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Mechanical Design 1

03 Assignment

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Mechanical Design 1

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Problem 1

Using a maximum allowable shear stress of 70 MPa, find the shaft diameter needed to transmit 40 kW when
(a) The shaft speed is 2500 rev/min
(b) The shaft speed is 250 rev/min

Solution:

For this question, we are asked to find the shaft diameter needed to transmit $40\ \mathrm{kW}$ when (a) the shaft speed is $2500\ \mathrm{rev/min}$ (b) the shaft speed is $250\ \mathrm{rev/min}$.

For the shaft speed is $2500\ \mathrm{rev/min}$,

$$\frac{16T}{\pi d_0^3} \le 70\ \mathrm{MPa}$$

$$\frac{16P_0}{\pi \eta d_0^3} \le 70\ \mathrm{MPa}$$

$$d_0 \ge \sqrt[3]{\frac{16P_0}{\pi\eta(70\ \mathrm{MPa})}} = \sqrt[3]{\frac{16 \times (40\ \mathrm{kW})}{\pi \times (2500\ \mathrm{rev/min}) \times (70\ \mathrm{MPa})}}$$

$$= \sqrt[3]{\frac{16 \times (40\ \mathrm{kW})}{\pi \times (2500\ \mathrm{rev/min}) \times \frac{\frac{2\pi}{60}\ \mathrm{rad/s}}{1\ \mathrm{rev/min}} \times (70\ \mathrm{MPa})}}$$

$$= \sqrt[3]{\frac{16 \times (40 \times 10^3\ \mathrm{W})}{\pi \times \left(2500 \times \frac{2\pi}{60}\ \mathrm{rad/s}\right) \times (70 \times 10^6\ \mathrm{Pa})}} = 0.02232\ \mathrm{m} = 22.32\ \mathrm{mm}$$

For the shaft speed is $250\ \mathrm{rev/min}$,

$$\frac{16T}{\pi d_0^3} \le 70 \text{ MPa}$$

$$\frac{16P_0}{\pi \eta d_0^3} \le 70 \text{ MPa}$$

$$d_0 \ge \sqrt[3]{\frac{16P_0}{\pi\eta(70 \text{ MPa})}} = \sqrt[3]{\frac{16 \times (40 \text{ kW})}{\pi \times (250 \text{ rev/min}) \times (70 \text{ MPa})}}$$

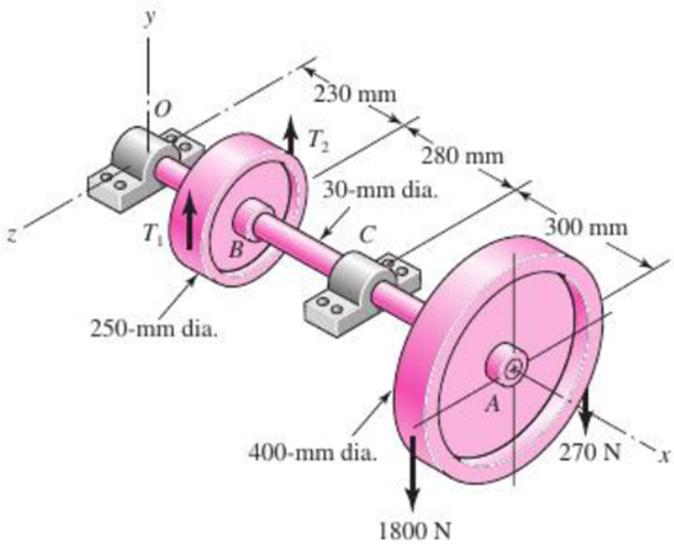
$$= \sqrt[3]{\frac{16 \times (40 \text{ kW})}{\pi \times (250 \text{ rev/min}) \times \frac{\frac{2\pi}{60} \text{ rad/s}}{1 \text{ rev/min}} \times (70 \text{ MPa})}}$$

$$= \sqrt[3]{\frac{16 \times (40 \times 10^3 \text{ W})}{\pi \times \left(250 \times \frac{2\pi}{60} \text{ rad/s}\right) \times (70 \times 10^6 \text{ Pa})}} = 0.04808 \text{ m} = 48.08 \text{ mm}$$

Problem 2

A countershaft carrying two V-belt pulleys is shown in the figure. Pulley *A* receives power from a motor through a belt with the belt tensions shown. The power is transmitted through the shaft and delivered to the belt on pulley *B*. Assume the belt tension on the loose side at *B* is 15 percent of the tension on the tight side.

a) Determine the tensions in the belt on pulley *B*, assuming the shaft is running at a constant speed
b) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports
c) Draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane
d) At the point of maximum bending moment, determine the bending stress and the torsional shear stress.
e) At the point of maximum bending moment, determine the principal stresses and the maximum shear stress using Mohr' circle



Solution:

(a) For this question, we are asked to determine the tensions in the belt on pulley B, assuming the shaft is running at a constant speed.

$$(1800\ \mathrm{N} - 270\ \mathrm{N}) \times \left(\frac{400\ \mathrm{mm}}{2}\right) + (T_2 - T_1) \times \left(\frac{250\ \mathrm{mm}}{2}\right) = 0$$

$$T_2 = 0.15 T_1$$

Therefore, I can know that

$$T_2 - T_1 = -2448\ \mathrm{N} \Rightarrow \begin{cases} T_1 = 2880\ \mathrm{N} \\ T_2 = 432\ \mathrm{N} \end{cases}$$

(b) For this question, we are asked to find the magnitudes of the bearing reaction forces.

$$+\circlearrowleft \sum M_O = 0$$

$$\Rightarrow F_C(230\ \text{mm} + 280\ \text{mm}) - (1800\ \text{N} + 270\ \text{N}) \times (230\ \text{mm} + 280\ \text{mm} + 300\ \text{mm}) + (2880\ \text{N} + 432\ \text{N}) \times (230\ \text{mm}) = 0 \Rightarrow M = 5500\ \text{N} \cdot \text{m}$$

Solving equation above yields that

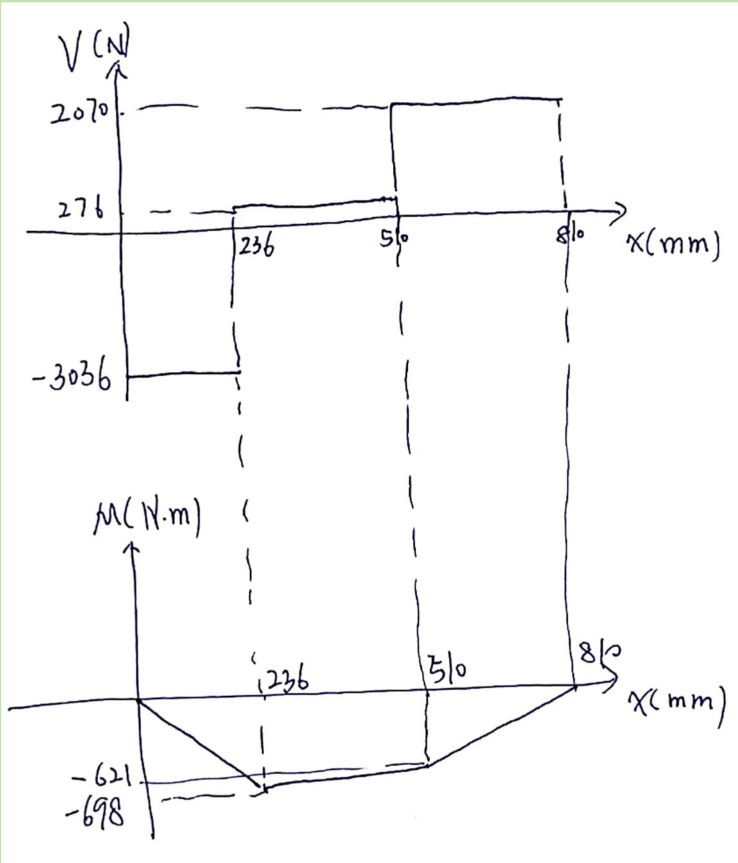
$$F_C = 1794\ \text{N} \uparrow$$

$$+\uparrow \sum F_y = 0 \Rightarrow -F_O + (2880\ \text{N} + 432\ \text{N}) - (1800\ \text{N} + 270\ \text{N}) + F_C = 0$$

Solving equation above yields that

$$F_O = 3036\ \text{N} \downarrow$$

(c) For this question, we are asked to draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.



(d) For this question, we are asked to determine the bending stress and the torsional shear stress.

$$\sigma_B = -\frac{M_B c}{I} = -\frac{M_B c}{\frac{\pi d^4}{64}} = -\frac{(-698.28\ \text{N} \cdot \text{m}) \times \frac{30\ \text{mm}}{2}}{\frac{\pi (30\ \text{mm})^4}{64}} = 263.43\ \text{MPa}$$

$$\tau_B = \frac{T_B c}{J} = \frac{T_B c}{\frac{\pi d^4}{32}} = \frac{(2448\text{ N}) \times \left(\frac{250\text{ mm}}{2}\right) \times \frac{30\text{ mm}}{2}}{\frac{\pi (30\text{ mm})^4}{32}} = 57.720\text{ MPa}$$

(e) For this question, we are asked to determine the principal stresses and the maximum shear stress.

Ignore the calculation below, I just use it to check.

$$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{(263.43\text{ MPa}) + 0}{2} + \sqrt{\left(\frac{(263.43\text{ MPa}) - 0}{2}\right)^2 + (57.720\text{ MPa})^2}$$

$$= 275.52\text{ MPa (T)}$$

$$\sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

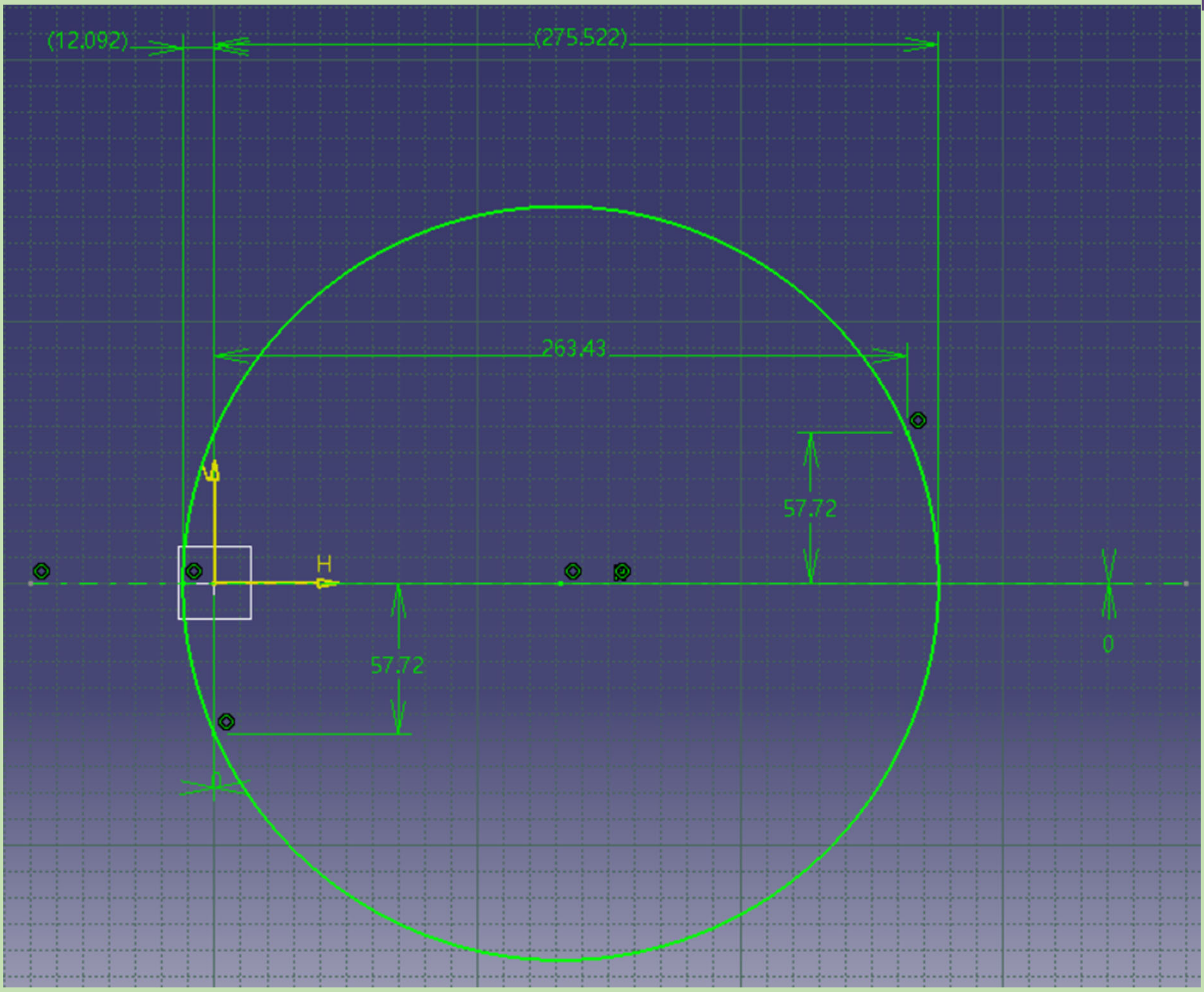
$$= \frac{(263.43\text{ MPa}) + 0}{2} - \sqrt{\left(\frac{(263.43\text{ MPa}) - 0}{2}\right)^2 + (57.720\text{ MPa})^2}$$

$$= -12.09\text{ MPa} = 12.09\text{ MPa (C)}$$

$$\sigma_{p3} = 0\text{ MPa}$$

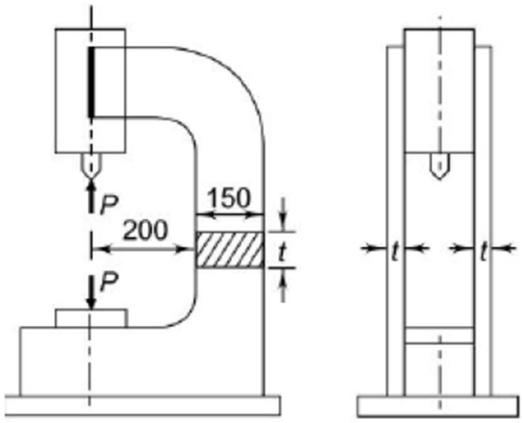
$$\tau_{max} = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \sqrt{\left(\frac{(263.43\text{ MPa}) - 0}{2}\right)^2 + (57.720\text{ MPa})^2} = 143.81\text{ MPa}$$

The Mohr circle is shown in figure below:


(12.092)
(275.522)
263.43
57.72
H
V
57.72
0

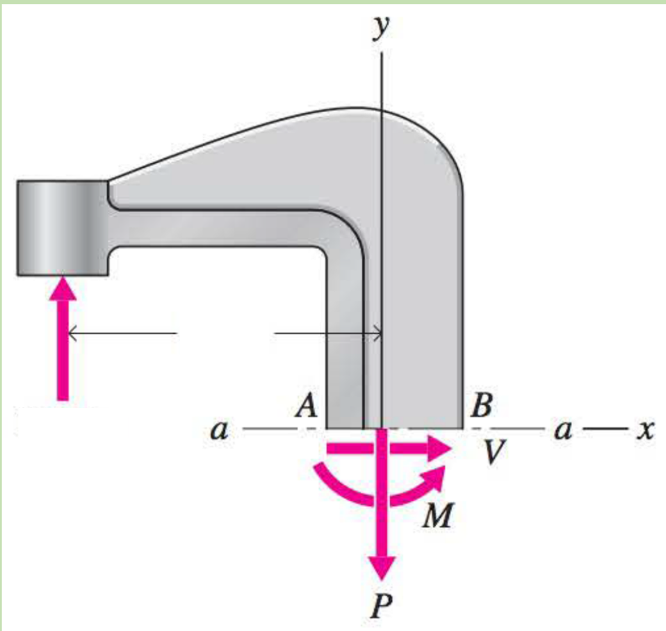
Problem 3

The frame of a hydraulic press consisting of two identical steel plates is shown. The maximum force P acting on the frame is 20 kN. The plates are made of steel 45C8 with tensile yield strength of 380 MPa. The factor of safety is 2.5. Determine the plate thickness



Solution:

For this question, we are asked to determine the plate thickness.



Since the load acts in a plane of symmetry, there are three independent equations of equilibrium. The internal forces at the section are found using the equations of equilibrium as follows:

$$+\rightarrow \sum F_x = 0 \Rightarrow V = 0$$

$$+\uparrow \sum F_y = 0 \Rightarrow P = 20 \text{ kN}$$

$$+\circlearrowleft \sum M_{c-c} = 0 \Rightarrow M - (20\ \mathrm{kN}) \times (275\ \mathrm{mm}) = 0 \Rightarrow M = 5500\ \mathrm{N \cdot m}$$

Therefore, I can know that

$$\frac{P}{A} + \frac{Mc}{I} = \frac{\sigma_{ts}}{FS}$$

$$\frac{20\ \mathrm{kN}}{2t(150\ \mathrm{mm})} + \frac{(5500\ \mathrm{N \cdot m}) \times \left(\frac{150\ \mathrm{mm}}{2}\right)}{\frac{2t \times (150\ \mathrm{mm})^3}{12}} = \frac{380\ \mathrm{MPa}}{2.5}$$

Solving the equation above yields that

$$t = 5.263\ \mathrm{mm}$$

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