

ME 1071: Applied Fluids

Lecture 5 External Incompressible Viscous Flow

Spring 2021

Weekly Study Plan





Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
5	Apr. 6	Exam I
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 27	Chapter 9: External Incompressible Viscous Flow
9	May. 4	Chapter 11: Flow in Open Channels
10	May. 11	Chapter 11/Exam II Review
11	May. 18	Exam II
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review

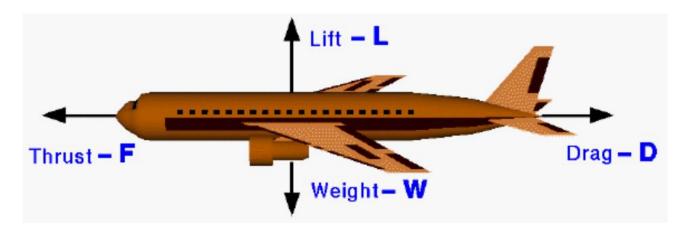
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Outlines





- Boundary Layer Review
- Drag
 - Flat Plate Parallel and Perpendicular to the Flow
 - Flow over a Sphere and Cylinder
 - Streamlining
- > Lift



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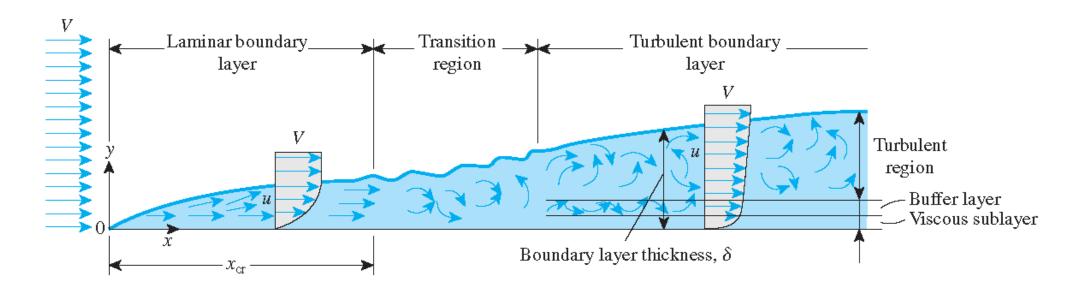




Regions for Boundary Layer Development over a Flat Plate

- The simplest possible boundary layer
- constant pressure field and zero pressure gradient
- Laminar from the leading edge and transits to turbulent downstream

$$ext{Re}_{cr}\!=\!rac{
ho V x_{cr}}{\mu}\!pprox\!500,000$$







Solving Blasius's equation

• Third-order, nonlinear, ordinary differential equation (can be solved using Runge-Kutta method)

$$2f''' + ff'' = 0$$

Boundary conditions:

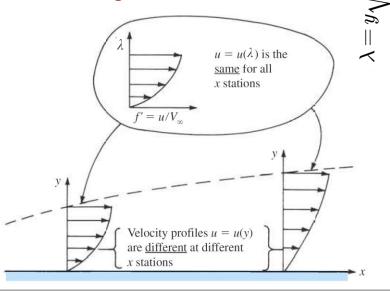
At
$$\lambda = 0$$
: $f = 0$, $f' = 0$; At $\lambda \rightarrow \infty$: $f' = 1$;

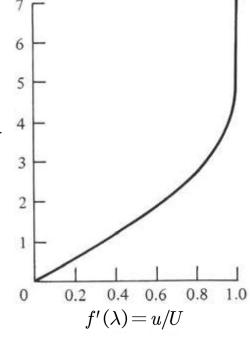
At
$$\lambda \to \infty$$
: $f' = 1$:

At $\lambda = 0$: f''(0) needs to be assumed for shooting method.

Self-similar solution

- Solutions are the same when plotted against a similarity variable λ .
- The governing equations can be reduced to one or more ODEs.
- Occurs only for certain types of flows.





Incompressible velocity profile for a flat plate; solution of the Blasius equation.





Self-similar solution

Local skin-friction coefficient

$$c_f\!=\!rac{ au_w}{rac{1}{2}
ho U^2}\!=\!rac{2\mu}{
ho U^2}\mu\!\left(\!rac{\partial u}{\partial y}\!
ight)_{y=0}\!=\!rac{2\mu}{
ho U^2}U\sqrt{rac{U}{
u x}}\,f''(0)$$

$$=2\sqrt{\frac{\mu}{\rho Ux}}f''(0)=\frac{2f''(0)}{\sqrt{\text{Re}_x}}$$

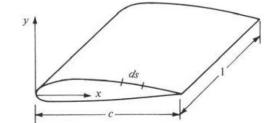
Skin friction drag coefficient (c = chord length)

$$C_f = rac{1}{c} \int_0^c c_f dx = rac{0.664}{c} \int_0^c x^{-1/2} dx = rac{1.328}{c} \sqrt{rac{\mu c}{
ho U}}$$

Boundary-layer thickness

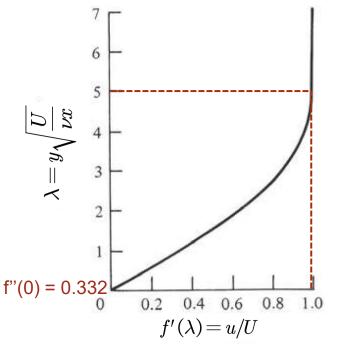
$$\lambda = y\sqrt{\frac{U}{\nu x}} = \delta\sqrt{\frac{U}{\nu x}} \approx 5.0$$
 $\delta = \frac{5.0x}{\sqrt{\mathrm{Re}_x}}$

$$\delta = \frac{5.0x}{\sqrt{\text{Re}_x}}$$



$$c_{\it f} = rac{0.664}{\sqrt{{
m Re}_{\it x}}}$$

$$au_w = rac{0.332
ho U^2}{\sqrt{\mathrm{Re}_x}}$$



Incompressible velocity profile for a flat plate; solution of the Blasius equation.

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Self-similar solution

Displacement thickness

$$\delta^* = \int_0^{y_1} \left(1 - \frac{u}{U}\right) dy = \sqrt{\frac{\nu x}{U}} \int_0^{\lambda_1} \left[1 - f'(\lambda)\right] d\lambda = \sqrt{\frac{\nu x}{U}} \left[\lambda_1 - f(\lambda_1)\right] \xrightarrow{\text{when } \lambda > 5} \delta^* = \frac{1.72x}{\sqrt{\text{Re}_x}}$$

Momentum thickness

$$\theta = \int_0^{y_1} \frac{u}{U} \left(1 - \frac{u}{U} \right) dy \qquad \begin{array}{c} \text{Variable transformation} \\ \theta = \sqrt{\frac{\nu x}{U}} \int_0^{\lambda_1} f'[1 - f'] d\lambda \end{array} \qquad \begin{array}{c} = 0.664 \\ \hline \theta = \frac{0.664x}{\sqrt{\text{Re}_x}} \end{array}$$

Relative relations

$$\delta = \frac{5.0x}{\sqrt{\mathrm{Re}_x}}$$

$$\delta^* = \frac{1.72x}{\sqrt{\mathrm{Re}_x}}$$

$$\theta = \frac{0.664x}{\sqrt{\mathrm{Re}_x}}$$

$$\theta = \frac{0.664x}{\sqrt{\mathrm{Re}_x}}$$

$$\delta^* = 0.34\delta$$

$$\theta = 0.13\delta$$

$$\theta_{x=c} = \frac{0.664c}{\sqrt{\mathrm{Re}_c}}$$

$$\theta_{x=c} = \frac{0.664c}{\sqrt{\mathrm{Re}_c}}$$
 The integrated skin-friction coefficient for the flat plate is proportional to θ at the trailing edge.





Flow with Zero Gradient

Laminar Flow More practical approximation

$$rac{u}{U} = 2 \left(rac{y}{\delta}
ight) - \left(rac{y}{\delta}
ight)^2 \hspace{0.5cm} au_w = rac{2 \mu U}{\delta} \hspace{0.5cm} rac{\delta}{x} = \sqrt{rac{30 \mu}{
ho U x}} = rac{5.48}{\sqrt{\mathrm{Re}_x}} \hspace{0.5cm} C_f = rac{0.730}{\sqrt{\mathrm{Re}_x}}$$

Turbulent Flow More practical approximation

$$rac{u}{U} = \left(rac{y}{\delta}
ight)^{1/7} \qquad rac{\delta}{x} = 0.382 \left(rac{
u}{Ux}
ight)^{1/5} = rac{0.382}{{
m Re}_x^{-1/5}} \qquad C_f = rac{0.0594}{{
m Re}_x^{-1/5}}$$





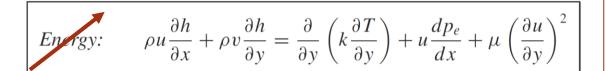
Incompressible boundary-layer equations

General 2-D steady equations

Continuity:
$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

x momentum:
$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right)$$

$$y momentum: \qquad \frac{\partial p}{\partial y} = 0$$



Incompressible flow over a flat plate

Constant
$$\rho$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Constant
$$\rho$$
, μ $d\rho / dx = 0$ $u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2}$ (运动黏度) $v \equiv \mu/\rho$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2}$$

Kinematic viscosity (运动黏度)
$$v \equiv \mu/\rho$$

$$\frac{\partial p}{\partial y} = 0$$

Energy equation is not considered in obtaining the Blasius' equation. The exact solution is different from the solution for real conditions.

Outlines





- Boundary Layer Review
- > Drag
 - Flat Plate Parallel and Perpendicular to the Flow
 - Flow over a Sphere and Cylinder
 - Streamlining
- > Lift





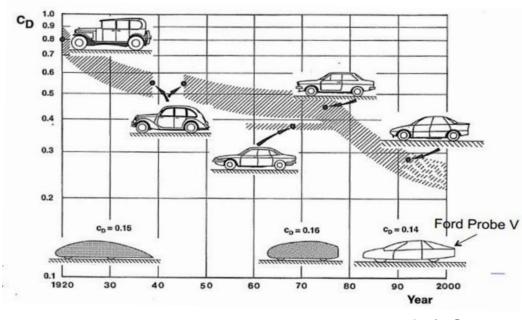
Drag Force

- Drag is a mechanical force generated by a solid object moving through a fluid.
- It acts parallel to the direction of relative motion.
- It depends on relative velocity V, the body shape and size, and the fluid properties.

• Drag Coefficient
$$C_D = rac{F_D}{rac{1}{2}
ho V^2 A}$$
 $C_D = f(\mathrm{Re})$

When considering compressibility and free-surface effect

$$C_D = f(\mathrm{Re,Fr,M}) \qquad \qquad \mathrm{M} = rac{V}{c}, \; \mathrm{Fr} = rac{v}{\sqrt{lg}}$$





Drag Coefficient

A dimensionless quantity that is used to quantify the drag or resistance of an object in a fluid environment.

$$Drag\ Force\ \emph{\emph{F}}_{\emph{D}} = Pressure\ Drag \ + Friction\ Drag + Induced\ Drag$$

$$C_{\scriptscriptstyle D}\!=\!rac{F_{\scriptscriptstyle D}}{rac{1}{2}
ho V^2 A}$$

The specific aera A depends on type of C_D

- Car projected frontal area of the vehicle
- Airfoil the nominal wing area

Shape and Flow	Pressure Drag	Skin Friction
	≈100%	≈0%
	≈90%	≈10%
	≈60%	≈40%
	≈10%	≈90%
	≈0%	≈100% 五纳百川 有容乃大



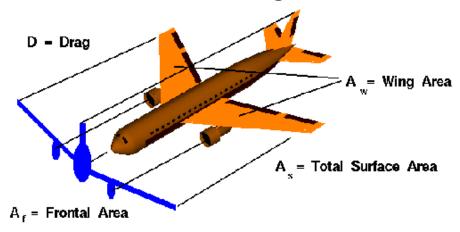


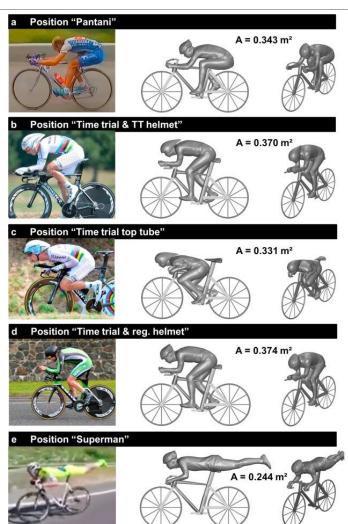
Drag Coefficient

$$C_D\!=\!rac{F_D}{rac{1}{2}
ho V^2 A}$$

The specific aera A depends on type of C_D

- Car projected frontal area of the vehicle
- Airfoil the nominal wing area





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Pure Friction Drag: Flow over a Flat Plate

Where *A* is the total surface area in contact with the fluid, i.e., the wetted area.

Laminar Flow

$$C_{f} = \frac{\tau_{w}}{\frac{1}{2}\rho U^{2}} = \frac{0.664}{\sqrt{\text{Re}_{x}}} \longrightarrow C_{D} = \frac{1}{A} \int_{A} 0.664 \ Re_{x}^{-0.5} dA = \frac{1}{bL} \int_{0}^{L} 0.664 \left(\frac{V}{\nu}\right)^{-0.5} x^{-0.5} b dx$$
$$= \frac{0.664}{L} \left(\frac{\nu}{V}\right)^{0.5} \left[\frac{x^{0.5}}{0.5}\right]_{0}^{L} = 1.33 \left(\frac{\nu}{VL}\right)^{0.5}$$





Pure Friction Drag: Flow over a Flat Plate

Laminar Flow

$$C_f = rac{ au_w}{rac{1}{2}
ho U^2} = rac{0.664}{\sqrt{\mathrm{Re}_x}} \ C_D = rac{1.33}{\sqrt{\mathrm{Re}_L}}$$

Turbulent Flow

$$C_f = rac{ au_w}{rac{1}{2}
ho U^2} = rac{0.0594}{ ext{Re}_x^{1/5}} \ C_D = rac{0.0742}{ ext{Re}_L^{1/5}}$$

$$(5 \times 10^5 < \text{Re}_L < 10^7)$$

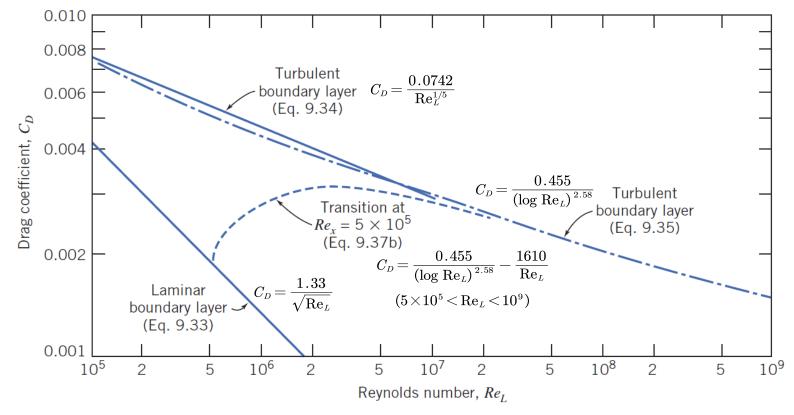


Fig. 9.8 Variation of drag coefficient with Reynolds number for a smooth flat plate parallel to the flow.



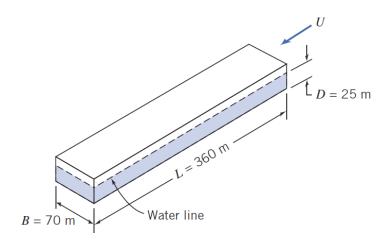


Pure Friction Drag: Flow over a Flat Plate Example

A supertanker is 360 m long and has a beam width of 70 m and a draft of 25 m. Estimate the force and power required to overcome skin friction drag at a cruising speed

of 6.69 m/s in seawater at 10°C.

$$C_D = rac{0.455}{\left(\log \, \mathrm{Re}_L
ight)^{\,2.58}} - rac{1610}{\mathrm{Re}_L}$$



$$Re_{L} = \frac{UL}{\nu} = 6.69 \frac{m}{s} \times 360 \text{ m} \times \frac{s}{1.37 \times 10^{-6} \text{m}^{2}} = 1.76 \times 10^{9}$$

$$C_{D} = \frac{0.455}{(\log 1.76 \times 10^{9})^{2.58}} - \frac{1610}{1.76 \times 10^{9}} = 0.00147$$

$$F_{D} = C_{D}A \frac{1}{2}\rho U^{2}$$

$$= 0.00147 \times (360 \text{ m})(70 + 50) \text{m} \times \frac{1}{2} \times 1020 \frac{\text{kg}}{\text{m}^{3}} \times (6.69)^{2} \frac{\text{m}^{2}}{\text{s}^{2}} \times \frac{\text{N} \cdot \text{s}^{2}}{\text{kg} \cdot \text{m}}$$

$$F_{D} = 1.45 \text{ MN} \longleftarrow F_{D}$$
The corresponding power is
$$\mathcal{P} = F_{D}U = 1.45 \times 10^{6} \text{N} \times 6.69 \frac{\text{m}}{\text{s}} \times \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}}$$

$$\mathcal{P} = 9.70 \text{ MW} \longleftarrow \mathcal{P}$$

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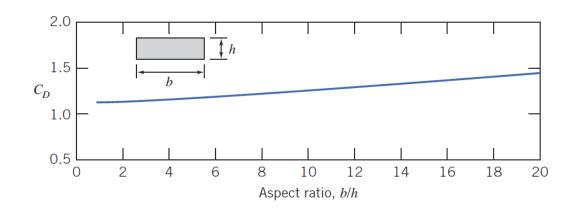
Pure Pressure Drag (Form Drag): Flow Normal to a Flat Plate

Pressure Drag Force

$$F_{D}\!=\!\int_{plate\; surface} \!pdA$$

Drag Coefficient (empirical results only)

 $C_D \approx 1.18 \ for \ aspect \ ratio \ b/h = \ 1 \ , \ \mathrm{Re} \gtrsim 1000$





Drag Coefficient Data for Selected Objects $(Re \gtrsim 10^3)^4$

Object	Diagram		$C_D(Re \gtrsim 10^3)$
Square prism	b	$b/h = \infty$ $b/h = 1$	2.05 1.05
Disk			1.17
Ring			1.20^{b}
Hemisphere (open end facing flow)			1.42
Hemisphere (open end facing downstream)			0.38
C-section (open side facing flow)			2.30
C-section (open side facing downstream)			1.20

^a Data from Hoerner [16].



^b Based on ring area

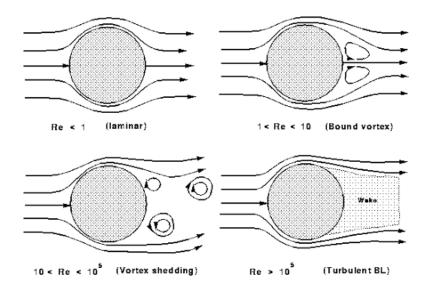


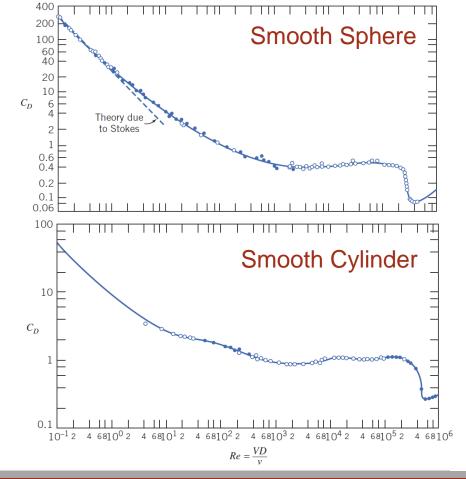


Friction and Pressure Drag: Flow over a Sphere and Cylinder

Laminar Case (Re < 1)

$$F_D = 3\pi\mu Vd \quad \ C_D = rac{24}{\mathrm{Re}}$$







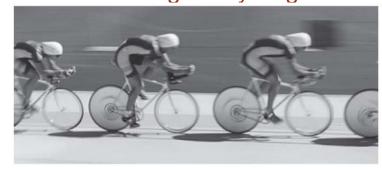


Friction and Pressure Drag

Drag can be significantly reduced by interaction and neighbors



Drafting on Cycling



Energy Savings Drafting formation 27 ± 7% 39±6% **300**



V formation of geese flying







Friction and Pressure Drag

Drag can be significantly reduced by interaction



The fish swim most energetically when they swim not one after the, but at an offset from the swimming direction of the leader. At such locations they harnesses the vortices generated by the leader by intercepting them with their head, splitting the vortex into fragments, that they then guide down their bodies. The progress of these fragmented vortices supplies the fish with thrust without robbing the leader of energy.

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Friction and Pressure Drag Example

A dragster weighing 7120 N attains a speed of 430 km/h in the quarter mile. Immediately after passing through the timing lights, the driver opens the drag chute, of area $A = 2.3 \text{m}^2$. Air and rolling resistance of the car may be neglected. Find the time required for the vehicle to decelerate to 160 km/h in standard air.

$$\sum F_{x} = ma = m\frac{dV}{dt} = -F_{D} = -\frac{1}{2}C_{D}\rho V^{2}A \qquad \qquad -\frac{\frac{1}{2}C_{D}\rho A}{m}\int_{0}^{t}dt = \int_{V_{0}}^{V_{f}}\frac{dV}{V^{2}}$$

$$t = \frac{(V_0 - V_f)}{V_f V_0} \frac{2m}{C_D \rho A} \rightarrow \rightarrow C_D = 1.42 \ (Table \ 9.3)$$

$$t = \frac{(430 - 160)}{(160)(430)} \times \left(\frac{3600 \frac{s}{h}}{1000 \frac{m}{km}}\right) \times \frac{2\left(\frac{7120}{9.81}\right)}{(1.42)(1.227)(2.3)} \qquad t = 5.12 s$$

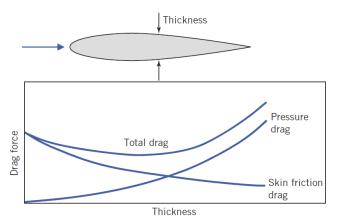
$$Re = \frac{DV}{v} = \frac{\sqrt{\frac{4A}{\pi}}(160)}{1.46 \times 10^{-5}} \times \left(\frac{1000}{3600}\right) = 5.21 \times 10^{6}$$





Streamlining

- Streamlining reduces the effects of boundary layer separation by tapering off the shape on the back end.
- This decreases the pressure gradient and in turn reduces pressure induced drag.
- The overall surface area the flow travels over increases, so skin friction will also increase.
- Airfoil design are determined to minimize the total drag and are used in aircraft and automobiles.







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Outlines





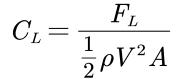
- Boundary Layer Review
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- > Lift

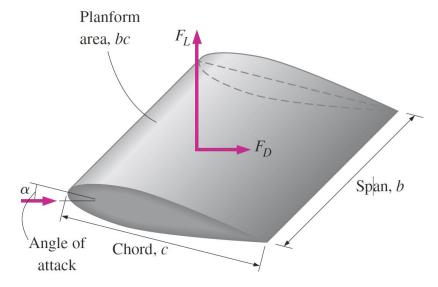


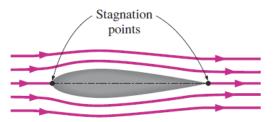


Definition

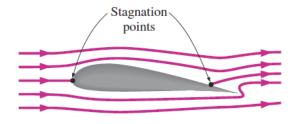
 Lift is defined as the component of the net force (due to viscous and pressure forces) that is perpendicular to the flow direction, and the lift coefficient was expressed as



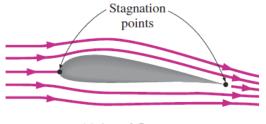




(a) Irrotational flow past a symmetrical airfoil (zero lift)



(b) Irrotational flow past a nonsymmetrical airfoil (zero lift)



(c) Actual flow past a nonsymmetrical airfoil (positive lift)

The Kutta Condition

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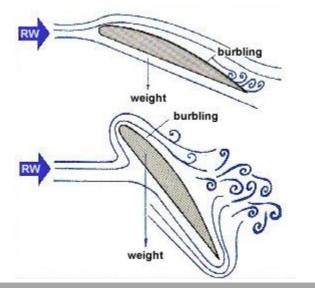


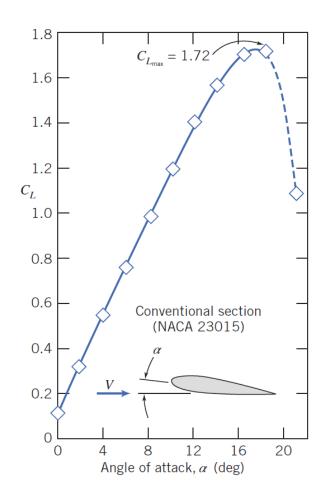


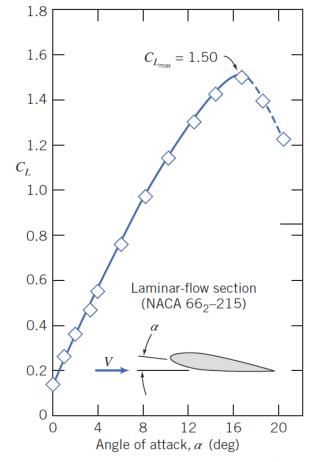
Lift vs Angle of Attack Stall

• C_L decreases as α increases

$$C_{\scriptscriptstyle L} = rac{F_{\scriptscriptstyle L}}{rac{1}{2}
ho V^2 A}$$







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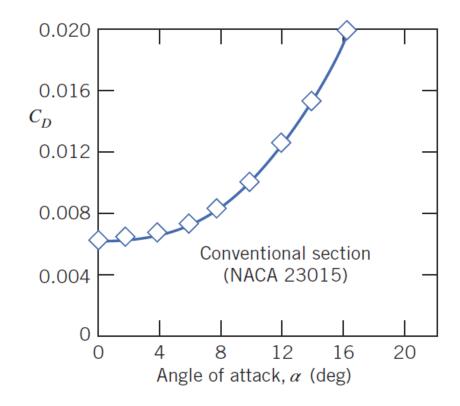




Lift vs Drag

 The drag coefficient increases with the angle of attack, often exponentially.

 Therefore, large angles of attack should be used sparingly for short periods of time for fuel efficiency.



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Induced Drag

• The effects of the finite aspect ratio can be characterized as a reduction $\Delta\alpha$ in the effective angle of attack.

$$\Delta lpha pprox rac{C_L}{\pi AR}, \;\; AR = rac{wingspan}{planform \; area} = rac{b^2}{A_p}$$

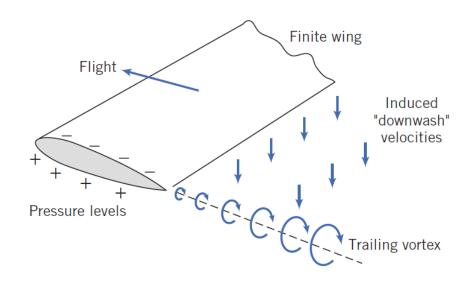




Fig. 9.20 Schematic representation of the trailing vortex system of a finite wing.

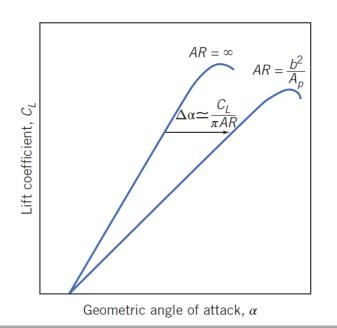


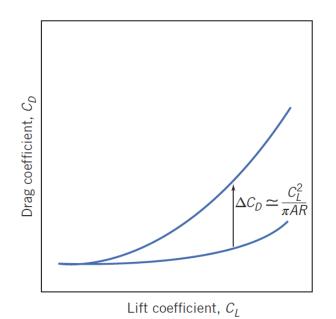


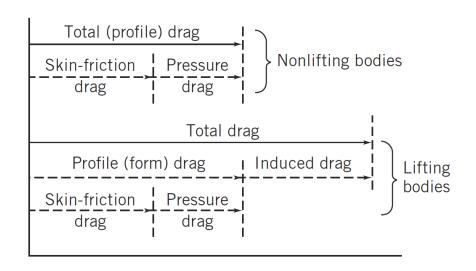
Induced Drag

 The effects of the finite aspect ratio can be characterized as a reduction Δα in the effective angle of attack.

$$\Delta lpha pprox rac{C_L}{\pi A R}, \;\; \Delta C_D \!pprox \!C_L \Delta lpha \!pprox \!rac{C_L^2}{\pi A R}, \;\; C_D \!=\! C_{D,\infty} \!+\! C_{D,i} \!=\! C_{D,\infty} \!+\! rac{C_L^2}{\pi A R}$$





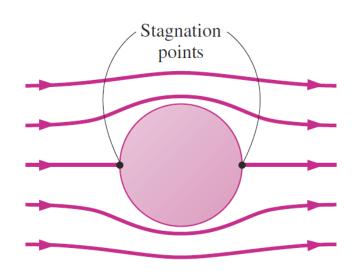




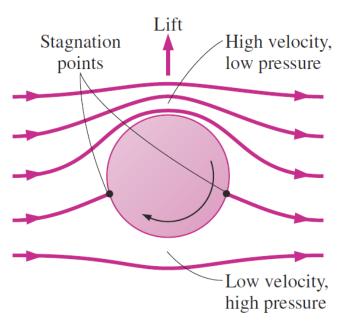


Magnus effect

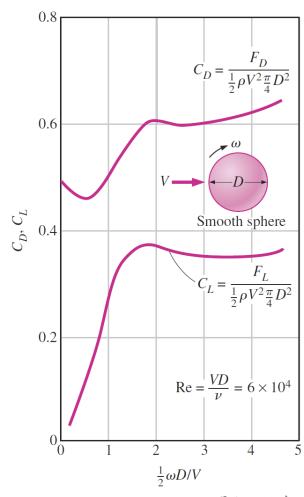
 The phenomenon of producing lift by the rotation of a solid body.



(a) Potential flow over a stationary cylinder



(b) Potential flow over a rotating cylinder



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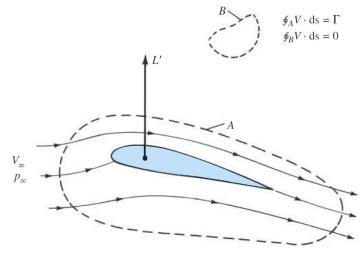


The Generation of Lift

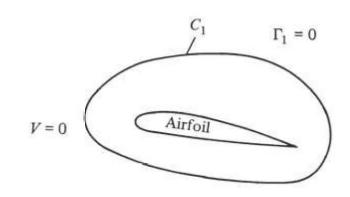
Kutta-Joukowski Theorem

$$L' =
ho V \Gamma$$
 The circulation $\Gamma \equiv \oint_A {f V} \cdot {f ds}$

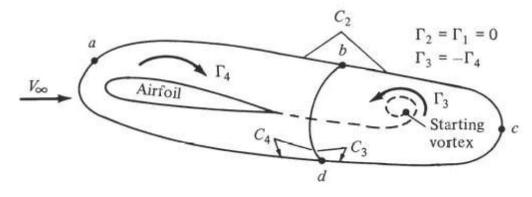
 The Kutta-Joukowski theorem states that lift per unit span on a two-dimensional body is directly proportional to the circulation around the body.



Circulation around a lifting airfoil.



(a) Fluid at rest relative to the airfoil



(b) Picture some moments after the start of the flow

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Wind Tunnel Investigations

- Minimizing Drag
- Optimum Cruising Velocities
- Lift Requirements









Problem 9.68

A fishing net is made of 0.75 mm diameter nylon thread assembled in a rectangular pattern. The horizontal and vertical distances between adjacent thread centerlines are 1 cm. Estimate the drag on a 2m \times 12 m section of this net when it is dragged (perpendicular to the flow) through 15°C water at 16 knots. What is the power required to maintain this motion?

$$C_{D}\!=\!rac{F_{D}}{rac{1}{2}
ho V^{2}A} \ C_{D}\!
ightarrow\!Fig.\,\,9.13$$

$$C_D \rightarrow Fig. \ 9.13$$





Problem 9.81

An F-4 aircraft is slowed after landing by dual parachutes deployed from the rear. Each parachute is 3.7 m in diameter. The F-4 weighs 142,400 N and lands at 160 m/s. Estimate the time and distance required to decelerate the aircraft to 100 m/s, assuming that the brakes are not used and the drag of the aircraft is negligible.

$$\sum F_{x} = ma = m\frac{dV}{dt} = -2F_{D} = -C_{D}\rho V^{2}A$$

$$-\frac{C_{D}\rho A}{m} \int_{0}^{t} dt = \int_{V_{0}}^{V_{f}} \frac{dV}{V^{2}}$$

$$\sum F_{x} = ma = mV\frac{dV}{dx} = -2F_{D} = -C_{D}\rho V^{2}A$$

$$-\frac{C_{D}\rho A}{m} \int_{0}^{x} dx = \int_{V_{0}}^{V_{f}} \frac{dV}{V}$$





Problem 9.116

An aircraft is in level flight at 225 km/h through air at standard conditions. The lift coefficient at this speed is 0.45 and the drag coefficient is 0.065. The mass of the aircraft is 900 kg. Calculate the effective lift area for the craft, and the required engine thrust and power.

$$\sum F_{y} = 0 = F_{L} - mg$$

$$C_{L} = \frac{F_{L}}{\frac{1}{2}\rho AV^{2}}$$

$$\sum F_{x} = 0 = T - F_{D}$$

$$C_{D} = \frac{F_{D}}{\frac{1}{2}\rho AV^{2}}$$





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Problem 9.122

Assume the Boeing 727 aircraft has wings with NACA 23012 section, planform area of 150 m², double-slotted flaps, and effective aspect ratio of 6.5. If the aircraft flies at 77.2 m/s in standard air at 778,750 N gross weight, estimate the thrust required to maintain level flight.

$$\sum F_{y} = 0 = F_{L} - mg$$

$$C_{L} = \frac{F_{L}}{\frac{1}{2}\rho AV^{2}} = \frac{mg}{\frac{1}{2}\rho AV^{2}}$$

$$\sum F_{x} = 0 = T - F_{D}$$

$$C_{D} = \frac{F_{D}}{\frac{1}{2}\rho AV^{2}} \rightarrow Figure \ 9.23 \rightarrow Eq. \ 9.43$$





This assignment is due by 6pm on April 22nd.

- Upload your solution to BB.
- You can either type your solution out using a word editor like Microsoft Word or clearly hand write the information and then copy/scan your solution to a digital file.