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Mechanical Design II Homework 03

Problem 1

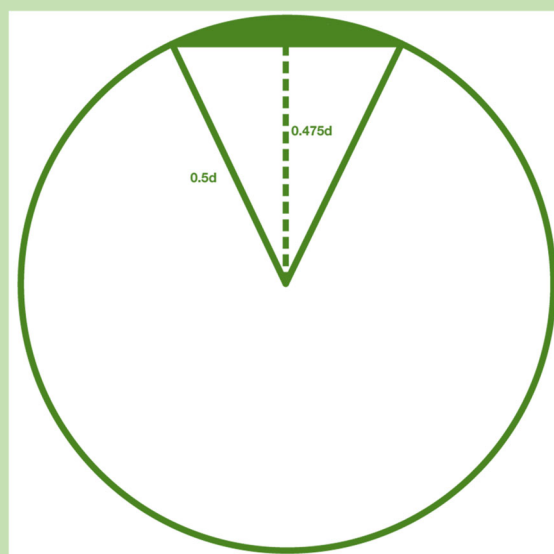
Table 6-3 states that effective diameter d_e of a nonrotating bar is $0.370d$. Use 95 critical stress area method to validate this effective diameter.



Solution:

For this question, we are asked to use 95 critical stress area method to validate this effective diameter d_e of a nonrotating bar.

The effective area is equal to the twice of the dark green area as shown in Figure below.



The area of the sector is equal to

$$A_s = \cos^{-1} \left(\frac{0.475d}{0.5d} \right) \times (0.5d)^2 = 0.07939d^2$$

The area of the triangle is equal to

$$A_t = \sqrt{(0.5d)^2 - (0.475d)^2} \times (0.475d) = 0.07416d^2$$

Therefore, the 95% effective area is equal to

$$A_{0.95\sigma} = 2A_s - 2A_t = 2 \times 0.07939d^2 - 2 \times 0.07416d^2 = 0.01046d^2$$

Hence, we can get the equation below:

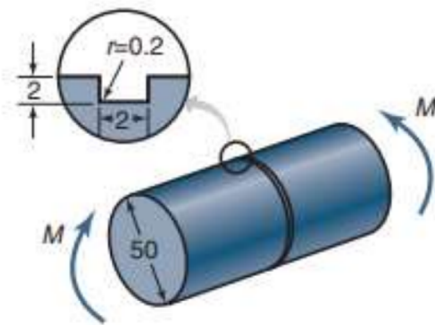
$$0.01046d^2 = \frac{\pi}{4} [d_e^2 - (0.95d_e)^2]$$

$$d_e = 0.370d$$

Problem 2

A rotating round shaft with a flat groove used to seat a retaining ring is shown in the figure below. AISI 1020 steel (quenched and tempered at 870°C) is used for the shaft, which is machined to its final dimensions. Endurance limit of the AISI 1020 steel is 200 MPa and ultimate strength is 395 MPa.

- Estimate the modified endurance limit for the shaft if the shaft is operating under room temperature.
- Calculate the allowable bending moment using a safety factor of 5.0. Use a reliability of 99% and no thermal or miscellaneous effects.



Solution:

- For this question, we are asked to estimate the modified endurance limit for the shaft if the shaft is operating under room temperature.

Surface Condition (machined):

$$k_a = 4.51 \times 395^{-0.265} = 0.9249$$

Size Effect ($d = 50$ mm):

$$k_b = \left(\frac{d}{7.62} \right)^{-0.107} = \left(\frac{50}{7.62} \right)^{-0.107} = 0.8177$$

Loading Effect (bending):

$$k_c = 1$$

Temperature Effect (room temperature):

$$k_d = 1$$

Reliability Effect (99%):

$$k_e = 0.814$$

Therefore, the modified endurance limit is equal to

$$S_e = k_a k_b k_c k_d k_e S'_e = 0.9249 \times 0.8177 \times 1 \times 1 \times 0.814 \times 200 \text{ MPa} \\ = 123.12 \text{ MPa}$$

- b. For this question, we are asked to calculate the allowable bending moment using a safety factor of 5.0. Use a reliability of 99% and no thermal or miscellaneous effects.

According to Figure A-15-16 ($\frac{a}{t} = 1$, $\frac{r}{t} = 0.1$), the stress concentration factors is equal to

$$K_t = 5.5$$

The original stress is equal to

$$\sigma_0 = \frac{32M}{\pi d^3} \\ \sqrt{a} = 0.246 - 3.08 \times 10^{-3} S_{ut} + 1.51 \times 10^{-5} S_{ut}^2 - 2.67 \times 10^{-8} S_{ut}^3 \\ = 0.246 - 3.08 \times 10^{-3} \times 57.29 + 1.51 \times 10^{-5} \times 57.29^2 \\ - 2.67 \times 10^{-8} \times 57.29^3 = 0.114 \\ \Rightarrow a = 0.0130$$

The fatigue stress concentration is equal to

$$K_f = 1 + \frac{(K_t - 1)}{1 + \sqrt{\frac{a}{r}}} = 1 + \frac{(5.5 - 1)}{1 + \sqrt{\frac{0.0130}{0.2}}} = 4.585$$

Therefore,

$$FS \cdot K_f \sigma_0 \leq S_e \\ FS \cdot K_f \frac{32M}{\pi d^3} \leq S_e \\ M \leq \frac{\pi d^3 S_e}{32 FS \cdot K_f} \\ M \leq \frac{\pi \times (46 \text{ mm})^3 \times (123.12 \text{ MPa})}{32 \times 5.0 \times 4.585} = 51.32 \text{ N} \cdot \text{m}$$

Problem 3

Calculate endurance limits and modification factors of a non-rotating rectangular cross section (width=40 mm, height=60mm). The beam is subjected to fully-reversed bending moment. As in Question 02, use AISI 1020 steel with a reliability of 99%, and no thermal or miscellaneous effects.

Solution:

For this question, we are asked to calculate endurance limits and modification factors of a non-rotating rectangular cross section (width=40 mm, height=60 mm). The beam is subjected to fully-reversed bending moment. As in Question 02, use AISI 1020 steel with a reliability of 99%, and no thermal or miscellaneous effects.

Surface Condition (machined):

$$k_a = 4.51 \times 395^{-0.265} = 0.9249$$

Size Effect ($b = 40$ mm, $h = 60$ mm):

$$d_e = 0.808\sqrt{hb} = 0.808\sqrt{(40 \text{ mm}) \times (60 \text{ mm})} = 39.58 \text{ mm}$$

$$k_b = \left(\frac{d_e}{7.62}\right)^{-0.107} = 0.8384$$

Loading Effect (bending):

$$k_c = 1$$

Temperature Effect (room temperature):

$$k_d = 1$$

Reliability Effect (99%):

$$k_e = 0.814$$

Therefore, the modified endurance limit is equal to

$$S_e = k_a k_b k_c k_d k_e S'_e = 0.9249 \times 0.8384 \times 1 \times 1 \times 0.814 \times 200 \text{ MPa} = 126.23 \text{ MPa}$$



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