Designing loot boxes: Implications for Profits and Welfare

ABSTRACT

A loot box is a probabilistic allocation of virtual products, the exact outcome of which is known to consumers only after purchase. Consumers sometimes purchase these goods multiple times until their preferred products are obtained. As loot boxes have been gaining enormous popularity in recent years, they are often criticized as exploitative and socially wasteful. In this study, we develop a stylized model to study the optimal design of loot boxes and its impact on profits and social welfare. We find that firms may assign asymmetric probabilities to ex ante symmetric products. Firms could use loot boxes to offer products at low prices to users who would not buy these products under the traditional pricing strategy. Loot boxes enable firms to earn higher profits due to better price discrimination and market expansion. Contrary to the widespread criticism of loot boxes as socially harmful, our analysis reveals that the loot box strategy can improve social welfare. Some platforms

promise that consumers can obtain their preferred products with no more than a certain

number of purchases. Contrary to conventional wisdom, our analysis reveals that such a

strategy can increase firm's profits while reducing consumer welfare.

Keywords: Loot boxes, Pricing, Welfare.

1 Introduction

An increasing number of video game developers are using loot boxes to monetize in-game content. A loot box is a probabilistic allocation of virtual products, the exact outcome of which is known to consumers only after purchase. Evolving from free-to-play games, loot boxes expanded and diversified into almost every available genre, including many big-budget "AAA" titles, bringing in an estimated \$15 billion revenue for the video game industry in 2021 and an estimated \$20 billion by 2025 (Juniper Research 2021).

Loot boxes can be viewed as a type of probabilistic selling in which firms only offer probabilistic goods. While existing literature has examined the role of buyer uncertainty and its impact on probabilistic selling, prior studies have focused on situations in which probabilistic goods are bought only once and they are offered in addition to deterministic goods (Fay and Xie 2008; Zhang, Joseph, and Subramaniam 2015; Jerath, Netessine, and Veeraraghavan 2010; Huang and Yu 2014). This is very different from the context which we study, where firms offer only the loot box and exclude deterministic products and consumers often buy multiple loot boxes. One exception is Chen, Elmachtoub, Hamilton, and Lei (2021) who compare the asymptotic loot box revenues with and without allowed duplicates, i.e., whether or not the firm guarantees that subsequent loot boxes will generate outcomes different from previous ones.

Although repeated purchases of loot boxes have gained enormous popularity, there is little research that has studied the optimal design of loot boxes. More specifically, the firm needs to decide both the price and the allocation probabilities before selling loot boxes to consumers. We examine why, when, and how a firm can benefit from the loot box strategy. We provide answers to several important questions: How should firms design loot boxes that contain horizontally differentiated products? What are the conditions under which loot boxes improve profits relative to the traditional pricing strategy? What are the conditions under which the firm can maximize profits by offering only loot box and exclude deterministic products? Are loot boxes predatory and socially wasteful? The answers to these questions are important from both managerial and public policy perspectives. From a managerial perspective, the analysis can inform firms about optimal pricing and loot

box design decisions. From a public policy perspective, such analysis would help public policymakers assess the impact of loot boxes on consumer and social welfare.

To address these issues, we develop a model in which a monopolist offers two symmetric products in the market. We consider rational consumers to study the profitability of loot box strategy under the conservative assumption that loot box purchases are not driven by psychological biases such as addiction. The market consists of two loyal segments, one for each product, and a segment of consumers who are indifferent between the two products. Under the traditional pricing strategy, the firm decides two prices for two distinct products, whereas under the loot box strategy, the seller decides the price and allocation probabilities. Notably, the product line consists solely of loot boxes. Consumers can buy one or multiple loot boxes.

In contrast to previous studies on probabilistic goods that argue for the optimality of uniform allocation probabilities (Fay and Xie 2008; Chen et al. 2021), our results show that it may be optimal for firms to allocate asymmetric probabilities to ex ante symmetric products. This is because asymmetric probabilities can increase the total number of purchases made by loyal consumers and therefore improve profits. Thus, our model is better at explaining the common practice whereby one product has a much lower allocation probability relative to other symmetric products in loot boxes. For example, the Emma Secret Forest Series Box has 9 regular figures and 1 hidden edition. The chance to win a hidden edition is about 10 times lower than the chance to get any regular figure. We show that the loot box strategy can sometimes generate higher profits than the traditional pricing strategy. This is because loot boxes enable the firm to price discriminate more effectively on the time dimension and expand the market with lower prices. Our results show that even rational consumers may purchase loot boxes repeatedly and end up paying more than their valuation for their desired products. While such multiple purchases in excess of the valuation could be viewed as evidence of addiction or gambling, our results show that this is not necessarily the case. Contrary to the widespread criticism in mass media that loot boxes exploit consumers, we show that the loot box strategy could improve consumer welfare because loot boxes enable the firm to offer products at low prices to consumers who would not buy them under the

¹https://www.amazon.com/Figures-Action-Secret-Forest-Blind/dp/B096KFHBDG?th=1

traditional pricing strategy. We also show that in many situations, the firm can maximize profits by selling only loot boxes. We establish conditions under which consumers may find it optimal to purchase multiple loot boxes, which is consistent with observation but has not been explored in prior literature. In particular, we show that if consumers with weak preferences have lower average valuations then it may be optimal to offer only loot box. Thus, despite the fact that, in the probabilistic selling literature, the profitability advantage relies on the expansion of the menu of prices, our results show that it is sometimes optimal for the firm to reduce the menu of prices under the loot box strategy even if it is not hard for the firm to add deterministic products to the menu. This result is consistent with practice.

Loot boxes that generate multiple purchases have attracted bitter criticism from mass media and legislatures. According to the research commissioned by the GambleAware charity, around 5% of players spend more than \$100 per month buying loot boxes, contributing to nearly half of the video game industry revenue (Close and Lloyd 2021). BBC News reported that four children spent nearly £500 in three weeks buying player packs in FIFA and still never got their favorite player. A 32-year old FIFA player spent \$10,000 on *Ultimate Team* in two years (EuroGamer 2018). The Norwegian Consumer Council, backed by 18 European countries, claimed that loot boxes exploit consumers with deceptive designs, skewed probabilities, and aggressive marketing. Video game developers and platforms in US have been accused of violating gambling laws by selling loot boxes which contain chance-based rewards (e.g., Taylor v. Apple, Coffee v. Google, G.G. v. Valve Corp.). The Belgian Gaming Commission examined popular video games in April 2018 and required the removal of loot box transactions with real money. The Dutch gaming authority examined ten popular video games and determined that four contravened its Betting and Gaming Act.² To examine addiction and consumer protection, we consider the case where consumers have a higher purchase propensity as they purchase more loot boxes. This addictive nature makes the loot box strategy even more attractive for the firm. In addition, we find that even in such situations, loot boxes can improve social welfare. Thus, public policies restricting loot

²Loot boxes have also been used and criticized in non-digital settings beyond the online entertainment industry. For example, KFC China launched its blind box promotion which offers a family meal with a loot box containing one of six Dimoo toy figures. The Chinese Consumer Association criticized KFC China for inducing excessive consumption and food waste, so contravening the sustainability initiatives.

boxes may be counterproductive.

Our model shows that higher production costs make the loot box strategy less attractive and harder to outperform the traditional pricing strategy. This result explains why loot boxes are more popular for virtual goods and less common for traditional goods. Some firms promise that consumers will receive their preferred products with no more than a certain number of purchases. Such a promise might be viewed as increasing consumer welfare since it potentially reduces excessive purchases needed for a consumer to receive her preferred product. However, our analysis reveals that such a promise can instead reduce consumer welfare and increase profits.

The remainder of this paper is organized as follows. In §2, we review the relevant literature. In §3, we describe our model and examine the implications of repeated purchases of loot boxes on relative profitability, consumer welfare, and social welfare. In §4, we extend the model to relax several assumptions and examine whether our model is robust to changes in assumptions and to derive additional results. §5 concludes the paper with discussion of managerial implications and directions for future research.

2 Literature Review

Our work is closely related to the probabilistic selling literature (see Jerath, Netessine, and Veeraraghavan 2009 for a comprehensive review). In their seminal paper, Fay and Xie (2008) define a probabilistic good as "an offer involving a probability of getting any one of a set of multiple distinct items" (p. 674) and examine whether it is profitable to add a probabilistic good to the existing product line. They show that the probabilistic selling strategy improves profits through both enhanced price discrimination and market expansion in horizontal markets. A variety of probabilistic selling applications are well documented in the existing literature. Zhang et al. (2015) investigate probabilistic selling in quality-differentiated markets and show its profit advantages from its better ability to dispose of excess capacity. Jerath et al. (2010) examine whether a firm operating in a horizontal duopoly market should dispose of excess capacity with last-minute selling or with probabilistic selling. The authors show that probabilistic selling dominates when consumer

valuations are low and there is high service differentiation. Ren and Huang (2022) consider the case when products are vertically differentiated and show that probabilistic selling can improve profits by softening inter-temporal cannibalization for high-quality products. Huang and Yu (2014) offer a behavioral perspective and show that probabilistic selling may soften price competition due to consumer bounded rationality. Prasad, Sheng, and Zhao (2021) study whether a retailer would offer probabilistic goods provided by an upstream manufacturer. They find that if the retailer could commit to allocation probabilities before the upstream manufacturer commits to wholesale prices, then asymmetric probabilities can be optimal. In our context, the manufacturer sells the product directly and we find that asymmetric allocation probabilities can be optimal even in such contexts.

The loot box strategy that we study differs from the existing literature on probabilistic selling in several ways. First, most of the probabilistic selling literature are studying situations where consumers buy at most once. This is a reasonable assumption in the case of hotel rooms or flights but cannot explain why we commonly observe the repeated purchase of probabilistic goods. Second, probabilistic selling as defined in Fay and Xie (2008) assumes that the firm offers the probabilistic good in addition to deterministic products. In contrast, in our context, the loot box is offered *instead* of deterministic goods. Thus, we model a situation in which a firm reduces the menu of prices. We show that reducing the menu of prices can lead to higher profits. In particular, we find that in situations where consumers with weak preferences have lower average valuations relative to consumers with stronger preferences, offering only loot box where some consumers purchase repeatedly can be optimal.³ To the extent that there are menu costs or preferences for simpler choice context, our approach may be preferred by firms and is consistent with the widespread adoptions of loot box strategies in practice. Our analysis shows that it may be optimal to use asymmetric probabilities for ex ante symmetric products in contrast to the current literature which argues for the optimality of uniform allocation probabilities (Fay and Xie 2008; Chen et al. 2021).

Despite its popularity, *repeated* purchases of loot boxes have been rarely discussed in the literature. One exception is Chen et al. (2021) who examine the price and design of loot

³This situation is implicitly ruled out by the Hotelling model used by Fay and Xie (2008).

boxes. They assume that consumer valuations for the N distinct items potentially in loot boxes are i.i.d. randomly drawn from the same distribution with known mean and variance. In contrast, our model assumes that products are horizontally differentiated and for some consumers, product valuations are negatively correlated. Our work complements Chen et al. (2021) who assume that each item in the loot box has an equal probability and defend this simplifying assumption using asymptotic optimality. When the sellers offer a sufficiently large variety of alternatives in the loot box (i.e. long product line), we also find that it is optimal to use uniform allocation probabilities. However, when there are relatively few potential outcomes (i.e., short product line), we find that asymmetric allocation probabilities can be optimal, which is consistent with practice.

3 Model

Firm Behavior. We consider a monopolist selling two symmetric products. For example, video game providers offer virtual add-ons such as character costumes which have almost identical functions but different appearances (Chen et al. 2021). We assume the production cost to be zero for virtual add-ons as it is generally costless for video game developers to provide an additional unit of virtual products. We relax this assumption in §4.3. While more than two variants are often offered by firms in practice, we consider two products here as this simplification allows us to cleanly highlight the intuition. In §4.4, we study the case of more than two alternatives offered on a longer product line.

Buyer Behavior. There are three segments of consumers in our model as summarized in Table 1. The first segment, labeled as Segment A, prefers product 1. In particular, this segment values product 1 at v and product 2 at w where v > w. Symmetrically, there is another segment, labeled as Segment B, who values product 2 at v and product 1 at w. We also allow for a third segment, labeled as Segment M, where the consumers are indifferent between the two products and have a value β for each product. Loyal consumers (i.e., Segment A and B) value their preferred product more than switchers (i.e., Segment M). In other words, we assume that $\beta < v$. Note that our model, however, does not restrict the

⁴ While it is possible that switchers can have a higher valuation than loyals i.e., $\beta > v$, the loot box strategy would be be suboptimal and dominated by the traditional pricing strategy. Therefore, we rule out

parameter β relative to w. In particular, we allow both $w < \beta$ and $w > \beta$. When $w > \beta$, the loyals' willingness to pay for their less preferred product is higher than the switchers' willingness to pay. On the other hand, when $w < \beta$, loyals have lower willingness to pay for their less preferred products than the switchers.

We assume that the size of Segment A and B is α and therefore Segment M has a size $1-2\alpha$. This assumption of equally sized loyal segments facilitates the discussion of symmetric products. Consistent with past research, we assume that the consumer's valuation of multiple products is the maximum single-product valuation. For example, consider the case where a soccer fan prefers a (virtual) red jersey to a blue jersey. Our model assumes that her valuation of a red jersey plus a blue jersey is the same as her valuation of a red jersey alone. In other words, given that she has already owned a red jersey, an additional jersey, whether red or blue, generates no additional valuation, as she would always (virtually) wear the same non-perishable red jersey. Consequently, the consumer only prefers to purchase once when both products are priced separately. The assumption is consistent with most models which allow consumers to make only a single purchase. For example, consider the typical Hotelling model. In such models, even if the consumer gets positive utility from both products, she consumes only the product that gives her the maximum utility and therefore implicitly the model assumes that the additional utility from any other product is 0, which is consistent with our assumption. Also note that in our context, this assumption is conservative in the sense that if we were to relax this assumption and allow consumers to receive additional utility from their less preferred products, then loot boxes, as a pricing strategy, would become even more attractive for the firm.

[Insert Table 1 here.]

Our formulation is similar in spirit to the commonly used models such as Narasimhan (1988) and Fay and Xie (2008). More specifically, Segment A could be seen as loyal to product 1, Segment B as loyal to product 2 while Segment M is indifferent between the two products. Each consumer's valuation is her private information, and therefore the firm cannot price discriminate on the basis of consumer segments.

the case where $\beta > v$ to simplify the presentation. We note that for this case (see footnote 8).

3.1 Traditional Pricing

We first consider the case where the firm sets the prices for each product separately. In this regard, there are three benchmark cases that the firm can employ, dependent upon whether the firm wants to serve all three segments or only partially serve the market. We have the following lemma.

Lemma 1. When the firm sets prices for each product separately, then the optimal prices are:

a. If
$$\alpha(v+w) \leq \beta \leq w$$
, then $p_1^* = v - (w-\beta)$ and $p_2^* = \beta$.

b. If
$$\beta > \max\left\{w, \frac{\alpha v}{1-\alpha}\right\}$$
, then $p_1^* = v$ and $p_2^* = \beta$.

c. Otherwise, the firm only sells to the loyal segments and sets a price v for both $(p_1^* = p_2^* = v)$.

To see the intuition for the lemma, suppose the firm charges p_1 for product 1 and p_2 for product 2. Without loss of generality, assume that $p_1 \geq p_2$. The individual rationality (IR) constraint for the switchers' purchase implies that at least one price has to be lower than the switchers' valuation i.e., $p_2 \leq \beta$. In order to make Segment A self-select into product 1 and Segment B into product 2, we have the following two incentive-compatibility (IC) constraints:

$$v - p_1 \ge w - p_2 \tag{1}$$

$$v - p_2 \ge w - p_1 \tag{2}$$

The constraint in (1) ensures that consumers from Segment A choose to purchase their preferred product (i.e., product 1) while the constraint in (2) ensures that consumers from segment B buy product 2.

Figure 1 depicts the optimal traditional pricing strategies under three different cases where profits are shaded. The first sub-figure corresponds to Lemma 1a where both loyal

⁵There is another symmetric solution with $p_1^* = \beta$ and $p_2^* = v - (w - \beta)$

segments who have weak product preferences are left with positive surplus. When the firm sets p_2 to be β to attract switchers, they have to lower p_1 by $w - \beta$ so that Segment A would not turn to buy their less preferred product (i.e., product 2). In comparison, Lemma 1b (shown in the second sub-figure) discusses the case where the loyal segments have strong product preferences. Their valuation of the less preferred product, w, is lower than β . In this case, the firm maximizes its profit by setting p_1 to be v under which the loyal segments still buy their preferred products. It is interesting to note that Lemma 1a and 1b show that the price and market share of the two ex-ante symmetric products will end up being asymmetric in order for the firm to price discriminate. Besides these two cases where the market is fully covered, the firm could choose to serve the two loyal segments only, as depicted in the third sub-figure and shown in Lemma 1c. This happens when the switchers have either low valuation or a small market share.

3.2 Loot box

Firm Behavior. Under the loot box strategy, the firm only sells a probabilistic good at a price p with a pre-announced allocation probability ϕ for product 1 and $1-\phi$ for product 2. We focus on the case when the firm only sells the loot box and does not offer products for direct purchase for several reasons. First, this represents how loot boxes are often sold in virtual markets. Furthermore, this formulation allows us to make meaningful comparisons between traditional pricing and loot boxes. If loot boxes are offered in addition to traditional pricing, this strategy must weakly increase profits. Instead of increasing the menu of prices, we are interested in the strategy of reducing the menu of prices by offering only loot boxes. In §4.5 we consider the scenario where the firm offers deterministic products in addition to loot boxes, and discuss when it is sufficient to only offer loot boxes. We also assume that the firm does not practice dynamic pricing, where the price offered may vary based on factors such as whether the consumer is making a repeat purchase. This would only increase the profitability of loot boxes, so we make the conservative assumption that the firm does not charge prices based on the consumer's purchase history.

We assume that the pre-announced allocation probability is true as required by

compliance guidelines in some countries.⁶ Both Apple and Google app stores require that games with loot boxes must declare the allocation probabilities. In addition, even if the disclosure of the true allocation probabilities is not mandatory, firms have been actively adopting blockchain technologies to convince consumers that the announced allocation probabilities are authentic, transparent and trustworthy (Carvalho 2021). As is standard in the literature (Fay and Xie 2008; Zhang et al. 2015), consumers are assumed to be rational and forward-looking. We relax the assumption of full rationality in §4.1.

To model repeated purchases of loot boxes, we assume that each Buyer Behavior. consumer maximizes her utility by sequentially deciding whether to purchase a loot box given the current realized outcome. Consumers take prospective repurchases of loot boxes into consideration when deciding whether to purchase their first loot box. When the price is lower than the expected incremental payoff, consumers would choose to purchase repeatedly until they get their ideal product. For example, consider a consumer from Segment A who purchases her first loot box and receives product 2. If her second loot box contains product 1, she would have an incremental payoff, v-w. If the realized outcome of her second loot box is again product 2, she would have no incremental payoff. Given this, the consumer decides whether to purchase the loot box again or exit. This decision-making process repeats itself. Since the switchers have zero incremental payoff from the second loot box, we only consider repeated purchases of loot boxes for the two loyal segments. In order to derive the conditions under which a consumer will repurchase, consider the case when consumer in Segment A, purchases once and receives product 2. She has two choices, either to purchase the loot box again or keep product 2 and exit. Let the value function for a Segment A consumer holding product 2 at t=2 be denoted by V_2^A . We therefore have:

$$V_2^A = \max(w, -p + \phi v + (1 - \phi)V_2^A)$$
(3)

where the first term in the bracket represents the valuation when the Segment A consumer keeps product 2 and exits. When the consumer purchases another loot box, she incurs price p and gets her preferred product 1 with probability ϕ . With probability $(1 - \phi)$, she will receive the continuation payoff V_2^A . If the optimal action is to repeat purchase, then from

(3), it follows that:

$$V_2^A = v - \frac{p}{\phi} \tag{4}$$

Using (3) and (4), it follows that the condition for the consumer to repeat purchase is:

$$p \le \phi(v - w) \tag{5}$$

Assuming (5) is satisfied, the consumer repeat purchases and therefore V_2^A is given by (4). In the same spirit, we denote the value function for a Segment A consumer at t=1 as $V_1^A = \max\left(0, -p + \phi v + (1-\phi)V_2^A\right)$ The consumer purchases her first loot box only when

$$-p + \phi v + (1 - \phi)V_2^A \ge 0 \tag{6}$$

Therefore, assuming (5) is satisfied, we use (4) to derive the condition for the Segment A consumer to purchase her first loot box as:

$$p \le \phi v \tag{7}$$

Clearly, (7) is a weaker condition than (5). Therefore, as long as (5) is satisfied, the Segment A consumer purchases repeatedly until she gets product 1.7

Using the same logic, the condition for the Segment B consumer to purchase repeatedly is given by:

$$p \le (1 - \phi)(v - w) \tag{8}$$

If (7) or (8) is violated, consumers would purchase the loot box once when the expected average payoff is greater than price. To be precise, if $p \in (\phi(v-w), \phi v + (1-\phi)w)$, then Segment A consumers would buy only once. For Segment M to buy, we require that $p \leq \beta$.

When we discuss the profit advantage of the loot box strategy, first note that the loot box strategy could possibly generate higher revenue than the traditional pricing strategy only in situations where both loyal segments purchase repeatedly and the switchers only purchase once. To see this, first consider the case where Segment M is not served at all. The firm

The condition for the Segment A consumer to purchase the first loot box becomes $p \leq \phi v + (1-\phi)w$. This result implies that the Segment A consumer would purchase her first loot box when the price is lower than the expected payoff. Therefore, when $\phi(v-w) , the Segment A consumer will purchase only once.$

gets zero profit from the switchers, for which the loot box strategy is no better than setting $p_1 = p_2 = v$ from the traditional pricing strategy.

Now, let's consider the case where Segment M will purchase. Each loyal segment could purchase once or repeatedly. If both purchase only once, then all three segments will effectively pay the same price. In this case, the loot box strategy would be inferior to the profits under Lemma 1a or 1b from the traditional pricing strategy. If one loyal segment purchases once and another purchases repeatedly, then without loss of generality, assume that Segment A purchases repeatedly and B once. Then we require that:

$$p \le \phi(v - w) \tag{9}$$

$$p > (1 - \phi)(v - w) \tag{10}$$

If the firm serves Segment M under the loot box strategy, then the expected total profit is:

$$\frac{\alpha p}{\phi} + (1 - \alpha)p \le \alpha(v - w) + (1 - \alpha)\beta < \alpha \min(v - w + \beta, v) + (1 - \alpha)\beta \tag{11}$$

where the first inequality follows since $p \leq \phi(v - w)$ for Segment A to purchase repeatedly and $p \leq \beta$ for Segment M to purchase. Using the results from Lemma 1a and 1b it follows that in this case profits under the loot box strategy is lower than traditional pricing.

If the firm only serves Segment A and B under the loot box strategy and only Segment A repurchases, then the expected total profit is:

$$\frac{\alpha p}{\phi} + \alpha p \le \alpha (v - w) + \alpha v < 2\alpha v \tag{12}$$

Thus, the loot box strategy would again generate lower profits compared with Lemma 1a or 1b from the traditional pricing strategy. Thus, the necessary condition for the loot box strategy to generate higher profits than the traditional pricing strategy is that both loyal segments purchase repeatedly and switchers purchase once. This then enables us to simplify the profit maximization problem under the loot box strategy as:

$$\max_{\phi,p} \quad \pi(\phi,p) = \frac{\alpha p}{\phi} + \frac{\alpha p}{1-\phi} + (1-2\alpha)p \tag{13}$$

s. t.
$$p \le \phi(v - w)$$
 (14)

$$p \le (1 - \phi)(v - w) \tag{15}$$

$$p \le \beta \tag{16}$$

We have the following result:

Proposition 1. The optimal loot box strategy when both loyal segments purchase repeatedly and switchers purchase once is given by

a. If
$$\beta > \frac{v-w}{2}$$
, the firm sets $\phi^* = \frac{1}{2}$ and a price $p^* = \frac{v-w}{2}$.

b. If
$$\beta \leq \frac{v-w}{2}$$
, the firm sets $\phi^* = \frac{\beta}{v-w}$ and a price $p^* = \beta$.

The firm will decide both the unit price (p^*) and the probability allocation rule (ϕ^*) under the loot box strategy. Figure 2 depicts the optimal loot box strategies under two different cases where the revenue is depicted by the shaded region. Since the optimal loot box strategy should satisfy the conditions that the two loyal segments will purchase repeatedly whereas the switchers buy once, the optimal price satisfies the following condition:

$$p^* = \min \{ \phi(v - w), (1 - \phi)(v - w), \beta \}$$

In choosing the optimal price and allocation probabilities, the firm can use two broad strategies. When the switchers' valuation β is relatively large (i.e., $\beta \geq \frac{v-w}{2}$), it follows that $p^* < \beta$. In other words, the inequality constraint for Segment M is slack. Therefore, the firm leaves positive surplus to the switcher segment but extracts all the incremental surplus (i.e., v-w) from the two loyal segments. On the other hand, when β is small, the firm is able to extract all the surplus from the switchers but must leave some surplus to the two loyal segments.

Proposition 1a considers this situation when the switchers' valuation β is large (i.e., $\beta \geq \frac{v-w}{2}$). In this case, the optimal solution is such that the consumer is equally likely to get both products with $\phi^* = \frac{1}{2}$. The firm leaves a surplus of w to both loyal segments and $\beta - \frac{v-w}{2}$ for switchers. Both loyal segments are expected to purchase the loot boxes twice whereas the switchers buy once, as shown in Figure 2a. This result is consistent with Chen

⁸From this it is immediate that if $\beta > v$ then the traditional pricing strategy will dominate the loot box strategy. To see this, note that when $\beta > v$, the firm can make at least v under the traditional pricing strategy. Under the loot box strategy, the switchers will only buy once and the analysis for Proposition 1 still applies. However, the profits when $\beta > v$ for loot box is given by $\left(\frac{v-w}{2}\right)\left(\alpha + \frac{1}{2}\right) < v$.

There is another symmetric solution with $\phi^* = 1 - \beta$ and $p^* = \frac{\beta}{v-w}$.

et al. (2021) who show the optimality of uniform allocation probabilities. However, our work considers the more practical case of a small and finite number of products.

[Insert Figure 2 here.]

Now consider the case when the switchers' valuation β is small. In this case, at least one of the two loyal segments has positive surplus. The firm will then set $p^* = \beta$ to extract all the surplus from the switcher segment. At the same time, the firm sets $\phi^* = \frac{\beta}{v-w}$ to extract all the incremental surplus from Segment A. However, since $\frac{\beta}{v-w} < \frac{1}{2}$, the chance for the Segment B consumer to get product 2 is higher than $\frac{1}{2}$ (see Figure 2).

Proposition 1b shows that when β is low, the optimal allocation probabilities are polarized, i.e., $\phi^* = \frac{\beta}{v-w} < \frac{1}{2}$ and $1 - \phi^* > \frac{1}{2}$). This result is different than the finite product result in Fay and Xie (2008) and the asymptotic result in Chen et al. (2021) where both authors find it optimal to use symmetric allocation probabilities. When uniform probability allocation is optimal (i.e., $\phi^* = \frac{1}{2}$), both loyal segments will make an average of two purchases. When the firm uses polarized probability allocations (i.e., $\phi^* < \frac{1}{2}$), however, Segment 1 has to purchase, on average, more than twice while Segment 2, on average, has to purchase less than twice. The magnitude of the increase in unit sales to Segment 1 exceeds the magnitude of the decrease in unit sales to Segment 2.¹⁰ The higher aggregate unit sales motivate the firm to polarize the probability allocations.

It is useful to contrast our result with prior literature on probabilistic goods. Similar to prior literature, loot boxes introduce buyer uncertainty in product assignments. However, the loot box strategy enables price discrimination using the time dimension whereas the probabilistic selling strategy creates a new product and adds it to the existing product line (Fay and Xie 2008; Zhang et al. 2015). It is also useful to note that in our context rational consumers may end up paying more to get their desired product than its worth. For example, consider the case when w = 0 and $\phi^* = \frac{1}{2}$. In this case, the loyal consumer will pay $\frac{v}{2}$ for a loot box until she has her desired product. Therefore, 25% of the loyal consumers will pay more than v whereby both of their first two loot boxes generate the less preferred products. Similarly, about 6.25% of loyal consumers will pay more than 2v, which is twice the valuation

To see this, note that the demand from the loyals is given by $\frac{1}{\phi} + \frac{1}{1-\phi} = \frac{1}{\phi(1-\phi)}$ which is decreasing in ϕ for $\phi < \frac{1}{2}$.

of their desired product. Such rational behavior could be interpreted as evidence of gambling and addiction, leading to calls for regulations. Our results show that one should be careful in equating repeated purchases with evidence of addiction.

Profit Comparison

Now we explore the conditions under which the loot box strategy generates higher profits than the traditional pricing strategy.

Proposition 2. When the consumer valuation for their less preferred product (w) is low, and both the size of the loyal segment (α) and the switchers valuation for the product (β) are in intermediate range, the loot box strategy generates higher profits than the traditional pricing strategy.

The exact conditions for the critical values of w and β are complicated functions which we present in Appendix A. Figure 3, shows the region in which loot box dominates traditional pricing.

[Insert Figure 3 here.]

To understand the intuition for the proposition, first note that the loot box strategy creates three effective prices: p for the probabilistic good if the consumers purchases only once, $\frac{p}{\phi}$ for product 1 if she purchases repeatedly until she gets product 1, and $\frac{p}{1-\phi}$ for product 2 if she purchases repeatedly until she gets product 2. There are two mechanisms by which loot boxes can be more profitable than traditional pricing: first, loot boxes can lead to better price discrimination; second, loot boxes can lead to more consumers being served. Under the traditional pricing strategy, the firm extracts all consumer surplus from the switchers or chooses not to serve this segment. When the market is fully served, the firm charges β from both the switchers and one loyal segment under traditional pricing strategy. However, the loot box strategy can offer more flexibility in the sense that the loyal segments and the switchers pay different effective prices. For example, when $\beta \leq \frac{v-w}{2}$, the switchers will pay β , which equals to the price paid under the traditional pricing strategy, whereas the effective price paid by the two loyal segments is 2β . The second reason why loot boxes can be more profitable is that the loot box strategy enables the firm to charge a low price and

serve the switchers even when their low valuation would have stopped the seller from serving the switchers under the traditional pricing strategy. This is because even with a low loot box price, the firm is able to adjust winning probabilities and charge high effective prices from two loyal segments.

Now, we will discuss the impact of various parameters on the relative profitability of the loot box strategy. First, consider the impact of the loyal consumer's valuation of the less preferred product (w). Under the loot box strategy, the firm has to leave at least w to each loyal segment so as to induce loyal segments to purchase repeatedly. In contrast, under the traditional pricing strategy, the firm extracts all consumer surplus from at least one loyal segment. Consequently, when w is large, the traditional pricing will outperform the loot box strategy.

Then we will examine the impact of the switchers' valuation (β) . When β is low, under the traditional pricing strategy, the firm foregoes selling to the switchers but extracts all consumer surplus from two loyal segments. Under the loot box strategy, nevertheless, the firm will serve the switchers at a low price of β . At the same time, the seller has to forego a relatively large surplus from one loyal segment (which is given by $v - \frac{\beta}{1-\phi}$). Therefore, the traditional pricing strategy will dominate for small β (i.e., when the switchers' willingness to pay is low). Conversely, for large values of β , under the traditional pricing strategy, the firm leaves the surplus $v - \beta$ to one loyal segment. Therefore, as β increases, the profitability of the traditional pricing strategy will increase. However, under the loot box strategy, the firm charges a price $\frac{v-w}{2}$, which is lower than β , and thus leaves surplus to all three segments. In particular, the seller is able to extract v - w from each of two loyal loyal segments and $\frac{v-w}{2}$ from the switchers. Consequently, when β is large enough (i.e., $\beta > \frac{v-(1+2\alpha)w}{2(1-\alpha)}$), the traditional pricing strategy will dominates the loot box strategy. In summary, only for intermediate values of β , the loot box strategy outperforms the traditional pricing strategy, as shown in Figure 3.

Finally, we will discuss the impact of the market share of each loyal segment (α) . Note that when there are sizable loyal consumers (i.e., $\alpha \geq \frac{\beta}{v+\beta}$), as α increases, the traditional pricing strategy will extract more surplus from two bigger loyal segments and forego selling to the smaller switchers' segment. This implies that for large values of α , the traditional pricing

strategy would dominate. Similarly, when there are only few loyal consumers (i.e., $\alpha < \frac{\beta}{v+\beta}$), the firm serves all three segments and leaves surplus $\alpha(v-\beta)$ to only one loyal segment under the traditional pricing strategy. Therefore, for small values of α , the traditional pricing strategy would dominate because the firm leaves less profit to the loyal segment as α decreases. To summarize, the loot box strategy is more profitable than the traditional pricing strategy when the share of loyal segments (α) is in intermediate range.

Welfare Analysis

Now we examine how loot boxes affect welfare. We have the following result:

Proposition 3. Relative to the traditional pricing strategy, the loot box strategy weakly improves social welfare and can also improve consumer welfare.

In mass media, loot boxes have been widely criticized as socially harmful (Abarbanel 2018; Li, Mills, and Nower 2019; BBC 2019). However, our analysis shows that the loot box strategy weakly improves social welfare. The reason is that loot boxes enable the firm to serve consumers who would be excluded under traditional pricing. In particular, the switcher segment is more likely to be excluded under traditional pricing than under the loot box strategy. This improvement in social welfare due to market expansion is robust whether the polarized or uniform allocation probabilities are used. Thus, loot boxes can lead to a win-win situation in which the firm, the consumer and the society all benefit. This result supports the recent decision by the Entertainment Software Rating Board (ESRB) on the legality of loot box usage. We note, nevertheless, that some of the concerns about the socially harmful impact are primarily related to the idea that loot boxes are addictive, which we have not considered so far. In §4.1, we will discuss this case and show that the main results are robust.

Absent market expansion, consumers can be worse off. This is because the firm will offer loot boxes to extract more consumer surplus with better price discrimination. For example, under the tradition pricing strategy, the consumers from Segment B pay β when the seller serves all three segments. Yet, under the loot box strategy, when the seller uses the polarized

¹¹https://bagogames.com/the-big-debate-about-loot-boxes-are-they-good-or-bad/

allocation probabilities, the effective price for Segment B consumers becomes $\frac{\beta}{1-\phi}$, which is higher than β . This finding justifies complaints on the mass media that loot boxes can hurt consumers. However, unlike the discussion in the media which associates loot boxes with addiction, our analysis does not rely on consumer irrationality. We explore the case of addictive loot boxes in §4.1 and consider its impact on consumer welfare.

4 Model Extensions

In this section, we extend the base model to relax several assumptions. This allows us to examine the validity of our results in the base model as well as to provide new insights. In the base model, we assume that consumers are rational. However, much of the criticism against loot boxes have been that they can be addictive and reduce welfare. We explore this issue in $\S4.1$. The base model also assumes that consumers will continue to pay for their product until they receive their preferred product. In practice, it is possible that consumers have a budget and they will not spend more than this budget. In $\S4.2$ we examine this case and see whether the insights from the base model survive in this formulation. In the base model, we assume that marginal costs are zero. This assumption is reasonable in the context of digital goods but less so in the context of physical goods. In $\S4.3$ we relax this assumption. In $\S4.4$, we examine the situation where the firm offers n products. In $\S4.5$, we allow the firm to offer deterministic products along with loot boxes and find the conditions under which it suffices for the firm to offer only loot boxes and not offer individual products for sale. In $\S4.6$, we examine the optimal design of winning promises where the firm can guarantee that the consumer will get her desired product with no more than a certain number of purchases.

4.1 Lootboxes are addictive.

In our base model, we assumed that consumers are rational and show that the loot box strategy in such context can be welfare enhancing and may even improve consumer welfare. However, loot boxes have been criticized as addictive when consumers purchase a large number of loot boxes. We explore this case in this section. In particular, we model a situation in which a consumer has an increasing propensity to purchase a loot box after a

higher number of failed trials to get their preferred product. In order to model this addictive nature in the simplest possible way, we assume that the consumer's desire to purchase another loot box after making t purchases increases by $\gamma(t) \geq 0$ and $\gamma'(t) > 0$. Consider a consumer in Segment A who has made t purchases and has not received her preferred product. She will make another purchase in period t + 1 if:

$$\phi v - p + \gamma(t) + (1 - \phi)V_{t+2} \ge w$$
 (17)

where V_t is the continuation value at period t. Note that unlike the base case, the consumer is more likely to purchase another loot box after she has already made a higher number of purchases. This parsimoniously captures the idea that buying loot boxes can be addictive.¹²

For a consumer to make purchase decisions, we assume that the consumer recognizes that she will experience this increased desire to purchase in the future. As is common in the literature on biases, we assume that the consumer has rational expectations about her future behavior. Alternatively, we could assume that consumers are partially sophisticated and underestimate their additional utility after t purchases as $\eta \gamma(t)$ where $0 < \eta < 1$. As long as $\eta > \frac{w}{\gamma(1)}$ our results continue to hold. Details are in Appendix A.

In Appendix A, we show that if a consumer purchases the product in period t then she will continue to purchase in period t+1. Thus, if (17) is satisfied then the consumer expects to purchase in the $(t+1)^{st}$ occasion, then $V_{t+2} = v - \frac{p}{\phi}$. Therefore, the consumer will purchase in $(t+1)^{st}$ period if:

$$p \le \phi \left[(v - w) + \gamma(t) \right] \tag{18}$$

Proceeding in this manner, we can show that for the consumer to make the first purchase

 $^{^{12}}$ Alternatively, consumers who have already invested t times in the loot box may feel that there is a need for them to recoup their investment due to sunk-cost bias. Such a formulation can be captured by assuming that not buying the product again gives an expected utility of $w - t\gamma p$ where p is the price paid each time. The key results with this alternate formulation are similar.

¹³Note that from the long-term perspective the consumer does not take into account $\gamma(t+j), j \geq 1$ into account when making her decisions. She does however recognize that $\gamma(t+j)$ affects her decisions in the subsequent periods. This is similar to the literature on self-control where consumers recognize that they over-weight current consumption but while making their decisions for the future, consider this additional utility received due to self-control bias only to the extent that it influences decisions in the future (see for example, Jain (2012)). If we were to view the additional utility that consumers receive from repeatedly buying the loot box, then this would be welfare enhancing and make loot boxes even more profitable. Our formulation is consistent with the idea that the addictive nature of loot boxes is not desirable from a welfare perspective.

we need that:

$$p \le \phi v + \phi \min(0, \gamma(1) - w) \tag{19}$$

This condition is weaker than the base case and therefore the firm could charge higher prices. In Appendix A, we derive the optimal prices and allocation probabilities as:

$$(p^*, \phi^*) = \begin{cases} \left(\beta, \frac{\beta}{v + \min(0, \gamma(1) - w)}\right) & \text{if } \frac{\beta}{v + \min(0, \gamma(1) - w)} \le \frac{1}{2} \\ \left(\frac{v}{2}, \frac{1}{2}\right) & \text{if } \frac{\beta}{v + \min(0, \gamma(1) - w)} > \frac{1}{2} \end{cases}$$

$$(20)$$

Note that the prices are weakly higher than those in the base case. Furthermore, the asymmetric allocation probabilities that we observe in the base case also exist in this case. In fact, since $\frac{\beta}{v+\min(0,\gamma(1)-w)}$ is weakly decreasing in $\gamma(1)$, asymmetric allocation probabilities are more likely when loot boxes are addictive.

Proposition 4. As loot boxes become more addictive i.e., as $\gamma(1)$ increases, the profit under the loot box strategy increases. Furthermore, relative to the traditional pricing strategy, social welfare under the loot box strategy is weakly higher while consumer welfare can be lower under the loot box strategy.

The first part of the proposition is intuitive. An increase in $\gamma(t)$ leads the consumers to purchase loot boxes under weaker conditions. This allows the firm to charge higher prices and make higher profits. The second part of the proposition shows that the earlier result of enhanced welfare continues to hold. The intuition is that even when loot boxes are addictive, they increase market coverage and therefore social welfare. In fact, in this situation the firm could offer loot boxes solely because consumers are addictive (i.e., $\gamma(t) > 0$). In other words, the firm would rather not use loot boxes when consumers are not addictive (i.e., $\gamma(t) = 0$). Consumer surplus could be negatively impacted since consumers pay higher prices.

4.2 Consumers have budget constraint.

The base model assumes that consumers will continue to pay for their preferred product until they receive the product. In some cases, consumers could limit their purchases by setting a maximum budget such that if they exceed the budget, they will not purchase the product.¹⁴

¹⁴ This requires that consumers are able to commit to not spending over the budget. This can be achieved through either self-control or through government regulations. For example, the National Press

We now consider the implications of such budgets. Assume that a consumer sets a budget $B \geq v$ so that the maximum number of times the consumer will purchase the loot box will be $\frac{B}{n}$, where for simplicity we ignore the integer constraint.

In Appendix A, we show that the constraints for profit maximization remain the same as in the base case and show that the firm's profit-maximization problem is:

$$\max_{p,\phi} \Pi_0 = \alpha p \left[\frac{1 - (1 - \phi)^{\frac{B}{p}}}{\phi} + \frac{1 - \phi^{\frac{B}{p}}}{1 - \phi} \right] + (1 - 2\alpha)p \tag{21}$$

subject to the constraints as before. Note that it follows from the Envelope theorem that when the budget constraint becomes tighter (i.e., a lower B), the loot box strategy becomes less profitable.

$$\frac{d\Pi_0}{dB} = -\frac{\alpha(1-\phi)^{\frac{B}{p}}\log(1-\phi)}{\phi} - \frac{\alpha\phi^{\frac{B}{p}}\log(\phi)}{1-\phi} > 0$$
(22)

Since the budget constraint (B) does not affect traditional pricing, as is intuitive, the loot box strategy becomes relatively less attractive when a consumer sets a budget relative to the base case of no budget constraint.

Unfortunately, the maximization problem for the general case is intractable. However, we are able show that a sufficient (but not necessary) condition for the loot box strategy to be more profitable than the traditional pricing strategy is that v > 5w, $\alpha \le \frac{2(v-w)}{5v+3w}$, $\frac{v-w}{2} \le \beta \le \frac{(2-\alpha)v-(3\alpha+1)w}{4(1-\alpha)}$, $B \ge \frac{3(v-w)}{2}$ (see Appendix A for details). For example, if $v=1, w=0, \beta=0.6, \alpha=0.4, B=2$ then we can numerically show that the profit under the loot box strategy with $p^*=\phi^*=0.5$ is 0.85, which is greater than the profit under the traditional pricing strategy (0.8). For another example, consider the case when $v=1, w=0, \beta=0.4, \alpha=0.3, B=2$. In this case, we can show numerically that the optimal loot box strategy is to use polarized winning probabilities ($p^*=\phi^*=0.4$). The profit under the loot box strategy is 0.634. For the same parameters, under the traditional pricing strategy, the firm will not serve the switchers. The traditional pricing profit is lower at 0.6. Note this example shows that even under budget constraints, asymmetric winning probabilities can remain optimal. In addition, this numerical example reveals that the profit under the loot box strategy can

and Publication Administration in China mandates that all video games must limit the amount of money that users can add to their game accounts and notify games of "irrational consumption behavior" through a pop-up window. If consumers are not able to commit to limiting purchases then the analysis reduces to the base case.

still be higher with the profit under the traditional pricing strategy. Furthermore, social welfare is also higher under the loot box strategy (0.7336 versus 0.6) since the switchers are served *only* under the loot box strategy.¹⁵ Therefore, the key insights from the base model still hold in this case.

4.3 Marginal costs are strictly positive.

In the base model, we set the production cost to be 0. In this extension, we examine the loot box strategy in the context of non-digital products where production costs are positive. For example, $Pop\ Mart$, a toy maker, creates and distributes loot boxes of toys. In this case, the marginal production cost is non-negligible. We assume that the marginal cost for the product is c > 0. To ensure that the firm can have a positive demand at a price above marginal costs, we assume that $c < \beta < v$ (Fay and Xie 2008).

Lemma 2. When the firm sets prices for each product separately, then the optimal prices are:

a. If
$$\alpha(v+w) + (1-2\alpha)c \le \beta \le w$$
, then $p_1^* = v - (w-\beta)$ and $p_2^* = \beta$. 16

b. If
$$\beta > \max\left\{w, \frac{\alpha v + (1-2\alpha)c}{1-\alpha}\right\}$$
, then $p_1^* = v$ and $p_2^* = \beta$.

c. Otherwise, the firm only sells to the loyals and sets a price v for both $(p_1^* = p_2^* = v)$.

Compared with Lemma 1, Lemma 2 implies that the firm is more likely to serve only the two loyal segments as production costs increase. Now consider loot box. The profit-maximization problem under the loot box strategy is therefore:

$$\max_{\phi, p} \quad \pi(\phi, p) = \frac{\alpha(p - c)}{\phi} + \frac{\alpha(p - c)}{1 - \phi} + (1 - 2\alpha)(p - c) \tag{23}$$

under the same constraints as in the base case, i.e., (14)-(16). We show in the Appendix that the optimal loot box strategy in this case is the same as that in the base case.¹⁷ However, the

¹⁵Social welfare is still lower than the case with no budget constraint since there is a probability that two loyal segments will not get their preferred products when they run out of their budgets. This probability is $(1-\phi)^{\frac{B}{p}}$ for segment A and $\phi^{\frac{B}{p}}$ for Segment B.

¹⁶There is another symmetric solution with $p_1^* = \beta$ and $p_2^* = v - (w - \beta)$

¹⁷ Intuitively, we would expect that the asymmetric allocation probabilities are less likely to be profitable as c increases. However, the optimal solution are determined by the constraints (14) and (16) which are independent of c.

profitability of loot box strategy depends on the production cost c, which will be incurred whenever the firm sells one loot box.

Proposition 5. When the consumer valuation for their less preferred product (w) is low, and both the switchers' valuation for the product (β) and the size of the loyal segment (α) are in intermediate values, the loot box strategy generates higher profits than the traditional pricing strategy. However, as the production cost (c) increases, the range in which the loot box strategy is more profitable will be smaller.

The first part of the proposition shows that the main result is robust. However, the second part shows that the loot box strategy is less attractive for non-digital products. To highlight the intuition, assume w=0 and first consider the case of $\beta=\frac{v}{2}$. We have shown in the previous section that loot boxes can extract all surplus from all three consumer segments by setting $p=\frac{v}{2}$ and $\phi=\frac{1}{2}$. However, as the two loyal segments purchase loot boxes twice, the firm incurs the production costs for each sold unit. Notably, production costs cannot be transferred to consumers as $p^*=\min\{\phi(v-w),(1-\phi)(v-w),\beta\}$. Therefore, a higher production cost reduces the range of β where the loot box strategy improves profit relative to the traditional pricing strategy. In the Appendix, we show that a necessary condition for loot boxes to improve profits is $c<\min\left(\frac{v-3w}{4},\frac{(1-2\alpha)v-(1+2\alpha)w}{2}\right)$. As virtual goods typically have low production costs, Proposition 5 explains why loot boxes are more common for virtual goods.

Now we examine the effect of positive marginal costs on welfare.

Proposition 6. The loot box strategy can reduce social welfare. This reduction is larger when $\beta \leq \frac{v-w}{2}$.

Proposition 6 shows that loot boxes may be socially wasteful as consumers may end up purchasing loot boxes that contain their less preferred products. In contrast, under the traditional pricing scheme, consumers only purchase their most preferred products. Given that polarized allocation probabilities drive higher sales, the negative impact of loot boxes on social welfare is particularly pronounced when $\beta \leq \frac{v-w}{2}$.

4.4 The firm sells more than 2 products.

In this extension, we consider the case of long product lines. More specifically, the firm sells a loot box which contains one of multiple possible products. As shown in Table 2, we assume there are n symmetric products and n+1 consumer segments. There is one loyal segment for each product who values this particular product at v and the other n-1 products at w where v>w. The switchers, labeled as Segment M, are indifferent between the n alternatives and have valuation β for each product. To facilitate the comparison between product lines of different lengths, we assume that the market share of each loyal segment to be α/n , and therefore Segment M has a size $1-\alpha$. This assumption keeps the size of the segment independent of n. This could represent situations where the switchers segment consists of consumers who are not affected by the variety offered and have lower willingness to pay for any product.¹⁸

Under the traditional pricing strategy, the firm can offer separate prices for each deterministic product.¹⁹ Under the loot box strategy, the firm only offers a loot box with the pre-announced level of probabilities ϕ_i for product i = 1, 2, ...n. The profit-maximization problem under the loot box strategy is formulated as:

$$\max_{\phi_1,...,\phi_n,p} \quad \pi(\phi_1,...,\phi_n,p) = \sum_{i=1}^n \frac{\alpha p}{n\phi_i} + (1-\alpha)p$$
 (24)

s. t.
$$p \le \phi_i(v - w) \quad \forall i$$
 (25)

$$p \le \beta \tag{26}$$

$$0 \le \phi_i \le 1 \quad \forall i \tag{27}$$

$$\sum_{i=1}^{n} \phi_i = 1 \tag{28}$$

where (25) ensures that the loyal consumers repeat purchase and (26) ensures that the

¹⁸We also examine another model where the market share of each loyal segment is α and the switchers have a size of $1 - n\alpha$ where $n \leq \frac{1}{\alpha}$. In this model, a longer product line is able to match more consumers with their preferred products. Therefore, the remaining switchers have a smaller market size as the product line become longer. We find that the seller is incentivized to include the maximum amount of alternatives in the loot box. In other words, the optimal product line is as long as possible.

¹⁹In Appendix A, we show that profits under traditional pricing do not depend on n if the firm only sells to only the loyals and increase in n if the firm sells to all consumers.

switchers purchase the loot box. We have the following result:

Proposition 7. The optimal loot box strategy when all loyal segments purchase repeatedly and switchers purchase only once is given by:

a. If
$$\beta \geq \frac{v-w}{n}$$
, the firm sets $\phi = \frac{1}{n}$ and a price $p = \frac{v-w}{n}$.

b. If
$$\beta < \frac{v-w}{n}$$
, the firm sets $\phi_1 = \phi_2 = ... = \phi_{n-1} = \frac{\beta}{v-w}$, $\phi_n = 1 - \frac{(n-1)\beta}{v-w}$ and a price $p = \beta$.²⁰

Proposition 7a shows that the firm should offer n alternatives with equal probabilities when the switchers' valuation is high. Note from (25) and (26) that the optimal price $p^*leq \min \{\phi_1(v-w), \phi_2(v-w), \dots, \phi_n(v-w), \beta\}$. When $\beta \geq \frac{v-w}{n}$, it follows that $p^* < \beta$. In this case, it is optimal for the firm to leave a surplus w to all n loyal segments in order to induce repeat purchase. When $\beta < \frac{v-w}{n}$, nevertheless, the optimal allocations probabilities are polarized. Proposition 7b reveals that the firm allocates probability $\frac{\beta}{v-w}$ to any n-1 products and leaves the remaining probability $1 - \frac{(n-1)\beta}{v-w}$ to the last product. In general, Proposition 7 implies that as the product line gets longer, the uniform allocation probabilities are more commonly used because it is easier to satisfy the condition for Proposition 7a (i.e., $\beta > \frac{v-w}{n}$) as n increases. This finding is consistent with the asymptotic optimality of uniform allocation probabilities reported by Chen et al. (2021).

We now consider how the profitability of the loot box strategy changes with product line length (n).

Proposition 8. The profitability of the loot box strategy has an inverted-U shaped relationship with the product line length.

When the product line is relatively short (i.e., $n \leq \frac{v-w}{\beta}$), the firm uses polarized allocation probabilities under the loot box strategy. In this case, the firm extracts all surplus from the switchers segment and leaves surplus to the loyal segments. As n increases, the firm receives higher profits. First, note that the firm gives positive surplus $\left(v - \frac{(v-w)\beta}{1-(n-1)\beta}\right)$ to only one

²⁰The solution is not unique since there are another (n-1) symmetric solutions with $\phi_i = 1 - \frac{(n-1)\beta}{v-w}$, $\phi_{-i} = \frac{\beta}{v-w}$ and $p = \frac{\beta}{v-w}$ $\forall i = 1, 2, ..., n-1$. However, the firm's profits under all of these solutions are the same and therefore our results are the same.

loyal segment, whose market share $(\frac{1}{n})$ is lower as the product line becomes longer. Second, the optimal allocation probability $\phi_n = 1 - \frac{(n-1)\beta}{v-w}$ decreases with n. Therefore, the seller gains more profits by leaving less surplus to the loyal segments as the product line gets longer.

However, when the product line is long $(i.e., n > \frac{v-w}{\beta})$, the firm uses uniform probabilities to extract all consumer surplus from loyal segments and leaves surplus to the switchers. As the product line gets longer, the price $(\frac{v-w}{n})$ is lower, and therefore the seller needs to leave more surplus to the switchers $(i.e., \beta - \frac{v-w}{n})$ is higher). In summary, as shown in Figure 4, profits first increase and then decrease as n increases. This implies that the optimal product line length is $n^* = \frac{v-w}{\beta}$.²¹

[Insert Figure 4 here.]

4.5 The firm offers both loot boxes and deterministic products.

Now we consider the case when the firm can expand the menu of prices by offering deterministic products along with loot boxes. In general, offering deterministic products along with loot boxes must weakly improve profits relative to the traditional pricing strategy. However, it does provide more choices for consumers, leading to higher menu costs for firms and higher information processing costs for consumers.

Suppose the firm offers product 1 at a price p_1 , product 2 at a price p_2 and the loot box at price p with a probability ϕ to win product 1 and $1 - \phi$ to win product 2. For all the three prices to be used, we can show that each consumer segment only purchases once. For Segment A to use p_1 , we have the individual rationality condition as:

$$p_1 \le v \tag{29}$$

Then given that Segment A could choose to buy one or multiple loot boxes, we have the incentive-compatibility condition for Segment A to use p_1 as

$$v - p_1 \ge v - \frac{p}{\phi} \tag{30}$$

$$v - p_1 \ge \phi v + (1 - \phi)w - p$$
 (31)

²¹ This assumes that there are no development costs for producing a longer product line. If the firm incurs a fixed cost of $\xi h(n)$ where $\xi > 0, h'(n) > 0, h''(n) \ge 0$ then $n^* < \frac{v-w}{\beta}$ and decreases as ξ increases.

Summarizing (29), (30) and (31), the equilibrium price p_1 should satisfy

$$p_1 \le \min\left\{v, \frac{p}{\phi}, p + (1 - \phi)(v - w)\right\} \tag{32}$$

By symmetry, we have the condition for equilibrium price p_2 as:

$$p_2 \le \min\left\{v, \frac{p}{1-\phi}, p + \phi(v - w)\right\} \tag{33}$$

For Segment M to purchase the loot box we need that $p \leq \beta$. Then we formulate the firm's profit-maximization problem for the case when the firm uses all three prices as:

$$\max_{p_1, p_2, \phi, p} \pi(p_1, p_2, \phi, p) = \alpha(p_1 + p_2) + (1 - 2\alpha)p \tag{34}$$

s. t.
$$p_1 \le \min \left\{ v, \frac{p}{\phi}, p + (1 - \phi)(v - w) \right\}$$
 (35)

$$p_2 \le \min\left\{v, \frac{p}{1-\phi}, p + \phi(v - w)\right\} \tag{36}$$

$$p \le \beta \tag{37}$$

We will compare the profits from this case with the case when the firm only offers the loot box. We have the following result.

Proposition 9. The firm will only offer the loot box if $\alpha \in (\alpha_1^*, \alpha_2^*), \beta \in (\beta_1^*, \beta_2^*)$, and $w \leq w^*$, where α is the market share of each loyal segment, β is the switchers' valuation, and w is the loyal consumers' valuation for their less preferred products.

The expressions for $\alpha_1^*, \alpha_2^*, \beta_1^*, \beta_2^*, w^*$ are complicated and are available in Appendix A. Figure 5 shows the region in which the firm will offer *only* loot box or certain combination of loot box and deterministic product(s). Recall that Proposition 2 shows that in the intermediate ranges of α and β and low w, the firm finds it more profitable to use the loot box strategy rather than the traditional pricing strategy. Proposition 9 shows that in the intermediate ranges of α and β and low w, it may be optimal to offer *only* the loot box. This explains why firms often offer *only* loot boxes even if it is not difficult for firms to offer deterministic products in addition to loot boxes. If the firm prefers simpler menu of prices or if loot boxes are addictive then the range for which loot boxes are optimal will be even larger. To see the intuition, first note that when $\beta > \frac{v+w}{2}$, the firm finds it profitable to

offer one probabilistic product (i.e., loot box) and two deterministic products as in Fay and Xie (2008). In this situation, loot boxes are not purchased multiple times. However, such three-product scheme (i.e., one probabilistic product plus two deterministic products) can never be optimal when the effective price for Segment A consumers $(\frac{p}{\phi})$ is the same as the price for the preferred deterministic product (p_1) . If so, the firm could reduce the menu of products and still charge de facto different prices for different segments.

Interestingly, the result shows that for intermediate values of α and β , the shortest possible menu, which contains *only* the loot box, would suffice. To see this, recall that when $\beta \leq \frac{v-w}{2}$, the seller will set $p^* = \beta$ and use polarized allocation probabilities ($\phi^* = \frac{\beta}{v-w}$) and leave positive surplus to both loyal segments under the loot box strategy.

However, when the firm adds a deterministic products to the product line, it could potentially extract all consumer surplus from one loyal segment. Without loss of generality, assume Segment A pays $p_1^* = v$. By adding one deterministic product (e.g., Product 1) in addition to the loot box, the seller extracts more surplus from Segment A. When the firm extracts all the surplus from Segment A, it must set the allocation probability ϕ to be low enough so that this segment buys the deterministic product rather than the loot box. This however means that $1 - \phi$ is larger and therefore there is more surplus left to Segment B when she buys the loot box. The extra profits gained from Segment A is αw , which is independent of the switchers' valuation (β) . However, the surplus left to Segment B increases with β . Therefore, there exists β_1^* such that the seller would offer both the loot box and one deterministic product when $\beta \in (w, \beta_1^*)$ but rather offer only the loot box when $\beta \in (\beta_1^*, \frac{v-w}{2})$.

When $\beta \in (\frac{v-w}{2}, \frac{v+w}{2})$, the seller will set $p^* = \frac{v-w}{2}$ and use uniform allocation probabilities $(\phi^* = \frac{1}{2})$ and give positive surplus to all three segments under the loot box strategy. Therefore, when the seller adds one deterministic product on top of the loot box, he faces a trade-off between gaining extra profits from Segments A and M versus giving more surplus to Segment B. As β increases, the incremental gain from Segment A increases whereas the incremental surplus given to Segment B decreases. Therefore, there exists β_2^* such that the seller would offer both the loot box and one deterministic product when $\beta \in (\beta_2^*, \frac{v+w}{2})$ but rather offer only the loot box when $\beta \in (\frac{v-w}{2}, \beta_2^*)$.

In summary, when one loyal segment has a positive expected surplus from buying loot boxes repeatedly, the seller could remove the corresponding deterministic product for the sake of a lower menu cost. Proposition 9 characterizes the condition under which a simple product line containing *only* loot boxes could be optimal. Of course, to the extent that there are menu costs and information overload (i.e., consumers are more likely to be overwhelmed when there are more prices to process), the range for the loot box *only* strategy would be even larger.

4.6 The firm promises the preferred product with no more than a certain number of purchases.

Loot boxes are often seen as wasteful and exploitative (BBC 2019; Close and Lloyd 2021; Juniper Research 2021) as consumers may end up purchasing loot boxes many times, in particular when the winning probability ϕ is low. In order to reduce excessive purchases, some firms offer a winning promise whereby consumers will get their preferred product with no more than a certain number of purchases. For example, *Genshin Impact* promises that buyers can get any five-star virtual figure within 90 draws. In this extension, we study the implications of winning promises on profits and consumer welfare.

We assume that the firm promises that the consumer will receive their preferred product after at most N purchases. More specifically, during the first (N-1) purchase incidences, consumers can win product 1 with probability ϕ . At the N^{th} purchase incidence, consumers can win their preferred products for sure. Therefore, the winning promise reduces consumers' expected number of purchases needed to acquire their preferred products.

Using backward induction, we first consider a Segment A consumer's decision for her N^{th} purchase. Given that she has not received product 1 from her previous N-1 purchases, she has the valuation of w from product 2 if she does not purchase her N^{th} loot box. Instead, if she purchases her N^{th} loot box, she will gain the valuation of v-p. Therefore, denote V_N as the continuation value for her N^{th} purchase and we have $V_N = \max(w, v-p)$. Then the corresponding pricing condition for a Segment A consumer to purchase at the N^{th} period is $p \le v - w \equiv p_N$.

Assume for now that $p \leq p_N$ and consider her $(N-1)^{st}$ purchase incidence. If she

purchases her $(N-1)^{st}$ loot box, she receives product 1 with probability ϕ and wins another product 2 with probability $1-\phi$. Therefore, her valuation at the $(N-1)^{st}$ period is $V_{N-1} = \max(w, -p + \phi v + (1-\phi)V_N) = \max(w, v - (2-\phi)p)$. Then the pricing condition for Segment A to purchase her $(N-1)^{st}$ loot box is that $p \leq \frac{v-w}{2-\phi} \equiv p_{N-1}$.

We see that the continuation valuation at the k^{th} $(2 \le k \le N)$ purchase incidence is:

$$V_k = \max(w, -p + \phi v + (1 - \phi)V_{k+1}) \quad 2 \le k \le N$$
(38)

The pricing condition for Segment A to purchase at the $(N-k)^{th}$ period is:

$$p \le p_k = \frac{v - w}{\sum_{j=0}^{N-k} (1 - \phi)^j} \quad 2 \le k \le N$$
 (39)

Note that the condition becomes stringent as k increases given $2 \le k \le N$.

$$p_2 < p_3 < \dots < p_N \tag{40}$$

The intuition is that as k gets closer to the promise N, it is easier for the buyer to win her desirable product. Therefore, a rational consumer is willing to pay a higher loot box price as k approaching the winning promise N. When the Segment A consumer makes her first purchase decision, however, she has not acquired any product yet. Therefore, her valuation is 0 if she does not purchase her first loot box. The continuation valuation at the 1^{st} purchase incidence becomes:

$$V_1 = \max(0, -p + \phi v + (1 - \phi)V_2) \tag{41}$$

The corresponding pricing condition for Segment A to purchase her first loot box is:

$$p_1 = \frac{\phi v}{1 - (1 - \phi)^N} \tag{42}$$

Therefore, the seller has to satisfy both (40) and (42) such that Segment A will purchase loot boxes repeatedly. Therefore, we need:

$$p \le \min_{1 \le i \le N} p_i = \min\left(\frac{\phi v}{1 - (1 - \phi)^N}, \frac{\phi(v - w)}{1 - (1 - \phi)^{N-1}}\right)$$

Next, we calculate the expected number of loot boxes purchased by Segments A and B

given the promise N, denoted as ψ_A and ψ_B , respectively:

$$\psi_A = \phi \sum_{j=1}^N j (1 - \phi)^{j-1} = \frac{1 - (1 - \phi)^N}{\phi}$$

$$\psi_B = (1 - \phi) \sum_{j=1}^N j \phi^{j-1} = \frac{1 - \phi^N}{1 - \phi}$$
(43)

Therefore, we formulate the firm's profit-maximization problem as:

$$\max_{\phi,p,N} \quad \pi(\phi,p,N) = \alpha p \left(\psi_A + \psi_B\right) + \left(1 - 2\alpha\right) p$$
s. t.
$$p \le \min\left(\frac{\phi v}{1 - (1 - \phi)^N}, \frac{\phi(v - w)}{1 - (1 - \phi)^{N-1}}\right)$$

$$p \le \min\left(\frac{(1 - \phi)v}{1 - \phi^N}, \frac{(1 - \phi)(v - w)}{1 - \phi^{N-1}}\right)$$

$$p \le \beta$$
(44)

We have the following result:

Proposition 10. Suppose $\beta > \frac{v-w}{2}$ where β is the switcher's valuation, v(w) is the loyal consumers' valuation for their desired (less preferred) products. Then the firm strictly improves profits by offering a promise that consumers will get their preferred product within a finite number of purchases. Furthermore, such promises can reduce consumer welfare.

As the winning promise limits purchase incidences beyond N, one might conjecture that introducing a winning promise would reduce profits and improve consumer welfare. However, Proposition 10 shows that the firm could use the winning promise as a strategic tool to increase profits with better price discrimination. More specifically, when $\beta \geq \frac{v-w}{2}$, without the winning promise, the firm must provide surplus to the switchers. However, by limiting excessive purchases beyond N for loyal segments, the winning promise enables the firm to set a higher loot box price and extract more surplus from the switchers segment. In addition, Proposition 10 reveals that consumers may be worse off after the firm offers the seemingly generous winning promise. This counter-intuitive finding arises as consumers will pay a higher loot box price in equilibrium, which outweighs the direct benefit from avoiding excessive purchases.

5 Conclusion

As repeated purchases of loot boxes are increasingly popular, consumer protection groups and public policymakers have expressed concerns about their potentially harmful impact. We build an analytical model to examine the optimal price and design of loot boxes and then characterize the conditions under which the loot box strategy is more profitable than the traditional pricing strategy. We then investigate the implications of loot boxes on consumer and social welfare. In addition, we examine whether it can be optimal for firms to offer only loot boxes and exclude deterministic products for sale. We also study the impact of winning promises (i.e., consumers could obtain their preferred product with no more than a finite amount of purchases). Our results provide insights into the following questions.

- 1. Does the loot box strategy generate higher profits than the traditional pricing strategy?

 Our results show that the loot box strategy may improve profits for intermediate values of the switchers' valuation and the market share of the loyal segments. Loot boxes enable firms to better price discriminate on the time dimension by allowing some consumers to purchase repeatedly while others purchase only once. In addition, the firm could use loot boxes for market expansion by attracting light consumers who could not afford the products under the traditional pricing strategy.
- 2. What is the optimal loot box design in terms of allocation probabilities? We find that firms may allocate asymmetric probabilities to symmetric products. While previous studies have argued for uniform allocation probabilities (see for example, Fay and Xie 2008; Chen et al. 2021), our study reveals the firm's incentives to polarize allocation probabilities and increase sales. Therefore, our model is better at explaining the common practice whereby one loot box sometimes contains a product with a much lower probability relative to other symmetric counterparts. For example, the Emma Secret Forest Series Box has 9 regular figures and 1 hidden edition.²² The chance to win a hidden edition is 10 times lower than any regular figure.
- 3. Does the use of loot boxes negatively impact consumer and social welfare? Our results

²²https://www.amazon.com/Figures-Action-Secret-Forest-Blind/dp/B096KFHBDG?th=1

suggest that loot boxes may benefit consumers and society, particularly when the switchers are not served under the traditional pricing strategy. We also examine the case when loot boxes can be addictive. Even in this situation, loot boxes can increase market coverage and therefore increase social welfare. This finding implies that policymakers should be cautious when considering banning loot boxes, as such actions may harm social welfare by driving firms to stop serving the light users and charge high prices for the heavy users.

- 4. Why are loot boxes more common for virtual goods and less common for traditional goods? The loot box strategy incurs higher production costs as consumers may end up buying loot boxes containing their less preferred products. Simultaneously, a higher production cost makes it harder for the firm to use loot boxes and achieve profit advantages. Our finding suggests that policymakers should pay attention to regulating loot boxes containing traditional goods.
- 5. Can it be profitable to offer loot boxes with no option of direct purchase? It is commonly observed that firms offer only loot boxes and exclude deterministic products for sale. Our results show that sometimes it suffices for the firm to offer only loot boxes. This happens even when consumers are rational.
- 6. Can winning promises improve profits and welfare? Our results show that offering a guarantee that consumers will receive their preferred product within a certain number of purchase, can lead to improved price discrimination and higher profits. While this strategy seems to offer some protection to consumers and may be perceived as welfare enhancing, our results suggest that it can instead lead to lower consumer welfare.

We examine the robustness of our findings by relaxing several model assumptions. Our model assumes that each consumer segment follows the same strategy. In practice, heterogeneous consumers may stop buying loot boxes after different numbers of purchases. In Appendix B, we show that even if consumers are heterogeneous in terms of the maximum number of purchases that they can afford, the optimal pricing and allocation probabilities remain the same as in the base model. Therefore, the key result continues to hold. Our base

model assumes that the two component products are equally attractive in the sense that the two corresponding loyal segments have equal market shares. In Appendix C, we consider the case of asymmetric products where one product attracts more loyal consumers than another and find that while loot boxes can still be optimal, the region of profit advantage shrinks as the degree of product asymmetry increases. Our model also does not allow a resale market. In Appendix D, we relax this assumption of no resale market and allow for a salvage system where consumers can resell any unwanted items to the firm. Interestingly, we show that such salvage systems can increase profits at the expense of consumer surplus. Finally, our model focuses on the loot boxes containing horizontally differentiated products. In many contexts, loot boxes may offer products of different qualities. In Appendix E, we briefly explore this case and show that the key results from our base model continues to hold in this case. In our model, we consider the case of a monopolist. As competition could also affect loot box strategy, future research can incorporate competitive firms. In our paper, we assume that the seller sells only one loot box. Future research could consider the case where the firm could offer multiple loot boxes. There is little empirical research which has examined the profitability of loot boxes. Future research can test the degree to which the loot box strategy outperforms the traditional pricing strategy.

	Segment A	Segment M	Segment B
Valuation of Product 1 Valuation of Product 2	$v \\ w$	β	$\frac{w}{v}$
Market Size	α	$\frac{\beta}{1-2\alpha}$	α

Table 1: Consumer Segmentation with two products.

	Segment A_1	Segment A_2	 Segment A_n	Segment M
Valuation of Product 1	v	w	 w	β
Valuation of Product 2	w	v	 w	eta
Valuation of Product n	w	w	 v	β
Market Size	α/n	α/n	 α/n	$1-\alpha$

Table 2: Consumer Segmentation with n products.

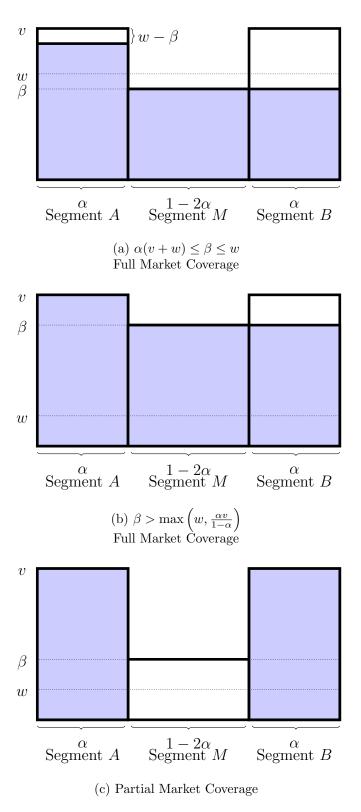
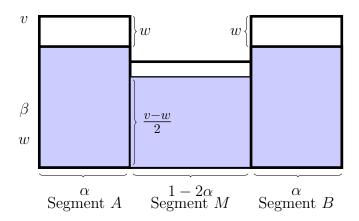
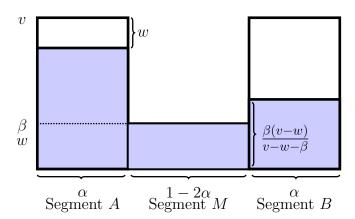


Figure 1: Optimal Traditional Pricing Strategy



(a) Uniform Allocation Probabilities $(\beta \leq \frac{v-w}{2})$



(b) Polarized Allocation Probabilities $(\beta < \frac{v-w}{2})$

Figure 2: Optimal Loot Box Strategy

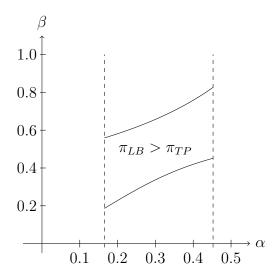


Figure 3: Region of Profitability Advantage (v=1, w=0.05)

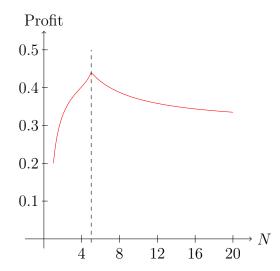


Figure 4: Optimal Product Line Length ($v=1, w=0, \alpha=0.3, \beta=0.2$)

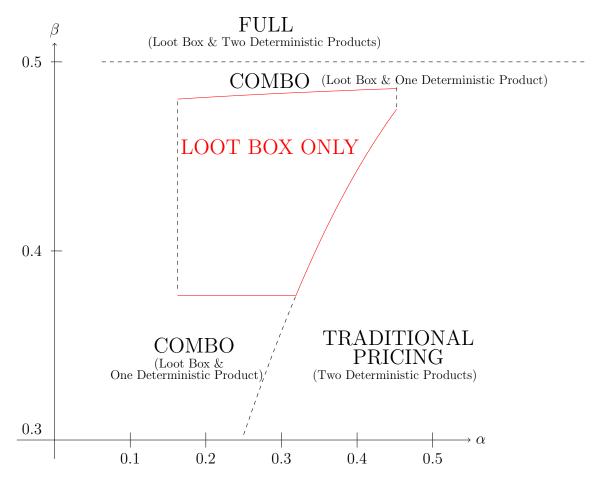


Figure 5: Region of Optimal Schemes (v=1, w=0.05)

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