Ve215 Electric Circuits

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Chapter 8 (Part 2)

Second-Order Circuits

8.6 Parallel *RLC* Circuit with Step Input

Consider the circuit in Fig. 8.22. We want to find i due to a sudden application of a dc current. For $t > 0$,

$$I_s = \frac{v}{R} + i + C\frac{dv}{dt}, \quad v = L\frac{di}{dt}$$

$$LC\frac{d^2 i}{dt^2} + \frac{L}{R}\frac{di}{dt} + i = I_s$$

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i = \frac{1}{LC}I_s$$

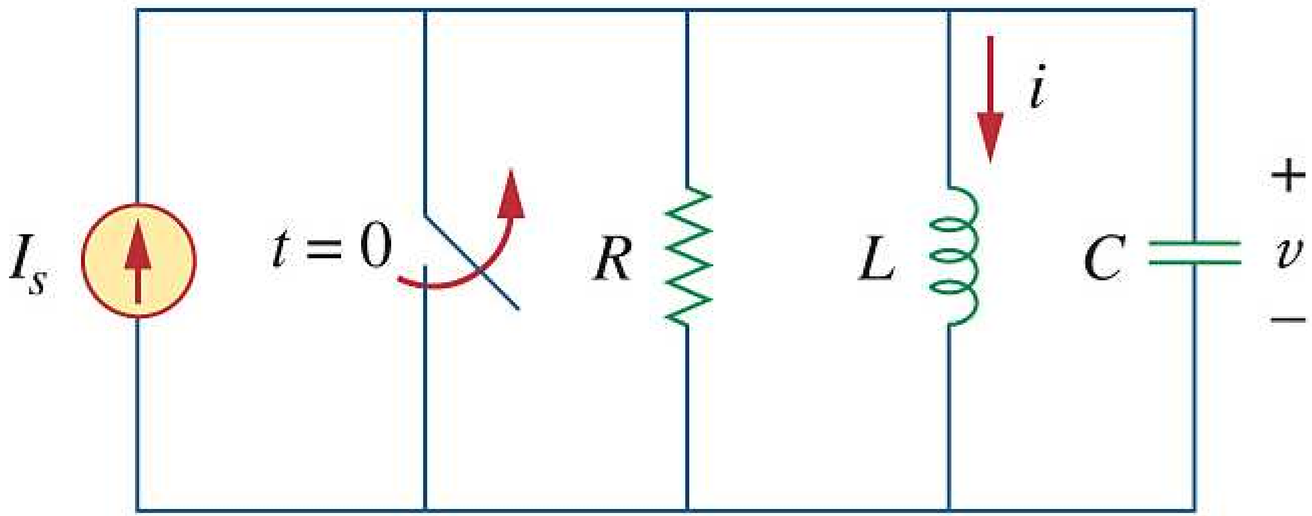

Figure 8.22 Parallel RLC circuit with an applied current.

It can be shown that the solution has three possible forms:

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + I_s$$

(Overdamped)

$$i(t) = (A_1 + A_2 t)e^{-\alpha t} + I_s$$

(Critically damped)

$$i(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) + I_s$$

(Underdamped)

Practice Problem 8.8 Find $i(t)$ and $v(t)$ for $t > 0$ in the circuit of Fig. 8.24.

Solution :

Step1

$$i(0^+) = i(0^-) = 0$$

$$v(0^+) = v(0^-) = 0$$

Step2

$$v(0^+) = 5\frac{di(0^+)}{dt} \Rightarrow i'(0^+) = \frac{1}{5}v(0^+) = 0$$

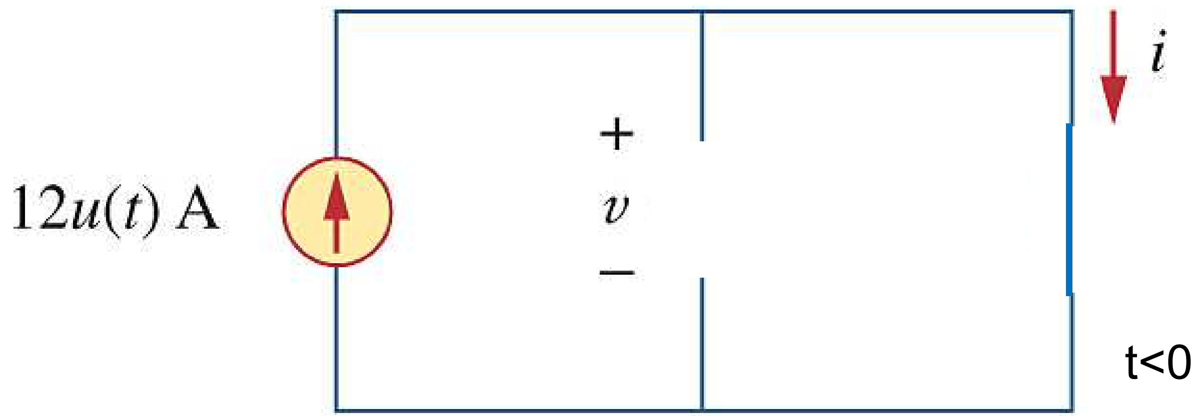

Figure 8.24 An LC circuit.

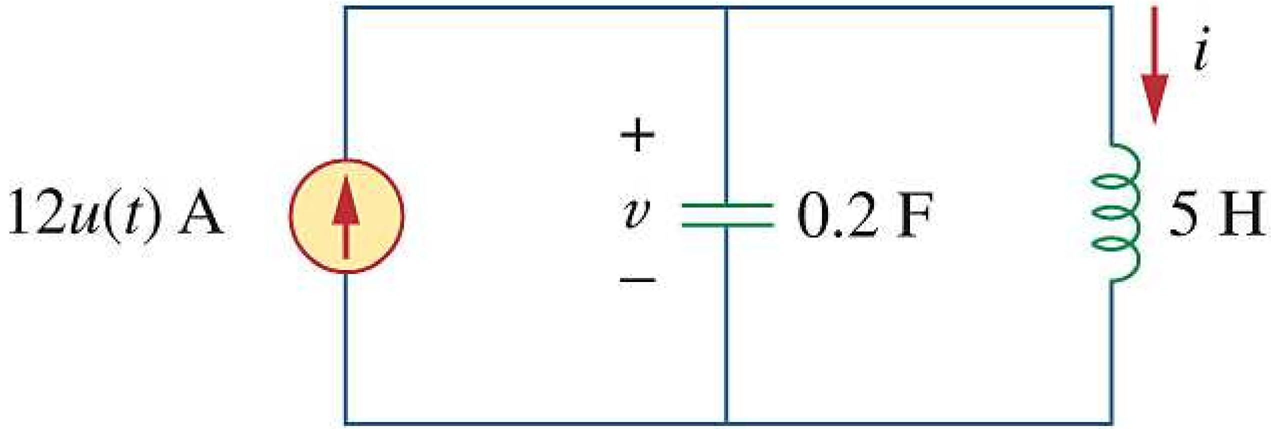

Figure 8.24 An LC circuit.

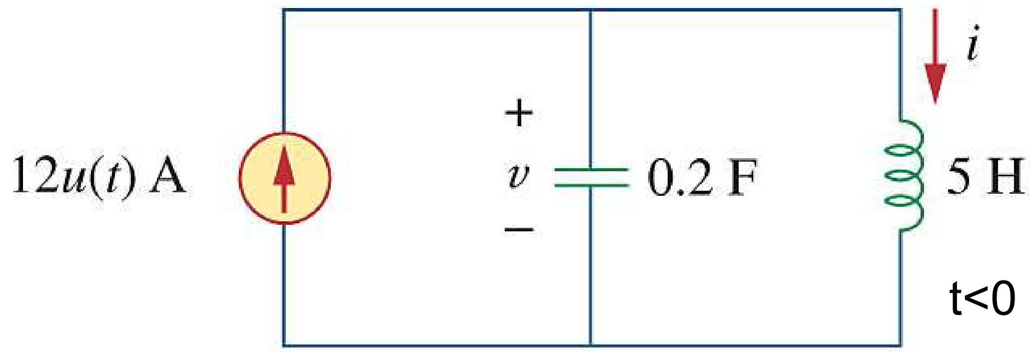

Figure 8.24 An LC circuit.

Step3

$$12 = 0.2\frac{dv}{dt} + i, \quad v = 5\frac{di}{dt}$$

$$\frac{d^2 i}{dt^2} + i = 12$$

$$s^2 + 1 \Rightarrow s_{1,2} = \pm j$$

$$i_n(t) = A_1 \cos t + A_2 \sin t$$

Step4

$$i_p(t) = 12$$

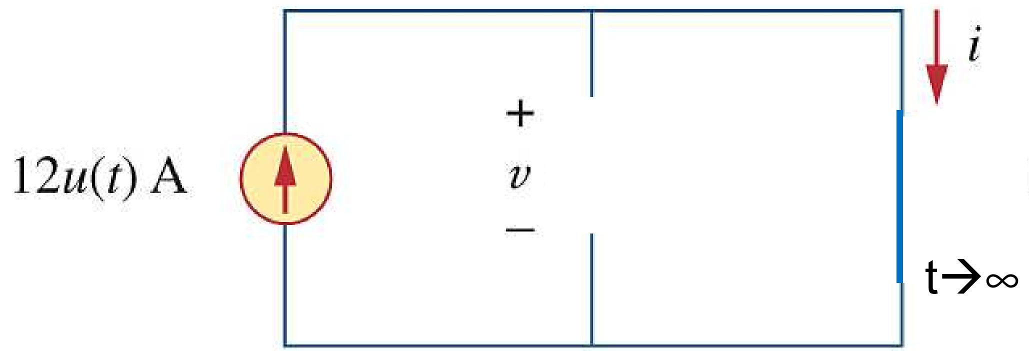

Figure 8.24 An LC circuit.

$$i(t) = i_n(t) + i_p(t) = A_1 \cos t + A_2 \sin t + 12$$

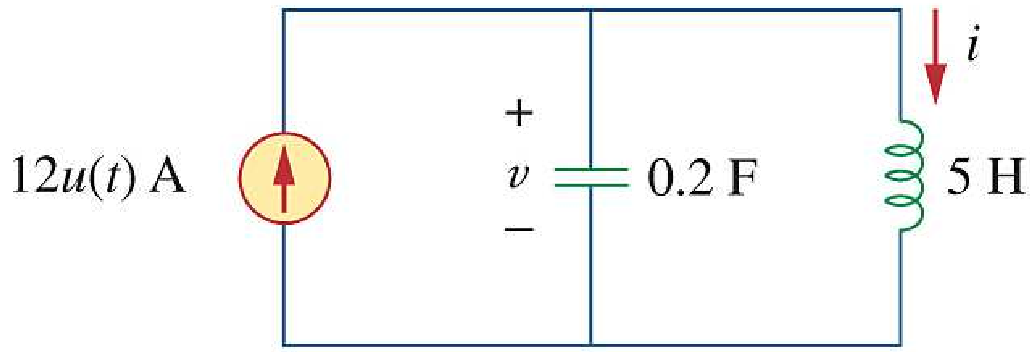

Figure 8.24 An LC circuit.

Step5

$$i(0^+) = A_1 + 12 = 0$$

$$i'(0^+) = -A_2 = 0$$

$$A_1 = -12,\ A_2 = 0$$

$$i(t) = -12\cos t + 12 = 12(1-\cos t)\ \text{(A)}$$

$$v(t) = 5\frac{di(t)}{dt} = 60\sin t\ \text{(V)}$$

8.7 General Second-Order Circuits

Practice Problem 8.10 For $t > 0$, obtain $v_o(t)$ in the circuit of Fig. 8.32. (*Hint :* First find v_1 and v_2.)

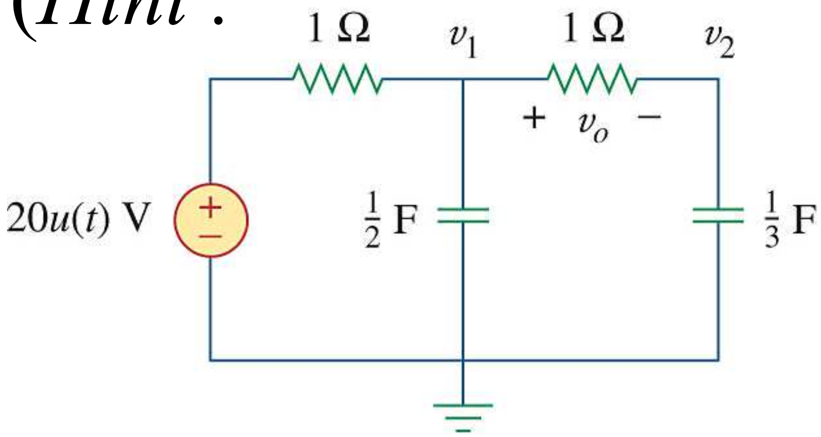

Figure 8.32 An *RCC* circuit.

Solution :

Step1

$$v_1(0^+) = v_2(0^+) = 0$$

Step2

$$\frac{20 - v_1(0^+)}{1} = \frac{1}{2}\frac{dv_1(0^+)}{dt} + \frac{v_1(0^+) - v_2(0^+)}{1}$$

$$v_1'(0^+) = 2[20 - 2v_1(0^+) + v_2(0^+)] = 40 \text{ (V/s)}$$

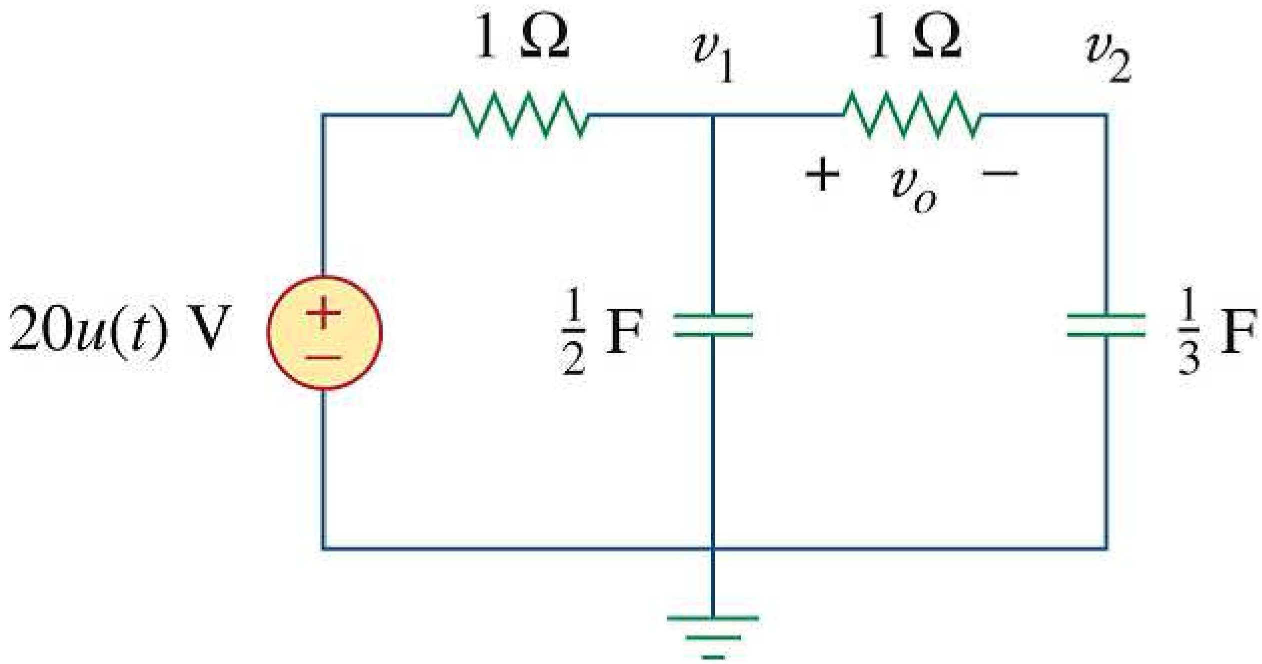

Figure 8.32 An RCC circuit.

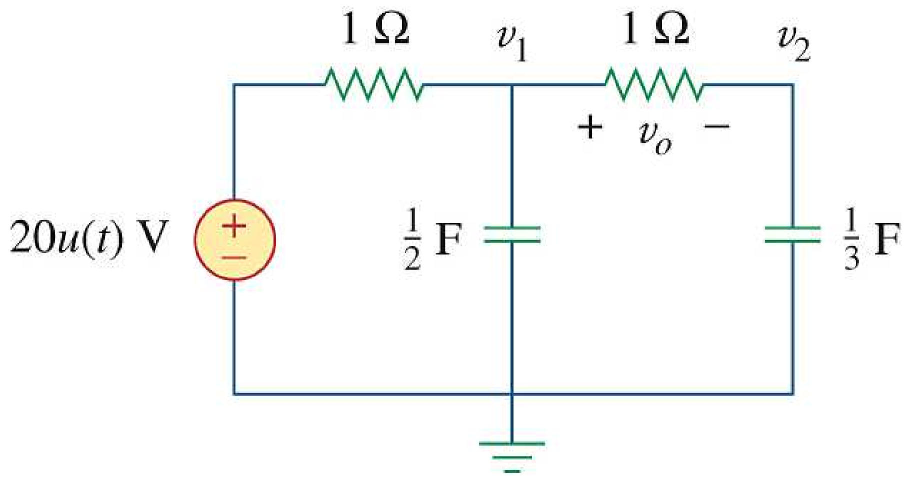

Figure 8.32 An *RCC* circuit.

Step4

$$v_1(\infty) = v_2(\infty) = 20 \text{ (V)}$$

Step3

$$\frac{20 - v_1}{1} = \frac{1}{2}\frac{dv_1}{dt} + \frac{v_1 - v_2}{1}, \quad \frac{v_1 - v_2}{1} = \frac{1}{3}\frac{dv_2}{dt}$$

$$\frac{d^2 v_1}{dt^2} + 7\frac{dv_1}{dt} + 6v_1 = 120$$

$$s^2 + 7s + 6 = 0 \Rightarrow s_1 = -1,\ s_2 = -6$$

$$v_{1h}(t) = A_1 e^{-t} + A_2 e^{-6t}$$

$$v_{1p}(t) = 20$$

Step5

$$v_1(t) = A_1 e^{-t} + A_2 e^{-6t} + 20$$

$$v_1(0^+) = A_1 + A_2 + 20 = 0$$

$$v_1'(0^+) = -A_1 - 6A_2 = 40$$

$$A_1 = -16,\ A_2 = -4$$

$$v_1(t) = -16e^{-t} - 4e^{-6t} + 20$$

Solve $v_2(t)$ using similar procedure from $v_2(t) = B_1e^{-t}+B_2e^{-6t}$ and steps 4, 5

$$v_2(t) = -24e^{-t} + 4e^{-6t} + 20$$

$$v_o(t) = v_1(t) - v_2(t) = 8e^{-t} - 8e^{-6t}\ \ (\mathrm{V})$$

8.10 Duality

- The concept of duality is a time-saving, effort-effective measure of solving circuit problems.
- Two circuits are said to be duals of one another if they are described **by the same characteristic equations with dual pairs interchanged**.
- Dual pairs are shown in Table 8.1.

TABLE 8.1 Dual Pairs

Resistance	Conductance
Inductance	Capacitance
Voltage	Current
Voltage source	Current source
Node	Mesh
Series path	Parallel path
Open circuit	Short circuit
KVL	KCL
Thevenin	Norton

Given a palnar circuit, we construct the dual circuit by taking the following steps:

1. Place a node at the center of each mesh of the given circuit. Place the reference node of the dual circuit outside the given circuit.
2. Draw lines between the nodes such that each line across an element. Replace the element by its dual.

3. To determine the polarity of voltage sources and direction of current sources, follow this rule: A voltage source that produces a positive (clockwise) mesh current has as its dual a current source whose reference direction is from the ground to the nonreference node.

In case of doubt, one may verify the dual circuit by writing the nodal or mesh equations.

Example 8.14 Construct the dual of the circuit in Fig. 8.44.

Solution : See Fig. 8.45.

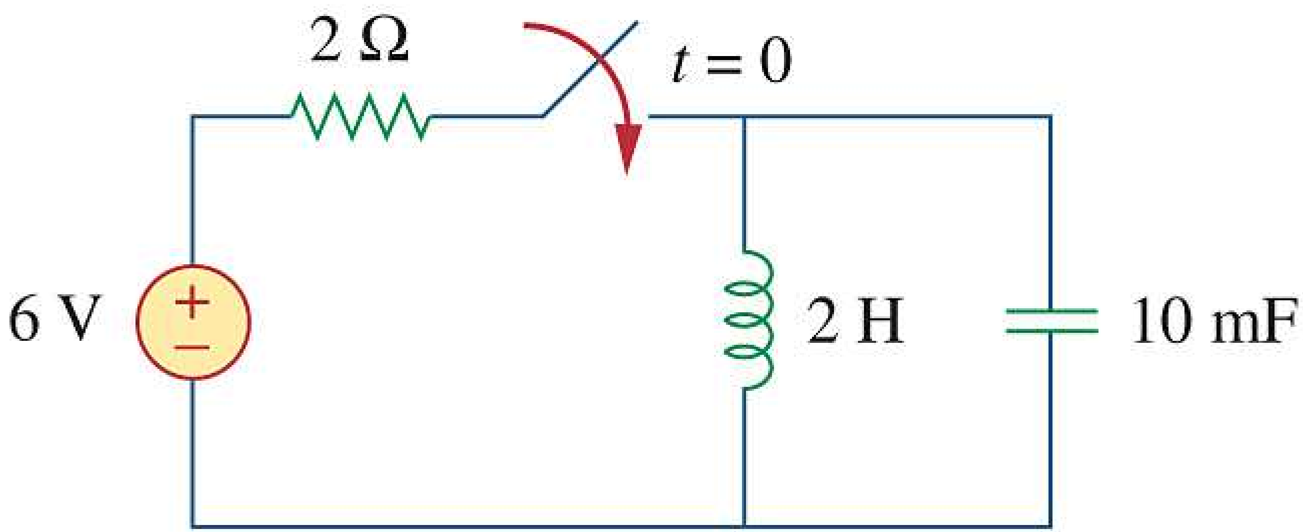

Figure 8.44

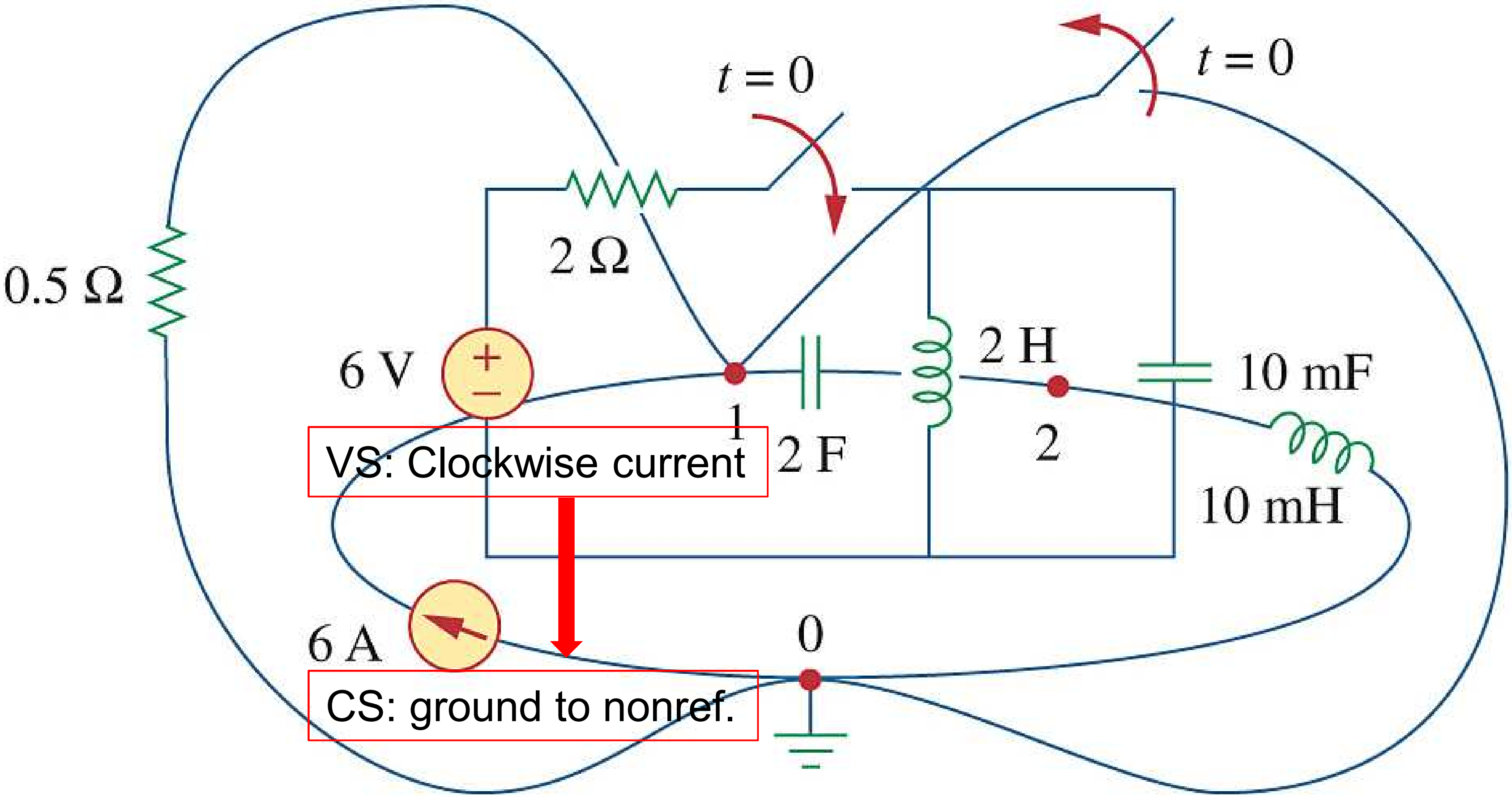

Figure 8.45(a) Construction of the dual circuit of Fig. 8.44.

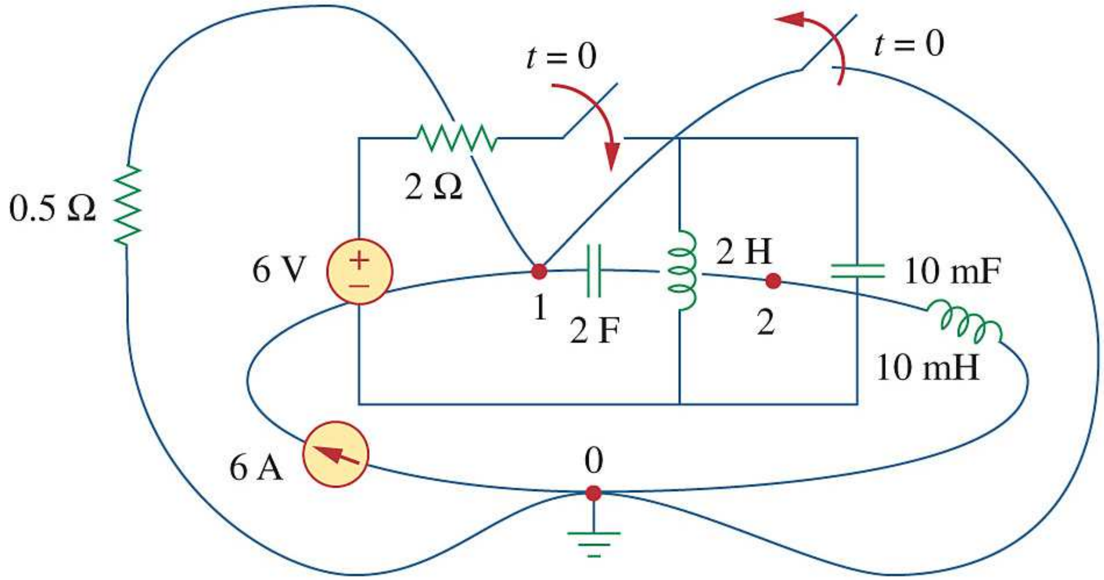

Figure 8.45(a) Construction of the dual circuit of Fig. 8.44.

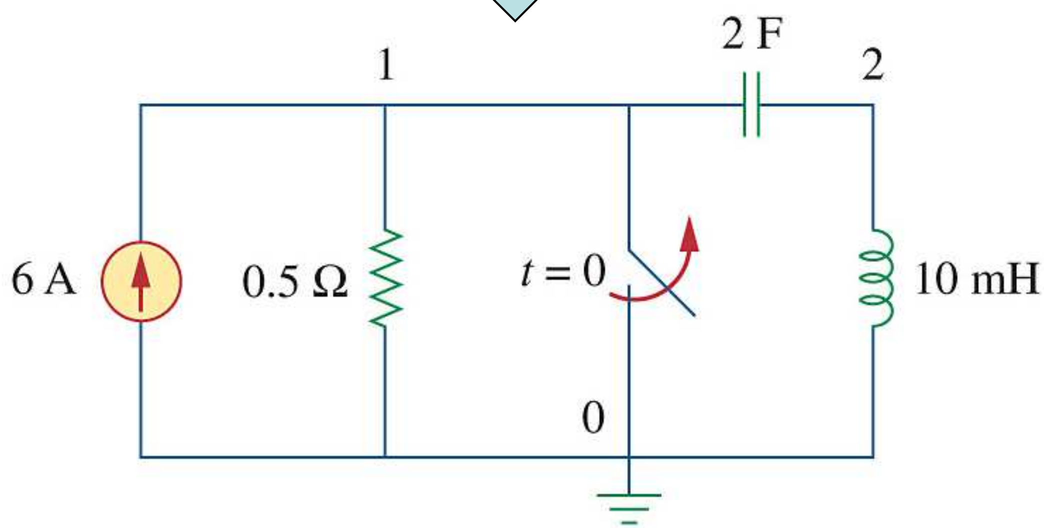

Figure 8.45(b) Dual circuit redrawn.

Example 8.15 Obtain the dual of the circuit in Fig. 8.48.

Solution : See Fig. 8.49.

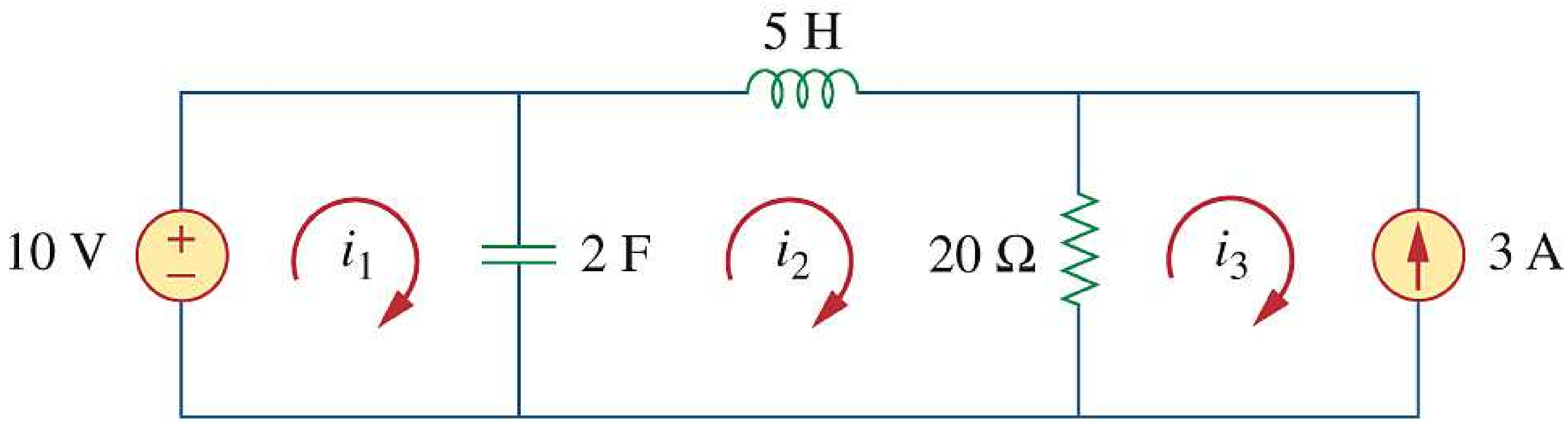

Figure 8.48

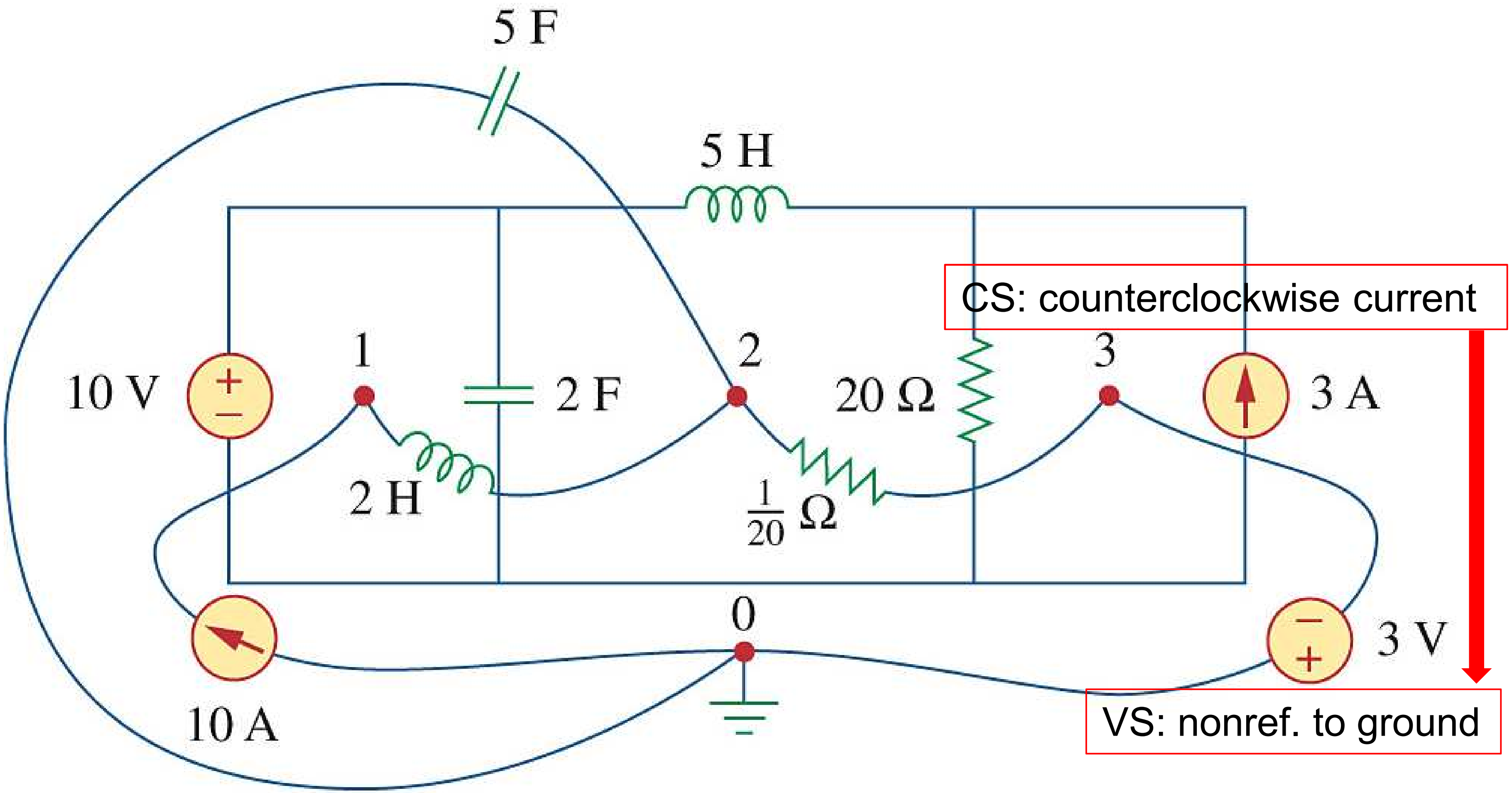

Figure 8.49(a) Construction of the dual circuit of Fig. 8.48.

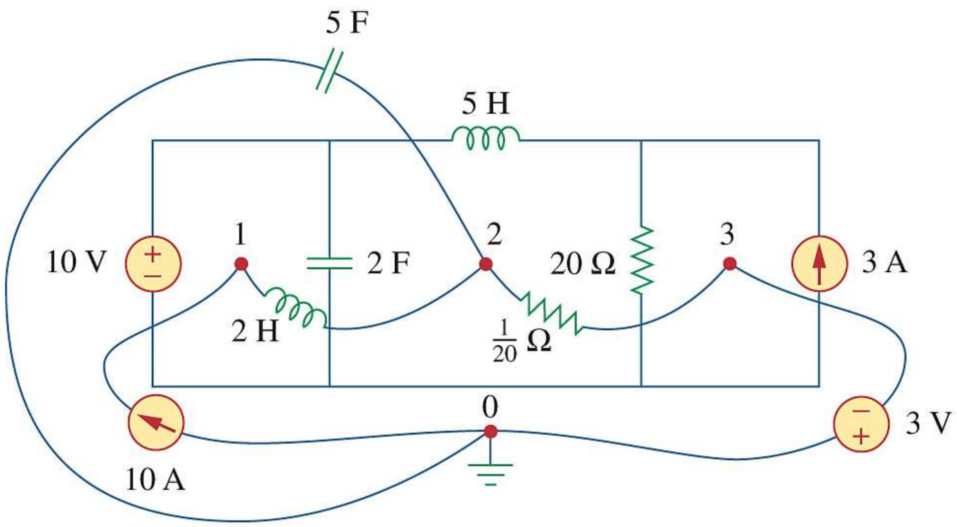

Figure 8.49(a) Construction of the dual circuit of Fig. 8.48.

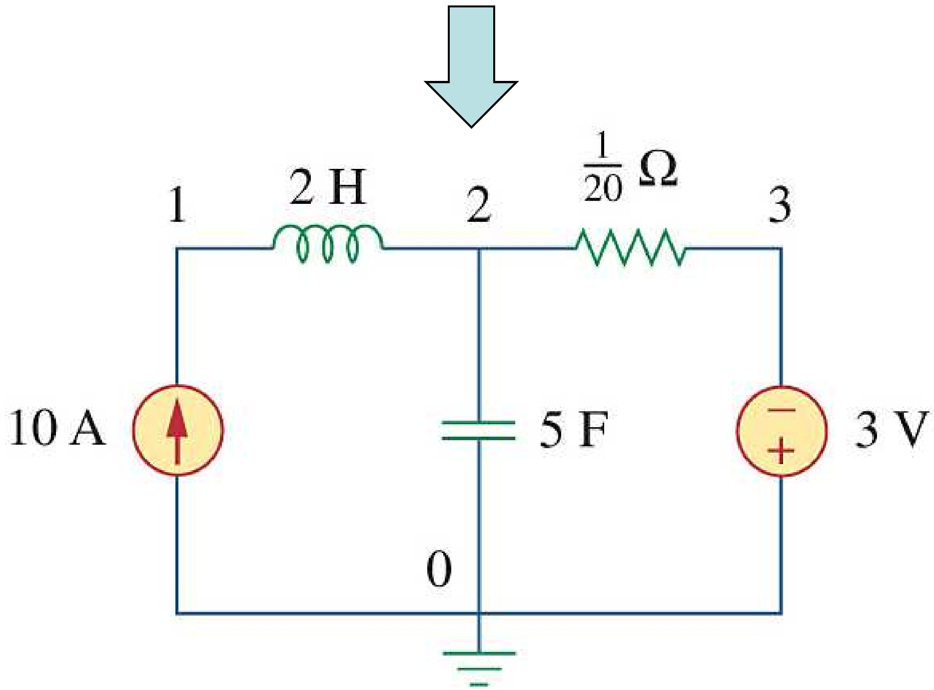

Figure 8.49(b) Dual circuit redrawn.