UM–SJTU Joint Institute
Ve215

Laboratory Report

Exercise 5
Filter Lab

Name: Jin Minhao ID: 516370910116
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1 Introduction

1.1 Objectives

1. Learn about four types of filters Low-Pass, High-Pass, Band-Pass, and Band-reject.
2. Learn about transfer functions.
3. Predict the theoretical result and make comparison with lab data.

1.2 Theoretical Background

1.2.1 Filters

Filters are everywhere in our lives. The circuits built to operate on signals usually apply filters. For example, telephone lines pass the sounds at frequencies between about 100Hz and 3kHz and practically blocks all other frequencies.

1.2.2 Transfer Function

Mathematically, the transfer function is used to analyze what the circuit did to the signal:

$$\text{Transfer function} = \frac{Output\ signal}{Input\ signal}$$

This function can also be expressed as

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}$$

The magnitude of the transfer function is called voltage gain, often measured as the ratio of the peak-to-peak (ppk) voltages:

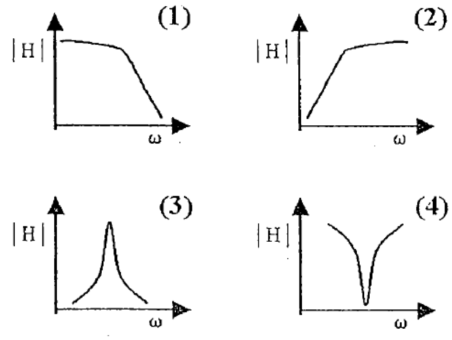
$$|H(\omega)| = \left|\frac{V_{out}(\omega)}{V_{in}(\omega)}\right| = \frac{V_{out,\ ppk}(\omega)}{V_{in,\ ppk}(\omega)}$$

It is convenient to express and plot the magnitude of the transfer function on the logarithmic scale using decibels:

$$|H(\omega)|_{dB} = 20 \cdot \log_{10}\left(\frac{V_{out,\ ppk}(\omega)}{V_{in,\ ppk}(\omega)}\right)$$

Since both ppk voltages are always positive, the transfer function magnitude is positive and thus can always be converted to decibels. The use of decibels allows us to review data over a broad range.

Types of filters



In the figure above are the four main families of filters:
(1): Low-Pass; (2): High-Pass; (3): Band-Pass; (4): Band-reject (also called band-stop or notch)

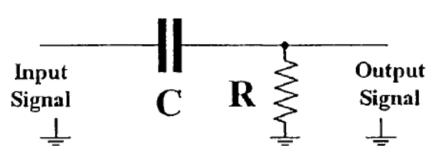
Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.

Filter circuits, which you are going to build in this lab, contain resistors, capacitors, and inductors. They are all passive filters.

1.2.3 High-Pass Filter

The high-pass filter we are going to build uses a capacitor and a resistor.

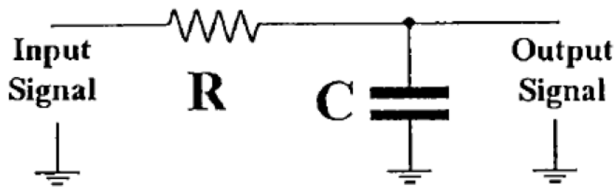


For the high-pass filter, $H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega RC}{1+j\omega RC}$.

Note that H(0) = 0, H(∞) = 1. Hence, it would only let high frequency pass.

1.2.4 Low-Pass Filter

The low-pass filter we are going to build uses a capacitor and a resistor.

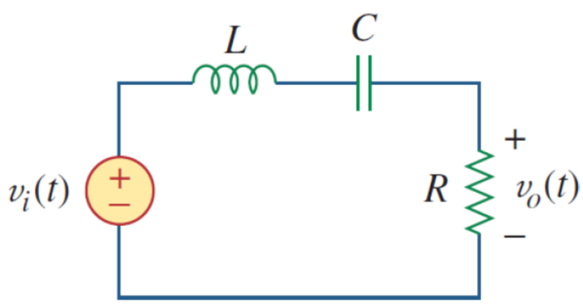


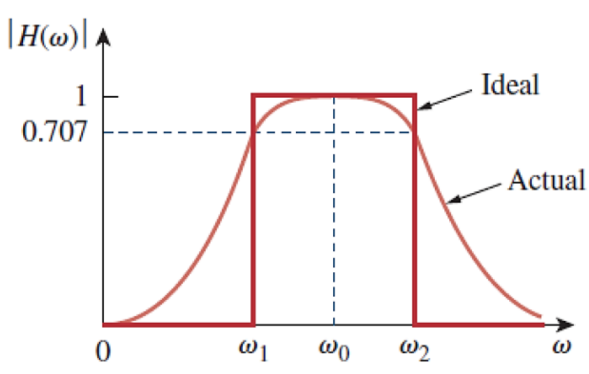
For the low-pass filter, $H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega RC}$.

Note that H(0) = 1, H(∞) = 0. It would only let low frequency pass.

1.2.5 Band-Pass Filter

The band-pass filter we are going to build uses a capacitor, an inductor and a resistor.



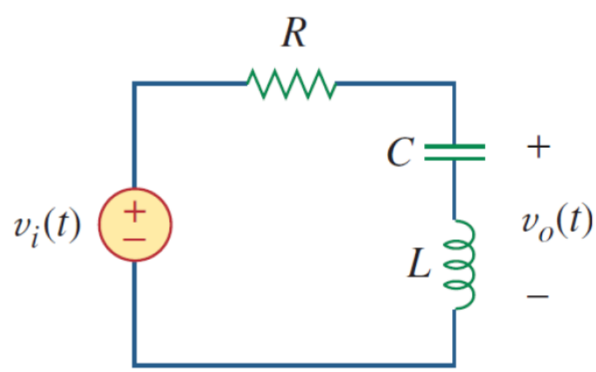


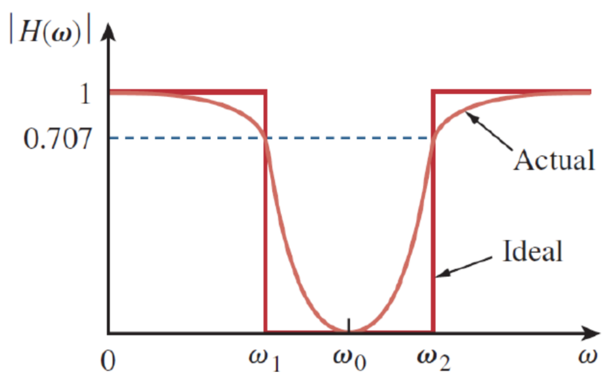
For the band-pass filter, $H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R+j(\omega L-\frac{1}{\omega C})}$.

Note that H(0) = 0, H(∞) = 0. The band-pass filter passes a band of frequencies centered on the center frequency ω_0, which is given by $\omega_0 = 1/\sqrt{LC}$.

1.2.6 Band-Stop Filter

The band-stop filter we are going to build uses a capacitor, an inductor and a resistor.





For the band-reject filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

Note that H(0)=0, H(∞)=0. The band-stop filter rejects a band of frequencies centered on the center frequency ω_0, which is given by $\omega_0 = \frac{1}{\sqrt{LC}}$.

2 Procedures

1. What is decibel value? (You may give a typical example to help your explanation.) What is the advantage of dB scale?

2. How to calculate the band width of rejection of a band-reject filter? (You may refer to Chapter 14.7 of the textbook.)

3. Predict the theoretical result of this lab. You need to estimate the Expected transfer function magnitude $|H(\omega)|$ and Expected transfer function magnitude in dB $|H(\omega)|_{dB}$ of all of the four types of filter. Fill in the tables in the Data Sheet to show the expected results (which will not be collected during the lab). Also, make a table for each type of filter to show the respected results (which will not collected during the lab as pre-lab assignment). We are using Resister of $R = 982\Omega$; Capacitor of $C = 0.1\mu F$; Inductor of L = 1mH.

e.g. For high-pass filter

Frequency	1MHz	100kHz	50kHz	10kHz	5kHz	1kHz
$\lvert H(\omega)\rvert$						
$\lvert H(\omega)\rvert_{dB}$						

Tip: *You may use MATLAB or Mathematica program to help you calculate this result.*

3 Results

3.1 Low-pass Filter

Frequency	Input signal amplitude V_{ppk}	Output signal amplitude (m)V_{ppk}	Transfer function generator	Expected transfer function magnitude	Transfer function magnitude in dB	Expected transfer function magnitude in dB
1MHz	5.000	20	0.0040	0.0016	-47.9588	-55.8085
100kHz	5.000	145	0.0290	0.0162	-30.7520	-35.8070
50kHz	5.000	253	0.0506	0.0324	-25.9170	-29.7898
10kHz	5.000	1050	0.2100	0.1600	-13.5556	-15.9184
5kHz	5.000	1950	0.3900	0.3083	-8.1787	-10.2191
1kHz	5.000	4540	0.9080	0.8510	-0.8383	-1.4010
500Hz	5.000	4980	0.9960	0.9556	-0.0348	-0.3948

The error and the relative error is :

Frequency	Error about transfer function amplitude	Relative error about transfer function amplitude	Error about transfer function magnitude in dB	Relative error about transfer function magnitude in dB
1MHz	0.0024	150%	7.8497	14.07%
100kHz	0.0128	79.01%	5.0550	14.12%
50kHz	0.0182	56.17%	3.8728	13.00%
10kHz	0.0500	31.25%	2.3628	14.84%
5kHz	0.0817	26.50%	2.0404	19.97%
1kHz	0.0570	6.698%	0.5627	40.17%
500Hz	0.0404	4.23%	0.3600	91.18%

3.2 High-pass Filter

Frequency	Input signal amplitude Vppk	Output signal amplitude, (m)Vppk	Transfer function magnitude	Expected transfer function magnitude	Transfer function magnitude in dB	Expected transfer function magnitude, in dB
1MHz	5.00	4.86	0.9720	0.9999	-0.2467	-0.0011
100kHz	5.00	4.86	0.9720	0.9999	-0.2467	-0.0724
50kHz	5.00	4.82	0.9640	0.9995	-0.3185	-0.0046
10kHz	5.00	4.70	0.9400	0.9871	-0.5374	-0.1126
5kHz	5.00	4.46	0.8920	0.9513	-0.9927	-0.4339
1kHz	5.00	2.25	0.4500	0.5251	-6.9357	-5.5952
500Hz	5.00	1.25	0.2500	0.2948	-12.0412	-10.6096
100Hz	5.00	0.285	0.0570	0.0616	-24.8825	-24.2107

Frequency	Error about transfer function amplitude	Relative error about transfer function amplitude	Error about transfer function amplitude in dB	Relative error about transfer function amplitude
1MHz	0.0279	2.7903%	0.2456	22300%
100kHz	0.0279	2.7903%	0.1743	240%
50kHz	0.0355	3.5518%	0.3139	682%
10kHz	0.0471	4.7716%	0.4248	377%
5kHz	0.0593	6.2336%	0.5588	129%
1kHz	0.0751	14.3020%	6.3405	107%
500Hz	0.0448	15.1967%	1.4316	13.49%
100Hz	0.0046	7.4675%	0.6718	2.77%

3.3 Band-pass Filter

Frequency	Input signal amplitude Vppk	Output signal amplitude (m) Vppk	Transfer function magnitude	Expected transfer function magnitude	Transfer function magnitude in dB	Expected transfer function magnitude in dB
1MHz	5.00	193	0.0386	0.1545	-28.2683	-16.2240
500kHz	5.00	1160	0.2320	0.2986	-12.6902	-10.4796
100kHz	5.00	4060	0.8120	0.8485	-1.8089	-1.4267
50kHz	5.00	4660	0.9320	0.9611	-0.6117	-0.3449
10kHz	5.00	4780	0.9560	0.9952	-0.3908	-0.7261
1kHz	5.00	2270	0.4540	0.5266	-6.8589	-5.5703
500Hz	5.00	1250	0.2500	0.2921	-12.0412	-10.6018

Frequency	Error about transfer function amplitude	Relative error about transfer function amplitude	Error about transfer function amplitude in dB	Relative error about transfer function amplitude
1MHz	0.1159	75.01%	12.04	74.23%
500kHz	0.0666	22.30%	2.21	21.09%
100kHz	0.0365	4.30%	0.3822	26.79%
50kHz	0.0291	3.03%	0.2668	77.35%
10kHz	0.0392	3.93%	0.3353	46.17%
1kHz	0.0726	13.79%	1.2886	23.13%
500Hz	0.0421	14.41%	1.4394	13.57%

3.4 Band-reject Filter

Frequency	Input signal amplitude Vppk	Output signal amplitude (m) Vppk	Transfer function magnitude	Expected transfer function magnitude	Transfer function magnitude in dB	Expected transfer function magnitude in dB
1MHz	5.00	5000	1	0.9880	0	-0.1049
500kHz	5.00	4980	0.5960	0.9544	-4.4951	-0.4056
300kHz	5.00	4660	0.9320	0.8863	-0.6117	-1.0481
200kHz	5.00	4140	0.8280	0.7860	-1.6394	-2.0911
100kHz	5.00	2790	0.5580	0.5292	-5.0673	-5.5282
50kHz	5.00	1420	0.2840	0.2763	-10.9336	-11.1721
5kHz	5.00	704	0.1408	0.0976	-17.0279	-20.2092
10kHz	5.00	1790	0.3580	0.2804	-8.9223	-11.0435
5kHz	5.00	4540	0.9080	0.8501	-0.8383	-1.4105
500Hz	5.00	4940	0.9980	0.9555	-0.1049	-0.3956

Frequency	Error about transfer function amplitude	Relative error about transfer function amplitude	Error about transfer function amplitude in dB	Relative error about transfer function amplitude
1MHz	0.0120	1.21%	0.1049	100%
500kHz	0.3584	37.55%	4.0895	100%
300kHz	0.0457	5.15%	0.4364	41.64%
200kHz	0.0420	5.34%	0.4517	21.60%
100kHz	0.0288	5.44%	0.4609	8.33%
50kHz	0.0077	2.78%	0.2385	2.13%
10kHz	0.0432	44.26%	3.1813	15.74%
5kHz	0.0776	27.67%	2.1212	19.20%
1kHz	0.0579	6.81%	0.5722	40.57%
500Hz	0.0325	3.40%	0.2907	73.49%

4 Conclusion

Through this lab, I learned about four types of filters Low-Pass, High-Pass, Band-Pass, and Band-reject, transfer functions and predicted the theoretical result and make comparison with lab data.

For the low-pass filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

For the high-pass filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

For the band-pass filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

For the band-reject filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

These are the method to calculate the theoretical value.

It seems that some of the data is not accurate enough. The relative error is very big. I think the reason is that the function ln an make an error when calculation. I find that when x is near 1, lnx may have a relatively big change, and if our result has a little bit difference with the theoretical value, the result after calculation may have a huge difference. Therefore, I don't think that our experiment has some very big mistakes, but it is because the equation that results the huge error. But actually, it is still a successful experiment. Most of the data is very close to the theoretical value and their relative error is also not very big. This is a very meaningful experiment.