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Mechanical Design 1

08 Assignment

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Mechanical Design 1

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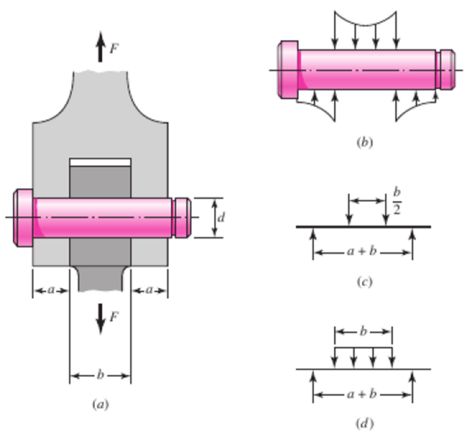
Problem 1

The clevis pin shown in the figure is 12 mm in diameter and has the dimensions a = 12 mm and b = 18 mm. The pin is machined from AISI 1018 hot-rolled steel (S_y = 220 MPa) and is to be loaded to no more than 4.4 kN. Determine whether or not the assumed loading of figure c yields a factor of safety any different from that of figure d. Use the maximum-shear-stress theory. Repeat the analysis using the distortion-energy theory



Solution:

For this question, we are asked to determine whether or not the assumed loading of figure c yields a factor of safety any different from that of figure d. Use the maximum-shear-stress theory. Repeat the analysis using the distortion-energy theory.

Maximum-shear-stress theory:

For figure c:

$$V(x)=\begin{cases}\frac{F}{2},\frac{a}{2}\le x<a+\frac{b}{4}\\ 0,a+\frac{b}{4}\le x\le a+\frac{3}{4}b\\ -\frac{F}{2},a+\frac{3}{4}b<x\le\frac{3}{2}a+b\end{cases}$$

$$M(x)=\begin{cases}\frac{F}{2}\left(x-\frac{a}{2}\right),\frac{a}{2}\le x<a+\frac{b}{4}\\ \frac{F}{2}\left(\frac{a}{2}+\frac{b}{4}\right),a+\frac{b}{4}\le x\le a+\frac{3}{4}b\\ -\frac{F}{2}\left(x-\frac{3}{2}a-b\right),a+\frac{3}{4}b<x\le\frac{3}{2}a+b\end{cases}$$

Therefore, $M_{max}=\frac{F}{2}\left(\frac{a}{2}+\frac{b}{4}\right)$.

$$\sigma=\frac{My}{I}=\frac{M\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}=\frac{32M}{\pi d^3}=\frac{32\times\frac{F}{2}\left(\frac{a}{2}+\frac{b}{4}\right)}{\pi d^3}=\frac{16\times(4.4\ \text{kN})\times\left(\frac{12\ \text{mm}}{2}+\frac{18\ \text{mm}}{4}\right)}{\pi\times(12\ \text{mm})^3}$$
$$=136.2\ \text{MPa}$$

$$\tau=\frac{VQ}{It}=\frac{4}{3}\frac{V}{A}=\frac{4}{3}\frac{\frac{F}{2}}{\pi r^2}=\frac{4}{3}\frac{\frac{F}{2}}{\pi\left(\frac{d}{2}\right)^2}=\frac{8}{3}\frac{F}{\pi d^2}=\frac{8}{3}\times\frac{(4.4\ \text{kN})}{\pi\times(12\ \text{mm})^2}=25.94\ \text{MPa}$$

$$\tau_{max}=\sqrt{\left(\frac{\sigma}{2}\right)^2}=\sqrt{\left(\frac{136.2\ \text{MPa}}{2}\right)^2}=68.1\ \text{MPa}$$

$$n=\frac{\frac{S_y}{2}}{\tau_{max}}=\frac{\frac{220\ \text{MPa}}{2}}{68.1\ \text{MPa}}=1.615$$

For figure d:

$$M_{max}=\frac{F}{2}\left(\frac{a+b}{2}\right)-\frac{F}{4}\left(\frac{b}{2}\right)^2=\frac{F}{2}\left(\frac{a}{2}+\frac{b}{4}\right)$$

$$\sigma_x=\frac{My}{I}=\frac{M\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}=\frac{32M}{\pi d^3}=\frac{32\times\frac{F}{2}\left(\frac{a}{2}+\frac{b}{4}\right)}{\pi d^3}=\frac{16\times(4.4\ \text{kN})\times\left(\frac{12\ \text{mm}}{2}+\frac{18\ \text{mm}}{4}\right)}{\pi\times(12\ \text{mm})^3}$$
$$=136.2\ \text{MPa}$$

$$\sigma_y = -\frac{F}{A} = -\frac{F}{bd} = -\frac{(4.4\ \text{kN})}{(18\ \text{mm}) \times (12\ \text{mm})} = -20.37\ \text{MPa}$$

$$\Rightarrow \tau_{max} = \frac{\sigma_x - 0}{2} = 68.1\ \text{MPa}$$

$$n = \frac{\frac{S_y}{2}}{\tau_{max}} = \frac{\frac{220\ \text{MPa}}{2}}{68.1\ \text{MPa}} = 1.615$$

Therefore, the assumed loading of figure c yields a factor of safety not different from that of figure d using the maximum-shear-stress theory.

Distortion-energy theory:

For figure c:

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{\frac{1}{2}} = (\sigma_x^2)^{\frac{1}{2}} = 136.2\ \text{MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{220\ \text{MPa}}{136.2\ \text{MPa}} = 1.615$$

For figure d:

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{\frac{1}{2}}$$

$$= [(136.2\ \text{MPa})^2 - (136.2\ \text{MPa}) \times (20.37\ \text{MPa}) + (20.37\ \text{MPa})^2]^{\frac{1}{2}}$$

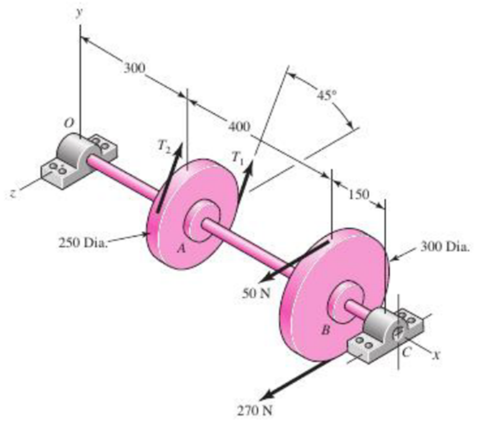
$$= 127.2\ \text{MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{220\ \text{MPa}}{127.2\ \text{MPa}} = 1.729$$

Therefore, the assumed loading of figure c yields a factor of safety different from that of figure d using the distortion-energy theory.

Problem 2

The figure is a schematic drawing of a countershaft that supports two V-belt pulleys. For each pulley, the belt tensions are parallel. For pulley A consider the loose belt tension is 15 percent of the tension on the tight side. A cold-drawn UNS G10180 steel shaft (S_y = 370 MPa) of uniform diameter is to be selected for this application. For a static analysis with a factor of safety of 3.0, determine the minimum preferred size diameter. Use the distortion-energy theory. Repeat using maximum shear stress



Solution:

For this question, we are asked to determine the minimum preferred size diameter using the distortion-energy theory and maximum-shear-stress theory.

Distortion-energy theory:

$$(270\ \text{N} - 50\ \text{N}) \times (150\ \text{mm}) = (T_1 - 0.15T_1) \times (125\ \text{mm})$$

$$\Rightarrow T_1 = 310.6\ \text{N}$$

$$T_2 = 0.15T_1 = 46.6\ \text{N}$$

$$-R_{oxz} \times (850\ \text{mm}) + (T_1 + T_2)\sin\theta \times (550\ \text{mm}) = 0$$

$$\Rightarrow R_{oxz} = 163.42\ \text{N}$$

$$\Rightarrow R_{cxz} = 89.14\ \text{N}$$

$$-R_{oxy} \times (850\ \text{mm}) + (T_1 + T_2)\sin\theta \times (550\ \text{mm}) - (270\ \text{N} + 50\ \text{N}) \times (150\ \text{mm}) = 0$$

$$\Rightarrow R_{oxy} = 107.0\ \text{N}$$

$$\Rightarrow R_{cxy} = 174.4\ \text{N}$$

$$M_A = (300\ \text{mm}) \times \sqrt{(163.42\ \text{N})^2 + (107.0\ \text{N})^2} = 58.59\ \text{N} \cdot \text{m}$$

$$M_B = (150\ \text{mm}) \times \sqrt{(89.14\ \text{N})^2 + (174.4\ \text{N})^2} = 29.38\ \text{N} \cdot \text{m}$$

$$\Rightarrow \sigma_x = \frac{32M_{max}}{\pi d^3} = \frac{32 \times (58.59\ \ \mathrm{N \cdot m})}{\pi d^3}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16 \times (270\ \ \mathrm{N} - 50\ \ \mathrm{N}) \times (150\ \ \mathrm{mm})}{\pi d^3}$$

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{\frac{1}{2}}$$

$$= \left[\left(\frac{32 \times (58.59\ \ \mathrm{N \cdot m})}{\pi d^3}\right)^2 + 3 \right.$$

$$\left. \times \left(\frac{16 \times (270\ \ \mathrm{N} - 50\ \ \mathrm{N}) \times (150\ \ \mathrm{mm})}{\pi d^3}\right)^2\right]^{\frac{1}{2}} = \frac{370\ \ \mathrm{MPa}}{3.0}$$

$$\Rightarrow d = 17.5\ \ \mathrm{mm}$$

$$\Rightarrow d = 18\ \ \mathrm{mm}$$

Maximum-shear-stress theory:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{\frac{32 \times (58.59\ \ \mathrm{N \cdot m})}{\pi d^3}}{2}\right)^2 + \left(\frac{16 \times (270\ \ \mathrm{N} - 50\ \ \mathrm{N}) \times (150\ \ \mathrm{mm})}{\pi d^3}\right)^2}$$

$$= \frac{\frac{S_y}{2}}{n} = \frac{\frac{370\ \ \mathrm{MPa}}{2}}{3.0}$$

$$\Rightarrow d = 17.7\ \ \mathrm{mm}$$

$$\Rightarrow d = 18\ \ \mathrm{mm}$$

— Christopher King —