.33)$$

or

$$\mu \geq \tan \lambda \quad (12.34)$$

It should be noted that Eqs 12.31 to 12.34 are for a statically loaded screw. Dynamic loading, such as vibration, can reduce the effective friction and cause a screw to back-drive.

In the design of a power screw, consideration should also be given to buckling of the screw and choice of material for the screw and nut.

Example 12.3

A self-locking power screw is required for a screw jack. An initial proposal is to use a single start 1.25–5 Acme power screw. The axial load is 4000 N and collar mean diameter is 1.75 in. Determine the lifting and lowering torques, the efficiency of the power screw and whether the design proposal is self-locking.

Solution

Single start thread, so the lead, L , is equal to the pitch, P .

$$N = 5 \text{ teeth per inch}$$

$$P = \frac{1}{N} = \frac{1}{5} = 0.2''$$

$$L = 0.2'' = 5.08 \text{ mm}$$

$$d_p = 1.15'' = 29.21 \text{ mm}$$

$$d_c = 1.75'' = 44.45 \text{ mm}$$

Assume sliding friction, $\mu = 0.15$.

The torque to lift the load is given by Eq. 12.23.

$$\begin{aligned} T &= \frac{4000 \times 0.02921}{2} \\ &\times \frac{0.15\pi 0.02921 + 0.00508 \cos 14.5}{\pi 0.02921 \cos 14.5 - 0.15 \times 0.00508} \\ &+ 0.15 \times 4000 \frac{0.04445}{2} \\ &= 58.42 \frac{0.01376 + 4.9182 \times 10^{-3}}{0.08884 - 7.62 \times 10^{-4}} + 13.34 \\ &= 12.39 + 13.34 = 25.73 \text{ N m} \end{aligned}$$

The torque to lower the load is given by Eq. 12.24.

$$T = 58.42 \frac{0.01376 - 4.9182 \times 10^{-3}}{0.08884 + 7.62 \times 10^{-4}} + 13.34 \\ = 5.765 + 13.34 = 19.11 \text{ Nm}$$

$$\eta = \frac{PL}{2\pi T}$$

$$\eta_{\text{screw}} = \frac{4000 \times 0.00508}{2\pi 12.39} = 0.261$$

$$\eta_{\text{both}} = \frac{4000 \times 0.00508}{2\pi 25.73} = 0.126$$

The design will be self-locking if

$$\mu \geq \frac{L}{\pi d_p} \cos \alpha = \frac{0.00508}{\pi 0.02921} \cos 14.5 \\ = 0.05359$$

$\mu = 0.15$, so the design is self-locking.

12.4 Rivets

Rivets are non-threaded fasteners that are usually manufactured from steel or aluminium. They consist of a preformed head and shank, which is inserted into the material to be joined and the second head that enables the rivet to function as a fastener is formed on the free end by a variety of means known as setting. A conventional rivet before and after setting is illustrated in Figure 12.12.

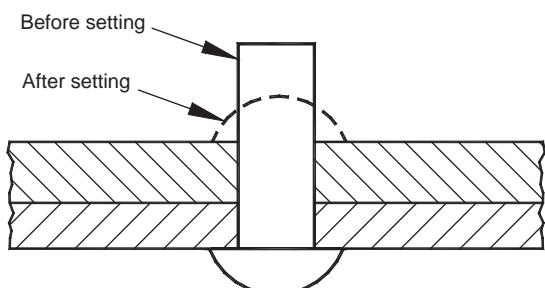


Figure 12.12 Conventional rivet before and after setting.

Rivets are widely used to join components in aircraft, boilers, ships and boxes and other enclosures. Rivets tend to be much cheaper to install than bolts and the process can be readily automated with single riveting machines capable of installing thousands of rivets an hour.

Rivets can be made from any ductile material, such as carbon steel, aluminium and brass. A variety of coatings is available to improve corrosion resistance. Care needs to be taken in the selection of material and coating in order to avoid the possibility of corrosion by galvanic action. In general, a given size rivet will be not as strong as the equivalent threaded fastener.

The two main types of rivet are tubular and blind and each type are available in a multitude of varieties. The advantage of blind rivets is that they require access to only one side of the joint. A further type of rivet with potentially many overall advantages, from the production perspective, is self-piercing rivets that do not require predrilled holes. The rivet is driven into the target materials with high force, piercing the top sheets and spreading outwards into the bottom sheet of material under the influence of an upsetting die to form the joint.

Factors in the design and specification of rivets include the size, type and material for the rivet, the type of join, and the spacing between rivets. There are two main types of riveted joint: lap-joints and butt-joints (Figure 12.13). In lap joints the components to be joined overlap each other, while for butt joints an additional piece of material is used to bridge the two components to be joined which are butted up against each other. Rivets can fail by shearing through one cross-section known as single shear, shearing through two cross-sections known as double shear, and crushing. Riveted plates can fail by shearing, tearing and crushing.

For many applications, the correct use of rivets is safety critical and their use is governed by construction codes. For information and data concerning joints for pressure vessels reference to the appropriate standards should be made, such as the ASME boiler code.

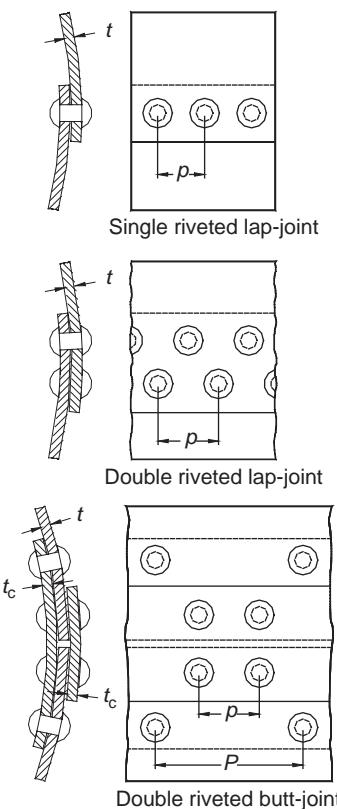


Figure 12.13 Some types of riveted joints.

Riveted joints can be designed using a simple procedure (Oberg *et al.*, 1996) assuming that:

- the load is carried equally by the rivets;
- no combined stresses act on a rivet to cause failure;
- the shearing stress in a rivet is uniform across the cross-section;
- the load that would cause failure in single shear would have to be double to cause failure in double shear;
- the bearing stress of the rivet and plate is distributed equally over the projected area of the rivet;
- the tensile stress is uniform in the section of metal between the rivets.

The allowable stress for a rivet is generally defined in the relevant standard. For example, the

ASME boiler code lists an ultimate tensile stress for rivets of 379 MPa, ultimate shearing stress of 303 MPa and an ultimate compressive or bearing stress of 655 MPa. Design stresses are usually 20 per cent of these values, i.e. for tensile, shear and bearing stresses the design limits are 75, 60 and 131 MPa, respectively.

For a single lap joint, the safe tensile load based on shear is given by

$$L = nA_r\tau_d \quad (12.35)$$

For a single lap joint, the safe tensile load based on compressive or bearing stress is given by

$$L = nA_b\sigma_c \quad (12.36)$$

$$A_b = td \quad (12.37)$$

For a single lap joint, the safe tensile load based on tensile stress is given by

$$L = A_p\sigma_t \quad (12.38)$$

where L is load (N); A_r , cross-sectional area of rivet (m^2); A_b , projected bearing area of rivet (m^2); A_p , cross-sectional area of plate between rivet holes (m^2); t , thickness of plate (m); d , diameter of rivet (m); τ_d , allowable shear stress (N/m^2); σ_c , allowable bearing or compressive stress (N/m^2); σ_t , allowable tensile stress (N/m^2).

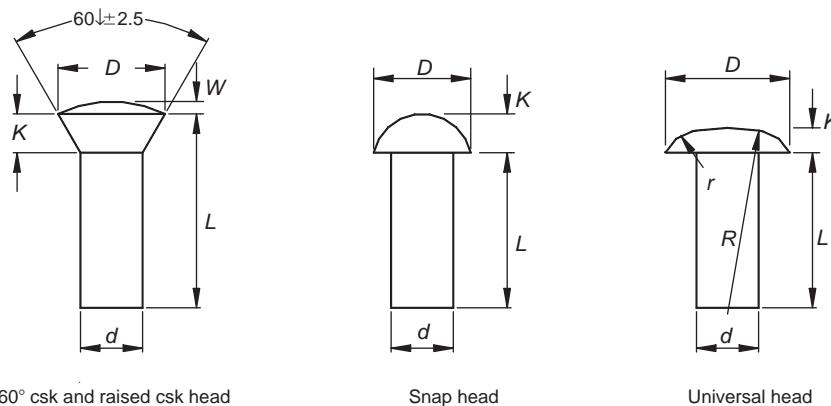
The efficiency of a riveted joint is given by

$$\eta = \frac{\text{least safe load}}{\text{ultimate tensile strength of unperforated section}} \quad (12.39)$$

A selection of rivets specified in BS 4620:1970 and ANSI B18.1.2-1972 is included in Tables 12.7 to 12.10 as examples. Rivets, however, are available as stock items from specialist manufacturers and suppliers in a much wider variety than the small selection presented in Tables 12.7 to 12.10. For any given application the relevant standard should be referenced and a range of manufacturers' products considered.

Table 12.7 British Standard hot-forged rivets for general engineering purposes

Hot-forged rivets



Head dimensions									
Nom shank diam d	Tol on diam d	60° csk and raised csk head		Snap head		Universal head			
		Nom diam D	Height of raise W	Nom diam D	Nom depth K	Nom diam D	Nom depth K	Rad R	Rad r
(14)		21	2.8	22	9	28	5.6	42	8.4
16	± 0.43	24	3.2	25	10	32	6.4	48	9.6
(18)		27	3.6	28	11.5	36	7.2	54	11
20		30	4.0	32	13	40	8.0	60	12
(22)	± 0.52	33	4.4	36	14	44	8.8	66	13
24		36	4.8	40	16	48	9.6	72	14
(27)		40	5.4	43	17	54	10.8	81	16
30		45	6.0	48	19	60	12.0	90	18
(33)	± 0.62	50	6.6	53	21	66	13.2	99	20
36		55	7.2	58	23	72	14.4	108	22
(39)		59	7.8	62	25	78	15.6	117	23

Extracted from BS 4620:1970. Note see the standard for full ranges. All dimensions are in millimetres. Sizes shown in parentheses are non-preferred.

Example 12.4

Determine the safe tensile, shear and bearing loads and the efficiency for a 300 mm section of single-riveted lap joint made from 1/4" plates using six 16-mm diameter rivets. Assume that the drilled holes are 1.5 mm larger in diameter than the rivets. The values for the design limits for tensile, shear and bearing stress can be taken as 75, 60 and 131 MPa, respectively.

Solution

The safe tensile load, L , based on shear of the rivets is given by

$$L = nA_r \tau_d = 6\pi \frac{0.016^2}{4} \times 75 \times 10^6 = 90.48 \text{ kN}$$

The safe tensile load based on bearing or compressive stress is given by

$$L = 6 \times 0.016 \times 6.35 \times 10^{-3} \times 131 \times 10^6 \\ = 79.86 \text{ kN}$$

Table 12.8 British Standard cold-forged rivets for general engineering purposes

Cold-forged rivets										
Nom shank diam d	Tol on diam d	Head dimensions								
		90° csk head		Snap head		Universal head				Flat head
Nom shank diam d	Tol on diam d	Nom diam D	Nom diam D	Nom depth K	Nom diam D	Nom depth K	Rad R	Rad r	Nom diam D	Nom depth K
1	± 0.07	2	1.8	0.6	2	0.4	3.0	0.6	2	0.25
1.2		2.4	2.1	0.7	2.4	0.5	3.6	0.7	2.4	0.3
1.6		3.2	2.8	1.0	3.2	0.6	4.8	1.0	3.2	0.4
2		4	3.5	1.2	4	0.8	6.0	1.2	4	0.5
2.5		5	4.4	1.5	5	1.0	7.5	1.5	5	0.6
3		6	5.3	1.8	6	1.2	9.0	1.8	6	0.8
(3.5)	± 0.09	7	6.1	2.1	7	1.4	10.5	2.1	7	0.9
4		8	7	2.4	8	1.6	12	2.4	8	1.0
5		10	8.8	3.0	10	2.0	15	3.0	10	1.3
6		12	10.5	3.6	12	2.4	18	3.6	12	1.5
(7)	± 0.11	14	12.3	4.2	14	2.8	21	4.2	14	1.8
8		16	14	4.8	16	3.2	24	4.8	16	2
10		20	18	6.0	20	4.0	30	6	20	2.5
12	± 0.14	24	21	7.2	24	4.8	36	7.2	—	—
(14)		—	25	8.4	28	5.6	42	8.4	—	—
16		—	28	9.6	32	6.4	48	9.6	—	—

Extracted from BS 4620:1970. Note see the standard for full ranges.

All dimensions are in millimetres. Sizes shown in parentheses are non-preferred.

The safe load based on tensile load is given by

$$L = A_p \sigma_t$$

The area of the plate between the rivet holes, A_p is given by

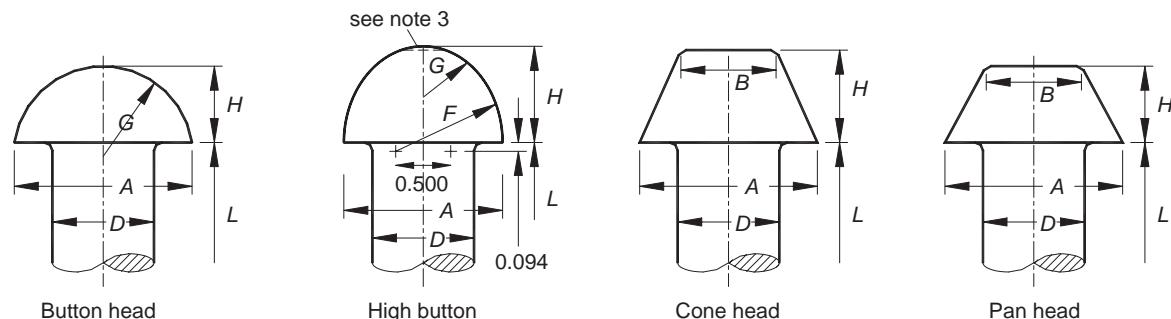
$$\begin{aligned} A_p &= 0.00635 (0.3 - 6(16 \times 10^{-3}) \\ &\quad + 1.5 \times 10^{-3})) \\ &= 1.238 \times 10^{-3} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} L &= A_p \sigma_t = 1.238 \times 10^{-3} \times 131 \times 10^6 \\ &= 162.2 \text{ kN} \end{aligned}$$

The safe tensile load would be the least of the three values determined, i.e. $L = 79.86 \text{ kN}$.

$$\eta = \frac{79.86 \times 10^3}{0.00635 \times 0.3 \times 75 \times 10^6} = 0.56$$

The efficiency is 56 per cent.

Table 12.9 Selected American National Standard large button, high button, cone and pan head rivets

Nom body diam D^a	Head diam A (in)		Height H (in)		Head diam A (in)		Height H (in)	
	Mfd note 1	Driven note 2	Mfd note 1	Driven note 2	Mfd note 1	Driven note 2	Mfd note 1	Driven note 2
Button head								
1/2	0.875	0.922	0.375	0.344	0.781	0.875	0.500	0.375
5/8	1.094	1.141	0.469	0.438	0.969	1.062	0.594	0.453
3/4	1.312	1.375	0.562	0.516	1.156	1.250	0.688	0.531
7/8	1.531	1.594	0.656	0.609	1.344	1.438	0.781	0.609
1	1.750	1.828	0.750	0.688	1.531	1.625	0.875	0.688
1 1/8	1.969	2.062	0.844	0.781	1.719	1.812	0.969	0.766
1 1/4	2.188	2.281	0.938	0.859	1.906	2.000	1.062	0.844
1 1/8	2.406	2.516	1.031	0.953	2.094	2.188	1.156	0.938
1 1/2	2.625	2.734	1.125	1.031	2.281	2.375	1.250	1.000
1 1/8	2.844	2.969	1.219	1.125	2.469	2.562	1.344	1.094
1 1/4	3.062	3.203	1.312	1.203	2.656	2.750	1.438	1.172
Cone head								
1/2	0.875	0.922	0.438	0.406	0.800	0.844	0.350	0.328
5/8	1.094	1.141	0.547	0.516	1.000	1.047	0.438	0.406
3/4	1.312	1.375	0.656	0.625	1.200	1.266	0.525	0.484
7/8	1.531	1.594	0.766	0.719	1.400	1.469	0.612	0.578
1	1.750	1.828	0.875	0.828	1.600	1.687	0.700	0.656
1 1/8	1.969	2.063	0.984	0.938	1.800	1.891	0.788	0.734
1 1/4	2.188	2.281	1.094	1.031	2.000	2.094	0.875	0.812
1 1/8	2.406	2.516	1.203	1.141	2.200	2.312	0.962	0.906
1 1/2	2.625	2.734	1.312	1.250	2.400	2.516	1.050	0.984
1 1/8	2.844	2.969	1.422	1.344	2.600	2.734	1.138	1.062
1 1/4	3.062	3.203	1.531	1.453	2.800	2.938	1.225	1.141
High button head (acorn)								
1/2	0.875	0.922	0.500	0.438	0.781	0.875	0.500	0.375
5/8	1.094	1.141	0.594	0.547	0.969	1.062	0.594	0.453
3/4	1.312	1.375	0.688	0.656	1.156	1.250	0.688	0.531
7/8	1.531	1.594	0.769	0.719	1.344	1.438	0.781	0.609
1	1.750	1.828	0.875	0.828	1.531	1.625	0.875	0.688
1 1/8	1.969	2.063	0.984	0.938	1.719	1.812	0.969	0.766
1 1/4	2.188	2.281	1.094	1.031	1.906	2.000	1.062	0.844
1 1/8	2.406	2.516	1.203	1.141	2.094	2.188	1.156	0.938
1 1/2	2.625	2.734	1.312	1.250	2.281	2.375	1.250	1.000
1 1/8	2.844	2.969	1.422	1.344	2.469	2.562	1.344	1.094
1 1/4	3.062	3.203	1.531	1.453	2.656	2.750	1.438	1.172
Pan head								
1/2	0.875	0.922	0.438	0.406	0.800	0.844	0.350	0.328
5/8	1.094	1.141	0.547	0.516	1.000	1.047	0.438	0.406
3/4	1.312	1.375	0.656	0.625	1.200	1.266	0.525	0.484
7/8	1.531	1.594	0.766	0.719	1.400	1.469	0.612	0.578
1	1.750	1.828	0.875	0.828	1.600	1.687	0.700	0.656
1 1/8	1.969	2.063	0.984	0.938	1.800	1.891	0.788	0.734
1 1/4	2.188	2.281	1.094	1.031	2.000	2.094	0.875	0.812
1 1/8	2.406	2.516	1.203	1.141	2.200	2.312	0.962	0.906
1 1/2	2.625	2.734	1.312	1.250	2.400	2.516	1.050	0.984
1 1/8	2.844	2.969	1.422	1.344	2.600	2.734	1.138	1.062
1 1/4	3.062	3.203	1.531	1.453	2.800	2.938	1.225	1.141

Extracted from ANSI B18.1.2–1972, R1989. Note see the standard for full ranges.

All dimensions are given in inches.

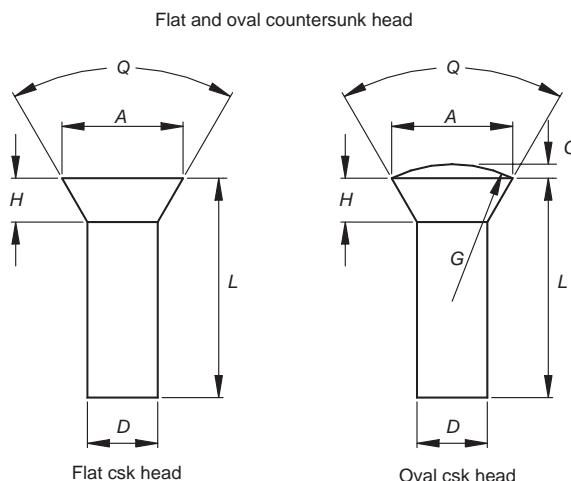
^a Tolerance for diameter of body is plus and minus from nominal and for 1/8-in. size equals +0.020, -0.022; for sizes 1/8 to 1-in, incl., equals +0.030, -0.025; for sizes 1 1/8 and 1 1/4-in equals +0.035, -0.027; for sizes 1 1/8 and 1 1/4-in equals +0.040, -0.030; for sizes 1 1/8 and 1 1/4-in equals +0.040, -0.037.

Note 1: Basic dimensions of head as manufactured.

Note 2: Dimensions of manufactured head after driving and also of driven head.

Note 3: Slight flat permissible within the specified head-height tolerance.

The following formulae give the basic dimensions for manufactured shapes: Button head, $A = 1.750D$; $H = 0.750D$; $G = 0.885D$. High button head, $A = 1.500D + 0.031$; $H = 0.750D + 0.125$; $F = 0.750D + 0.281$; $G = 0.750D - 0.281$. Cone head, $A = 1.750D$; $B = 0.938D$; $H = 0.875D$. Pan head, $A = 1.600D$; $B = 1.000D$; $H = 0.700D$. Length L is measured parallel to the rivet axis, from the extreme end to the bearing surface plane for flat bearing surface head type rivets, or to the intersection of the head top surface with the head diameter for countersunk head type rivets.

Table 12.10 Selected American National Standard large flat and oval countersunk rivets

Body diameter D (in)			Head diam A (in)		Head depth H (in)	Ref.	Oval crown height ^a C (in)	Oval crown radius ^a G (in)
Nominal ^a	Max.	Min.	Max. ^b	Min. ^c				
1/2	0.500	0.520	0.478	0.936	0.872	0.260	0.095	1.125
5/8	0.625	0.655	0.600	1.194	1.112	0.339	0.119	1.406
3/4	0.750	0.780	0.725	1.421	1.322	0.400	0.142	1.688
7/8	0.875	0.905	0.850	1.647	1.532	0.460	0.166	1.969
1	1.000	1.030	0.975	1.873	1.745	0.520	0.190	2.250
1 1/8	1.125	1.160	1.098	2.114	1.973	0.589	0.214	2.531
1 1/4	1.250	1.285	1.223	2.340	2.199	0.650	0.238	2.812
1 1/8	1.375	1.415	1.345	2.567	2.426	0.710	0.261	3.094
1 1/2	1.500	1.540	1.470	2.793	2.652	0.771	0.285	3.375
1 1/8	1.625	1.665	1.588	3.019	2.878	0.831	0.309	3.656
1 1/4	1.750	1.790	1.713	3.262	3.121	0.901	0.332	3.938

Extracted from ANSI B18.1.2-1972, R1989. Note see the standard for full ranges.

All dimensions are given in inches.

^a Basic dimension as manufactured. For tolerances see table footnote on Table 12.9.

^b Sharp edged head.

^c Rounded or flat edged irregularly shaped head (heads are not machined or trimmed).

Example 12.5

Determine the maximum safe tensile load that can be supported by a 1 m section of double riveted butt joint with 15 mm thick main plates and two 8 mm thick cover plates. There are six rivets in each of the outer rows and seven rivets in each of the inner rows. The rivets are all 20 mm in diameter. Assume that the drilled holes are 1.5 mm larger in diameter

than the rivets. The values for the design limits for tensile, shear and bearing stress can be taken as 75, 60 and 131 MPa, respectively.

Solution

In analysing a double riveted joint, it is only necessary to analyse one side due to symmetry.

The safe tensile load based on double shearing of the rivets is equal to the number of rivets

times the number of shearing planes per rivet times the cross-sectional area of the rivet times the allowable shearing stress.

$$L = n \times 2 \times A_r \tau_d$$

$$n = 6 + 7 = 13$$

$$L = 13 \times 2 \times \pi \frac{0.02^2}{4} \times 60 \times 10^6 \\ = 490.1 \text{ kN}$$

The safe tensile load based on bearing stress is given by $L = nA_b\sigma_c$

$$L = 13 \times 0.02 \times 0.015 \times 131 \times 10^6 \\ = 510.9 \text{ kN}$$

The cover plates have a combined thickness greater than that of the main plates, so detailed analysis need not be considered here.

The safe tensile load based on tensile stress, $L = A_p\sigma_t$

$$A_p = 0.015(1 - 6(0.02 + 0.0015)) \\ = 0.01307 \text{ m}^2$$

$$L = 0.01307 \times 75 \times 10^6 = 980.3 \text{ kN}$$

To complete the analysis it is necessary to consider the sum of the load that would cause tearing between rivets in the inner section plus the load carried by the rivets in the outer section. The sum of the loads is used because if the joint is to fail it must fail at both sections simultaneously.

$$L = nA_b\sigma_c = 6 \times 0.02 \times 0.015 \times 131 \times 10^6 \\ = 235.8 \text{ kN}$$

The safe tensile load based on the tensile strength of the main plate between the holes in the inner section is

$$L = A_p\sigma_t \\ = 0.015(1 - 7(0.02 + 0.0015)) \times 75 \times 10^6 \\ = 955.7 \text{ kN}$$

The total safe tensile load based on the sum of the above two values is 1.191 MN.

The safe tensile load for the riveted joints would be the least of the values calculated, i.e. 490.1 kN.

The efficiency of the riveted joint is given by

$$\eta = \frac{490.1 \times 10^3}{0.015 \times 1 \times 75 \times 10^6} = 0.436 = 43.6\%$$

12.5 Adhesives

Adhesive bonding involves the joining of similar or dissimilar materials using natural or synthetic substances, which form a rigid or semi-rigid interface without the use of mechanical fastening. Adhesives can be used to bond metals to non-metals, such as plastics, rubber, glass and other ceramics and in applications where welding would cause adverse effects on material properties. The advantages of adhesives in comparison to discrete fastening mechanisms include more uniform distribution of stresses over the bonded area, stiffer structures, smooth surface finishes, fatigue resistance, low temperature fabrication, connection of dissimilar materials and sealing. The limitations of adhesives include strength, decreasing strength of the joint with increasing temperature, oxidation or other forms of chemical reaction reducing the strength of the adhesive joint over time, necessity to cure and/or clamp components during assembly and bonded structures are not easy to dismantle.

The range of adhesives is extensive including acrylics, cyanoacrylates, epoxies, anaerobics, silicones, phenolics, plastiols, polyester hot melts, polyvinyl acetates and polyurethane. The merits of a variety of adhesives are listed in Table 12.11. Adhesives can be purchased from specialist manufacturers and suppliers, however, it should be noted that no one manufacturer is likely to produce all the different types available. The choice of which adhesive to use for a given application depends on

Table 12.11 Types of adhesives and principal characteristics

Adhesive	Principal characteristics	Typical applications
Anaerobics	Cures in the absence of oxygen	Locking, and sealing of close fitting components such as threaded fasteners
Cyanoacrylates	Cures through reaction with moisture held on surfaces to be bonded. Flows easily between close fitting components. Usually solidify within seconds	Small plastic components, metals, ceramics, rubber and wood
Acrylics	Fast curing. High strength. Tough. Supplied in two parts, resin and catalyst	Metals and plastics
Ceramics	High temperature capability	Metals and ceramics
Epoxies	Consist of an epoxy resin and a hardener. Form strong and durable bonds	Metals, glass, plastics and ceramics
Hot melts	Based on thermoplastics and epoxy and polyurethane thermosetting adhesives	Fast assembly of lightly loaded components
Phenolics	Require heat and pressure to cure bond	Metal to metal, metal to wood and metal to brake-liners
Plastisols	Modified PVC dispersion that require heat to harden. Resilient and tough	
Polyurethanes	Two part and fast curing. Form strong, resilient joints that are resistant to impact	Glass-fibre reinforced plastic. Certain thermoplastics
Polyvinyl acetates (PVA)		Porous materials such as paper or wood and general packaging
Rubber adhesives	Based on solutions or latexes, they solidify through loss of the solvent or water medium	
UV curable adhesives	Can be cured rapidly by exposure to ultraviolet radiation	
Silicones	Flexible adhesive with good high temperature performance (up to approximately 200°C)	Metals, glass, plastics, rubber, wood and ceramics

the type of materials to be joined, the type of joint to be used, the environmental factors for the application, such as temperature and chemicals, the method of production and costs.

Bonded joints can fail by means of tensile, compressive, shear or peel stresses or some combination of these (Figure 12.14). Bonded joints tend to perform poorly under peel or cleavage loading and generally a bonded joint needs to be designed, so that the loading is along the lines of the adhesive's strongest bond. Examples of joint designs and their relative performance are illustrated in Figure 12.15 and practical assemblies in Figure 12.16. Adhesives can be useful for adding stiffeners to a sheet of material or adding strength to an aperture as illustrated in Figures 12.17 and 12.18.

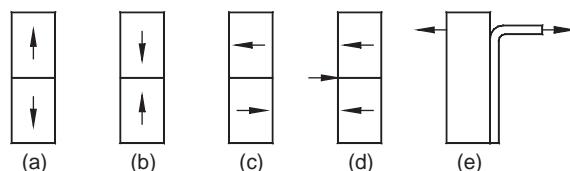


Figure 12.14 Loading of adhesive joints. (a) Tension. (b) Compression. (c) Shear. (d) Cleavage. (e) Peel.

The strength of a lap joint depends on the nature of the materials, the type of adhesive, the thickness of the sheets and the area of the overlap. Test data are available for a variety of combinations of these variables as illustrated in Figure 12.19. The tensile stress for the sheet material is given by

$$\sigma = \frac{P}{tw} \quad (12.40)$$

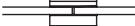
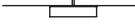
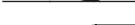
Stepped lap joint		Very good
Double lap joint		Very good
Tapered double strap joint		Excellent
Double strap joint		Very good
Strap joint		Fair
Butt joint		Poor for thin sheets
Scarf joint		Excellent
Tapered lap joint		Very good
Simple lap joint		Good

Figure 12.15 Relative performance of a variety of joint designs.

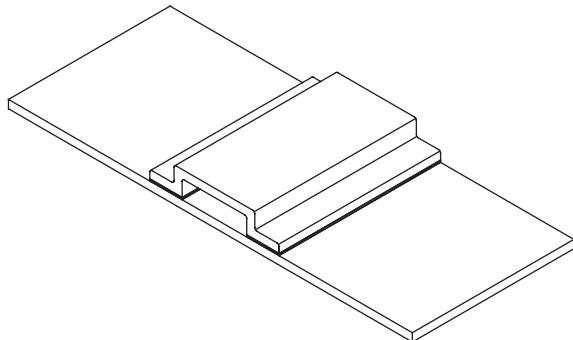


Figure 12.17 Stiffening of sheets.

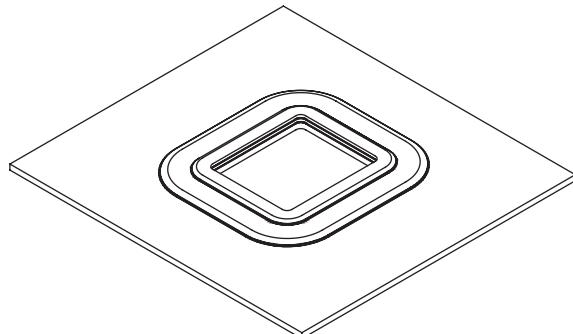
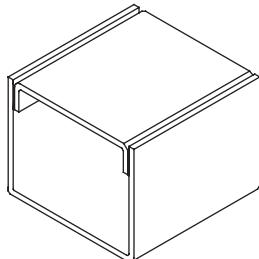


Figure 12.18 Strengthening an aperture.

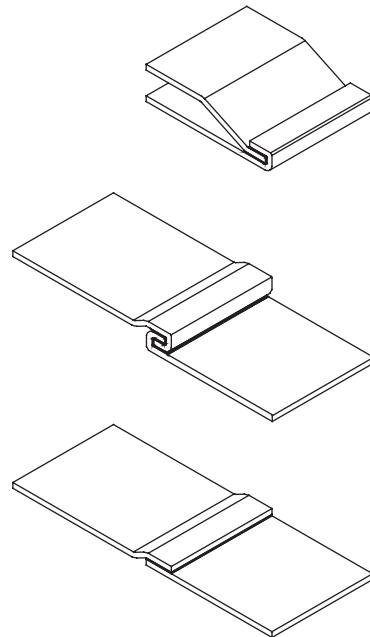


Figure 12.16 Practical joins for sheet materials.

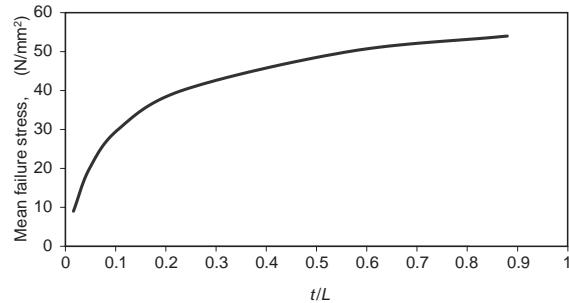


Figure 12.19 Shear strength versus t/L for simple lap joints. Data relate to Araldite bonded BS 1470-HS30 lap joints (after Ciba-Geigy, 1995).

The shear stress in the joint is given by

$$\tau = \frac{P}{Lw} \quad (12.41)$$

Substituting for the load gives

$$\tau = \sigma \frac{t}{L} \quad (12.42)$$

This equation can be used to determine the overlap required for a given sheet thickness, load and joint strength.

A substantial source of data and design hints for the use of adhesives is given in the *Loctite World Design Handbook* (1998).

Example 12.6

Determine the optimum overlap for a simple lap joint for 1/16" sheet metal if the maximum load is 250 N/mm width. Use the araldite for which data are given in Figure 12.19.

Solution

Selecting a safety factor of two to accommodate uncertainty in the loading conditions, the design load becomes 500 N/mm. The tensile stress in the metal is given by

$$\sigma = \frac{P}{t} = \frac{500 \times 10^3}{(1/16) \times 25.4 \times 10^{-3}}$$

$$= 315.0 \text{ MPa}$$

$$\alpha = \frac{\tau}{t/L} = 315 \text{ MPa}$$

$$\tau = 315t/L$$

Plotting this line on Figure 12.19 gives the intercept shear stress of approximately 28 MPa and $t/L = 0.09$.

With $t = 1.59 \text{ mm}$, $L = 17.6 \text{ mm}$.

The optimum overlap is therefore 17.6 mm.

Example 12.7

The araldite for which data are given in Figure 12.19 is to be used for joining two sheets of metal. The joint should support loads of up to 180 N/mm width. If an overlap of 10 mm can be accommodated, determine the optimum sheet thickness.

Solution

Selecting a safety factor of two to accommodate uncertainty in the loading conditions, the design load becomes 360 N/mm.

The failure stress in the adhesive is given by

$$\tau = \frac{F/w}{L} = \frac{360 \times 10^3}{0.01} = 36 \text{ MPa}$$

Examination of Figure 12.19 gives $t/L = 0.16$ for $\tau = 36 \text{ MPa}$. If $L = 0.01 \text{ m}$, then $t = 1.6 \times 10^{-3} \text{ m} = 1.6 \text{ mm}$.

12.6 Welding

Welding can be described as the process of joining material together by raising the temperature of the surfaces to be joined so that they become plastic or molten.

For metals welding involves the metallurgical bonding of components usually by application of heat by an electric arc, gas flame or resistance heating under heavy pressure. There are numerous types of welding including tungsten inert gas welding (TIG), metal inert gas welding (MIG), manual metal arc welding (MMA), submerged arc welding (SAW), resistance welding and gas welding (see Swift and Booker, 1997). A wide number of types of join are possible with welding a few of which are illustrated in Figure 12.20. The design of component joins is considered by Sampath (1997).

Thermoplastics can also be welded. Heat can be applied by means of a hot gas, which is usually inert. Other types of welding for plastics include inertia, ultrasonic and vibration welding (see Oberg *et al.*, 1996).

Soldering and brazing involve the joining of heated metals while the components are in their solid state by means of molten filler metals (solders).

12.7 Snap fasteners

Snap fasteners comprise a feature that deforms during assembly and then positively locates against

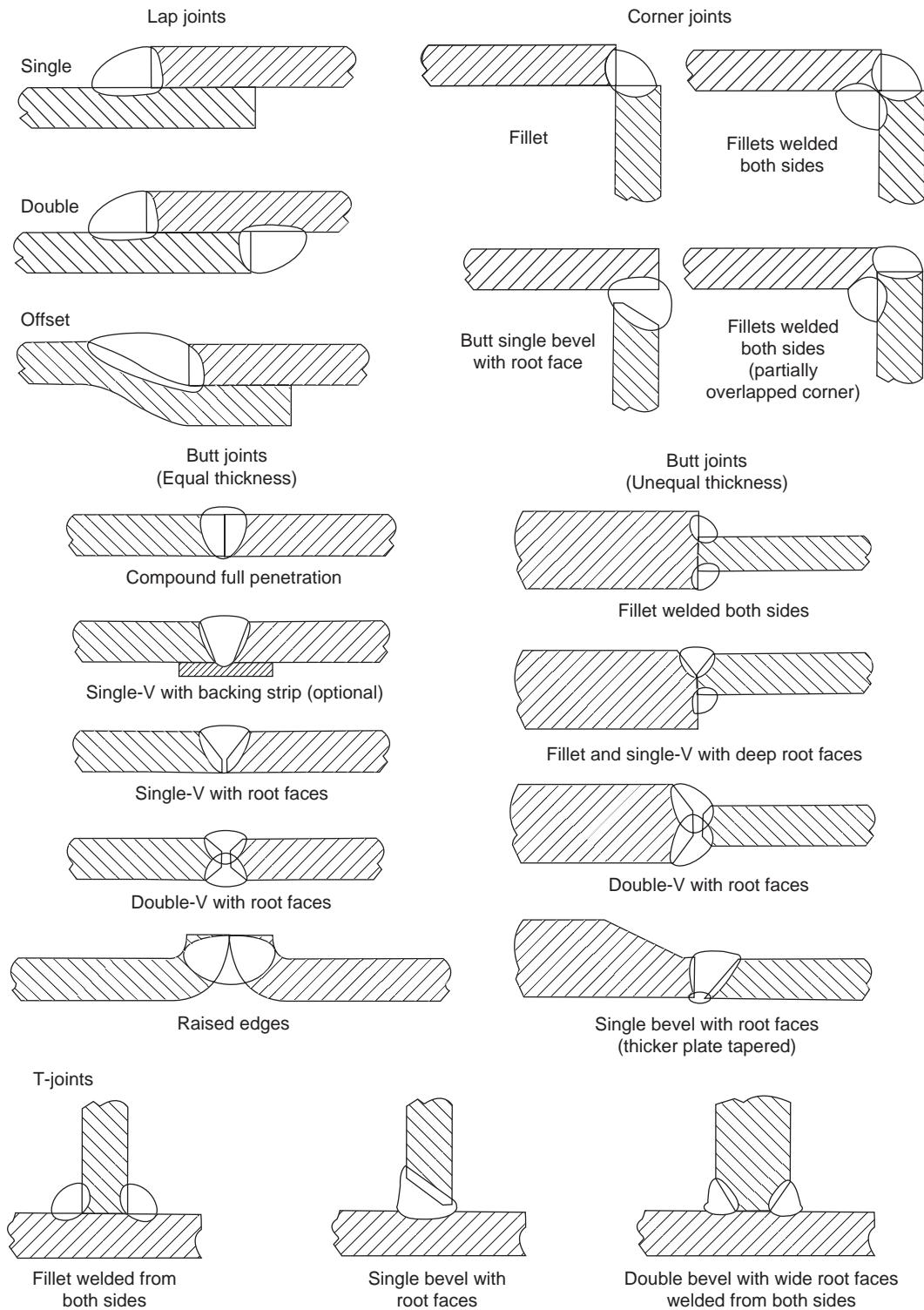


Figure 12.20 Weld joint design (reproduced from Swift and Booker, 1997).

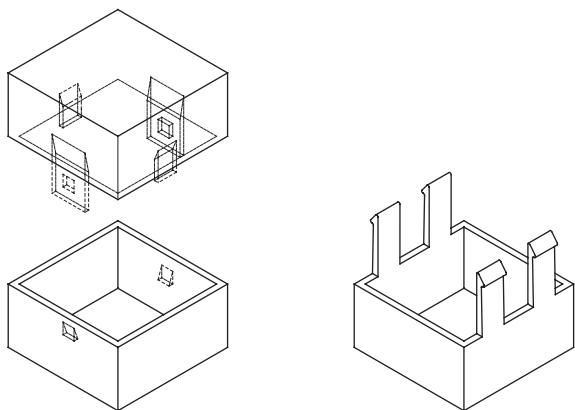


Figure 12.21 Examples of snap fasteners integrated onto mouldings.

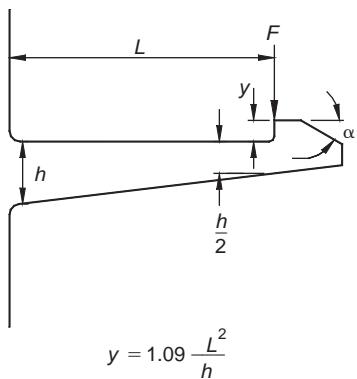


Figure 12.22 Permissible deflection for a rectangular-section cantilever snap joint.

a feature. Various examples are illustrated in Figure 12.21. Snap fasteners are extremely popular because they can be moulded directly into a component reducing part count and the need to purchase separate fasteners and the ease of assembly and associated savings in costs.

The analysis of snap fasteners is complex due to the nonlinear elastic behaviour of the polymers typically used in the mouldings concerned (see Hamrock *et al.*, 1999) for further information. The permissible deflection for a rectangular-section cantilever snap joint is illustrated in Figure 12.22.

12.8 Conclusions

The use of fasteners is critical to the majority of machine design. Their selection and design is

frequently left to too late in the design process. This can result in the use of too many fasteners or inconvenient locations with resultant component and machining costs. Instead the means of fastening components should be considered within the total design activity and the interrelationships with other components identified and appropriate fastening methods selected at an early stage in the design process. The range of fastening techniques available to the design is extensive. Frequently they can be obtained as a stock item with accompanying economies. This chapter has provided an overview of some of the fastening techniques available. It is, however, merely an introduction to an important subject.

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- British Standards Institution. BS 4464. *Metric washers.* BSI
- EN 24032/3/5. *ISO metric hexagon bolts, screws and nuts.*
- EN ISO 2009. *Slotted head bolts.*
- EN ISO 7046-1. *Recessed head set screws.*

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At the time of going to press, the internet contained useful information relating to this chapter at the following sites.

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- www.apexfasteners.com
- www.associatedfastening.com
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- www.boltscience.com
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- www.seac.uk.com
- www.sinofastener.com
- www.skybolt.com
- www.ssfast.co.uk
- www.toleeto.com
- www.vaughanjones.co.uk
- www.watai.com
- www.yerd.com.hk

Nomenclature

Generally, preferred SI units have been stated.

A_r	cross-sectional area of rivet (m^2)
A_b	projected bearing area of rivet (m^2)
A_p	cross-sectional area of plate between rivet holes (m^2)
A_t	tensile stress area of the bolt (m^2)
csk	countersunk
d	diameter (m)
d_c	diameter of the thrust collar (m)
f	frictional force (N)
F	force (N)
F_b	final force in the bolt (N)
F_c	final force on the clamped components (N)
F_e	externally applied load (N)
F_i	preload (N)
h	maximum cantilever thickness (m)
K	constant
k_b	stiffness of the bolt (N/m)
k_c	stiffness of the clamped components (N/m)
L	length (m), safe load (N)
MIG	metal inert gas
MMA	manual metal arc
n	number
N	reaction (N)
P	pitch (m)
r, R	radius (m)
SAW	submerged arc welding
t	thickness (m)
T	torque (N m)
T_c	torque required to turn the thrust collar (N m)
T_d	torque for lowering a load (N m)
T_s	screw torque (N m)
T_u	torque for lifting a load (N m)
TIG	tungsten inert gas
UNC	unified coarse pitch thread series
UNF	unified fine pitch thread series
W	work done (N m)
γ	deflection (m)
α	angle
ε	permissible strain
η	efficiency
λ	angle
μ	coefficient of friction
μ_c	coefficient of friction in the thrust bearing
σ	stress (N/m^2)
σ_p	proof strength of the bolt (N/m^2)
σ_t	tensile stress (N/m^2)

σ_y	yield stress (N/m^2)	τ_d	allowable shear stress (N/m^2)
τ	shear stress (N/m^2)		

Learning objectives achievement

Can you determine the preload and tightening torque for a threaded fastener? ✓ ✗ Section 12.2

Can you determine the lifting and lowering torques for a power screw? ✓ ✗ Section 12.3

Can you determine the safe load for a variety of riveted joints? ✓ ✗ Section 12.4

Can you determine the thickness or overlap for a simple adhesive lap joint? ✓ ✗ Section 12.5

13

FRAMES, CASINGS AND ENCLOSURES

Machine elements have to be supported and this is the function of an enclosure or frame. The aim of this chapter is to introduce the principal considerations in the design of frames, casings and enclosures.

13.1 Introduction

All machine elements operate within an overall structure or framework, such as a casing or enclosure. Examples include gears that are supported on shafts located on bearings, which are housed within a box, and brake calipers, which are attached to a stationary mounting plate. The design principles and procedures presented in books such as this may assist in the design or selection of individual machine elements. These must, however, be located within an overall machine. In addition to cost and aesthetic considerations, functional requirements may include:

- rigidity
- service access
- security and safety
- assembly considerations
- clearance of critical components
- life
- corrosion resistance and environmental considerations
- noise
- size and weight.

The purpose of the total design model is to provide an overall framework within which design can occur. It probably is not possible to produce a

design procedure specific to casings and enclosures that will provide the optimum design for all applications. Each application invariably has its own unique set of constraints and peculiarities precluding the development of a generalized approach. However, there are a number of principles that are useful in the design of frames and casings for supporting machine elements. These include consideration of:

- Forces exerted by the machine elements through mounting points, such as bearings, pivots and brackets;
- Support for the frame itself;
- Allowable deflections of components;
- The environment in which the machine will be installed;
- Production quantities and facilities available for production;
- Design skills and tools available.

Common types of support and enclosures include baseplates, truss frames, shells, C frames, O frames and housings as illustrated in Figures 13.1 to 13.6. Baseplates are typically flat plates with mounting holes or inverted T slots. Ribs can be included on the underside to increase rigidity. Open truss frames are widely used to support machine elements because they offer good strength and rigidity for moderate weight. They can be fabricated from rectangular or round tubes, or rod material and can be pinned, glued or welded. Shell structures typically incorporate stiffeners to prevent the skin from buckling and can sometimes be useful to protect components mounted within the shell. C and O frames provide a simple and effective

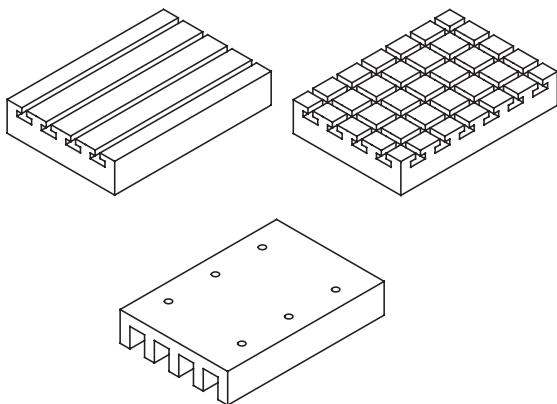


Figure 13.1 Baseplates.

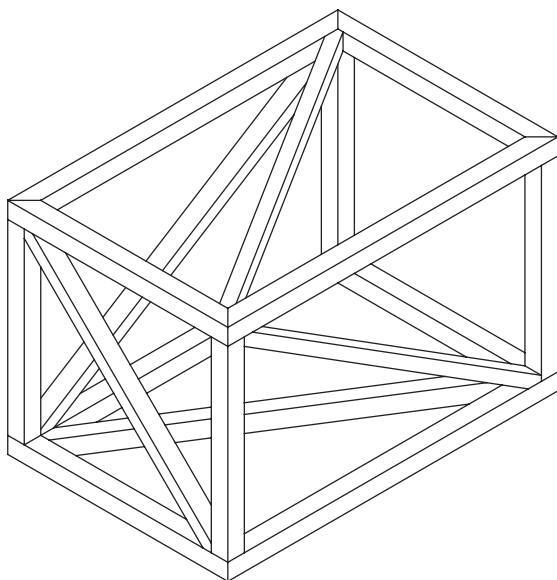


Figure 13.2 Truss frame.

means of support for some types of machinery and are commonly used for machine tools. Housings tend to be the more expensive type of enclosure and can be implemented by casting or mouldings.

13.2 Designing to resist bending

A typical requirement for a machine structure is to limit deflection due to bending to some maximum

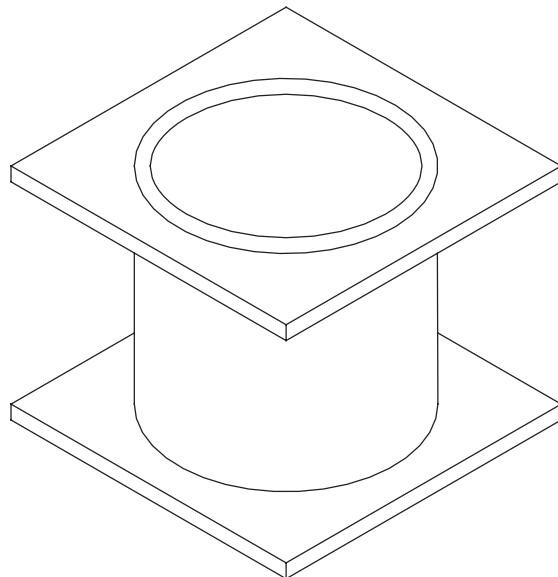


Figure 13.3 Shell.

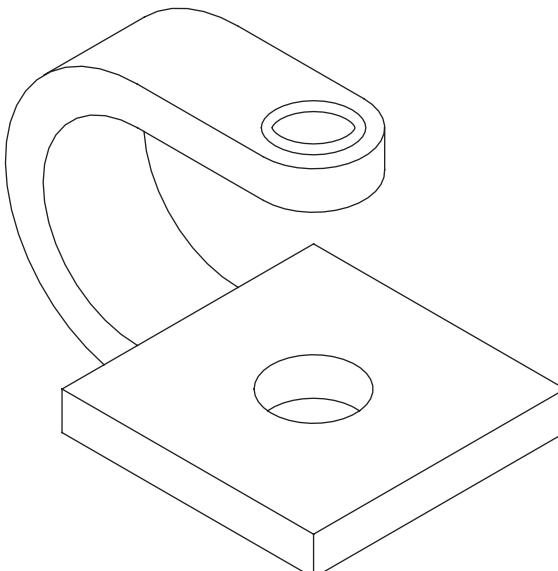


Figure 13.4 C frame.

acceptable value. This will be governed by the specific application under consideration. Common elements in frames and structures are beams such as those illustrated in Figure 13.7. The maximum deflection for a mid-loaded simply supported beam

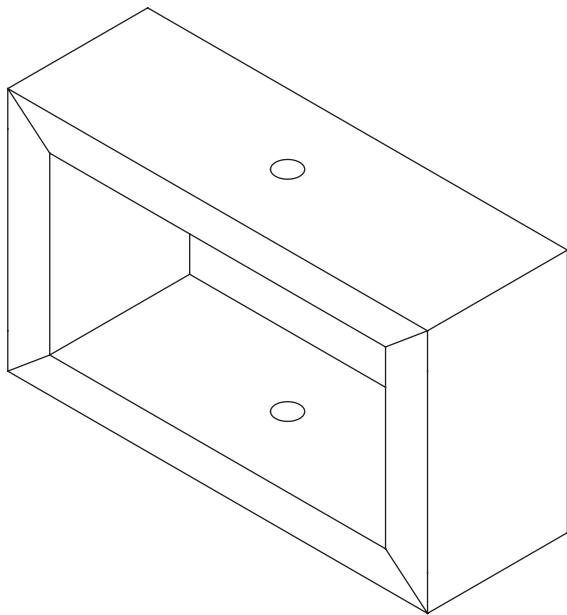


Figure 13.5 O frame.

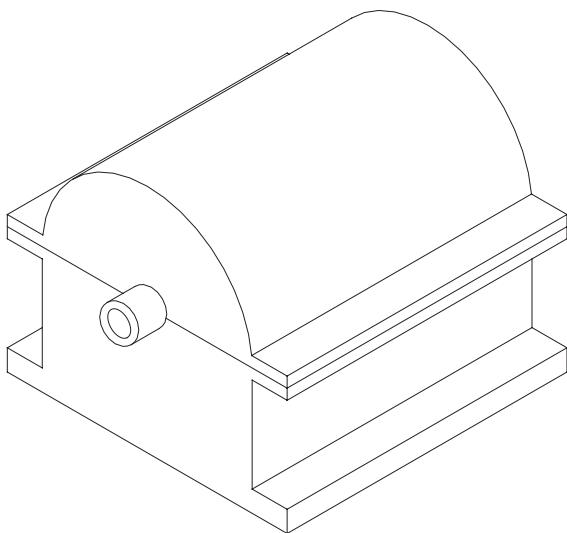


Figure 13.6 Housing.

is given by

$$\delta = \frac{FL^3}{48EI} \quad (13.1)$$

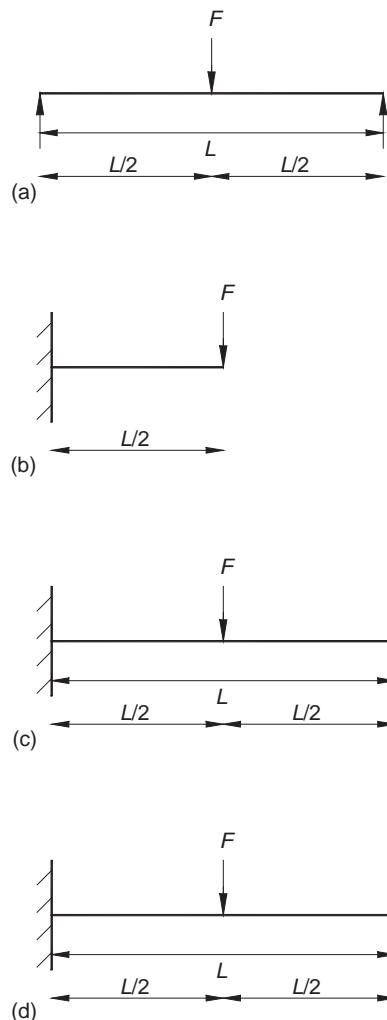


Figure 13.7 Typical beams. (a) Simply supported. (b) Cantilever. (c) Supported cantilever. (d) Fixed-end.

The maximum deflection for an end-loaded cantilever beam is given by

$$\delta = \frac{FL^3}{24EI} \quad (13.2)$$

where F is force (N); L , length (m); E , Young's modulus (N/m²); I , second moment of area (m⁴).

The maximum deflection for a mid-loaded supported cantilever beam is given by

$$\delta = \frac{FL^3}{107EI} \quad (13.3)$$

Table 13.1 Equations for the second moment of area for a variety of cross-sections

Cross-section	Second moment of area, I (m^4)
Circular	$\pi D^4/64$
Hollow cylinder	$\pi(D^4 - d^4)/64$
Square	$b^4/12$
Rectangular	$bd^3/12$
Triangular	$bh^3/36$
Semi-circular	$0.007D^4$
Regular hexagon	$0.06D^4$

The maximum deflection for a mid-loaded fixed end beam is given by

$$\delta = \frac{FL^3}{192EI} \quad (13.4)$$

Consideration of Eqs 13.1 to 13.4 for the deflection of the simply supported beams shows that the deflection is typically given by

$$\delta = \frac{FL^3}{KEI} \quad (13.5)$$

where the value for the constant K depends on the actual form of support. Some general principles can be developed from examination of Eq. 13.1. In order to minimize deflections, loads and lengths should be kept small and the Young's modulus for the beam material and the second moment of area should be maximized. The most critical of these is length as the deflection is a cubic function of the length. As an example for a given load, reducing the length by a factor of two reduces the deflection by a factor of eight. In order to maximize the second moment of area for a beam, as much material needs to be located as far away from the neutral axis of bending as possible. Examples of this include hollow rectangular sections and wide-flange beams. Equations for the second moment of area for a variety of cross-sections are given in Table 13.1.

13.3 Designing to resist torsion

As well as resisting bending, frames and structures also need to resist torsion in some applications.

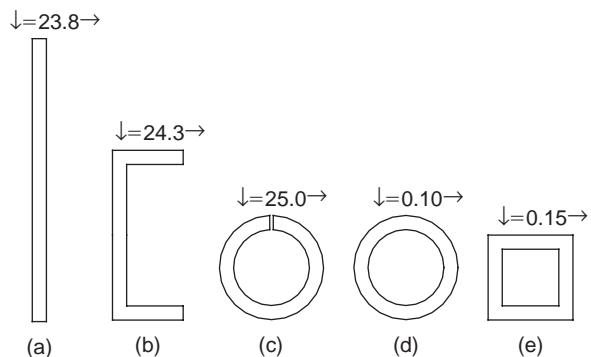


Figure 13.8 Comparison of torsional deformation as a function of shape for a constant thickness and perimeter section. (a) Thin strip. (b) Channel. (c) Open hollow cylinder. (d) Hollow cylinder. (e) Square tube (after Mott, 1999).

In general the torsional deflection of a beam can be modelled by

$$\theta = \frac{TL}{GR} \quad (13.6)$$

where T is applied torque (N m); L , length (m); G , shear modulus of elasticity (N/m^2); R , torsional rigidity constant.

The general principles that can be determined from consideration of Eq. 13.6 include:

- Use closed sections wherever possible, such as solid bars with large cross-sections and hollow tubes.
- Avoid use of open sections made from thin material.
- For frames and bases use diagonal braces at 45° to the frame.
- Use rigid connections where possible by, for instance, welding panels together.

Figures 13.8 and 13.9 illustrate the dramatic effect of shape on torsional deflection. In Figure 13.8 each section represents the use of the same thickness and perimeter of material, yet the torsional deflection of the closed hollow round tube is just 0.1° , in comparison with 24° for the C channel shape.

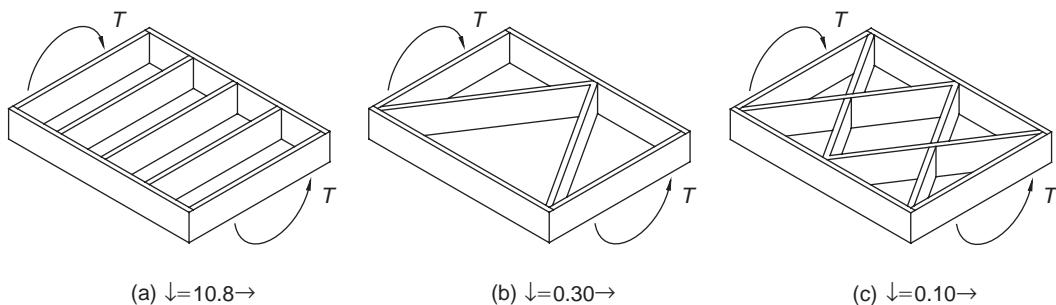


Figure 13.9 Comparison of torsional deformation as a function of shape for a box frame with differing bracing. (a) Cross bracing. (b) Single diagonal bracing. (c) Double diagonal bracing (after Mott, 1999).

13.4 Designing to provide adequate ventilation

Many machines generate heat that must be dissipated in order to ensure that components do not reach temperatures where, for instance, the material strength drops below a critical value or clearances reduce too much such that components touch causing machine failure. Examples of components that generate heat include the self-heating effect of electricity in electric windings, rotating shafts and discs causing frictional windage heating of the surrounding fluid, meshing teeth in gearboxes, belt drives, clutches and brakes and compression of fluids.

In order to evaluate whether a machine will need to be cooled it will be necessary to estimate the range of temperatures that a machine will operate at during its life. This can be a substantial task as it may involve modelling both solid components and the surrounding fluid and even in some cases the distant environment if, for instance, heat transfer by thermal radiation is significant.

Heat transfer occurs due to temperature differences. It is convenient to distinguish heat transfer according to the dominant physical mechanism. If a temperature gradient exists in a stationary solid or fluid then the term used to describe the mode of heat transfer is ‘conduction’. Heat conduction can be determined using Fourier’s law, which for a one-dimensional slab of material under steady state

conditions is given by

$$q = -k \frac{dT}{dx} \quad (13.7)$$

where q is heat flux (W/m^2); k , thermal conductivity ($\text{W}/(\text{m K})$); T , temperature (K); x , location (m).

Heat flux is the heat transfer rate in a given direction per unit area perpendicular to the direction. The negative sign in Eq. 13.7 is used to signify that heat is transferred in the direction of decreasing temperature. The thermal conductivity is a transport property of the material concerned. In the form given in Eq. 13.7, it is assumed that the thermal conductivity is independent of temperature. This is a valid assumption for most materials provided the temperature range is not very large and that a value for the thermal conductivity at an average temperature for the application is used. Data for the thermal conductivity can be obtained in summary form from textbooks on heat transfer such as Incropera and Dewitt (1996), or from data books such as Touhoukian *et al.* (1970a,b) and Kaye and Laby (1986).

If a temperature gradient exists in a fluid then it is possible for heat transfer to occur due to both random molecular motion and bulk macroscopic motion of the fluid and this mode of heat transfer is described as convection. Convection from a surface under steady state conditions can be described by Newton’s law of cooling, Eq. 13.8.

$$q = h(T_s - T_f) \quad (13.8)$$

where q is heat flux normal to the surface (W/m^2); h , the heat transfer coefficient ($\text{W/m}^2 \cdot \text{K}$); T_s , surface temperature (K or $^\circ\text{C}$); T_f , bulk fluid temperature (K or $^\circ\text{C}$).

The value of the heat transfer coefficient depends principally on conditions in the boundary layer which are influenced by the surface geometry, flow condition and thermodynamic and transport properties.

All matter will emit electromagnetic radiation due to its temperature. Heat transfer due to radiation between a small object and its surroundings, can be modelled by

$$q = \varepsilon\sigma(T_s^4 - T_\infty^4) \quad (13.9)$$

where q is the heat flux (W/m^2); ε , total surface emissivity; σ , the Stefan–Boltzmann constant ($5.67 \times 10^{-8}\text{W/(m}^2\text{ K}^4)$); T_s , temperature of the surface (K); T_∞ , temperature of surroundings (K).

The modelling of heat transfer can involve considerably more complex analysis than that indicated by Eqs 13.7–13.9. The temperature distribution may involve gradients in more than one dimension, in which case two- or three-dimensional equations are necessary, or the temperature at a given location may not be constant with time and a transient term will need to be included. The reader is referred to the texts by Bejan (1993), Kreith and Bohn (1997), Incropera and Dewitt (1996), Chapman (1987) and Long (1999) for a thorough treatment of the subject of heat transfer. Nevertheless Eqs 13.7–13.9 can be highly useful in identifying the characteristics of a heat transfer application and, for example, the magnitude of the disturbance caused by the use of a particular feature.

In electrical applications, an estimate of the heat generated in a machine can be made from the self-heating equation

$$\text{Power} = I^2R \quad (13.10)$$

where I is current (A); R , resistance (Ω).

In rotating machinery one of the significant heat generation mechanisms will be windage. Other significant sources of heat include dissipation of

electrical power in windings, compression of gases and combustion. For the components considered in this book, equations for the heat generated by discs and cylinders are useful.

For a rotating disc the moment on one side of a disc is given by

$$M = \frac{1}{2}\rho\omega^2r^5C_m \quad (13.11)$$

where M is moment (N m); ρ , density of fluid (kg/m^3); r , radius (m); C_m , moment coefficient.

The moment coefficient for one side of a rotating disc is given by

$$C_m = 0.491(\log_{10} Re)^{-2.58} \quad (13.12)$$

where Re is the Reynolds number and is defined by

$$Re = \frac{\rho\omega r^2}{\mu} \quad (13.13)$$

where ω is angular velocity (rad/s); μ , viscosity (Pa s).

For a rotating cylinder, the external surface windage drag can be estimated from

$$\frac{1}{\sqrt{C_f}} = 4.07 \log_{10} Re \sqrt{C_f} - 0.6 \quad (13.14)$$

where C_f is a friction coefficient which is defined by

$$C_f = \frac{\tau}{0.5\rho r^2\omega^2} \quad (13.15)$$

The moment coefficient is given by

$$C_m = \frac{2\pi LC_f}{r} \quad (13.16)$$

The temperature rise of a system due to the dissipation of power, \dot{Q} , can be determined using the first law of the thermodynamics, by

$$\dot{Q} = \dot{m}c_p\Delta T \quad (13.17)$$

where \dot{Q} is power (J/s); \dot{m} , mass flow through system (kg/s); c_p , specific heat capacity of fluid (J/(kg K)); ΔT , temperature rise (K).

13.5 Safety

A primary obligation for the designer and manufacturer is to make a product safe. Safety can be defined as the freedom from danger, injury or damage. This will involve estimating or measuring the risk or judging the acceptability of the estimated risk.

Safeguards should:

- prevent operator contact with dangerous parts;
- make safeguard removal or tampering difficult;
- protect against falling objects or rollover;
- create no new hazards;
- allow for safe maintenance.

A designer should typically proceed according to Collins, 2003:

1. Endeavouring to design all hazards out of a product.
2. If it is not possible to design all hazards out of a product then provide devices to eliminate the danger.
3. If it is not possible to provide complete protection, provide appropriate instructions on safe usage and post clear warnings.

Types of guard include fixed, interlocked, adjusting and self-adjusting. Devices and techniques that can be used to implement these include (Kutz, 1986):

- photoelectric detection
- capacitance
- electromechanical
- pullback
- restraints
- safety trips
- pressure sensitive bars
- two-handed control
- gates
- automatic feeds
- automatic ejection.

13.6 Conclusions

Machine elements have to be supported adequately in order to function and this is the purpose of a

frame, casing or enclosure. Typical requirements are that of rigidity, ventilation and safety and the principal considerations in providing for these have been introduced in this chapter. An enclosure for a machine provides not only the support for the machine, but also gives form to a product. Attention to visual attractiveness is not always given the attention it warrants by designers of machine elements. However, we produce our components to be purchased and current understanding indicates that visual perception is dominant amongst the senses. The shape of our products is therefore important and we should design accordingly.

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Nomenclature

Generally, preferred SI units have been stated.

- C_f friction coefficient
 C_m moment coefficient

c_p	specific heat capacity of fluid (J/(kg K))	T	applied torque (N m), temperature (K)
E	Young's modulus (N/m ²)	T_∞	temperature of surroundings (K)
F	force (N)	T_f	bulk fluid temperature (K or °C)
G	shear modulus of elasticity (N/m ²)	T_s	surface temperature (K or °C)
h	heat transfer coefficient (W/m ² K)	x	location (m)
I	second moment of area (m ⁴), current (A)	ε	total surface emissivity
k	thermal conductivity (W/m K)	θ	torsional deflection (rad)
L	length (m)	μ	viscosity (Pa s)
\dot{m}	mass flow through system (kg/s)	ρ	density of fluid (kg/m ³)
M	moment (N m)	σ	Stefan–Boltzmann constant ($5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \text{ K}^4)$)
q	heat flux (W/m ²)	ω	angular velocity (rad/s)
\dot{Q}	power (J/s)	ΔT	temperature rise (K)
r	radius (m)		
R	torsional rigidity constant, resistance (Ω)		

14

SENSORS AND ACTUATORS

Many products rely upon the careful control of a range of parameters for efficient and effective operation. This is typically achieved by means of measurement, processing of the information obtained and subsequent action using some form of actuator to influence the process concerned. The aim of this chapter is to introduce the principles involved in control, and a range of sensors and actuators commonly used in mechanical applications.

LEARNING OBJECTIVES

At the end of this chapter you should be more familiar with:

- the generic control loop;
- the range of sensing technologies available;
- the range of actuators available;
- the scope for sensing and control in machine design.

14.1 Introduction

If a power plant, such as an internal combustion engine is considered, its operation is dependent on the monitoring and control of a wide number of parameters. Machine elements such as bearings and gears are essential in the functioning of an engine, but it is only by measuring and adjusting parameters in the moment-by-moment operation of an engine that competitive or even legal engines, in terms of their emissions, can be produced.

A generic control loop structure is illustrated in Figure 14.1. This comprises the outputs of interest, factors that disturb the system, measuring

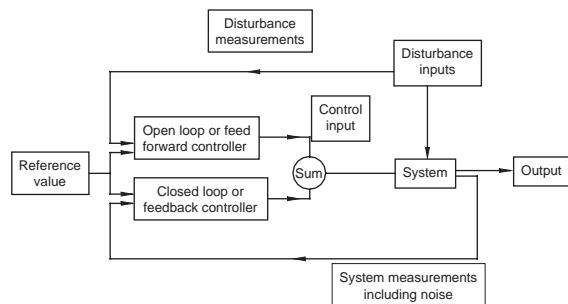


Figure 14.1 Generic control loop structure.

equipment, mechanisms to adjust the operation of the system and the ability to make controlling adjustments to the system. In the case of an internal combustion engine the output of interest may be engine power. The factors that disturb the system include engine load, temperature, humidity and fuel quality. Measurements of the system include the driver's sensations such as touch, feel and hearing and control adjustments are determined by the driver's brain and are achieved by use of the accelerator. A basic engine control system is illustrated in Figure 14.2.

As can be seen from Figures 14.1 and 14.2, control relies on the ability to make measurements, process this information and then affect the process. Measurements can be achieved by means of sensors, which are introduced in Section 14.2. Once a measurement has been taken then the information obtained needs to be processed and an action, if necessary undertaken. This processing can be achieved by means of a mechanism, electronic circuit or computer, which compares a signal with say a reference signal to determine whether a particular

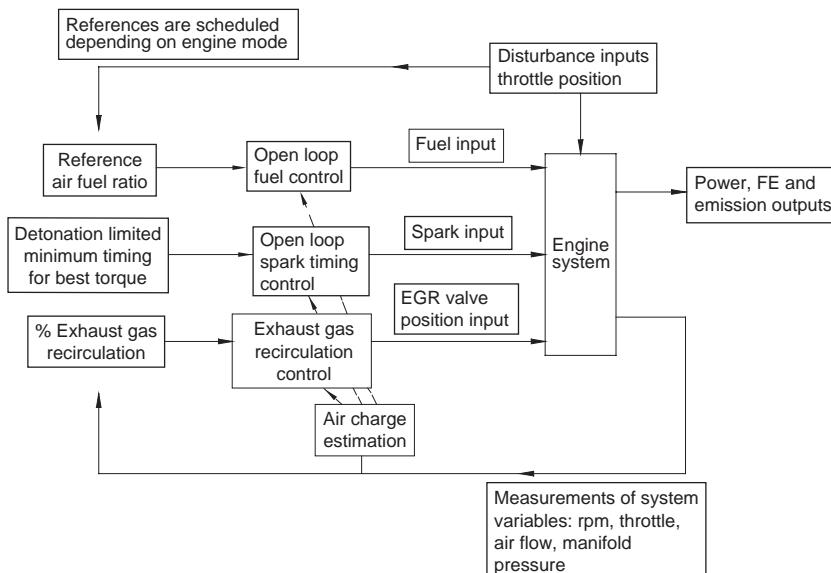


Figure 14.2 Basic engine control (after Forster, 2002).

parameter needs to be increased or decreased. Once a decision has been reached then an action needs to be taken and this can be achieved by means of an actuator. A range of common actuators is introduced in Section 14.3. The aim in this chapter is to introduce the technologies available in order to expose the reader to the scope of devices and mechanisms available. If further information is required then the references provided can be used as a starting point for more in-depth information.

- eddy current
- flow
- force
- gradient
- Hall effect
- chemical composition
- position
- pressure
- speed
- strain
- torque
- vibration.

Here consideration is limited to position, speed, force, torque, temperature, pressure, and fluid flow in Sections 14.2.1 to 14.2.7, respectively.

14.2 Sensors

The terms ‘sensor’ and ‘transducer’ are commonly used in discussions on instrumentation. The moniker sensor is used here to describe the measuring device as a whole, while the term transducer is used to define the part of the sensor that converts changes in a physical parameter to a measurable quantity.

Sensors are available for a wide range of physical parameters such as

- acceleration
- dirt

14.2.1 Position

It is frequently necessary to determine the position of a component. An example is the radial clearance between the tips of rotor blades and the corresponding casing in a turbine. This clearance may vary with operating conditions and is likely to be substantially different to that when the machine is cold and stationary. Another example is to determine

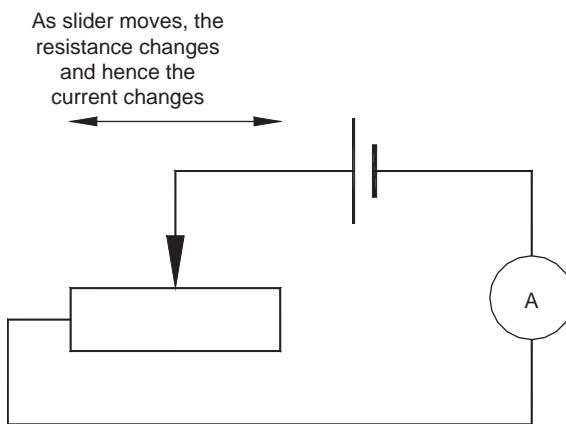


Figure 14.3 Potentiometer.

the true position of an actuator, such as a valve. Position can be measured by a variety of techniques including, in addition to the traditional metrology techniques, interferometry, potentiometers, linear variable differential transformers, capacitance, radar and lasers.

Potentiometers (Figure 14.3) offer a simple method of determining location. A sliding contact on a resistance element is used as the transducer. As the slider moves, the electrical path and hence the resistance changes. Potentiometers can consist of linear or rotating potentiometers.

An example of a practical application of position measurement is the throttle position sensor (TPS) in an automobile (Figure 14.4). This uses a rotary potentiometer with a hard-wearing resistive element compatible with the total number of cycles of use expected during a vehicle's lifetime. The unit is packaged within a plastic housing in order to exclude dirt from modifying the resistance-angular position characteristics.

Linear variable differential transformers (LVDTs) consist of inductive coils wound around a moveable core (Figure 14.5). The movement of the core alters the voltage levels and phases of the voltages across the outer coils and these voltages can be converted in a direct current output by a phase sensitive detector. LVDTs are robust devices and widely available from specialist manufacturers and suppliers.

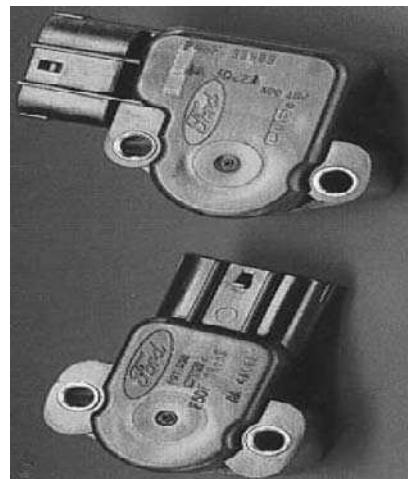


Figure 14.4 Throttle position sensor (courtesy Visteon, 2002).

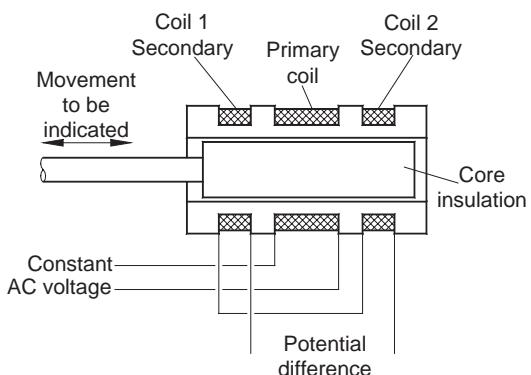


Figure 14.5 Construction of a linear variable differential transformer (LVDT).

14.2.2 Speed

The linear velocity of a component can be determined by monitoring its position with time. This can be exploited in the case of small displacements by resistance and inductive devices. However, if the distances involved are too large for a displacement sensor to be mounted then an optical, acoustic or electromagnetic wave technique may be more suitable.

Angular velocity can be conveniently measured by a number of methods, including optical or electromagnetic techniques. In the case of an optical system, a disc with a number of slits rotating on

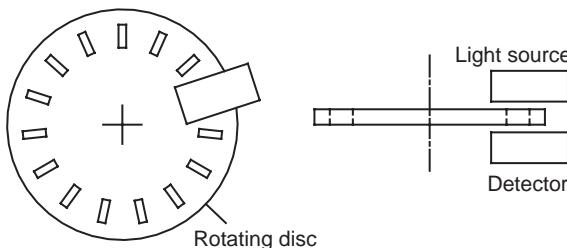


Figure 14.6 Optical system for angular velocity.

the component of interest (Figure 14.6) can be used to produce a light signal that is proportional to the speed of rotation. A light source is located on one side of the disc and an optically sensitive transducer such as a photocell on the other. The frequency of the light received by the photocell will be proportional to the speed of rotation and number of slits in the disc. The signal can be read directly on an oscilloscope, logged or, more appropriately for a control system, converted into an analogue signal by a frequency-to-voltage converter.

A magnetic or capacitive sensor located near a toothed wheel rotating at the same speed as the application can be used to measure angular velocity. In the case of a capacitive system the change in capacitance between adjacent teeth can be used to develop a signal that is proportional to the period it takes for a tooth to pass a sensor. A ferromagnetic toothed wheel and magnetic pickup are illustrated in Figure 14.7. Here the pickup comprises a magnet and coil. The permittivity in the teeth is different to that in the air or other fluid in the gap between the teeth. Alternatively the gap can be filled with another material, such as an epoxy, with a different permittivity. As the toothed wheel rotates an emf is produced in the coil as a result of changes in the magnetic field. The voltage in the coil can be evaluated from Eq. 14.1. The angular velocity can be found from the frequency or amplitude of the output signal.

$$V = CN\omega \sin(N\omega t) \quad (14.1)$$

where V is voltage (V); C , a proportionality constant; N , number of teeth in the rotating toothed wheel; ω , angular velocity (rad/s); t , time (s).

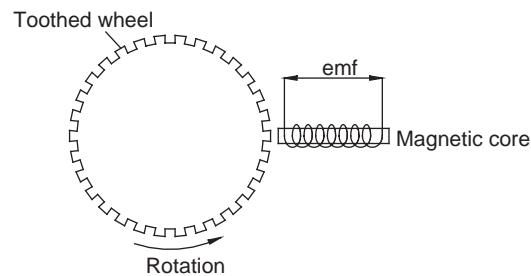


Figure 14.7 Ferromagnetic toothed wheel and magnetic pick-up.

14.2.3 Force

Force is commonly measured by a number of types of load cells, such as strain gauge, piezoelectric and proving ring load cells. Load cells typically consist of an elastic member that deflects with force and a sensor to detect the deflection. Principles exploited for the detection of deflection include capacitance, resistance and piezoelectricity. Strain gauge load cells consist of a column or beam with a strain gauge or number of strain gauges attached that detect the deflection of the beam with force. A proving ring consists of a cylindrical section with a central beam. Strain gauges are attached to the cylindrical surfaces along with a displacement transducer on the central beam. Proving rings are less robust than strain gauge load cells and their use tends to be limited to laboratory applications. Piezoelectric materials develop a charge when deformed. Piezoelectric load cells are characterized by high frequency response up to 15 kHz and sensitivities from 0.05 to 10 mV/N.

14.2.4 Torque

Torque can be measured by locating strain gauges on the rotating shaft and transmitting signals by means of telemetry or slip ring units. Alternatively an optical system can be employed monitoring the angle of twist between two locations of a rotating shaft.

Table 14.1 Examples transducer properties and the physical property measured for some temperature sensors

Sensor	Transducer property	Physical quantity measured
Liquid-in-glass thermometer	Thermal expansion	Length
Constant volume gas thermometer	Thermal expansion	Pressure
Constant pressure gas thermometer	Thermal expansion	Volume
Bimetallic strip	Thermal expansion	Length
Thermocouple	Seebeck effect	Voltage
Platinum resistance thermometer	Electrical resistance	Resistance
Thermistor	Electrical resistance	Resistance
Transistor	Electrical resistance	Voltage
Capacitance thermometers	Electric permittivity	Capacitance
Noise thermometers	Johnson noise	Power
Thermochromic liquid crystals	Reflection	Colour
Thermographic phosphors	Fluorescence	Intensity
Heat sensitive crayons and paints	Chemical	Colour
Pyrometric cones	Chemical	Shape
Infrared thermometer	Thermal radiation	Radiation intensity
Thermal imager	Thermal radiation	Radiation intensity
Absorption spectroscopy	Absorption	Wavelength
Emission spectroscopy	Emission	Wavelength
Rayleigh scattering	Elastic scattering	Wavelength
Raman scattering	Inelastic scattering	Wavelength
CARS (coherent anti-Stokes Raman scattering)	Scattering	Wavelength
Degenerative four-wave mixing	Scattering	Wavelength
Laser-induced fluorescence	Emission	Time
Acoustic thermography	Sound velocity	Time

Reproduced with modifications from Childs, 2002.

14.2.5 Temperature measurement

Temperature cannot be measured directly. Instead its measurement requires the use of a transducer to convert a measurable quantity to temperature. Examples of temperature sensors and the associated transducer property used and the measured quantity are listed in Table 14.1. As can be seen from this table a wide variety of sensors is available and within each type listed a further selection is usually available.

The selection of a specific sensor for a temperature measurement can become a complex activity requiring consideration of a number of aspects including:

- uncertainty
- temperature range
- thermal disturbance
- level of contact
- size of the sensor

- transient response
- sensor protection
- availability
- cost.

The range of application, defined as the span from the minimum temperature of operation to the maximum temperature for a particular measurement technique or for a specific instrument is a critical parameter in selection. Some types of measurement techniques such as thermocouples are suitable for use over a wide temperature range, in this case from approximately -273°C to 3000°C , but this can only be achieved using several different types of thermocouples as no single thermocouple is suitable for the entire range. Figure 14.8 illustrates the approximate temperature range capability for a variety of methods.

Before selecting a temperature measuring system a number of aspects have to be considered.

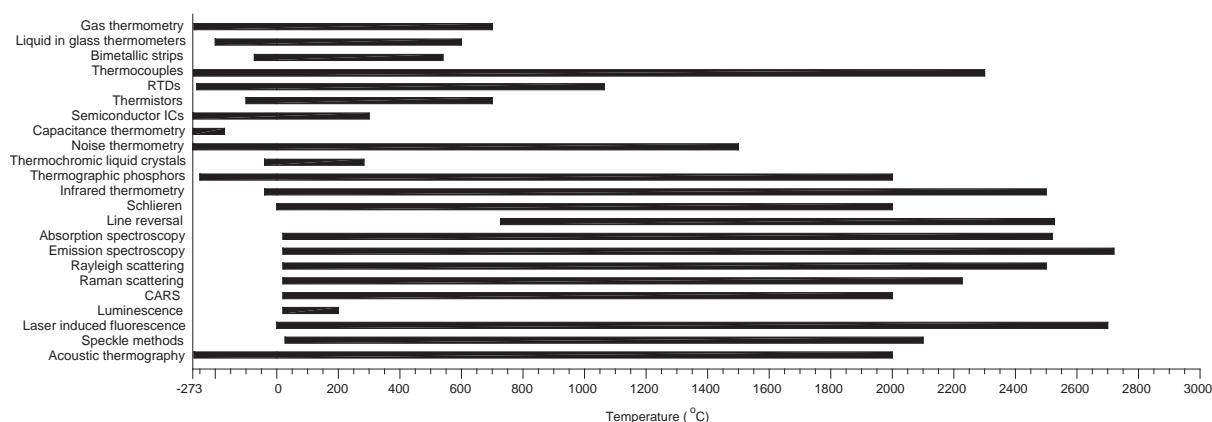


Figure 14.8 Approximate temperature ranges for various temperature measurement techniques.

Some of the issues that might have to be addressed are given below (see ESDU Item 02006).

1. What size and shape of sensor will be required?
If there is no special requirement, most types of invasive sensor are available as cylindrical probes, which usually incorporate a protective cover. These may range from about 0.25 to 15.0 mm in diameter.
2. Will an invasive, or non-invasive method be the most suitable? In most circumstances, an invasive sensor can be used and this may provide the most economic solution. However, with some more difficult measuring situations, non-invasive methods are essential and any disadvantages such as complexity, cost and increased uncertainty have to be accepted.
3. What is the temperature range? Some techniques provide a wide temperature range with a single sensor, some a wide overall temperature range but individual sensors are limited, and with others the temperature range possible with the technique itself may be quite limited.
4. What uncertainty can be accepted? Is the temperature needed with low uncertainty in absolute terms or are temperature differences or changes in temperature really required? With many types of sensor the gradient of the output versus temperature characteristic may remain

more stable with time than the apparent origin. If low uncertainty is required, the sensors will need to be calibrated against known standards before use. If this uncertainty has to be maintained, regular substitution by new calibrated sensors may be needed.

5. Are steady state or transient phenomena to be measured? Some types of invasive sensor are better suited to fast response, e.g. fine thermocouple or resistance wires.
6. Will the sensor need protection? Sensors may be fragile and also susceptible to contamination. Many sensors can be fitted in tubes, sleeves or pockets, but by so doing their uncertainty and response characteristics may be impaired.
7. Will the sensor take up the required temperature?
8. Will the presence of the sensor modify the local temperature significantly? If this is likely to be a problem it may be better to consider a non-invasive method.
9. Can a standard, catalogued system be employed? Although special sensors and systems can be designed and manufactured they are inevitably likely to be more expensive.
10. Is it a multi-channel scheme? If so, thermocouple or resistance temperature detectors are generally more suitable.

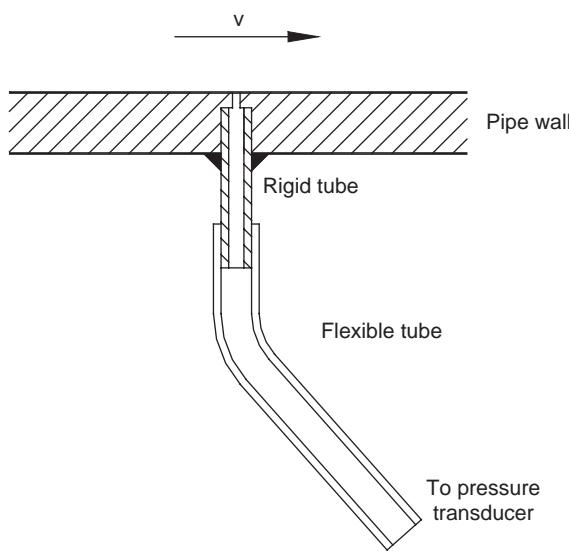


Figure 14.9 Measuring static pressure.

14.2.6 Pressure

Pressure can be measured by a wide range of devices such as McLeod gauges, barometers, manometers which are suitable for laboratory use, Bourdon gauges and bellows which are suitable for some industrial applications and piezoelectric transducers that are useful for both scientific measurements and industrial control and monitoring.

The static pressure of a fluid flowing parallel to a surface can be measured by forming a small hole perpendicular to a solid wall (Figure 14.9). The hole must be perpendicular to the flow and free of burrs and must not be chamfered or have a fillet radius.

Piezoelectric pressure transducers convert an applied pressure into an electrical signal. This signal is proportional to the applied pressure and in many cases nearly linear. The output electrical signal can be used in control, displayed on a panel meter or read by a data logger or voltmeter. Standard output signals available from transducer suppliers are a 4–20 mA two wire current loop or a voltage signal, such as 0–10 V direct current. All transducers require an excitation or supply voltage in order to power the internal circuitry typically

in the range of ± 30 V DC, although any voltage within the published range for a particular transducer is acceptable.

Most pressure transducers are designed for a fixed range of pressure. The range should be equal to or greater than the highest working pressure expected for that particular application. There are four basic pressure references.

- Vacuum pressure transducers are vented to atmosphere and produce a 20 mA output when no vacuum is applied. The output decreases as the vacuum increases.
- Gauge pressure transducers are referenced to atmospheric pressure. With no pressure applied, the output is 4 mA or 0 V DC.
- Compound ranges are vented to atmosphere and measure vacuum or pressure relative to atmospheric pressure.
- Absolute pressure transducers are referenced to a sealed vacuum chamber located within the sensor. These transducers sense changes in barometric pressure as the applied pressure is compared to a known vacuum. Absolute units are used where barometric changes affect the operation of the system being measured.

The maximum pressure defines the safety margin of pressure the transducer can tolerate for short periods of time without significant damage. Burst pressure defines the maximum pressure causing permanent damage or destruction of the transducer.

General purpose and industrial grade transducers are available as packaged units comprising amplification and conditioning circuitry and a plug connector, and are usually fitted with threads, depending on the country and application concerned such as metric threads, British standard pipe fittings, 316 stainless steel NPT (national pipe thread) process connections. These transducers are for use with non-clogging media only. When measuring the pressure of a medium that is highly viscous, crystallizing, or contains particulates, thread connection is not suitable because the connecting orifices may clog. In such applications, a

flush-mounted transducer should be used. Alternatively a chemical diaphragm seal can be used to protect the transducer.

14.2.7 Fluid flow

Fluid flow measurements are necessary in a very wide range of applications from the control of fuel flow in engine management systems to the regulation of drug delivery in ventilators. Fluid flow measurements involve determination of the flow velocity, the mass flow rate or volumetric flow rate.

The one-dimensional continuity equation for mass flow of a fluid is given by

$$\dot{m} = \rho \bar{u} A \quad (14.2)$$

where \dot{m} is the mass flow rate (kg/s); ρ , density (kg/m^3); \bar{u} , average flow speed (m/s); A , cross-sectional area of flow channel (m^2).

Dividing through by density gives the volumetric flow rate

$$\dot{Q} = \bar{u} A \quad (14.3)$$

where \dot{Q} is the volumetric flow rate (m^3/s).

From Eqs 14.2 and 14.3, measurements of the average flow speed can be used to determine the mass flow, if the density and cross-sectional area are known. Alternatively if the mass flow is known the average flow speed can be determined.

Fluid velocity can be measured by a variety of techniques including Pitot tubes, thin film gauges and laser Doppler anemometry. Pitot tubes consist of an L-shaped tube inserted into the flow so that the end stagnates the flow (Figure 14.10). The flow speed is given by

$$u = \sqrt{\frac{2(p_o - p)}{\rho}} \quad (14.4)$$

where u is speed (m/s); p_o , stagnation pressure (N/m^2); p , static pressure (N/m^2); ρ , density (kg/m^3).

A number of devices measure mass flow based on the pressure loss across a constriction or flow feature. These include orifice plates and Venturi

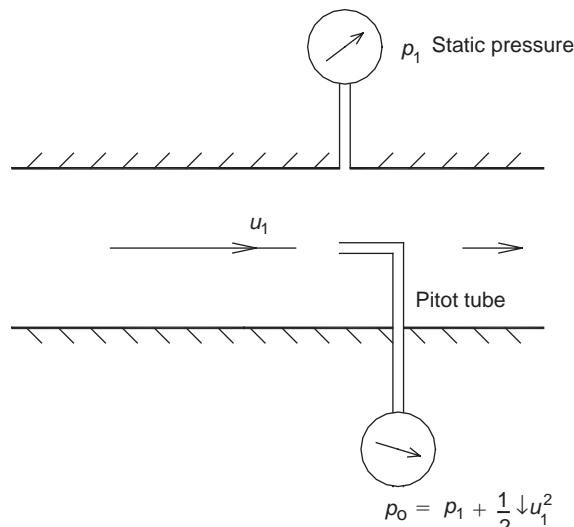


Figure 14.10 Velocity measurement with a Pitot tube.

meters. The installation and use of orifice plates is documented in ISO 5167-1:1991(E) (also BS1042: Section 1.1:1992). The information given here has been sourced from this document and refers to devices with static pressure tappings at a distance of one diameter upstream of the plate and half a diameter downstream only (known as D and $D/2$ tappings). The mass flow rate through an orifice plate installation can be determined from:

$$q_m = \frac{C}{\sqrt{1 - \beta^4}} \varepsilon_1 \frac{\pi}{4} d^2 \sqrt{2\Delta p \rho_1} \quad (14.5)$$

where q_m , is mass flow (kg/s); C , coefficient of discharge; β , diameter ratio (d/D); d , diameter of orifice (m); D , upstream internal pipe diameter (m); ε_1 , expansibility factor; Δp , differential pressure (Pa); ρ_1 , density of the fluid (kg/m^3); q_m , mass flow rate (kg/s).

$$L_1 = D \pm 0.1D$$

$$L_2 = 0.5D \pm 0.02D \text{ for } d/D \leq 0.6$$

$$L_2 = 0.5D \pm 0.01D \text{ for } d/D > 0.6.$$

The density can be evaluated from conditions at the upstream pressure tapping ($\rho_1 = P_1/RT$). The

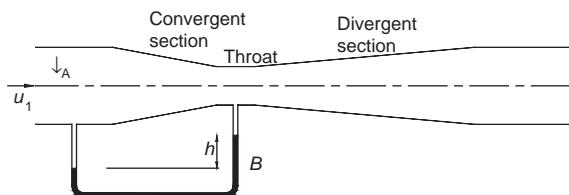


Figure 14.11 Convergent–divergent nozzle (Venturi meter).

temperature should be measured downstream of the orifice plate at a distance of between $5D$ and $15D$. The temperature of fluid upstream and downstream of the orifice plate is assumed constant. The minimum straight lengths of pipe upstream and downstream of the orifice plate are listed in ISO 5167-1:1991(E). For a wide range of conditions for air ϵ_1 and C can be approximated by $\epsilon_1 = 0.99$, $C = 0.61$.

A common device for flow measurement is the convergent–divergent nozzle or Venturi meter (see Figure 14.11). This device involves the contraction of the fluid and subsequent expansion. In comparison to other flow measurement devices such as an orifice plate, the smooth contraction and expansion does not cause a significant pressure loss as fluid streamlines are able to conform to the geometry.

Assuming the flow is incompressible flow (ρ is constant) and that pressure variations due to area change dominate the effects of friction,

$$Q = u_1 A_1 = A_1 \sqrt{\frac{2(p_1 - p_t)}{\rho(A_1/A_t)^2 - 1}} \quad (14.6)$$

Hot film anemometry is used as the basis of mass flow measurement in automotive engine applications. A thin film of nickel is maintained at a constant temperature on a cylinder or alternative mounting located in the fluid stream (Figure 14.12). A constant temperature is achieved by an electronic circuit, which rapidly varies the current supplied, depending on the flow conditions. The convection heat transfer characteristics for a cylinder in cross flow are well understood or alternatively

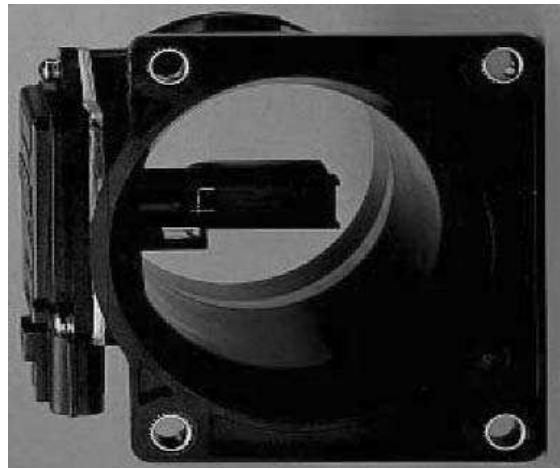


Figure 14.12 Hot film air mass flow sensor.

a calibration can be undertaken for the specific geometry concerned to relate the current flow to the local mass flow. Such sensors can be manufactured fit-for-purpose and with care the total cost minimized, as there is only a single measurement that needs be made.

14.3 Actuators

An actuator is a device that puts a parameter into action or physical motion. In the case of machine elements actuators include

- pneumatics
- hydraulics
- magnets
- electromagnets and solenoids
- electric motors
- piezoelectric devices.

14.3.1 Pneumatics and hydraulics

Fluid power actuators extract energy from a fluid and convert it to a mechanical output to perform a useful task. Fluid actuators can be classified as hydraulic or pneumatic depending on whether a liquid or gas is used as the working fluid. Fluid actuators include devices for producing either

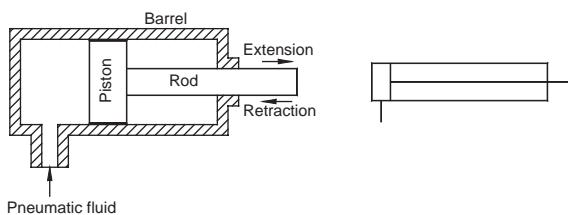


Figure 14.13 Single-acting hydraulic cylinder and corresponding symbolic representation.

linear motion, such as hydraulic cylinders, which extend and retract, or rotary motion, such as hydraulic motors.

The simplest type of linear actuator is the single acting hydraulic or pneumatic cylinder as illustrated in Figure 14.13. This consists of a piston inside a cylindrical housing, called a barrel. Attached to one end of the piston is a rod that extends through a hole and seal in the cylinder. At the other end is a port for the admission and exit of hydraulic fluid, such as oil. Single acting cylinders can only provide actuation in the extending direction. Retraction is achieved by means of gravity or by including a compression spring in the rod end.

The double acting hydraulic cylinder, illustrated in Figure 14.14, can be extended and retracted under hydraulic power. Pneumatic and hydraulic cylinders are available in a wide variety of sizes and end configurations and the rod end is usually threaded so that it can be connected directly to a load, a clevis, yoke or some other mating device.

The force in the extension stroke from a double-acting cylinder is given by

$$F = pA \quad (14.7)$$

where F is force (N); P , pressure (N/m^2); A , piston area (m^2).

The velocity is given by

$$V = \frac{Q}{A} \quad (14.8)$$

where Q is volumetric input flow (m^3/s).

In the retraction stroke the force and speed are moderated by the effects of the rod area:

$$F = p(A - A_{\text{rod}}) \quad (14.9)$$

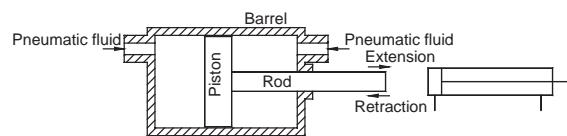


Figure 14.14 Double-acting hydraulic cylinder and corresponding symbolic representation.

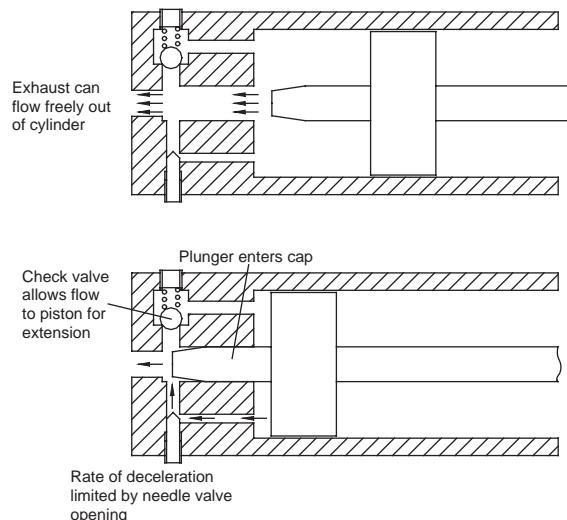


Figure 14.15 Cylinder cushioning.

where A_{rod} is the cross-sectional area of the rod (m^2)

$$V = \frac{Q}{A - A_{\text{rod}}} \quad (14.10)$$

The power delivered by a hydraulic cylinder can be determined from

$$\text{Power} = FV = pQ \quad (14.11)$$

Some double-acting cylinders include cylinder cushions to decelerate the piston near the end of the stroke. These typically comprise channels and associated pressure relief valves as illustrated in Figure 14.15.

The building blocks for a hydraulic or pneumatic circuit are the pump or compressor, valves, the hydraulic actuators, sensors and the control circuit. Valves are used to control the flow direction,

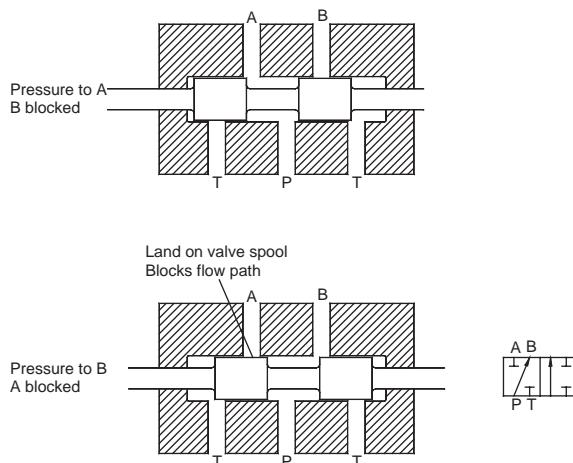


Figure 14.16 Spool positions inside a two-way valve.

pressure and flow rate. Figure 14.16 illustrates a two-way valve along with the corresponding symbolic representation. In two-way valves the flow path can be controlled by the position of a spool. Two-way valves are useful for directing flow to either of two different parts of a flow circuit depending on the spool position. The diagrams in Figure 14.16 illustrate the flow path options. In the first position flow can pass from P to A. In the second position, flow can pass from P to B. A very common method of actuating a spool valve is by using a solenoid.

Pressure control is usually used to limit the pressure in a system to a specified maximum value by diverting the pump flow back to the main tank. The operation of a simple pressure relief valve is illustrated in Figure 14.17.

14.3.2 Electric motors

Electric motors are the common choice as prime movers for many machines because of their ease of operation and convenience, lack of harmful exhaust fumes in comparison to many combustion engines at the location of use and, for machines of under approximately 1 kW, small size in comparison to other prime movers. The two major classifications of electric motors are alternating current

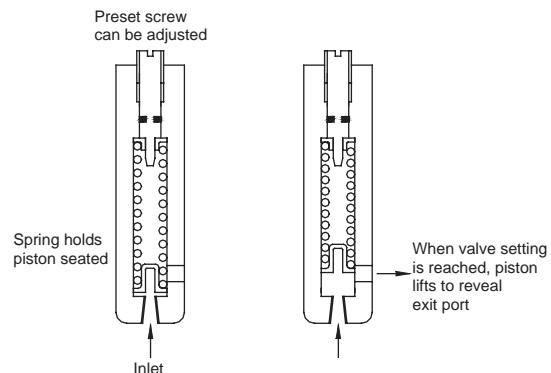


Figure 14.17 Pressure relief valve.

(AC) and direct current (DC), although some machines can operate on either supply.

Important parameters in the selection of an electric motor include

- Motor power supply: DC, single-phase AC, three-phase AC, etc.
- Operating voltage and frequency
- Current limitations
- Power rating
- Rotational speed
- Starting torque
- Load variations expected
- Shaft loading
- Shaft coupling
- Frame size
- Mounting details
- Duty cycle: how often the machine will be started and stopped
- Environmental considerations
- Operating costs
- Initial and service costs

Electric motors are commonly classified by their power, either as a horsepower rating or in W or kW. The conversion equation that can be used is

$$1 \text{ hp} = 746 \text{ W} = 0.746 \text{ kW} \quad (14.12)$$

Alternating current power is produced by electric utilities and distributed in a variety of forms. In

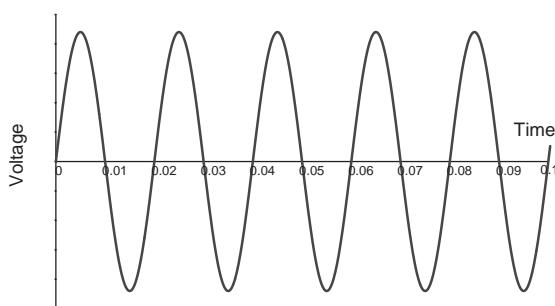


Figure 14.18 Single-phase AC power.

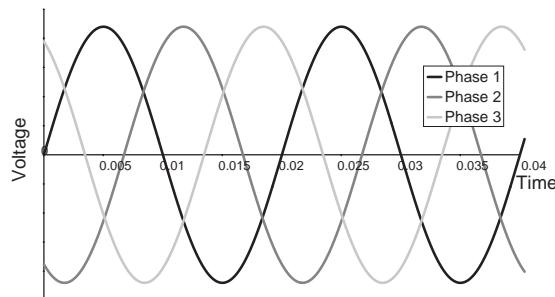


Figure 14.19 Three-phase AC power.

Europe, AC power has a frequency of 50 Hz, while in the USA the power is distributed with a frequency of 60 Hz. AC power is distributed as single-phase or three-phase (Figures 14.18 and 14.19). In the case of single-phase AC the voltage varies as a continuous sine wave at the frequency of the distributed power supply and can be carried by two conductors plus a ground lead. Triple phase or three-phase power comprises three distinct waves of the same frequency and amplitude, but with each offset from the next by 120°. Although extra cables are necessary, smaller motors are possible with three-phase power and it tends to have economic advantages for larger installations.

AC motors have an even number of poles, typically from two to 12. An AC motor under no load would tend to operate at or near to its synchronous speed, which is related to the number of poles and the frequency of the power supply and is

Table 14.2 Induction motor synchronous speeds

Poles	Synchronous speed (50 Hz)	Full-load speed (rpm)	Synchronous speed (60 Hz)	Full-load speed (rpm)
2	3000	2850	3600	3450
4	1500	1425	1800	1725
6	1000	950	1200	1140
8	750	713	900	850
10	600	570	720	690
12	500	475	600	575

given by

$$n_s = \frac{120f}{p} \text{ rpm} \quad (14.13)$$

However, induction motors operate at a speed lower than the synchronous speed with the difference being called slip. An approximate generalization for slip is to assume that an induction motor will operate at 95 per cent of the synchronous speed. Typical operating speeds for induction motors are listed in Table 14.2. Synchronous motors are designed to operate at their synchronous speed without slip.

The functional performance of electric motors can be displayed as a graph of torque against speed (Figure 14.20). The torque at the bottom of the graph is the starting torque and is the torque initially available to accelerate the machine. If the torque from the load is greater than this quantity then the motor will not spin. In such applications a clutch or gearbox may be necessary. Types of AC electric motors include motors designated as design B, C and D by the National Electrical Manufacturers Association (NEMA), round rotor motors, synchronous motors and universal motors that are also capable of operating on a DC supply. Common types of single-phase AC motors include the split-case, capacitor start, permanent split capacitor and shaded pole. Each has a unique torque speed characteristic, as illustrated in Figure 14.21, and the choice depends on the selection criteria for a particular application. For example, capacitor start motors have the highest starting torque,

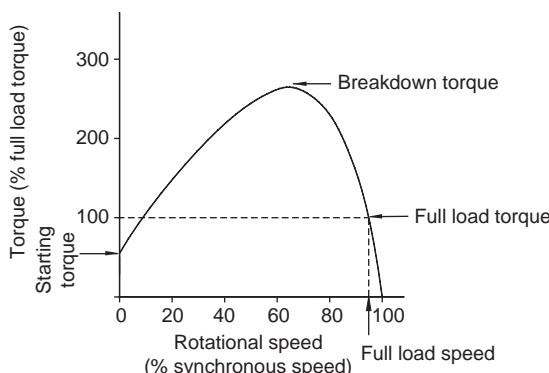


Figure 14.20 Typical torque speed characteristic for a three-phase AC motor.

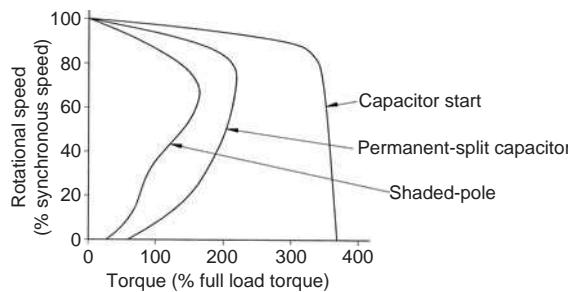


Figure 14.21 Performance characteristics for three types of single-phase AC motor.

but the inclusion of a large enough capacitor results in a bulky overall package in comparison to other motor types.

Motors are available in a variety of packages and frames such as

- Foot-mounted, where the frame has integral feet with a standard hole pattern for bolting the motor to the machine;
- Cushion base, where the motor is provided with a resilient base to reduce vibration and noise;
- C face mounting, where a machined face is provided with a standard pattern of machined tapped holes;
- D flange mounting, where a machined face is provided with a standard pattern of machined clearance holes for attaching the motor to the driven machine;

- Vertical mounting, where the motor is mounted with its axis vertically in order to reduce loads on bearings;
- Unmounted, where the motor is supplied without a casing and the purchaser can build it straight into the application.

A number of options are typically available for electric motor enclosures such as

- Open, where a sheet metal housing is provided around the stator with end plates to support the bearings and shaft.
- Protected, where the housing is designed to prevent dripping water from entering the motor. Ventilation holes are provided on the lower half of the housing.
- Totally enclosed non-ventilated, where there are no openings at all in the housing to protect the motor from harmful atmospheres and particulates. Cooling of the motor is enhanced by the addition of fins cast into the motor housing.
- Totally enclosed, fan-cooled. These are similar to totally enclosed non-ventilated motors, except that a fan is included mounted on one end of the shaft to draw air over the cooling fins.
- Explosion proof. This design is similar to that for totally enclosed fan-cooled motors except that special provision is given to electrical connections to prohibit fire or explosion in hazardous environments.

Stepper motors consist of specially wound armatures, which allow the precise angular positioning of the rotor. Several step angles are commercially available such as 1.8° , 3.6° , 7.5° , 15° , 45° and 90° , although they can be mounted with a gearbox in which case the angles available will be a function of both pole configuration and the gear ratio. Stepper motors need to be powered and controlled by special circuits which deliver the power in pulses. When the pulses stop, the motor will stop and hold its position with a particular holding torque. It is important that the holding torque

is sufficient for the application to ensure that the application is held in an appropriate position.

14.3.3 Electromagnets and solenoids

When a current flows through a wire, an electromagnetic field is set up. If a wire is formed into a coil, the magnetic field becomes many times stronger. Although a magnetic field can be formed in air it flows much more easily through iron or steel. This is the principle in adding an iron path or C stack around the coil, which concentrates the field in order to give high power densities and therefore more powerful and compact devices. If an iron path, known as a T plunger, is also added at the centre of the coil, this will concentrate the magnetic field even further. Because iron provides a more attractive path for the magnetic field than air, a moveable T plunger will be drawn in further to the coil so that the maximum possible magnetic flux can pass through the plunger.

In order to limit stray eddy currents in an electromagnet, the C stack and plunger can be made of laminated sheets of alternate conducting and insulating material. The magnetic field can still flow through its path, but the eddy currents are limited to a two-dimensional plane. Unfortunately for electromagnets, although increasing the current will increase the power of the magnetic field, the heat generated in a solenoid increases approximately with the square of the current. Double the current will increase the heat generated by a factor of four. Each time an electromagnet is cycled, its temperature rises and its force decreases. If cycled too fast it will overheat and burn out.

A solenoid is a specially designed type of electromagnet comprising a coil and plunger connected to an application of interest. They are used very widely in industry with applications, including the actuation of valves and locks.

14.3.4 Piezoelectric actuators

When a voltage is applied to certain materials, such as the polycrystalline ferroelectric ceramics

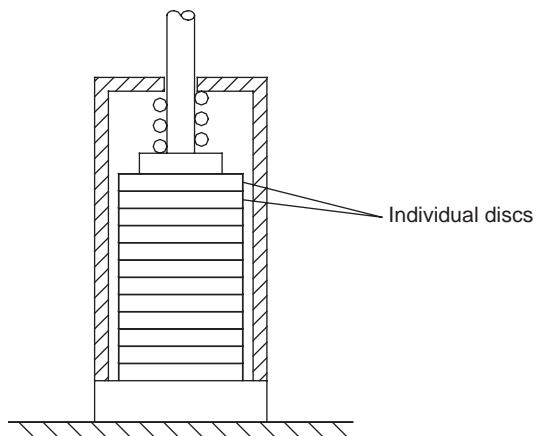


Figure 14.22 Disc stack actuator.

BaTiO_3 and lead zirconate titanate (PZT), the shape of the material will change with growth in the dimensions aligned with the electric field and a contraction along the axes normal to the electric field. This is the principle exploited in piezoelectric actuators, which are devices that produce a small displacement when a voltage is applied. There are many potential applications of piezoelectric actuators, such as precise positioning and the generation and control of high forces or pressures.

The configuration of a piezoelectric actuator depends on the application. Actuators are available in a number of configurations including discs, plates, rings and blocks. Stack actuators are produced by stacking up a number of piezoelectric discs or plates along their central axis. A disc stack actuator is illustrated in Figure 14.22.

The primary parameters in the specification of a piezoelectric actuator are the displacement, force and operating voltage. It is also necessary to consider the stiffness, resonant frequency and capacitance. Although PZT ceramic material can withstand stresses up to 250 MPa before fracture, this value must not be approached as depolarization occurs at pressures of the order of 20 per cent of this mechanical limit. Standard PZT stack actuators can generate forces of several thousand newtons with a range of more than 100 μm , with

time constants of the order of microseconds and low power consumption, making them exceptionally versatile and a device whose potential is not yet fully realized. PZT actuators must be handled with care because the ceramic materials involved are fragile.

Applications have included scientific instruments such as image stabilization, scanning microscopy, autofocussing, fibre-optic alignment, laser tuning, disc drives, needle valve actuation, micro-engraving and dispensing devices.

14.4 Conclusions

In the effective and efficient operation of machinery, it is often necessary to monitor and control a number of parameters. Control often requires monitoring of the key parameters in order to know which need modifying. This chapter has introduced a range of measuring techniques and sensors commonly found in mechanical machines along with a number of actuators. The information given here has been introductory in nature and the reader is recommended to review the references given in the individual sections for further details.

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www.detroitcoil.com
www.geindustrial.com
www.leeson.com
www.pneumatic-source.com
www.reliance.com
www.siemens.com
www.tic.toshiba.com
www.usmotors.com

Nomenclature

Generally, preferred SI units have been stated.

A	area (m^2)
A_{rod}	cross-sectional area of the rod (m^2)
AC	alternating current
C	a proportionality constant, coefficient of discharge
d	diameter of orifice (m)
D	upstream internal pipe diameter (m)
DC	direct current
F	force (N)
l	length (m)
LVDT	linear variable differential transformer
m	mass flow (kg/s)
n_s	synchronous speed (rpm)
N	number of teeth in the rotating toothed wheel
NPT	national pipe thread
p_o	stagnation pressure (N/m^2)
p	pressure (N/m^2)
q_m	mass flow (kg/s)
\dot{Q}	volumetric flow (m^3/s)
R	characteristic gas constant ($\text{J}/\text{kg K}$)
TPS	throttle position sensor
T	time (s)
U	speed (m/s)

\bar{u} average flow speed (m/s)
 V voltage (V), velocity (m/s)
 β diameter ratio
 ε_1 expansibility factor

ρ density (kg/m^3)
 ρ_1 density of the fluid (kg/m^3)
 ω angular velocity (rad/s)
 Δp differential pressure (Pa)

Learning objectives achievement

Can you outline a generic control loop?	<input type="checkbox"/>	✓	x	Section 14.1
Are you aware of the range of measurement techniques available?	<input type="checkbox"/>	✓	x	Section 14.2
Are you aware of the range of technologies available for actuators?	<input type="checkbox"/>	✓	x	Section 14.3

15

ENGINEERING TOLERANCING

The aims of this chapter are to introduce the concepts of component and process variability and the allocation of component tolerances. The ability to select suitable tolerances for the assembly of components should be developed, along with an understanding of geometric tolerancing and the capability to determine the quantitative variability of a process or assembly.

LEARNING OBJECTIVES

At the end of this section you should:

- be able to select a tolerance band for different practicable applications;
- be able to determine an interference fit torque capacity;
- be able to recognize a geometric tolerance symbol;
- be able to determine the sure fit variability of a process;
- be able to determine statistical variability of a process.

15.1 Introduction

A solid is defined by its surface boundaries. Designers typically specify a component's nominal dimensions such that it fulfils its requirements. In reality, components cannot be made repeatedly to nominal dimensions due to surface irregularities and the intrinsic surface roughness. Some variability in dimensions must be allowed to ensure manufacture is possible. However, the variability permitted must not be so great that the performance of the assembled parts is impaired. The allowed

variability on the individual component dimensions is called the 'tolerance'.

The term tolerance applies not only to the acceptable range of component dimensions produced by manufacturing techniques, but also to the output of machines or processes. For example, the power produced by a given type of internal combustion engine varies from one engine to another. In practice the variability is usually found to be modelled by a frequency distribution curve, for example the normal distribution (also called the Gaussian distribution). The modelling and analysis of this kind of variable output is outlined in Section 15.3. One of the tasks of the designer is to specify a dimension on a component and the allowable variability on this value that will give acceptable performance. A general introduction to the specification of component tolerances is given in Section 15.2.

15.2 Component tolerances

Control of dimensions is necessary in order to ensure assembly and interchangeability of components. Tolerances are specified on critical dimensions that affect clearances and interference fits. One method of specifying tolerances is to state the nominal dimension followed by the permissible variation. So a dimension could be stated as 40.000 ± 0.003 mm. This means that the dimension should be machined so that it is between 39.997 and 40.003 mm. Where the variation can vary either side of the nominal dimension, the tolerance is called a bilateral tolerance. For a unilateral tolerance, one tolerance is zero, e.g. $40.000^{+0.006}_{0.000}$ mm.

Table 15.1 Example tolerance bands and typical applications

Class	Description	Characteristic	ISO code	Assembly	Application
Clearance	Loose running fit	For wide commercial tolerances	H11/c11	Noticeable clearance	IC engine exhaust valve in guide
	Free running fit	Good for large temperature variations, high running speeds or heavy journal pressures	H9/d9		Multiple bearing shafts, hydraulic piston in cylinder, removable levers, bearings for rollers
	Close running fit	For running on accurate machines and accurate location at moderate speeds and journal pressures	H8/f7	Clearance	Machine tool main bearings, crankshaft and connecting rod bearings, shaft sleeves, clutch sleeves, guide blocks
					Push on gear wheels and clutches, connecting rod bearings, indicator pistons
	Sliding fit	When parts are not intended to run freely, but must move and turn and locate accurately	H7/g6	Push fit without noticeable clearance	
Transition	Location clearance fit	Provides snug fit for location of stationary parts, but can be freely assembled	H7/h6	Hand pressure with lubrication	Gears, tailstock sleeves, adjusting rings, loose bushes for piston bolts and pipelines
		For accurate location (compromise between clearance and interference fit)			Pulleys, clutches, gears, flywheels, fixed hand-wheels and permanent levers
	Location transition fit	For more accurate location	H7/n6	Needs pressure	Motor shaft armatures, toothed collars on wheels
Interference	Locational interference fit	For parts requiring rigidity and alignment with accuracy of location	H7/p6	Needs pressure	Split journal bearings
	Medium drive fit	For ordinary steel parts or shrink fits on light sections	H7/s6	Needs pressure or temperature difference	Clutch hubs, bearing bushes in blocks, wheels, connecting rods. Bronze collars on grey cast iron hubs

Most organizations have general tolerances that apply to dimensions when an explicit dimension is not specified on a drawing. For machined dimensions a general tolerance may be ± 0.5 mm. So a dimension specified as 15.0 mm may range between 14.5 and 15.5 mm. Other general tolerances can be applied to features such as angles, drilled and punched holes, castings, forgings, weld beads and fillets.

When specifying a tolerance for a component, reference can be made to previous drawings or general engineering practice. Tolerances are typically specified in bands as defined in British or ISO (International Organisation for Standardisation) standards. Table 15.1 gives a guide for the general applications of tolerances. For a given tolerance, e.g.

H7/s6 a set of numerical values is available from a corresponding chart for the size of component under consideration. The section following gives specific examples of this for a shaft or cylindrical spigot fitting into a hole.

15.2.1 Standard fits for holes and shafts

A standard engineering task is to determine tolerances for a cylindrical component, e.g. a shaft, fitting or rotating inside a corresponding cylindrical component or hole. The tightness of fit will depend on the application. For example, a gear located on to a shaft would require a 'tight' interference fit where the diameter of the shaft is actually slightly

greater than the inside diameter of the gear hub in order to be able to transmit the desired torque. Alternatively, the diameter of a journal bearing must be greater than the diameter of the shaft to allow rotation. Given that it is not economically possible to manufacture components to exact dimensions, some variability in sizes of both the shaft and hole dimension must be specified. However, the range of variability should not be so large that the operation of the assembly is impaired. Rather than having an infinite variety of tolerance dimensions that could be specified, national and international standards have been produced defining bands of tolerances, examples of which are listed in Table 15.1, e.g. H11/c11. To turn this information into actual dimensions corresponding tables exist defining the tolerance levels for the size of dimension under consideration (see Tables 15.2 and 15.3). In order to use this information the following list and Figure 15.1 give definitions used in conventional tolerancing. Usually the hole-based system is used (data sheet BS 4500A, Table 15.2) as this results in a reduction in the variety of drill, reamer, broach and gauge tooling required within a company.

- Size: A number expressing in a particular unit the numerical value of a dimension.
- Actual size: The size of a part as obtained by measurement.
- Limits of size: The maximum and minimum sizes permitted for a feature.
- Maximum limit of size: The greater of the two limits of size.
- Minimum limit of size: The smaller of the two limits of size.
- Basic size: The size by reference to which the limits of size are fixed.
- Deviation: The algebraic difference between a size and the corresponding basic size.
- Actual deviation: The algebraic difference between the actual size and the corresponding basic size.
- Upper deviation: The algebraic difference between the maximum limit of size and the corresponding basic size.

- Lower deviation: The algebraic difference between the minimum limit of size and the corresponding basic size.
- Tolerance: The difference between the maximum limit of size and the minimum limit of size.
- Shaft: The term used by convention to designate all external features of a part (including parts that are not cylindrical).
- Hole: The term used by convention to designate all internal features of a part.

BS4500 data sheets 4500A and 4500B list selected ISO clearance, transitional and interference fits for internal and external components. When using the Standard, upper case letters refer to the hole and lower case letters to the shaft.

Example 15.1

Find the shaft and hole dimensions for a loose running fit with a 35 mm diameter basic size.

Solution

From Table 15.1 the ISO symbol is 35H11/c11 (the 35 designating a 35 mm nominal tolerance). From data sheet BS4500A (Table 15.2) the hole tolerance, for a 35 mm hole, is $^{+0.16}_{-0.00}$ mm and the shaft tolerance, for a 35 mm shaft, is $^{-0.12}_{-0.28}$ mm.

So the hole should be given dimensions $35.00^{+0.16}_{-0.00}$ mm and the shaft $35.00^{-0.12}_{-0.28}$ mm.

Example 15.2

Find the shaft and hole dimensions for a medium drive fit using a basic hole size of 60 mm.

Solution

From Table 15.1 the ISO symbol is 60H7/s6. From data sheet BS4500A (Table 15.2) the hole tolerance is $^{+0.030}_{-0.000}$ mm and the shaft tolerance is $^{+0.072}_{-0.053}$ mm.

So the hole should be given dimensions $60.00^{+0.030}_{-0.000}$ mm and the shaft $60.00^{+0.072}_{-0.053}$ mm.

Table 15.2 Selected ISO fits. Holes basis

Clearance fits												Transition fits				Interference fits					
HOLES	H11	H9	H9	H8	H7	H7	H7	k6	H7	n6	H7	p6	H7	s6							
	c11	d10	e9	f7	g6	h6															
Nominal sizes (mm)	Tolerance (0.001 mm)																				
Over	To	H11	c11	H9	d10	H9	e9	H8	f7	H7	g6	H7	h6	H7	k6	H7	n6	H7	p6	H7	s6
—	3	+60 0	—60 —120	+25 0	—20 —60	+25 0	—14 —39	+14 0	—6 —16	+10 0	—2 —8	+10 0	—6 0	+10 0	+6 0	+10 0	+10 +4	+10 0	+12 +6	+10 0	+20 +14
3	6	+75 0	—70 —145	+30 0	—30 —78	+30 0	—20 —50	+18 0	—10 —22	+12 0	—4 —12	+12 0	—8 0	+12 0	+9 +1	+12 +1	+16 0	+12 +12	+20 0	+12 +19	
6	10	+90 0	—80 —170	+36 0	—40 —98	+36 0	—25 —61	+22 0	—13 —28	+15 0	—5 —14	+15 0	—9 0	+15 0	+10 +1	+15 +1	+19 +10	+15 0	+24 +15	+15 0	+32 +23
10	18	+110 0	—95 —205	+43 0	—50 —120	+43 0	—32 —75	+27 0	—16 —34	+18 0	—6 —17	+18 0	—11 0	+18 0	+12 +1	+18 0	+23 +12	+18 0	+29 +18	+18 0	+39 +28
18	30	+130 0	—110 —240	+52 0	—65 —149	+52 0	—40 —92	+33 0	—20 —41	+21 0	—7 —20	+21 0	—13 0	+21 0	+15 +2	+21 0	+28 +15	+21 0	+35 +22	+21 0	+48 +35
30	40	+160 0	—120 —280	+62 0	—80 —180	+62 0	—50 —112	+39 0	—25 —50	+25 0	—9 —25	+25 0	—16 0	+25 0	+18 +2	+25 0	+33 +17	+25 0	+42 +26	+25 0	+59 +43
40	50	+160 0	—130 —290	+62 0	—80 —180	+62 0	—50 —112	+39 0	—25 —50	+25 0	—9 —25	+25 0	—16 0	+25 0	+18 +2	+25 0	+33 +17	+25 0	+42 +26	+25 0	+59 +43
50	65	+190 0	—140 —330	+74 0	—100 —220	+74 0	—60 —134	+46 0	—30 —60	+30 0	—10 —29	+30 0	—19 0	+30 0	+21 +2	+30 0	+39 +20	+30 0	+51 +32	+30 0	+72 +53
65	80	+190 0	—150 —340	+74 0	—100 —220	+74 0	—60 —134	+46 0	—30 —60	+30 0	—10 —29	+30 0	—19 0	+30 0	+21 +2	+30 0	+39 +20	+30 0	+51 +32	+30 0	+78 +59
80	100	+220 0	—170 —390	+87 0	—120 —260	+87 0	—72 —159	+54 0	—36 —71	+35 0	—12 —34	+35 0	—22 0	+35 0	+25 +3	+35 0	+45 +23	+35 0	+59 +37	+35 0	+93 +71
100	120	+220 0	—180 —400	+87 0	—120 —260	+87 0	—72 —159	+54 0	—36 —71	+35 0	—12 —34	+35 0	—22 0	+35 0	+25 +3	+35 0	+45 +23	+35 0	+59 +37	+35 0	+101 +79
120	140	+250 0	—200 —450	+100 0	—145 —305	+100 0	—84 —185	+63 0	—43 —83	+40 0	—14 —39	+40 0	—25 0	+40 0	+28 +3	+40 0	+52 +27	+40 0	+68 +43	+40 0	+117 +92
140	160	+250 0	—210 —460	+100 0	—145 —305	+100 0	—84 —185	+63 0	—43 —83	+40 0	—14 —39	+40 0	—25 0	+40 0	+28 +3	+40 0	+52 +27	+40 0	+68 +43	+40 0	+125 +100
160	180	+250 0	—230 —480	+100 0	—145 —305	+100 0	—84 —185	+63 0	—43 —83	+40 0	—14 —39	+40 0	—25 0	+40 0	+28 +3	+40 0	+52 +27	+40 0	+68 +43	+40 0	+133 +108
180	200	+290 0	—240 —530	+115 0	—170 —355	+115 0	—100 —215	+72 0	—50 —96	+46 0	—15 —44	+46 0	—29 0	+46 0	+33 +4	+46 0	+60 +31	+46 0	+79 +50	+46 0	+151 +122
200	225	+290 0	—260 —550	+115 0	—170 —355	+115 0	—100 —215	+72 0	—50 —96	+46 0	—15 —44	+46 0	—29 0	+46 0	+33 +4	+46 0	+60 +31	+46 0	+79 +50	+46 0	+159 +130
225	250	+290 0	—280 —570	+115 0	—170 —355	+115 0	—100 —215	+72 0	—50 —96	+46 0	—15 —44	+46 0	—29 0	+46 0	+33 +4	+46 0	+60 +31	+46 0	+79 +50	+46 0	+169 +140
250	280	+320 0	—300 —620	+130 0	—190 —400	+130 0	—110 —240	+81 0	—56 —108	+52 0	—17 —49	+52 0	—32 0	+52 0	+36 +4	+52 0	+66 +34	+52 0	+88 +56	+52 0	+190 +158
280	315	+320 0	—330 —650	+130 0	—190 —400	+130 0	—110 —240	+81 0	—56 —108	+52 0	—17 —49	+52 0	—32 0	+52 0	+36 +4	+52 0	+66 +34	+52 0	+88 +56	+52 0	+202 +170
315	355	+360 0	—360 —720	+140 0	—210 —440	+140 0	—125 —265	+89 0	—62 —119	+57 0	—18 —54	+57 0	—36 0	+57 0	+40 +4	+57 0	+73 +37	+57 0	+98 +62	+57 0	+226 +190
355	400	+360 0	—400 —760	+140 0	—210 —440	+140 0	—125 —265	+89 0	—62 —119	+57 0	—18 —54	+57 0	—36 0	+57 0	+40 +4	+57 0	+73 +37	+57 0	+98 +62	+57 0	+244 +208
400	450	+400 0	—440 —840	+155 0	—230 —480	+155 0	—135 —290	+97 0	—68 —131	+63 0	—20 —60	+63 0	—40 0	+63 0	+45 +5	+63 0	+80 +40	+63 0	+108 +68	+63 0	+272 +232
450	500	+400 0	—480 —880	+155 0	—230 —480	+155 0	—135 —290	+97 0	—68 —131	+63 0	—20 —60	+63 0	—40 0	+63 0	+45 +5	+63 0	+80 +40	+63 0	+108 +68	+63 0	+292 +252

After BS4500 data sheet 4500A.

Table 15.3 Selected ISO fits. Shafts basis

Clearance fits												Transition fits					Interference fits															
HOLES		C11		D10		E9		F8		G7		H7		K7																		
	SHAFTS		h11		h9		h9		h7		h6		h6		h6	N7	h6	P7	h6	S7												
Nominal sizes (mm)	Over	To	Tolerance (0.001 mm)	h11	C11	Tolerance (0.001 mm)	h9	D10	Tolerance (0.001 mm)	h9	E9	Tolerance (0.001 mm)	h7	F8	Tolerance (0.001 mm)	h6	G7	Tolerance (0.001 mm)	h6	H7	Tolerance (0.001 mm)	h6	K7	Tolerance (0.001 mm)	h6	N7	Tolerance (0.001 mm)	h6	P7	Tolerance (0.001 mm)	h6	S7
—	3	—60	0 +120 —25 +60	0 —25	+60 +20	0 —25	+39 +14	0 —10	+20 +6	0 —6	+12 +2	0 —6	+10 0	0 —6	0 —10	0 —6	—4 —14	0 —6	—6 —16	0 —6	—6 —24											
3	6	—75	0 +145 +70 —30	0 +78 +30 —30	+50 +20	0 —12	+28 +10	0 —8	+16 +4	0 —8	+12 0	0 —8	+3 —9	0 —8	—4 —16	0 —8	—8 —20	0 —8	—8 —27													
6	10	—90	0 +170 +80 —36	0 +98 +40 —36	+61 +25	0 —15	+35 +13	0 —9	+20 +5	0 —9	+15 0	0 —9	+5 —10	0 —9	—4 —19	0 —9	—9 —24	0 —9	—9 —32													
10	18	—110	0 +205 +95 —43	0 +120 +50 —43	+75 +32	0 —18	+43 +16	0 —11	+24 +6	0 —11	+18 0	0 —11	+6 —12	0 —11	—5 —23	0 —11	—11 —29	0 —11	—21 —39													
18	30	—130	0 +240 +110 —52	0 +149 +65 —52	+92 +40	0 —21	+53 +20	0 —13	+28 +7	0 —13	+21 0	0 —13	+6 —15	0 —13	—28 —13	0 —13	—14 —35	0 —13	—27 —48													
30	40	—160	0 +280 +120 —62	0 +180 +80 —62	+112 +50	0 —25	+64 +25	0 —16	+34 +9	0 —16	+25 0	0 —16	+7 —18	0 —16	—8 —33	0 —16	—17 —42	0 —16	—34 —59													
40	50	—160	0 +290 +130 —62	0 +80 +80 —62	+112 +50	0 —25	+64 +25	0 —16	+34 +9	0 —16	+25 0	0 —16	+7 —18	0 —16	—8 —33	0 —16	—17 —42	0 —16	—34 —59													
50	65	—190	0 +330 +140 —74	0 +220 +100 —74	+134 +60	0 —30	+76 +30	0 —19	+40 +10	0 —19	+30 0	0 —19	+9 —21	0 —19	—9 —39	0 —19	—21 —51	0 —19	—42 —72													
65	80	—190	0 +340 +150 —74	0 +220 +100 —74	+134 +60	0 —30	+76 +30	0 —19	+40 +10	0 —19	+30 0	0 —19	+9 —21	0 —19	—9 —39	0 —19	—21 —51	0 —19	—42 —78													
80	100	—220	0 +390 +170 —87	0 +260 +120 —87	+159 +72	0 —35	+90 +36	0 —22	+47 +12	0 —22	+35 0	0 —22	+10 —25	0 —22	—10 —45	0 —22	—24 —59	0 —22	—58 —93													
100	120	—220	0 +400 +180 —87	0 +260 +120 —87	+159 +72	0 —35	+90 +36	0 —22	+47 +12	0 —22	+35 0	0 —22	+10 —25	0 —22	—10 —45	0 —22	—24 —59	0 —22	—66 —101													
120	140	—250	0 +450 +200 —100	0 +305 +145 —100	+185 +85	0 —40	+106 +43	0 —25	+54 +14	0 —25	+40 0	0 —25	+12 —28	0 —25	—12 —52	0 —25	—28 —68	0 —25	—77 —117													
140	160	—250	0 +460 +210 —100	0 +305 +145 —100	+185 +85	0 —40	+106 +43	0 —25	+54 +14	0 —25	+40 0	0 —25	+12 —28	0 —25	—12 —52	0 —25	—85 —123	0 —25	—123 —133													
160	180	—250	0 +480 +230 —100	0 +305 +145 —100	+185 +85	0 —40	+106 +43	0 —25	+54 +14	0 —25	+40 0	0 —25	+12 —28	0 —25	—12 —52	0 —25	—85 —133	0 —25	—93 —133													
180	200	—290	0 +530 +240 —115	0 +355 +170 —115	+215 +100	0 —46	+122 +50	0 —29	+61 +15	0 —29	+46 0	0 —29	+13 —33	0 —29	—14 —60	0 —29	—33 —79	0 —29	—105 —151													
200	225	—290	0 +550 +260 —115	0 +355 +170 —115	+215 +100	0 —46	+122 +50	0 —29	+61 +15	0 —29	+46 0	0 —29	+13 —33	0 —29	—14 —60	0 —29	—33 —79	0 —29	—113 —159													
225	250	—290	0 +570 +280 —115	0 +355 +170 —115	+215 +100	0 —46	+122 +50	0 —29	+61 +15	0 —29	+46 0	0 —29	+13 —33	0 —29	—14 —60	0 —29	—33 —79	0 —29	—123 —169													
250	280	—320	0 +620 +300 —130	0 +400 +190 —130	+240 +110	0 —52	+137 +56	0 —32	+62 +17	0 —32	+52 0	0 —32	+16 —36	0 —32	—14 —66	0 —32	—36 —88	0 —32	—138 —190													
280	315	—320	0 +650 +330 —130	0 +400 +190 —130	+240 +110	0 —52	+137 +56	0 —32	+62 +17	0 —32	+52 0	0 —32	+16 —36	0 —32	—14 —66	0 —32	—36 —88	0 —32	—150 —202													
315	355	—360	0 +720 +360 —140	0 +440 +210 —140	+265 +125	0 —57	+151 +62	0 —36	+75 +18	0 —36	+57 0	0 —36	+17 —40	0 —36	—16 —73	0 —36	—41 —98	0 —36	—169 —226													
355	400	—360	0 +760 +400 —140	0 +440 +210 —140	+265 +125	0 —57	+151 +62	0 —36	+75 +18	0 —36	+57 0	0 —36	+17 —40	0 —36	—16 —73	0 —36	—41 —98	0 —36	—187 —244													
400	450	—400	0 +840 +440 —155	0 +480 +230 —155	+290 +135	0 —63	+165 +68	0 —40	+83 +20	0 —40	+63 0	0 —40	+18 —45	0 —40	—17 —80	0 —40	—45 —108	0 —40	—209 —272													
450	500	—400	0 +880 +480 —155	0 +480 +230 —155	+290 +135	0 —63	+165 +68	0 —40	+83 +20	0 —40	+63 0	0 —40	+18 —45	0 —40	—17 —80	0 —40	—45 —108	0 —40	—229 —292													

After BS4500 data sheet 4500B.

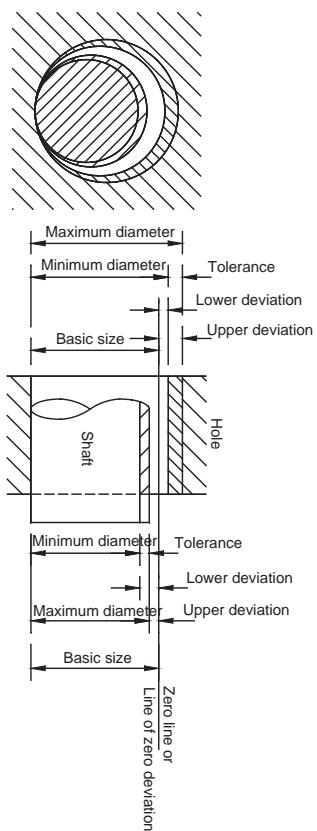


Figure 15.1 Definitions of terms used in conventional tolerancing (after BS4500).

15.2.2 Interference fits

Interference fits are those for which, prior to assembly, the inside component is larger than the outside component. There is some deformation of the components after assembly and a pressure exists at the mating surfaces. This pressure is given Eq. 15.1 if both components are of the same material, or by Eq. 15.2 if the two components are of differing materials (see Figure 15.2 for definition of radii).

$$p = \frac{E\delta}{2b} \frac{(c^2 - b^2)(b^2 - a^2)}{2b^2(c^2 - a^2)} \quad (15.1)$$

$$\delta = 2b \left(\frac{1}{E_o} \frac{c^2 + b^2}{c^2 - b^2} + \mu_o + \frac{1}{E_i} \frac{b^2 + a^2}{b^2 - a^2} - \mu_i \right) \quad (15.2)$$

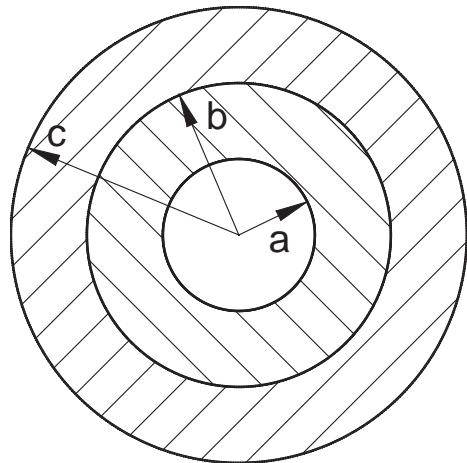


Figure 15.2 Terminology for cylindrical interference fit.

where p is the pressure at the mating surface (N/m^2); δ , total diametral interference (m); a , is the internal radius of inner cylinder (m); b , outer radius of inner cylinder (m); c , outer radius of outer cylinder (m); E , Young's modulus (N/m^2); μ , Poisson's ratio.

The subscripts i and o refer to the inner and outer components, respectively.

The tensile stress in the outer component, σ_o , can be calculated from Eq. 15.3 and the compressive stress in the inner component, σ_i , from Eq. 15.4.

$$\sigma_o = p \frac{\downarrow c^2 + b^2}{\downarrow c^2 - b^2} \quad (15.3)$$

$$\sigma_i = -p \frac{\downarrow b^2 + a^2}{\downarrow b^2 - a^2} \quad (15.4)$$

The increase in diameter of the outer component, δ_o , due to the tensile stress can be calculated from Eq. 15.5 and the decrease in diameter of the inner component due to the compressive stress, δ_i , from Eq. 15.6.

$$\delta_o = \frac{2bp}{E_o} \frac{\downarrow c^2 + b^2}{\downarrow c^2 - b^2} + \mu_o \quad (15.5)$$

$$\delta_i = -\frac{2bp}{E_i} \frac{\downarrow b^2 + a^2}{b^2 - a^2} - \mu_i \quad (15.6)$$

The torque which can be transmitted between cylindrical components by an interference fit can be estimated by

$$T = 2fp\pi b^2 L \quad (15.7)$$

where T is the torque (N m); f , coefficient of friction; p , interference pressure (N/m^2); b , interface radius (m); L , length of interference fit (m).

Example 15.3

What interference fit is required in order to transmit 18 kN m torque between a 150 mm diameter steel shaft and a 300 mm outside diameter cast iron hub which is 250 mm long. (Take Young's modulus for the steel and cast iron to be 200 and 100 GPa, respectively, assume the coefficient of friction is 0.12 and that Poisson's ratio is 0.3 for both materials.)

Solution

The torque is given by $T = 2fp\pi b^2 L$ and this equation can be rearranged to evaluate the pressure required to transmit the torque:

$$p = \frac{T}{2f\pi b^2 L} = \frac{18 \times 10^3}{2 \times 0.12\pi(0.075)^2 \times 0.25} = 16.97 \times 10^6 \text{ N/m}^2.$$

The interference required to generate this pressure is given by:

$$\begin{aligned} \delta &= 2bp \frac{1}{E_o} \frac{\downarrow c^2 + b^2}{b^2 - a^2} + \mu_o + \frac{1}{E_i} 1 - \mu_i \\ &= 2 \times 0.075 \times 16.97 \times 10^6 \\ &\times \frac{1}{100 \times 10^9} \frac{\downarrow 0.15^2 + 0.075^2}{0.15^2 - 0.075^2} + 0.3 \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{200 \times 10^9} (1 - 0.3) \\ &= 5.89 \times 10^{-5} \text{ m} \end{aligned}$$

So the required interference fit is 0.059 mm. This could be specified using the H7/s6 tolerance band with the hole diameter dimension as $150.000^{+0.04}_{-0}$ mm and the shaft diameter as $150.000^{+0.125}_{-0.100}$ mm.

15.2.3 Machine capability

A given manufacturing technique can only produce a component within a range of accuracy around the specified nominal dimension. For instance, centre lathes are capable of machining 10 mm diameter components to tolerances of 0.01 mm as illustrated in Figure 15.3. In addition different manufacturing techniques produce differing levels of surface roughness as shown in Table 15.4. Given that drilling cannot be normally used to give a surface roughness of better than 0.8 μm , tolerances for drilled components should not be specified smaller than 0.0008 mm. When defining a tolerance, the designer should have in mind the 'total design' of the product. This encompasses efficient operation of the product, the ability of the company to manufacture the component and total costs (design, manufacture, servicing and performance, etc.). Different manufacturing techniques cost different amounts as illustrated in Figure 15.4 and this should be considered.

15.2.4 Geometric tolerancing

The location for a drilled hole has been specified in Figure 15.5 by means of a dimension with bilateral tolerances. The dimensions given imply a square rectangular tolerance zone as identified in Figure 15.6(a), anywhere within which it would be permissible to set the hole centre. Examination of the tolerance zone shows that the maximum deviation in location of the hole centre actually occurs along the diagonals as shown in Figure 15.6(b).

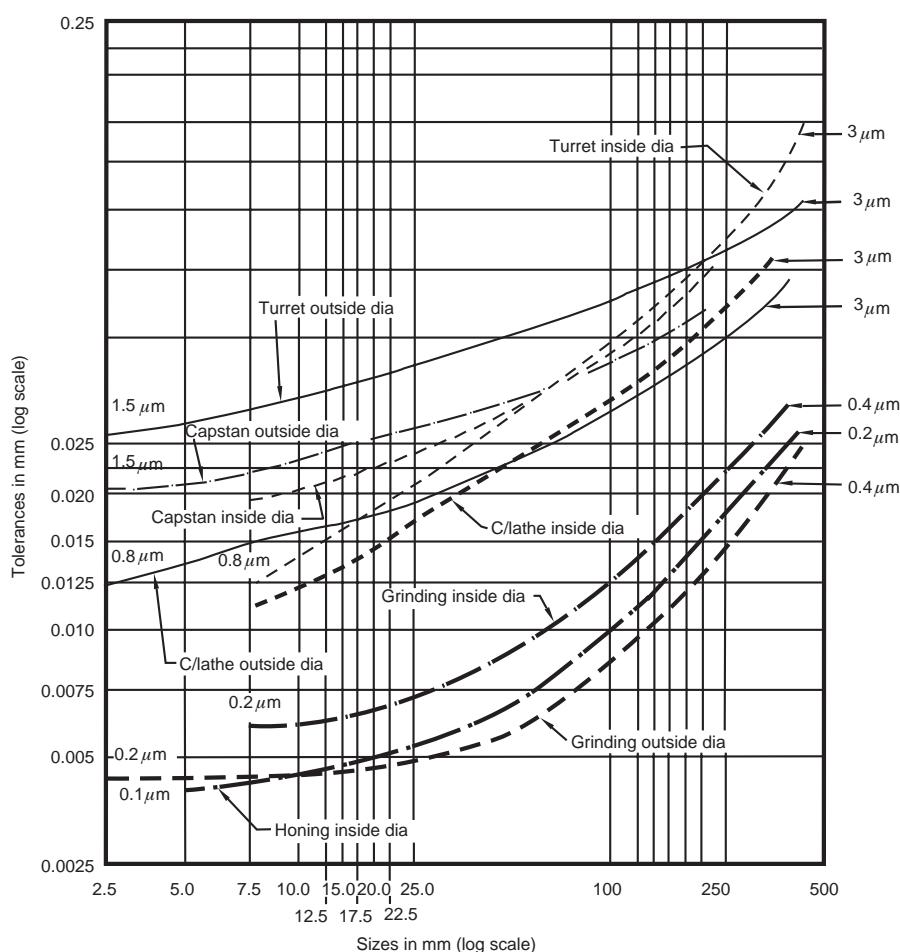


Figure 15.3 Accuracy to be expected from machining operators using various machines (BSI, 1991).

Thus although the drawing has superficially specified the maximum tolerance deviation as ± 0.1 mm or 0.2 mm, the maximum actual deviation within the specification of the drawing is 0.28 mm. If the deviation of 0.28 mm provides correct functioning of the device in the diagonal directions, it is likely to be acceptable in all directions. If this is the case then a circular tolerance could be specified with a diameter of 0.28 mm as shown in Figure 15.7. The area of this circular zone is 57 per cent larger than the square tolerance zone and could have substantial implications for cost reduction in the manufacturing process.

The specification of a circular tolerance zone can be achieved by specifying the exact desired location of the hole and a permissible circular tolerance

zone centred at the exact location, as illustrated in Figure 15.8. Here the exact location of the hole is indicated by the boxed dimensions and the tolerance zone is indicated by the 'crossed circle' symbol within the rectangular box. The diameter of the tolerance zone is specified to the right of the symbol.

The symbol used for the positional tolerance is one of a series used in engineering drawing as illustrated in Table 15.5 and their use is referred to as 'geometric tolerancing'. Geometric tolerancing allows the engineer to specify control over feature locations and deviation of form such as flatness, parallelism and concentricity. The various symbols used and a brief description are given in Table 15.5. Figure 15.9 illustrates their use for defining the

Table 15.4 Surface roughness values produced by common production processes

Process	Roughness value, Ra (μm)												
	50 to 25	25 to 12.5	12.5 to 6.3	6.3 to 3.2	3.2 to 1.6	1.6 to 0.8	0.8 to 0.4	0.4 to 0.2	0.2 to 0.1	0.1 to 0.05	0.05 to 0.025	0.025 to 0.0125	
	25	12.5	6.3	3.2	1.6	0.8	0.4	0.2	0.1	0.05	0.025	0.0125	
Flame cutting													
Sawing													
Planing, shaping													
Drilling													
Electric discharge machining													
Milling													
Broaching													
Reaming													
Boring, turning													
Roller burnishing													
Grinding													
Honing													
Polishing													
Lapping													
Superfinishing													
Sand casting													
Hot rolling													
Forging													
Permanent mould casting													
Investment casting													
Extruding													
Cold rolling, drawing													
Die casting													

After BSI, 1991.

[] , Average application; [] , less frequent application.

locating radius and shoulder for a centrifugal compressor impeller. In order to avoid significant out of balance forces and correct clearance between the compressor blade tips and the stationary shroud, it is critical that the impeller is located concentrically on the shaft and the back face is perpendicular to the shaft. Control over the shaft radial dimensional variation has been specified by means of the runout symbol, which defines the limits of variability for the cylindrical surface and the squareness symbol for the shoulder.

The *Manual of British Standards in Engineering Drawing and Design* defines the standard use of engineering drawing methods and symbols and the texts by Spotts (1983) and Drake (1999) provide very useful information on tolerancing of components.

15.3 Statistical tolerancing

Many processes produce outputs, such as component dimensions or power, which follow a frequency distribution such as the 'normal' or Gaussian distribution illustrated in Figure 15.10. The average value of the output, say a component dimension, is called the mean. The spread of the output values around the mean represents the variability of the process and is represented by a quantity called the standard deviation. The normal distribution is defined such that 68.26 per cent of all the output is within one standard deviation of the mean, 95.44 per cent of all the output is within two standard deviations of the mean and 99.73 per cent of all the output is within three standard deviations of the mean.

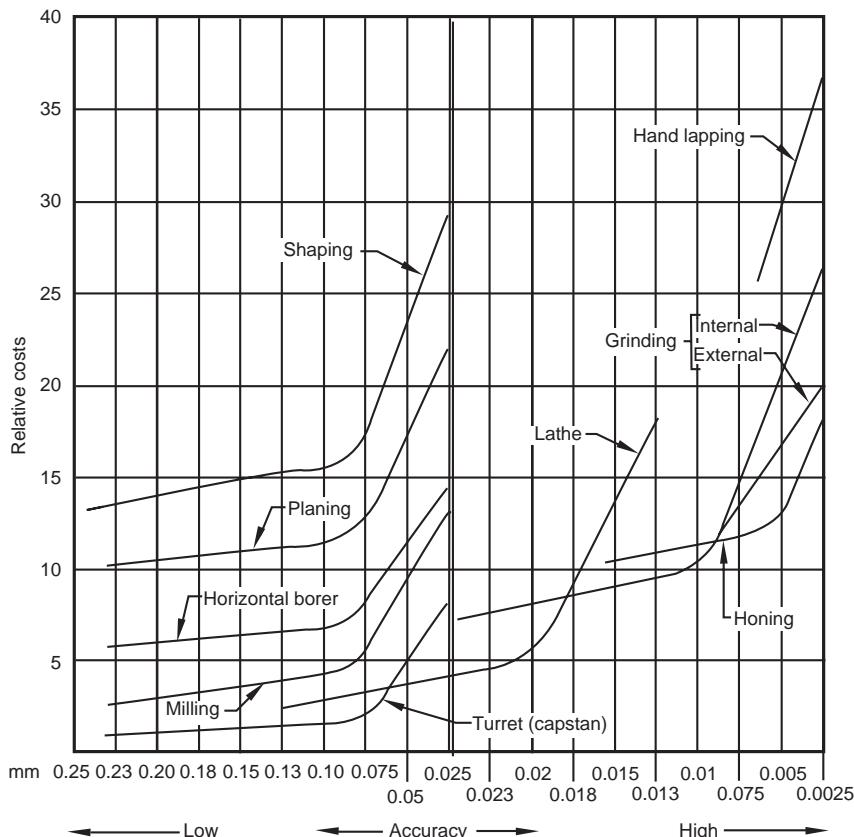


Figure 15.4 Relative cost of various machine and hand processes for achieving set tolerances (BSI, 1991).

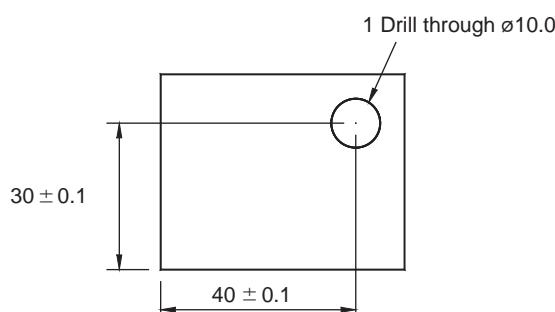
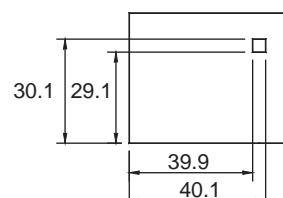


Figure 15.5 Drilled hole location.

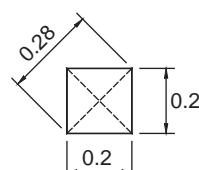
The normal distribution is defined by the equation:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{1}{2} \frac{(x - \mu)^2}{\sigma^2} \quad (15.8)$$

$-\infty < x < \infty$



(a)



(b)

Figure 15.6 Implied tolerance zone.

where $f(x)$ is the probability density function of the continuous random variable x ; μ is the mean; and σ is the standard deviation.

Equation 15.8 can be solved numerically or by use of a table of solutions (as given in Table 15.6). As an illustration of use of the normal table with $z = 3$, gives $F(z) = 0.998650$. So $2 \times (1 - 0.998650) = 0.0027 = 0.27$ per cent of all items can be expected to fall outside of the

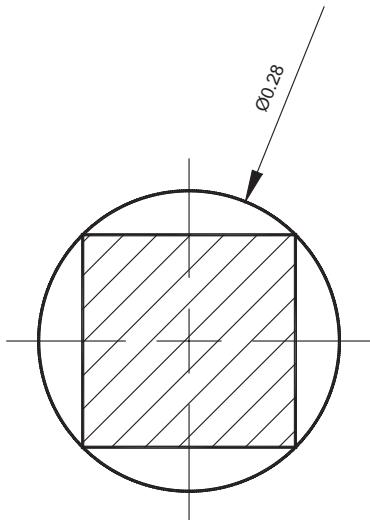


Figure 15.7 Circular tolerance zone.

tolerance limits. In other words 99.73 per cent of all items produced will have dimensions within the tolerance limits.

In practice 80 per cent of all processes, e.g. many machined dimensions and engine power outputs, are well modelled by the normal distribution (BSI, 1983). Given a set of data, a statistical test can be undertaken to determine whether the data are normally distributed. If measurements of a dimension for a particular component are found to be normally distributed with mean μ and standard deviation σ then the tolerance limits are often taken as $\mu \pm 3\sigma$. Tolerance limits defined as \pm three standard deviations from the mean are called ‘natural tolerance limits’. So if the mean and standard deviation of a component are 8.00 and 0.01, respectively, the dimension and bilateral tolerances would be defined as $8.00^{+0.03}_{-0.03}$ (i.e. the natural tolerance = $6\sigma = 6 \times 0.01 = 0.06$, so the bilateral tolerance is $0.06/2 = 0.03$).

15.3.1 Sure-fit or extreme variability

The sure-fit or extreme variability of a function of several uncorrelated random variables could be found by substituting values for the variables to find the maximum and the minimum value of the

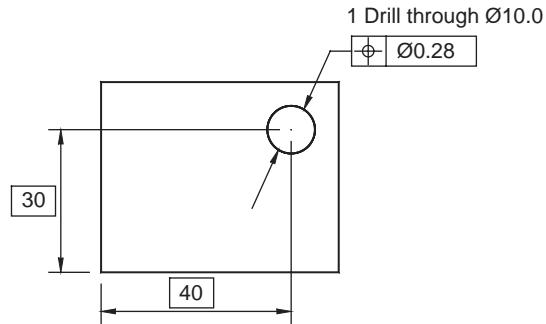


Figure 15.8 Circular tolerance zone specification.

function, e.g for the function:

$$\gamma = \frac{x_1 + 2x_2}{x_3^3}$$

The maximum value of γ_{\max} is given by:

$$\gamma_{\max} = \frac{x_{1,\max} + 2x_{2,\max}}{x_{3,\min}^3}$$

Similarly for the minimum value:

$$\gamma_{\min} = \frac{x_{1,\min} + 2x_{2,\min}}{x_{3,\max}^3}$$

The sure-fit or extreme variability can then be determined as:

$$\Delta\gamma = \gamma_{\max} - \gamma_{\min}$$

If $x_1 = 12 \pm 0.1$, $x_2 = 15 \pm 0.2$, $x_3 = 2 \pm 0.15$, then $x_{1,\max} = 12.1$, $x_{1,\min} = 11.9$, $x_{2,\max} = 15.2$, $x_{2,\min} = 14.8$, $x_{3,\max} = 2.15$, $x_{3,\min} = 1.85$, $\bar{\gamma} = 5.25$:

$$\begin{aligned}\gamma_{\max} &= \frac{x_{1,\max} + 2x_{2,\max}}{x_{3,\min}^3} \\ &= \frac{12.1 + (2 \times 15.2)}{1.85^3} = 6.712\end{aligned}$$

Table 15.5 Geometric tolerancing symbols

Symbol	Definition	Description
— — —	Straightness	This defines the straightness tolerance for an edge, axis of revolution or line on a surface. The tolerance zone is either the area between two parallel straight lines in the plane containing the toleranced edge or surface line or a cylindrical region for an axis of revolution
	Flatness	This is the condition of a surface having all elements in one plane
	Roundness	This is the condition on a surface of revolution (cylinder or cone or sphere) where all the points of the surface intersected by any plane (i) perpendicular to a common axis (cylinder/cone); (ii) passing through a common centre (sphere), are equidistant from the centre
	Cylindricity	A cylindricity tolerance zone is the annular space between two coaxial cylinders. The tolerance zone is the radial separation of the two cylinders
	Profile of a line	The profile of a line is the condition permitting a uniform amount of profile variation, either unilaterally or bilaterally, along a line element of a feature
	Profile of a surface	This defines the tolerance zone of variability for a surface relative to the perfect or exact form
	Parallelism	This is the condition of a surface, line or axis which is equidistant at all points from a datum plane or axis
	Squareness	The squareness tolerance defines the angular tolerance for perpendicularity (90°)
	Angularity	This defines the allowable variability for angular location. The angular tolerance zone is defined by either two parallel lines or surfaces
	Position	This defines a zone within which the axis or centre plane of a feature is permitted to vary from the true, theoretically correct, position
	Concentricity	A concentricity tolerance limits the deviation of the centre or axis of a feature from its true position
	Symmetry	This typically defines the area between two parallel lines or the space between two parallel surfaces, which are symmetrically located about a datum feature
	Runout	This is the deviation in position of a surface of revolution as a part is revolved about a datum axis

$$\begin{aligned}\gamma_{\min} &= \frac{x_{1,\min} + 2x_{2,\min}}{x_{3,\max}^3} \\ &= \frac{11.9 + (2 \times 14.8)}{2.15^3} = 4.176\end{aligned}$$

$\Delta y = \gamma_{\max} - \gamma_{\min} = 6.712 - 4.176 = 2.536$, so the variable $y = \bar{y} \pm (\Delta y/2) = 5.25 \pm 1.268$.

Alternatively the expression given in Eq. 15.13 can be used to approximate this extreme variability.

$$\Delta y = \left| \frac{\gamma}{x_1} \right| \Delta x_1 + \left| \frac{\gamma}{x_2} \right| \Delta x_2 + \left| \frac{\gamma}{x_3} \right| \Delta x_3$$

where

$$\Delta x_1 = x_{1,\max} - x_{1,\min}$$

$$\Delta x_2 = x_{2,\max} - x_{2,\min}$$

$$\Delta x_3 = x_{3,\max} - x_{3,\min}$$

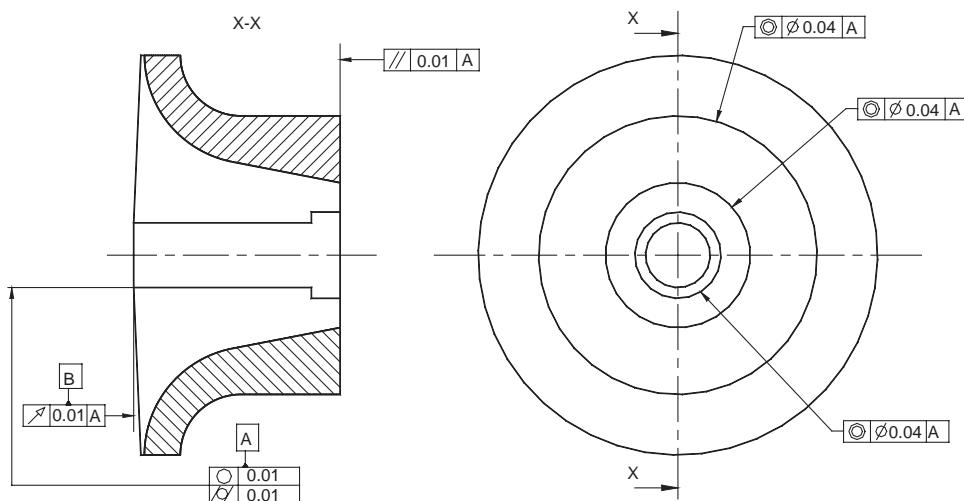


Figure 15.9 Example of the use of geometric tolerancing for specifying control over a centrifugal compressor shaft dimensions.

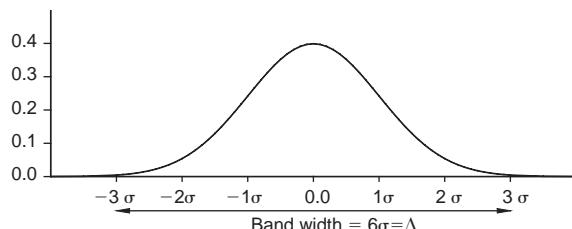


Figure 15.10 The normal distribution.

For the numerical example:

$$\gamma / x_1 = 1/x_3^3 = 1/2^3 = 0.125$$

$$\gamma / x_2 = 2/x_3^3 = 2/2^3 = 0.25$$

$$\gamma / x_3 = -3(x_1 + 2x_2)/x_3^4 = 7.875$$

$$\Delta x_1 = 0.2, \Delta x_2 = 0.4, \Delta x_3 = 0.3$$

$$\begin{aligned} \Delta y_{\text{approx}} &= 0.125 \times 0.2 + 0.25 \times 0.4 \\ &\quad + 7.875 \times 0.3 = 2.4875 \end{aligned}$$

So $\gamma = 5.25 \pm 1.244$ (a value within a few per cent of the value calculated in the above exact method).

15.3.2 Linear functions or tolerance chains

Often we require to know the overall tolerance in a dimension chain or knowledge of which

dimensions can be slackened without a deleterious effect on performance or which tolerances need to be tightened to improve the overall tolerance.

Taking a simple linear chain of components of overall dimension z :

$$z = x_1 + x_2 + x_3 + \dots + x_n \quad (15.9)$$

The expected value of z is given by:

$$E(z) = \mu_1 + \mu_2 + \dots + \mu_n = \mu_z \quad (15.10)$$

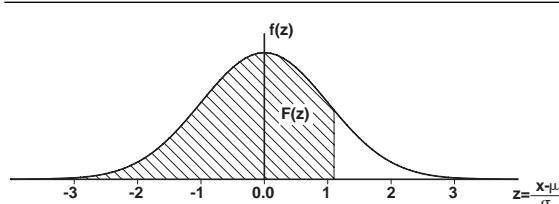
and the variance by:

$$\begin{aligned} \text{variance}(z) &= \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots + \sigma_n^2 \\ &= \sigma_z^2 \end{aligned} \quad (15.11)$$

If the response variable is a linear function of several measured variables, each of which is normally distributed then the response variable will also be normally distributed.

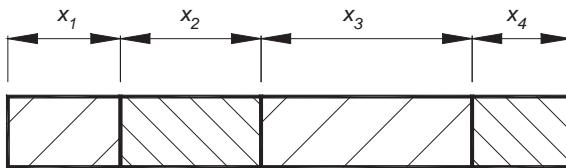
Example 15.4

A product is made by aligning four components (see Figure 15.11), whose lengths are x_1, x_2, x_3 and x_4 . The overall length is denoted by z . The tolerance limits of the lengths are known to

Table 15.6 The normal distribution

z	$f(z)$	$F(z)$
0.0	0.398942	0.5
0.1	0.396952	0.539827
0.2	0.391043	0.559260
0.3	0.381388	0.617911
0.4	0.368270	0.655422
0.5	0.352065	0.691465
0.6	0.333225	0.725747
0.7	0.312254	0.758036
0.8	0.289692	0.788145
0.9	0.266085	0.815940
1.0	0.241971	0.841345
1.1	0.217852	0.864334
1.2	0.194186	0.884930
1.3	0.171369	0.903195
1.4	0.149727	0.919243
1.5	0.129518	0.933193
1.6	0.110921	0.945201
1.7	0.094049	0.955435
1.8	0.078950	0.964069
1.9	0.065616	0.971284
2.0	0.053991	0.977250
2.1	0.043984	0.982136
2.2	0.035475	0.986097
2.3	0.028327	0.989276
2.4	0.022395	0.991803
2.5	0.017528	0.993791
2.6	0.013583	0.995339
2.7	0.010421	0.996533
2.8	0.007915	0.997495
2.9	0.005952	0.998134
3.0	0.004432	0.998650
3.5	0.000873	0.999768
4.0	0.000134	0.999968
4.5	0.000016	0.999996
5.0	0.0000015	0.9999997

be 8.000 ± 0.030 , 10.000 ± 0.040 , 15.000 ± 0.050 and 7.000 ± 0.030 mm, respectively. If the lengths of the four components are independently normally distributed, estimate the tolerance limits for the length of the overall assembly.

**Figure 15.11** Dimension chain.

Solution

Assuming $\pm 3\sigma$ natural tolerance limits:

$$\sigma_1 = 0.03/3 = 0.01 \text{ mm}$$

$$\sigma_2 = 0.04/3 = 0.0133333 \text{ mm}$$

$$\sigma_3 = 0.05/3 = 0.0166667 \text{ mm}$$

$$\sigma_4 = 0.03/3 = 0.01 \text{ mm}$$

$$\begin{aligned}\sigma_z^2 &= \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 \\ &= 0.01^2 + 0.013333^2 + 0.0166667^2 \\ &\quad + 0.01^2 = 6.555 \times 10^{-4}\end{aligned}$$

$$\sigma_z = 0.0256$$

$$\text{Overall variability} = 6\sigma_z$$

$$\text{Tolerance limits} = \pm(6\sigma_z/2) = \pm 3\sigma_z = \pm 0.077$$

The mean length of the overall assembly is given by:

$$\begin{aligned}\mu_z &= \mu_{x_1} + \mu_{x_2} + \mu_{x_3} + \mu_{x_4} \\ &= 8 + 10 + 15 + 7 = 40 \text{ mm}\end{aligned}$$

$$z = 40.000 \pm 0.077 \text{ mm}$$

So 99.73 per cent of all items would have an overall dimension within these tolerance limits. (By comparison the sure fit or extreme variability tolerance limits would be $z = 40.000 \pm 0.150$ mm, i.e. 100 per cent of all items would be within this tolerance limit.)

The basic normal model indicated a better, or more attractive (certainly for marketing purposes), tolerance band. If the sure-fit model was used and an assembly tolerance of ± 0.077 mm was desired the tolerances on the

individual components would need to be 8.000 ± 0.019 , 10.000 ± 0.019 , 15.000 ± 0.019 and 7.000 ± 0.019 mm. This is a very fine or tight tolerance and would be costly to achieve. As the statistical model takes account of 99.73 per cent of the assemblies, use of the sure-fit approach only gives 0.27 per cent of the assemblies with better quality and would require considerable extra effort and expense.

Example 15.5

Suppose the statistical tolerance bandwidth for the overall length of the previous example (± 0.077 mm) is too large for the particular application. An overall statistical tolerance of ± 0.065 mm is required. How can this be achieved.

Solution

This can be achieved in several different ways. One way would be to set the tolerance (and hence the standard deviation) equal, on all the components:

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma$$

Set

$$\sigma_z^2 = (0.065/3)^2 = 4\sigma^2$$

Hence

$$\sigma = 0.0108 \text{ mm}$$

Hence the tolerance limits for each individual component would be $\pm 3\sigma = \pm 0.033$ mm.

Alternatively the tolerance limits on say components 1 and 4 could be kept the same and the tolerance limits on components 2 and 3 could be tightened.

Suppose we decide that the tolerance limits on items 2 and 3 should be equal ($\sigma_2 = \sigma_3$).

$$\begin{aligned}\sigma_z^2 &= \sigma_1^2 + \sigma_4^2 + 2\sigma_2^2 \\ 2\sigma_2^2 &= 2.69444 \times 10^{-4} \\ \sigma_2 &= 0.0116 \text{ mm}\end{aligned}$$

So the tolerance limits on components 2 and 3 would need to be ± 0.035 mm in order for the overall tolerance of the assembly to be ± 0.065 mm.

Example 15.6

Which of the two following methods of manufacturing a product 150 mm in length would give the best overall tolerance:

1. An assembly of 10 components, each of length 15.000 mm with tolerance limits of ± 0.050 mm.
2. A single component of 150 mm also with tolerance limits of ± 0.050 mm.

Solution

1. $\sigma_1 = 0.05/3$
 $\sigma_z^2 = 10\sigma_1^2$
 $\sigma_z = 0.0527$

Tolerance limits for overall length $\pm 3\sigma = \pm 0.158$ mm.

$$\begin{aligned}\mu_z &= 10\mu_1 = 150.000 \text{ mm} \\ z &= 150.000 \pm 0.158 \text{ mm}\end{aligned}$$

2. $z = 150.000 \pm 0.050$ mm. As this method gives smaller tolerance limits, this is the most suitable way of achieving the tighter tolerance.

15.3.3 Several independent, uncorrelated random variables

Statistical analysis of tolerances can be applied to calculate the variation in physical phenomena such

as stress, fluid flow rates, compression ratio, etc., as well as component dimension chains. BS5760 states that the normal distribution model can be considered to be an appropriate distribution for modelling 80 per cent or more cases of distributional analysis in non-life circumstances. As such this analysis has wide ranging capability.

The theoretical preliminaries that enable analysis of functions of several independent, uncorrelated random variables are given below (after Furman, 1981).

Let γ be a function of several independent, uncorrelated random variables.

$$\gamma = f(x_1, x_2, \dots, x_j, \dots, x_n) \quad (15.12)$$

The sure-fit or extreme variability of the variable γ is given by

$$\delta\gamma = \sqrt{\sum_{j=1}^n \frac{f}{x_j} \delta x_j} \quad (15.13)$$

The statistical mean and standard deviation of a function of several independent, uncorrelated random variables are given by:

$$\mu_\gamma = f(\mu_1, \mu_2, \dots, \mu_n) + \frac{1}{2} \sum_{j=1}^n \frac{f}{x_j^2} \sigma_{x_j}^2 \quad (15.14)$$

$$\sigma_\gamma^2 = \sum_{j=1}^n \left(\frac{f}{x_j} \right)^2 \sigma_{x_j}^2 + \frac{1}{2} \sum_{j=1}^n \left(\frac{f}{x_j^2} \right)^2 \sigma_{x_j}^4$$

$$\sigma_\gamma^2 = \sum_{j=1}^n \left(\frac{f}{x_j} \right)^2 \sigma_{x_j}^2 \quad (15.15)$$

These equations may appear complex, however their application is procedural and various examples are given to show their use.

Example 15.7

The stiffness of a helical compression spring (see Figure 15.12) can be calculated from:

$$k = \frac{d^4 G}{8D^3 n}$$

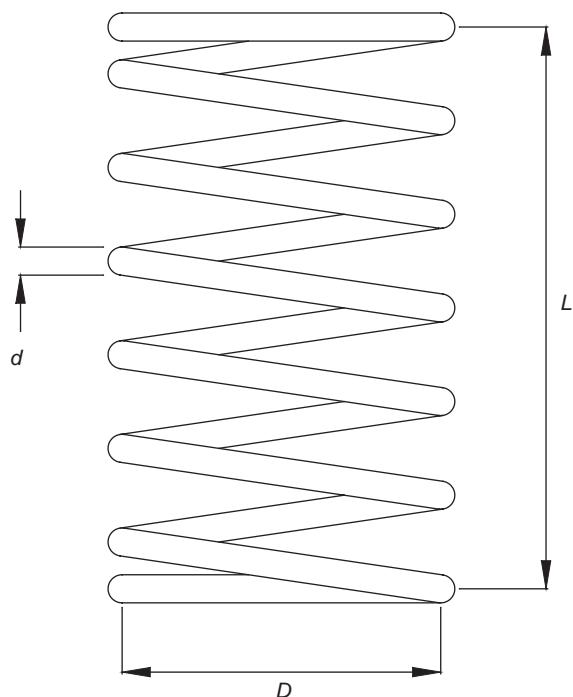


Figure 15.12 Helical compression spring.

where d is wire diameter; G , modulus of rigidity; D , coil diameter; n , number of coils; k , spring stiffness.

The mean and standard deviation for each variable are known:

$$\mu_d = 2.34 \text{ mm}, \mu_D = 16.71 \text{ mm},$$

$$\mu_G = 79.29 \times 10^3 \text{ N/mm}^2,$$

$$\mu_n = 14 \text{ coils},$$

$$\sigma_d = 0.010 \text{ mm}, \sigma_D = 0.097 \text{ mm},$$

$$\sigma_G = 1.585 \times 10^3 \text{ N/mm}^2,$$

$$\sigma_n = 0.0833 \text{ coils}.$$

Calculate the sure-fit extreme tolerance limits and the statistical basic normal tolerance limits for the spring stiffness. Assume $\pm 3\sigma$ natural tolerance limits: i.e. 99.73 per cent of measurements lie within ± 3 standard deviations.

Solution

Substituting values into the equation for k to give the average value of the spring constant:

$$\bar{k} = \frac{2.34^4 \times 79.29 \times 10^3}{8 \times 16.71^3 \times 14} = 4.549 \text{ N/mm.}$$

$$\sigma_d = 0.01 \text{ mm,}$$

$$\Delta d = 6 \times \sigma_d = 0.06 \text{ mm,}$$

$$\sigma_G = 1.585 \times 10^3,$$

$$\Delta G = 6 \times \sigma_G = 9510 \text{ N/mm}^2$$

$$\sigma_D = 0.097,$$

$$\Delta D = 6 \times \sigma_D = 0.582 \text{ mm}$$

$$\sigma_n = 0.0833 \text{ coils,}$$

$$\Delta n = 6 \times \sigma_n = 0.5$$

$$\frac{k}{d} = \frac{4d^3G}{8D^3n} = \frac{2.34^3 \times 79.29 \times 10^3}{2 \times 16.71^3 \times 14} = 7.776.$$

$$\frac{k}{G} = \frac{d^4}{8D^3n} = 5.7374 \times 10^{-5}.$$

$$\frac{k}{D} = -\frac{3d^4G}{8D^4n} = -8.1673 \times 10^{-1}.$$

$$\frac{k}{n} = -\frac{d^4G}{8D^3n^2} = -3.2494 \times 10^{-1}.$$

$$\begin{aligned}\Delta k_{\text{sure-fit}} &= \left| \frac{k}{d} \right| \Delta d + \left| \frac{k}{G} \right| \Delta G \\ &\quad + \left| \frac{k}{D} \right| \Delta D + \left| \frac{k}{n} \right| \Delta n \\ &= 7.776 \times 0.06 + 5.7374 \times 10^{-5} \\ &\quad \times 9510 + 0.81673 \times 0.582 \\ &\quad + 0.32494 \times 0.5 \\ &= 0.46656 + 0.5456 \\ &\quad + 0.4753 + 0.16247 \\ &= 1.65 \text{ N/mm.}\end{aligned}$$

This figure ($\Delta k_{\text{sure-fit}}$) gives the overall worst case variability of the spring stiffness (i.e. the

total tolerance). The bilateral tolerance can be obtained by dividing Δk by two (i.e. $1.65/2 = 0.825$).

So the spring stiffness can be stated as $k = 4.549 \pm 0.825 \text{ N/mm}$

Calculation of the basic normal variability:

$$\begin{aligned}\Delta k_{\text{basic normal}}^2 &= \left| \frac{k}{d} \right|^2 \Delta d^2 + \left| \frac{k}{G} \right|^2 \Delta G^2 \\ &\quad + \left| \frac{k}{D} \right|^2 \Delta D^2 + \left| \frac{k}{n} \right|^2 \Delta n^2 \\ &= 0.21768 + 0.297708 \\ &\quad + 0.22594 + 0.026397 \\ &= 0.7677.\end{aligned}$$

$$\Delta k_{\text{basic normal}} = 0.8762 \text{ N/mm.}$$

The overall variability in the spring stiffness is calculated as 0.8762 N/mm using the basic normal method. The bilateral tolerance can be calculated by dividing this value in two, i.e. the spring stiffness $k = 4.549 \pm 0.4381 \text{ N/mm}$.

The basic normal variation model implies a narrower band width of variation $k = 4.549 \pm 0.438 \text{ N/mm}$ versus $k = 4.549 \pm 0.825 \text{ N/mm}$ for sure-fit. This is more attractive from a sales perspective and accounts for 99.73 per cent (i.e. nearly all) of all items.

If we wanted to decrease the variation in k , we would need to tighten the individual tolerances on d , G , D and n with particular attention to d and D as these are controllable and influential on the overall variability.

The standard deviation of the spring stiffness can be calculated, if required, by $\sigma_k = \Delta k/6 = 0.146$.

Example 15.8

In the design work for a model single cylinder reciprocating engine, the work done during the expansion stroke is to be represented as:

$$W = P\pi D^2 C(1 - Q^{n-1})/(4(n-1))$$

where

$$Q = C/(C + 2R)$$

and

$$C = M - R - L - H$$

The quantities are defined as: W , work done during the expansion stroke; P , cylinder pressure at start of stroke; D , piston diameter; C , length of the clearance volume; n , expansion stroke polytropic exponent; R , crank radius; M , distance from crank centre to underside of cylinder head; L , connecting rod length; H , distance from gudgeon pin to piston crown; Q , ratio of clearance to clearance-plus-swept volumes.

It is recognized that the independent quantities are subject to variability and it is desired to determine the consequent extreme and probable variability in the work done.

If

$$P = 20 \times 10^5 \pm 0.5 \times 10^5 \text{ N/m}^2,$$

$$n = 1.3 \pm 0.05,$$

$$D = 4 \times 10^{-2} \pm 25 \times 10^{-6} \text{ m},$$

$$R = 2 \times 10^{-2} \pm 50 \times 10^{-6} \text{ m},$$

$$M = 10.5 \times 10^{-2} \pm 50 \times 10^{-6} \text{ m}$$

$$H = 2 \times 10^{-2} \pm 25 \times 10^{-6} \text{ m},$$

$$L = 6 \times 10^{-2} \pm 50 \times 10^{-6} \text{ m}$$

determine the extreme and probable (normal model) limits of the work (Ellis, 1990).

To help with your solution, certain partial derivatives have already been calculated.

$$\frac{W}{R} = -2599.46$$

$$\frac{W}{M} = 2888.39$$

$$\frac{W}{H} = -2888.39$$

$$\frac{W}{L} = -2888.39$$

Note: It may be useful to recall that if $y = a^x$ then $\frac{dy}{dx} = x \log_e(a)$.

Solution

The sure-fit 'extreme' limit of the work done is given by

$$\begin{aligned}\Delta W_{\text{sure-fit}} &= \left| \frac{W}{P} \right| \Delta P + \left| \frac{W}{D} \right| \Delta D \\ &\quad + \left| \frac{W}{M} \right| \Delta M + \left| \frac{W}{R} \right| \Delta R \\ &\quad + \left| \frac{W}{L} \right| \Delta L + \left| \frac{W}{H} \right| \Delta H \\ &\quad + \left| \frac{W}{n} \right| \Delta n\end{aligned}$$

The average values of C , Q , and W are:
 $\bar{C} = 0.005$, $\bar{Q} = 0.1111$, $\bar{W} = 20.22$.

$$\begin{aligned}\frac{W}{P} &= \frac{\pi D^2 C (1 - Q^{n-1})}{4(n-1)} \\ &= \pi 0.04^2 0.005 \frac{1 - 0.1111^{0.3}}{4 \times 0.3} \\ &= 1.011035 \times 10^{-5}.\end{aligned}$$

$$\Delta P = 2 \times 0.5 \times 10^5 = 1 \times 10^5.$$

$$\left| \frac{W}{P} \right| \Delta P = 1.011035.$$

$$\begin{aligned}\frac{W}{D} &= 2P\pi DC \frac{1 - Q^{n-1}}{4(n-1)} \\ &= 2\pi 20 \times 10^5 \times 0.04 \\ &\quad \times 0.005 \frac{1 - 0.1111^{0.3}}{4 \times 0.3} \\ &= 1011.035.\end{aligned}$$

$$\Delta D = 50 \times 10^{-6}.$$

$$\left| \frac{W}{D} \right| \Delta D = 0.0505517.$$

$$\frac{W}{M} = 2888.39 \quad \Delta M = 100 \times 10^{-6}.$$

$$\begin{aligned}\left| \frac{W}{M} \right| \Delta M &= 0.288839. \\ \frac{W}{R} &= -2599.46 \quad \Delta R = 100 \times 10^{-6}. \\ \left| \frac{W}{R} \right| \Delta R &= 0.259946. \\ \frac{W}{L} &= -2888.39 \quad \Delta L = 100 \times 10^{-6}. \\ \left| \frac{W}{L} \right| \Delta L &= 0.288839. \\ \frac{W}{H} &= -2888.39 \quad \Delta H = 50 \times 10^{-6}. \\ \left| \frac{W}{H} \right| \Delta H &= 0.1444195.\end{aligned}$$

Let

$$\begin{aligned}W &= K_3 \frac{\downarrow_{1-Q^{n-1}}}{\rightarrow n-1} \quad K_3 = \frac{P\pi D^2 C}{4}. \\ \left| \frac{W}{n} \right| &= K_3 \frac{\downarrow_{(n-1)} - \frac{Q^{n-1}}{n} - (1-Q^{n-1})1}{\rightarrow (n-1)^2} \\ &= K_3 \frac{\downarrow_{(n-1)} Q^{n-1} \ln Q - (1-Q^{n-1})1}{\rightarrow (n-1)^2},\end{aligned}$$

which on substituting for K_3 gives

$$\begin{aligned}&= \frac{W(n-1)}{1-Q^{n-1}} \\ &\quad \times \frac{\downarrow_{(n-1)} Q^{n-1} \ln Q - (1-Q^{n-1})}{\rightarrow (n-1)^2} \\ \frac{W}{n} &= -\frac{W}{n-1} \frac{\downarrow_{(n-1)} Q^{n-1} \ln Q}{\rightarrow 1-Q^{n-1}} + 1 \\ &= \frac{20.22}{0.3} \\ &\quad \times \frac{\downarrow_{0.3} \times 0.1111^{0.3} \ln 0.1111}{\rightarrow 1-0.1111^{0.3}} + 1 \\ &= 19.7909.\end{aligned}$$

$$\begin{aligned}\Delta n &= 2 \times 0.05 = 0.1. \\ \left| \frac{W}{n} \right| \Delta n &= 1.979. \\ \Delta W_{\text{sure-fit}} &= 1.011035 + 0.0505517 \\ &\quad + 0.288839 + 0.259946 \\ &\quad + 0.288839 + 0.1444195 \\ &\quad + 1.979 = 4.022. \\ W &= \bar{W} \pm (\Delta W/2) = 20.22 \pm 2.011 \text{ J}.\end{aligned}$$

Statistical tolerance limits:

$$\begin{aligned}\Delta W_{\text{normal}}^2 &= \frac{\downarrow W}{\rightarrow P}^2 \Delta P^2 + \frac{\downarrow W}{\rightarrow D}^2 \Delta D^2 \\ &\quad + \frac{\downarrow W}{\rightarrow M}^2 \Delta M^2 + \frac{\downarrow W}{\rightarrow R}^2 \Delta R^2 \\ &\quad + \frac{\downarrow W}{\rightarrow L}^2 \Delta L^2 + \frac{\downarrow W}{\rightarrow H}^2 \Delta H^2 \\ &\quad + \frac{\downarrow W}{\rightarrow n}^2 \Delta n^2 = 5.196. \\ \Delta W &= 2.2796.\end{aligned}$$

So

$$W = \bar{W} \pm (\Delta W/2) = 20.22 \pm 1.1398 \text{ J}.$$

If required

$$\sigma_w = 1.1398/3 = 0.38.$$

15.3.4 Statistical design techniques and quality assurance

The traditional approach to handling quality problems is based upon defining quality as conformance to specifications. For example, if the dimension and tolerance for a component was defined as 40.0 ± 0.1 mm, it would not matter whether the component dimension was 40.1, 40.5, 40.0 or 39.9 mm, the specification would have been satisfied. A Japanese engineer, Genichi Taguchi, has defined a new approach for solving quality problems based

on product uniformity around the target value. In the example above, the target value is 40.0 and the approach proposed by Taguchi is to endeavour to ensure as many components as possible are near to this value. Essentially Taguchi methods are based on the premise that quality is the avoidance of financial loss to society after the article is shipped. Within this definition quality is related to quantifiable monetary loss, not emotional gut reaction. Taguchi techniques of ensuring good quality require a three-step approach to product or process design consisting of system, parameter and tolerance design. These are defined below.

- System design. First, all the possible systems that can perform the required functions must be considered including new ones that have not yet been developed.
- Parameter design. During this stage the appropriate system parameters should be specified to improve quality and reduce costs.
- Tolerance design. Decisions concerning the tolerance specifications for all the components and appropriate grades of materials must be made. The objective of tolerance design is to decide trade-offs between quality levels and cost in designing new systems.

As a qualitative illustration of this technique, a case study is outlined of the production of the same design of an automotive transmission by a multi-national corporation but at two different sites, one in Japan and one in the USA. Both factories manufactured the transmission within a tolerance band as specified by the designer but in market surveys it was found that American customers preferred the Japanese manufactured transmissions. Various factors were investigated including the dimensional tolerance chain which is shown for the two factories in Figure 15.13. Too tight a tolerance chain results in high wear or binding of the transmission and too loose a tolerance chain results in high noise and component impact levels. The Japanese manufactured transmission was found to be approximately normally distributed about the target value, with 0.27 per cent of items

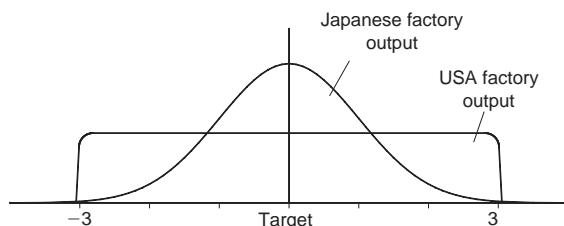


Figure 15.13 Distribution of automotive transmission tolerance.

falling outside the tolerance bandwidth. The output from the US factory was rectangularly distributed with all the items within the tolerance band. So the product from the US factory was always within the tolerance specifications unlike the output from the Japanese factory which was occasionally outside the tolerance specification. However, the market surveys showed that customers had preferred the product that had originated from the Japanese factory. This initially surprising result, considering the on-target nature of the US product, can be identified as being due to proportionally more customers getting a higher quality, or nearer the target, product if supplied from the Japanese factory. The customers' perception of quality responded to the Japanese factory paying more attention to meeting the target, as opposed to the USA factory's approach of meeting the tolerances.

Application of Taguchi methods requires an understanding of statistical information along with budgetary information for each stage of the design, production and marketing process. Further information on these methods can be found in the texts by Taguchi (1989), Taguchi *et al* (1993) and Phadke (1989).

Customers require products that have characteristics that satisfy their needs and expectations. Because of the competitive nature of business, as well as the desire to improve service, organizations are continually driven to improve their products and processes. The ISO 9000 and 9001 series of standards describe the fundamentals of quality management systems and the requirements where an organization needs to demonstrate its ability to

provide products that fulfil customer and regulatory requirements. An increasing proportion of business has embraced such approaches and the reader is referred to the standards themselves for full details or Hoyle (1998).

15.4 Conclusions

Tolerances have to be applied to components in order to define acceptable limits for manufacture purposes. Generally the larger the tolerance, the easier it is to ensure that components will conform. The subject of tolerances is, however, not limited to components and assemblies of components, but also applies to processes. This chapter has introduced the subject of tolerances and introduced methods to model the overall tolerance for an assembly or process based on statistical methods. In addition the subject of quality assurance has been briefly introduced.

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Nomenclature

Generally, preferred SI units have been stated.

- a internal radius of inner cylinder (m)
- b outer radius of inner cylinder (m), interface radius (m)
- c outer radius of outer cylinder (m)
- C length of the clearance volume (m)
- d wire diameter (m)
- D coil diameter (m), piston diameter (m)
- E Young's modulus (N/m^2)
- f coefficient of friction
- $f(x)$ probability density function of the continuous random variable x
- G modulus of rigidity (N/m^2)

<i>H</i>	distance from gudgeon pin to piston crown (m)
<i>k</i>	spring stiffness (N/m)
<i>L</i>	length of interference fit (m), connecting rod length (m)
<i>M</i>	distance from crank centre to underside of cylinder head (m)
<i>n</i>	number of coils, expansion stroke polytropic exponent
<i>p</i>	pressure (N/m ²)
<i>P</i>	cylinder pressure at start of stroke (N/m ²)
<i>Q</i>	ratio of clearance to clearance-plus-swept volumes
<i>R</i>	crank radius (m)
<i>T</i>	torque (N m)
<i>W</i>	work done during the expansion stroke (N m)
δ	total diametral interference (m)
μ	Poisson's ratio, mean
σ	standard deviation

Worksheet

- 15.1 A bronze bushing is to be installed in a steel shaft. The bushing has an inner diameter of 50 mm and a nominal outer diameter of 62.5 mm. The steel sleeve has a nominal inner diameter of 62.5 mm and an outer diameter of 87.5 mm.
- Specify the limits of size for the outer diameter of the bushing and the inner diameter of the sleeve so as to provide a medium drive fit.
 - Determine the limits of interference that would result.
 - For the maximum resulting interference calculate the pressure generated between the bushing and the sleeve, the stresses in both components and the deformations.
- Take $E = 200 \text{ GPa}$ for steel and $E = 113 \text{ GPa}$ for bronze. Take $\mu = 0.27$ for both materials.
- 15.2 Determine the interference fit required to transmit 6 kW of power at 30 rpm between a 40 mm diameter steel shaft and a 100 mm outer diameter cast iron hub which is 80 mm long. Take $E = 207 \text{ GPa}$ for steel and $E = 100 \text{ GPa}$ for cast iron, the coefficient of friction as 0.12 and Poisson's ratio as 0.3 for both materials.
- 15.3 Describe the differences between clearance, transition and interference fits and outline examples of usage.
- 15.4 A device is assembled by combining three components in line whose lengths are 40 ± 0.05 ,

20 ± 0.05 and 35 ± 0.03 mm. If the lengths of the three components are independently normally distributed, calculate the statistical and sure-fit tolerance limits.

If the tolerance limit for the overall length is deemed too wide and the tolerance limit for the third length cannot be varied find the reduced statistical tolerance limits for length one (and length two if necessary) such that the overall tolerance limits are ± 0.07 mm.

- 15.5 Three measured variables x_1 , x_2 and x_3 have small independent errors. The precisions are $\sigma_{x1} = 0.02$, $\sigma_{x2} = 0.03$, $\sigma_{x3} = 0.04$. The response variable is given by:

$$z = \frac{x_1 + x_2}{4x_3}$$

Estimate the precision of z when the measurements of x_1 , x_2 and x_3 are 2.49, 3.05, 2.05, respectively.

- 15.6 The torque capacity, T , for a multiple disc clutch can be calculated using the relationship

$$T = \frac{2}{3} \mu F_a N \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}$$

where μ is coefficient of friction; F_a , axial clamping force; N , number of disc faces; r_o , outer radius of clutch ring; r_i inner radius of clutch ring.

Determine the sure-fit and basic normal probable limits for the torque capacity if μ is 0.3 ± 0.03 ; $F_a = 4000 \pm 200 \text{ N}$; $N = 12$; $r_o = 60 \pm 0.5 \text{ mm}$, $r_i = 30 \pm 0.5 \text{ mm}$

Which design parameter should be reduced to create a maximum reduction in the variability of the torque capacity.

- 15.7 Describe the differences between the sure-fit (extreme variability) and the basic normal model for accounting for variability in component assemblies or processes.
- 15.8 For a standard series 'A' Belleville spring the spring rate or stiffness can be determined by the equation

$$k = \frac{4E}{(1 - \mu^2)} \times \frac{t^3}{K_1 D_e^2}$$

where E is the Young's modulus of the washer material, t is the washer thickness, μ is the Poisson's ratio, K_1 is a dimensionless constant and D_e is the external diameter.

Determine the sure-fit and probable limits for the spring stiffness stating any assumptions made if t is 2.22 ± 0.03 mm; D_e , 40.00 ± 0.08 mm; μ , 0.30 ± 0.003 ; E , $207 \times 10^9 \pm 2 \times 10^9$ N/m 2 ; K_1 , 0.69.

Comment on which tolerance could be altered to best reduce the overall variability.

Answers

15.1 H7/s6 hole $62.50_{-0}^{+0.03}$ mm, bushing $62.50_{-0.053}^{+0.072}$ mm, $\delta = 0.072$ mm, $p = 21.06$ MPa, $\sigma_o = 64.9$ MPa, $\sigma_i = 94.94$ MPa, $\delta_o = 0.022$ mm, $\delta_i = 0.05$ mm.

15.2 $64 \mu\text{m}$.

15.4 Sure-fit 95 ± 0.13 , basic normal 95 ± 0.077 , 40 ± 0.045 , 20 ± 0.045 .

15.5 $\sigma_z = 0.0139$

15.6 672 ± 108 N m, 672 ± 75 N m.

15.8 $k_{\text{sure-fit}} = 9017000 \pm 506700$ N/m, $k_{\text{basic normal}} = 9017000 \pm 378100$ N/m.

Learning objectives achievement

Can you select a tolerance band for different practicable applications?

✓ ✗ Section 15.2

Can you determine an interference fit torque capacity?

✓ ✗ Section 15.2.2

Can you recognize a geometric tolerance symbol?

✓ ✗ Section 15.2.4

Can you determine the sure-fit variability of a process?

✓ ✗ Sections 15.3.1, 15.3.2

Do you determine statistical variability of a process?

✓ ✗ Section 15.3.3

16

DESIGN MANAGEMENT AND CASE STUDY

The aim of this chapter is to explore design in its broad context. The management of design is considered and methodology for evaluating which design, out of a number of options, is the best. Aspects of machine elements introduced in individual chapters in this book are pulled together in a case study on a power transmission in order to illustrate the use of a wide range of design skills, the interactions between machine elements and decision-making.

LEARNING OBJECTIVES

At the end of this chapter you should:

- be able to describe the total design activity;
- be able to evaluate alternative concepts and make a selection;
- have an awareness of the range of information sources available to the designer;
- have developed additional experience of the application and interrelationships between machine elements.

16.1 Introduction

The term ‘design’ conveys different meanings to different people. To some design is the use of drawing tools, pen and paper or mouse and computer, to draw details of a component or device. To others design is a sophisticated process resulting in the detailed design of an overall system, such as a car or computer. The word design can be taken as either a verb or a noun and is used here in both forms.

The role of the designer is to create a new device or product. This involves the application of many skills and processes from the identification of the market need, product specification, the generation of concept solutions, detailed design, manufacture and marketing. These processes can be considered as forming the core of the design activity and their collective consideration is called ‘total design’ as discussed in Chapter 1. This design model and methodology has been adopted by many organizations as it provides a framework allowing the design process to be competently undertaken and exploited. Traditional design approaches are often not effective enough as products have become more sophisticated, consumer-driven and legislation influenced and as an increasing number of concepts are protected by patenting, precluding their economic usage. Products based on ‘blank sheet of paper’ ideas are not that likely to be able to compete with today’s global industries unless the originators have raised their comprehensive competence to such a level that they can undertake design from consideration of the system to detailed design, marketing and costing. A good example of this, is recounted by James Dyson (1997) with reference to his Dyson vacuum cleaners. Sustained success requires systematic thoroughness and meticulous attention to detail from the beginning to the end of the design process.

It is worth noting here the relevance of many of the subjects covered in modern design and engineering education. The qualities demanded of designers include detailed scientific knowledge, logical thought process, creative, conceptual

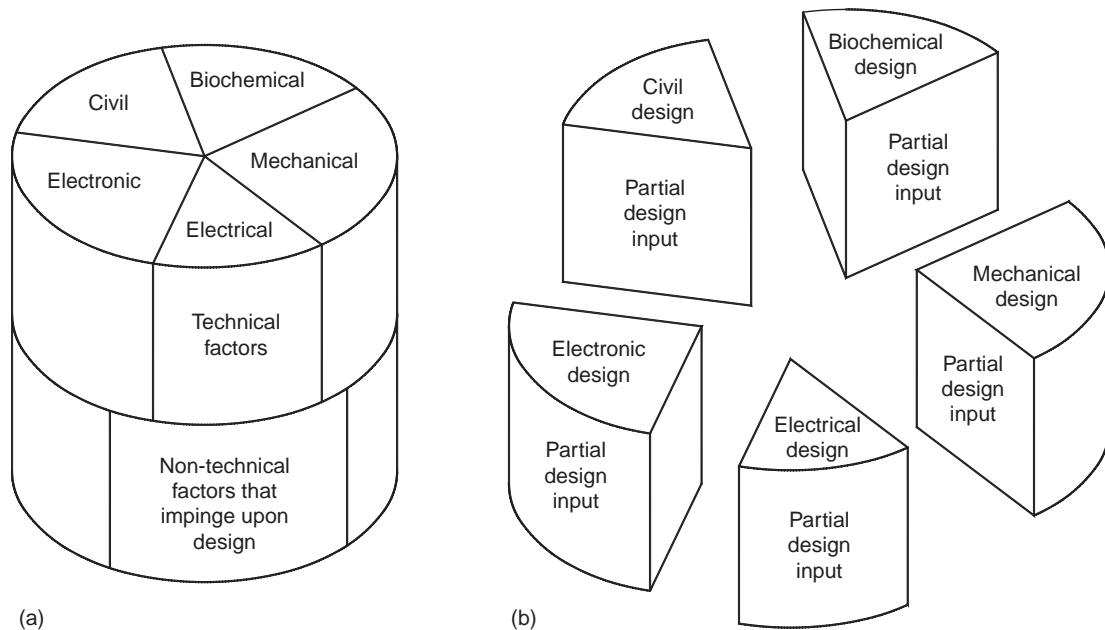


Figure 16.1 (a) Total design inputs. (b) Partial design input (after Pugh, 1990).

and innovative abilities and communication skills. Traditional engineering science courses such as thermodynamics, heat transfer and fluid mechanics, statics and dynamics, satisfy the demands for detailed technical scientific knowledge and development of logical thought process. In addition, the cohort of courses in management and technical communication satisfy requirements for organization and communication. The remaining demands must be satisfied by studies in design and personal professional development. This requires a degree of self-cultivation and provision of the environment for creative, innovative and conceptual skills to develop. This should be aided by general knowledge of what has been done, what is possible and scientifically feasible. Any product, for example a car or a jet engine, requires co-ordinated input from several different specialisms as illustrated in Figure 16.1a. Any one input considered in isolation can be regarded as partial design (see Figure 16.1b). If it occurred on its own, no product would result. Industry is principally concerned with total design or the sum of

the partial design inputs as this results in identifiable and marketable products. This principle applies to machine elements such as bearings and gears. In isolation they are of little use but combined together they can form useful machines.

Total design is the systematic activity from the identification of the market need to the selling of the product or process to satisfy this need. Total design can be regarded as having a central core of activities consisting of market need, product design specification, conceptual design, detailed design, manufacture and marketing and is illustrated again in Figure 16.2.

Design usually starts with a need or requirement which, when satisfied, will allow a product or process to be offered to an existing or new market. The market need may be to meet competition, a response to social changes, opportunities to reduce hazards, inconvenience or costs, or exploitation of a new market opportunity. This is the 'front end' of design. People with front-end skills, such as market researchers and information scientists, are fundamental to the design process.

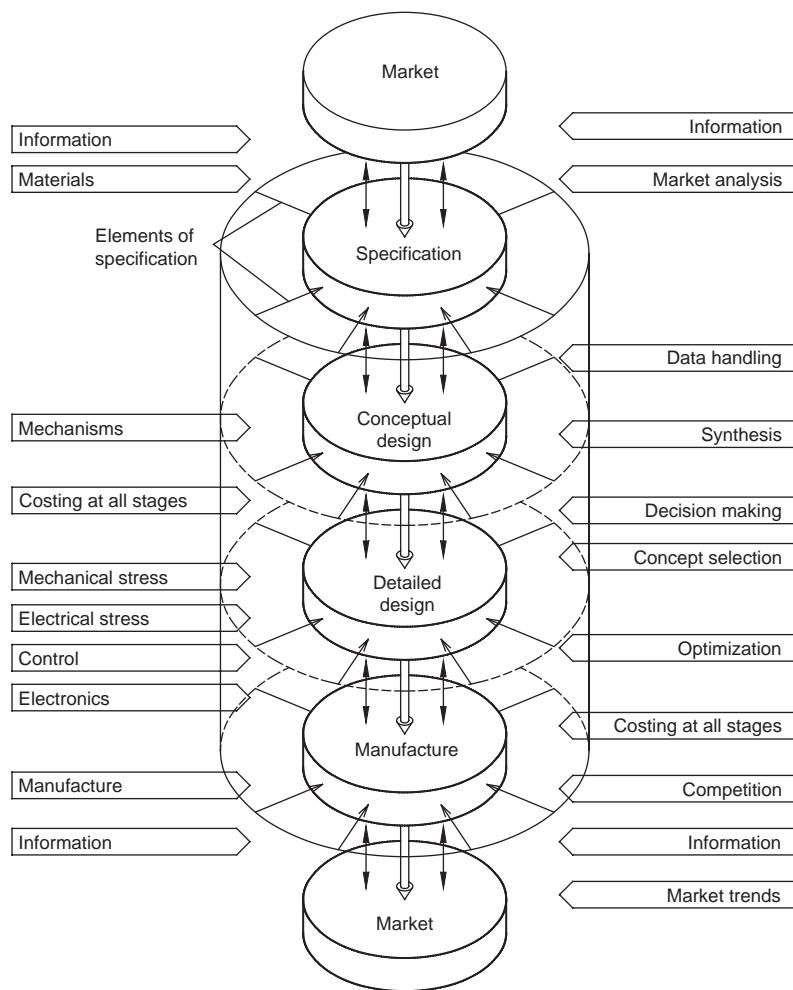


Figure 16.2 The total design process, showing the core, steps and associated inputs (after Pugh, 1990).

The starting point for any design should be the establishment of the market need situation. Common practice is to produce a document, called 'the brief', at this stage. The brief varies from a simple statement of the requirements to a comprehensive document that describes the users' true needs.

Specification is critical to effective design. Thorough specification provides limits defining the attributes that a design should have. The practice recommended here is to develop a product design specification (PDS) as outlined in Chapter 1. On completion of the specification, conceptual design can proceed. This is the most creative part of the

design process, where solutions to the specification are given form. Having generated the conceptual solutions, the next step is to express these so that they are communicable to all involved in the total design process. In practice, this may take the form of a drawn scheme or three-dimensional model, which may be physical or computer-generated. By the time the general scheme has been completed and calculations undertaken to determine the solution's compatibility to the product design specification, the basis will have been established for the detailed design phase to commence.

It is worth noting the importance of the conceptual design stage. It has been estimated that by

the time the conceptual design phase has been completed, approximately 70 per cent of the total cost of the design has been committed (Institution of Product Engineers, 1984). In addition time and applied effort spent at the conceptual stage can avoid serious mistakes. Although the Sinclair C5 three-wheeler vehicle is now an icon of 1980s' design, it was actually a commercial flop. It involved a significant quantity of detailed design, but the conceptual design was flawed. Sir Clive Sinclair admitted, in hindsight, that the driver was too close to the ground and overexposed to the elements, and that the product did not meet the aspirations and needs of enough customers at that time.

Conceptual design is concerned with finding solutions that meet the PDS. As convergence towards selection of one concept is made, the concept should be expanded in greater detail to give a better understanding and assure compatibility. The design core stages are highly iterative and interactive in practice. The phases are not necessarily distinct and run into each other and overlap. Detailed design is concerned with the technical design of subsystems and components which make up the concept or system and involves defining the layout, form, dimensions, tolerances and surface finishes, materials and machining process of the product components. The product will be made up of components, which to some degree will have been defined at the conceptual design phase, but the final make-up of these may vary considerably from this following the detailed design phase. Ion and McCracken (1993) describe the detail design phase as the final stage in the translation of a proposal into a fully detailed and tested product description, which can be manufactured. Detailed design can involve the division of a product or process into a subsystem, units assemblies and component parts, analysis and validation and refinement of the concept and its subelements, the design and selection of components, prototyping and test evaluation and preparation of the manufacturing information such as drawings,

computer models and machining coordinates. Common sense dictates that poor detail design can wreck a good concept and excellent detailed design can turn a marginal concept into a marketing success.

Detailed design requires knowledge of materials, analysis techniques, manufacture quantities, life prediction, aesthetic appeal, etc. The principal emphases in component design are local performance, local environment and local constraints. During the design of a component the context of the component is critical to its design. The context can be defined or quantified by the component design specification or CDS. If issues are neglected these will contribute towards shortened life, lower quality and customer dissatisfaction. Typical component design specification elements are:

- product performance
- company constraints
- local environment
- customer requirements
- testing
- quality
- maintenance
- weight
- manufacture
- standards
- component cost
- ergonomics
- safety and product liability
- aesthetics
- quantity
- materials.

The detailed design phase relates directly to the manufacturing phase as the decisions taken have implications for the manufacturing process. Manufacture considerations must be accounted for at all stages in a design programme from the market investigation phase and also during the specification, conceptual design and detailed design. The specification phase has particular implications for manufacture as aspects such as quality and reliability, weight, finish, quantity, cost

and size may be defined. There must be materials available for the design and manufacture and assembly must be technically and economically possible. The modern approach to material selection should include consideration of metals, polymers, ceramics or composites for any component. In the preparation of a detailed manufacturing drawing considerations should include (see Corbett, 1989):

- Can a less expensive material be selected?
- Can the quantity of material used for a product be reduced?
- Is a stock item or material available from a supplier that does not need machining?
- Must all the surfaces be machined?
- What tolerances are necessary?
- Can the components be held for manufacture?
- Is assembly possible?

Various strategies for consideration of manufacture throughout the design process exist such as design for manufacture (DFM), design for assembly (DFA) and concurrent engineering (McMahon and Browne, 1998). The principal aims of DFM are to minimize component and assembly costs, to minimize development cycles and enable higher quality products to be generated. DFM has two key constituent elements: design for assembly (DFA) and design for piece-part producibility (DFP). The essence of DFA is the selection of the assembly process, for example manual, high speed or robot assembly, and the subsequent design of the product for the chosen assembly process. DFP is concerned with the production of the individual constituent components. Many products are subject to cost reduction at some later stage in their commercial life. A key aim of DFM is to get the costs and quality of manufacture right initially. Some companies operate with distinct product design and manufacturing operations. Others have integrated design and manufacture teams from the outset of a project. Concurrent engineering is a systematic approach to the integrated simultaneous design and development of both the components and

their associated process, such as manufacture and testing.

During the marketing or sales phase of the total design model, the product is launched into the market place. The marketing technique will depend on whether a one-off product, a batch-produced product or a mass-produced product is being sold. Sales marketing will involve advertising, distribution, servicing and maintenance. Of direct relevance to the design process is feedback of information to the design team concerning customer reaction to the product, features that are liked or disliked, any component failures and requirements for redesign, any servicing or maintenance issues.

As outlined, the total design process requires consideration of the overall system and detailed constituent components with an interdisciplinary input of skills from marketing to system and component analysis. The total design process shown in Figure 16.2, illustrates as well as the design core, the detailed considerations at each level, which may be prioritized, and also the disciplinary skills required.

16.2 Management of design

Management of design is a complex activity, as it needs to encourage creativity whilst at the same time allowing work to be controlled, especially when defined timescales and fixed costs are involved. Good communication with all concerned is essential, including, for example, the manufacturing and commercial departments, to ensure that the specification is still being met in all respects, including the costs of design and manufacture. Good communication at the specification stage ensures 'ownership' of the decisions. A desire or need to change the design from the agreed specification must be agreed, particularly by the marketing department as it may affect the viability of the product in the market place. It cannot be assumed that the design team knows the capability of the manufacturing unit, or that

the factory will understand, as intended, the information, which they receive from the designers. A balance must be found between performance, availability and cost. This decision must be taken at an early stage since the whole philosophy of the design is affected by these choices, which again depend upon the needs of the actual or potential customers.

Design reviews can be an important method of auditing a design. A design review brings together representatives of all the people who have an interest in the new product. It should be a critical examination of the design. It will, because it takes time, increase the cost of design, but will increase the cost-effectiveness if the recommendations are put into practice. Participants should therefore include, in addition to the design team, representatives of production, quality, safety, purchasing and of course marketing. The number of reviews will depend on the complexity of the product, but an initial review at the proposal stage and another as the design approaches maturity would seem to be the minimum needed.

DEF STAN 05-67/1 suggests the following topics to be considered at a comprehensive design review for a major project.

- Interpretation of the requirement and its achievement with respect to safety, performance and reliability;
- Maintenance policy with respect to manpower, repair, tools and test equipment;
- Environmental conditions and methods of operational use;
- Installation, assembly and operational constraints;
- Standardization and interchangeability;
- Design philosophy, design concept, proposed solutions and alternatives;
- Distribution of work between departments and subcontractors;
- Requirements for research;
- Trials plans, collection and analysis of trials data;
- Quality, reliability, maintainability, non-destructive testing and safety plans;

- Failure mode and effect analysis;
- Stress and weight analysis;
- Production processes required;
- Production, inspection and test methods, test equipment;
- Release of drawings, software and configuration control;
- Value engineering and life cycle costing;
- Packaging;
- Handbooks;
- Post design services.

A most important responsibility of the design team is to certify that their design is suitable for use. This requires approval of the design information and stating that the design is correct for manufacture. This approval may be given at various stages as information becomes finalised, but should certainly be given at the point when quantity manufacture is about to be approved.

16.2.1 Project management

Project management typically occurs under the auspices of a project manager. The position, responsibilities and contribution of a project manager can vary widely from organization to organization. It can range from monitoring the performance of others, to controlling their performance, to having the overall technical and operational responsibility for a complete activity, or project, or business. Project managers may be extra to the present teams, or a present member may be given this additional task. In general, the project manager will be the reporting link to the person in charge of the complete project, but being just the reporter is a minimal responsibility, as this carries no authority for the information being passed. To make project management a worthwhile position, there must be some element of control, even if this is the result of planning of the various activities as part of a critical path analysis (CPA). Just planning the work must not be mistaken for control.

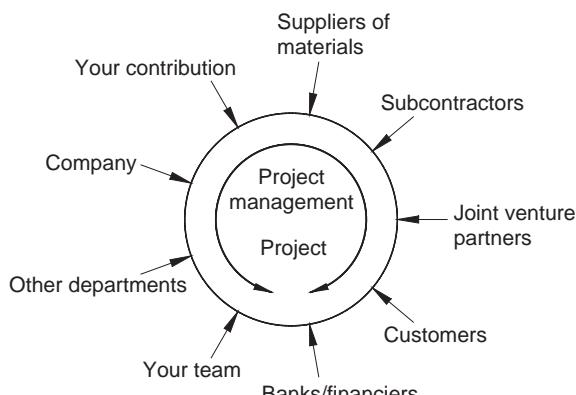


Figure 16.3 Essential links for a project manager.

A project manager can be the prime contact with the client, with suppliers and subcontractors. The project manager is in a position of considerable responsibility, with the ability to commit significant amounts of expenditure on behalf of the company. A full understanding of the specification is essential. Any doubts must be resolved at a very early stage of the project. Even if the project manager does not have direct control of the people on the project, the expectation is still to deliver on time. The project manager is the project champion and will be expected to promote the project with the company management in order to optimize the project performance. Figure 16.3 shows the links that are essential for any project manager.

In the real world a project manager may be faced with the following scenario:

- the specification is incomplete and ambiguous;
- the required resources are not available;
- the design is not correct first time;
- the production unit has made scrap;
- the supplies are late;
- the customer has changed his mind;
- the changes were not negotiated correctly due to a lack of understanding.

The project manager has to maintain his reputation throughout this situation and still convince all concerned that his requests are credible and important.

Project management can be assisted by a 12-step planning process (Leach, 1999).

- Step 1: Requirements capture and flow-down.
- Step 2: Define the deliverables.
- Step 3: Create the work breakdown structure.
- Step 4: Define the performance measures.
- Step 5: Identify and assess the risks.
- Step 6: Define the activities.
- Step 7: Identify the key dependencies.
- Step 8: Define the milestones.
- Step 9: Produce the schedule.
- Step 10: Estimate resource and cost requirements.
- Step 11: Assemble the plan.
- Step 12: Obtain commitment acceptance.

16.2.2 Evaluation

Having generated ideas and concepts, the next necessary step is to evaluate them and select the best concept. One method of evaluation is the use of tabular matrices as illustrated in Table 16.1. These consist of a series of criteria against which the concepts must be marked. The importance of the criteria can be weighted, if appropriate, and the most suitable concept is identified as the one with the highest overall mark. This method provides a structured technique of evaluation and makes it difficult for individuals within a design team to push their own ideas for irrational reasons.

An example of the use of an evaluation matrix is given in Table 16.2. Here four difference concepts for the pallet moving device, presented in Section 1.2.1, are being considered. The evaluation criteria considered important here are safety, positional accuracy, ease of control, ease of manufacture and durability. These criteria have been weighted with positional accuracy of the pallet-moving device deemed the most important. The scores are given with a maximum of 10. The marks in the columns for the weighted scores have been summed and the totals are given at the bottom of the table. The highest weighted score is for the linkage mechanism in this example, and this concept should therefore be selected and developed.

Table 16.1 Generic evaluation matrix

Selection criteria	Weight	Concepts		Concept B		Concept N	
		Concept A		Concept B		Concept N	
		Rating	Weighted score	Rating	Weighted score	Rating	Weighted score
Aspect 1	Percentage importance of aspect 1	Mark out of 10 for aspect 1 of concept A	Rating × weight for aspect 1 of concept A	Mark out of 10 for aspect 1 of concept B	Rating × weight for aspect 1 of concept B	Mark out of 10 for aspect 1 of concept N	Rating × weight for aspect 1 of concept N
Aspect 2	Percentage importance of aspect 2	Mark out of 10 for aspect 2 of concept A	Rating × weight for aspect 2 of concept A	Mark out of 10 for aspect 2 of concept B	Rating × weight for aspect 2 of concept B	Mark out of 10 for aspect 2 of concept N	Rating × weight for aspect 2 of concept N
...							
Aspect n	Percentage importance of aspect n	Mark out of 10 for aspect n of concept A	Rating × weight for aspect n of concept A	Mark out of 10 for aspect n of concept B	Rating × weight for aspect n of concept B	Mark out of 10 for aspect n of concept N	Rating × weight for aspect n of concept N
Total weighted score			Sum of weighted scores		Sum of weighted scores		Sum of weighted scores
Ranking							

Table 16.2 Evaluation tabular matrix for a pallet moving device

Selection criteria	Weight	Concepts		Hydraulic mechanism		Cable hoist		Linkage mechanism		Rack and pinion	
				Rating	Weighted score	Rating	Weighted Score	Rating	Weighted Score	Rating	Weighted Score
		Rating	Weighted score	Rating	Weighted Score	Rating	Weighted Score	Rating	Weighted Score	Rating	Weighted Score
Safety	20%	2	0.4	3	0.6	4	0.8	2	0.4		
Positional accuracy	35%	6	2.1	5	1.75	7	2.45	6	2.1		
Ease of control	10%	6	0.6	5	0.5	7	0.7	6	0.6		
Ease of manufacture	20%	7	1.4	8	1.6	5	1.0	6	1.2		
Durability	15%	7	1.05	6	0.9	5	0.75	6	0.9		
Total score			5.55		5.35		5.7		5.2		
Rank			2		3		1		4		
Continue?			No		No		Develop		No		

16.3 Costing

Estimating the cost of a product during design is notoriously difficult. It is however a very necessary activity. Without an estimate of the cost of a conceptual solution, it is difficult to evaluate whether

it meets cost aspects of the specification or whether it has cost advantages over other designs. The cost of a component, or system, is often significantly influenced by a few specific features. It is often stated that 20 per cent of a system usually equates to 80 per cent of its cost. Identification of the

expensive aspects can be useful in influencing the design process. It may be possible to meet the functional requirements of the specification by selecting alternatives to the expensive features concerned. Important figures in costing are the selling price and allowable costs. These values are described in Sections 16.3.1 and 16.3.2, respectively.

16.3.1 Selling price

The selling price is the amount charged to a purchaser. It is made up of the sum of the costs of making the product, plus the costs of running the business divided by the notional quantity, plus an amount for profit.

$$\frac{\text{direct costs} + \text{overheads}}{\text{quantity}} + \text{profit} = \text{selling price} \quad (16.1)$$

The only factor that is discretionary in this sum is the profit; all other components are determined by the design, the efficiency of the organization and the number of the articles being made.

16.3.2 Allowable cost

Having defined or agreed the selling price, the allowable cost of making the product, including its share of the overheads is inevitably determined. This provides a limitation, or control, on the time available to carry out the design and manufacture and on the cost of materials and processes. Costs come in two categories:

- 1 direct;
- 2 indirect (or overheads).

Direct costs are those costs which can be charged directly to the product, or are directly attributable to some aspect of the design, marketing or selling of the product. Direct costs are either time-related and correspond to the cost of the time spent by the various contributors to the product and the cost of materials used, or to the expenditure on labour and materials in carrying out the design and preparation

for manufacture. Direct costs can be divided into fixed direct costs and variable direct costs:

- Fixed direct costs are those that are not related to the volume or number of items produced;
- Variable direct costs are those that change depending upon the quantity to be made.

Fixed direct costs include time spent in:

- preparing the specification;
- making trial designs;
- calculating costs;
- carrying out the detailed design;
- producing manufacturing information;
- production engineering;
- tool design;
- training staff in this product;
- labour costs, other than manufacture;
- negotiations with suppliers and subcontractors;
- selling costs directly attributable;
- cost of materials used in carrying out the design;
- cost of tools for this project only (some may be recurring).

Variable direct costs include:

- manufacturing labour;
- inspection and test;
- cost of materials used in manufacture;
- cost of subcontracted work;
- cost of bought-out components;
- packaging materials.

Indirect costs or overheads cover any cost that cannot be directly allocated to the specific product unless a decision is taken to spread the project costs across the whole business. Indirect costs or overheads can be divided into two categories:

- those associated with design and manufacture;
- the cost of running the business, sometimes known as commercial overheads.

Overheads associated with design and manufacture include:

- the administration of the design organization,
- the administration of the manufacturing organization,

- production planning,
- services and utilities used by these departments.

Overheads may include new tools or machinery, which although applicable to the project, may be of general use. The cost is then distributed across all the business.

For example, consider the justification for a new machine costing £30 000. A case for the purchase has been made on the basis that it increases efficiency of a process. The present system using existing plant manufactures the product for £5.00 each. The new machine would do the same task for £3.00 each. The submission has to demonstrate how the machine will provide a cost reduction, which is not present with existing techniques, and that the cost of the machine will be 'recovered' in a defined time, related to the life of the machine and any product manufactures that will use it. The 'recovery' can also be set at an amount to fund the replacement of the machine at the end of its life. The calculation will need to include some form of discounted cash flow, the interest on a bank loan taken for the purchase and related to the depreciation in value over life. Simplistically, over a run of 500 000 items the saving would be

$$(\text{£}5 - \text{£}3) \times 500\,000 = \text{£}1.0 \text{ million}$$

This figure indicates that a new machine is worthwhile, to which would be added the yield from the sale of the existing machine, less the cost of installation and training on the new one. It is important to get the estimates correct! The case needs direct participation by the marketing department as well as manufacturing, in order that the justification be owned by all concerned. In a justification of this sort it is crucial to ensure that the sales estimates are realistic. In this example, for instance, is the figure of 500 000 items correct?

The cost of running a business, sometimes known as commercial overheads, includes:

- rent
- rates
- telephone, electricity, gas, water
- banking

- cleaning
- stationery
- insurance
- computer systems
- salaries of personnel staff
- salaries of office staff
- salaries of marketing staff
- general sales and marketing activities
- entertainment of prospective clients
- commercial functions
- salaries of accounts staff
- salaries of purchasing staff
- management salaries
- quality management
- salaries of plant engineers
- general maintenance activities
- security
- transport
- agents
- catering
- drinks machines
- and many more items, depending upon the organization, such as scrap, rework and contingencies.

Some of the items listed as commercial overheads may be divided or shared with those associated with design and manufacture. The overheads have to be recovered from customers by some means. Many methods are used, from adding a percentage to the direct costs, which may not be easy to justify, to more sophisticated systems, to produce a works cost price (WCP).

One method uses a 'local' overhead for each department contributing to the design or manufacture. The WCP that results, then represents a basic figure for the actual costs that have been incurred in making the product. Having obtained a WCP, it is then possible to add a further percentage, to cover the overheads for the cost of running the business. The overhead percentage for design and works labour is usually large and figures in excess of 200 per cent are not untypical. Material overheads are usually much less, reflecting the cost of the Goods-In department, at say 130 per cent.

Commercial overheads vary over a large range, depending upon the type of business and the size of the various activities. A figure of 125 per cent is not untypical for a large operation.

A typical works cost price may be made up as follows:

The recurring costs:

$$\begin{aligned} & \text{Cost of works labour} \times L\% + \text{cost} \\ & \text{of works materials} \times M\% \\ & = \text{Works cost price} \end{aligned} \quad (16.2)$$

where $L\%$ is labour overhead; and $M\%$ is materials overhead.

The non-recurring costs: The cost of design would then be

$$\begin{aligned} & \text{Design labour} \times D\% + \text{design} \\ & \text{material} \times DM\% \end{aligned} \quad (16.3)$$

where $D\%$ is design overhead; $DM\%$ is design material overhead.

The selling price would be:

$$\begin{aligned} & \text{Profit} + \frac{\downarrow \text{works costs price}}{\rightarrow} + \frac{\downarrow \text{cost of design}}{\rightarrow \text{quantity}} \\ & \times \% \text{ commercial overheads} \end{aligned} \quad (16.4)$$

The percentages are established from the need for the company to at least break even, even if an item was sold at zero profit. Breaking down the costs into departmentally related amounts, allows each area to use an overhead, which balances its costs, without undue loading from another area. It also gives each departmental manager the total responsibility for local costs and hence encourages ownership of the results.

It would be possible to have a different overhead for each part of the organization. Accountants tend to like this, as it would permit a detailed analysis of the performance of each department. However this can get out of hand. It tends to increase accounting costs and would be unlikely to improve the control of the business in proportion to the extra effort involved. For example, the overhead percentage on works labour would be calculated

to recover the cost of the manufacturing management, its office staff, the cost of the manufacturing floor space and such commonly used activities, such as quality assurance and machine maintenance. In some cases, contrary to the definition of 'fixed', the fixed overheads may well change with significant changes in the volume of sales.

16.3.3 Cost estimating

Estimating is a difficult task at any time. In many cases an estimate is needed to allow a response to an invitation to tender. In the case of a new company product, similar problems will arise, as there will be a shortage of accurate information available. There will typically be inadequate breakdown of the work detail, failure to provide information on major cost items or areas of high risk and there are likely to be omissions. At the start of a new product, by definition, the design has not been done. In these circumstances, experience counts for a great deal, especially as it is likely that one of the most significant aspects will be the time taken to design the product. This in itself is a notoriously difficult aspect to assess.

An example of estimating the cost of designing and making a one-off special fastener is given in Table 16.3. The table itemizes activities, the time estimated for each activity, the rate and overhead for each activity and the cost.

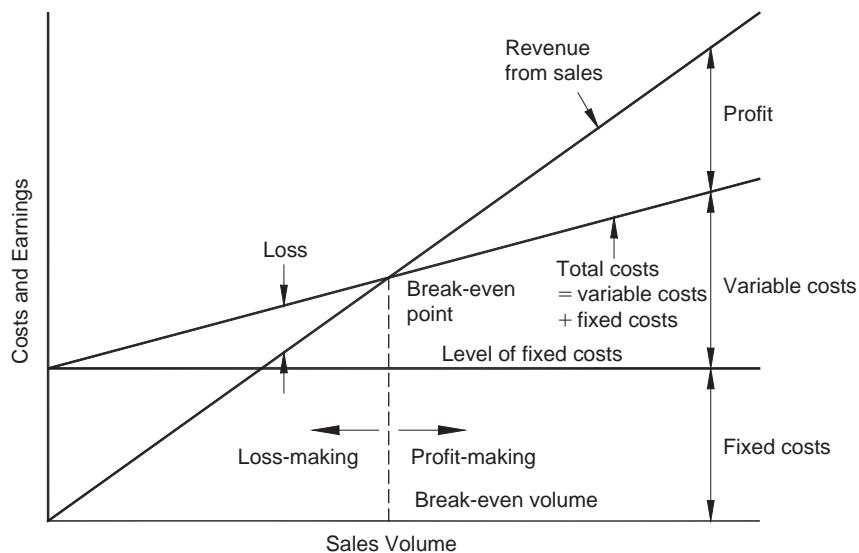
Alarming as the values in Table 16.3 are, and note that a low overhead percentage has been used, they are relatively realistic and show that it is crucial to ensure that a large quantity is to be produced, or the one-off design will be prohibitively expensive.

A characteristic break-even chart, giving costs and earnings as a function of sales for a typical product, is given in Figure 16.4. It has been assumed that income or revenue commences at the same time as the start of the fixed costs. This is unlikely in practice, as some costs will usually be incurred well before there is any income.

It is important to separate out the recurring (variable) from the non-recurring (fixed) costs and to ensure that the appropriate techniques are used

Table 16.3 Estimation of the cost of design and manufacture of a one-off special fastener

Activity	Time (h)	Rate and overhead	Cost
Understanding the specification	1.0	20/h + 100%	40
Select material	10.0	20/h + 100%	400
Trial design	20.0	20/h + 100%	800
Evaluate	15.0	20/h + 100%	600
Make prototype, including special tools	50.0	15/h + 150%	1875
Design test procedure	10.0	20/h + 100%	400
Arrange for test facilities	5.0	20/h + 100%	200
Test prototype	10.0	15/h + 150%	375
Consider results	5.0	20/h + 100%	200
Rework if necessary	???		???
Cost of procurement of materials	5.0	10/h + 100%	100
Cost of using test facilities	30.0	3000 single payment	3000
		Subtotal	7990
Contingency		2010 (say)	2010
		Total	10 000

**Figure 16.4** Break-even chart.

that relate to the quantity to be made. If some aspect of the design requires a high cost item or special tools, this should be investigated. Similarly any item that is high risk from the point of view of operation, or supply, must be considered carefully and an allowance made for the possibility of a failure occurring.

The estimating activity must be repeated as often as the specification changes, or the design

matures and as the manufacturing information is produced. The subsequent estimates should be used to affirm that the product is still viable.

16.3.4 Accounts

In order to assess the financial viability of a proposed project it is necessary to estimate the income and expenditure for the short- and long-term life

Table 16.4 Example accounts for development and production of a cordless screwdriver

Units sold @ £8.00	No. 15 000	£ 120 000	No. 50 000	£ 400 000	No. 250 000	£ 2 000 000
Profit & loss account year 2						
Cost of sales						
Materials and components @£3.00	45 000		150 000		750 000	
Labour cost 1.00 per item	15 000		50 000		250 000	
Total direct costs	60 000		200 000		1 000 000	
Gross margin	60 000		200 000		1 000 000	
Overheads (with design and tooling costs)	195 000		205 000		225 000	
Net profit/loss before tax	−135 000		−5000		775 000	
Profit & loss account year 3						
Cost of sales						
Materials and components @£3.00	45 000		150 000		750 000	
Labour cost 1.00 per item	15 000		50 000		250 000	
Total direct costs	60 000		200 000		1 000 000	
Gross margin	60 000		200 000		1 000 000	
Overheads (no design or tooling costs)	70 000		80 000		100 000	
Net profit/loss before tax	−10 000		120 000		900 000	

of the design. An example of accounts, are given in Table 16.4 for the second and third year of a new product. In the example the development is completed by the end of the second year. The figures are arbitrary, but relate to the possible manufacturing and design costs for a cordless screwdriver. The gross margin is the difference between the cost of manufacture (cost of sales) and the income from those sales. The net profit is the difference between the income from sales and the sum of the cost of sales and the total overheads (indirect costs). The figures illustrate the importance of sufficient sales to ensure timely profit. If the sales are only 120 000 a year for this scenario, then losses extend into the third year. This is unlikely to be acceptable.

Table 16.5 gives a typical, but simplified format for the calculation of some of the overheads that have to be used to run a company with a turnover of £10 million. The overhead figure is almost irrespective of the investment needed to design and sell products. In this example however, the funding of the design team has been included, as this would allow a new product to be launched without the handicap of having the 'recover' the cost of the design. In a large multiproduct company,

existing business can fund new development out of retained profits. The amounts for expenditure for all items are for example only, in order to show the mix of expenditure and profit.

An example of the calculation of a works cost price is given in Table 16.6. This example assumes that there will be an accurate assessment of the sales volume and of the time required to carry out the design. The estimated sales volume in this example is 100 000 units. The works labour is low, £6/hour, which is near the minimum legal wage in the UK at the time of writing and compatible with relatively unskilled labour. It is unlikely that this figure could be reduced. In the calculation of the works cost price, it has been assumed initially that the profit is zero. This gives a break-even value for the selling price.

From the figures given in Table 16.6, the selling price is given by:

$$\text{Selling price} =$$

$$\downarrow_{\text{WCP}} + \frac{\text{Cost of design}}{\rightarrow \text{Quantity}} \times \text{Commercial overhead} + \text{Profit}(\text{£})$$

$$= \downarrow_{9.2} + \frac{40250}{\rightarrow 100000} \times 125\% = £12.00$$

Table 16.5 Example calculation of overheads for a company with a turnover of £10 million

Overhead structure	£	%	Total
Fixed direct costs			
Including			
Design labour	150 000	225	337 500
Design material	5000	130	6500
Variable direct costs			
including			
Labour	500 000	225	1 125 000
Material including bought out	3 000 000	130	3 900 000
Total direct costs			5 369 000
Overheads			
Including			
Rent, rates, telephone, banking, etc.			1 000 000
Salaries of management			100 000
Salaries of office staff			50 000
Salaries other indirect staff			30 000
Paying salaries, etc.			25 000
Cost of plant			500 000
Maintenance			100 000
Commercial costs			100 000
Depreciation			226 000
Total overheads			2 131 000
Turnover	10 000 000		
Profit	2 500 000		
Costs and overheads	7 500 000		7 500 000

Table 16.6 Example calculation of a works cost price

	Rate £/h	Time h	Cost £	Overhead rate %	Total £
Works labour	6.00	0.20	1.20	225	2.7
Works material			5.00	130	6.5
WCP					9.2
Design cost	60.00	250.00	15 000.00	225	33 750.00
Design materials			5000.00	130	6500.00
Cost of design					40 250.00

If market analysis shows that the maximum selling price that the market will bear is £12, then no profit is possible with the costs detailed. Examination of the table shows that the WCP is dominated by works material. This is an obvious target for cost reductions. Although the hourly rate for works labour is low, it may be possible to decrease the time spent in labour by automation. The installation of machinery would, however, need to be carefully evaluated offsetting the cost of the machinery against the labour reduction.

16.4 A guide to design literature and sources of information

The sources of information for design are varied and extensive and include books, general media such as magazines, advertising, catalogues and websites, patents, standards and journals.

The textbooks relevant to mechanical engineering design broadly split into those on general

design and those covering machine design. The reader is referred to the following texts:

- *The Material of Invention* by Enzio Manzini (1989) which gives an introduction to the broad design and creation process.
- *Total Design* by S. Pugh (1990) and '*Product Design and Development*' by Ulrich and Eppinger (1995) which introduce the overall design process within a technical context.
- *Engineering Design* by Pahl and Beitz (1996), which gives an alternative design model to total design.
- *Conceptual Design for Engineers* by French (1985), which introduces approaches to developing conceptual ideas.
- *Engineering Design* by Dhillon (1996), which gives an overview of design techniques.

There have been many books on machine design and mechanical component design varying from engineering reference handbooks and encyclopaedias to introductory texts. One of the more comprehensive reference books is *The Standard Handbook of Machine Design* edited by Shigley and Mischke (1996). For an introduction to component and machine design the texts by Norton (1996), Juvinal and Marshek (2000), Hurst (1994), Shigley and Mischke (2001) and Mott (1998) are excellent.

The best source of specific information for mechanical components are the manufacturers' brochures, catalogues and design guides. Obtaining these can appear awesome to students, but companies are normally willing to post these out following a telephone call or web-based request. The trick is finding out the names, addresses and contact numbers. A good starting point is the business directories.

These are many academic and industrial journals on mechanical design and university and the national lending libraries are a good place to access these.

Professional designers must operate according to statutory standards. These can be obtained from national standard organizations, such as the

British Standards Institution (BSI), DIN, ISO, AGMA, SAE.

Electronic databases and communication are still in an evolutionary stage, but these can be good sources of information. Electronic information on design ranges from the descriptive and portals to proprietary databases.

16.5 Case study

In order to illustrate the intricacies of the design process, a case study will be considered here for the transmission of a compressor. The study draws together a number of subjects considered in this book and is typical of the design process in that it involves a wide range of skills and decision-making.

16.5.1 Brief

A small and medium enterprise (SME) company has investigated the potential market for a small electric motor driven air compressor. The market analysis has confirmed the requirement for an $80 \text{ m}^3/\text{h}$, 7 bar air supply driven by a 5 kW electric motor. Preliminary design has already resulted in the selection of an electric motor, with a synchronous speed of 2850 rpm, and the piston and cylinder and associated components for a reciprocating compressor. The compressor design speed is 1500 rpm. A general arrangement for the compressor concept is given in Figure 16.5. The choice of transmission and its detailed design is yet to be undertaken. The task here is to explore the specification, conceptual design and detailed design of the transmission for this compressor.

16.5.2 Specification

The brief given outlines the general requirement for a transmission for a reciprocating compressor. Not all the information necessary to develop the design is given in the brief and must be developed by means of liaison with the company and application of design skills and experience.

A specification for the transmission is developed below:

1. The transmission must transmit at least 5 kW.
2. The input is from an electric motor with a synchronous speed of approximately 2850 rpm. Details of the electric motor are not given in the brief. Contact with the company has

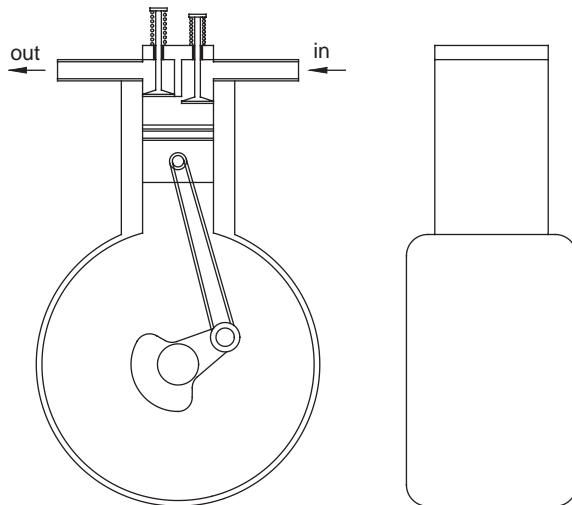


Figure 16.5 General arrangement for a 7 bar, $80 \text{ m}^3/\text{h}$ reciprocating compressor.

identified the critical interface motor dimensions, which are given in Figure 16.6.

3. The output from the transmission is to a reciprocating compressor running at 1500 rpm. The load is reciprocating and therefore constitutes medium to light shock.
4. Following contact with the preliminary design team, dimensions for the compressor shaft have been provided and are given in Figure 16.7.
5. The preliminary design team has confirmed that the electric motor torque speed

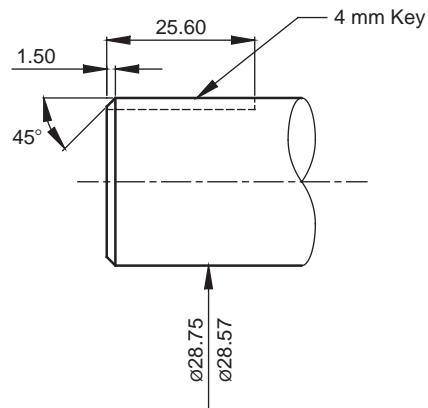


Figure 16.7 Principal dimensions for the compressor shaft connection.

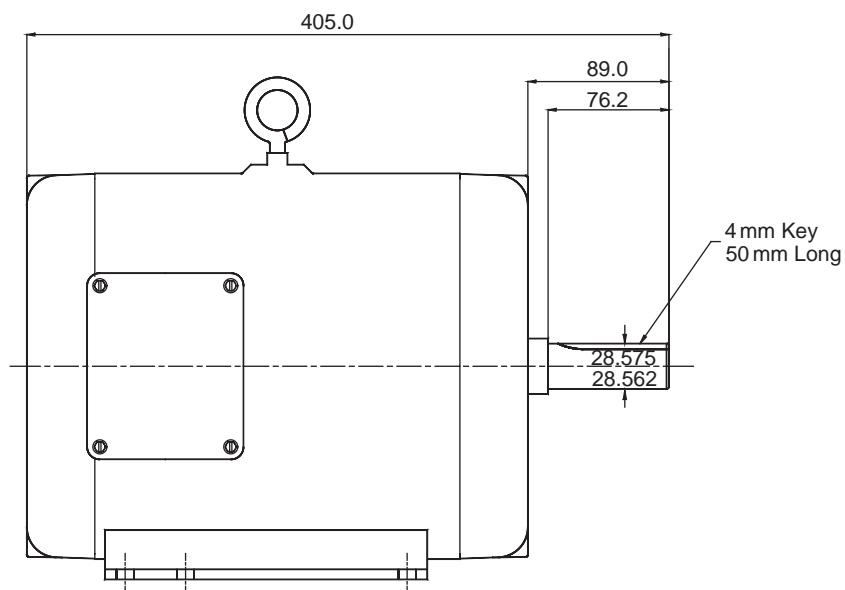


Figure 16.6 Principal dimensions for the 5 kW electric motor drive connection.

- characteristic is sufficient to start the compressor, provided the transmission gives the required ratio with reasonable efficiency.
6. A mechanical efficiency for the transmission has not been specified. However, a mechanical efficiency of at least 95 per cent should be achievable and is proposed for the specification.
 7. The means of connecting the motor and compressor to the transmission have not been detailed in the brief. Contact with the SME has confirmed that the connection of the transmission to the compressor and electric motors forms part of the brief. Flexible couplings permit some degree of misalignment and isolation of vibration and are therefore specified for connecting the input and output shafts of the transmission to the motor and compressor.
 8. Following discussion with the company and examination of the market analysis, compact size for the compressor and therefore the transmission is desirable. Limits have not been identified in the market analysis.
 9. The life of the compressor is not defined in the brief. Following discussion with the SME business team, the following information has been identified. The compressor is expected to operate for a maximum of 2 h a day on 200 days of the year. An 18-month trouble-free warranty life is desirable.
 10. The compressor will be enclosed to prevent casual contact with hot or moving components. However, the transmission should include means to support its functional components and retain all lubricant if present.
 11. The transmission will operate in an internal environment at temperatures between -10°C and 50°C . The design should be tolerant to dust and high humidity.
 12. Up to 1000 units a year will be manufactured.
 13. The cost of the transmission should be minimized. A maximum limit for the works cost price has been set as 600 Euros.
 14. The weight is not critical, but very high weights are deemed undesirable. A maximum

mass for the transmission has been set by the SME at 30 kg.

15. Ideally the transmission should use readily available components.
16. Unique transmission components should be manufactured locally to the SME, in order to ensure reliability of parts supply.
17. The design should take a maximum of 6 man-months and be undertaken over a 2-month period.
18. Although aesthetics are not a primary factor in the design, the design should augment the overall aesthetics of the overall product.
19. The transmission should comply with all relevant safety criteria and legislation.
20. The design should not violate any existing patents in order to protect against royalty payments.
21. The design should provide recommendations for future recycling of components in order to comply with expected legislation on sustainability.
22. The transmission must be robust enough for transportation on standard pallets by road.

The aspects of the specification listed do not yet provide sufficient bounds in order to enable conceptual or detailed design to proceed sensibly at this stage. Use of a pro-forma such as those illustrated in Section 1.4 might be helpful in order to further develop definite limits on the design parameters in order to ensure that the work effort is being sensibly directed towards the product need defined by the marketing analysis referred to in the brief. The product design specification is developed in Table 16.7.

As part of the specification, it is also important to identify the evaluation criteria before the design process proceeds. These may be altered as the design proceeds if aspects of the evaluation are deemed inappropriate. Evaluation criteria are necessary because it is possible that a number of competing designs may be identified, all of which meet the specification. The evaluation criteria can be used to determine which of the design solutions should be

Table 16.7 Product design specification for the compressor transmission

Aspect	Objective	Criteria	Test conditions
Performance	Transmit torque	Transmission must transmit 5 kW at 2850 rpm	Test transmission on a dynamometer
	Speed ratio	The transmission ratio between driving and driven shafts should be $2850/1500 = 1.9$	If positive drive employed then assume specified ratio or if a positive drive is not used measure speed of output shaft at design conditions
	Efficiency	At least 95%	Dynamometer test
		Must connect to specified 1.5"driving shaft and 1.5"driven shaft using flexible couplings	Use standard coupling methods that are rated for the maximum torque and speed with a factor of safety suitable for the application
		Temperature	Bench test prototype
	Humidity	Operation in an internal environment at temperatures between -10°C and 50°C	Bench test prototype
		Operate in an environment with a humidity of 100% at 20°C	
Size	Minimize	Must be smaller or similar to competition. Competition best transmission fits within an envelope space of $120 \times 250 \times 250$ mm	Measure
Cost	Minimize cost	The cost for the transmission per unit should be less than 600 Euros assuming 1000 units per year	Calculate works cost price
Life	Maximize	The warranty life of the machine should exceed $2850 \times 60 \times 2 \times 200 \times 1.5 = 1.02 \times 10^8$ cycles	Test design under accelerated stress conditions
Weight	Minimize	Must be less than 30 kg	Weigh it
Manufacture	Ensure ready supply of components	Use stock machine elements where possible	Identify components in stock supply inventories
Design time	Minimize	Design using manufacturing techniques that are readily available from a number of local plants	Use established suppliers
		Detailed design of the transmission must take less than a maximum of 6 man-months and occur over a 2-month period	Use standard project management methods to track progress and rectify excursions from the plan
Patents	Avoid	Do not infringe existing patents	Patent search
Transport	Robust design	The transmission must be robust enough for transportation on standard pallets by road	Vibration test
Aesthetics	Attractive design	Design should augment the overall aesthetics of the overall product	Use focus group on concept graphics model
Safety	High	Must comply with existing and envisaged legislation	Check national and international standards and relevant legislation
Recycling	Noise	Must be less than legal limits	Measure
	Compliant	The design should provide recommendations for future recycling of components in order to comply with expected legislation on sustainability	Check national and international standards and legislation
Maintenance	Easy	Provide access to components that might fail to enable their replacement	Use graphics model of design to verify maintenance access

adopted. Here the evaluation criteria that seem appropriate at this stage are:

- cost
- safety

- size
- reliability
- maintenance
- noise and vibration.

16.5.3 Conceptual solutions

There are a number of ways of meeting this specification including:

- belt drive
- chain drive
- gear drive
- belt and chain
- belt and gear
- gear and chain.

Alternatively the specification and brief could be challenged. For example, could a motor running at 1500 rpm, providing the required torque be used? The combination drives listed will be discounted here due to complexity. However, the three different types of machine element drive along with direct drive are all potential solutions that could be viable. The different concepts are illustrated as general arrangements in Figure 16.8.

All the conceptual solutions identified in Figure 16.8 appear viable and meet the specification. An evaluation matrix will be used in order to determine which concept should be developed. This is developed in Table 16.8.

The scores given in Table 16.8, out of 10, are quasi-subjective. They reflect the opinion and skills of the judge or judges concerned. Different individuals or groups could produce varying outcomes. The aim here, however, is to illustrate the design process and selection of one design, and hopefully the most appropriate, out of a number of

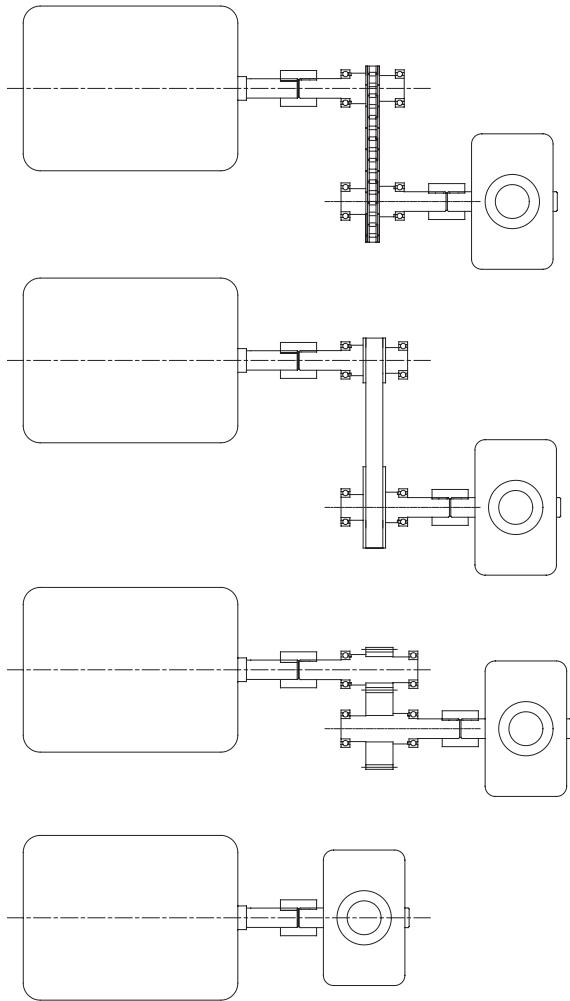


Figure 16.8 Conceptual arrangements for the transmission.

Table 16.8 Evaluation matrix

Criteria	Weighting (%)	Concept									
		Chain		Belt		Gear		Direct drive			
		Score	Weighted score	Score	Weighted score	Score	Weighted score	Score	Weighted score	Score	Weighted score
Safety	15	3	0.45	5	0.75	6	0.9	7	1.05		
Size	10	3	0.3	2	0.2	8	0.8	6	0.6		
Reliability	20	5	1.0	3	0.6	7	1.4	8	1.6		
Maintenance	10	3	0.3	6	0.6	3	0.3	7	0.7		
Noise and vibration	15	4	0.6	5	0.75	6	0.9	8	1.2		
Cost	30	7	2.1	8	2.4	6	1.8	2	0.6		
Total			4.75		5.3		6.1		5.75		

possibilities. The evaluation matrix scores in Table 16.8 indicate that the gear concept is the most suitable. This will therefore form the basis of the conceptual solution for detailed design.

16.5.4 Detailed design

For a reduction ratio of 1.9, a single pair of gears is possible. Spur, helical, bevel and worm gears are all possible. Helical gears are generally quieter and more compact than spur gears, but the thrust load they generate must be accommodated, bevel gears provide drive through right angles, which is not a requirement here and worm gears are too inefficient. Spur gears appear to be the most appropriate selection for the design.

Given that this design must be capable of operating for 2 h a day, 200 days a year with a warranty life of 18 months, detailed calculations based on the AGMA standards will be made for the gears. The size of the gears is unknown at this stage. The results of calculations for a range of modules and face widths are given in Table 16.9. These calculations are based on the procedures presented in Chapter 7. An overload factor of 1.25 has been assumed to account for the reciprocating load. A grade 1 steel with Brinell hardness of 240 and 200 has been assumed for the pinion and gear, respectively.

Examination of Table 16.9 shows that all the proposals have high factors of safety for bending.

Table 16.9 AGMA bending and contact safety factors for a variety of modules and face widths

Module	$m = 2$	$m = 2$	$m = 2.5$	$m = 3$
N_p	20	20	20	20
N_g	38	38	38	38
d_p (mm)	40	40	50	60
d_g (mm)	76	76	95	114
V (m/s)	5.97	5.97	7.46	8.95
F (mm)	20	50	30	40
W_t (N)	838	838	670	558
H_{BP}	240	240	240	240
H_{BG}	200	200	200	200
S_{FP}	1.6	3.7	3.6	6.6
S_{FG}	1.7	3.9	3.8	7.0
S_{HP}	0.76	1.1	1.1	1.5
S_{HG}	0.96	1.4	1.4	1.9

The highest threat to failure for all the proposals is due to wear on the pinion. The proposal with the highest factor of safety for contact stress on the pinion and acceptable values for the other parameters is $m = 3$. This design proposal has therefore been selected for the transmission.

The next step in the design is to size the shaft. This design experiences bending moments and torque so the ASME design code for transmission shafting given in Chapter 5 is appropriate. Figure 16.9 gives an approximate arrangement for the shaft illustrating proposed dimensions, shoulders, steps and key ways. In order to determine the shaft diameter using Equation 5.10, it is necessary to know the torque and bending moment and endurance limit for the material.

The transmitted load from the gears is 558 N.

For a spur gear the radial load is given by

$$W_r = W_t \tan \phi = 558 \tan 20 = 203 \text{ N}$$

The mass of the pinion is approximately 0.46 kg. The mass of the shaft is approximately 0.57 kg. Combining these values gives an approximate vertical load due to gravity of 10.1 N.

The mass of the gear is approximately 1.66 kg. The mass of the shaft is approximately 0.57 kg. Combining these values gives an approximate vertical load due to gravity of 21.9 N.

These various loads and the geometry proposed in Figure 16.9 are used to develop the shearing force and bending moment diagrams given in

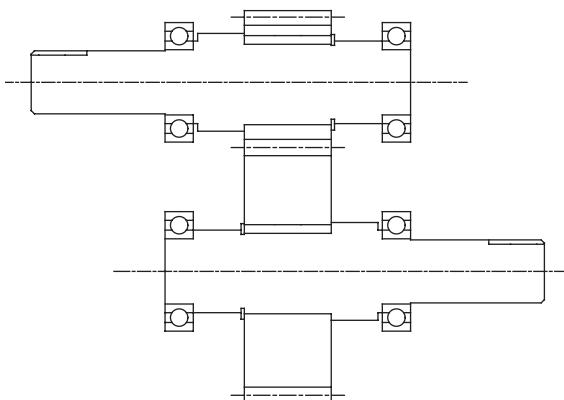


Figure 16.9 General arrangement proposal for the pinion and gear shafts.

Figure 16.10. The maximum resultant bending moments for the pinion and gear shafts are 15.1 N m and 15.4 N m, respectively.

The torque transmitted by the pinion shaft is given by

$$T = \frac{\text{power}}{\omega} = \frac{5000}{2850 \times \frac{2\pi}{60}} = 16.8 \text{ N m}$$

The torque transmitted by the gear shaft is given by

$$T = \frac{\text{power}}{\omega} = \frac{5000}{1500 \times \frac{2\pi}{60}} = 31.8 \text{ N m}$$

An estimate of the shaft diameter needs to be made in order to evaluate the endurance limit.

Here $d = 30 \text{ mm}$ will be assumed. 817M40 hot-rolled alloy steel will be proposed for the shaft. The ultimate tensile and yield strengths for this material are 1000 and 770 MPa, respectively. Taking $k_a = 0.405$, $k_b = 0.856$, $k_c = 0.814$, $k_d = 1$, $k_e = 1$, $k_t = 2$, $q = 0.72$, $k_f = 0.538$, $k_g = 1$ and $\sigma_e' = 504 \text{ MPa}$ gives $\sigma_e = 66.1 \text{ MPa}$.

The ASME equation for transmission shafting gives shaft diameters of 29.3 and 29.8 mm for the pinion and gear shafts, respectively. It should be noted that the diameter determined is particularly sensitive to the surface finish and the fatigue stress concentration factors.

It is necessary to check the critical frequency, deflection and slope characteristics of the shaft. The deflection of the shaft can be estimated by considering the shaft to be a simply supported beam. The deflection at mid-span of a simply

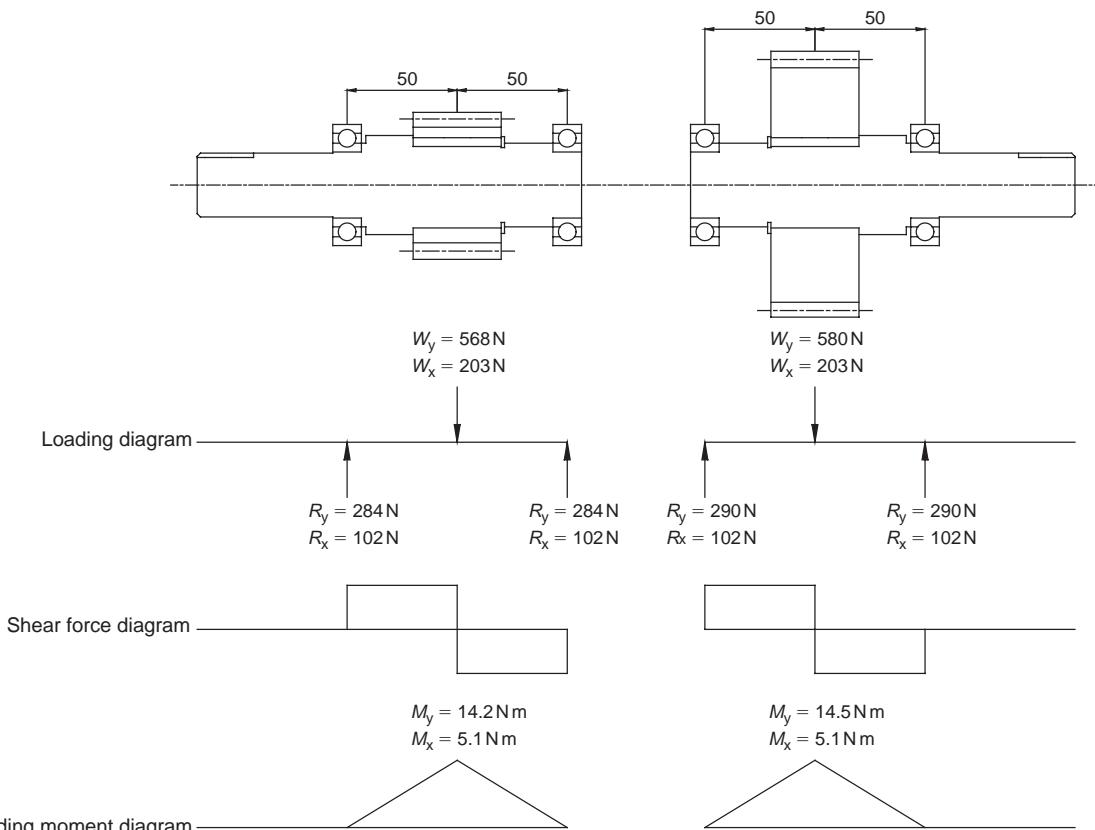


Figure 16.10 Shear and bending moment diagrams for the proposed shafts.

supported beam with a mid-mounted load is given by

$$\gamma = -\frac{WL^3}{48EI}$$

Taking the total mass of the gear and the shaft located at mid-span the deflection is

$$\begin{aligned}\gamma &= -\frac{29.1 \times 0.1^3}{48 \times 200 \times 10^9 \pi 0.03^4 / 64} \\ &= 7.624 \times 10^{-8} \text{ m}\end{aligned}$$

This is a very small deflection indicating that slope of the shaft, at the bearings, and deflection, are unlikely to be a problem in this design.

Substitution in the Rayleigh–Ritz equation gives a first critical frequency of

$$\begin{aligned}\omega_c &= \sqrt{g/\gamma} = \sqrt{9.81/7.624 \times 10^{-8}} \\ &= 11340 \text{ rad/s}\end{aligned}$$

This represents a rotating speed of 108 000 rpm, which is an order of magnitude higher than the operating speed. Even accounting for deflection in the bearings, the critical speed will still be significantly higher than the design speed of the gearbox. This is not surprising given the stocky nature of the shaft with its small span.

Deep groove ball bearings are proposed for the transmission as these bearings provide support for the radial load and axial alignment. The design life for the bearings is 1.02×10^8 revolutions. The maximum load on the bearings is approximately 310 N. The basic dynamic load rating required is given by

$$C = 310 \frac{\sqrt[1/3]{1.02 \times 10^8}}{\rightarrow 1 \times 10^6} = 1448 \text{ N}$$

A 30 mm bore bearing with a basic dynamic load rating greater than this value and a speed capability with grease filled bearings of more than 2850 rpm would be suitable. The bearings from a number of specialist bearing manufacturers should be considered. From Table 4.7, ISO designation

6006 would be suitable with a considerable margin of safety. It is worth considering worst case usage. If the machine was operated continuously for the warranty period and other components did not fail, the design life would be

$$\begin{aligned}2850 \times 60 \times 24 \times 365 \times 1.5 \\ = 2.25 \times 10^9 \text{ revolutions}\end{aligned}$$

Using this figure, the required basic dynamic load rating is 3798 N.

Selecting a low-carbon, cold drawn steel for the key with a yield strength of 350 MPa, assuming a square key of 4×4 mm and setting $N = 4$ for this application, the necessary key length is given by

$$\begin{aligned}L &= \frac{4TN}{DW\sigma_y} = \frac{4 \times 31.8 \times 4}{0.03 \times 0.004 \times 350 \times 10^6} \\ &= 0.0121 \text{ m}\end{aligned}$$

This length can readily be accommodated in the shaft and mating components. Because it is comparatively short, it would be possible to use the same type of key for the gears and couplings. This would have the advantage of a reduced parts inventory, simplifying tooling and economy of scale.

An almost finished engineering drawing for the pinion shaft is given in Figure 16.11. The drawing includes the results of a considerable number of decisions. The calculations outlined so far in this case study have resulted in the specification of some of the major dimensions. However, geometric tolerances need to be specified on certain features in order to control features such as location, form, profile, orientation and runout, and ensure adequate assembly and operation of the machine. In addition, particular surface finishes must be specified on critical regions such as mating surfaces and areas experiencing reversed bending. A typical surface finish for a ground surface is $0.4 \mu\text{m}$ and that for turning or milling is $1.6 \mu\text{m}$. Figure 16.11 should be examined by the reader and each dimension and feature scrutinized.

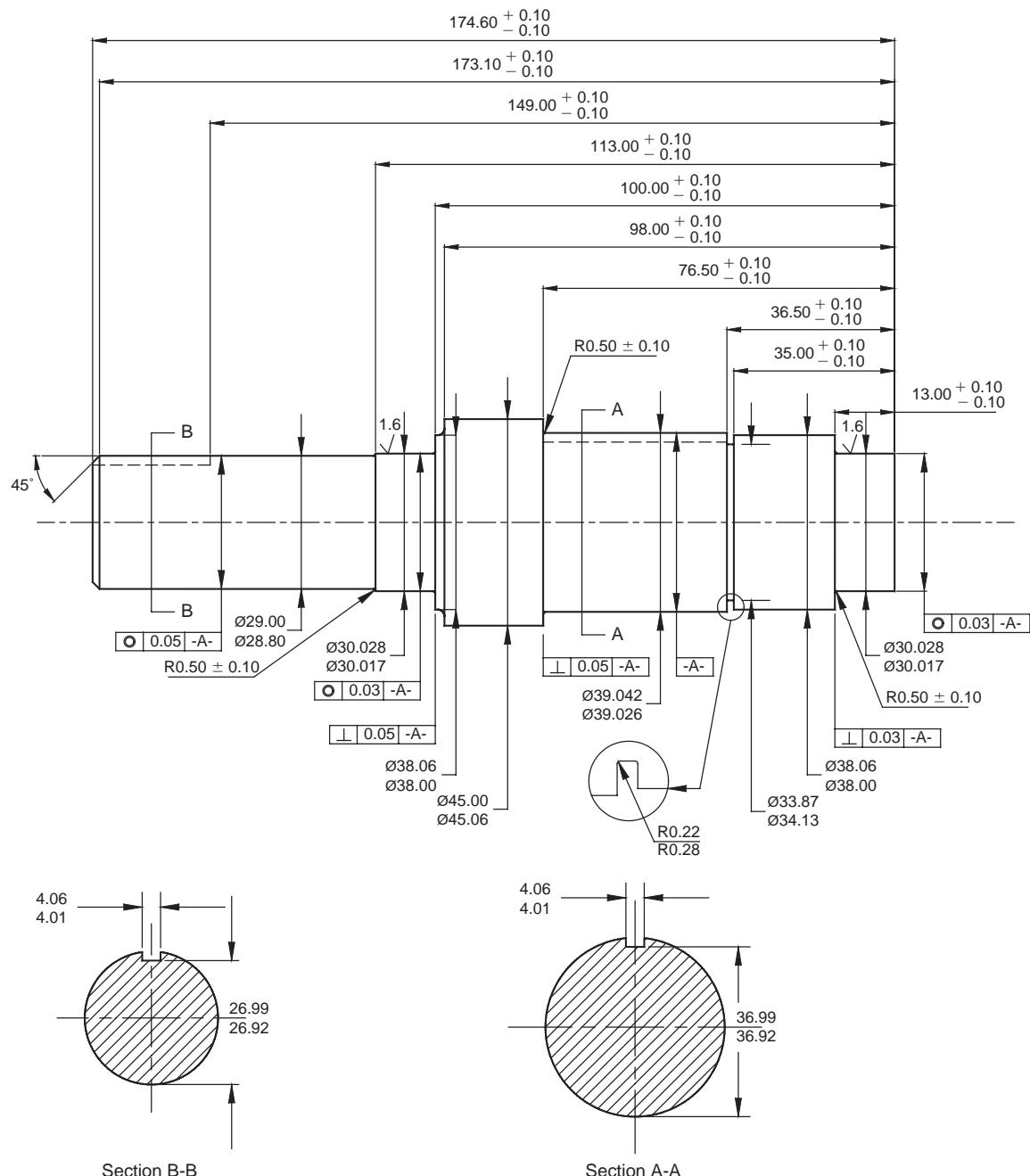


Figure 16.11 Pinion shaft.

The similarity in diameter required for the pinion shaft and gear shaft provides an opportunity to use a common design for both shafts. This has the advantage of reduced inventory, setting up costs

and economies of scale. This approach has been adopted for this design.

The options for the transmission support and enclosure realistically include fabricating, moulding

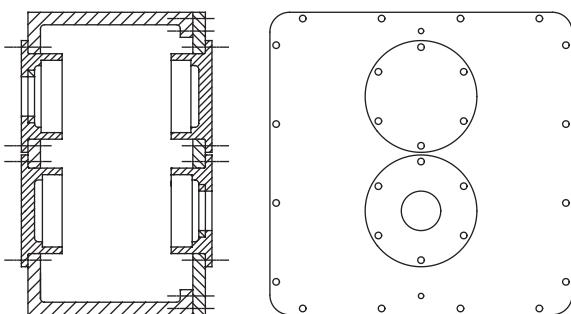


Figure 16.12 Casing.

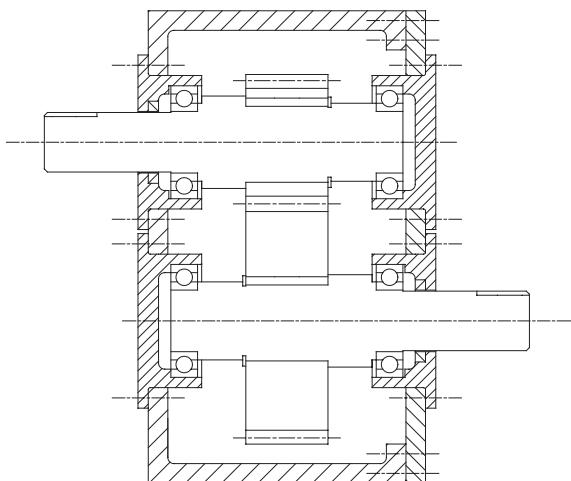


Figure 16.13 General arrangement for the compressor transmission.

or casting using metal or plastic. The temperature of operation indicates that a plastic material is a possibility provided rigidity, strength and cost requirements are met. Metals are, however, an obvious choice. In order to provide rigid support use of a two-part casting assembly is explored in Figure 16.12. Turned inserts are proposed for the stationary bearing mountings. Although this solution appears adequate, fabrication and use of a plastic moulding should also be explored to identify whether they can meet the specification and provide a cost advantage.

Engineering drawings, similar to that given in Figure 16.11, should also be produced for all the components in the transmission. Here, however, just a general arrangement for the overall transmission is given in Figure 16.13.

Although the design developed is probably far from optimal, an estimate of the works cost price should be developed in order to identify whether the proposal is likely to meet the specification in terms of price and which aspects are responsible for determining the majority of the costs. These can then be assessed and the use of expensive features justified or, if possible, removed from the design altogether. Estimating the cost of a product at the design stage is notoriously difficult. In the design proposal presented here, estimates for the costs of stock items such as the gears, bearings, keys and fasteners can be obtained by examination of price lists if published or by contacting the OEM or stock supplier concerned and obtaining a quotation. Ensure that the quotation provides the appropriate discount for the quantities concerned. For a production run of 1000 units per year, it is likely that the transmissions will be manufactured in batches. In this case a quotation for the necessary batch size would seem appropriate so that variations in demand will not tie up company capital unnecessarily. Estimates for the cost of the castings will need to be obtained by means of quotations from casting companies. In this design, machining of the shafts and finish machining of the castings needs to be performed either in-house or bought in. Obviously quotations from machine shops can be obtained to evaluate the possibility of buying the shafts directly. For in-house machining, reliance needs to be placed on experience or use of standard estimates for machining times. Standard estimates for a wide range of machining processes are outlined in Swift and Booker (1997).

From Table 16.10, taking the materials overhead as 130 per cent and the labour overhead as 225 per cent, the works cost price is given by

$$\begin{aligned} \text{Works cost price} \\ = (261 \times 1.3) + (156 \times 2.25) \\ = 690.3 \text{ Euros} \end{aligned}$$

The works cost price for the transmission system is close to the target price of 600 Euros. With careful control of procurement costs and in-house manufacturing costs it should be possible to

Table 16.10 Estimates for component costs based on a batch run of 100 gearboxes

Item	Number	Time	Unit cost (Euro)	Total cost (Euro)	Source of estimate
Pinion	1 off		12	12	Stock gear manufacturer
Gear	1 off		18	18	Stock gear manufacturer
Keys	6 off		0.5	3	Stock supplier
Shaft material	2 off		10	20	Metal supplier
Bearings	4 off		10	40	Stock supplier
Split rings	2 off		2	4	Stock supplier
Main casting	1 off		60	60	Quotation from casting specialist
Casting cover	1 off		20	20	Quotation from casting specialist
Casting seal	1 off		2	2	O ring cording supplier
Bearing mountings material	4 off		5	20	Stock supplier
Shaft seals	2 off		8	16	Seal manufacturer
Couplings	2 off		20	40	Coupling manufacturer
Fasteners	20 off		0.25	5	Fastener stock supplier
Gear hardening	Batch of 200		100	1 per transmission	Specialist service
Materials subtotal				261	
Key way machining	6 off	2 min each	60/h	12	Machine shop
Turning	2 off	15 min/shaft	60/h	30	In-house estimate
Grinding	2 off	10 min/shaft	60/h	20	In-house estimate
Casting machining	1 off	30 min	60/h	30	In-house estimate
Bearing mountings machining	4 off	10 min/mounting	60/h	40	In-house estimate
Assembly	1 off	30 min	40/h	20	In-house estimate
Packaging	1 off	4 min	4	4	In-house estimate
Labour subtotal				156	

get this total cost well below the 600 Euro target. As such, the design is likely to meet the specification.

A proposal for the compressor transmission has been developed. The design appears adequate in terms of function and compliance with the specification, although the works cost price has exceeded the target. It should, however, be possible to refine the design of the shaft and casing and bearing mounts to reduce costs and cheaper quotations for the stock items may be available. It is also possible that one of the alternative concepts, such as the belt drive, could be considerably cheaper to produce.

16.6 Conclusions

The design process can be modelled by the total design model comprising marketing, specification, conceptual design, detailed design, manufacturing and sales activities. Design is an iterative process involving repetition of activities and simultaneous

consideration of other phases. This book has outlined the total design process highlighting the importance of each area. Design skills have been encouraged in the areas of specification, concept generation, design assessment and detailed design. Specific to the detailed design activity, skills in the design, selection and specification of bearings, shafts, gears, belts, chains, clutches and brakes, springs and fasteners have been developed.

Design is a fulfilling profession allowing the design and creation of new and better products. It is hoped that the information presented here will encourage you to apply and develop your design skills.

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Websites

At the time of going to press, the internet contains useful information relating to this chapter at the following sites.

www.astm.org
www.bsi.org.uk
www.eevl.ac.uk
www.iso.ch
www.sae.org
www.co-design.co.uk

Nomenclature

Generally, preferred SI units have been stated.

C	basic dynamic load rating (N)
d	pitch diameter (m)
D	depth (m)
E	Young's modulus (N/m^2)
F	face width (m)
g	acceleration due to gravity (m/s^2)
H_{BG}	Brinell hardness of the gear
H_{BP}	Brinell hardness of the pinion
L	length (m)
m	module

Standards

- British Standards Institution. (1987). BS5750: Part 1: 1987. *Quality systems. Part 1. Specification for design/development, production, installation and servicing*. BSI.

N	design factor
N_G	number of teeth in the gear
N_p	number of teeth in the pinion
PDS	product design specification
S_F	AGMA factor of safety
S_H	AGMA factor of safety
V	velocity (m/s)
T	torque (N m)
W	load (N), width (m)
W_r	radial load (N)
W_t	transmitted load (N)
Y	deflection (m)
ϕ	pressure angle ($^\circ$)
ω	angular velocity (rad/s)
σ_y	yield strength (N/m 2)

Worksheet

- 16.1 Describe, in about 150 words, the total design activity.
- 16.2 What information should occur on a product design specification.
- 16.3 Three concepts are illustrated in Figure 16.14 for a lifting device to raise or lower a surface to reject items from a conveyor. Using an evaluation matrix identify which of the conceptual solutions should be selected for further consideration.
- 16.4 As a result of initial design work, three different conceptual solutions for digging trenches on a building site have emerged including:
 - (a) a standard single bucket mounted on a hydraulic driven linkage,
 - (b) a standard two jaw bucket mounted on a hydraulic driven linkage,
 - (c) a series of buckets mounted on a ladder boom.
 Using an evaluation matrix, determine which of the solutions should be selected for further consideration.
- 16.5 A small and medium enterprise (SME) company has investigated the potential market for an electric motor-driven air compressor. The market analysis has confirmed the requirement for a $360\text{ m}^3/\text{h}$, 7 bar air

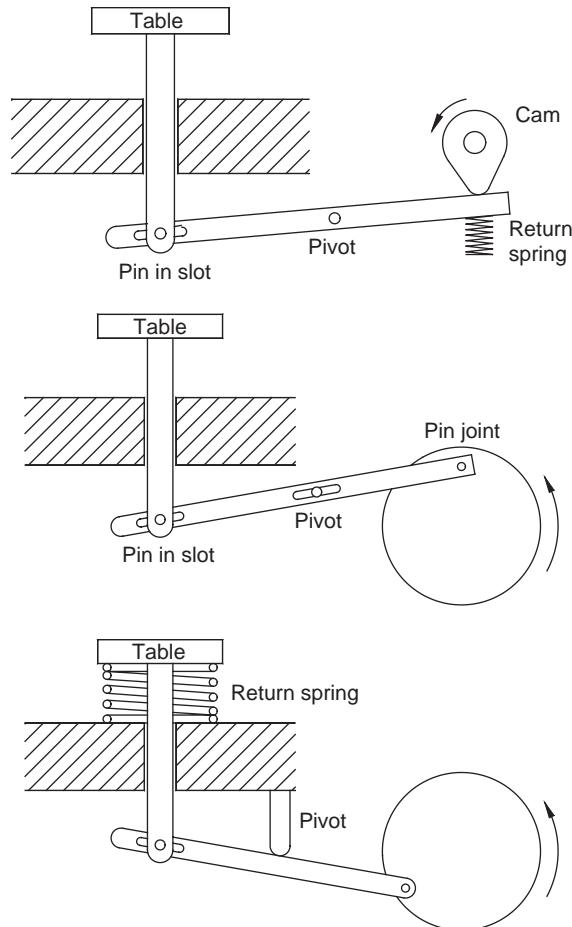


Figure 16.14 Lifting mechanisms (after Hawkes and Abinett, 1984).

supply driven by a 45 kW electric motor. Preliminary design has already resulted in the selection of an electric motor, with a synchronous speed of 1460 rpm, and the piston and cylinder and associated components for a reciprocating compressor. The compressor design speed is 700 rpm. The choice of transmission and its detailed design is yet to be undertaken. The task here is to explore the specification, conceptual design and detailed design of the transmission for this compressor.

- 16.6 A small and medium enterprise (SME) company has investigated the potential market

for a mechanical saw driven by a small internal combustion engine. Preliminary design has already resulted in the selection of the engine, with a design speed of 3000 rpm. The design speed of the saw is 460 rpm. The

choice of transmission and its detailed design is yet to be undertaken. The task here is to explore the specification, conceptual design and detailed design of the transmission for this machine.

Learning objectives achievement

Can you describe the total design activity?	<input type="checkbox"/>	✓	✗	Section 16.1
Can you evaluate alternative concepts and make a selection?	<input type="checkbox"/>	✓	✗	Section 16.2.2
Are you able to determine a works cost price?	<input type="checkbox"/>	✓	✗	Section 16.3.2
Do you have an awareness of the range of information sources available to the designer?	<input type="checkbox"/>	✓	✗	Section 16.4

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