Ve215 Electric Circuits

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Fall 2017

Chapter 3

Methods of Analysis

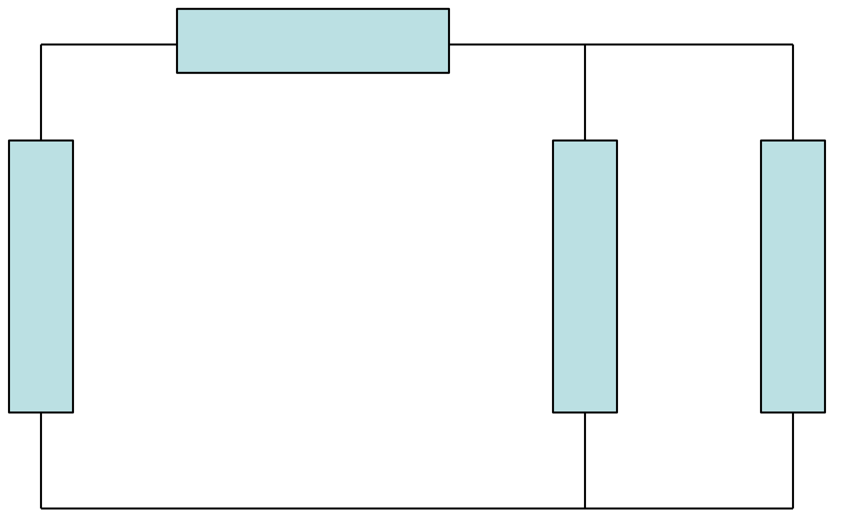
3.1 Introduction

- In this chapter, we develop two powerful techniques for circuit analysis: nodal analysis, which is based on a systematic application of KCL, and mesh analysis, which is based on a systematic application of KVL.

3.2 Nodal Analysis

- Nodal analysis provides a general procedure for analyzing circuits using *node voltages* as the circuit variables. **Choosing node voltages instead of element voltages** as circuit variables is convenient and reduce the number of equations one must solve simultaneously.

3 variables vs. 2 variables



Given a circuit with n nodes without voltage sources, the nodal analysis involves taking the following four steps:

1. Select a node as the reference node (commonly called the ground). We flag the chosen reference node with one of the symbols as in Fig. 3.1.

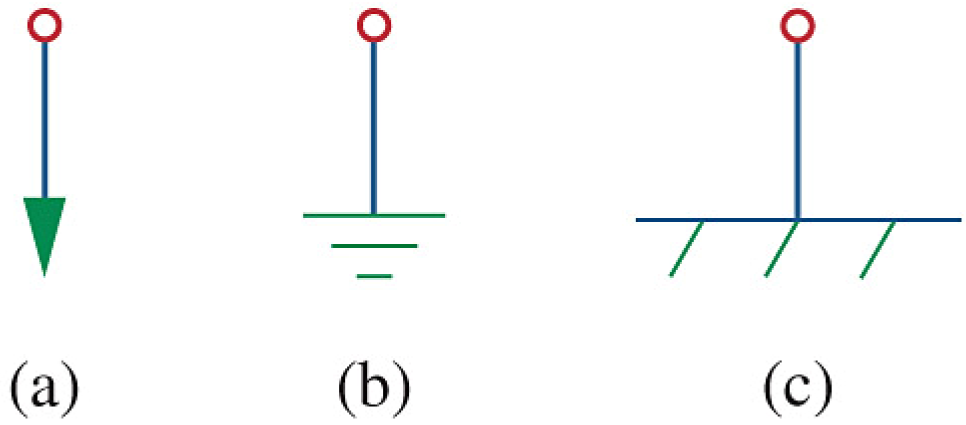


Figure 3.1 Common symbols for indicating a reference node: (a) common ground, (b) ground, (c) chassis ground.

2. Assign node voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining $n-1$ nodes. A node voltage is defined as the voltage rise from the reference node to a nonreference node.

3. Apply KCL to each of the nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example 3.1 Calculate the node voltages in the circuit shown in Fig. 3.3(a).

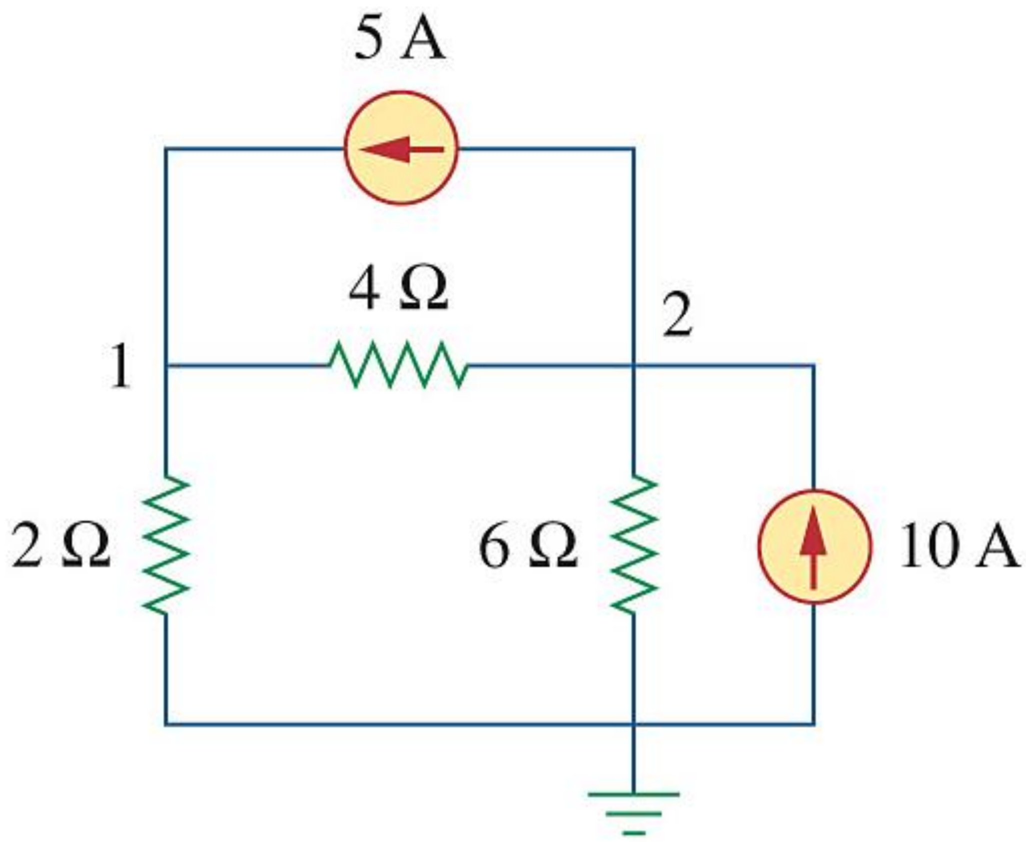


Figure 3.3 (a)

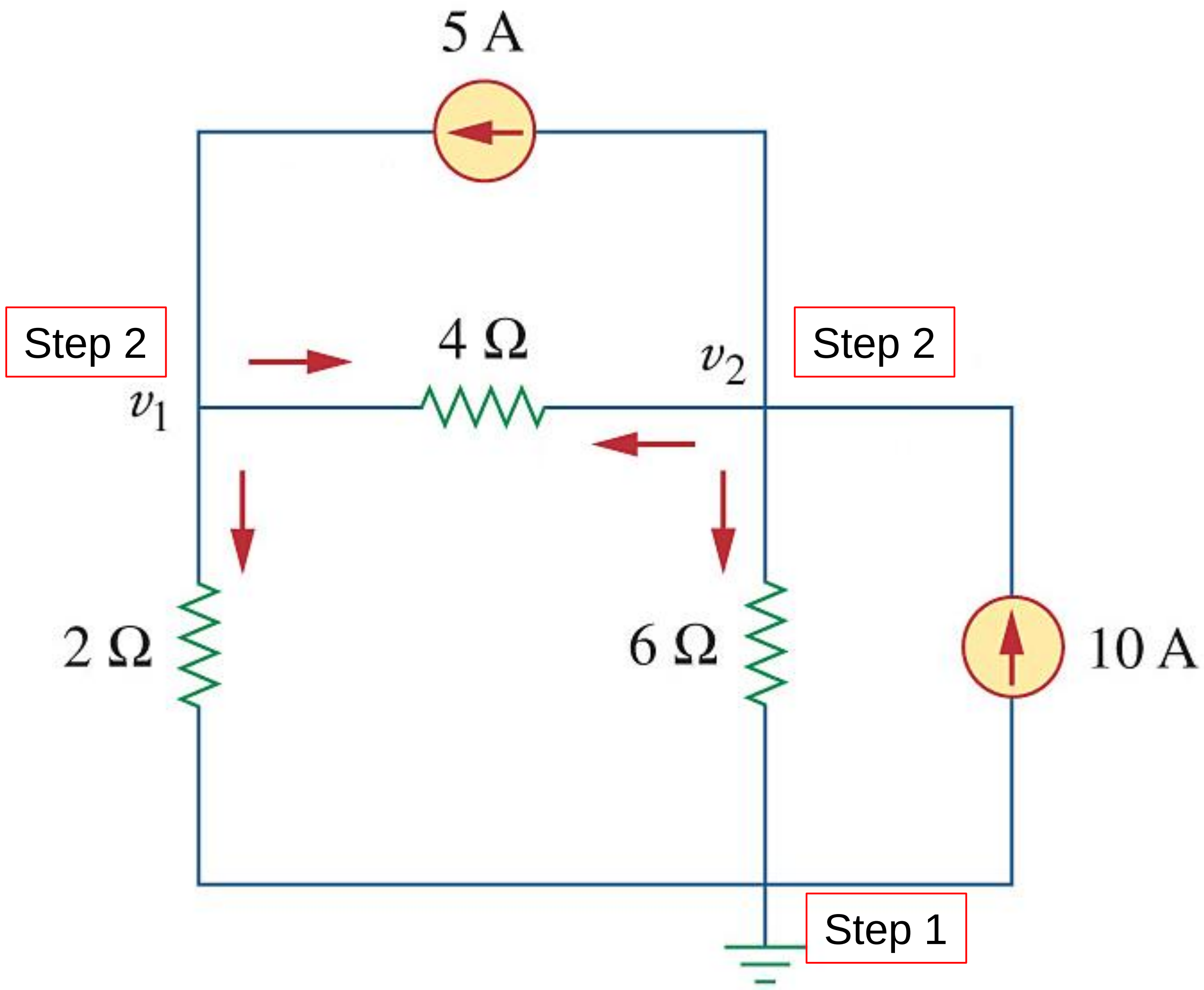


Figure 3.3(b)

Solution :

Step 3: KCL, Ohm's law

At node 1,

$$\frac{v_1}{2}+\frac{v_1-v_2}{4}-5=0$$

$$\left(\frac{1}{4}+\frac{1}{2}\right)v_1-\frac{1}{4}v_2=5 \qquad (1)$$

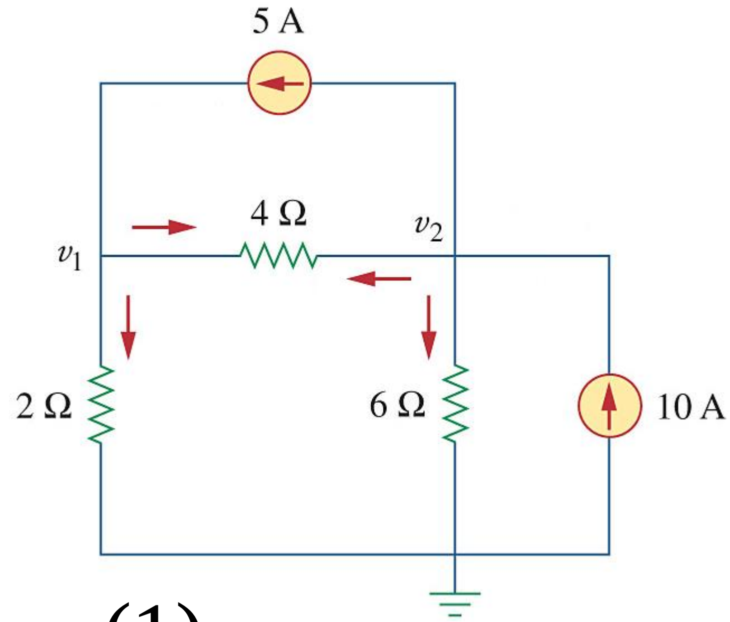


Figure 3.3(b)

At node 2,

$$\frac{v_2-v_1}{4}+\frac{v_2}{6}-10+5=0$$

$$-\frac{1}{4}v_1+\left(\frac{1}{4}+\frac{1}{6}\right)v_2=10-5 \qquad (2)$$

$\boxed{\text{Step 4}}$

$$\begin{cases} \left(\dfrac{1}{4}+\dfrac{1}{2}\right)v_1 - \dfrac{1}{4}v_2 = 5 \\ -\dfrac{1}{4}v_1 + \left(\dfrac{1}{4}+\dfrac{1}{6}\right)v_2 = 10-5 \end{cases}$$

$$\begin{bmatrix} \dfrac{1}{4}+\dfrac{1}{2} & -\dfrac{1}{4} \\ -\dfrac{1}{4} & \dfrac{1}{4}+\dfrac{1}{6} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10-5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

Use Cramer's rule,

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 3 \times 5 - (-1) \times (-3) = 12$$

$$\Delta_1 = \begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix} = 20 \times 5 - (-1) \times 60 = 160$$

$$\Delta_2 = \begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix} = 3\times 60 - 20\times(-3) = 240$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{160}{12} = \frac{40}{3} \approx 13.33 \text{ (V)}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{240}{12} = 20 \text{ (V)}$$

(If we need element voltages and/or currents, we can easily calculate them from the values of the nodal voltages.)

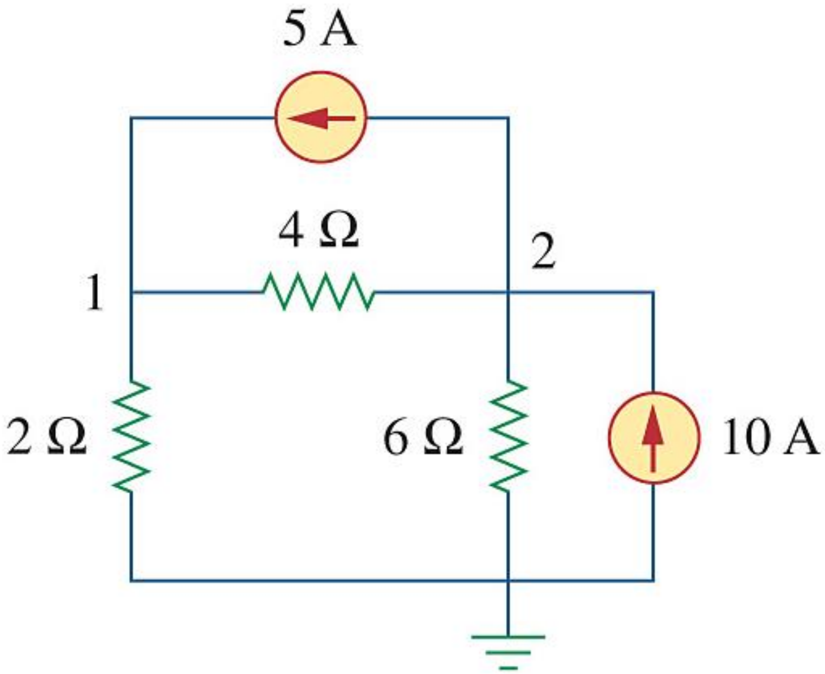
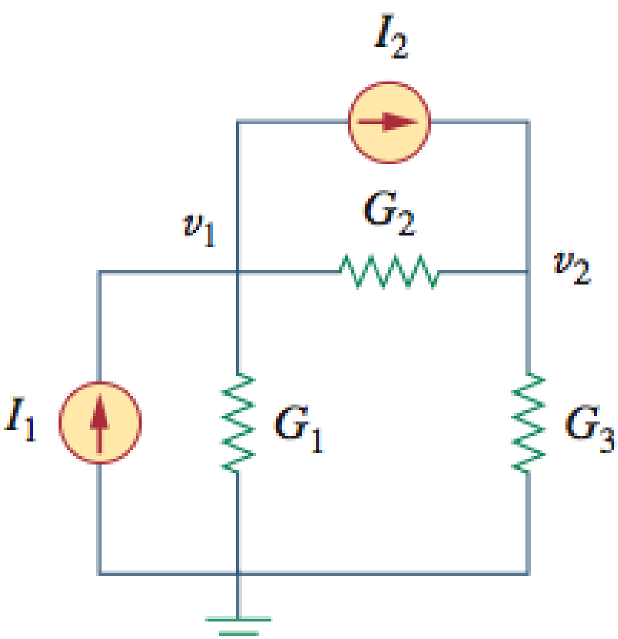


Figure 3.3 (a)

$$\begin{cases} \left(\dfrac{1}{4}+\dfrac{1}{2}\right)v_1 - \dfrac{1}{4}v_2 = 5 \\ -\dfrac{1}{4}v_1 + \left(\dfrac{1}{4}+\dfrac{1}{6}\right)v_2 = 10-5 \end{cases}$$

$$\begin{bmatrix} \dfrac{1}{4}+\dfrac{1}{2} & -\dfrac{1}{4} \\ -\dfrac{1}{4} & \dfrac{1}{4}+\dfrac{1}{6} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10-5 \end{bmatrix}$$



$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Nodal Analysis by Inspection (Section 3.6)

If a circuit with only independent current ? sources has N nonreference nodes, the node-voltage equations can be written as

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

or simply

$\mathbf{Gv} = \mathbf{i}$

where

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$.

v_k = Unknown voltage at node k

i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive

$\mathbf{G}$ is called the conductance matrix; $\mathbf{v}$ is the output vector; and $\mathbf{i}$ is the input vector.

Example 3.8 Write the node-voltage matrix equations for the circuit in Fig. 3.27 by inspection.

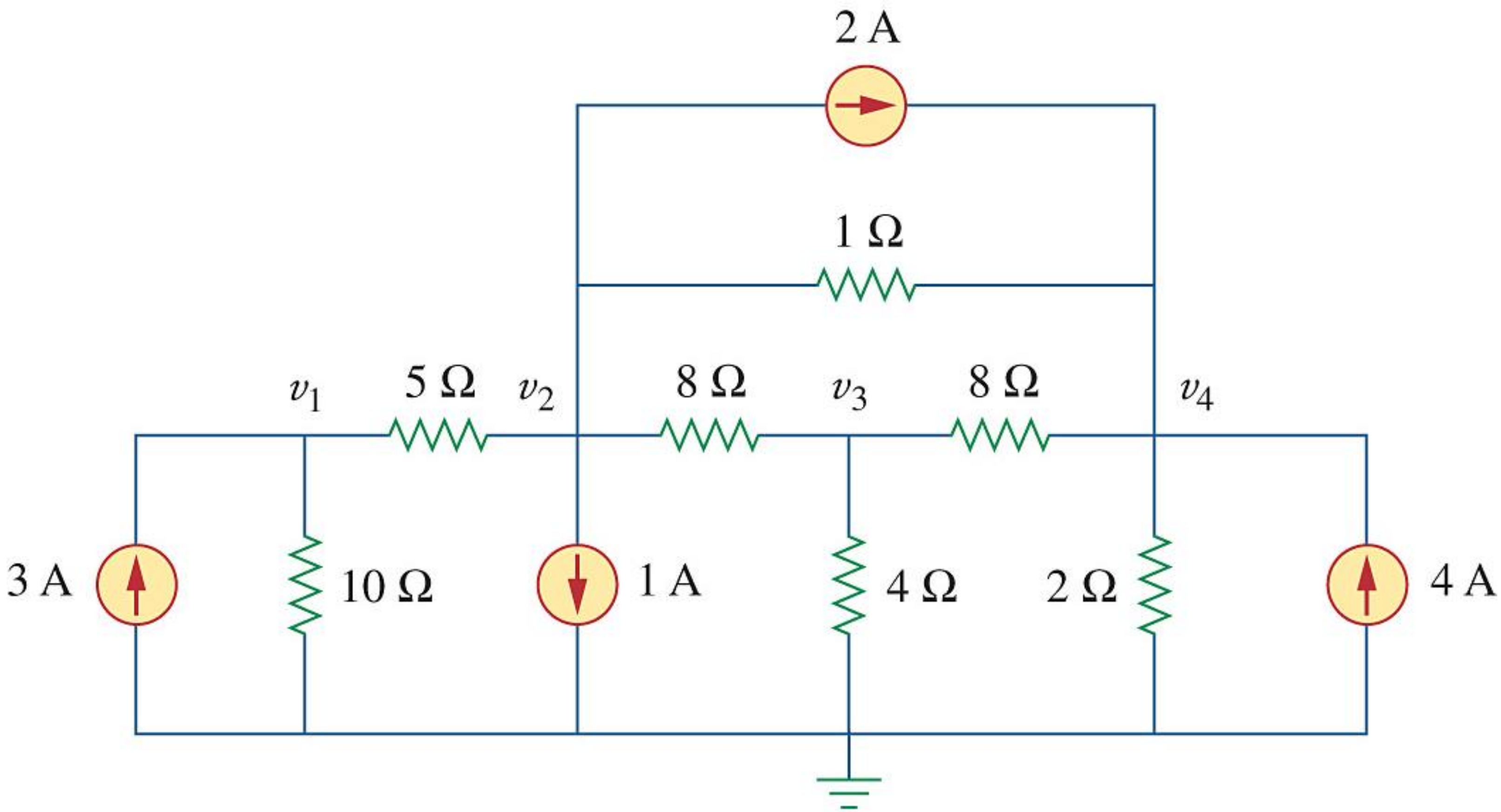


Figure 3.27

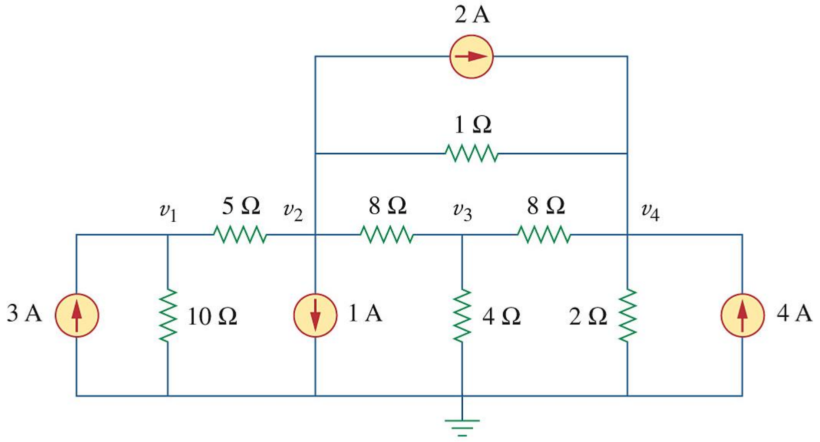


Figure 3.27

Solution :

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{10} & -\frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{8}+1 & -\frac{1}{8} & -1 \\ 0 & -\frac{1}{8} & \frac{1}{4}+\frac{1}{8}+\frac{1}{8} & -\frac{1}{8} \\ 0 & -1 & -\frac{1}{8} & 1+\frac{1}{8}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -1-2 \\ 0 \\ 2+4 \end{bmatrix}$$

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

which can be solved using MATLAB.

```
>> G=[0.3 -0.2 0 0; -0.2 1.325 -0.125 -1; 0 -0.125 0.5 -0.125; 0 -1 -0.125 1.625];
>>  I=[3 -3 0 6]';
>> V=inv(G)*I

V =

   13.8966
    5.8449
    3.3479
    7.5467
```

$$GV=I$$
$$G^{-1}GV=G^{-1}I$$
$$V=G^{-1}I$$

Example 3.2 Determine the voltages at the nodes in Fig. 3.5(a).

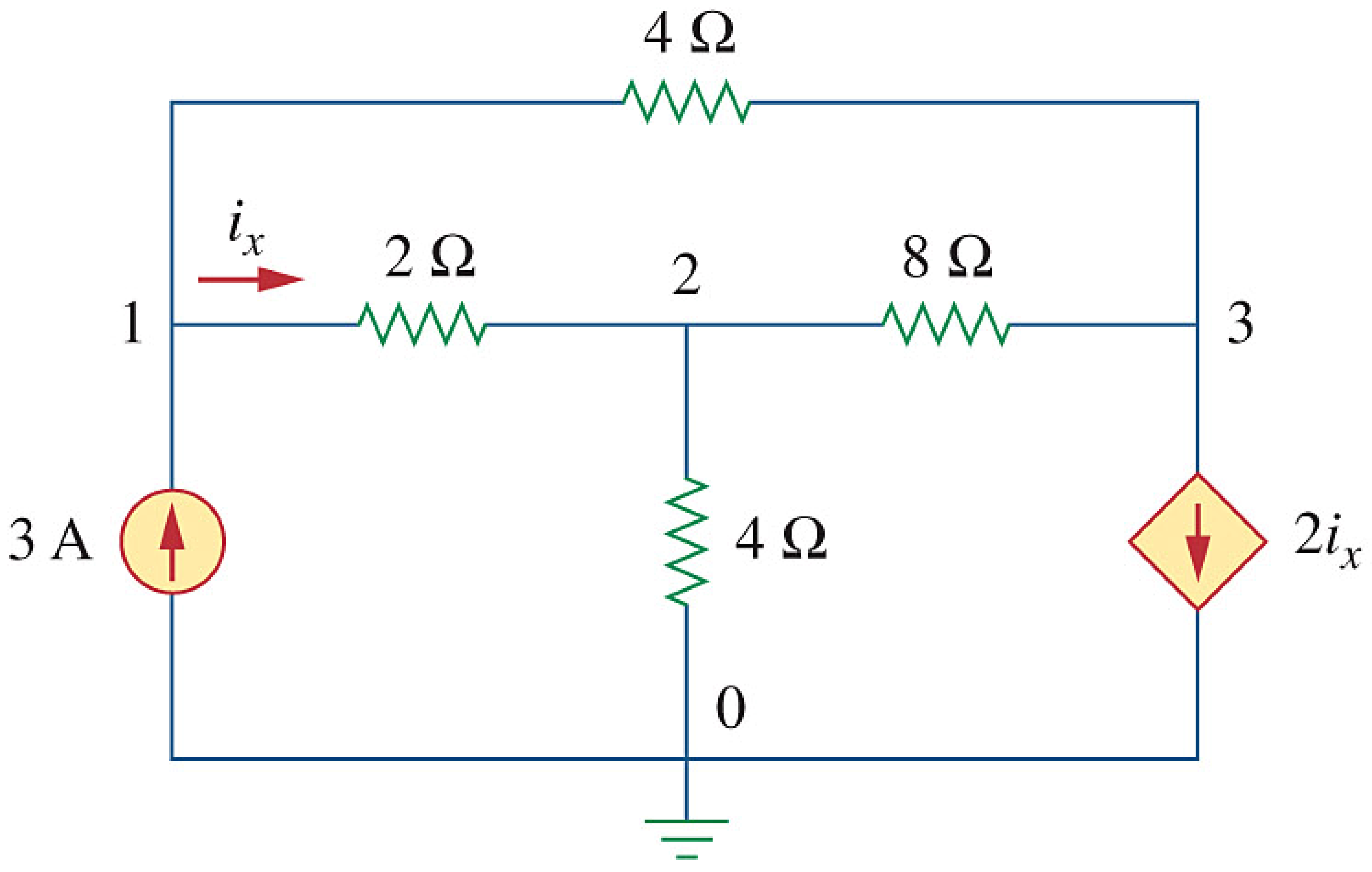


Figure 3.5(a)

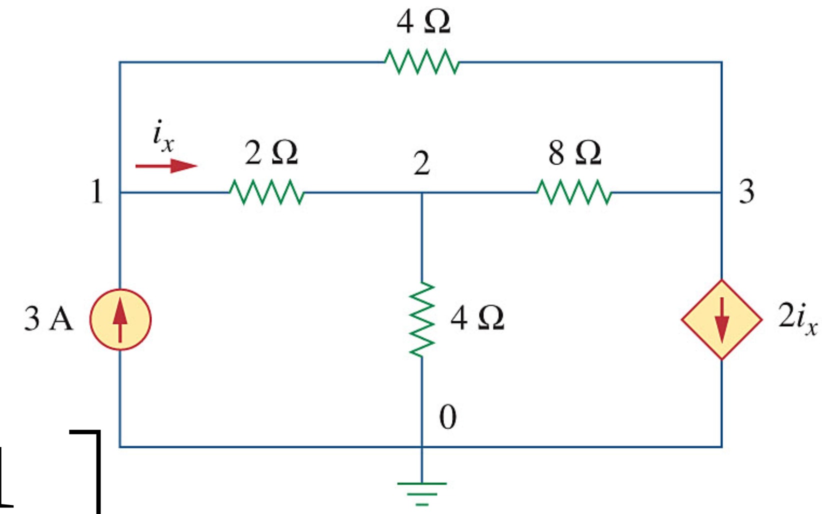


Figure 3.5(a)

Solution :

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{4}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{4} & -\frac{1}{8} & \frac{1}{4}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2i_x \end{bmatrix}$$

$$i_x = \frac{v_1 - v_2}{2}$$

Express i_x in terms of node voltages (i.e., the variables of input vector)

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{4}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{4} & -\frac{1}{8} & \frac{1}{4}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -v_1+v_2 \end{bmatrix}$$

$-v_1/4-v_2/8+(1/4+1/8)v_3 = -v_1+v_2$

$(-1/4+1)v_1+(-1/8-1)v_2+(1/4+1/8)v_3 = 0$

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{4}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{4}+1 & -\frac{1}{8}-1 & \frac{1}{4}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 6 & -9 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = 3\times 7\times 1 + (-4)\times(-3)\times(-1)$$

$$+2\times(-1)\times(-2) - (-1)\times 7\times 2 - (-1)\times(-3)\times 3$$

$$-1\times(-4)\times(-2) = 21 - 12 + 4 + 14 - 9 - 8 = 10$$

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix} = 12\times 7\times 1 + 0\times(-3)\times(-1)$$

$$+0\times(-1)\times(-2) - (-1)\times 7\times 0 - (-1)\times(-3)\times 12$$

$$-1\times(0)\times(-2) = 84 - 36 = 48$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix} = 3\times 0\times 1 + (-4)\times 0\times(-1)$$

$$+2\times(-1)\times 12 - (-1)\times 0\times 2 - (-1)\times 0\times 3$$

$$-1\times(-4)\times 12 = -24 + 48 = 24$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix} = 3\times 7\times 0 + (-4)\times(-3)\times 12$$

$$+2\times 0\times(-2) - 12\times 7\times 2 - 0\times(-3)\times 3$$

$$-0\times(-4)\times(-2) = 144 - 168 = -24$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8\ \text{(V)}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4\ \text{(V)}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4\ \text{(V)}$$

Method 2 Use MATLAB to solve the simultaneous equations.

```
>> G=[3 -2 -1; -4 7 -1; 2 -3 1];
>> I=[12 0 0]';
>> V=inv(G)*I

V =

    4.8000
    2.4000
   -2.4000
```

3.3 Nodal Analysis with Voltage Sources

Applying nodal analysis to circuits containing voltage sources (dependent or independent) is easier than what we encountered in section 3.2.

1. If a voltage source is connected between the reference node and a nonreference node, the node voltage is known. In Fig. 3.7, e.g., $v_1 = 10$ V

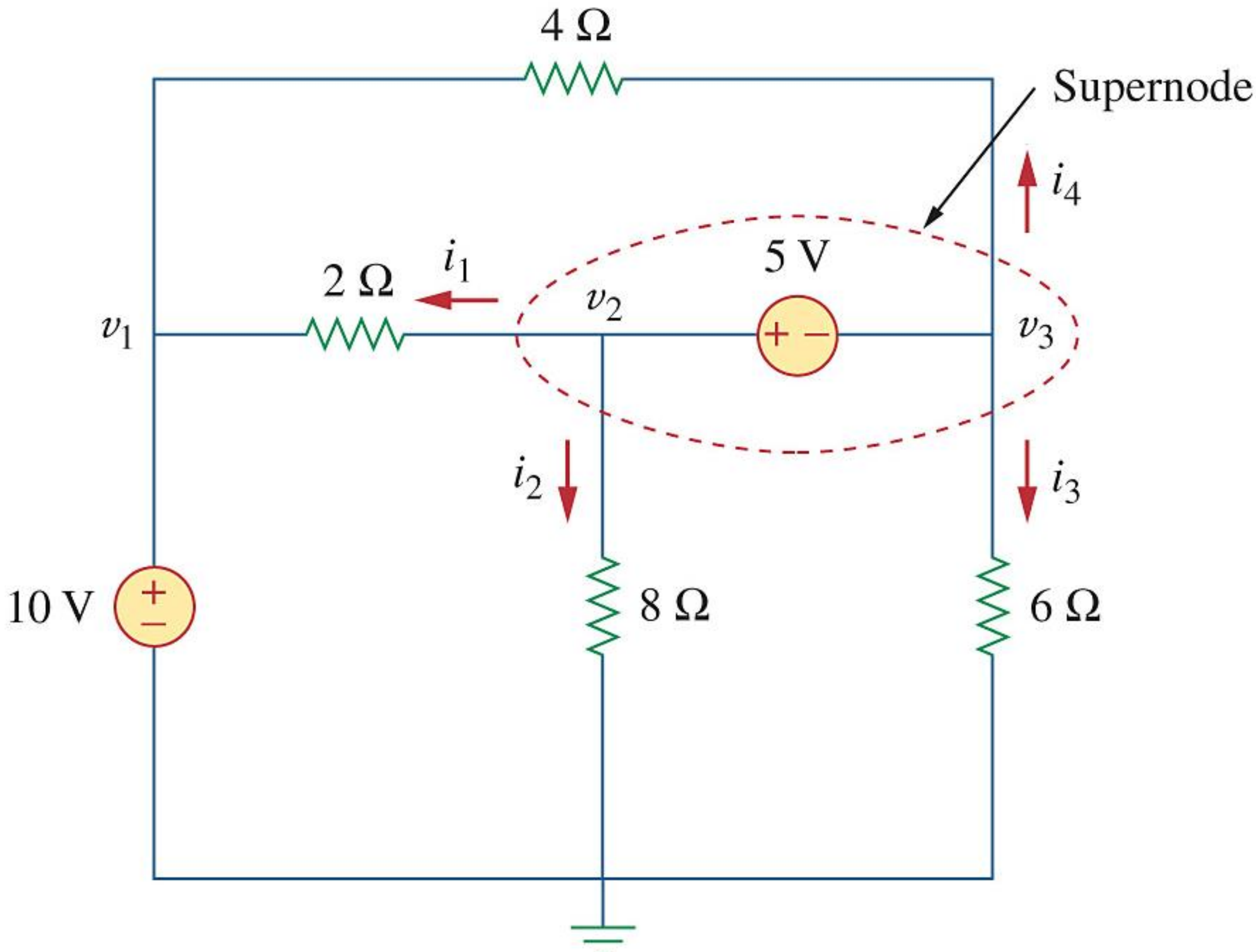


Figure 3.7

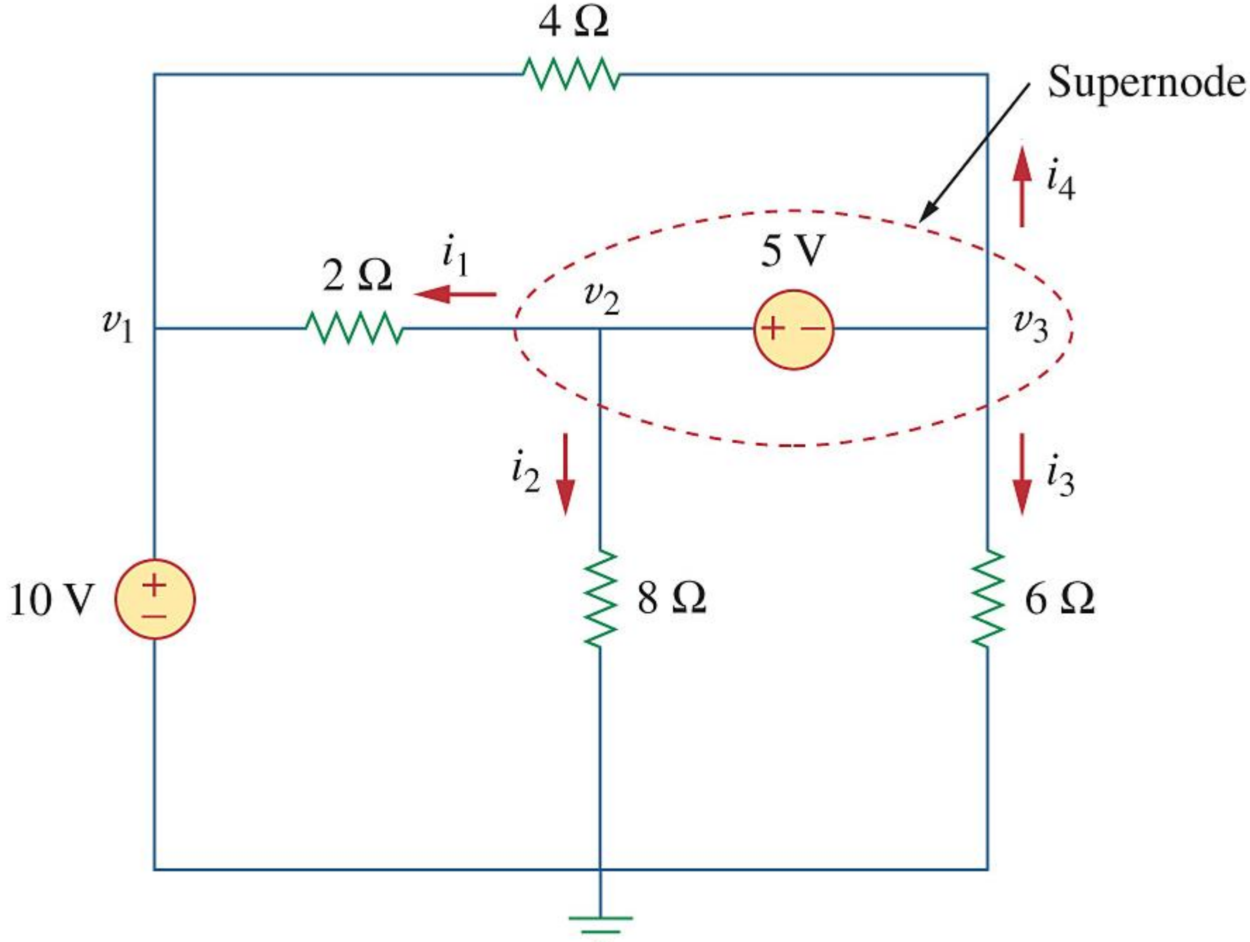
2. If a voltage source is connected between two nonreference nodes, the two nonreference nodes form a *generalized node* or *supernode*. The supernode provides a constraint on the two node voltages, in Fig. 3.7, e.g.,

$$v_2 - v_3 = 5$$

A supernode is formed by enclosing a voltage source between two nonreference nodes and any elements connected in parallel with it. A supernode has no voltage of its own. We apply KCL to the supernode (closed boundary),

$$\frac{v_2 - v_1}{2} + \frac{v_2}{8} + \frac{v_3}{6} + \frac{v_3 - v_1}{4} = 0 \quad \text{or}$$

$$-\left(\frac{1}{2} + \frac{1}{4}\right)v_1 + \left(\frac{1}{2} + \frac{1}{8}\right)v_2 + \left(\frac{1}{6} + \frac{1}{4}\right)v_3 = 0$$



$$\frac{v_2 - v_1}{2} + \frac{v_2}{8} + \frac{v_3}{6} + \frac{v_3 - v_1}{4} = 0 \quad \text{or}$$

$$-\left(\frac{1}{2} + \frac{1}{4}\right)v_1 + \left(\frac{1}{2} + \frac{1}{8}\right)v_2 + \left(\frac{1}{6} + \frac{1}{4}\right)v_3 = 0$$

Example 3.3 For the circuit shown in Fig. 3.9, find the node voltages.

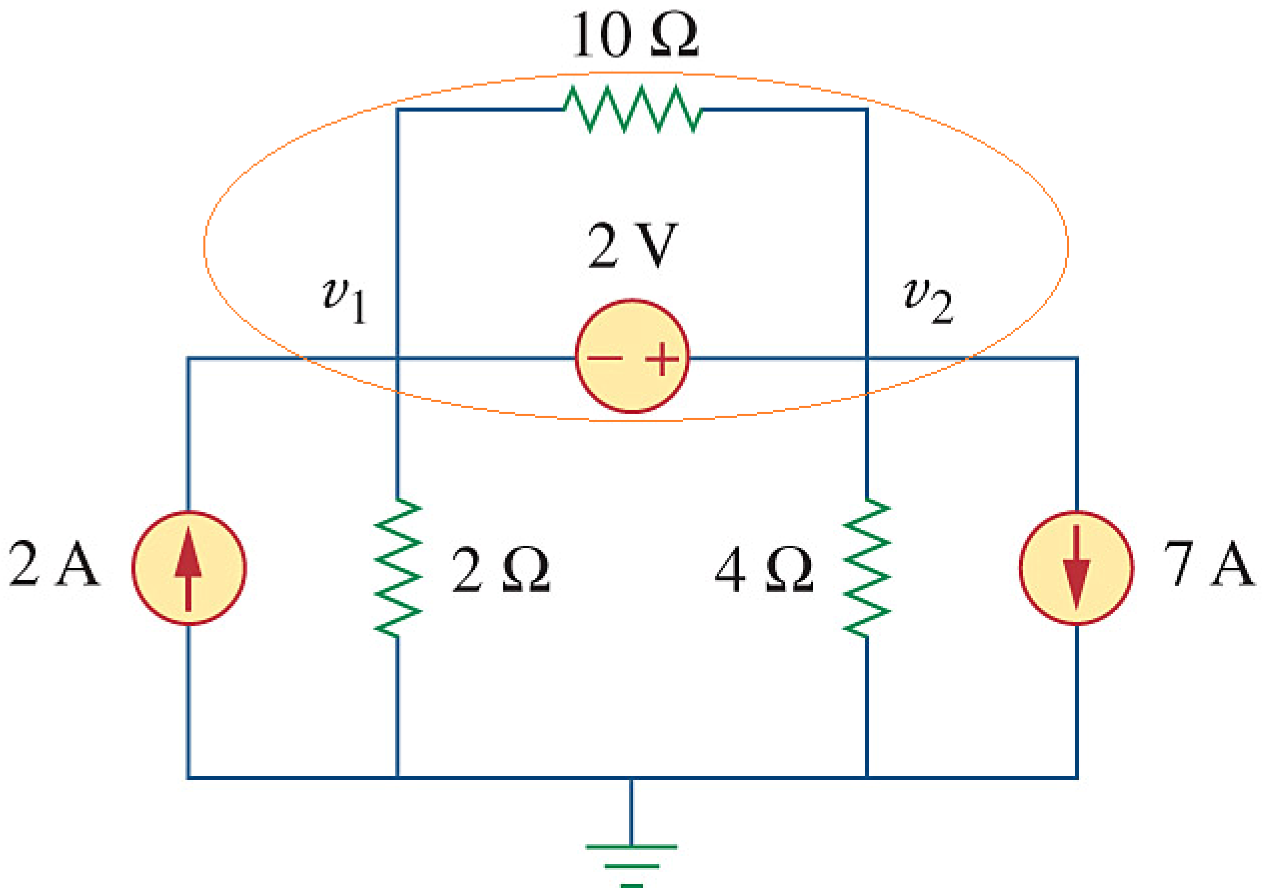


Figure 3.9

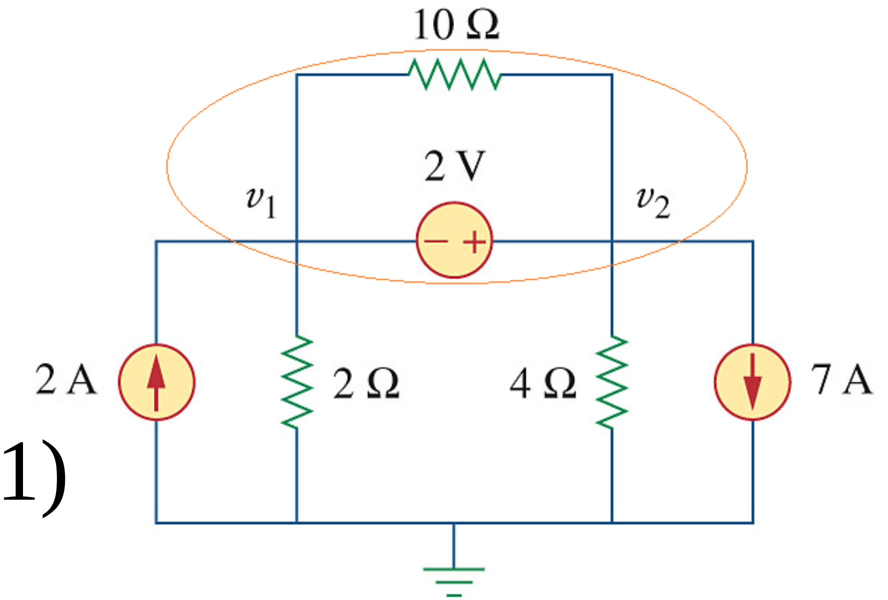


Figure 3.9

Solution :

$$v_1 - v_2 = -2 \qquad (1)$$

$$\frac{v_1}{2} + \frac{v_2}{4} = 2 - 7 \qquad (2)$$

$$\begin{cases} v_1 - v_2 = -2 \\ 2v_1 + v_2 = -20 \end{cases}$$

$$3v_1 = -22 \Rightarrow v_1 = -\frac{22}{3} \approx -7.33 \text{ (V)}$$

$$v_2 = v_1 + 2 = -5.33 \text{ (V)}$$

Practice Problem 3.3 Find v and i in the circuit of Fig. 3.11.

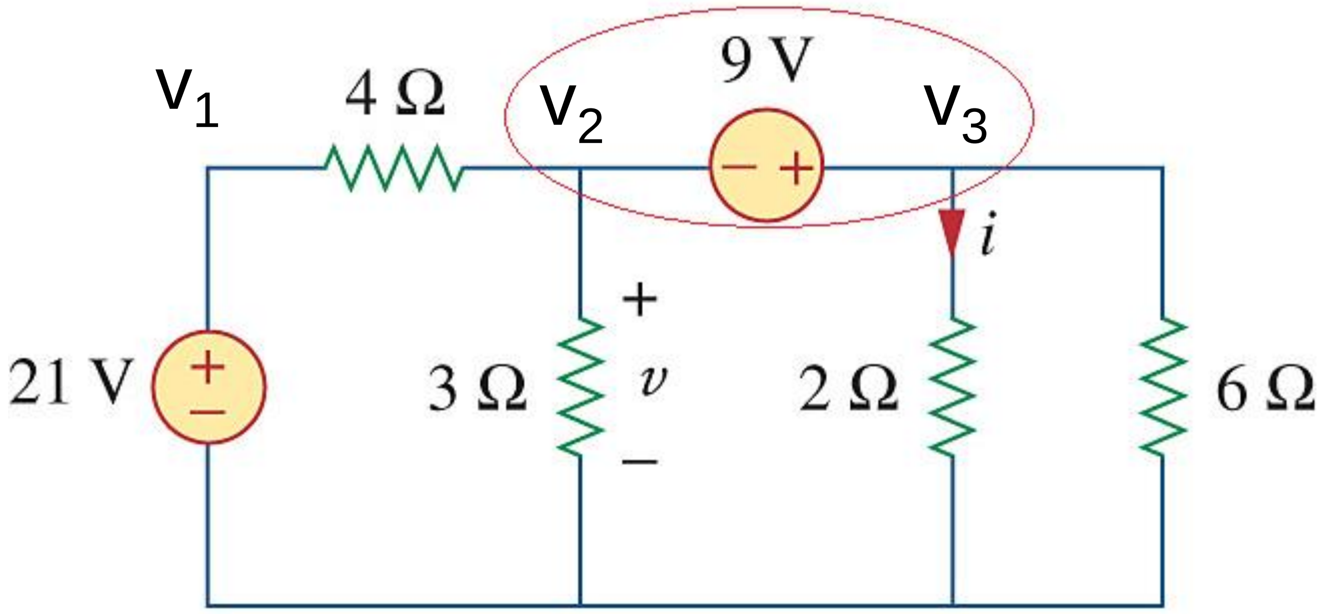


Figure 3.11

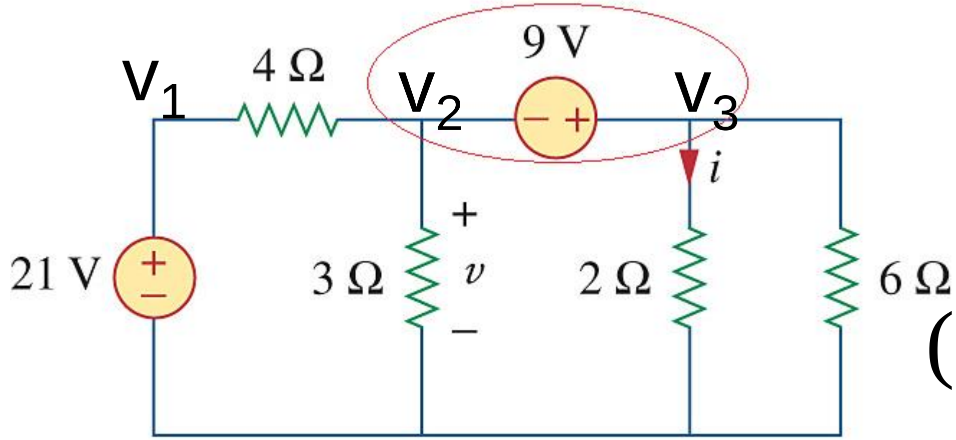


Figure 3.11

Solution :

$$v_1 = 21 \qquad (1)$$

$$v_2 - v_3 = -9 \qquad (2)$$

$$\frac{v_2 - v_1}{4} + \frac{v_2}{3} + \frac{v_3}{2} + \frac{v_3}{6} = 0$$

$$-3v_1 + 7v_2 + 8v_3 = 0 \qquad (3)$$

$$v_2 = -\frac{3}{5} = -0.6 \text{ (V)},\ v_3 = \frac{42}{5} = 8.4 \text{ (V)}$$

$$v = v_2 = -0.6 \text{ V},\ i = \frac{v_3}{2} = 4.2 \text{ (A)}$$

Example 3.4 Find the node voltages in the circuit of Fig. 3.12.

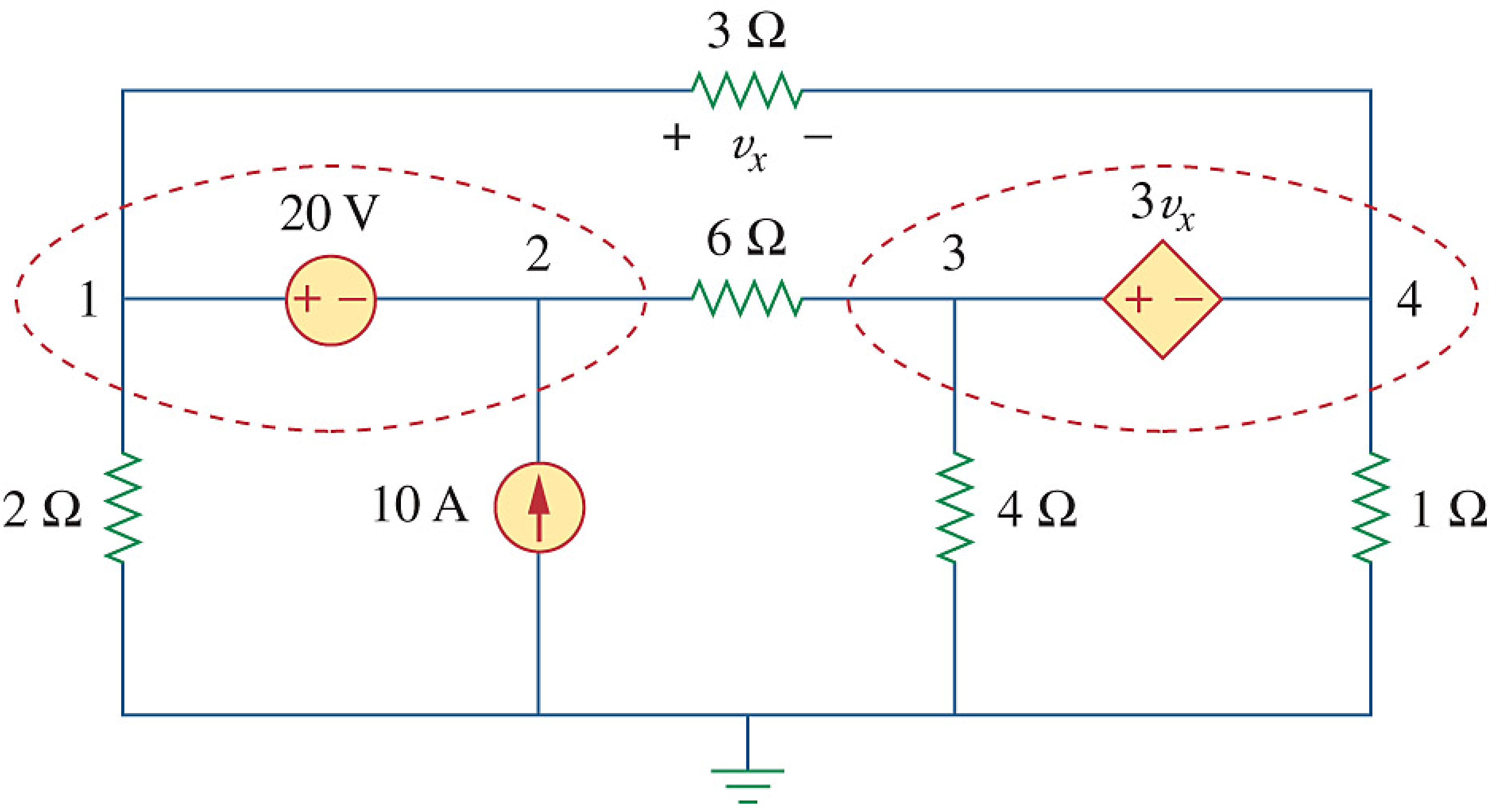


Figure 3.12

Solution :

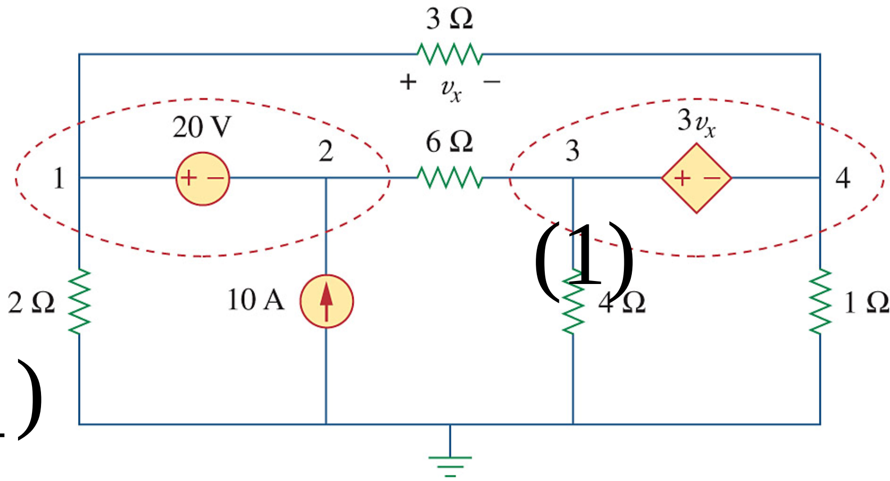


Figure 3.12

$$v_1 - v_2 = 20 \qquad (1)$$

$$v_3 - v_4 = 3v_x = 3(v_1 - v_4)$$

$$3v_1 - v_3 - 2v_4 = 0 \qquad (2)$$

$$\frac{v_1}{2} + \frac{v_1 - v_4}{3} + \frac{v_2 - v_3}{6} = 10$$

$$5v_1 + v_2 - v_3 - 2v_4 = 60 \qquad (3)$$

$$\frac{v_3 - v_2}{6} + \frac{v_3}{4} + \frac{v_4}{1} + \frac{v_4 - v_1}{3} = 0$$

$$-4v_1 - 2v_2 + 5v_3 + 16v_4 = 0 \qquad (4)$$

```
>> G=[1 -1 0 0; 3 0 -1 -2; 5 1 -1 -2; -4 -2 5 16];
>> I=[20 0 60 0]';
>> V=inv(G)*I

V =

   26.6667
    6.6667
  173.3333
  -46.6667
```

- Introduction to Nodal Analysis Part 1

http://www.youtube.com/watch?v=emat52YR1nQ

- Introduction to Nodal Analysis Part 2

http://www.youtube.com/watch?v=db89gfmlhKM

- Nodal Analysis Example-Independent Voltage Source (Harder)

http://www.youtube.com/watch?v=XdWJ_h463jQ

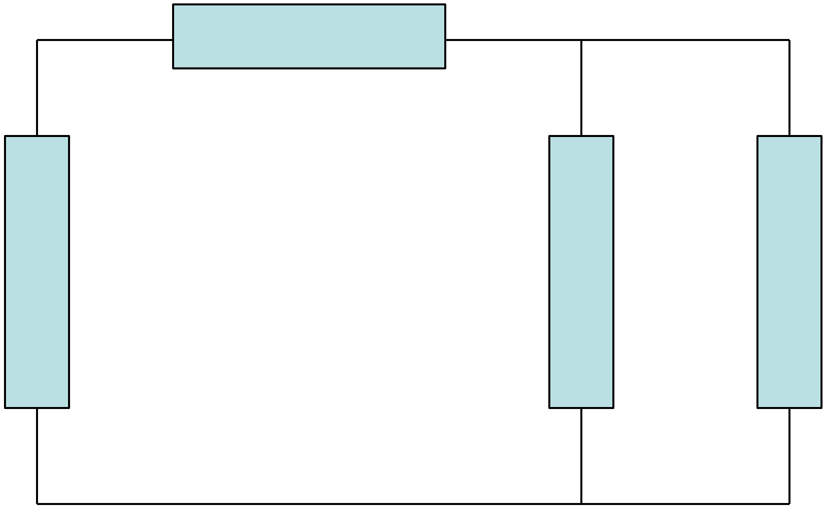
- Chapter 3: Supernode

http://www.youtube.com/watch?v=NA_zlZTDiKU

3.4 Mesh Analysis

- Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. **Using mesh currents instead of element currents as circuit variables** is convenient and reduces the number of equations that must be solved simultaneously.

3 variables vs. 2 variables



- Nodal analysis applies KCL to find unknown voltages, while mesh analysis applies KVL to find unknown currents.
- Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is *planar*. A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is *nonplanar*.

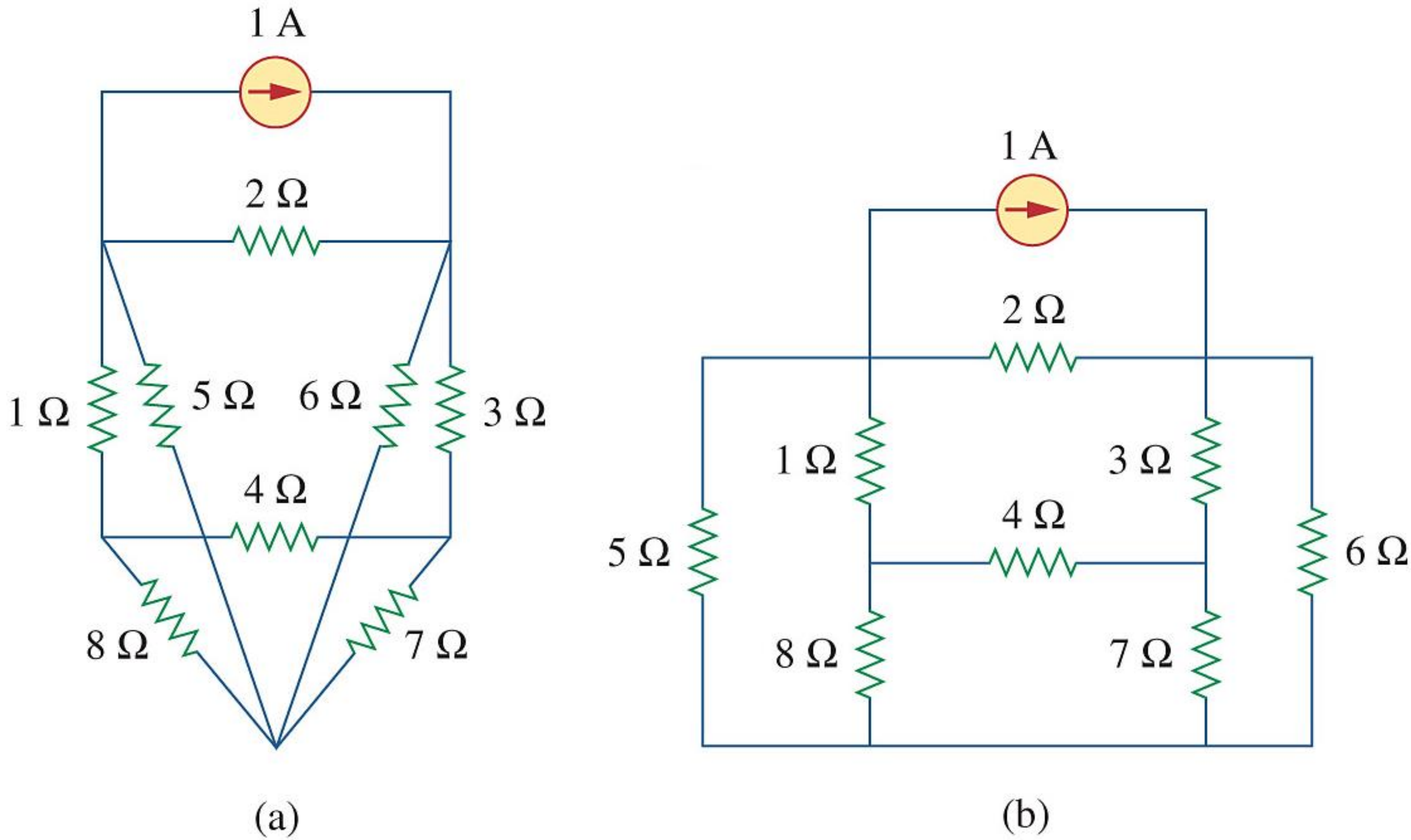


Figure 3.15 (a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

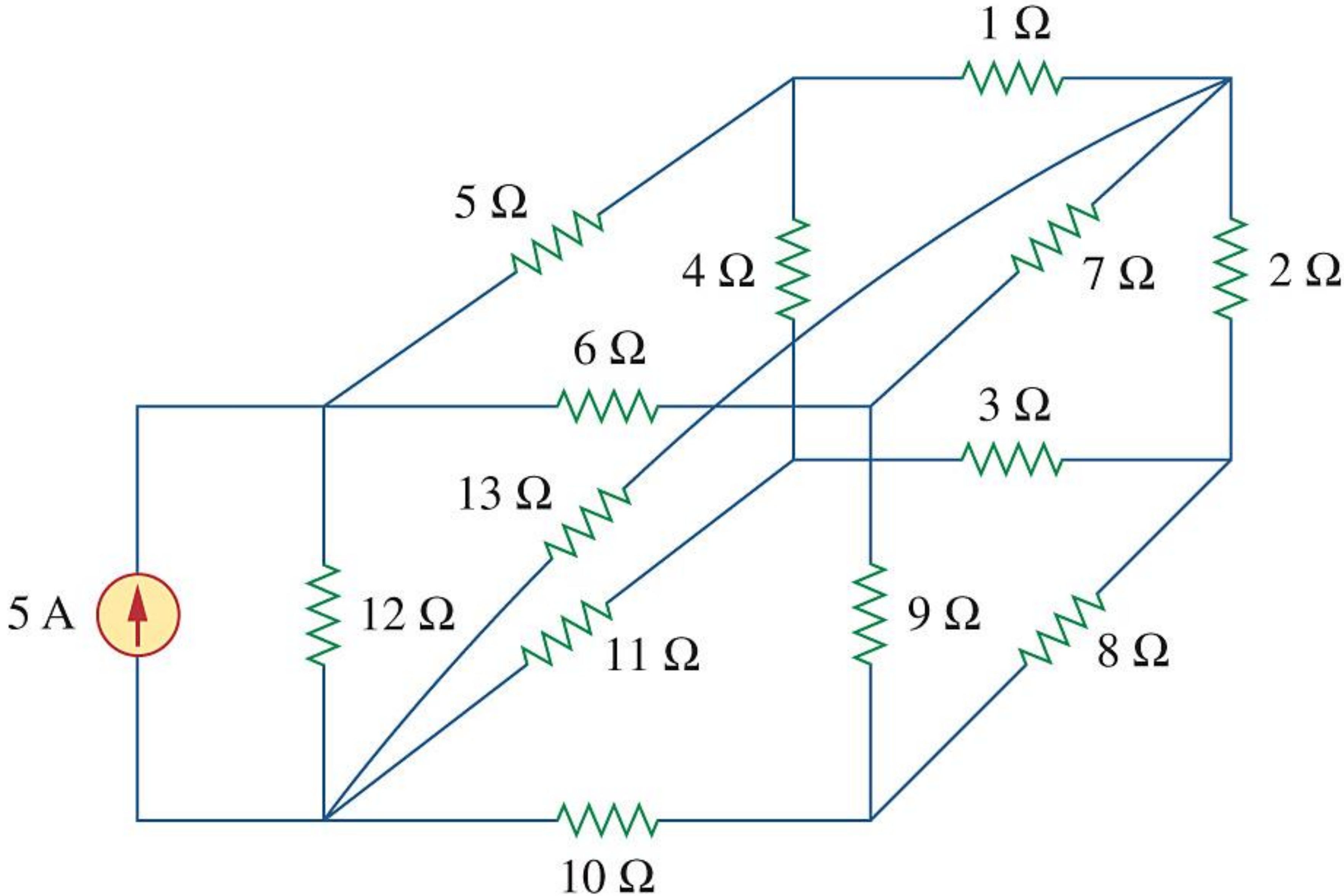


Figure 3.16 A nonplanar circuit.

Definitions on Independent loop and Mesh

- Independent loop: *A loop is said to be independent if it contains at least one branch which is* ***not a part of any other independent loop****. Independent loops or paths result in independent sets of equations.*

- Mesh: *A mesh is a loop that does not contain any other loop within it. (Smallest loops)*

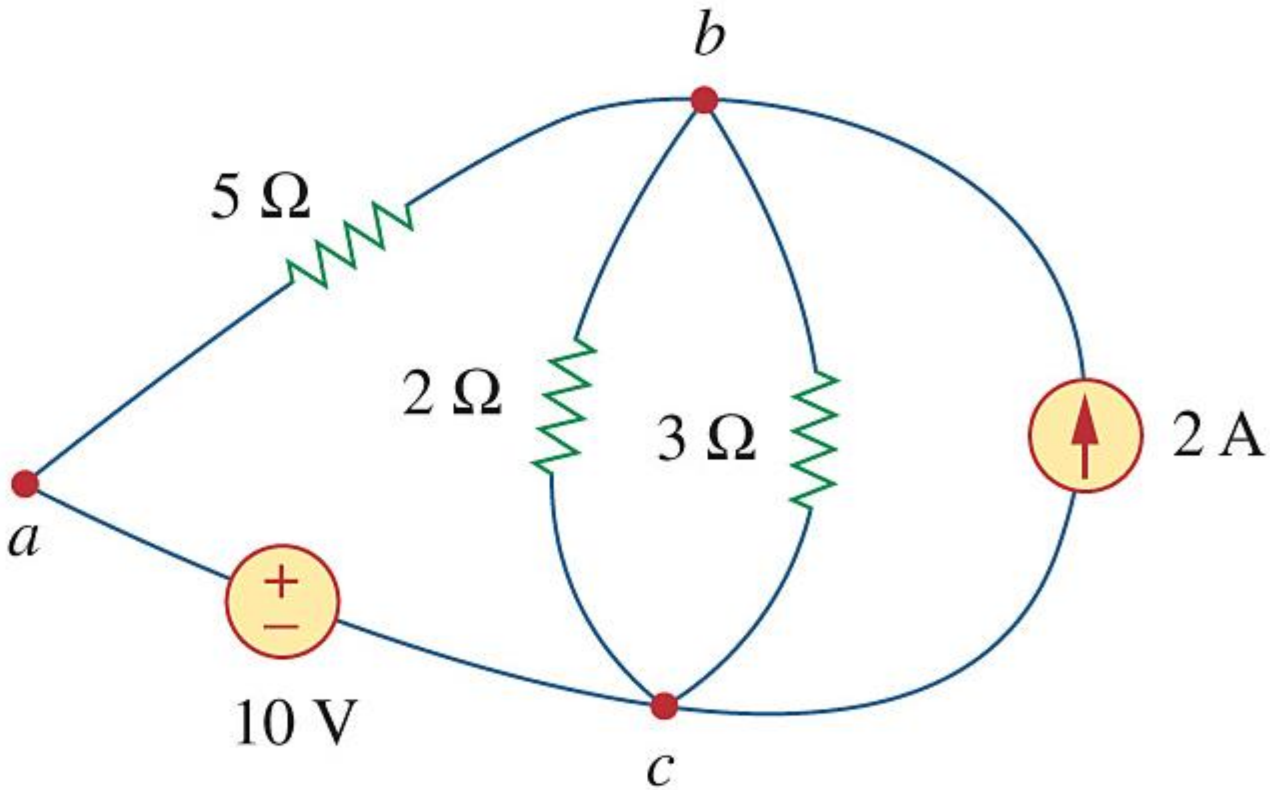


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

- # of meshs = # of **max.** independent loops
- Different choices of 3 independent loops

of max. independent loops

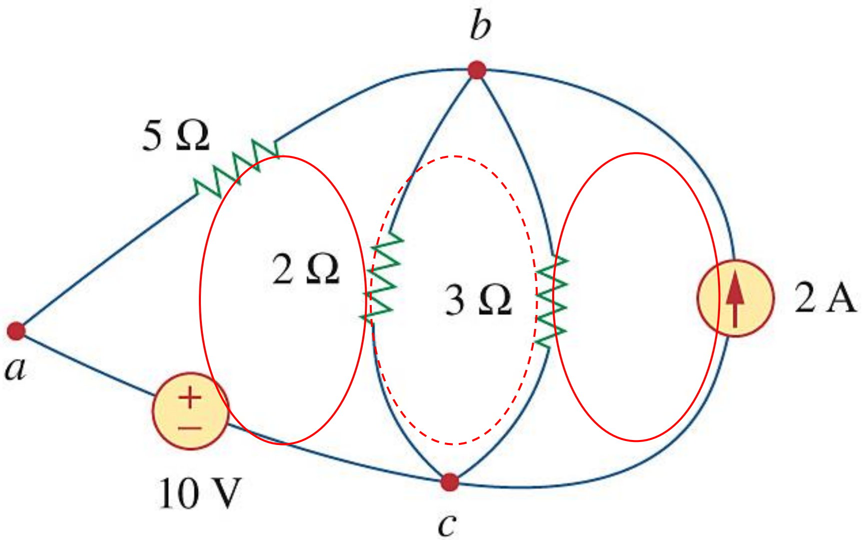


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

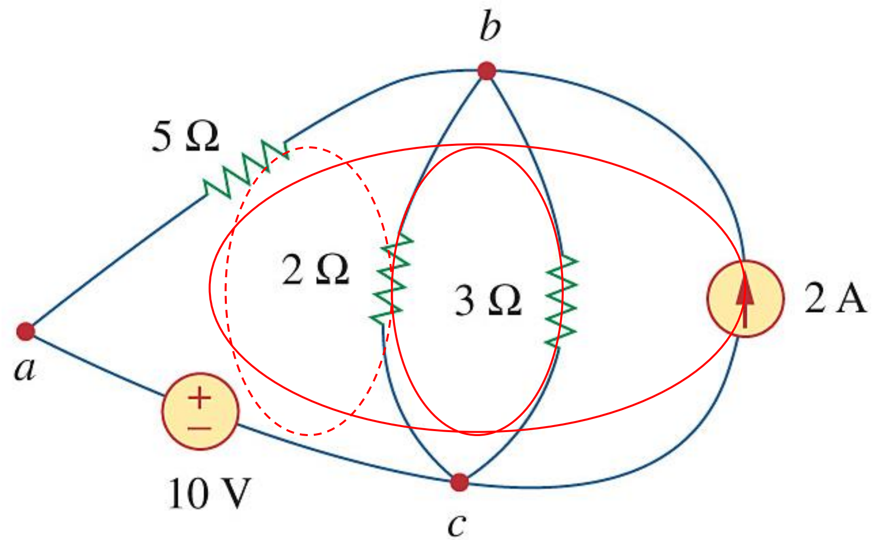


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

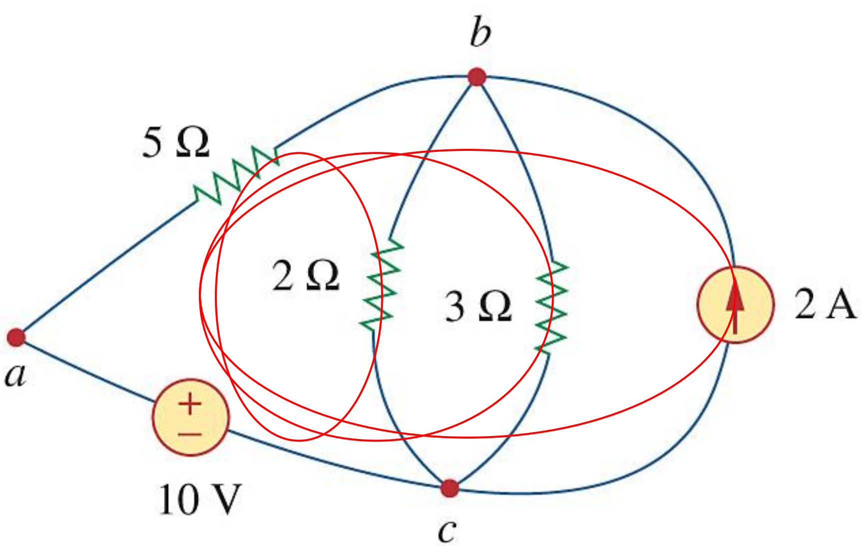


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

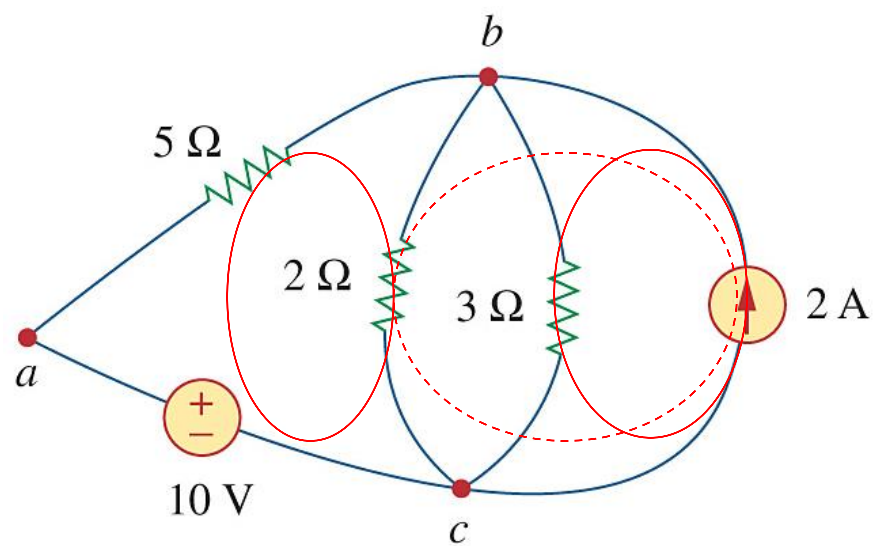


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

Mesh $\neq$ Independent Loop?

- On the text book page of 95 "*According to Eq. (2.12), if a circuit has n nodes, b branches, and l independent loops or meshes, then b = l + n - 1. Hence, l independent simultaneous equations are required to solve the circuit using mesh analysis.*"

Definitions are different
# of them will be same

Example

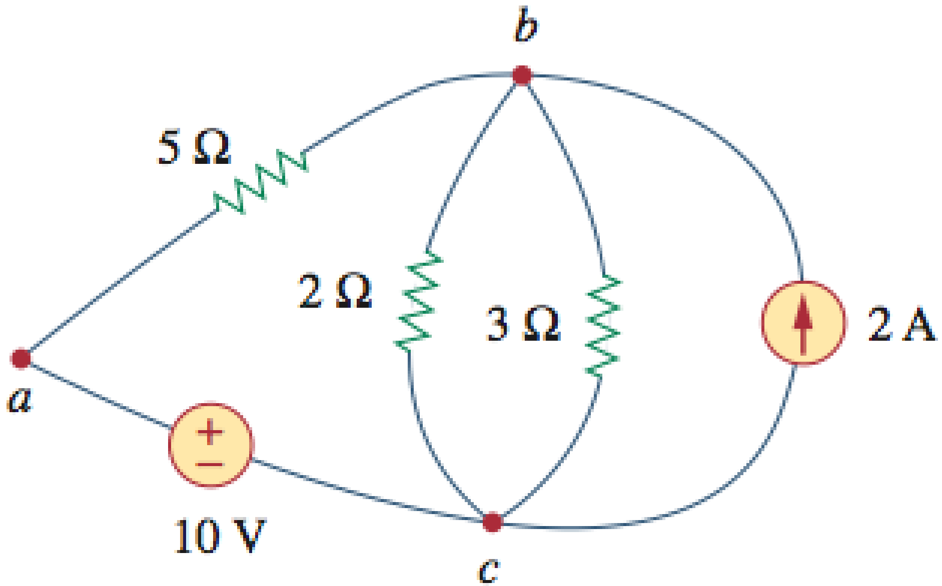


Figure 2.11
The three-node circuit of Fig. 2.10 is redrawn.

$$5 = I + 3 - 1$$
$$I = 3$$

Example

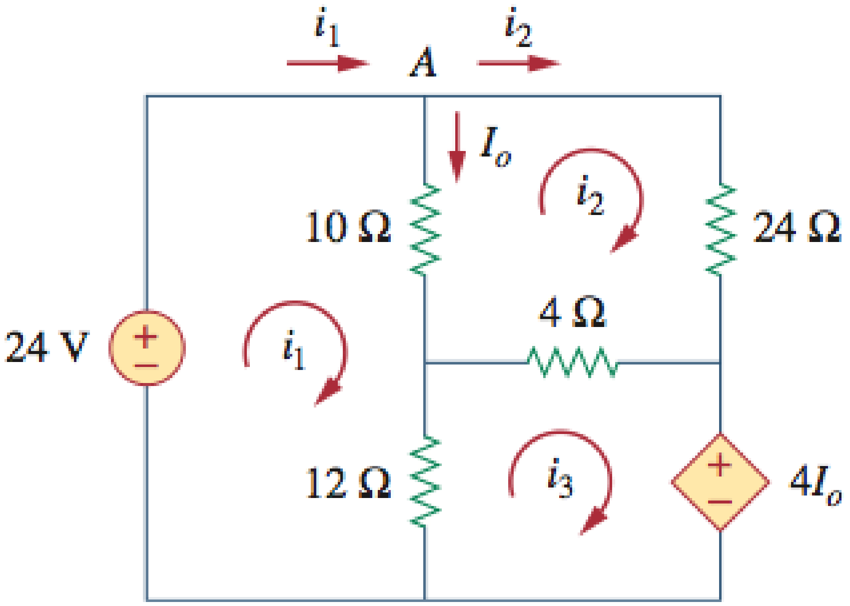


Figure 3.20
For Example 3.6.

$$6 = I + 4 - 1$$
$$I = 3$$

The current through a mesh is known as *mesh current*. In Fig. 3.17, for example, i_1 and i_2 are mesh currents. (compared with branch currents)

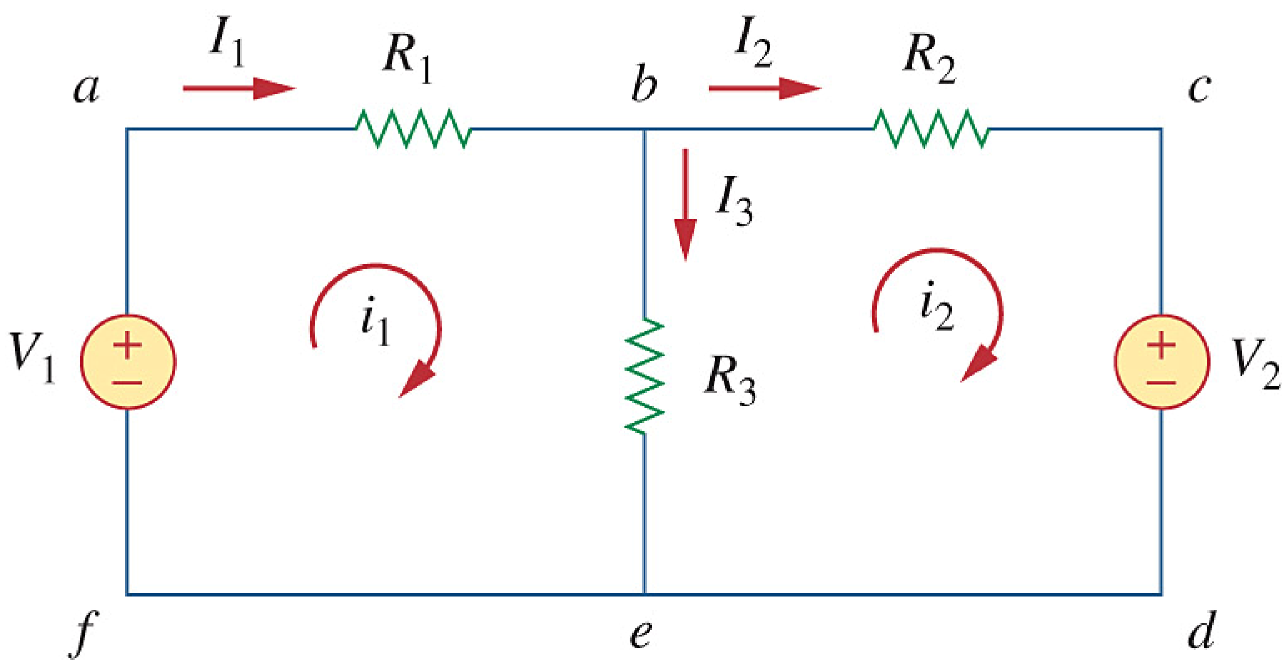


Figure 3.17 A circuit with two meshes. Paths *abefa* and *bcdeb* are meshes.

Steps to Determine Mesh Currents for Circuit with *n* Meshes without Current Sources :

1. Assign mesh currents $i_1, i_2, \cdots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Example 3.5 For the circuit in Fig. 3.18, find the branch currents I_1, I_2, and I_3 using mesh analysis.

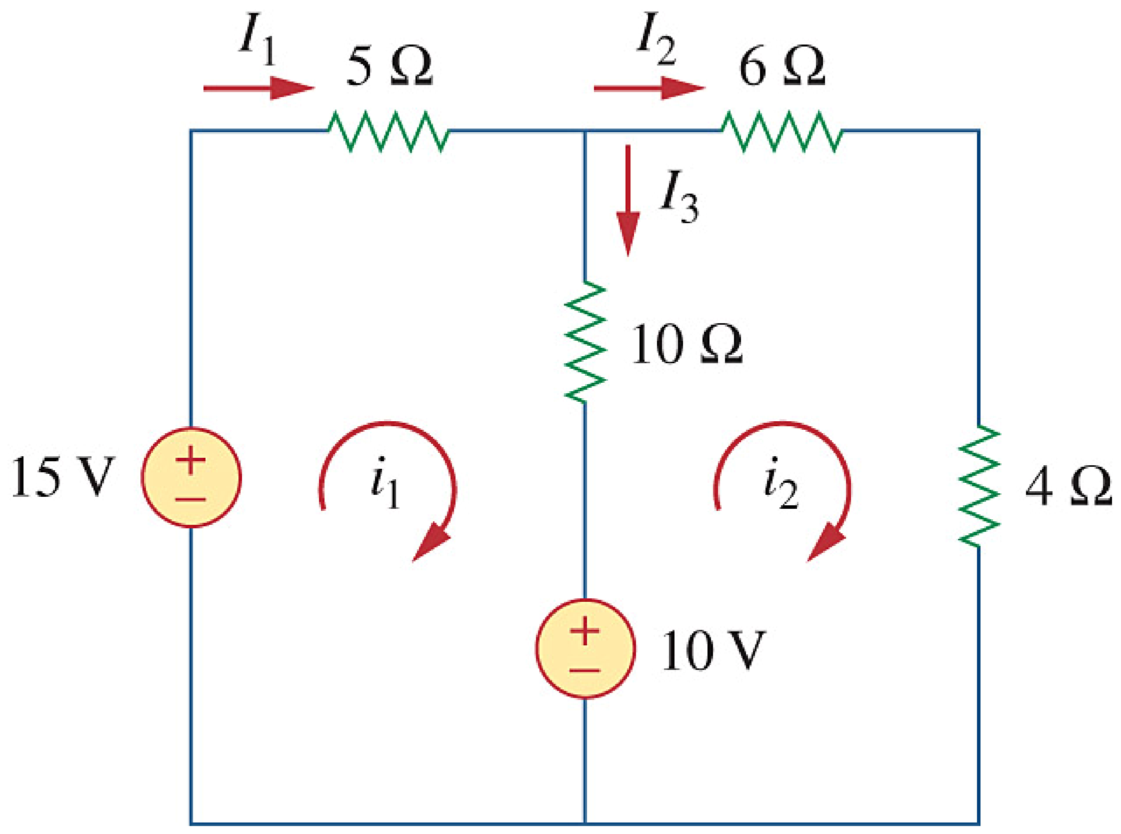


Figure 3.18

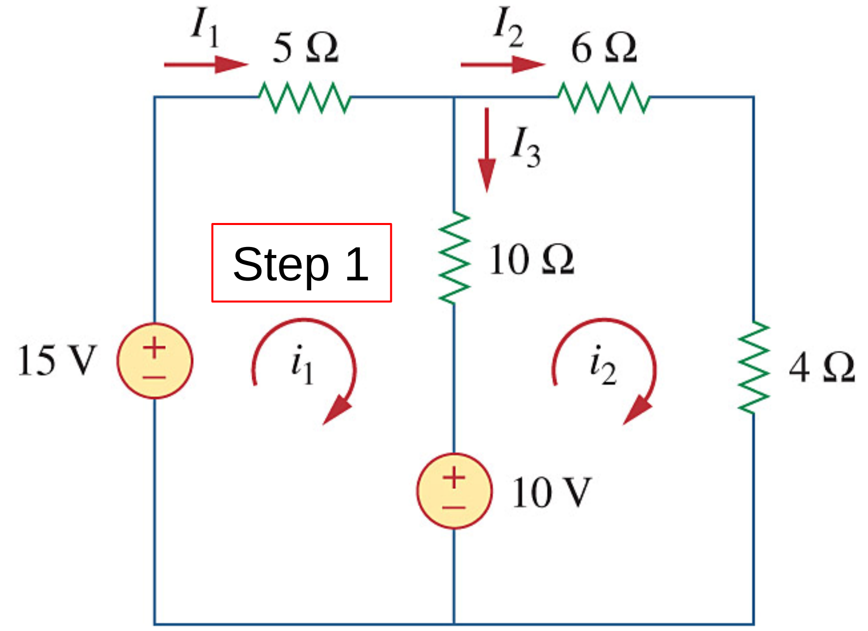


Figure 3.18

Solution :

Step 2: KVL

$$5i_1 + 10(i_1 - i_2) + 10 - 15 = 0$$

$$(5+10)i_1 - 10i_2 = -10 + 15 \qquad (1)$$

$$6i_2 + 4i_2 - 10 - 10(i_1 - i_2) = 0$$

$$-10i_1 + (6+4+10)i_2 = 10 \qquad (2)$$

Step 3

$$\begin{bmatrix} 5+10 & -10 \\ -10 & 6+4+10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -10+15 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 4, \; \Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 4$$

$$\Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 4$$

$$i_1 = \frac{\Delta_1}{\Delta} = 1\,(\mathrm{A})$$

$$i_2 = \frac{\Delta_2}{\Delta} = 1\,(\mathrm{A})$$

$$I_1 = i_1 = 1\ \mathrm{A},\ I_2 = i_2 = 1\ \mathrm{A},\ I_3 = i_1 - i_2 = 0$$

Mesh Analysis by Inspection (Section 3.6)

If a circuit with only independent voltage sources has N meshes, the node-current equations can be written as

$$\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

or simply

$$\mathbf{Ri} = \mathbf{v}$$

where

$R_{kk} =$ Sum of the resistances in mesh k

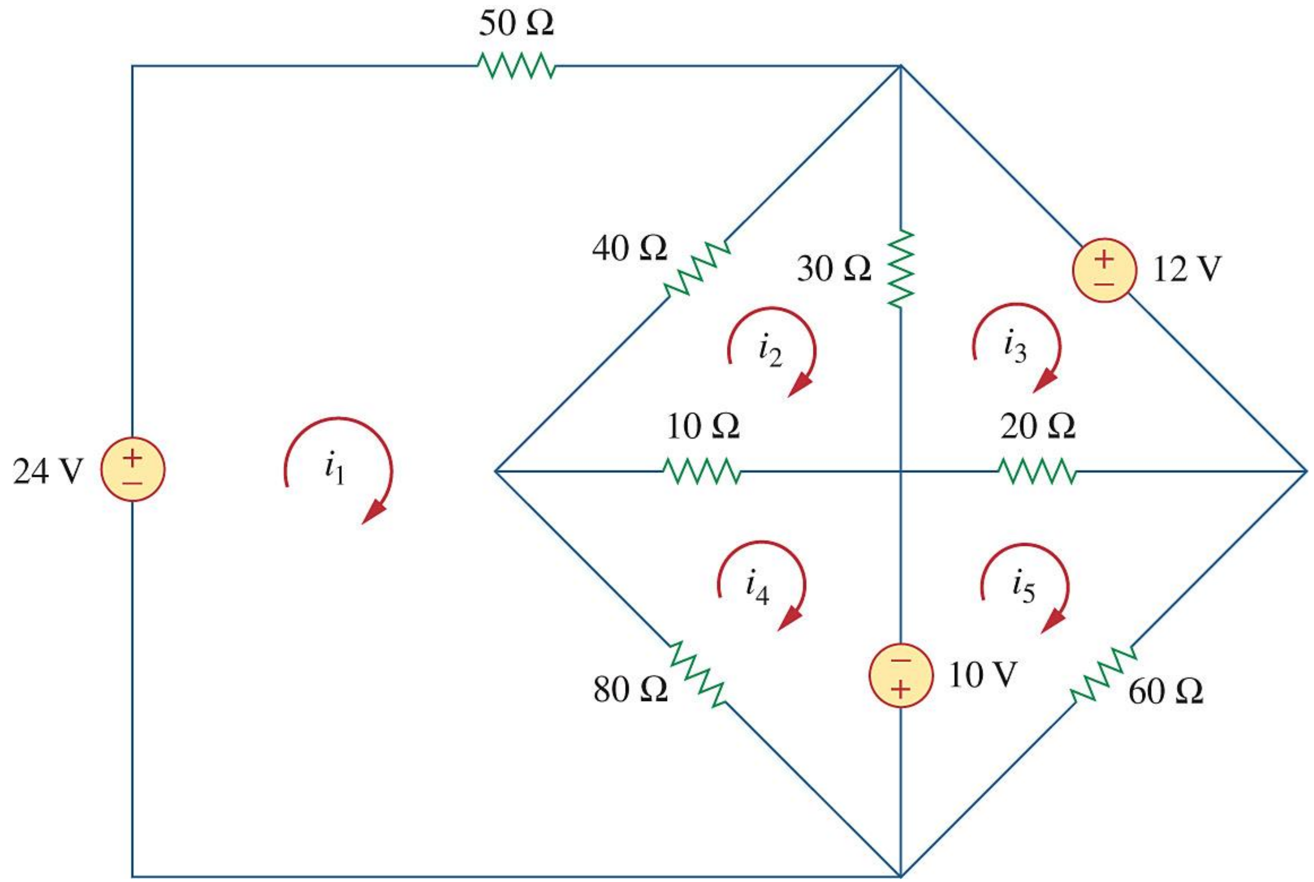
$R_{kj} = R_{jk} =$ Negative of the sum of the resistances in common with meshes k and $j, k \neq j.$

$i_k =$ Unknown current for mesh k in the clockwise direction

$v_k =$ Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

R is called the resistance matrix; **i** is the output vector; and **v** is the input vector.

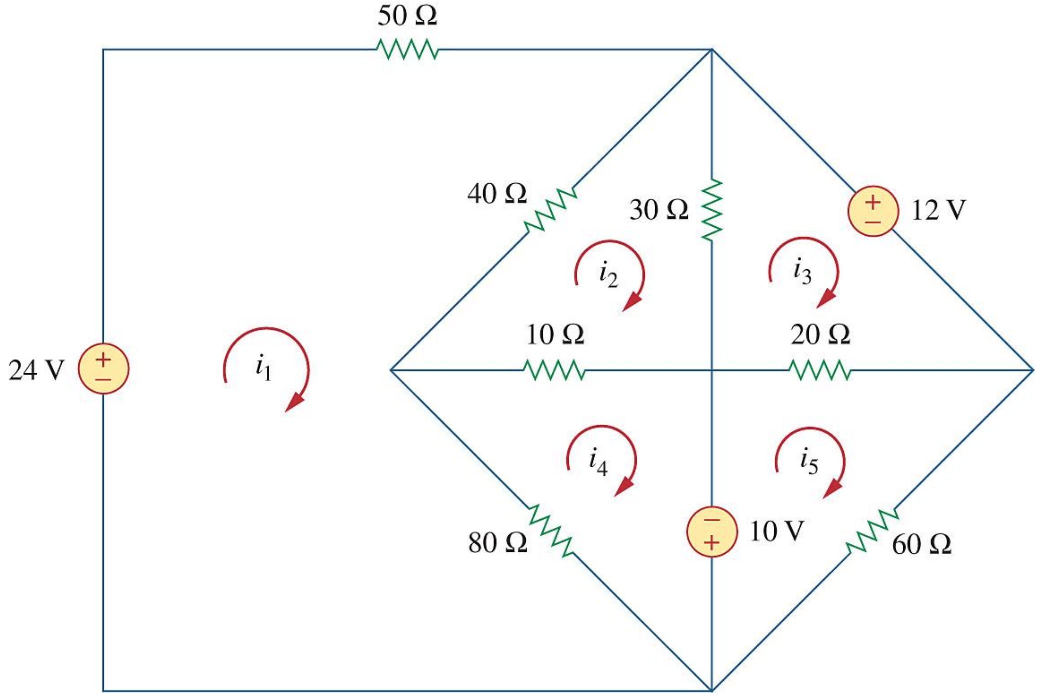
Practice Problem 3.9 By inspection, obtain the mesh-current equations forthe circuit in Fig. 3.30.



Solution :

$$\begin{bmatrix} 50+40+80 & -40 & 0 & -80 & 0 \\ -40 & 40+30+10 & -30 & -10 & 0 \\ 0 & -30 & 30+20 & 0 & -20 \\ -80 & -10 & 0 & 10+80 & 0 \\ 0 & 0 & -20 & 0 & 20+60 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

$$= \begin{bmatrix} 24 \\ 0 \\ -12 \\ 10 \\ -10 \end{bmatrix}$$



Example 3.6 Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

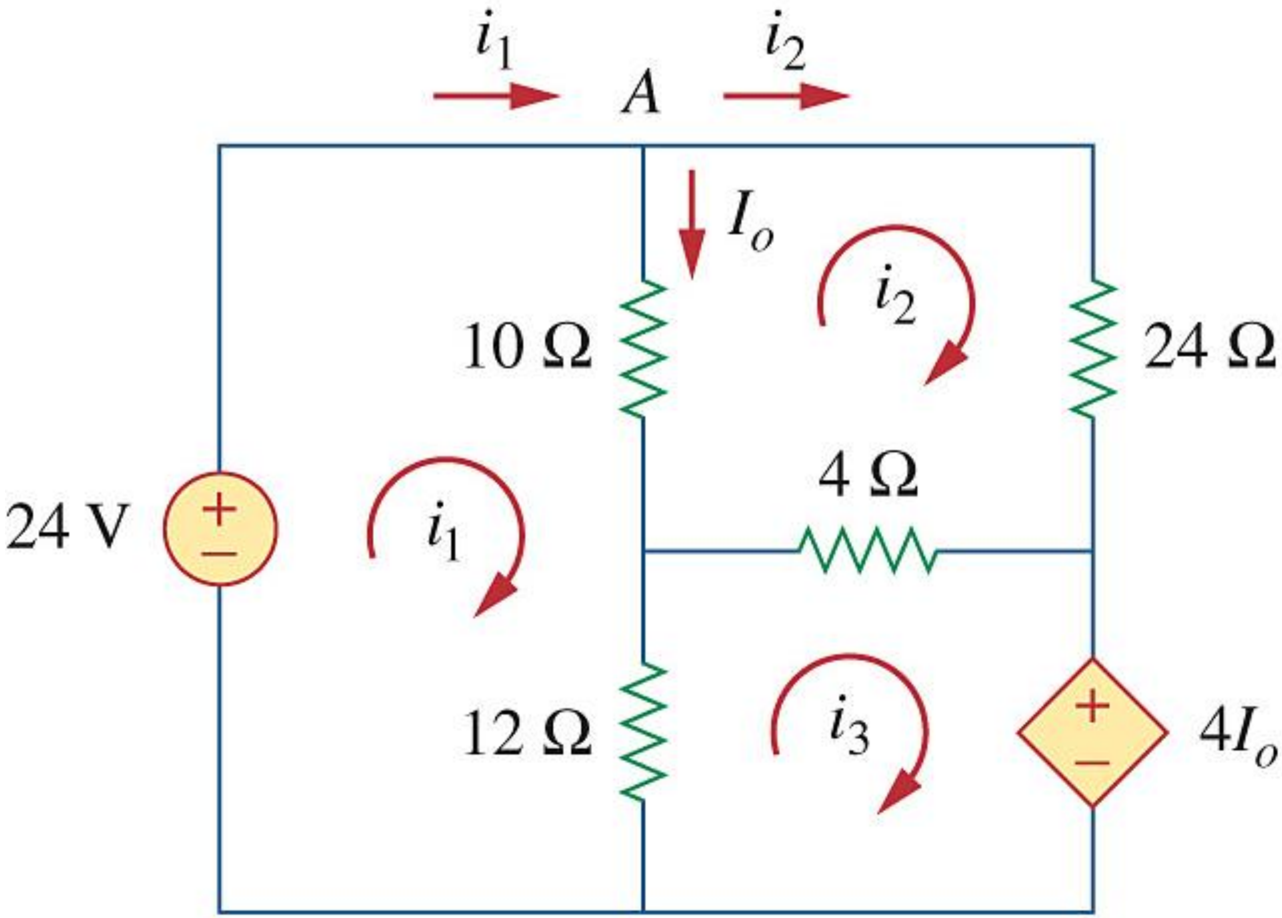


Figure 3.20

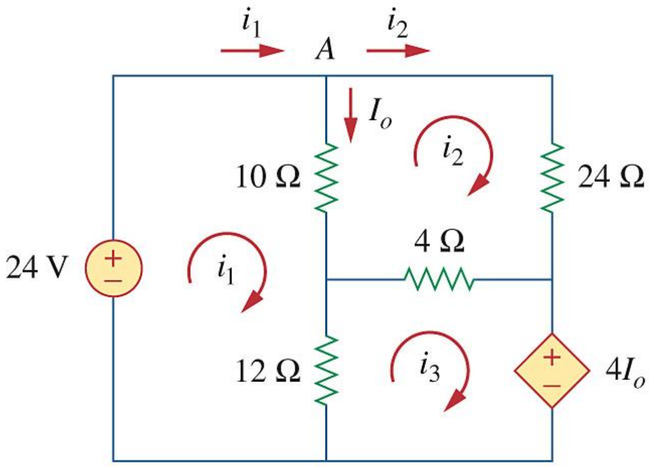


Figure 3.20

Solution :

$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 10+24+4 & -4 \\ -12 & -4 & 12+4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -4I_o \end{bmatrix}$$

$$I_o = i_1 - i_2$$

$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 10+24+4 & -4 \\ -12 & -4 & 12+4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -4(i_1 - i_2) \end{bmatrix}$$

$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 10+24+4 & -4 \\ -12+4 & -4-4 & 12+4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 22 & -10 & -12 \\ -10 & 38 & -4 \\ -8 & -8 & 16 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{vmatrix} = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \end{vmatrix} = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \end{vmatrix} = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \end{vmatrix} = 288$$

$$i_1 = \frac{\Delta_1}{\Delta} = 2.25\ (\mathrm{A}),\ i_2 = \frac{\Delta_2}{\Delta} = 0.75\ (\mathrm{A})$$

$$I_o = i_1 - i_2 = 1.5\ (\mathrm{A})$$

3.5 Mesh Analysis with Current Source

Applying mesh analysis to circuits containg current sources (dependent or independent) is easier than what we encountered in section 3.4.

1. When a current source exists only in one mesh, the mesh current is known. In Fig. 3.22, e.g.,

$$i_2 = -5 \text{ A}$$

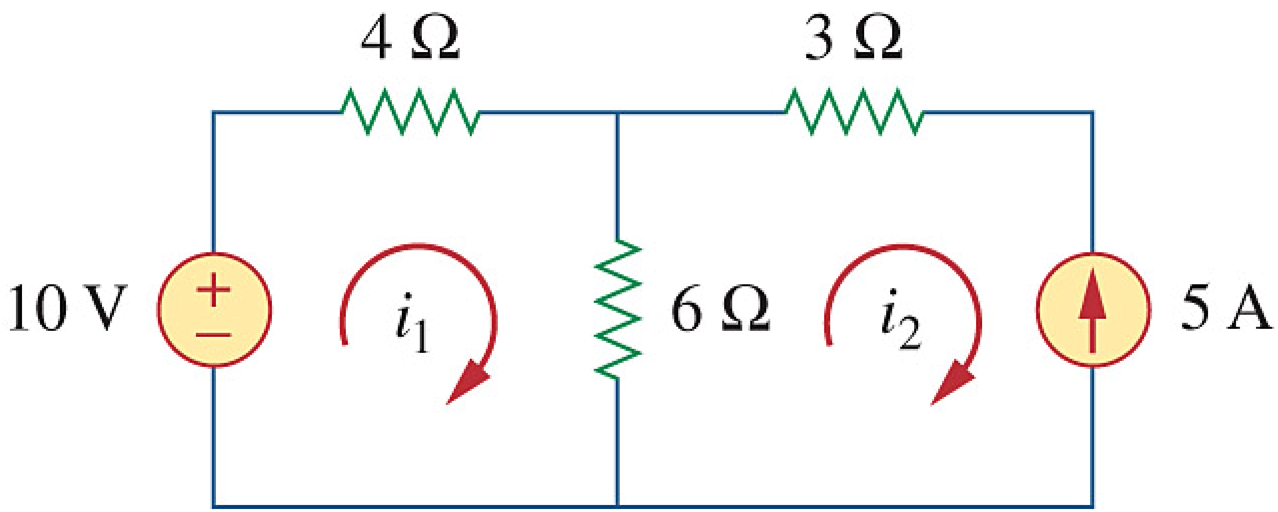


Figure 3.22

2. If a current source exists between two meshes, the two meshes form a *supermesh*. The supermesh provides a constraint on the two mesh currents. In Fig. 3.23(a), e.g.,

$$i_1 - i_2 = -6$$

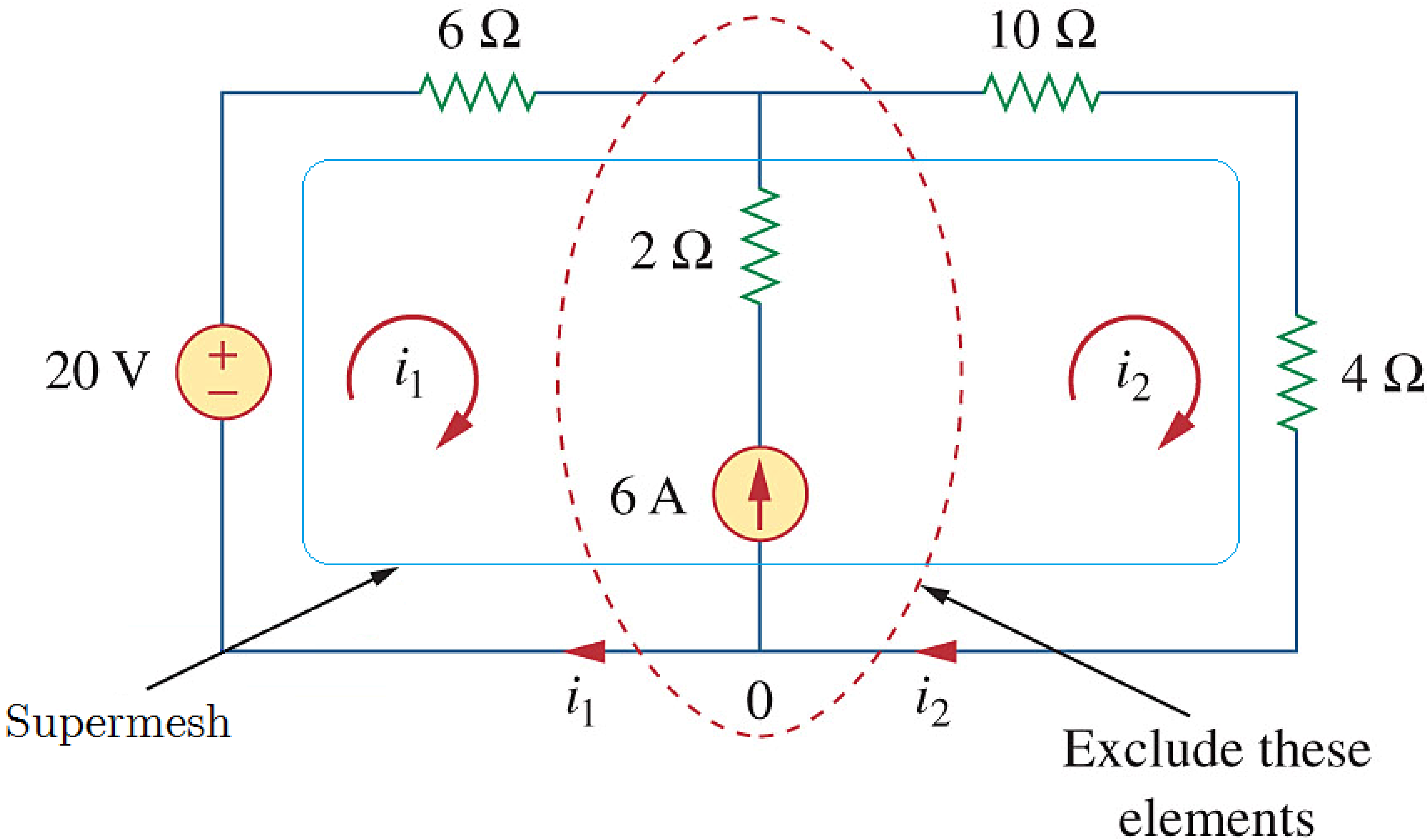

6 Ω
10 Ω
2 Ω
20 V
i_1
i_2
4 Ω
6 A
i_1
0
i_2
Supermesh
Exclude these elements

Figure 3.23 (a)

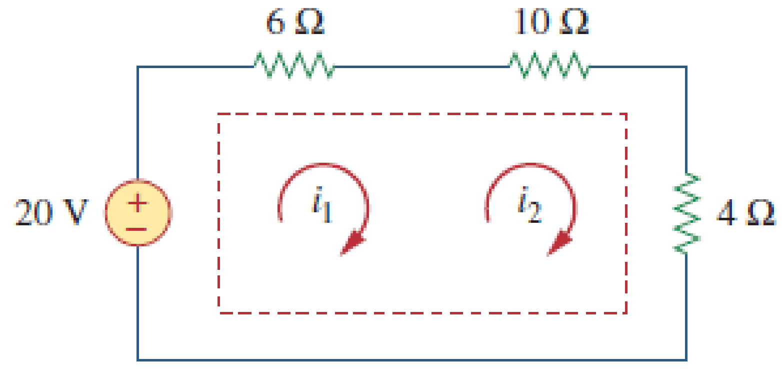
A supermesh is formed by excluding a current source between two meshes and any elements connected in series with it.

A supermesh has no current of its own.

We apply KVL to the supermesh, in Fig. 3.23, e.g.,

$$6i_1 + 10i_2 + 4i_2 - 20 = 0$$ or

$$6i_1 + (10+4)i_2 = 20$$



(b)

Example 3.7 For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis.

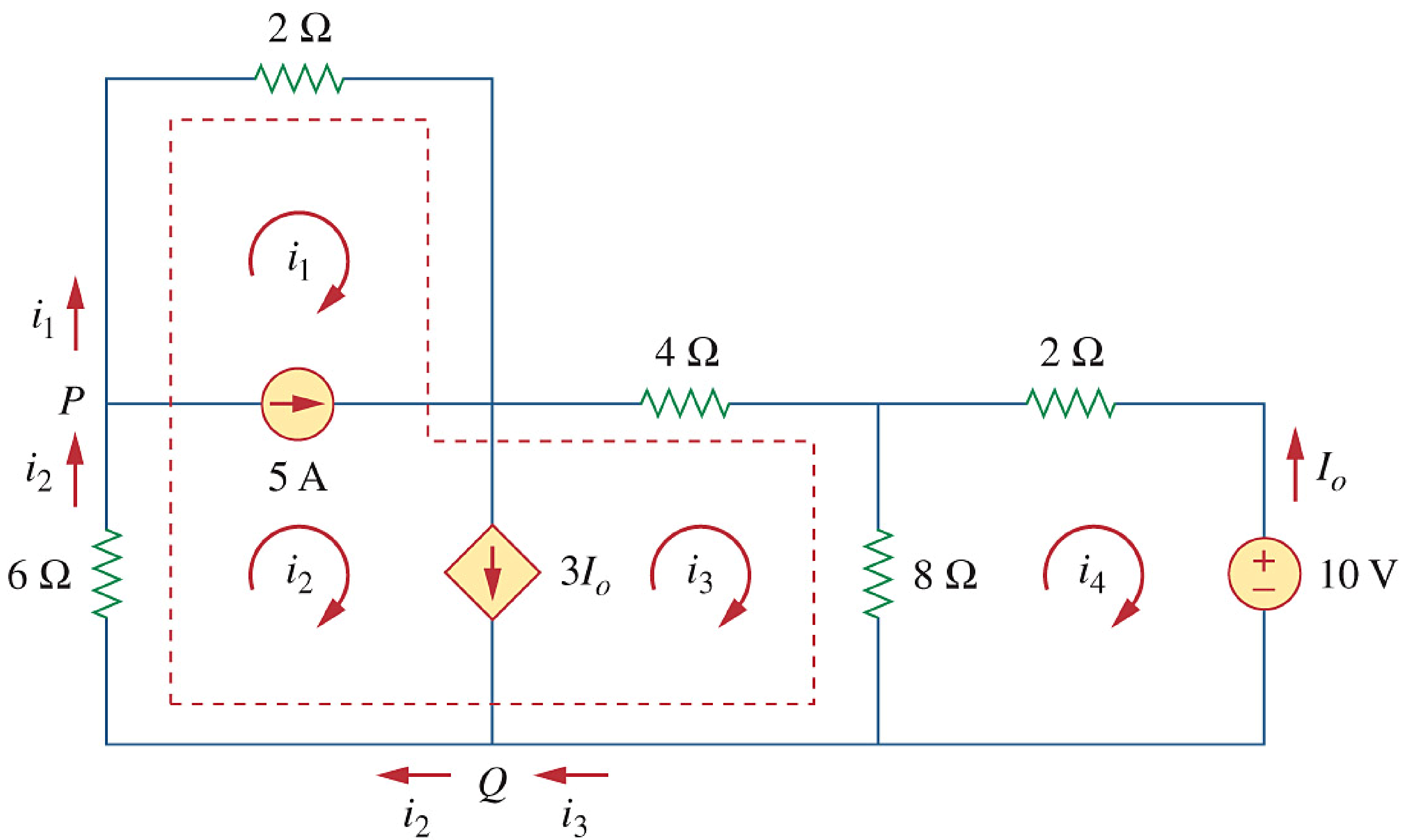


Figure 3.24

Solution :

Note that meshes 1 and 2 form a supermesh.

Also, meshes 2 and 3 form another supermesh.

The two supermeshes intersect. They should be combined to form a larger supermesh. For the combined supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0 \quad \text{or}$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \qquad (1)$$

For meshes 1 and 2,

$$-i_1 + i_2 = 5 \qquad (2)$$

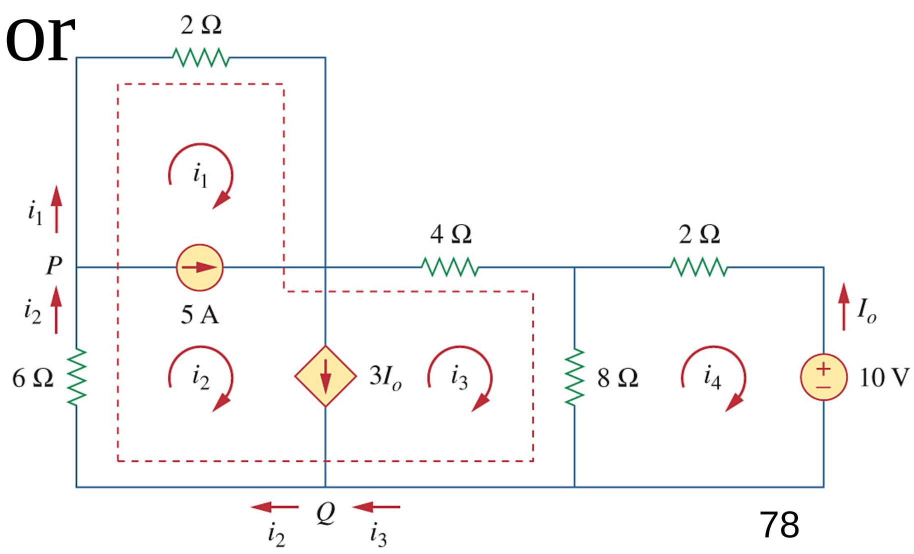


Figure 3.24

For meshes 2 and 3,

$$i_2 - i_3 = 3I_o$$

But $I_o = -i_4$, hence,

$$i_2 - i_3 + 3i_4 = 0 \qquad (3)$$

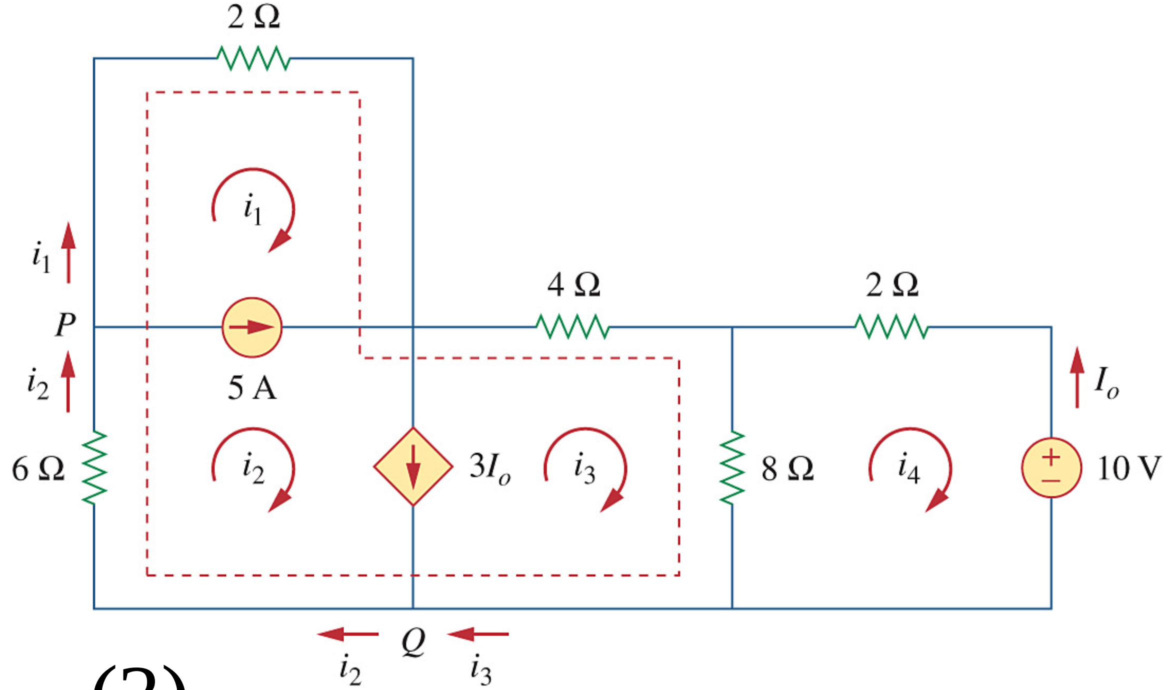


Figure 3.24

For mesh 4,

$$-8i_3 + (2+8)i_4 = -10 \quad \text{or}$$

$$-4i_3 + 5i_4 = -5 \qquad (4)$$

- 5 variables: i_1, i_2, i_3, i_4, I_o
- 5 equations:
  - 2 mesh (or KVL) equations
  - 2 supermesh constraints
  - 1 mesh current relation

From Eqs. (1) to (4),

$$i_1 = -7.5\ \text{A},\ i_2 = -2.5\ \text{A},\ i_3 \approx 3.93\ \text{A},\ i_4 \approx 2.14\ \text{A}$$

Nodal Versus Mesh Analysis

Nodal Analysis	Mesh Analysis
Contain many parallel-connected elements, current sources, or super-nodes	Contain many series-connected elements, voltage sources, or super-meshes
Fewer nodes than meshes	Fewer meshes than nodes
If node voltages are required	If branch or mesh currents are required

Question/Discussion:

-Why many parallel-connected elements?

-Why many current sources?

-Why many supernodes?

- Many parallel-connected elements → less nodes → less variables → less equations
- Many current sources → more know branch currents → simplified node equation (e.g., 3A, instead of $v_1/2$)
- Many supernodes → constraint equations are easier than node equations (e.g., $v_1-v_2=2$, instead of $(v_1-v_2)/2$ + bala bala…)

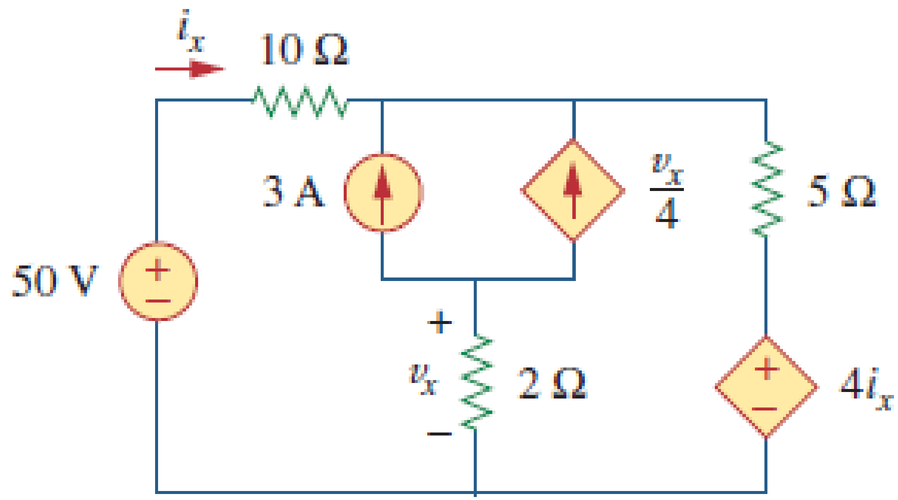
Notes

- Nodal analysis by Inspection can only be applied to the case without voltage sources (i.e., without supernodes)

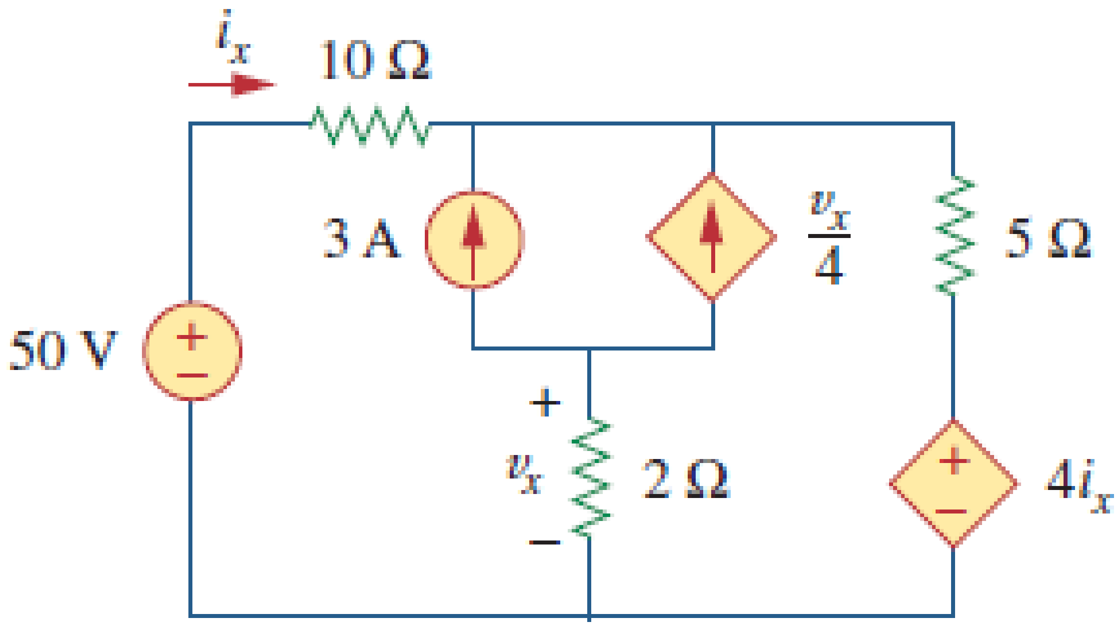
- Mesh analysis by Inspection can only be applied to the case without current sources (i.e., without supermeshes)

Practice Problem

- Find V_x and I_x?



1. Nodal analysis or mesh analysis?
2. If nodal analysis, the case with voltage source or not?; If mesh analysis, the case with current source or not?
3. Write down the equations and solve it



I. → Mesh analysis with supermeshes

II.

$I_1 – i_2 = –3$ (1)

$I_2 – i_3 = –v_x/4$ (2)

$10i_x + 5i_3 + 4i_x – 50 = 0$ (3)

III.

Check how many unknows: i_1, i_2, i_3, i_x, v_x → Need 2 more equations.

$i_x = i_1$

$v_x = (i_1 - i_3)2$

IV.

Substitute in eqs. (1)-(3), solve i_1, i_2, i_3

Then, i_x v_x can be solved.