Ve215 Electric Circuits

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Chapter 2

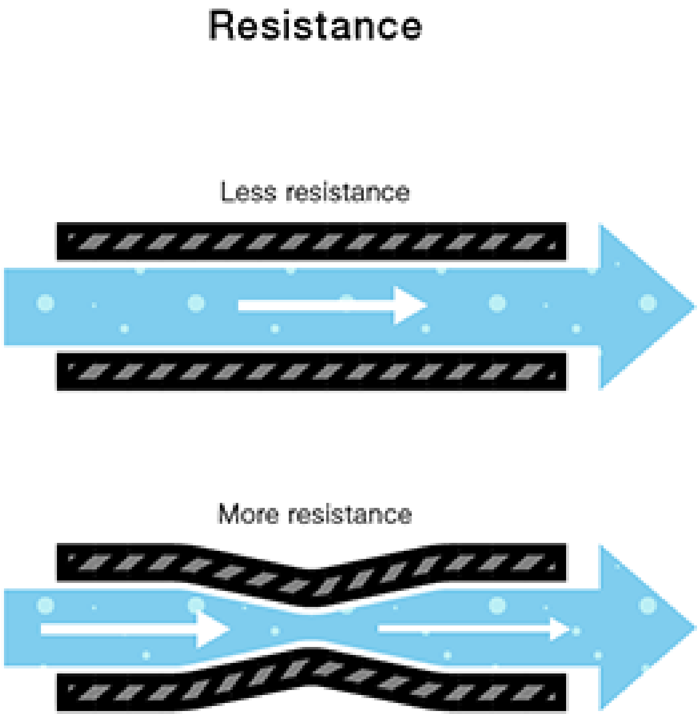
Basic Laws

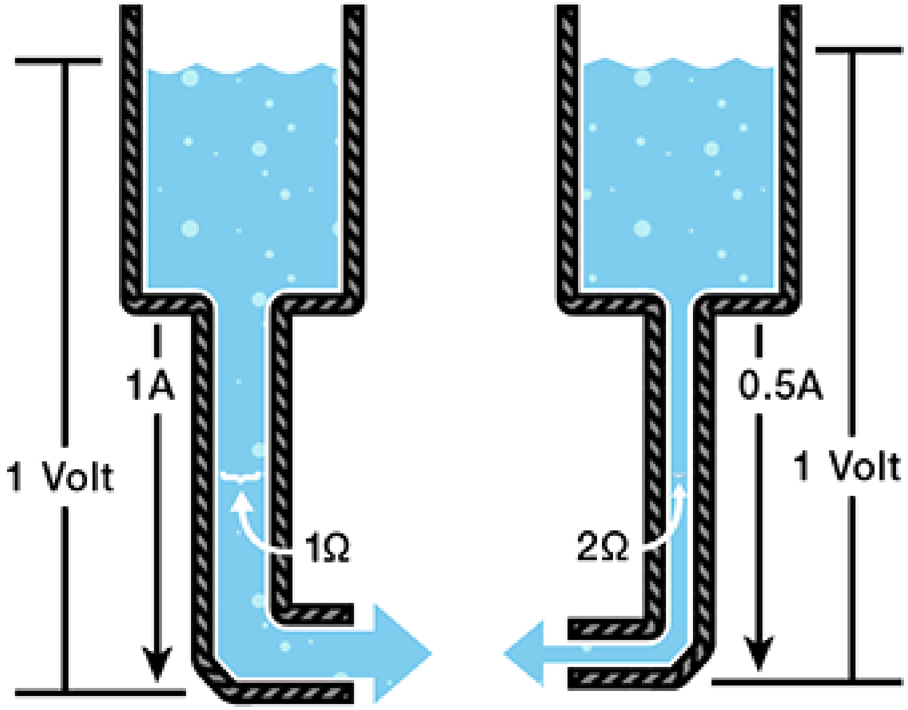
2.1 Introduction

- In this chapter, we study some fundamental laws that govern electric circuits, known as Ohm’s law and Kirchhoff’s laws, and discuss some techniques commonly applied in circuit analysis.

2.2 Ohm's Law

- Materials in general have a current-resisting behavior. This physical property is known as *resistance* and is represented by the symbol R.
- The element used to model the current-resisting behavior of a material is the *resistor*.





Analogy of water flow and electric current

Water volume ⇔ # of electrons

Water flow ⇔ electric current

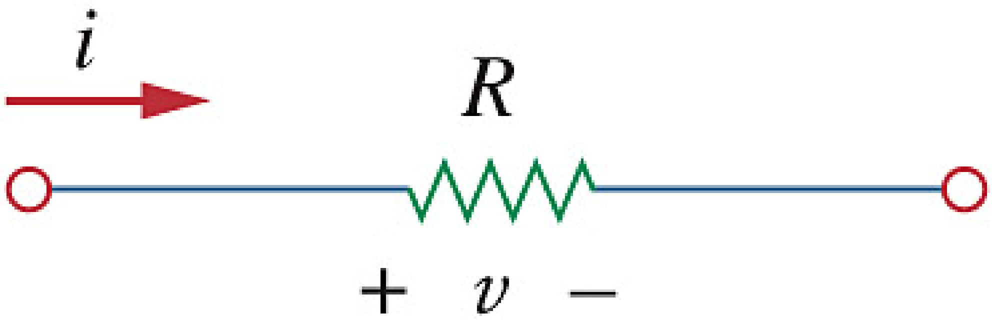


Figure 2.1(b) Circuit symbol for resistor.

Ohm's law states that the voltage v across a resistor and the current i through the resistor are related by

$v = iR$ for PSC

or

$v = -iR$ for ASC

where R is the resistance, measured in ohms (Ω).

PSC:
$p = +vi = +(iR)i = i^2R > 0$
➔ Absorbing power

v=0
i can be any value

i=0
v can be any value

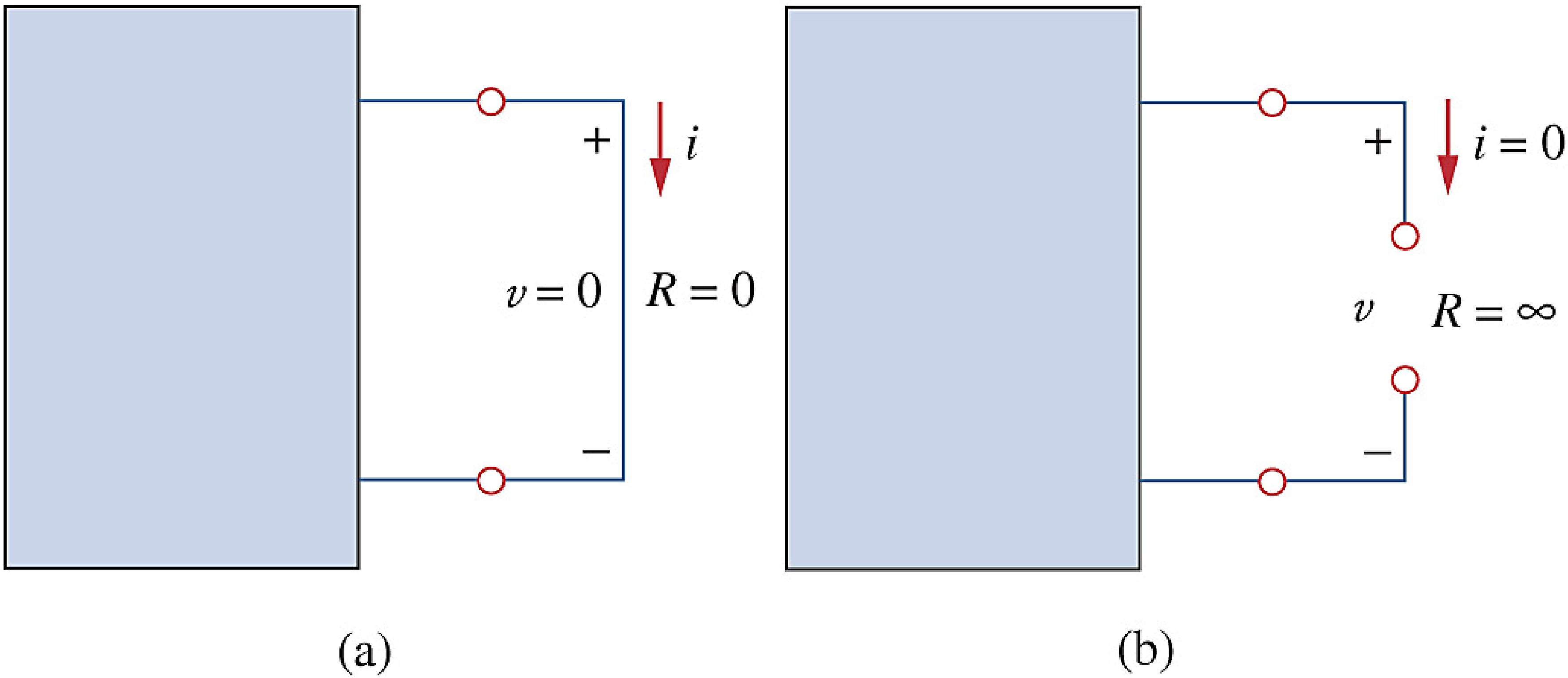
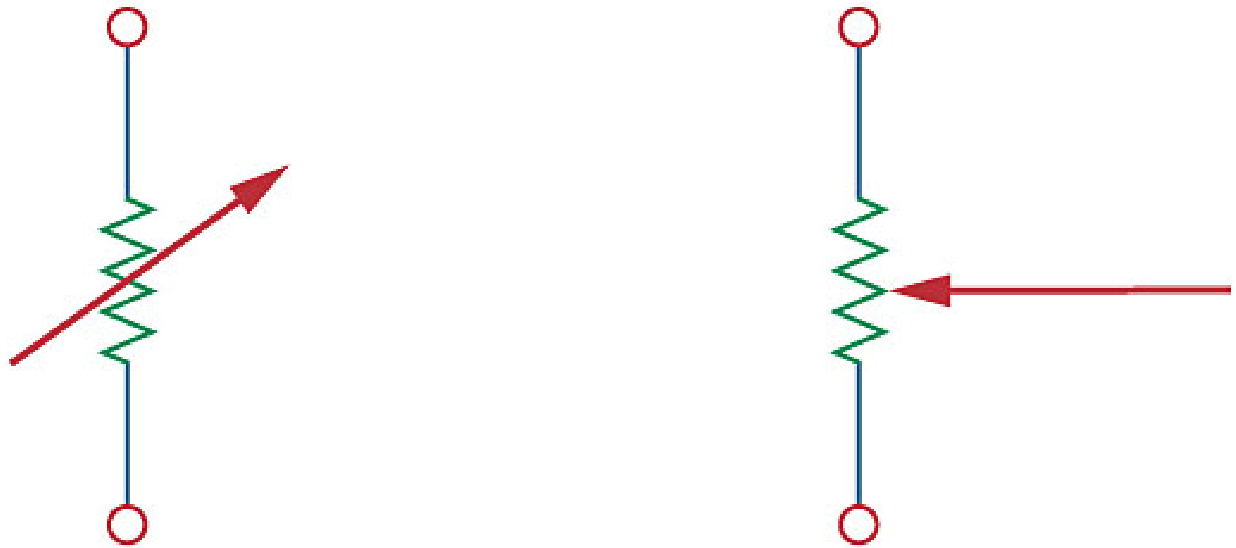


Figure 2.2 Two extreme possible values of R: (a) short circuit ($R = 0$), (b) open circuit ($R = \infty$).

- A short circuit is a circuit element with resistance approaching zero.
- An open circuit is a circuit element with resistance approaching infinity.

- A resistor is either fixed or variable.



(a) (b)

Figure 2.4 Circuit symbol for (a) a variable resistor in general, (b) a potentiometer.

- A *linear* resistor has a constant resistance and thus its current-voltage characteristic is a straight line passing through the origin.
- The resistance of a nonlinear resistor varies with current.

v=iR, R is a const.　　　　v=i*R(i)

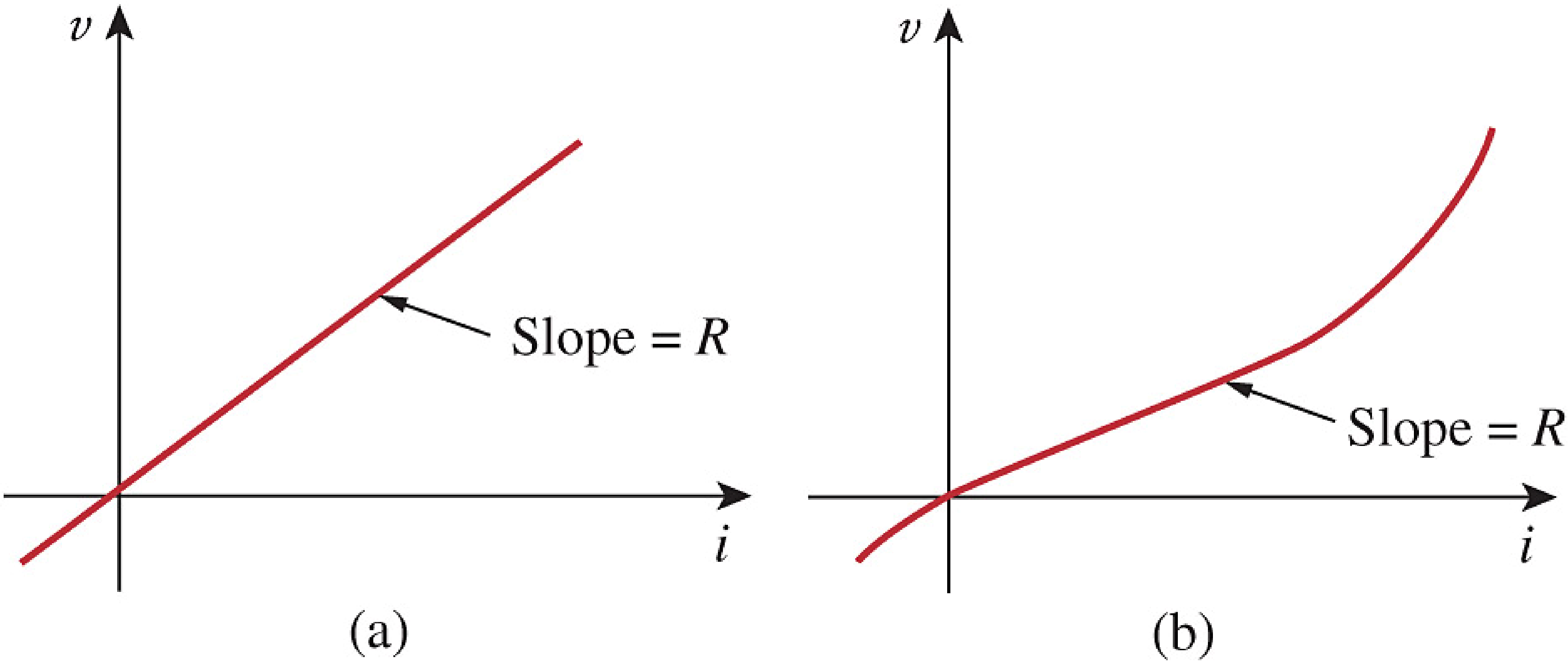


Figure 2.7 The i - v characteristic of (a) a linear resistor, (b) a nonlinear resistor.

A useful quantity in circuit analysis is the reciprocal of resistance *R*, known as conductance and denoted by *G*. Conductance is the ability of an element to conduct electric current. It is measured in mhos (℧) or siemens (S). The word mho is ohm spelled backward.

Wikipedia: **Ernst Werner Siemens**, **von Siemens** since 1888, (13 December 1816 – 6 December 1892) was a German inventor and industrialist. Siemens' name has been adopted as the SI unit of electrical conductance, the siemens. He was also the founder of the electrical and telecommunications company Siemens.

The power dissipated by a resistor can be expressed in terms of R or G.

$$p = vi = i^2 R = \frac{v^2}{R}$$

$$p = vi = \frac{i^2}{G} = v^2 G$$

2.3 Nodes, Branches, and Loops

- A circuit is also known as a *network*.
- A *branch* represents a single element such as a voltage source or a resistor. In other words, a branch represents any two-terminal element.
- A *node* is the point of connection between two or more branches.
- A *loop* is any closed path in a circuit.

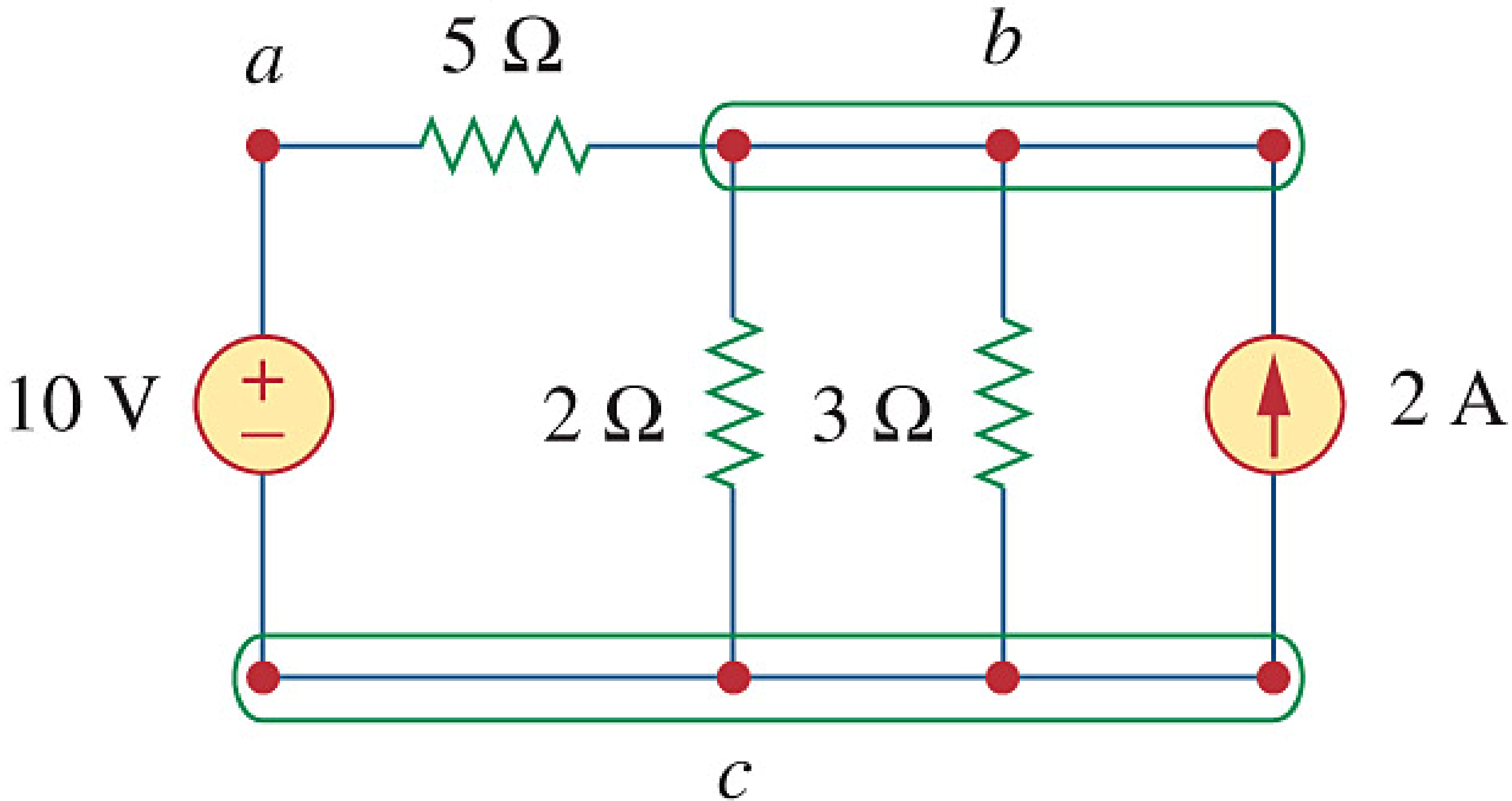


Figure 2.10 Nodes, branches, and loops.

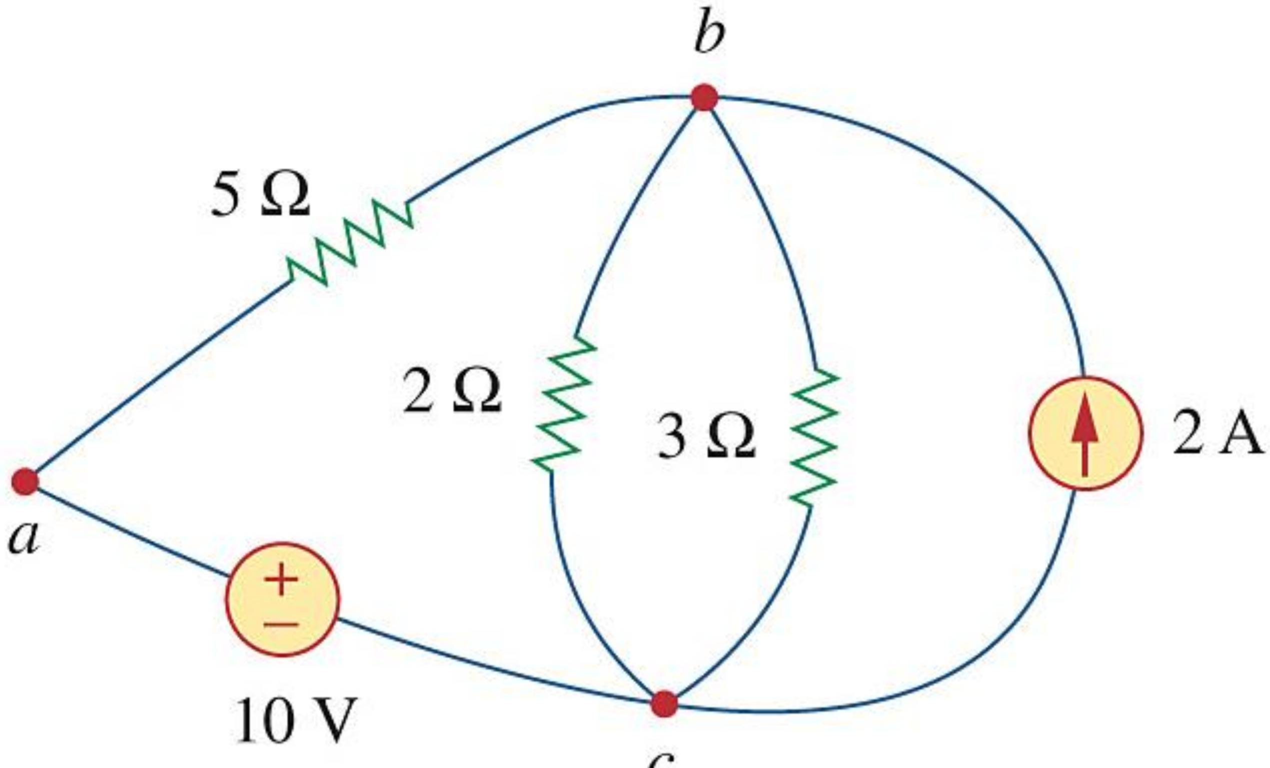


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

- A *mesh* is a loop that does not enclose any other loops. (i.e., smallest loop)
- Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage.

A network with b branches, n nodes, and m meshes will satisfy the fundamental theorem of network topology:

$$b = m + n - 1$$

Example 2.4 Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identity which elements are in series and which are in parallel.

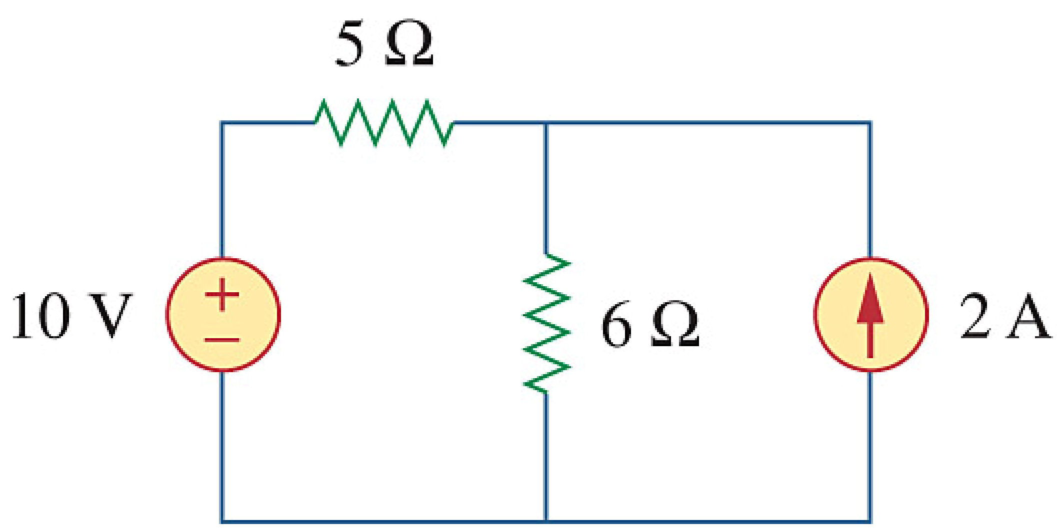


Figure 2.12

Solution :

Four branches and three nodes are identified in Fig. 2.13. ⋯

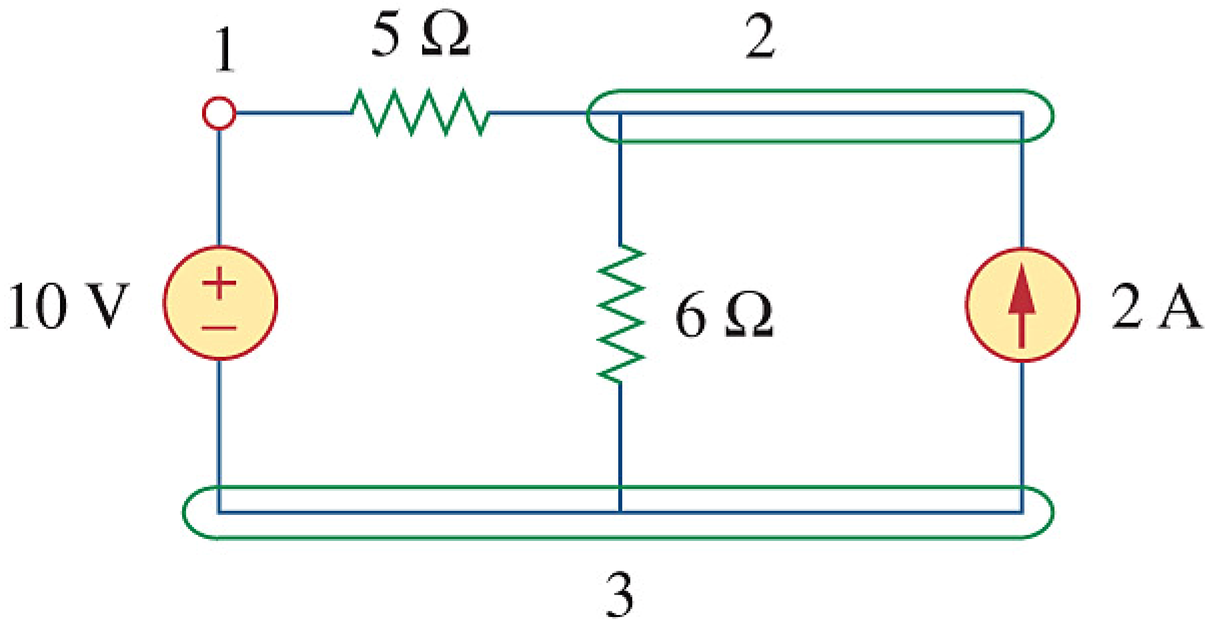


Figure 2.13

2.4 Kirchhoff’s Laws

- Kirchhoff’s current law (KCL) is based on the law of conservation of charge. It states that the algebraic sum of currents entering a node (or a closed boundary) is zero. In other words, the sum of the currents entering a node is equal to the sum of the currents leaving the node.

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node.

the law of conservation of charge

KCL

$$\sum Q_n = 0 \xrightarrow{d/dt} \sum i_n = 0$$

$\sum i_n = 0$

$i_1 - i_2 + i_3 + i_4 - i_5 = 0$

$i_1 + i_3 + i_4 = i_2 + i_5$

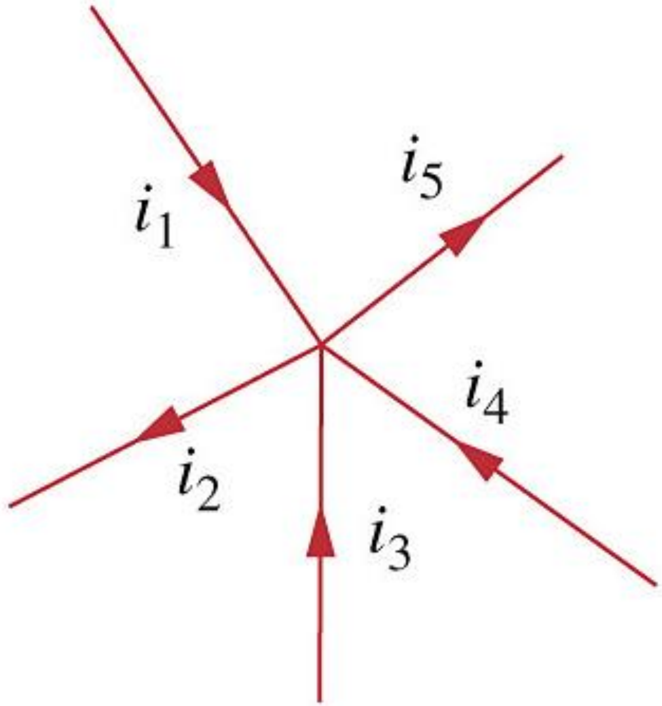


Figure 2.16 Current at a node illustrating KCL.

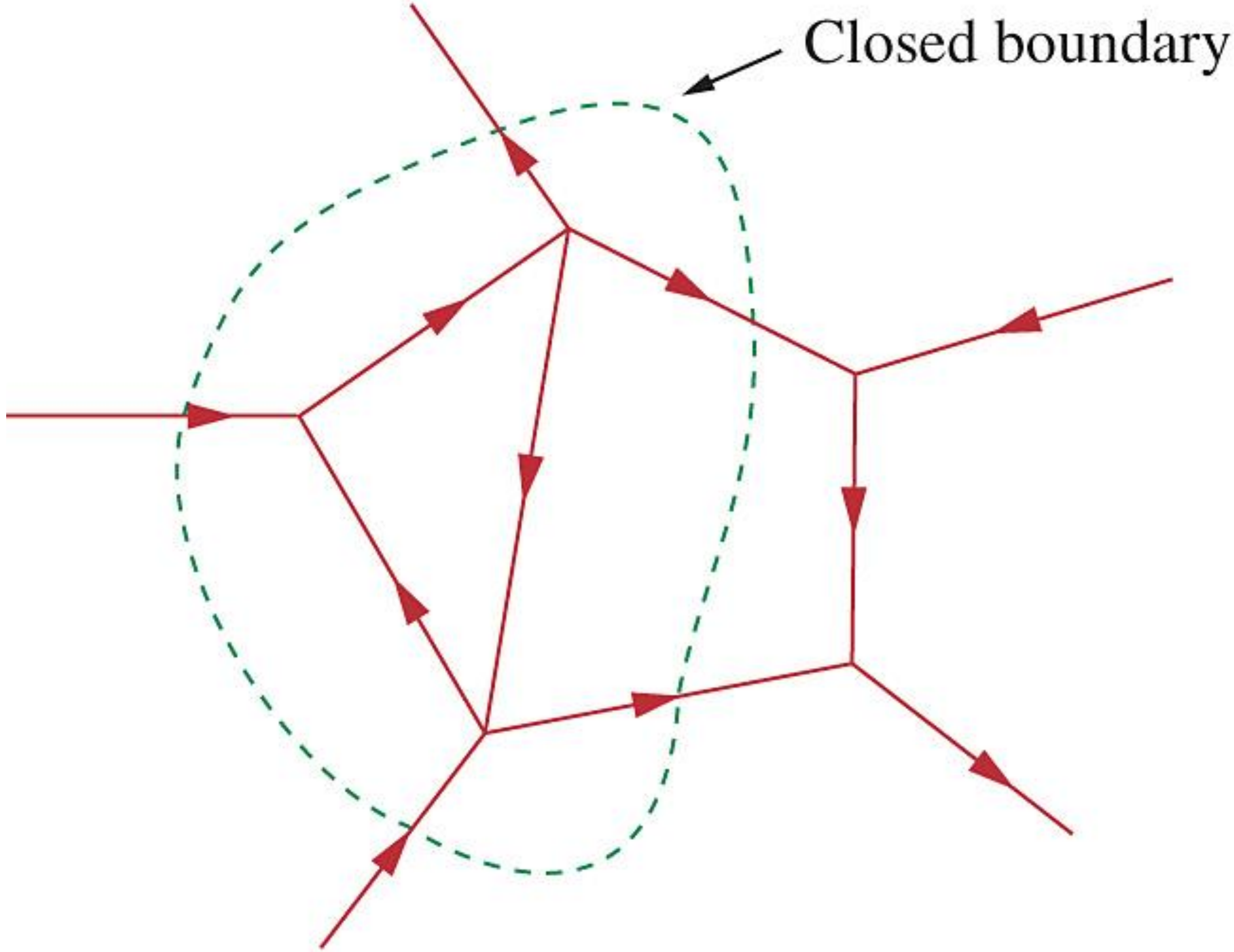
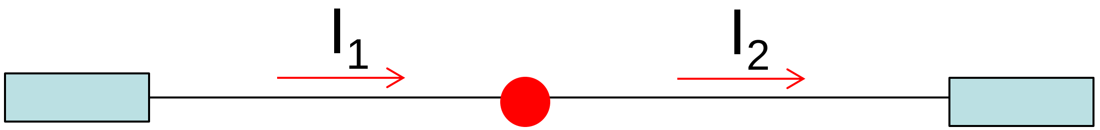


Figure 2.17 Applying KCL to a closed boundary.

- A simple application of KCL is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. See Fig. 2.18.
- A circuit cannot contain two different currents, I_1 and I_2, in series, unless $I_1 = I_2$; otherwise, KCL will be violated.



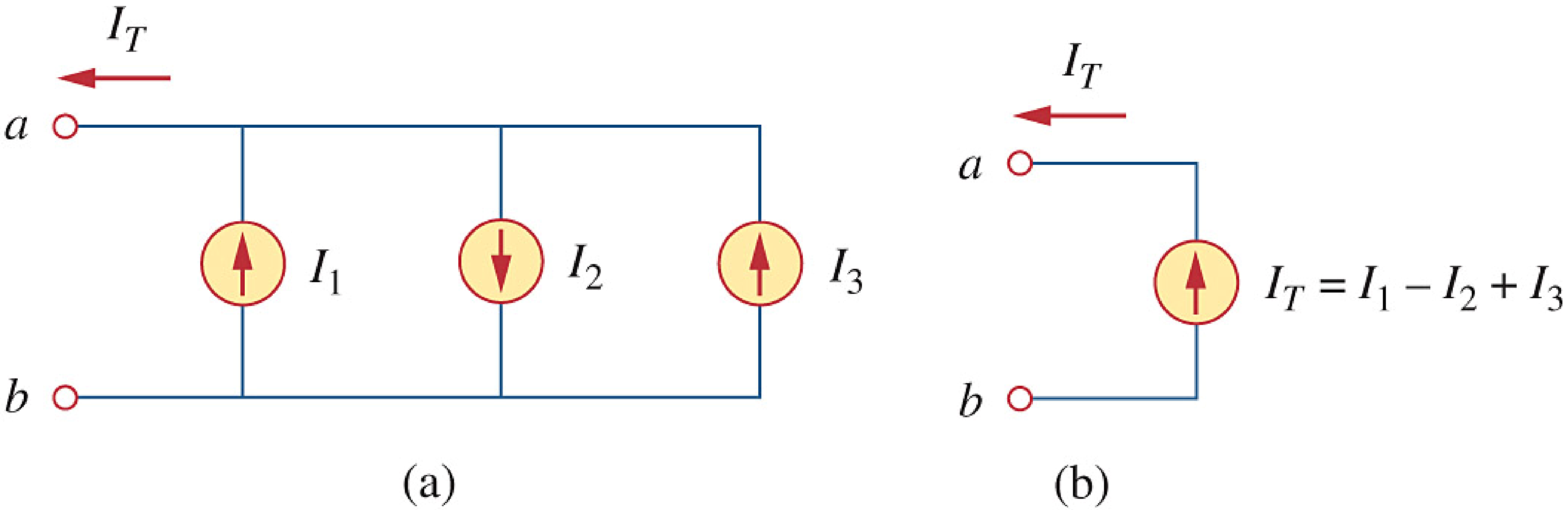


Figure 2.18 Current sources in parallel: (a) original circuit, (b) equivalent circuit.

Circuits are said to be equivalent if they have the same i - v relationship at a pair of terminals.

- Kirchhoff's voltage law (KVL) is based on the principle of conservation of energy.
- The University Physics: *Potential* is potential energy per unit charge.

$v=dw/dq$

- The University Physics: The potential difference V_{ab} equals the work done by the electric force when a unit charge moves from *a* to *b*.

Analogy: $U=mgh$ ➔ $(gh)=dU/dm$
U: potential energy
m: mass of the object
g: acceleration due to gravity
h: altitude of the object

- The NET work done on a charged particle in a closed path is zero.

- KVL states that the algebraic sum of all voltages around a closed path (or loop) is zero. In other words, the sum of voltage drops is equal to the sum of voltage rises.

Mathematically, KVL implies that

$$\sum_{m=1}^{M} v_m = 0$$

where M is the number of branches in the loop and v_m is the mth voltage drop (or rise) in the loop.

We can start with any branch and go around the loop either clockwise or counterclockwise.

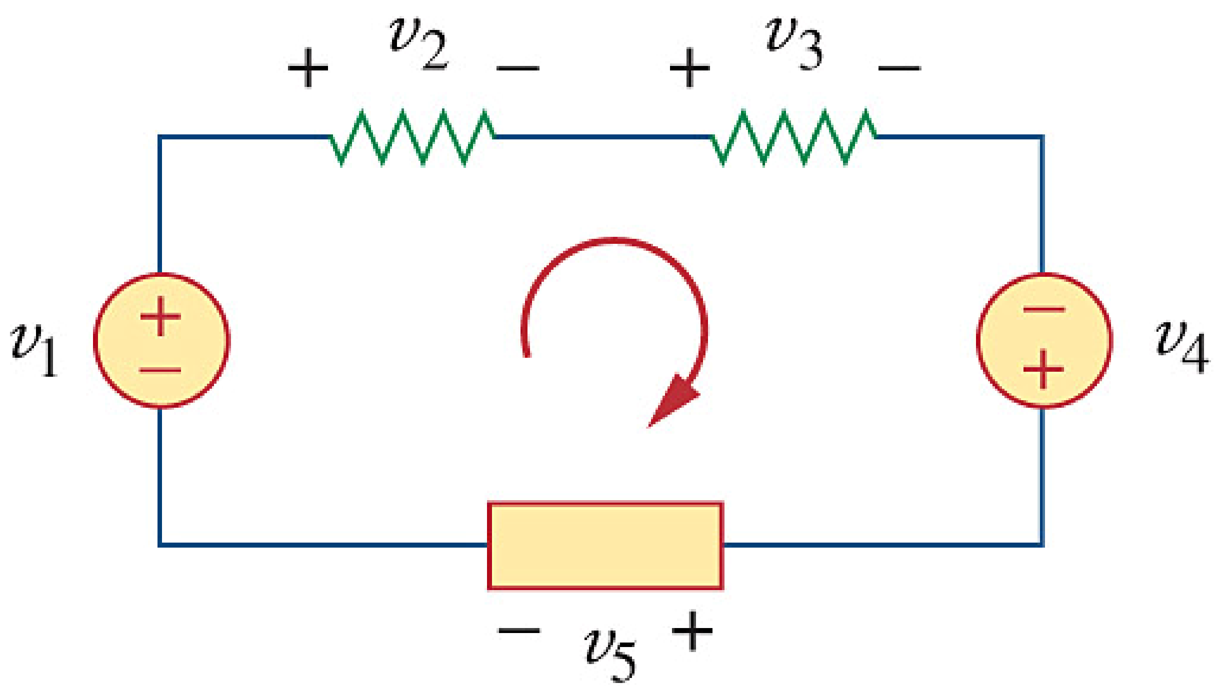
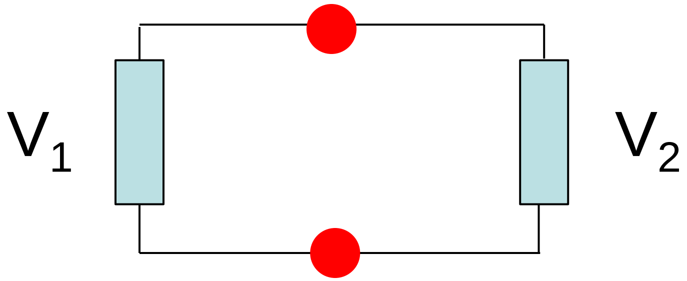


Figure 2.19 A single-loop circuit illustrating KVL.

$$-v_1+v_2+v_3-v_4+v_5 = 0$$

- A simple application of KVL is combining voltage sources in series. The combined voltage is the algebraic sum of the voltages supplied by the individual sources. See Fig. 2.20.
- A circuit cannot contain two different voltages, V_1 and V_2, in parallel, unless $V_1 = V_2$; otherwise, KVL will be violated.



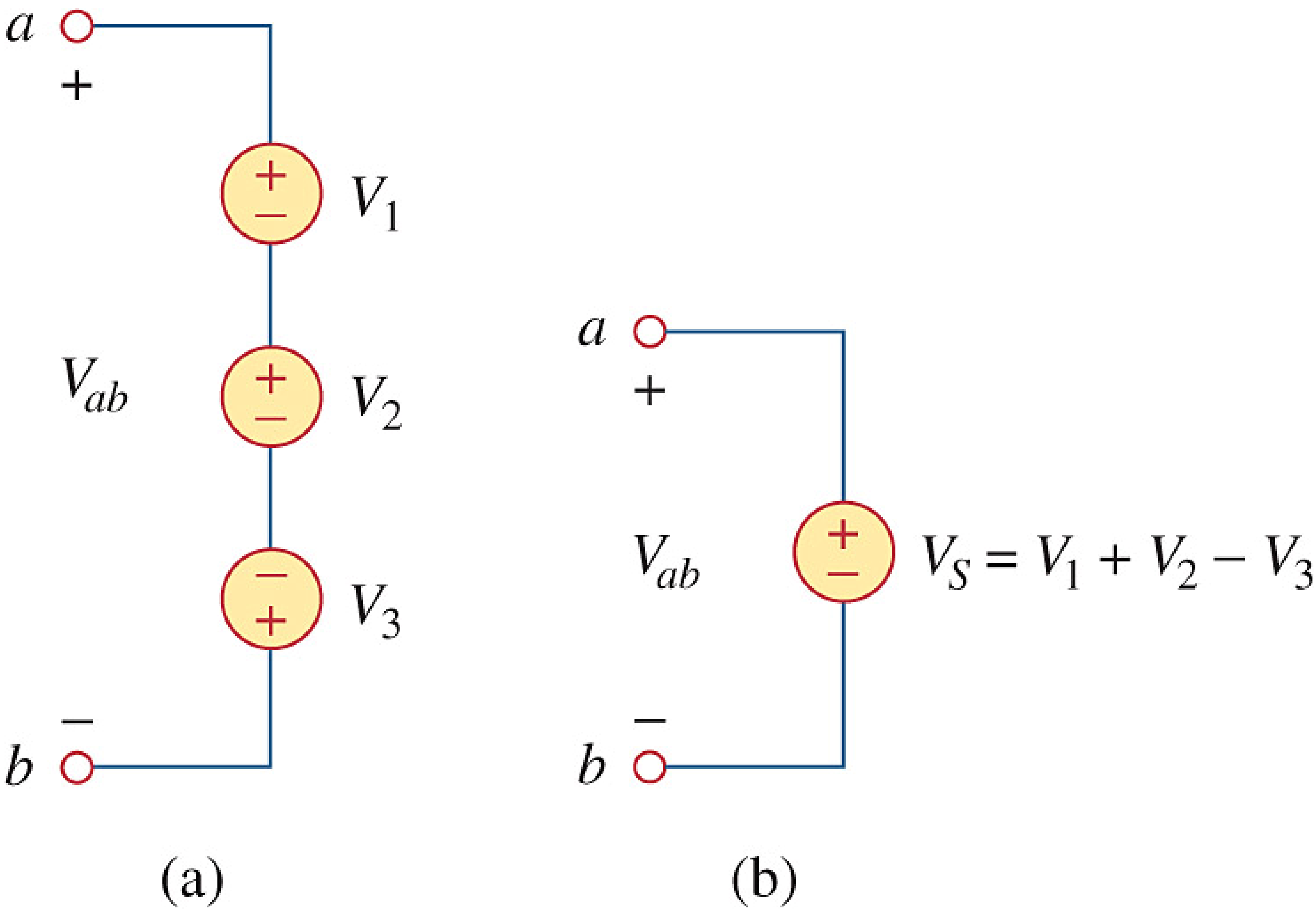


Figure 2.20 Voltage sources in series: (a) original circuit, (b) equivalent circuit.

Practice Problem 2.5 Find v_1 and v_2 in the circuit of Fig. 2.22.

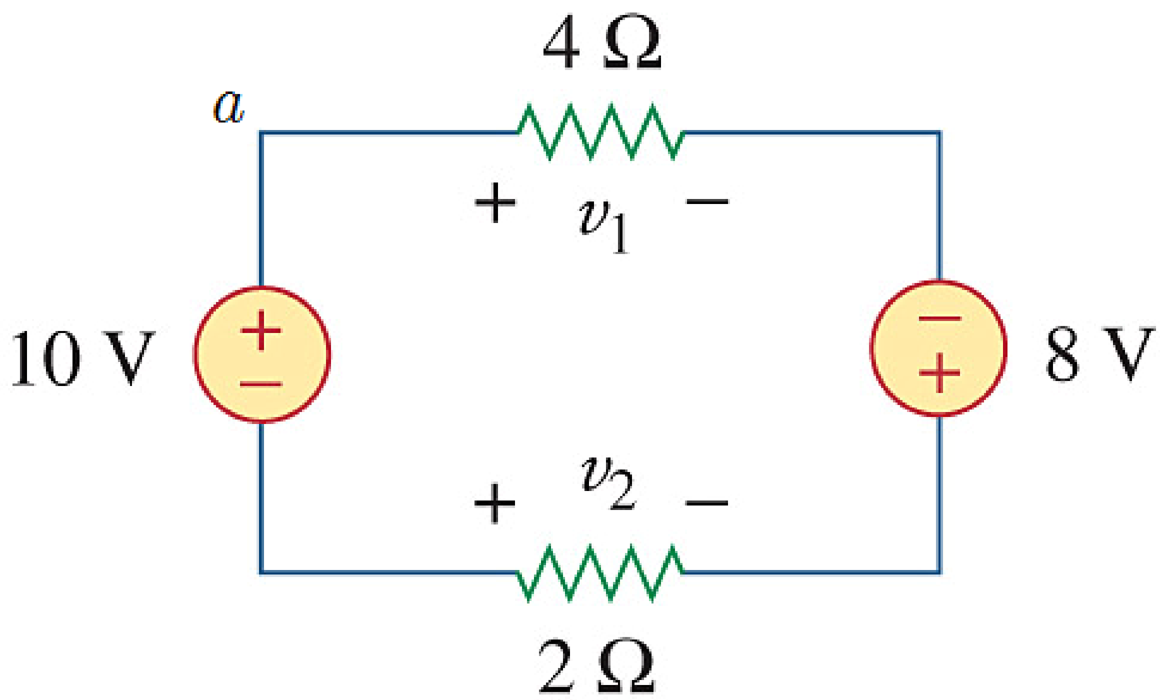


Figure 2.22

Solution : Apply KVL (Choose node a as the strating point, trace the loop clockwise, and assign a positive sign to a voltage drop)

$$v_1 - 8 - v_2 - 10 = 0 \qquad (1)$$

From Ohm's law,

$$i = \frac{v_1}{4} = -\frac{v_2}{2} \Rightarrow v_1 = -2v_2 \qquad (2)$$

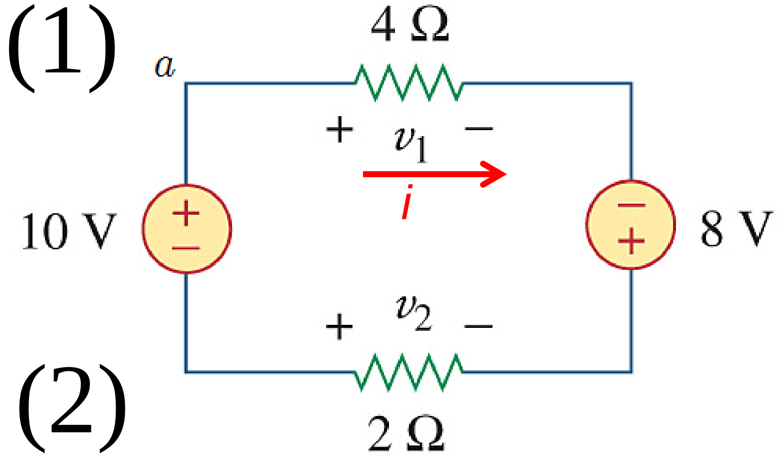


Figure 2.22

Slove the simultaneous equations, we have

$v_1 = 12$ V and $v_2 = -6$ V.

Example 2.6 Determine v_o and i in the circuit shown in Fig. 2.23(a).

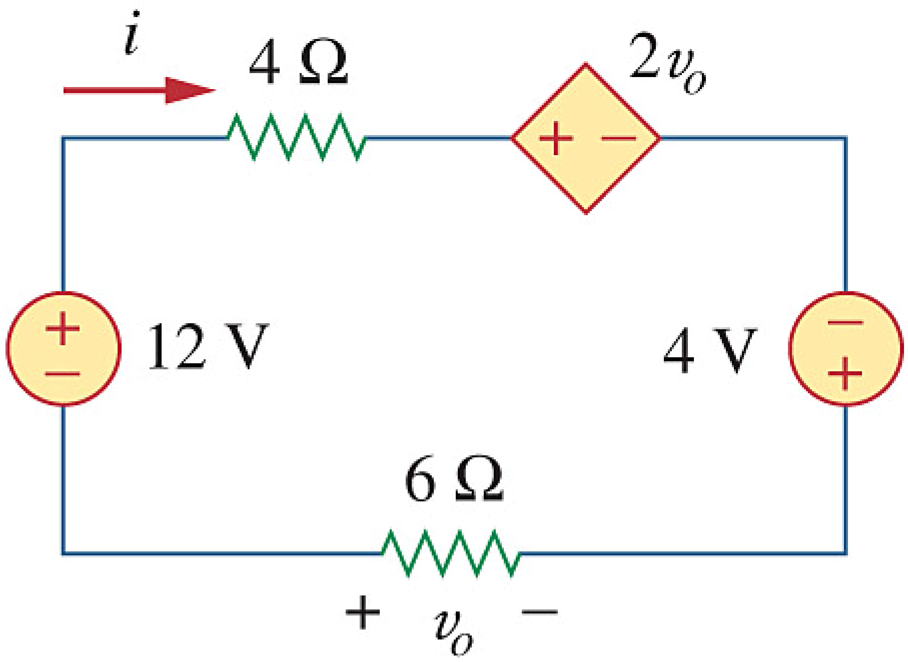


Figure 2.23 (a)

Answer : 48 V, −8 A.

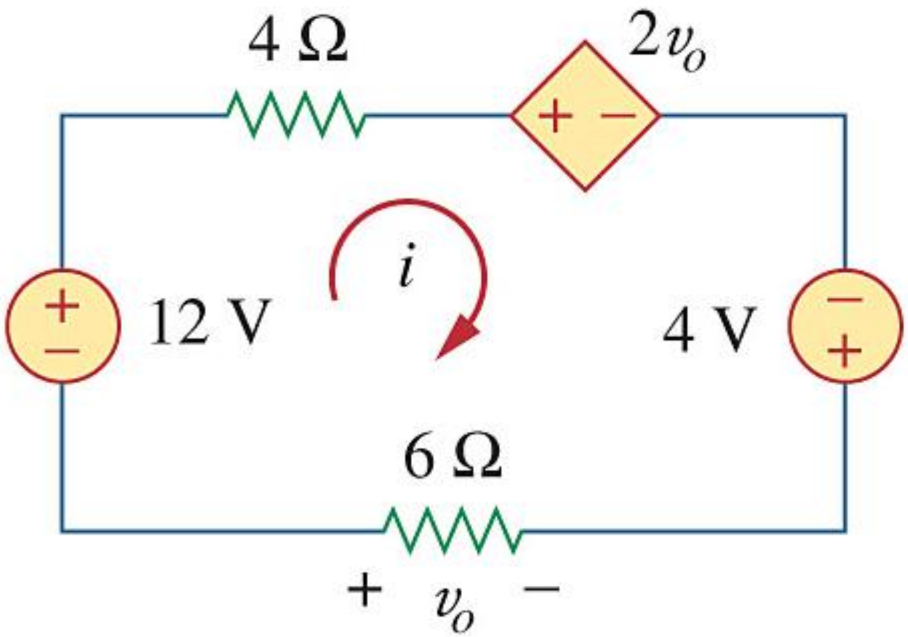


Figure 2.23 (b)

Series connection ➔ KVL

$-12+4i+2v_o-4-v_o=0$

$v_o=6(-i)$

$-12+4i-12i-4+6i=0$

$-2i=16$

$i=-8A$

$v_o=48V$

Practice Problem 2.7 Find v_o and i_o in the circuit of Fig. 2.26.

Answer : 8V, 4A.

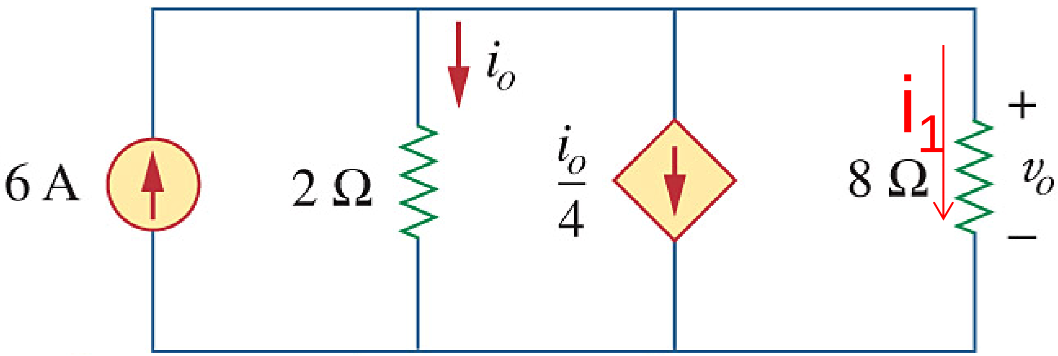


Figure 2.26

Parallel connection ➔ KCL

$i_1=v_o/8$; $v_o=2i_o$

$6=i_o+i_o/4+i_1$

$6=i_o+i_o/4+i_o/4$

$3i_o/2=6$

$i_o=4A$

$v_o=8V$

Example 2.8 Find currents and voltages in the circuit shown in Fig. 2.27(a).

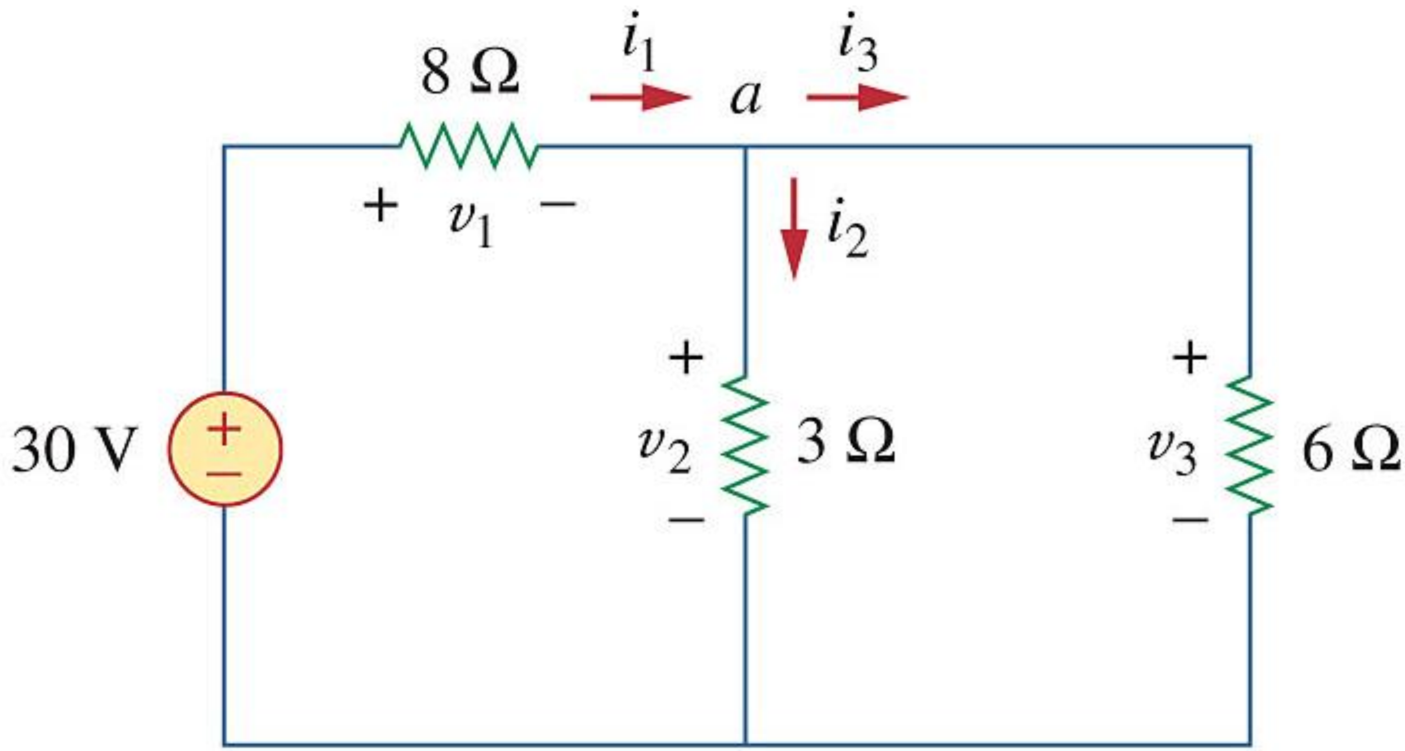


Figure 2.27(a)

Answer : $i_1 = 3$ A, $i_2 = 2$ A, $i_3 = 1$ A,

$v_1 = 24$ V, $v_2 = v_3 = 6$ V.

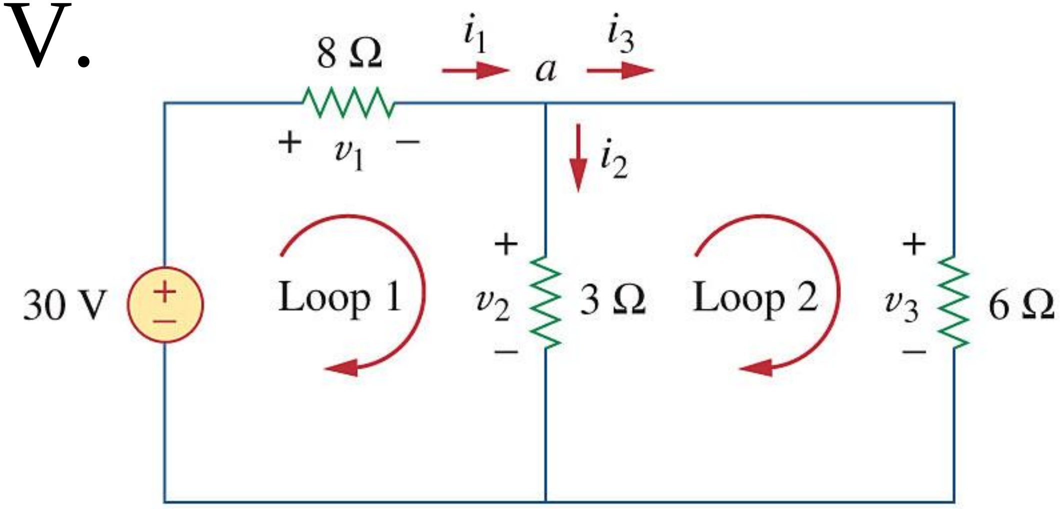


Figure 2.27 (b)

KVL for loop 1: $-30+v_1+v_2=0$ … (1)

KVL for loop 2: $-v_2+v_3=0 \rightarrow v_2=v_3$ … (2)

KCL at node a: $i_1=i_2+i_3$

Ohm's law: $v_1/8=v_2/3+v_3/6$ … (3)

$v_1+v_2=30$; $v_1/8=V_2/2$ or $V_1=4V_2$

$5v_2=30$, $v_2=6$V…

2.5 Series Resistors and Voltage Division

- The equivalent resistance of *N* resistors connected in series is the sum of their individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

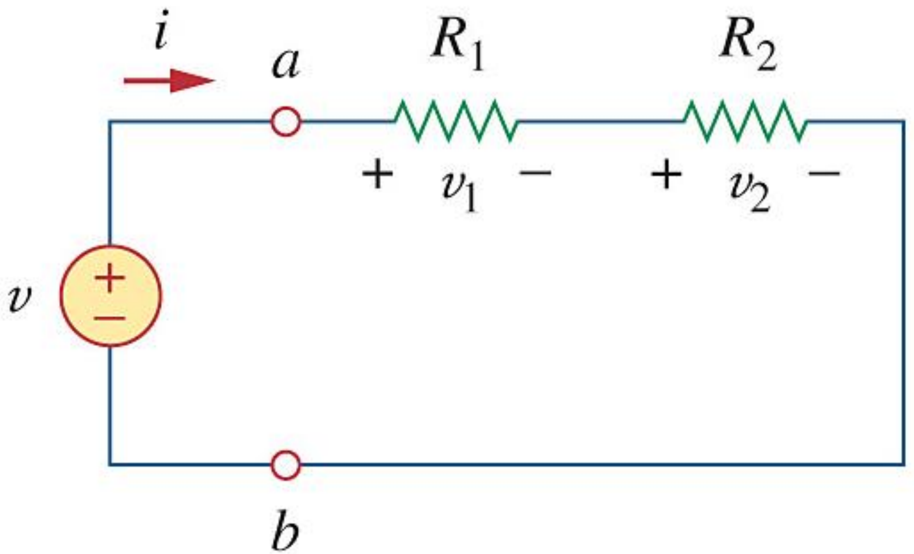


Figure 2.29 A single-loop circuit with two resistors in series.

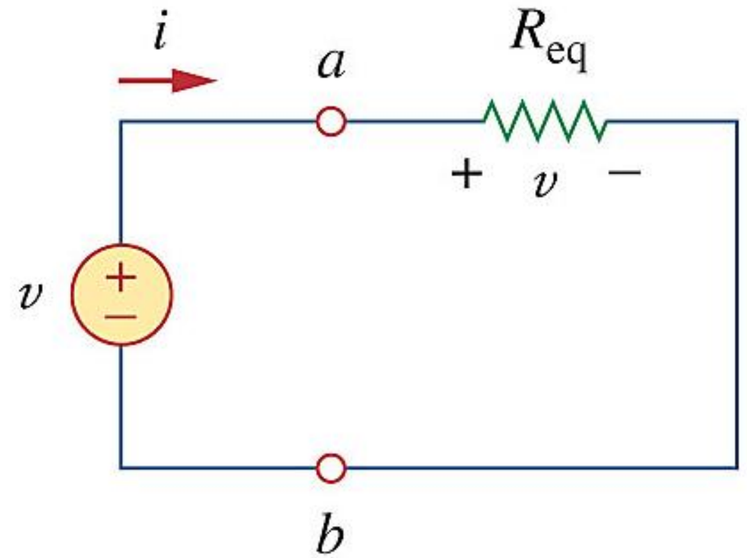


Figure 2.30 Equivalent circuit of the Fig. 2.29 circuit.

$$v = v_1 + v_2 = iR_1 + iR_2 = i \times (R_1 + R_2)$$

$$v = i \times R_{eq}$$

$$R_{eq} = R_1 + R_2$$

The voltage across each resistor is

$$v_n = \frac{R_n}{\sum_{n=1}^{N} R_n} v, \quad n = 1, 2, \ldots, N$$

In series ➔ same $i = v/\sum R_n$
$v_n = iR_n = (v/\sum R_n)R_n = (R_n/\sum R_n)v$

Notice that the source voltage *v* is divided among the resistors in direct proportion to their resistances. This is called the *principle of voltage division*, and the the circuit in Fig. 2.29 is called a *voltage divider*.

- The equivalent conductance of *N* resistors connected in parallel is the sum of their individual conductances.

$$G_{eq} = G_1 + G_2 + \cdots + G_N = \sum_{n=1}^{N} G_n$$

i.e.,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

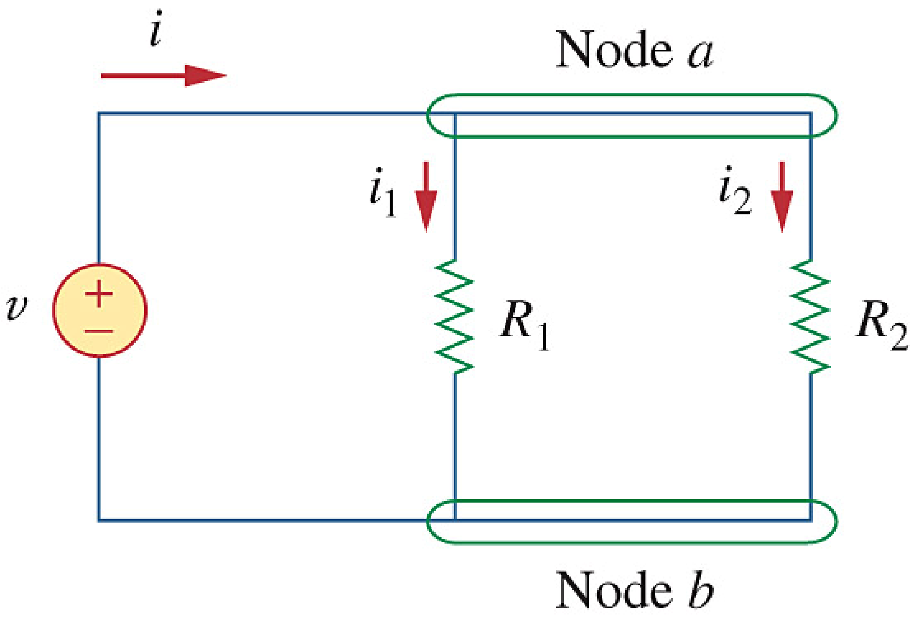


Figure 2.31 Two resistors in parallel

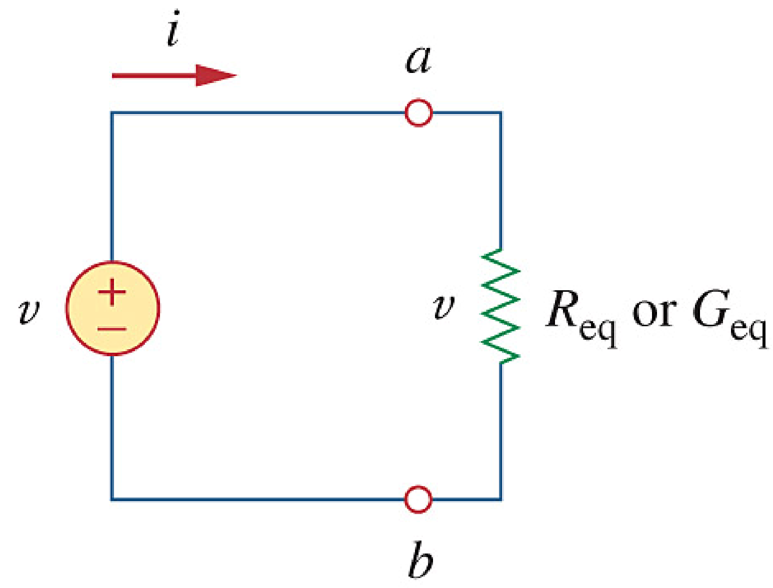


Figure 2.32 Equivalent circuit to Fig. 2.31

$$i = i_1 + i_2 = v/R_1 + v/R_2 = v \times (1/R_1 + 1/R_2)$$

$$I = v \times (1/\ R_{eq})$$

$$1/R_{eq} = 1/R_1 + 1/R_2$$

The current through each resistor is

$$i_n = \frac{G_n}{\sum_{n=1}^{N} G_n} i, \quad n = 1, 2, \ldots, N$$

In parallel ➔ same v = $i/\sum G_n$
$i_n = vG_n = (i/\sum G_n)G_n = (G_n/\sum G_n)i$

Notice that the source current i is divided among the resistors in direct proportion to their conductances. This is called the *principle of current division*, and the the circuit in Fig. 2.31 is called a *current divider*.

Example 2.9 Find R_{eq} for the circuit shown in Fig. 2.34.

Answer : 14.4 Ω.

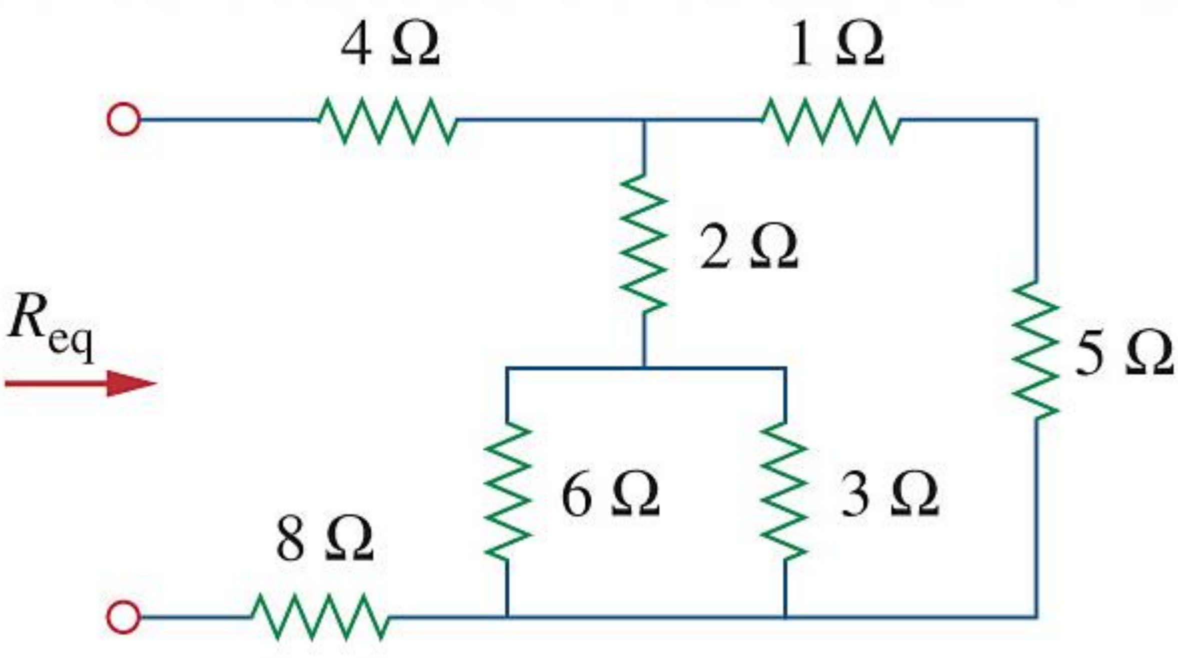


Figure 2.34

6||3=2

2+2=4

5+1=6

4||6=2.4

8+2.4+4=14.4

Example 2.10 Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

Answer : 11.2 Ω.

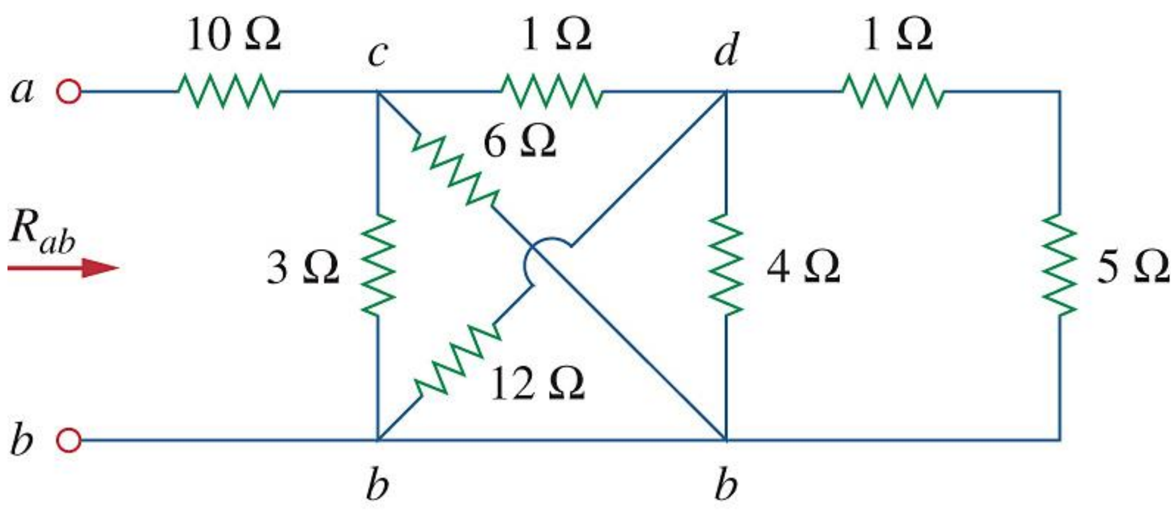


Figure 2.37

6||3=2

12||4=3

(1+5)||3=2

(1+2)||2=1.2

10+1.2=11.2

2.7 Wye-Delta Transformations

- Situations often arise in circuit analysis when resistors are neither in parallel nor in series.

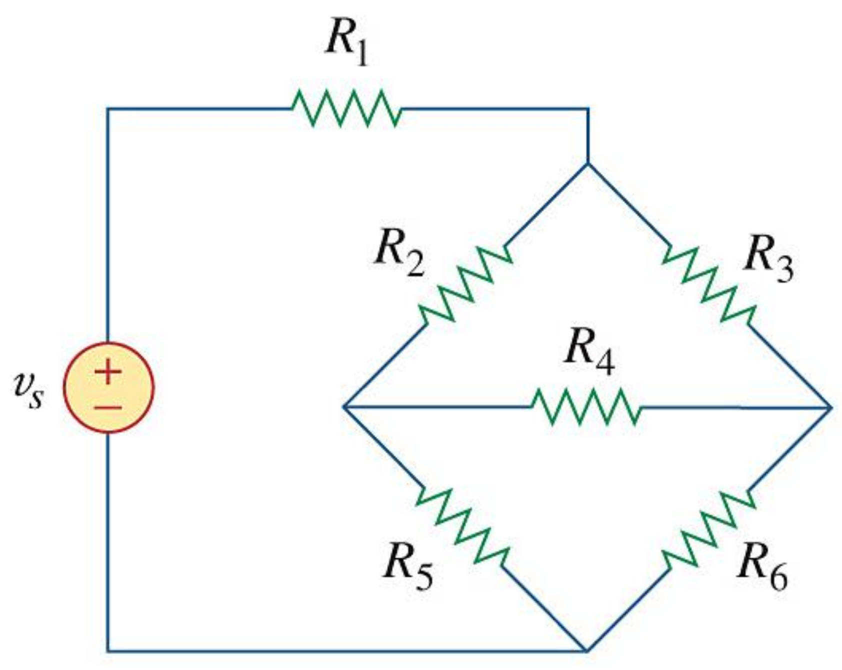


Figure 2.46 The bridge network.

- How do we combine resistors when the resistors are neither in parallel nor in series?
- Many circuits of the type shown in Fig 2.46 can be simplified by using three-terminal networks.
- These are the wye or tee network shown in Fig. 2.47 and the delta or pi network shown in Fig. 2.48.

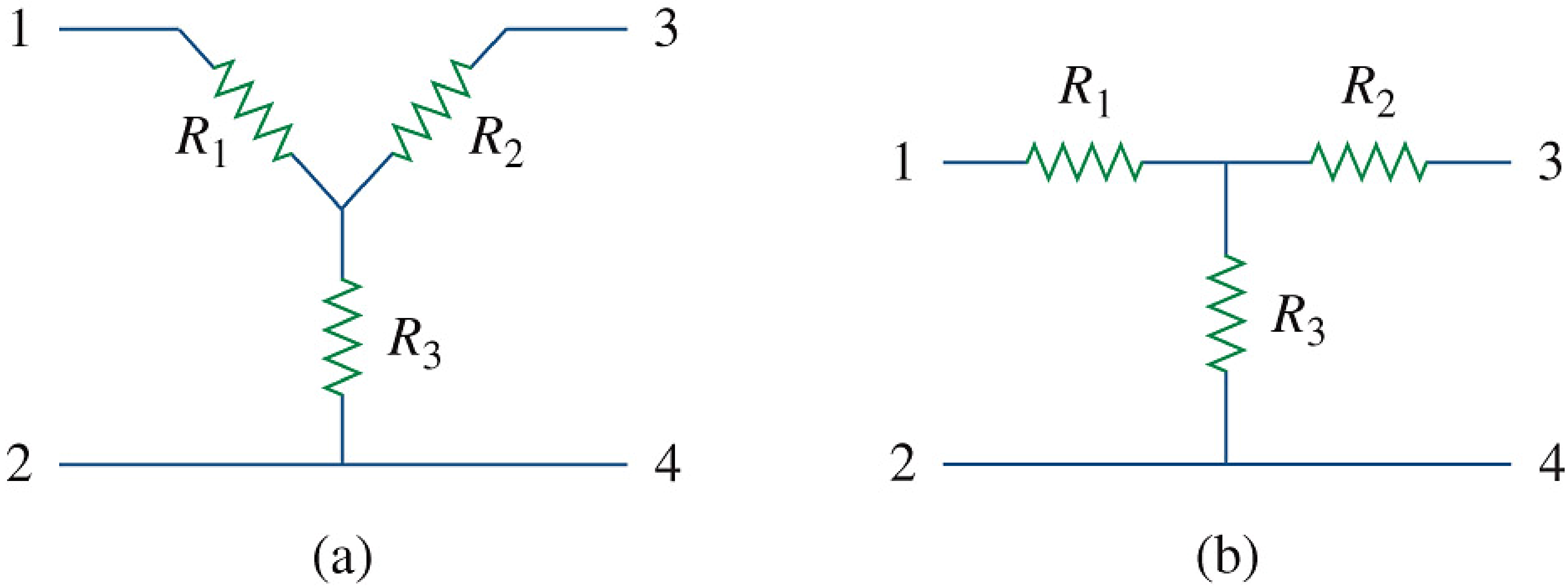


Figure 2.47 Two forms of the same network: (a) Y, (b) T.

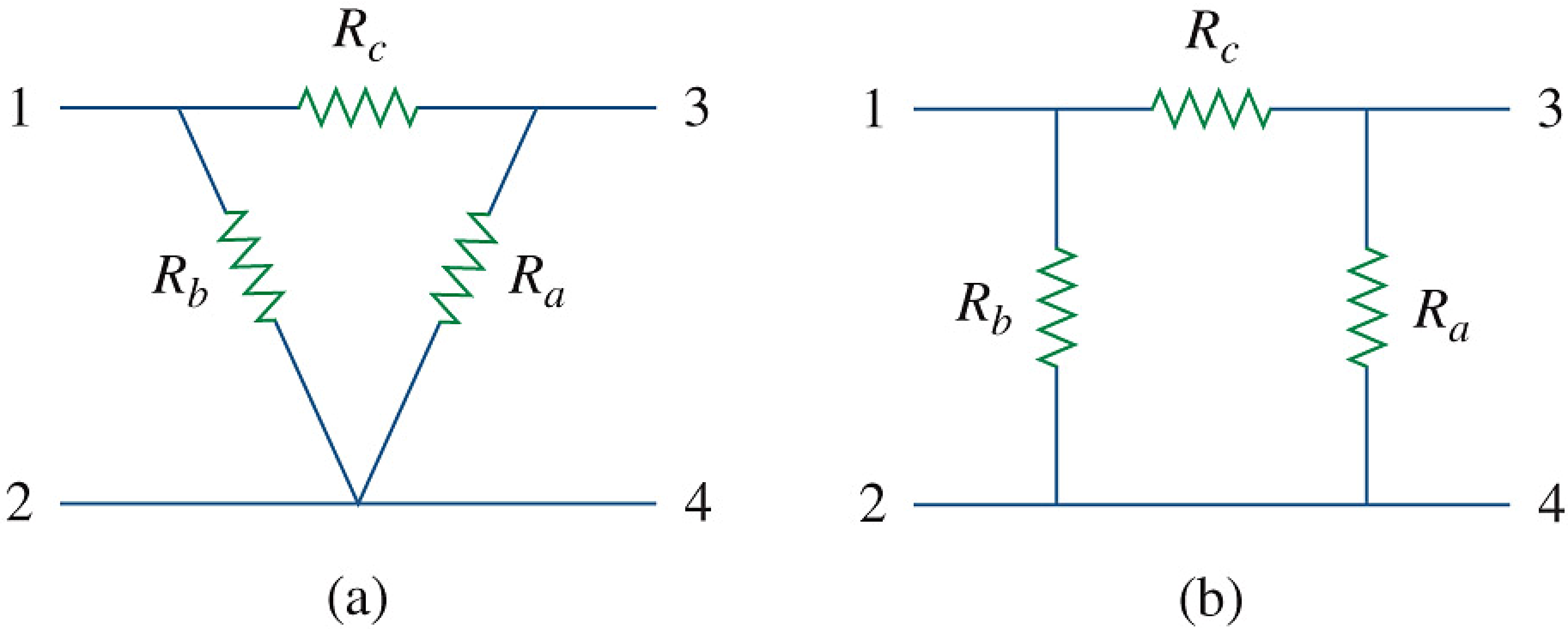


Figure 2.48 Two forms of the same network: (a) $\mathbf{\Delta}$, (b) $\mathbf{\Pi}$.

- The networks occur by themselves or as part of a larger network. Our major interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network. There are two types of transformation:
  - Delta to wye conversion
  - Wye to delta conversion

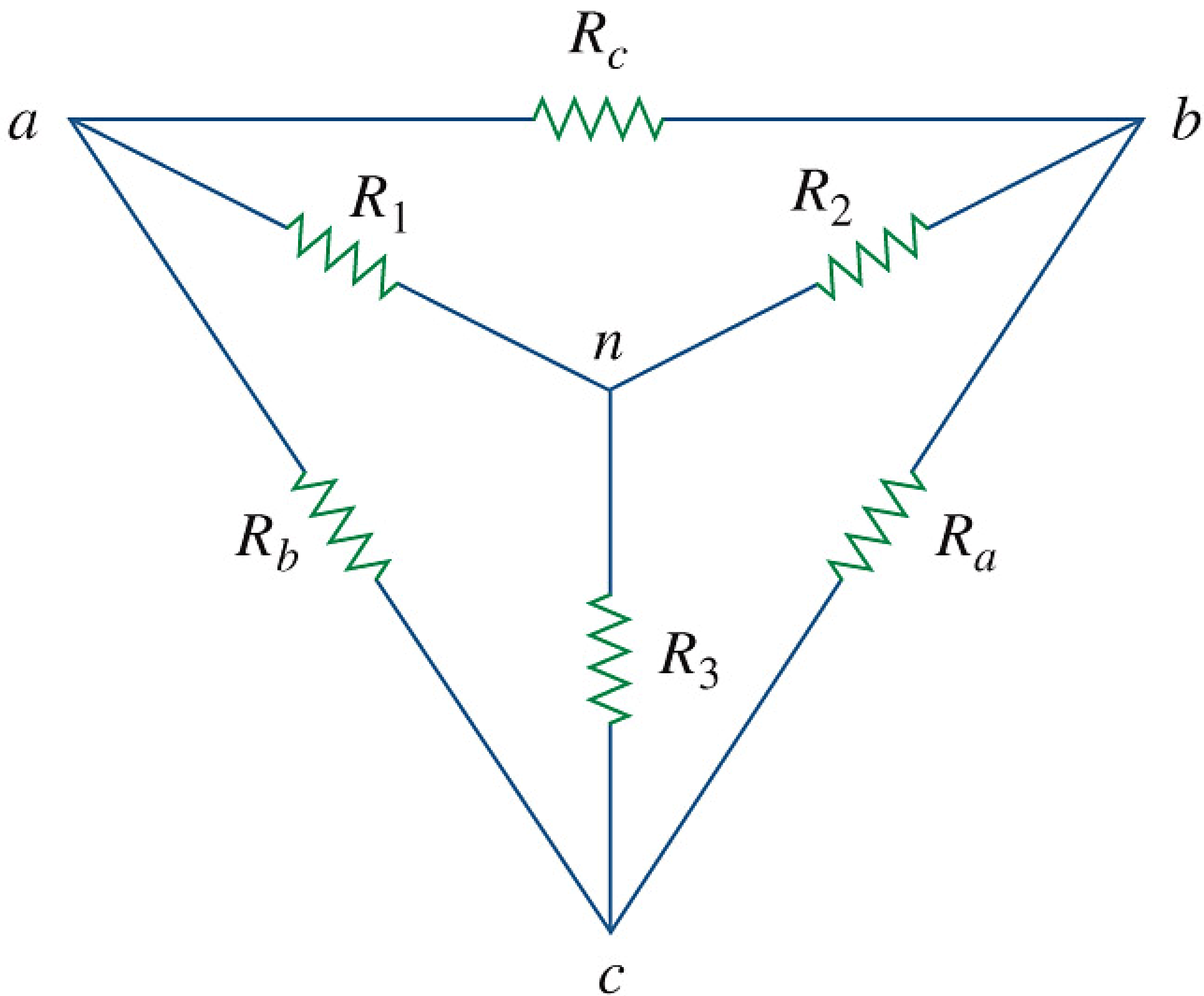


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Delta to Wye Conversion Each resistance in the Y network is the product of the resistances in the two adjacent Δ branches, divided by the sum of the three Δ resistances.

$$\begin{cases} R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c} \\ R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c} \\ R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c} \end{cases}$$

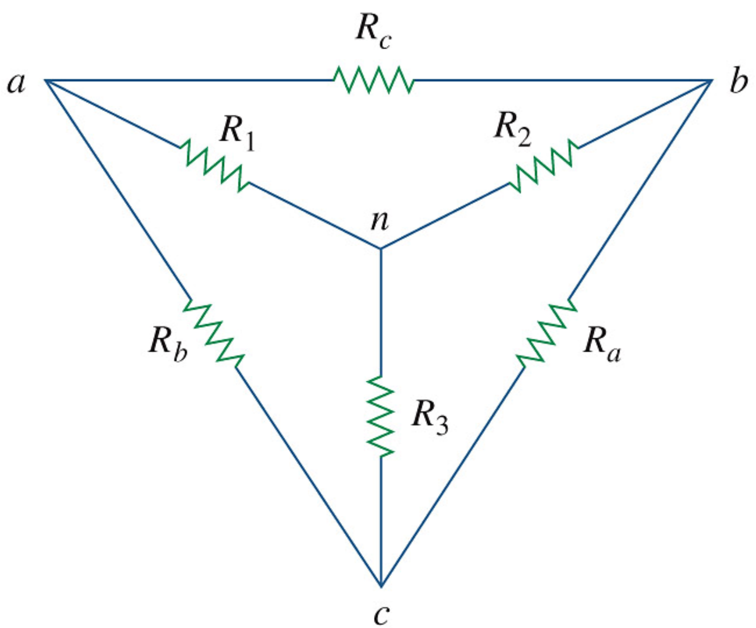


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Wye to Delta Conversion Each resistance in the Δ network is the sum of all possible products of Y resistances taken two at a time, divided by the opposite Y resistance.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

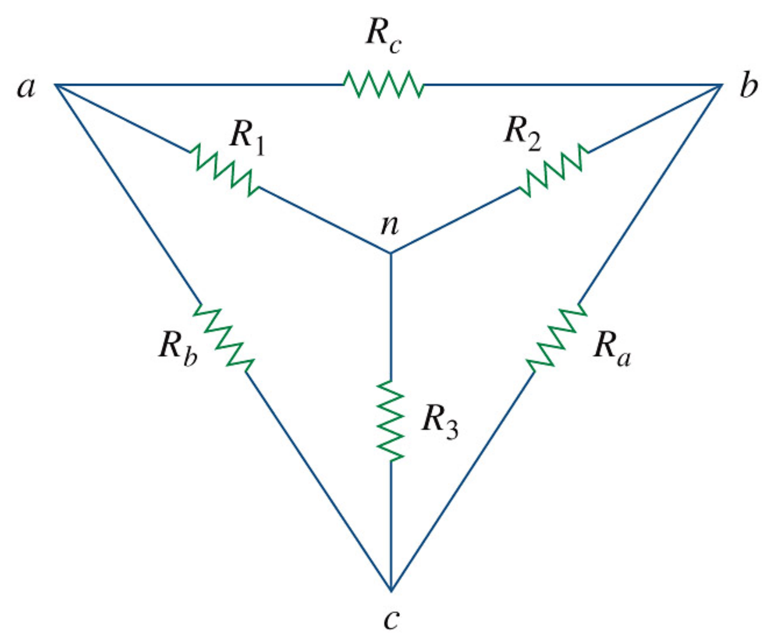


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

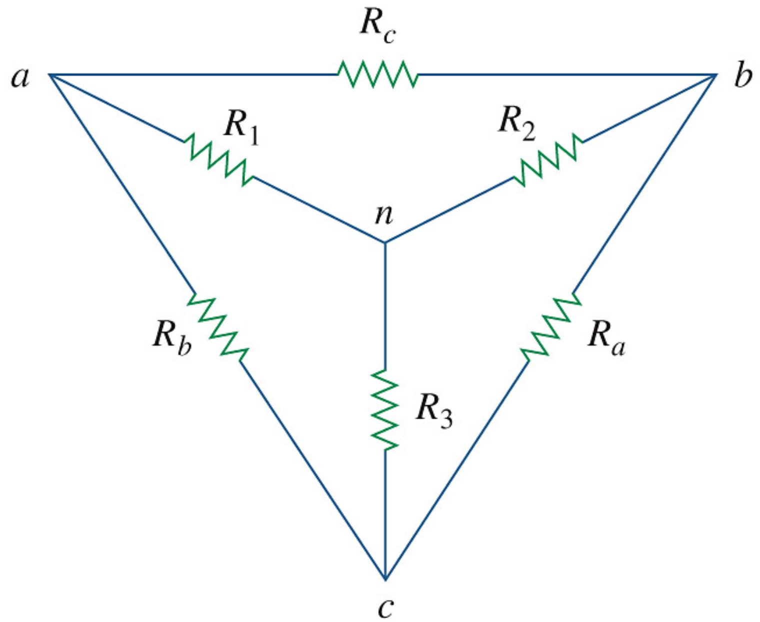
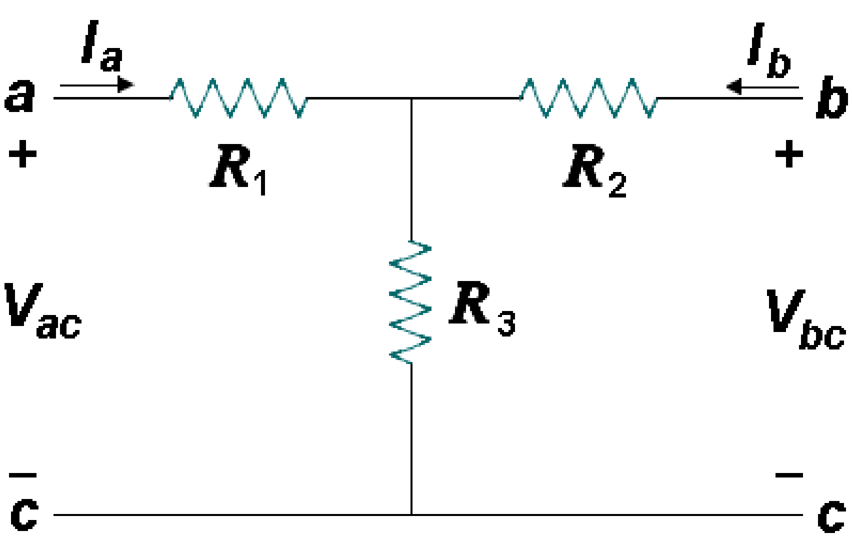
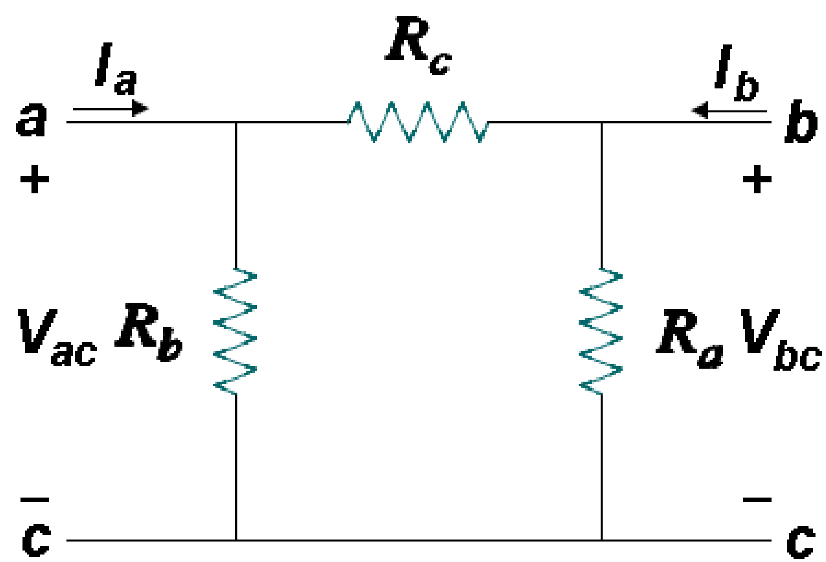


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.



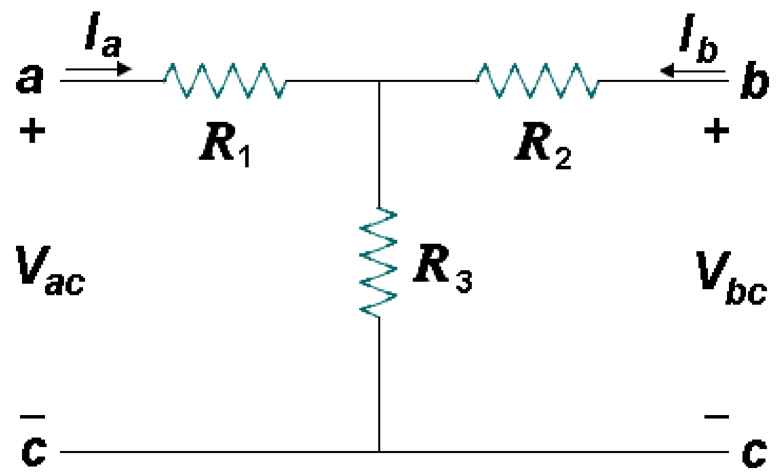


Proof : From the T network,

$\boxed{\text{KVL}}$

$$\begin{cases} I_a R_1 + (I_a + I_b) R_3 = V_{ac} \\ I_b R_2 + (I_a + I_b) R_3 = V_{bc} \end{cases}$$

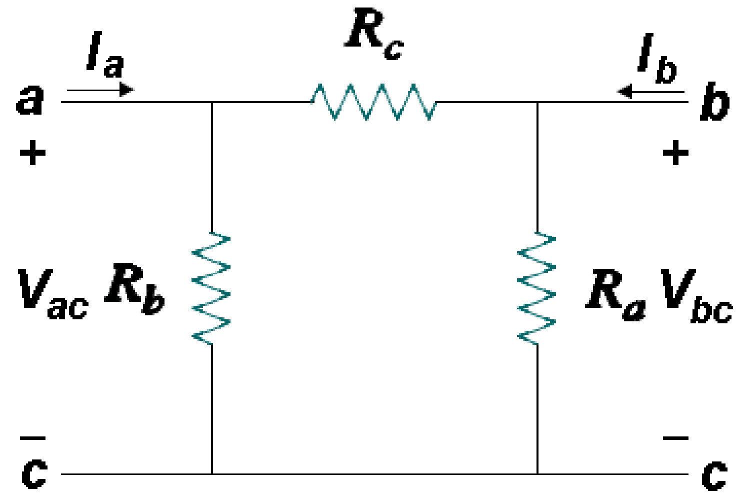
$$\begin{cases} (R_1 + R_3) I_a + R_3 I_b = V_{ac} \\ R_3 I_a + (R_2 + R_3) I_b = V_{bc} \end{cases} \qquad (1)$$



From the Π network,

$\boxed{\text{KCL}}$

$$\begin{cases} I_a = G_b V_{ac} + G_c (V_{ac} - V_{bc}) \\ I_b = G_a V_{bc} + G_c (V_{bc} - V_{ac}) \end{cases}$$



$$\begin{cases} I_a = (G_b + G_c)V_{ac} - G_c V_{bc} \\ I_b = -G_c V_{ac} + (G_a + G_c)V_{bc} \end{cases}$$

$$\begin{cases} V_{ac} = \dfrac{(G_a + G_c)I_a + G_c I_b}{G_a G_b + G_b G_c + G_c G_a} \\ V_{bc} = \dfrac{G_c I_a + (G_b + G_c)I_b}{G_a G_b + G_b G_c + G_c G_a} \end{cases} \qquad (2)$$

$$(R_1+R_3)I_a+R_3I_b=V_{ac}$$
$$R_3I_a+(R_2+R_3)I_b=V_{bc} \qquad (1)$$

$$V_{ac}=\frac{(G_a+G_c)I_a+G_cI_b}{G_aG_b+G_bG_c+G_cG_a}$$
$$V_{bc}=\frac{G_cI_a+(G_b+G_c)I_b}{G_aG_b+G_bG_c+G_cG_a} \qquad (2)$$

Comparing (1) and (2) yields

$$\begin{cases} R_1+R_3=\dfrac{G_a+G_c}{G_aG_b+G_bG_c+G_cG_a} \\ R_3=\dfrac{G_c}{G_aG_b+G_bG_c+G_cG_a} \\ R_2+R_3=\dfrac{G_b+G_c}{G_aG_b+G_bG_c+G_cG_a} \end{cases}$$

$$
\begin{cases}
R_1 = \dfrac{G_a}{G_aG_b + G_bG_c + G_cG_a} = \dfrac{R_bR_c}{R_a + R_b + R_c} \\
R_2 = \dfrac{G_b}{G_aG_b + G_bG_c + G_cG_a} = \dfrac{R_cR_a}{R_a + R_b + R_c} \\
R_3 = \dfrac{G_c}{G_aG_b + G_bG_c + G_cG_a} = \dfrac{R_aR_b}{R_a + R_b + R_c}
\end{cases} \quad (3)
$$

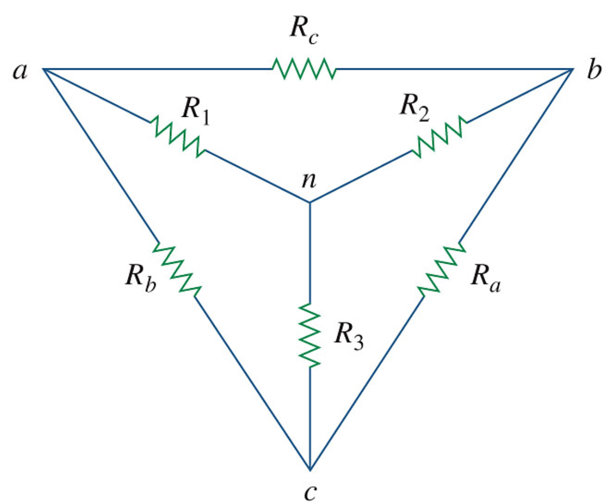


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

$\boxed{\Delta\text{-Y transformation}}$

From (3),

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c}{R_a + R_b + R_c} \qquad (4)$$

Dividing (4) by each of (3) leads to

$$\begin{cases} R_a = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \\ R_b = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ R_c = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \end{cases} \qquad (5)$$

Y-Δ transformation

Example 2.14 Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

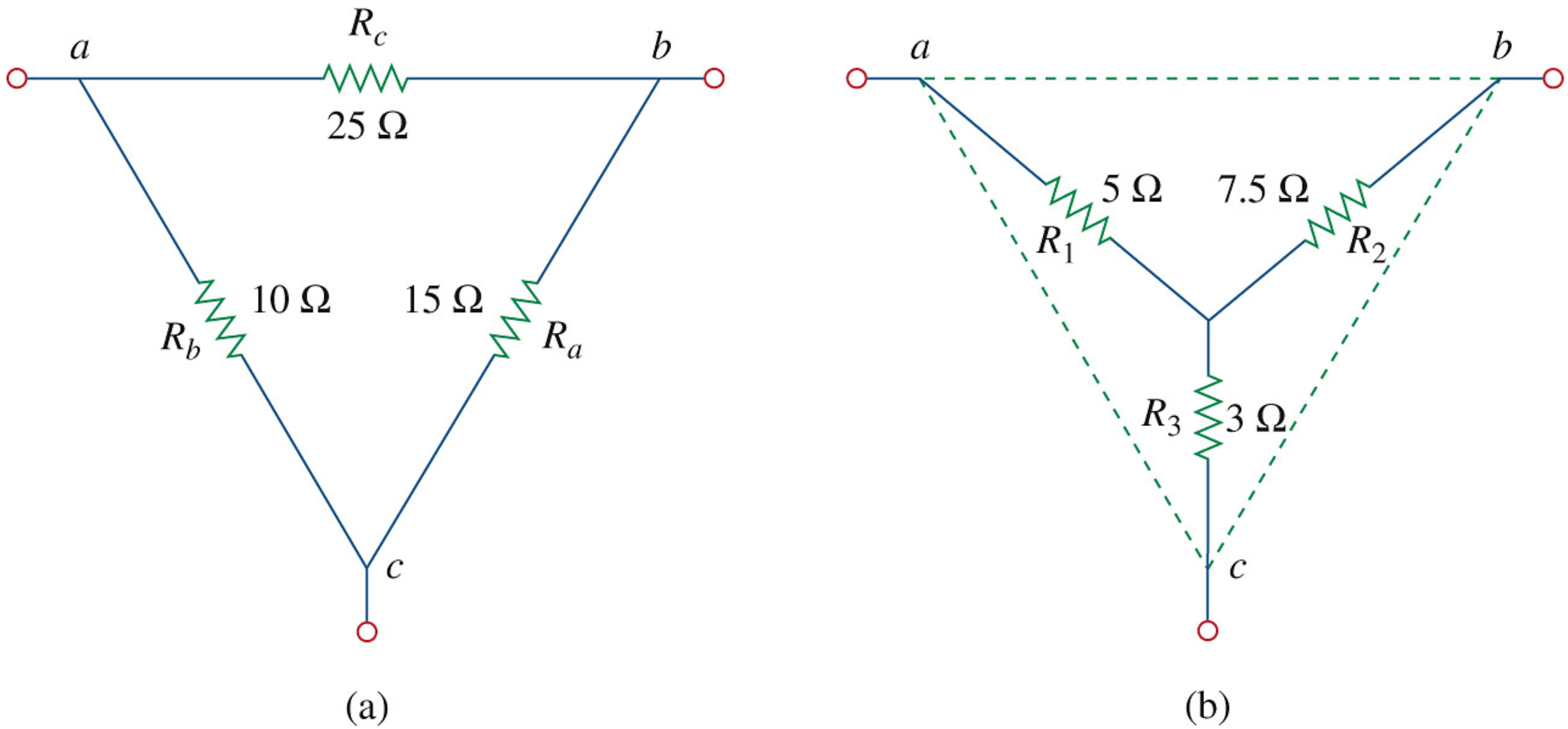
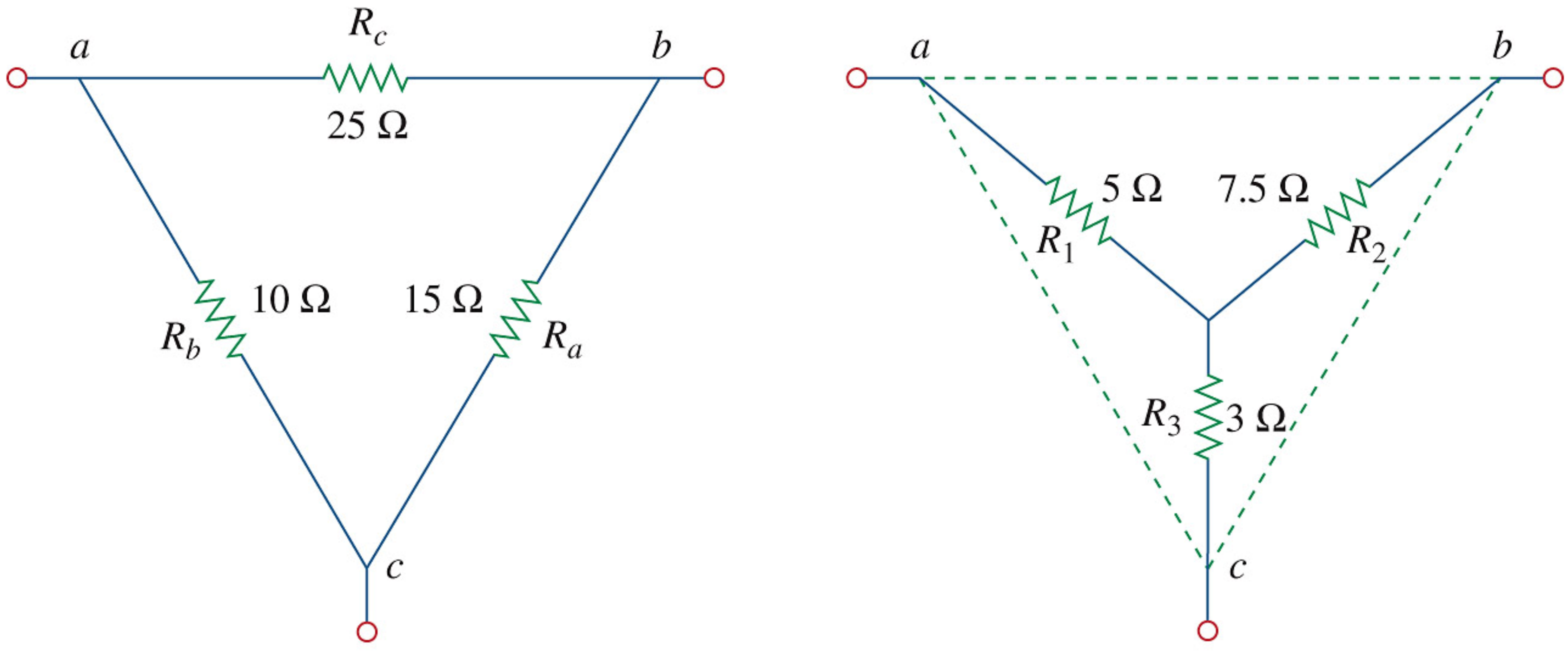


Figure 2.50

Q: why R in Δ branches larger?



(a) (b)

Figure 2.50

Y is like series; Δ is like parallel

Let $R_a=0$ (i.e., $R_{bc}=0$)

(a) $R_{ac}=R_b||R_c \Rightarrow R_{ac} < R_b$ or R_c

(b) $R_{ac}=R_1+(R_2||R_3) \Rightarrow R_1 < R_{ac}$

$\Rightarrow$ $R_1 < R_b$ or R_c

Practice Problem 2.14 Transform the wye network in Fig. 2.51 to a delta network.

Answer : $R_a = 140\ \Omega,\ R_b = 70\ \Omega,\ R_c = 35\ \Omega.$

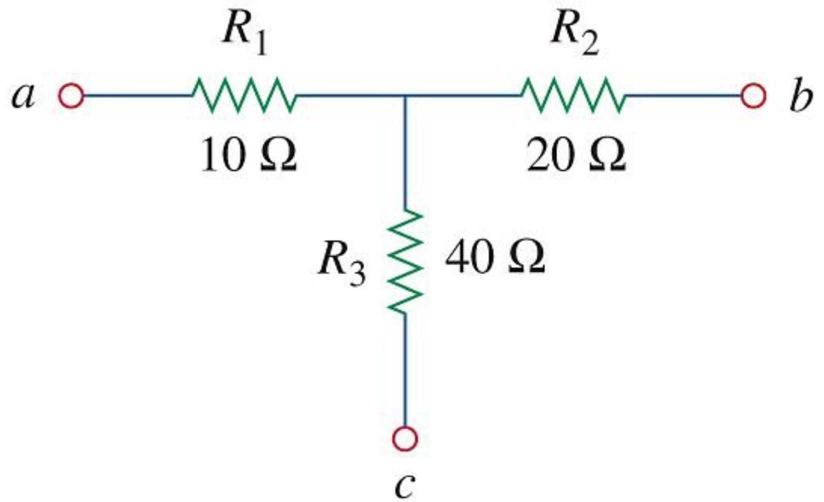


Figure 2.51

$R_a = R_{bc} = (R_1R_2 + R_2R_3 + R_3R_1)/R_1$
$= (200+800+400)/10 = 140$ ohm

Practice Problem 2.15 For the bridge network in Fig. 2.54, find R_{ab} and i.

Answer : $R_{ab} = 40\ \Omega$, $i = 2.5$ A.

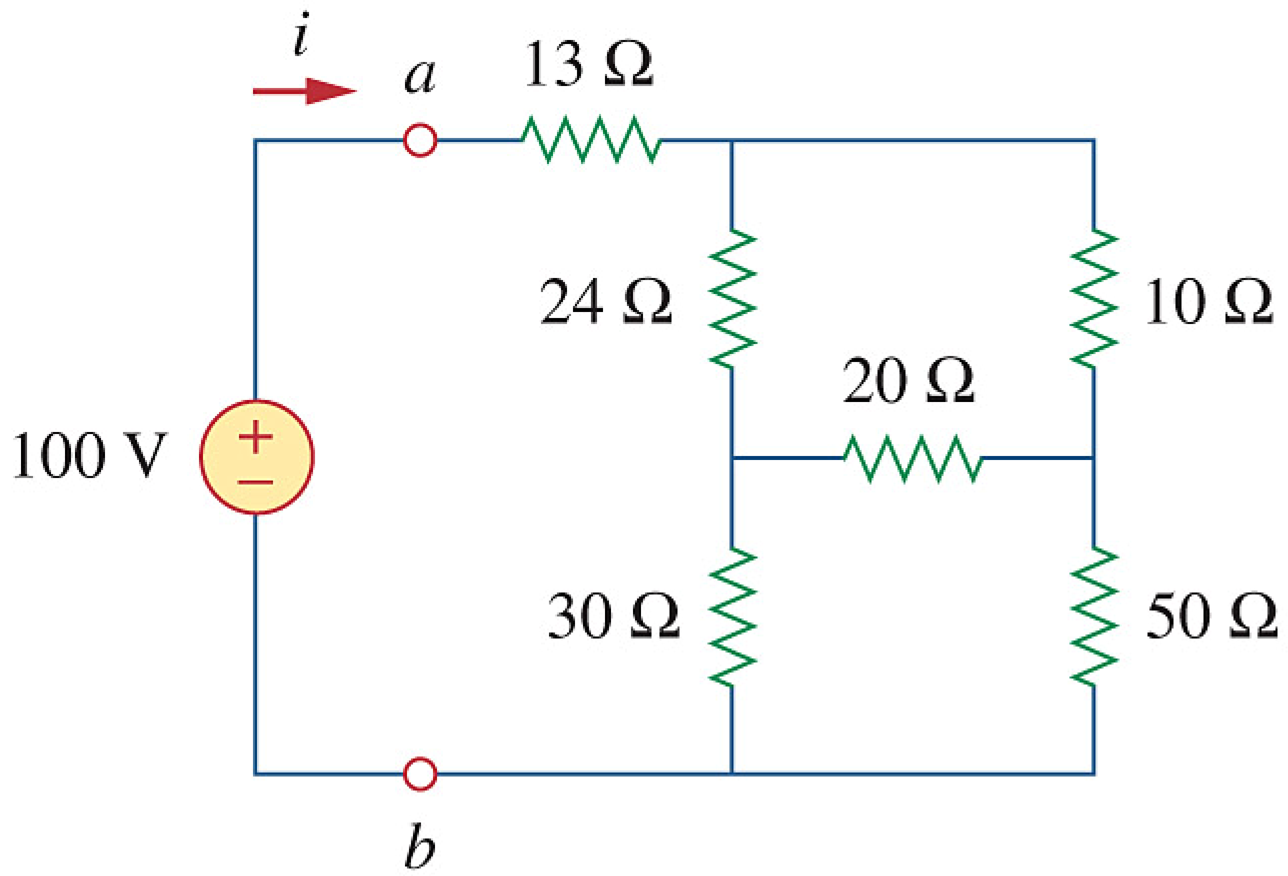


Figure 2.54

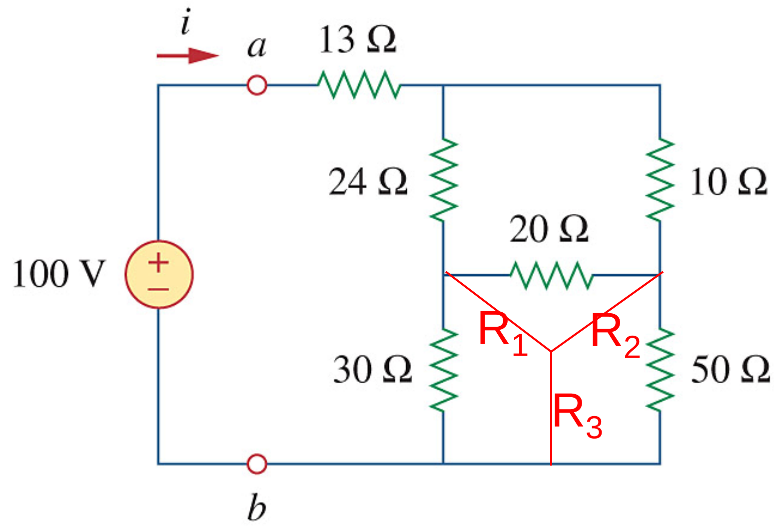


Figure 2.54

-Delta to Y, either convert the upper delta or the lower delta.

-For example, the lower delta (easier…)

R_1=600/100=6; R_2=10; R_3=15

15+(30||20)+13=40 ohm

i=V/R_{eq}=100/40=2.5A