MEMS1045 Automatic control

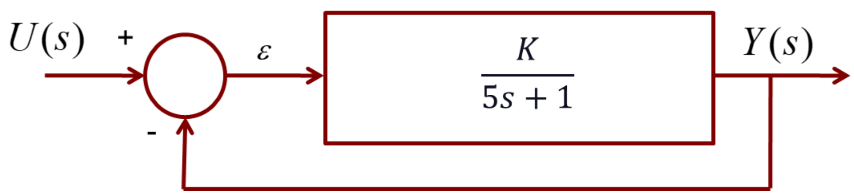
Lecture 13

Frequency response 3

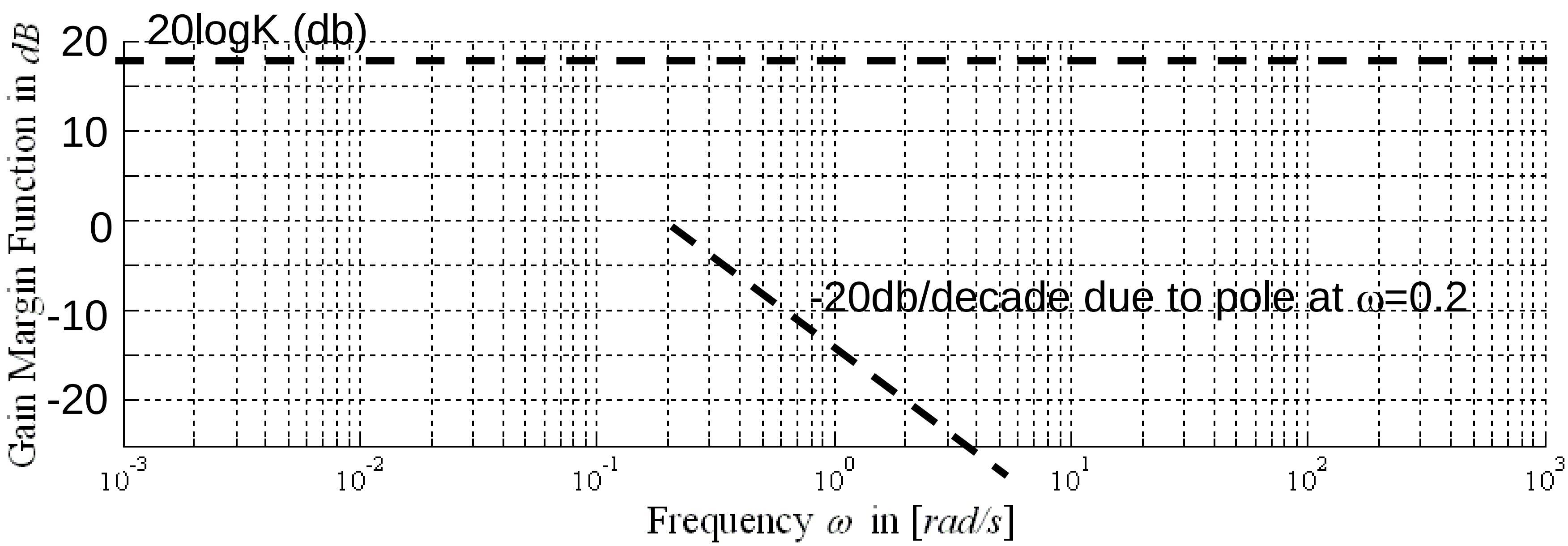
Objectives

- Design proportional, lag and lead compensators using frequency response of open-loop transfer function based on closed-loop response specifications

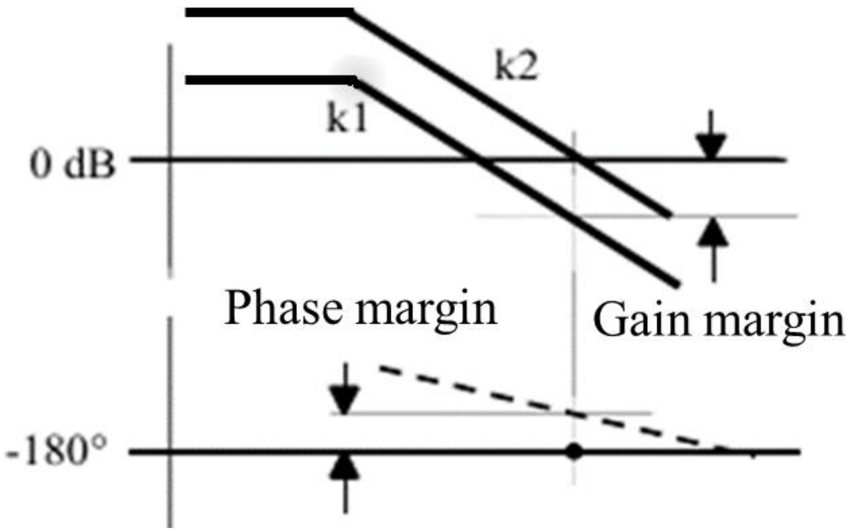
Gain adjustment



Consider the above system with a proportional gain K. How would the open-loop Bode plot change when the gain changes?



Gain adjustment



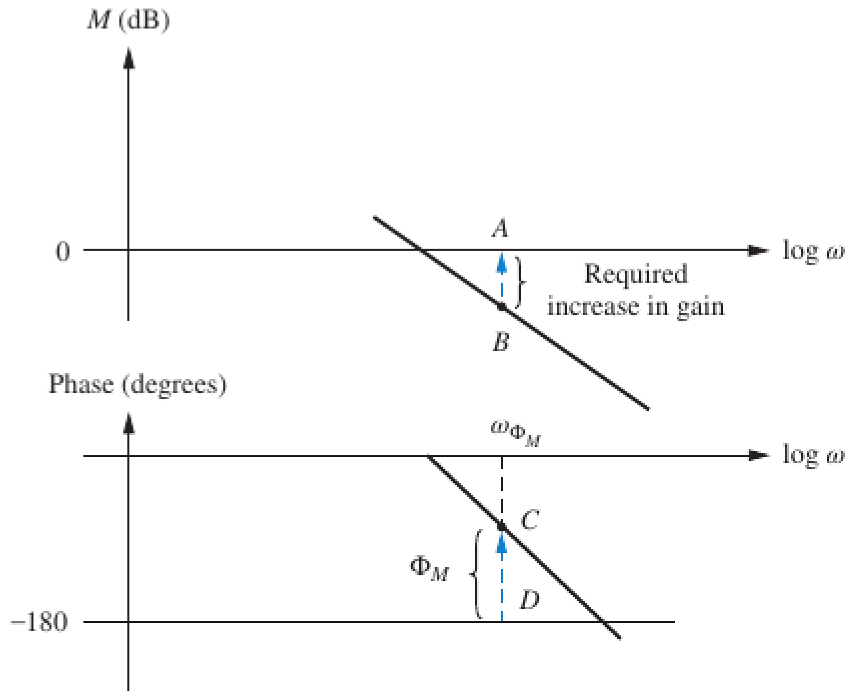
Gain adjustment will

- ❖ Move the magnitude ratio plot up and down but will not change the phase plot
- ❖ By moving the magnitude ratio plot vertically, the phase margin can be adjusted to meet the desired specs

- ❖ No change in the system type (it cannot reduce the steady state error to zero but can change the value of the steady state error)
- ❖ To change the shape of the open-loop Bode plot, controllers with poles and zero can be added

Procedure (P controller)

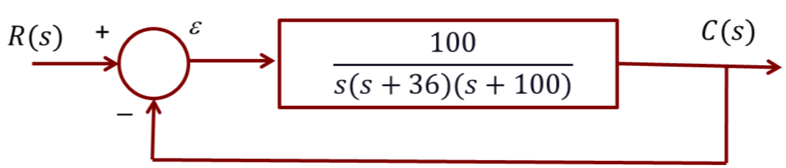
- Select a convenient value of gain
- Draw the Bode magnitude and phase plots for the gain
- If OS% is given, find the damping ratio and the required open-loop phase margin
- Find the frequency, $\omega_{\Phi M}$ on the Bode phase diagram that yields the desired phase margin, CD, as shown in figure

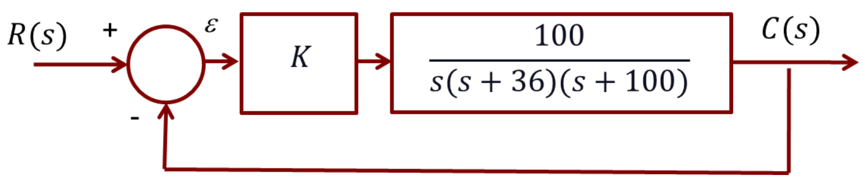


- Change the gain by an amount AB (shown in figure) to force the magnitude curve to go through 0 dB at $\omega_{\Phi M}$. The amount of gain adjustment is the addition or reduction in gain needed to produce the required phase margin.

Example 1

For the unity feedback system shown, use frequency response to design a proportional controller to yield a 9.5% overshoot in the transient response for a step input to the control system shown





For 9.5% overshoot in the transient response, the damping ratio is given by

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.6$$

Required phase margin needed:

$$PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} = 59.2°$$

Example 1

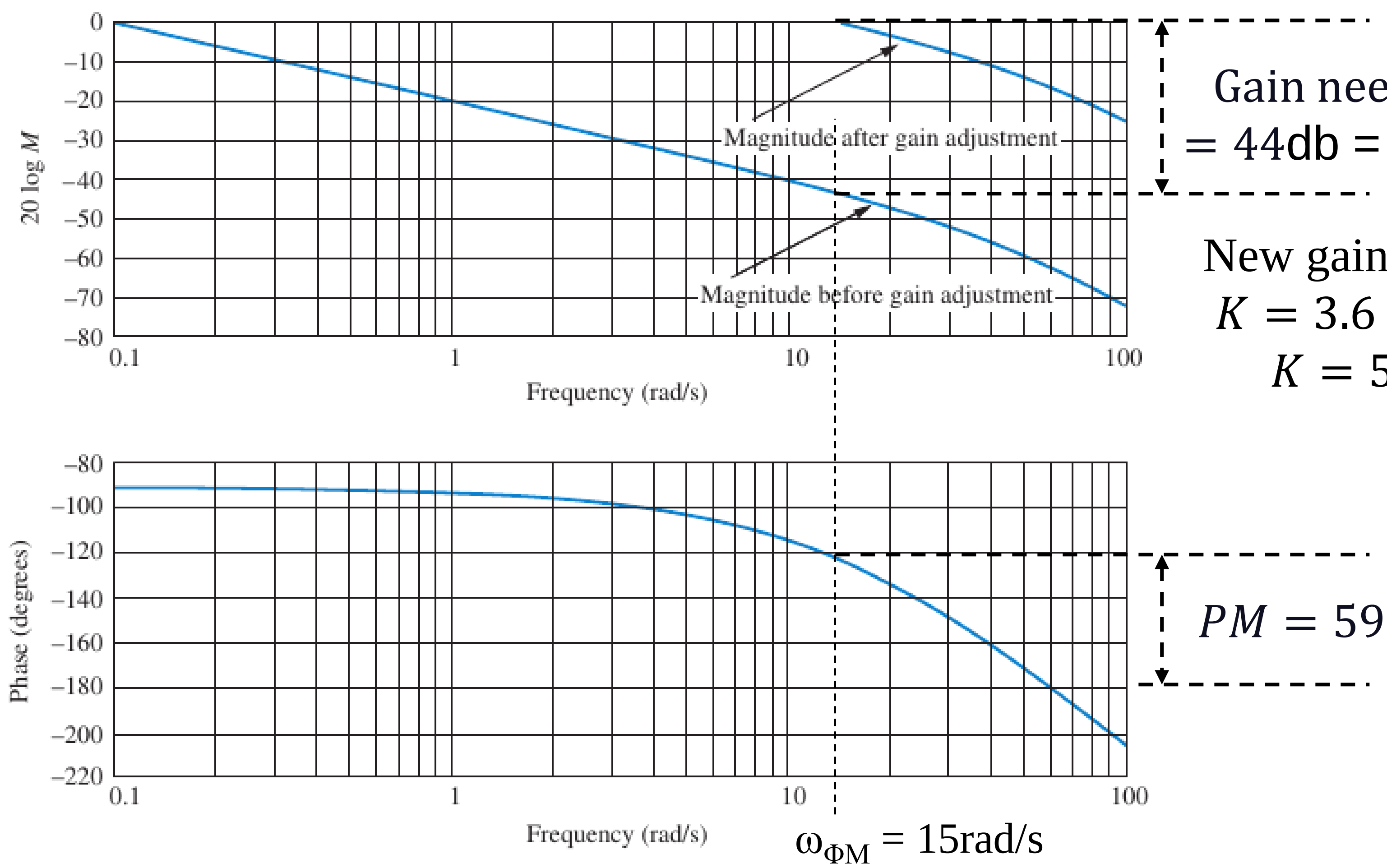
Choose $K = 3.6$ so that the open-loop transfer function becomes:

$$G(s) = \frac{0.1}{s(0.0278s + 1)(0.01s + 1)}$$

$$G(j\omega) = \frac{0.1}{j\omega(0.0278j\omega + 1)(0.01j\omega + 1)}$$

- Gain $20\log 0.1 = -20\text{db}$
- Poles at $\omega = 0$, 36, and 100
- Sketch the Bode diagrams
- Find the frequency, $\omega_{\Phi M}$ on the Bode phase diagram that yields the desired phase margin = $59.2°$
- With this frequency $\omega_{\Phi M}$ = 15 rad/s determine the new gain needed from the amplitude ratio plot
- New gain $K = 3.6 \times 159 = 570$
- Check response and refinement if needed

Example 1



Gain needed $= 44\text{db} = 159$

New gain

$K = 3.6 \times 159$

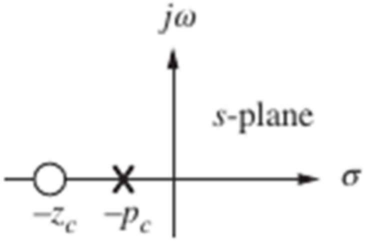
$K = 570$

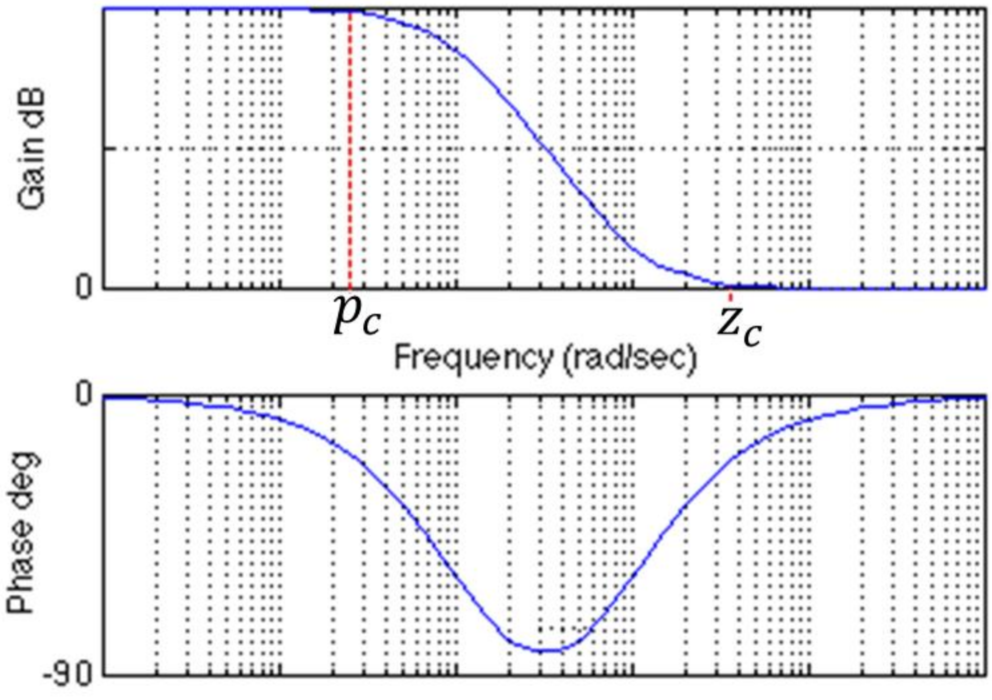
$PM = 59.2°$

$\omega_{\Phi M} = 15\text{rad/s}$

Lag compensation

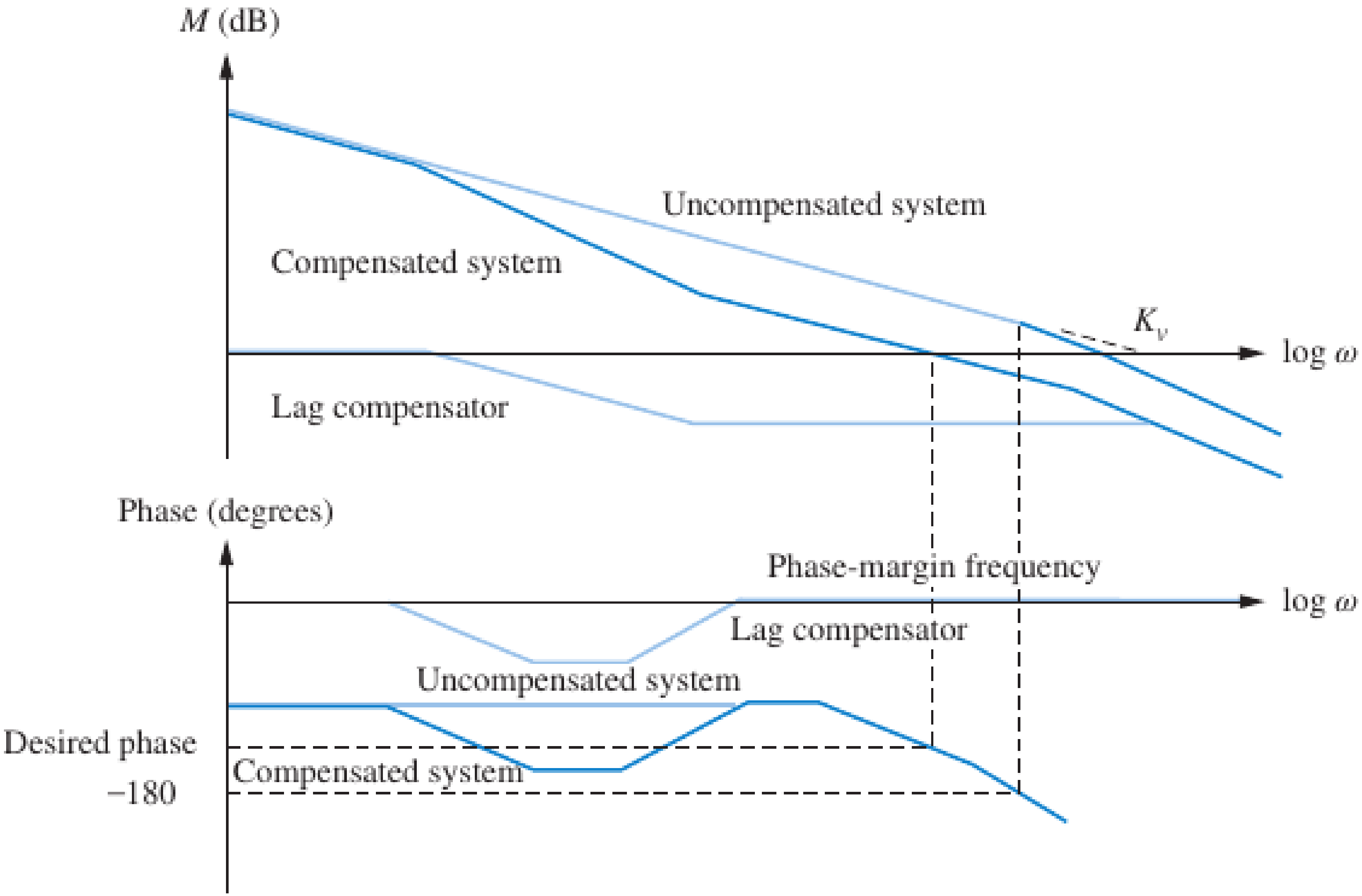
Lag compensator: $G_c(s) = \frac{(s+z_c)}{(s+p_c)}$ where $|z_c| > |p_c|$





A lag compensator can "push" the phase margin function "down" within a certain interval. Once the phase margin function is satisfactory, the gain margin function can be shifted upwards or downwards by changing the *K* parameter (controller gain) of the controller

Lag compensation

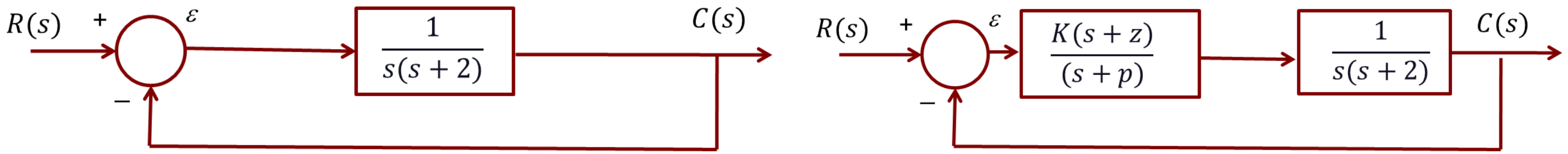


Procedure (lag compensator)

1) Set the gain, K, to the value that satisfies the steady-state error specification and plot the Bode magnitude and phase diagrams for this value of gain
2) Find the frequency where the phase margin is 5° to 12° greater than the phase margin that yields the desired transient response
3) Select a lag compensator whose magnitude response yields a composite Bode magnitude diagram that goes through 0 dB at the frequency found in Step 2 as follows: Draw the compensator's high-frequency asymptote to yield 0 dB for the compensated system at the frequency found in Step 2. Thus, if the gain at the frequency found in Step 2 is 20 log K_{PM}, then the compensator's high-frequency asymptote will be set at -20 log K_{PM}. Select the upper break frequency to be 1 decade below the frequency found in Step 2; select the low-frequency asymptote to be at 0 dB. Connect the compensator's high- and low-frequency asymptotes with a 20 dB/decade line to locate the lower break frequency
4) Reset the system gain, K, to compensate for any attenuation in the lag network in order to keep the static error constant the same as that found in Step 1

Example 2

For the unity feedback system shown, use frequency response to design a lag compensator such that static velocity error constant $K_v \geq 20$ and $\zeta \geq 0.45$



The system is type 1 and $K_v = \lim_{s \to 0} sG(s) = K/2 \geq 20$

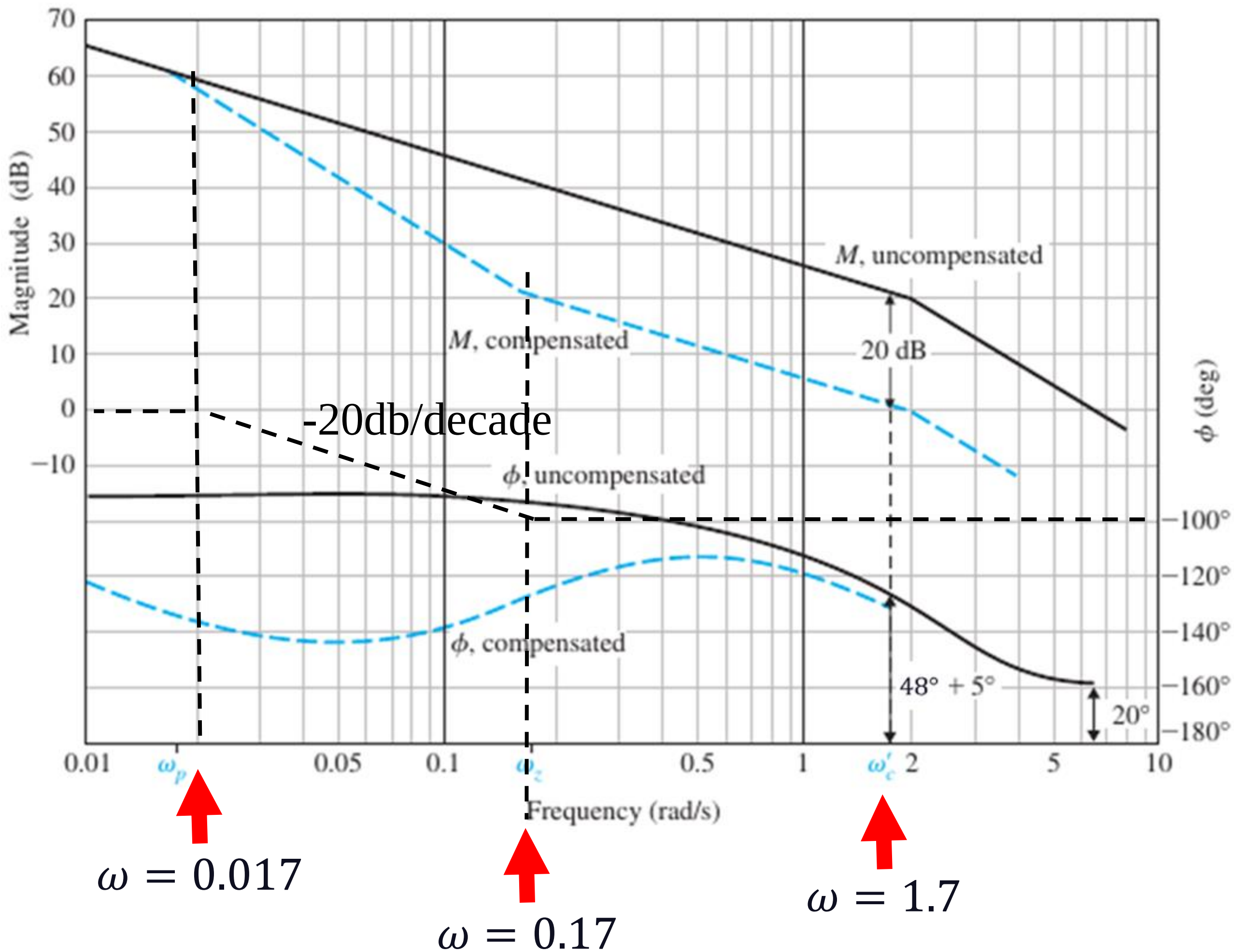
Hence set K = 40 and plot the uncompensated open-loop Bode diagram

Given $\zeta \geq 0.45$, required open-loop phase margin needed:

$$PM = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} = 48°$$

Add 5 deg to get $PM = (48 + 5)°$

Example 2



At $PM = (48 + 5)°$
$\omega = 1.7$ at M=20db
Upper break frequency 1 decade below 1.7 rad/s, i.e. zero at 0.17 rad/s
Draw -20db/decade to meet 0 db at frequency 0.017 rad/s i.e. pole at 0.017 rad/s
Lag controller:

$$G_c(s) = \frac{(s + 0.17)}{(s + 0.017)}$$

Example 2

The open-loop transfer function with compensator is

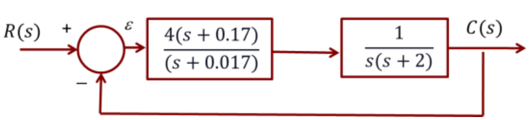
$$G_c(s)G(s) = \frac{(s+0.17)}{(s+0.017)}\frac{K}{s(s+2)}$$

Reset the gain based on steady state error:

$K_v = \lim_{s\to 0} sG(s) = 5K \geq 20$ or $K > 4$

Hence lag compensator is set at

$$G_c(s)G(s) = \frac{4(s+0.17)}{(s+0.017)}$$



Check response and refine if needed

Note:

After getting the zero at $\omega_z = 0.17$ rad/s

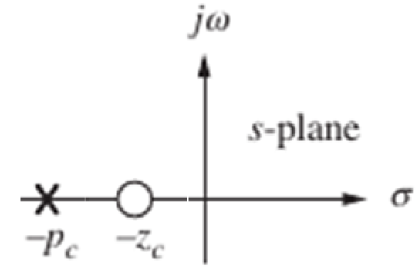
And M=20db at Bode *PM*, we can find solve for α where

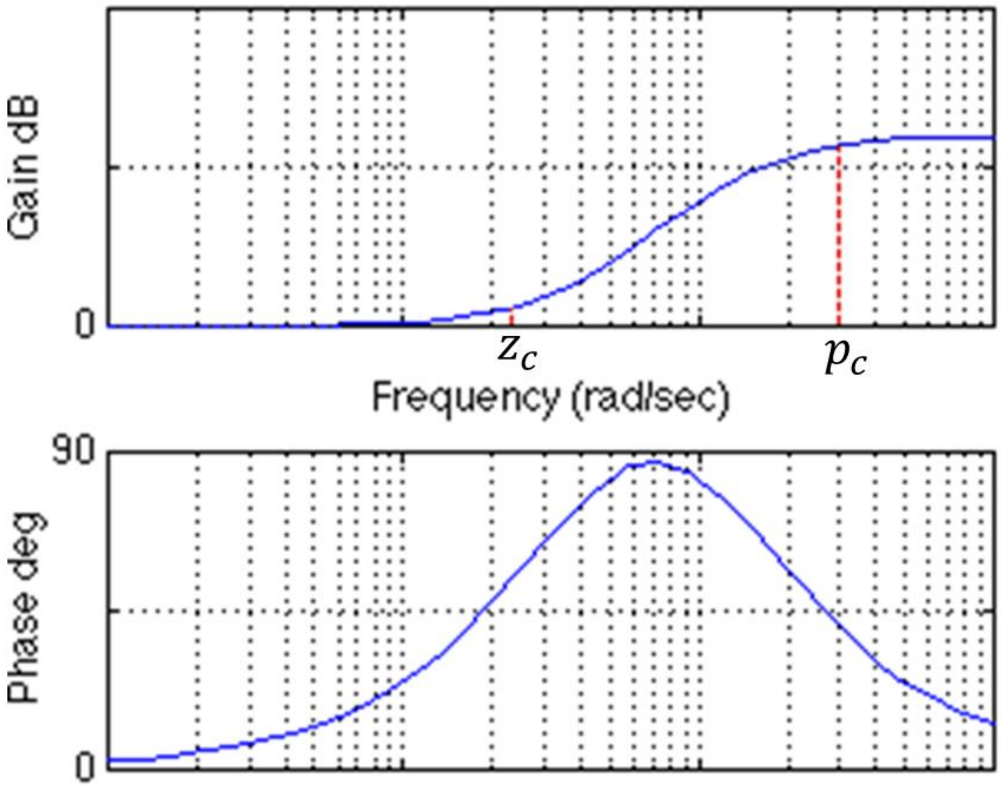
$20 \log \alpha = \mathrm{M}$; i.e. $\alpha = 10$

Find the pole location using $\omega_p = \frac{\omega_z}{\alpha} = \frac{0.17}{10} = 0.017$

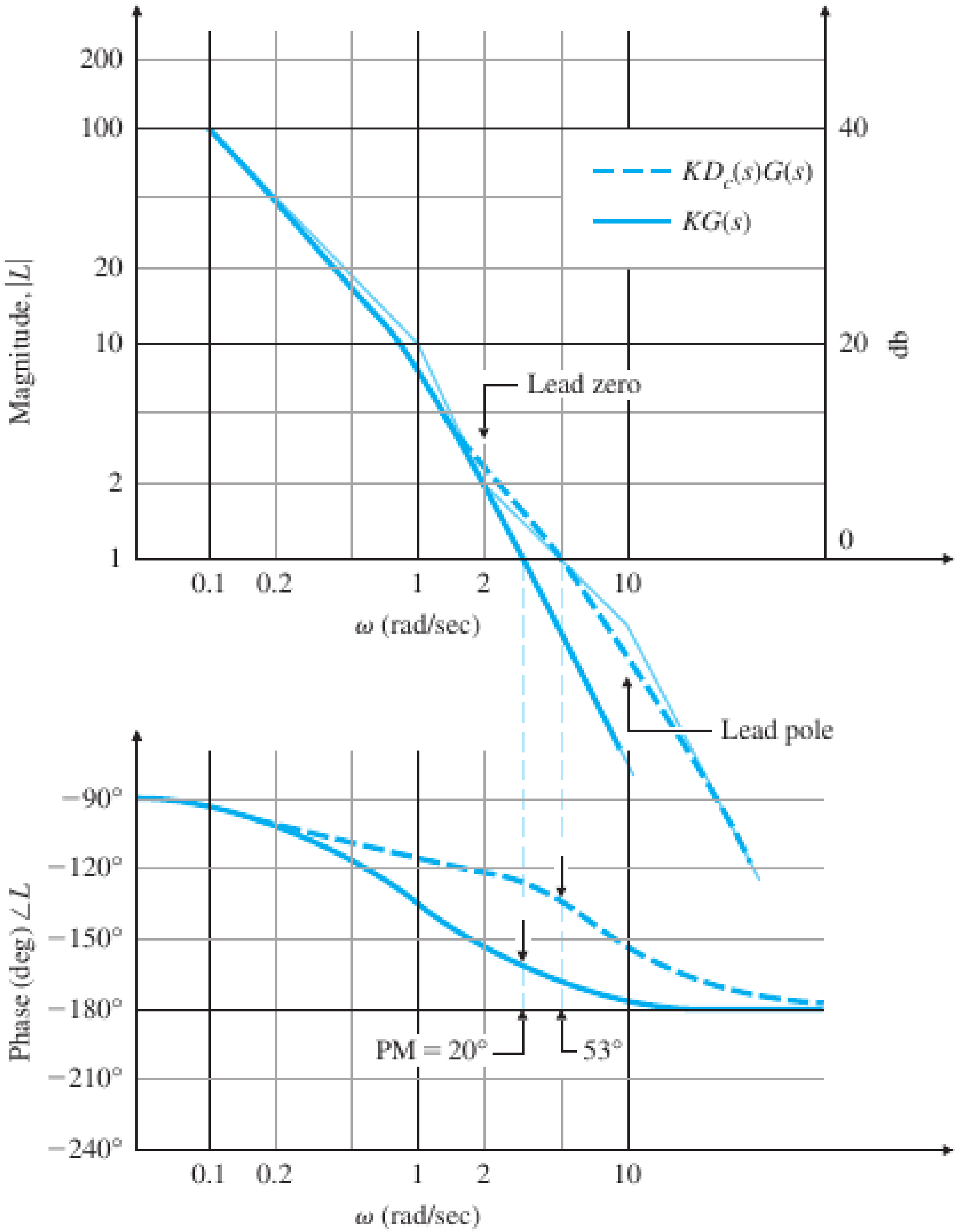
Lead compensation

Lead compensator: $G_c(s) = \frac{(s+z)}{(s+p)}$ where $|z| < |p|$





A lead compensator can "lift" the phase margin function within a certain interval. Once the phase margin function is satisfactory, the gain margin function can be shifted upwards or downwards by changing the K parameter (controller gain) of the controller.



200
100
20
10
2
1
40
20
0
db
Magnitude, |L|
$KD_c(s)G(s)$
$KG(s)$
Lead zero
Lead pole
0.1
0.2
1
2
10
ω (rad/sec)
−90°
−120°
−150°
−180°
−210°
−240°
Phase (deg) ∠L
PM = 20°
53°
0.1
0.2
1
2
10
ω (rad/sec)

Lead compensation

Lead compensator: $G_c(s) = \frac{(s+z)}{(s+p)}$ where $|z| < |p|$ can be reformatted as

$$G_c(s) = \frac{1}{\beta} \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$$

Where $0 < \beta < 1$

The maximum compensator phase shift is $\emptyset_{max} = \tan^{-1} \frac{1-\beta}{2\sqrt{\beta}} = \sin^{-1} \frac{1-\beta}{1+\beta}$

The maximum compensator phase shift occurs at frequency $\omega_{max} = \frac{1}{T\sqrt{\beta}}$

The compensator's magnitude at ω_{max} is $|G_c(\omega_{max})| = \frac{1}{\sqrt{\beta}}$

Procedure (lead compensator)

1) Assume the leads compensator ($0 < \beta < 1$):

$$G_c(s) = k_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$$

and define $K = k_c \beta$

Then

$$G_c(s) = K \frac{Ts + 1}{\beta Ts + 1}$$

The open-loop transfer function of the system is

$$G_c(s)G(s) = K \frac{Ts + 1}{\beta Ts + 1} G(s) = \frac{Ts + 1}{\beta Ts + 1} KG(s)$$

Use $KG(s)$ to determine the value of K to satisfy the steady state error and plot the Bode magnitude and phase diagrams for this value of gain

Procedure (lead compensator)

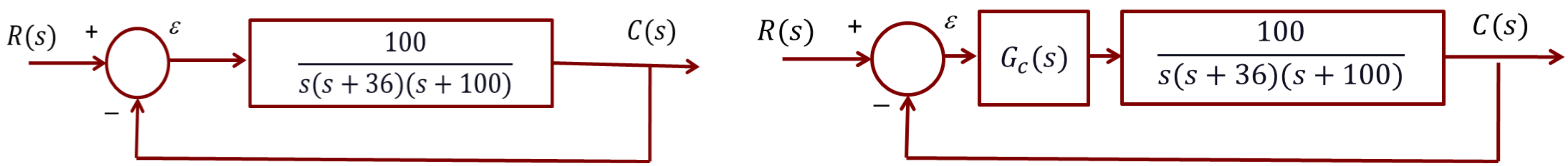
2) Find the desired phase margin from the specifications. Compare this with the phase margin obtained from the Bode plot to determine the amount needed and add another 5° to 12°. This will be $\emptyset_{max}$
3) Determine the factor β using $\emptyset_{max}$
4) Calculate $-20\log(1/\beta)$ and locate the frequency where this magnitude occurs on the Bode plot. This frequency is $\omega_{max} = \frac{1}{T\sqrt{\beta}}$
5) Determine the value of T using ω_{max} so that

    Zero of lead compensator is at $\omega = 1/T$

    Pole of lead compensator is at $\omega = 1/(\beta T)$
6) Using the value of K determined in step 1 and factor β in step 4 to calculate constant k_c from $k_c = K/\beta$
7) Check the response to be sure the specifications have been met. If not, repeat the design process by modifying the pole–zero location of the compensator until a satisfactory result is obtained.

Example 3

For the unity feedback system shown, use frequency response to design a lead compensator such that static velocity error constant $K_v = 40$ and a peak time of 0.1 second with 20% overshoot



For 20% overshoot in the transient response, the damping ratio is given by

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.46$$

Required phase margin needed:

$$PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} = 48.5°$$

Example 3

A peak time of 0.1 second will need a closed-loop bandwidth of

$$\omega_{BW} = \frac{\pi}{T_p\sqrt{1-\zeta^2}}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}} = 46.6\text{rad/s}$$

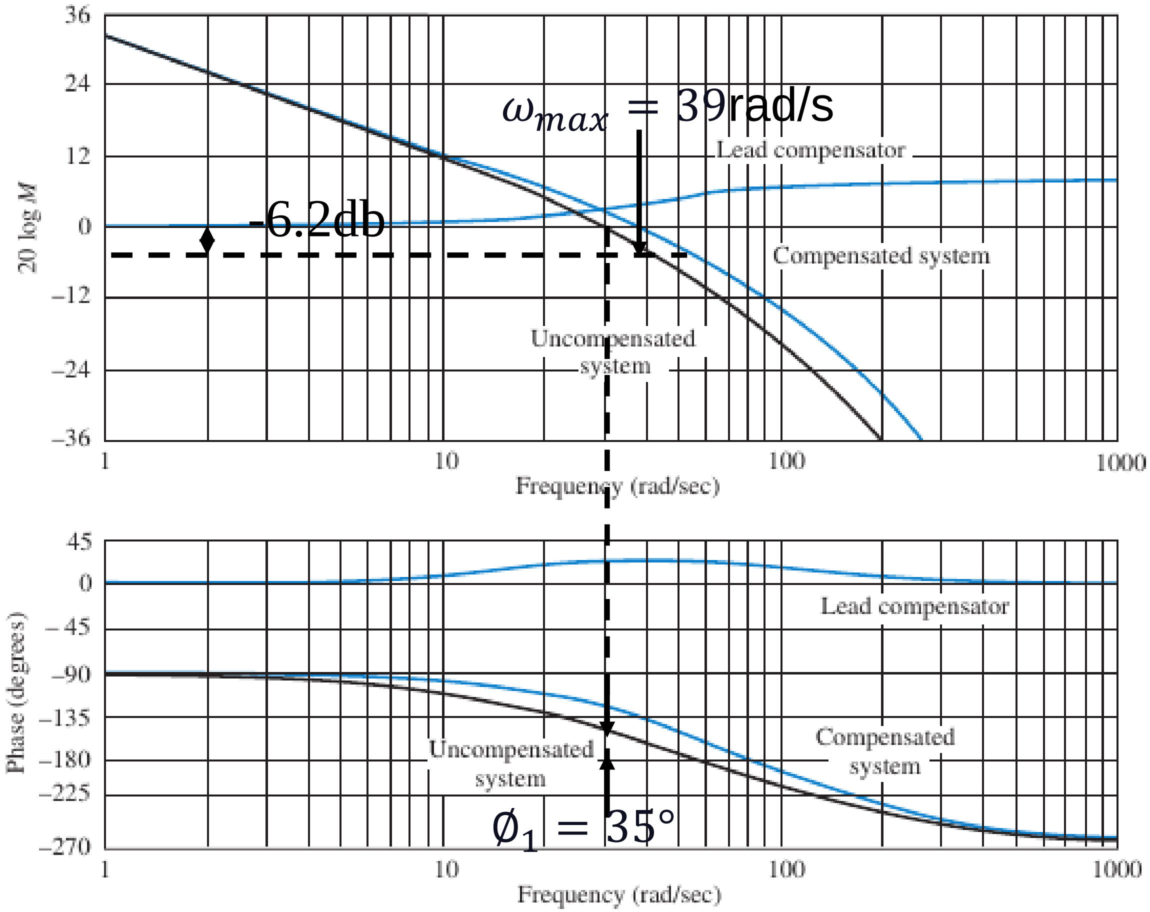
Use $KG(s)$ to determine the value of K to satisfy the steady state error:

$$KG(s) = \frac{100K}{s(s+36)(s+100)}$$

The system is type 1 and $K_v = \lim_{s\to 0} sKG(s) = K/36 = 40$ or $K = 1440$

Sketch the Bode diagram for

$$KG(s) = \frac{100K}{s(s+36)(s+100)} = \frac{144000}{s(s+36)(s+100)}$$



Phase margin from Bode plot:

$$\emptyset_1 = 35°$$

Required $PM = 48.5°$

Need to increase by about 13.5°

Add another 5° to 12°.

Try $\emptyset_{max} = 20°$

Find β using $\emptyset_{max} = 20°$

$$\emptyset_{max} = \sin^{-1}\frac{1-\beta}{1+\beta};$$

$\beta = 0.49$

Find $-20\log(1/\beta) = -6.2$db

Locate this on Bode plot to find the frequency $\omega_{max} \approx$ 39rad/s

Determine the value of T using $\omega_{max} = \frac{1}{T\sqrt{\beta}}$ so that

Zero of lead compensator is at $\omega = 1/T = 27.3$

Pole of lead compensator is at $\omega = 1/(\beta T) = 55.7$

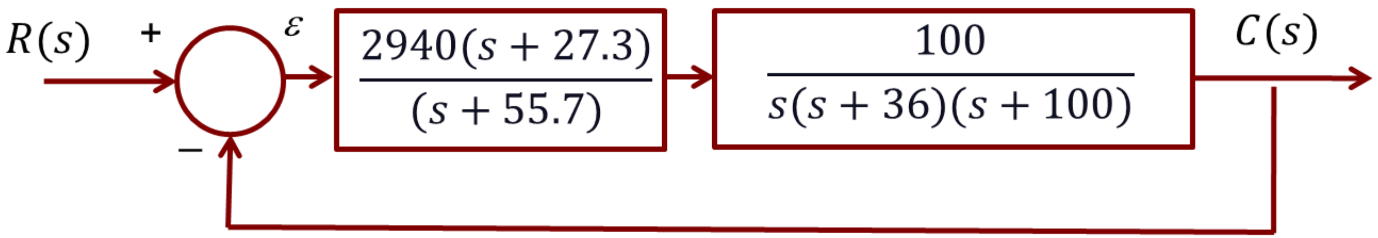
$$k_c = K/\beta = 1440/0.49 = 2940$$

Example 3

The lead compensator transfer function is

$$G_c = k_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = 2940\,\frac{s+27.3}{s+55.7}$$

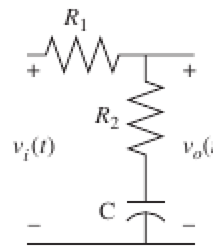
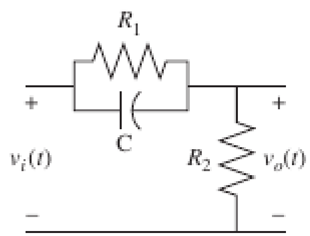
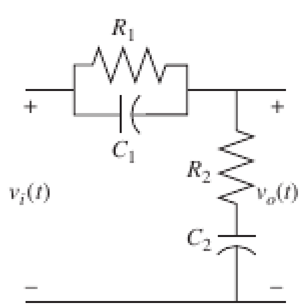
Check the response to be sure the specifications have been met. If not, repeat the design process by modifying the pole–zero location of the compensator until a satisfactory result is obtained



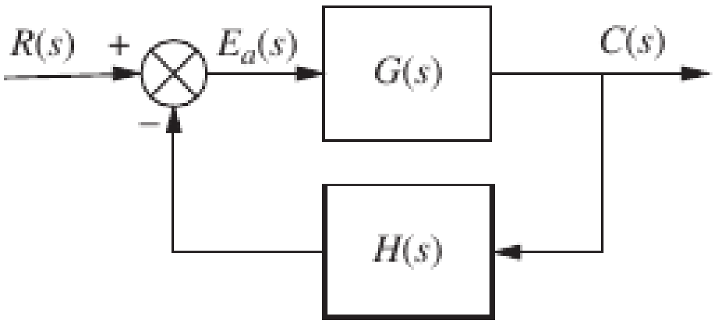
Realization of controllers

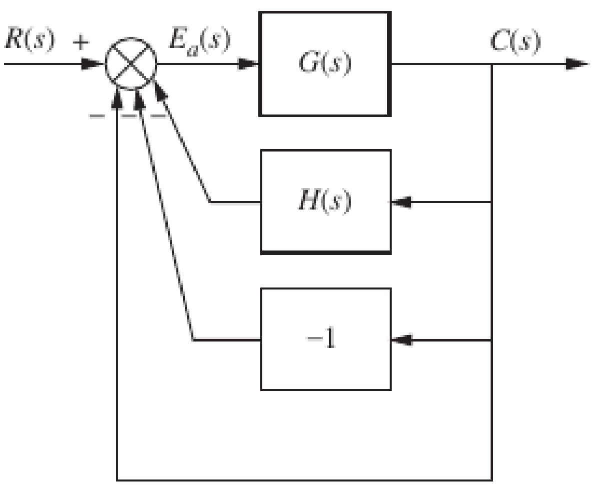
Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Gain	R_1	R_2	$-\frac{R_2}{R_1}$
Integration	R	C	$-\frac{\frac{1}{RC}}{s}$
Differentiation	C	R	$-RCs$
PI controller	R_1	R_2 C	$-\frac{R_2}{R_1}\frac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller	C R_1	R_2	$-R_2C\left(s+\frac{1}{R_1C}\right)$
PID controller	C_1 R_1	R_2 C_2	$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$

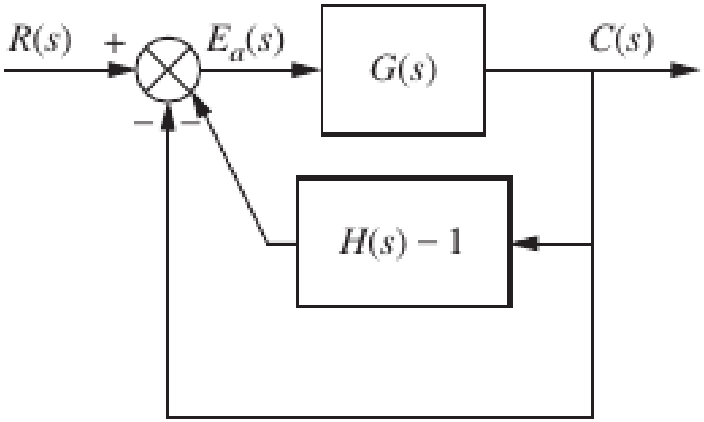
Realization of controllers

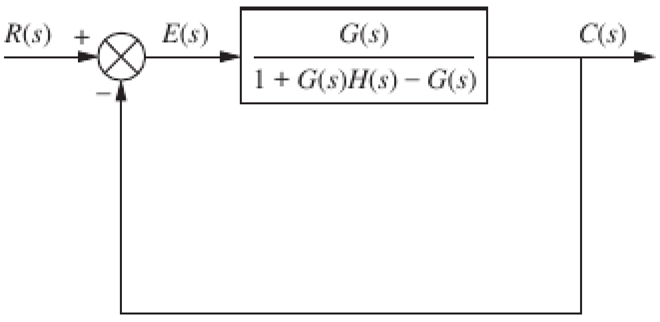
Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lag compensation		$\frac{R_2}{R_1+R_2}\,\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}}$
Lead compensation		$\frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$
Lag-lead compensation		$\frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$

Equivalent unity feedback









Course summary

No.	Topics
1	Review of differential equations and Laplace transform
2	Modeling of dynamic systems
3	Stability analysis of linear dynamic systems
4	Time response analysis
5	Stability analysis
6	Steady–state error analysis
7	Root locus techniques and design of feedback controller via Root locus
8	Frequency response techniques and design of feedback controllers via frequency response

Course learning outcomes

At the completion of this course, students will be able to:

- ❖ Develop mathematical models of dynamic systems
- ❖ Analyze the stability of the dynamic systems
- ❖ Quantify the response characteristics of these systems in the time and frequency domains
- ❖ Design feedback controllers to regulate the system performance that meets required specification