MEMS1045
Automatic control

Lecture 1

Introduction & revision

Objectives

- Describe the course including the class policy, topics, learning outcomes, etc.
- Explain the concepts of open and closed-loop control
- Revise ordinary differential equations and Laplace transform

Instructors’ & class information

- **Instructor:** S.C. Fok, PhD
- **Office:** Room 222 (Zone 4) Currently outside China
- **Office hours:** Wednesday 14:00 – 16:00;
  Thursday 10:00 – 12:00
- **Email:** saicheong.fok@scupi.cn
- **TAs:**
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  - Li Xiaomin, email: 1103489384@qq.com (section 2)

Lectures:

Section 1: in Zone 3-101 on Monday 8:15-11:00

Section 1: in Zone 3-101 on Tuesday 13:50-16:25

Learning resources

- **Textbook:**

Control System Engineering, 8th edition, Norman S. Nise, Wiley, ISBN – 978-1-119-59435-2

Additional references and supplementary notes (if needed) will be posted on Blackboard

Course objective

The aims of this course are:

- Introduce students to the modelling, analysis and design of control systems, including applications to electromechanical systems
- Enable students to appreciate how characteristics such as stability, transient response, and steady-state error can be changed through dynamic compensation
- Design feedback controllers for single-input, single-output, linear time-invariant systems based on classical control design techniques
- Utilize computer-aided tools in the modelling, analysis and design of feedback control systems

Skill Set
Design including analysis & communication; utilization of computer-aided tools in control system designs

Course overview

No.	Topics
1	Review of differential equations and Laplace transform
2	Modeling of dynamic systems
3	Stability analysis of linear dynamic systems
4	Time response analysis
5	Stability analysis
6	Steady–state error analysis
7	Root locus techniques and design of feedback controller via root locus
8	Frequency response techniques and design of feedback controllers via frequency response

Course learning outcomes

At the completion of this course, students will be able to:

- Analyze system dynamics using mathematical models
- Examine the stability of the dynamic systems
- Evaluate the characteristics of dynamic systems in the time and frequency domains
- Design feedback controllers to regulate the system performance that meets required specification

Assessments & Grading

Description	Percentage
Assignments, quizzes, & class participation	20%
Labs & project	20%
Midterm	30%
Final exam	30%

- Students must follow/satisfy the rules/requirements stated in the assessment items

Class policy

- Attendance at all scheduled class section is expected
- Students who are absent should inform the instructor in a timely manner. They are responsible to acquire class materials and assignment notes from their classmates
- All assignments must be neatly completed and submitted on time. Only in exceptional circumstances where supporting evidence is supplied and discussed with the instructor, in a timely manner, will

  (a) extensions be granted

  (b) late work be accepted without penalty (Penalty will be decided by the instructor based on the circumstances)
- Academic misconduct is not tolerated
- All disputes and appeal of grades must be filed through a written process

Class policy

Blackboard

- Important information concerning this unit of study is placed on Blackboard, accessible via https://learn.scupi.cn/
- It is your responsibility to access on a regular basis the Blackboard site for
  - Course materials,
  - Course announcements,
  - Online quizzes, assignments, projects, etc.
- You should also check your SCUPI email regularly

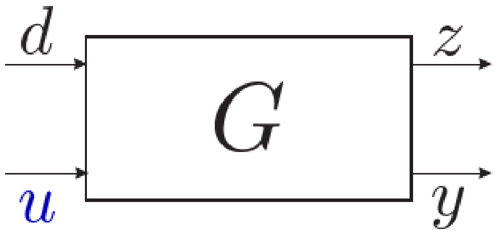
What is the course about?

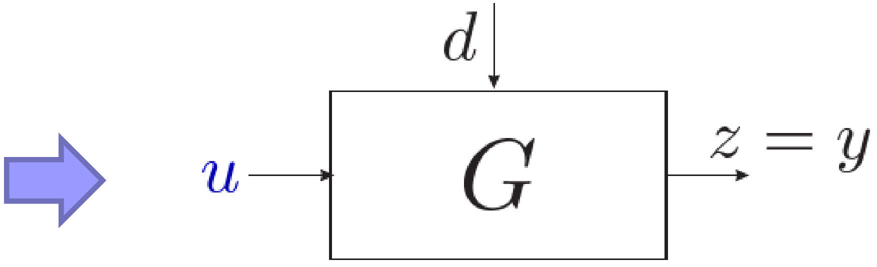
- This course deals with the mathematical modeling and response analysis of dynamic systems so that appropriate controllers can be designed to regulate their performances
- Systems covered include mechanical, electrical and electromechanical systems
- What will we do in this course:

1. Model – derive mathematical equations (ODE) to describe the system
2. Solve – determine the solution to the mathematical model
3. Analyse – evaluate the system performance characteristics
4. Control – design controllers to meet specific performance criteria

What is control?

Given a system G with some inputs for which we can set the values, we want to change its behavior by supplying the proper inputs (u)

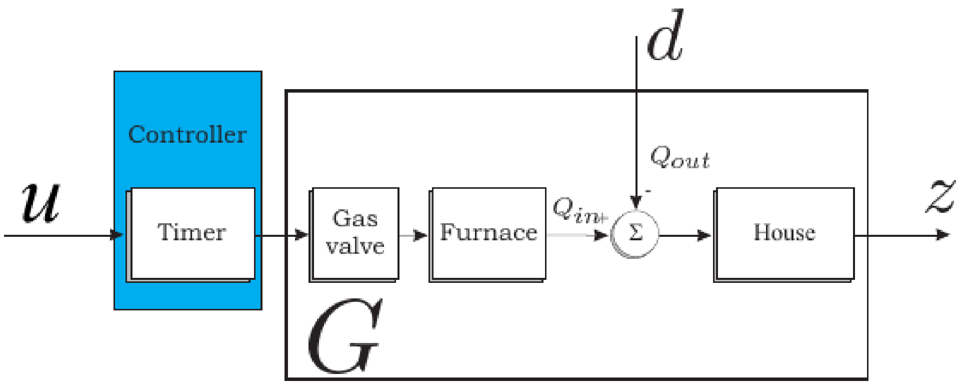


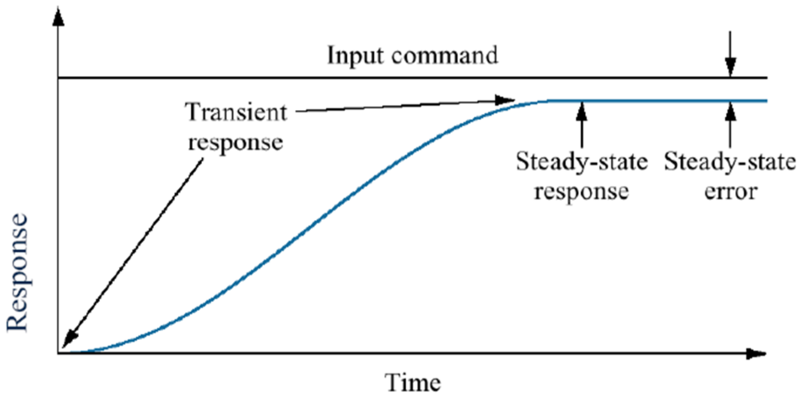


Often we consider $y = z$

Model of the system to be controlled = G
Controllable inputs = u
Disturbances (we cannot influence them) = d
Variable to be controlled to a certain value = z
Measurements available from the system = y

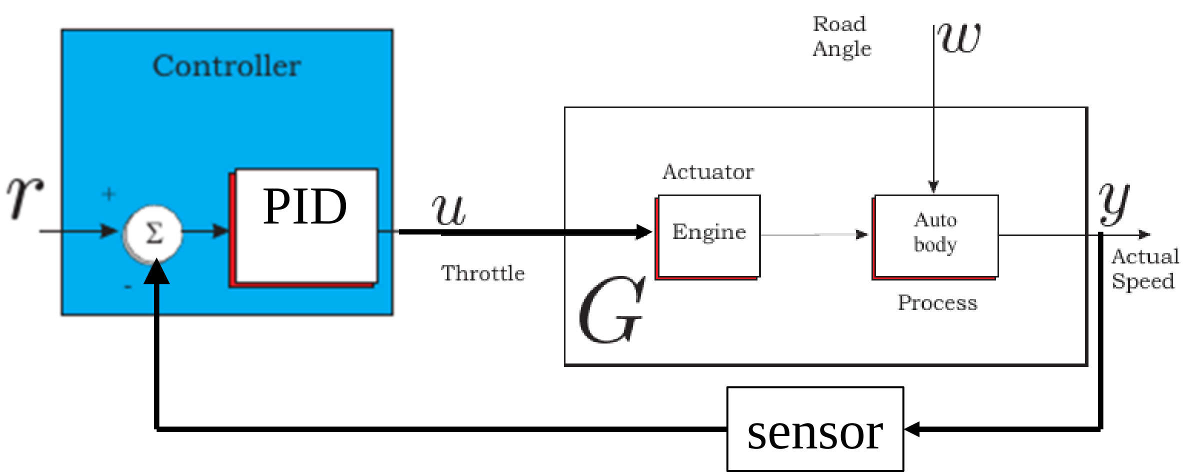
Example – temperature control

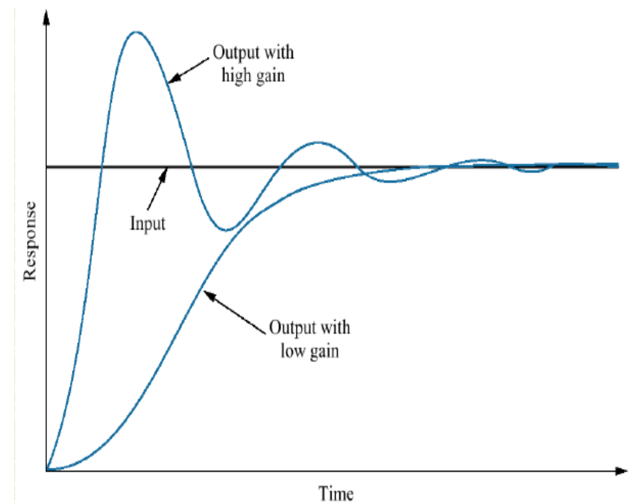




- The input *u* set the timer to automatically set the valve position to obtain a desired room temperature
- This is an example of open-loop control system (no feedback)
- The timer could open and close the valve at constant times like a central heating with no thermostat. It may not work properly: either you sweat or you freeze!!

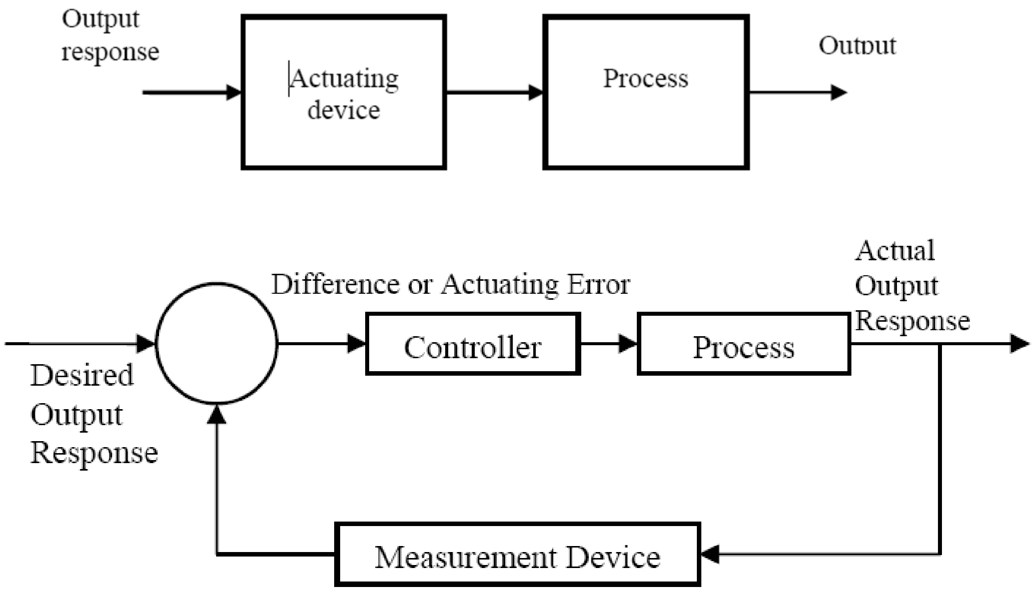
Example – cruise control





- This is an example of closed-loop control system (feedback)
- The reference input is *r* (the driver desired speed). With feedback, the actual speed *y* will become as close as possible to *r*

Open vs closed–loop control



- Feedback is a key tool that can be used to modify the behavior of a system.
- This behavior altering effect of feedback is a key mechanism that control engineers exploit deliberately to achieve the objective of acting on a system to ensure that the desired performance specifications are achieved

Applications

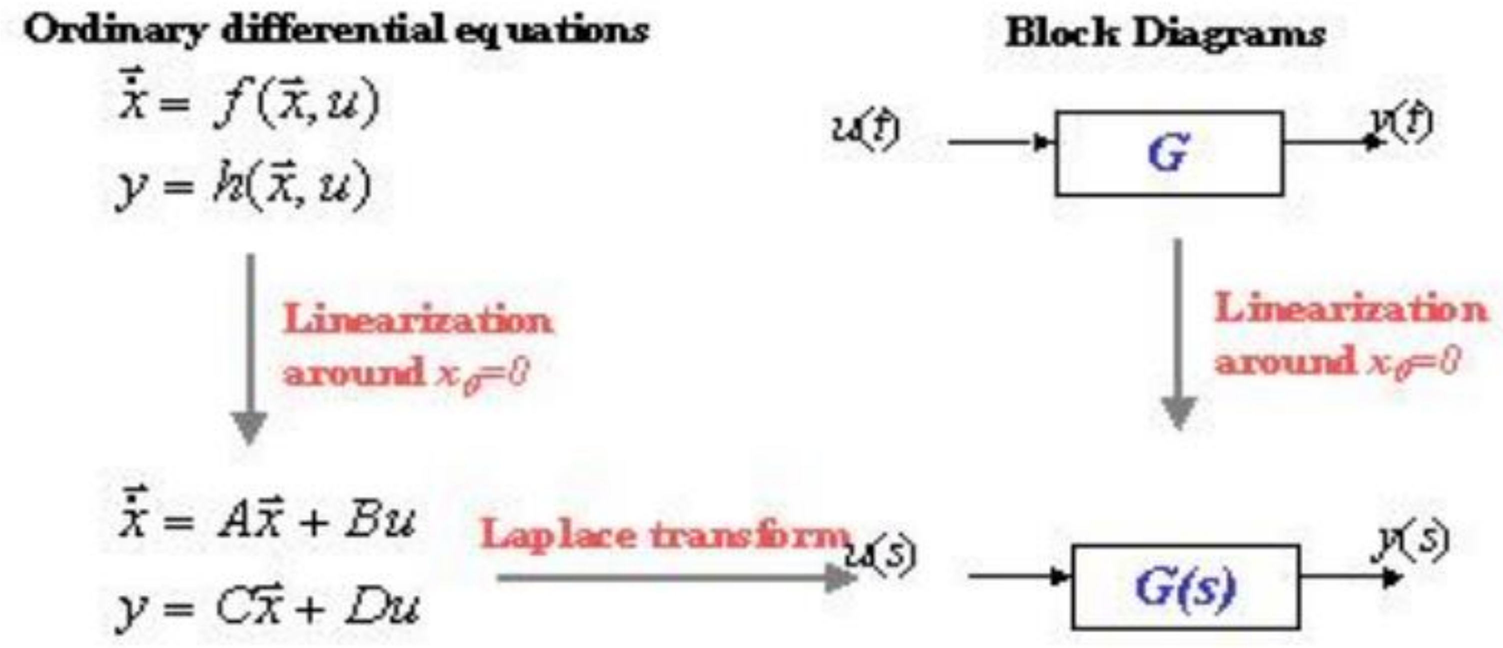
Highly maneuverable aircraft, like this X-29, often require sophisticated control systems to fly stably.

Underwater vehicle

System and models

For the design of the controller, we have to deal in general with the real system "*G*" and derive a "good" mathematical model of the real system within the operating conditions

- Systems can be represented by inputs *u*(*t*) and outputs *y*(*t*) :



Differential equations

An *n*-order linear ordinary differential equation (ODE) has the form:

$$a_0 \frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = u$$

- ❖ If the coefficients of all terms (i.e. a_i, i=0,…,n) are constant, it is called a linear time-invariant ODE; e.g. $2\ddot{y} + 3\dot{y}$ +5y=9
- ❖ It is called a time-varying linear ODE if any coefficient is dependent on time, e.g.

$$\ddot{y} + (1 - t)y = u$$

- ❖ A differential equation is called nonlinear if it is not linear, e.g.

$$\ddot{y} + (y^2 - 1)\dot{y} + y + y^3 = u$$

Differential equations

To solve an n-order linear ODE (with known coefficients)

$$a_0 \frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = u$$

- ❖ The input *u* has to be known and
- ❖ We need to know the *n* initial conditions (or states)

$$y(0), \left.\frac{dy}{dt}\right|_{t=0}, \left.\frac{d^2 y}{dt^2}\right|_{t=0}, \cdots, \left.\frac{d^{n-1} y}{dt^{n-1}}\right|_{t=0}$$

- ❖ Outputs y: Variables of interest to be calculated or measured Initial states and inputs u(t) completely determine future outputs

- E.g. to solve $2\ddot{y} + 3\dot{y}$ +5y=u, we need to know the input u, y(0), and $\dot{y}(0)$

Linear systems

Linearity property:

- Linear system: equations describing the system are linear and principle of superposition holds, e.g. $\frac{\mathrm{dy}}{\mathrm{dt}} + \mathrm{a(t)y} = \mathrm{u}$

$$\mathbf{u}_1(t) \longrightarrow \mathbf{y}_1(t), \text{ and } \mathbf{u}_2(t) \longrightarrow \mathbf{y}_2(t)$$

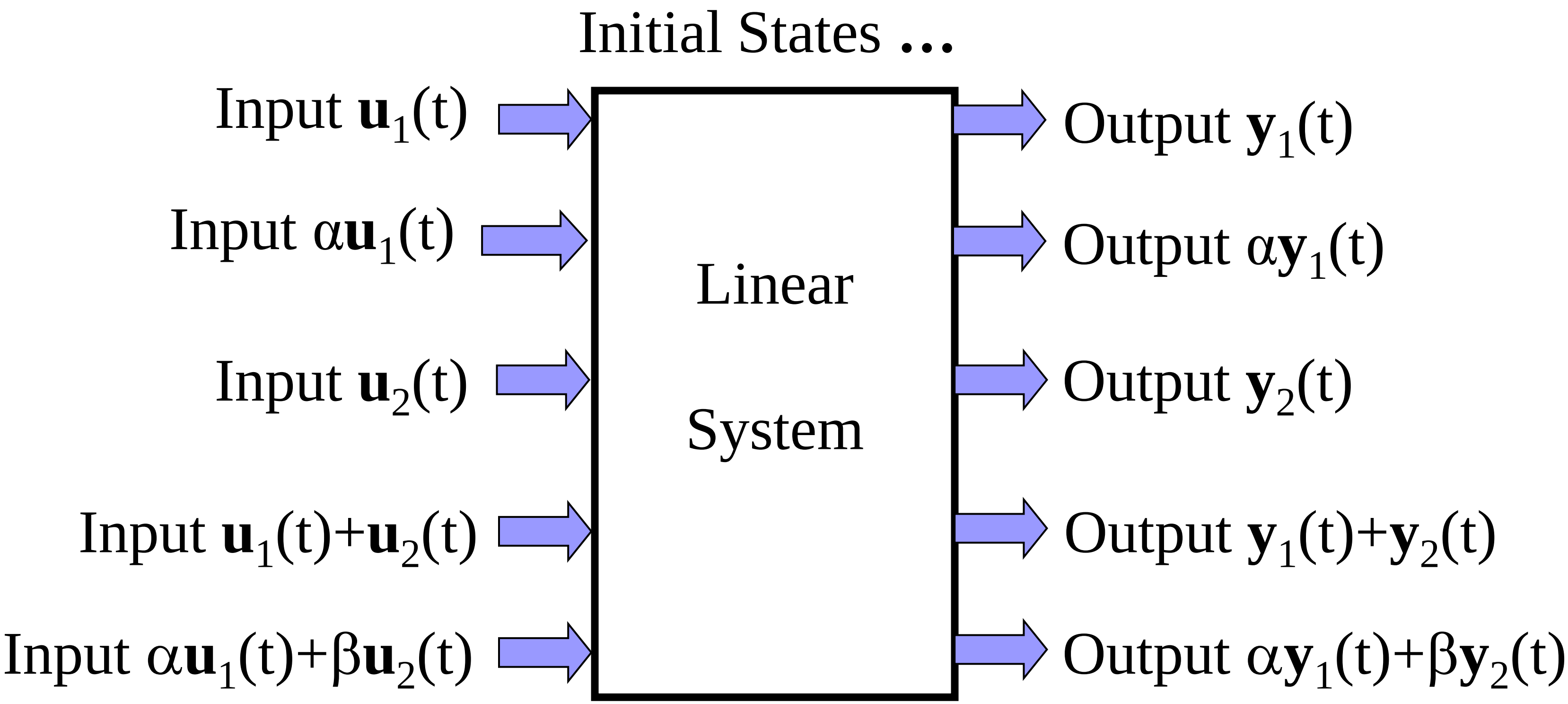
$$\text{Then } \mathbf{u}_1(t)+\mathbf{u}_2(t) \rightarrow \mathbf{y}_1(t)+\mathbf{y}_2(t)$$

$$\text{and } \mathbf{ku}_1(t) \rightarrow \mathbf{ky}_1(t)$$

- Nonlinear system: equations describing the system are nonlinear and principle of superposition does not hold, e.g.

$$\dot{y} + \mathrm{a(t)y} + \mathrm{y}^3 = \mathrm{u}$$

Principle of superposition



Revision – Laplace transform

- Used to solve linear ODEs
- Let $f(t)$ = a function such that $f(t) = 0$ for $t < 0$
- The Laplace transform of $f(t)$ is define as

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

where $s = \sigma + j\omega$ is a complex variable

- The inverse Laplace transform converts $F(s)$ back to $f(t)$:

$$\mathcal{L}^{-1}[F(s)\,] = f(t) = \frac{1}{2j\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds$$

Revision – Laplace transform

TABLE 2.1 Laplace transform table

Item no.	$f(t)$	$F(s)$	
1.	$\delta(t)$	1	Impulse
2.	$u(t)$	$\frac{1}{s}$	Step
3.	$tu(t)$	$\frac{1}{s^2}$	Ramp
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$	Exponential
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$	Sine
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$	Cosine

Note: $u(t) = 1$ for $t > 0$ and $u(t) = 0$ for $t < 0$

Revision – Laplace transform

TABLE 2.2 Laplace transform theorems

Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)]$	$= F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s)-f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s)-sf(0-)-f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s)-\sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.
[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Poles and zeros

Given a complex function $F(s)$ of the form:

$$F(s)=\frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)^3\cdots(s+p_n)}=\frac{\text{numerator}}{\text{denominator}}$$

- Points at which $F(s)$ equals infinity are called poles, i.e.

  $s=-p_1, s=-p_2, \dots, s=-p_n$ are called poles
- The poles can be real or complex
- If the denominator involves multiple factors $(s+p)^r$ then s=-p is called a multiple pole of order r (if r = 1, it is a simple pole)
- Points at which $F(s)$ = 0 are called zeros, i.e.

  $s=-z_1, s=-z_2, \dots, s=-z_m$ are called zeros
- $F(s)$ is a strictly proper rational function if $m < n$

Distinct real poles

If $F(s)$ is a strictly proper rational function with distinct real poles (in denominator), it can always be written as a sum of simple partial fractions:

$$F(s) = \frac{A_1}{(s+p_1)} + \frac{A_2}{(s+p_2)} + \cdots + \frac{A_n}{(s+p_n)}$$

where $A_1, A_2, \ldots, A_n$ are constants

- ❖ The value of coefficient A_k (for k=1, … n) can be found using:

$$A_k = (s+p_k)F(s)\Big|_{s=-p_k}$$

Since

$$\mathcal{L}^{-1}\left[\frac{A_k}{(s+p_k)}\right] = A_k e^{-p_k t}$$

$$f(t) = \mathcal{L}^{-1}[F(s)] = A_1 e^{-p_1 t} + A_2 e^{-p_2 t} + \cdots + A_n e^{-p_n t} \quad \text{for } t>0$$

Example 1

Solve $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 5u(t)$ given u = unit step and all initial conditions are zero

$$(s^2Y(s) - sy(0) - \dot{y}(0)) + 5(sY(s) - y(0)) + 4Y(s) = \frac{5}{s}$$

$$s^2Y(s) + 5sY(s) + 4Y(s) = (s+1)(s+4)Y(s) = \frac{5}{s}$$

$$Y(s) = \frac{5}{s(s+1)(s+4)} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+4}$$

$$A_1 = (s)Y(s)\Big|_{s=0} = \frac{5}{4}$$

$$A_2 = (s+1)Y(s)\Big|_{s=-1} = -\frac{5}{3}$$

$$A_3 = (s+4)Y(s)\Big|_{s=-4} = \frac{5}{12}$$

$$y(t) = \frac{5}{4} - \frac{5}{3}e^{-t} + \frac{5}{12}e^{-4t} \quad \text{for } t > 0$$

Repeated real poles

If $F(s)$ is a strictly proper rational function with some distinct real poles and some repeated real poles (in denominator), e.g.

$$F(s) = \frac{b_m s^m + \cdots + b_0}{(s+p_1)^r (s+p_{r+1})(s+p_{r+2})\cdots(s+p_n)}$$

the partial-fraction expansion of $F(s)$ is

$$\frac{B_1}{s+p_1} + \frac{B_2}{(s+p_1)^2} + \cdots + \frac{B_{r-1}}{(s+p_1)^{r-1}} + \frac{B_r}{(s+p_1)^r} + \frac{A_{r+1}}{s+p_{r+1}} + \frac{A_{r+2}}{s+p_{r+2}} + \cdots + \frac{A_n}{s+p_n}$$

- ❖ The coefficients of A_j (for distinct real poles) can be found as previously discussed

$$A_k = (s+p_k)F(s)\Big|_{s=-p_k}$$

Repeated real poles

$$\frac{B_1}{s+p_1} + \frac{B_2}{(s+p_1)^2} + \cdots + \frac{B_{r-1}}{(s+p_1)^{r-1}} + \frac{B_r}{(s+p_1)^r} + \frac{A_{r+1}}{s+p_{r+1}} + \frac{A_{r+2}}{s+p_{r+2}} + \cdots + \frac{A_n}{s+p_n}$$

- The coefficients of B_j (for $j=r$, r-1, …, 1) can be found by:

$$B_r = (s+p_1)^r F(s)\big|_{s=-p_1}$$

$$B_{r-1} = \frac{1}{1!}\left\{\frac{d}{ds}(s+p_1)^r F(s)\right\}\bigg|_{s=-p_1}$$

$$\ldots B_{r-k} = \frac{1}{k!}\left\{\frac{d^k}{ds^k}(s+p_1)^r F(s)\right\}\bigg|_{s=-p_1}$$

$$\ldots B_1 = \frac{1}{(r-1)!}\left\{\frac{d^{r-1}}{ds^{r-1}}(s+p_1)^r F(s)\right\}\bigg|_{s=-p_1}$$

Inverse transform for $t > 0$: $f(t) = \mathcal{L}^{-1}[F(s)]$

$$= \left[B_1 + B_2 t + \cdots + \frac{B_{r-1}}{(r-2)!}t^{r-2} + \frac{B_r}{(r-1)!}t^{r-1}\right]e^{-p_1 t} + A_{r+1}e^{-p_{r+1}t} + \cdots + A_n e^{-p_n t}$$

Example 2

Solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 2u(t)$ given u = unit step and all initial conditions are zero

Take Laplace transform: $s^2Y(s) + 4sY(s) + 4Y(s) = \frac{2}{s}$

$$(s+2)^2Y(s) = \frac{2}{s}$$

$$Y(s) = \frac{2}{s(s+2)^2} = \frac{A_1}{s} + \frac{B_2}{(s+2)^2} + \frac{B_1}{s+2}$$

where

$$A_1 = (s)Y(s)\Big|_{s=0} = \frac{1}{2}$$

$$B_2 = (s+2)^2Y(s)\Big|_{s=-2} = -1$$

$$B_1 = \frac{1}{1!}\left\{\frac{d}{ds}(s+2)^2F(s)\right\}\Bigg|_{s=-2} = \left\{\frac{d}{ds}\left(\frac{2}{s}\right)\right\}\Bigg|_{s=-2} = \left\{-\left(\frac{2}{s^2}\right)\right\}\Bigg|_{s=-2} = -\frac{1}{2}$$

$$y(t) = \frac{1}{2} - \frac{1}{2}e^{-2t} - te^{-2t} \quad \text{for } t > 0$$

Complex poles

- For unrepeated complex poles (in denominator), $F(s)$, a strictly proper rational function, can always be written as a sum of simple partial fractions:

$$F(s) = \frac{a_1 s + b_1}{(s+p_1)(s+\bar{p}_1)} + \frac{a_2 s + b_2}{(s+p_2)(s+\bar{p}_2)} + \cdots + \frac{a_n s + b_n}{(s+p_n)(s+\bar{p}_n)}$$

- Note: Complex poles in $F(s)$ always appear in complex conjugate pair

$p_1 = \sigma_1 + j\omega_1$ and $\bar{p}_1 = \sigma_1 - j\omega_1$ is the complex conjugate of p_1

$$(s+p_1)(s+\bar{p}_1) = s^2 + 2\sigma_1 s + \sigma_1^2 + \omega_1^2 = (s+\sigma_1)^2 + \omega_1^2$$

- Each of the term can be arranged as

$$\frac{a_1 s + b_1}{(s+p_1)(s+\bar{p}_1)} = \frac{A_1(s+\sigma_1)}{(s+\sigma_1)^2 + \omega_1^2} + \frac{B_1\omega_1}{(s+\sigma_1)^2 + \omega_1^2}$$

Complex poles

$$\mathcal{L}^{-1}\left[\frac{a_1 s + b_1}{(s+p_1)(s+\bar{p}_1)}\right] = \mathcal{L}^{-1}\left[\frac{A_1(s+\sigma_1)}{(s+\sigma_1)^2+\omega_1^2} + \frac{B_1\omega_1}{(s+\sigma_1)^2+\omega_1^2}\right]$$
$$= \{A_1\cos(\omega_1 t)\}e^{-\sigma_1 t} + \{B_1\sin(\omega_1 t)\}e^{-\sigma_1 t}$$

The value of coefficients A_1 and B_1 can be obtained by completing the squares in the denominator and comparing terms in the numerator

❖ Notes on complex numbers: $z = x + jy = |z|\angle\theta = |z|e^{j\theta}$

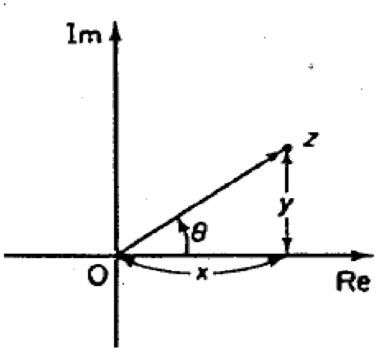
Magnitude: $|z| = \sqrt{x^2+y^2}$ and angle: $\angle z = \theta = \tan^{-1}\left(\frac{y}{x}\right)$

Real part: $x = |z|\cos(\theta)$ and imaginary part: $y = |z|\sin(\theta)$

Euler's theorem: $e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$ and $\alpha z_1 = \alpha x_1 + j\alpha y_1$

$z_1 z_2 = |z_1||z_2|\angle(\theta_1+\theta_2)$ and $\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}\angle(\theta_1-\theta_2)$



Example 3

Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3u(t)$ given all initial conditions are zero

Take Laplace transform: $s^2Y(s) + 2sY(s) + 5Y(s) = \frac{3}{s}$

$$(s^2 + 2s + 5)Y(s) = \frac{3}{s}$$

$$Y(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{3}{s(s^2 + 2\sigma_1 s + \sigma_1^2 + \omega_1^2)}$$

Note complex roots:

$(s^2 + 2s + 5) = (s + 1 + 2j)(s + 1 - 2j)$ or $\sigma_1 = 1$ and $\omega_1 = 2$;

Hence

$$Y(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{K_1}{s} + \frac{A_1(s + \sigma_1)}{(s + \sigma_1)^2 + \omega_1^2} + \frac{B_1\omega_1}{(s + \sigma_1)^2 + \omega_1^2}$$

For real distinct root:

$$K_1 = (s)Y(s)\Big|_{s=0} = \frac{3}{5}$$

Example 3

This reduces the Laplace function to:

$$Y(s) = \frac{3}{s(s^2+2s+5)} = \frac{3}{5s} + \frac{A_1(s+1)}{(s+1)^2+4} + \frac{2B_1}{(s+1)^2+4}$$

$$Y(s) = \frac{3}{s(s^2+2s+5)} = \frac{(3/5)}{s} + \frac{A_1(s+1)+2B_1}{(s^2+2s+5)}$$

$$\frac{3}{s(s^2+2s+5)} = \frac{(3/5)(s^2+2s+5) + A_1 s(s+1) + 2B_1 s}{s(s^2+2s+5)}$$

Comparing the terms on the LHS and RHS:

$A_1 + \frac{3}{5} = 0$ and $\frac{6}{5} + A_1 + 2B_1 = 0$ or $A_1 = -\frac{3}{5}$ and $B_1 = \frac{3}{10}$

$$F(s) = \frac{(3/5)}{s} + \frac{-(3/5)\,(s+\sigma_1)}{(s+\sigma_1)^2+\omega_1^2} + \frac{(3/10)\omega_1}{(s+\sigma_1)^2+\omega_1^2}$$

Inverse transform:

$$y(t) = (3/5) - \{(3/5)\cos(2t)\}e^{-t} + \{(3/10)\sin(2t)\}e^{-t} \text{ for } t > 0$$