

Introduction to Predictive Analytics

예측?

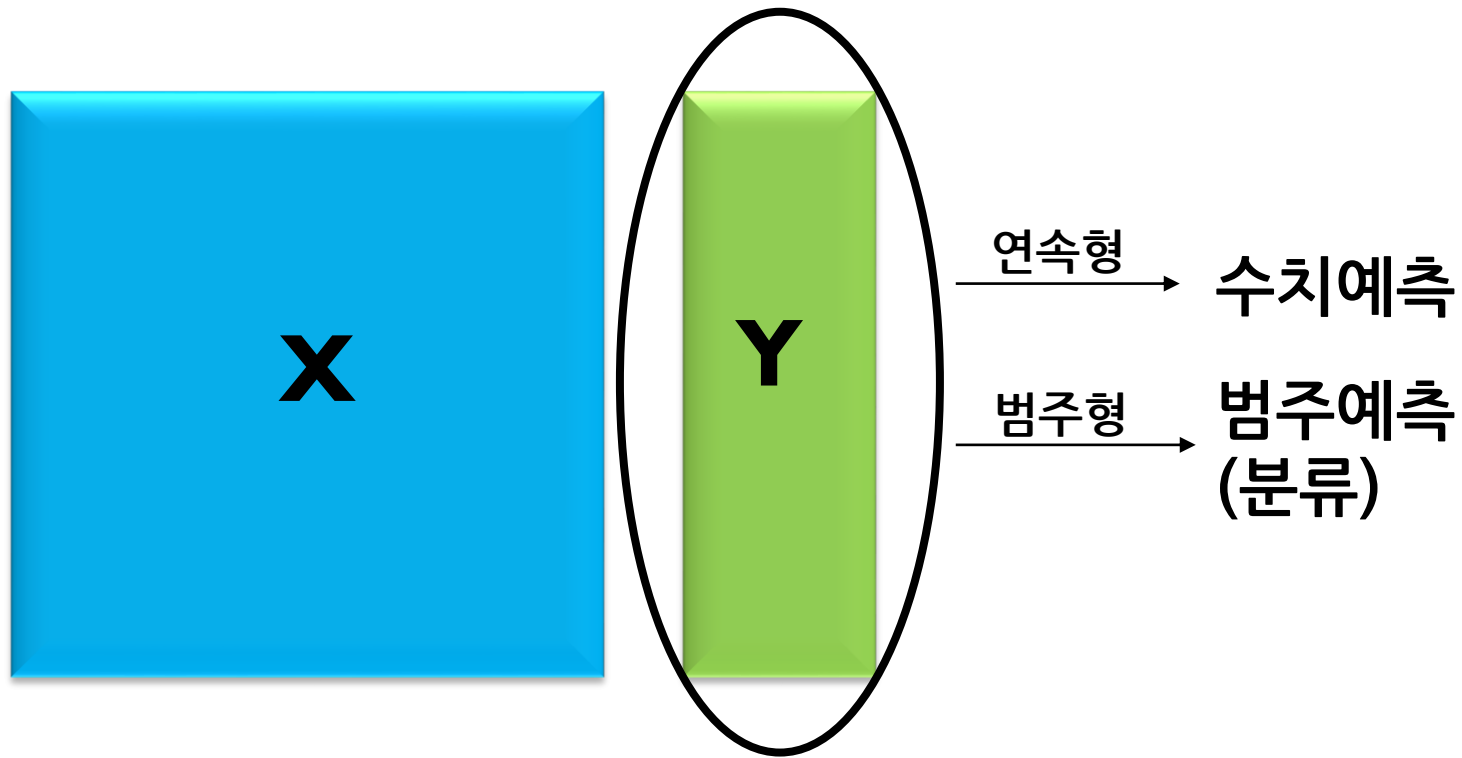
데이터

Y (결과): 종속변수, 반응변수, 출력변수

X (원인): 독립변수, 예측변수, 입력변수

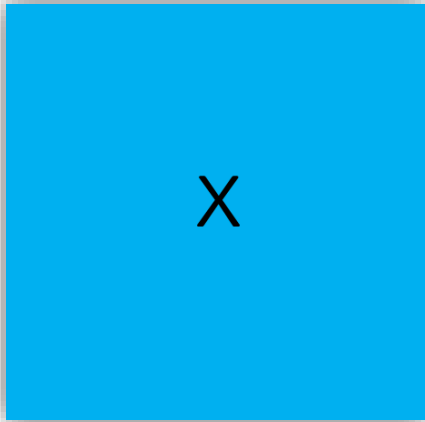
관측치 \ 변수	X (원인)						Y (결과)
	X_1	...	X_i	...	X_p		Y
N_1	x_{11}	...	x_{1i}	...	x_{1p}		y_1
N_2	x_{21}	...	x_{2i}	...	x_{2p}		y_2
...
N_{n-1}	x_{n-11}	...	x_{n-1i}	...	x_{n-1p}		y_{n-1}
N_n	x_{n1}	...	x_{ni}	...	x_{np}		y_n

수치예측 / 범주예측 (분류)



- 연속형 데이터: 데이터를 숫자로 표현
예) 가격, 길이, 압력, 두께, ...
- 범주형 데이터: 원칙적으로 숫자로 표시할 수 없는 데이터
예) 제품불량여부 (양품/불량), 보험사기여부(정상/비정상), ...

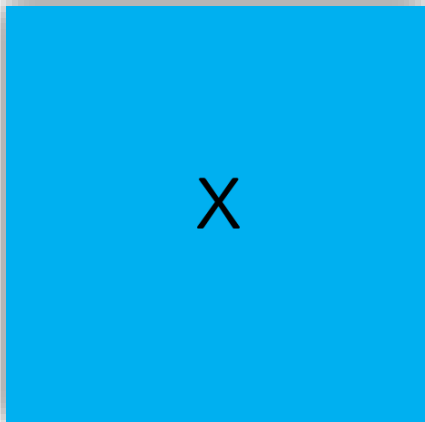
수치예측 데이터



연속형 → 수치예측 (Regression)

<div>인자 (변수)</div> <div>관측치</div>	X_1	...	X_i	...	X_p	Y
N_1	x_{11}	...	x_{1i}	...	x_{1p}	20.5
N_2	x_{21}	...	x_{2i}	...	x_{2p}	22.2
...
N_{n-1}	x_{n-11}	...	x_{n-1i}	...	x_{n-1p}	72.3
N_n	x_{n1}	...	x_{ni}	...	x_{np}	82.8

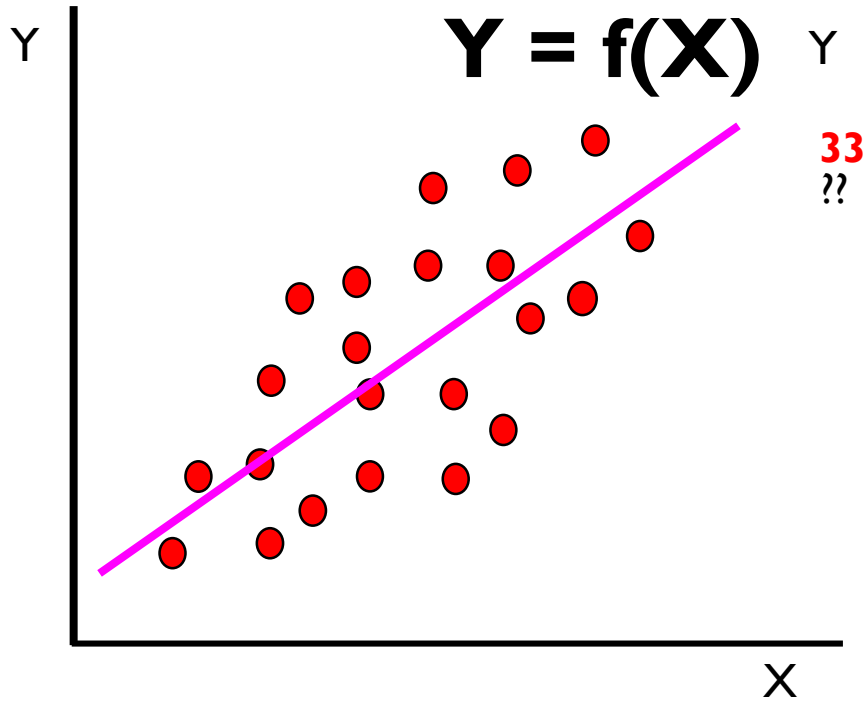
범주예측 데이터



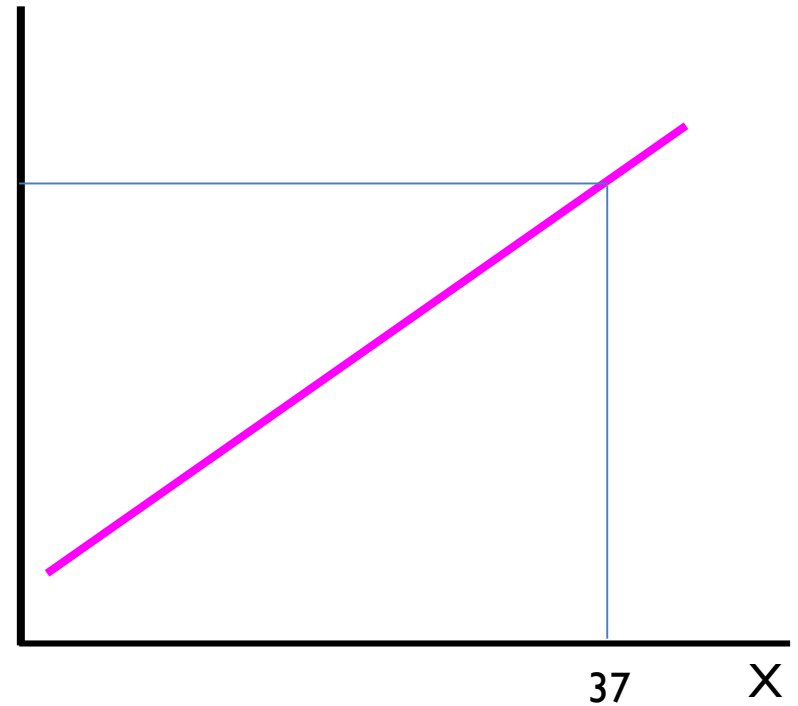
범주형 → 범주예측, 분류 (Classification)

<div>인자 (변수)</div> <div>관측치</div>	X_1	...	X_i	...	X_p	Y
N_1	x_{11}	...	x_{1i}	...	x_{1p}	0 (정상)
N_2	x_{21}	...	x_{2i}	...	x_{2p}	0 (정상)
...
N_{n-1}	x_{n-11}	...	x_{n-1i}	...	x_{n-1p}	1 (불량)
N_n	x_{n1}	...	x_{ni}	...	x_{np}	1 (불량)

수치예측 모델링 개요



Training (학습)



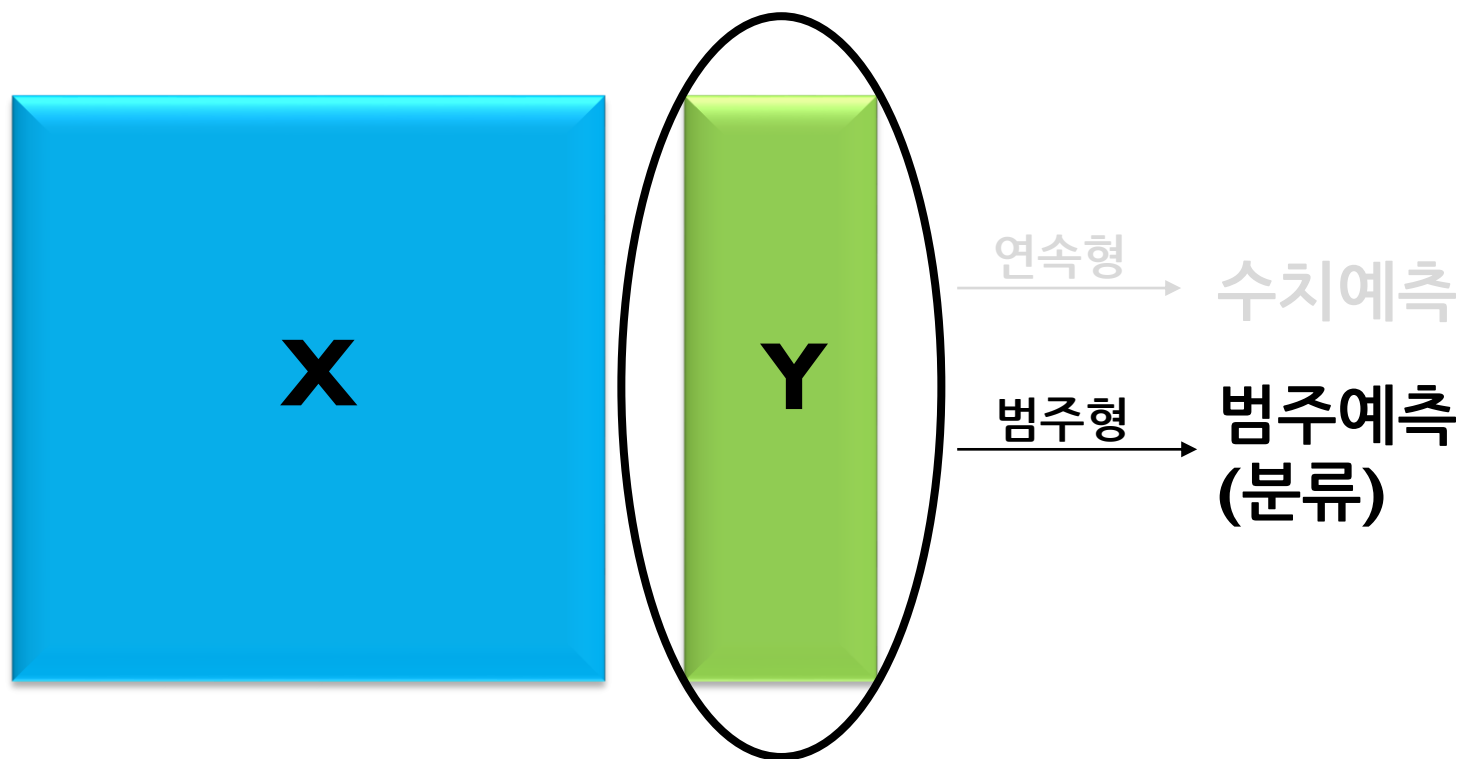
Testing, Inference (테스팅, 인퍼런스)

수치예측 예제 - 중고차 가격 예측

모델
TOYOTA Corolla 2.0 D4D HATCHB TERRA 2/3-Doors
TOYOTA Corolla 1800 T SPORT VVT I 2/3-Doors
TOYOTA Corolla 1.9 D HATCHB TERRA 2/3-Doors
TOYOTA Corolla 1.8 VVTL-i T-Sport 3-Drs 2/3-Doors
TOYOTA Corolla 1.8 16V VVTI 3DR T SPORT BNS 2/3-Doors
TOYOTA Corolla 1.6 VVTI Linea Terra Comfort 2/3-Doors
TOYOTA Corolla 1.6 16v L.SOL 2/3-Doors
TOYOTA Corolla 1.6 16V VVT I 3DR TERRA 2/3-Doors
TOYOTA Corolla 1.6 16V VVT I 3DR SOL AUT4 2/3-Doors
TOYOTA Corolla 1.6 16V VVT I 3DR SOL 2/3-Doors
TOYOTA Corolla 1.4 VVTI Linea Terra 2/3-Doors
TOYOTA Corolla 1.4 16V VVT I 3DR 2/3-Doors
TOYOTA Corolla 1.4 16V VVT I 3DR 2/3-Doors
TOYOTA Corolla 1.4 16V VVT I 3DR 2/3-Doors
TOYOTA Corolla 1.4 16V VVT I 3DR 2/3-Doors
TOYOTA Corolla 2.0 D4D 90 5DR TERRA COMFORT 4/5-Doors
TOYOTA Corolla 2.0 D4D 90 5DR TERRA COMFORT 4/5-Doors
TOYOTA Corolla 2.0 D4D 90 5DR SOL 4/5-Doors
TOYOTA Corolla 2.0 D4D 90 5DR TERRA 4/5-Doors
TOYOTA Corolla 1.4 16V VVT I 5DR TERRA COMFORT 4/5-Doors
TOYOTA Corolla 1.4 16V VVT I 5DR TERRA COMFORT 4/5-Doors

X			Y
주행거리	마력	용량 (CC)	가격
46986	90	2000	13500
19700	192	1800	21500
71138	69	1900	12950
31461	192	1800	20950
43610	192	1800	19950
21716	110	1600	17950
25563	110	1600	16750
64359	110	1600	16950
43905	110	1600	16950
56349	110	1600	15950
9750	97	1400	12950
27500	97	1400	14750
49059	97	1400	13950
44068	97	1400	16750
46961	97	1400	13950
110404	90	2000	16950
100250	90	2000	16950
84000	90	2000	19000
79375	90	2000	17950
75048	97	1400	15800
132151	110	1600	18200
			??????

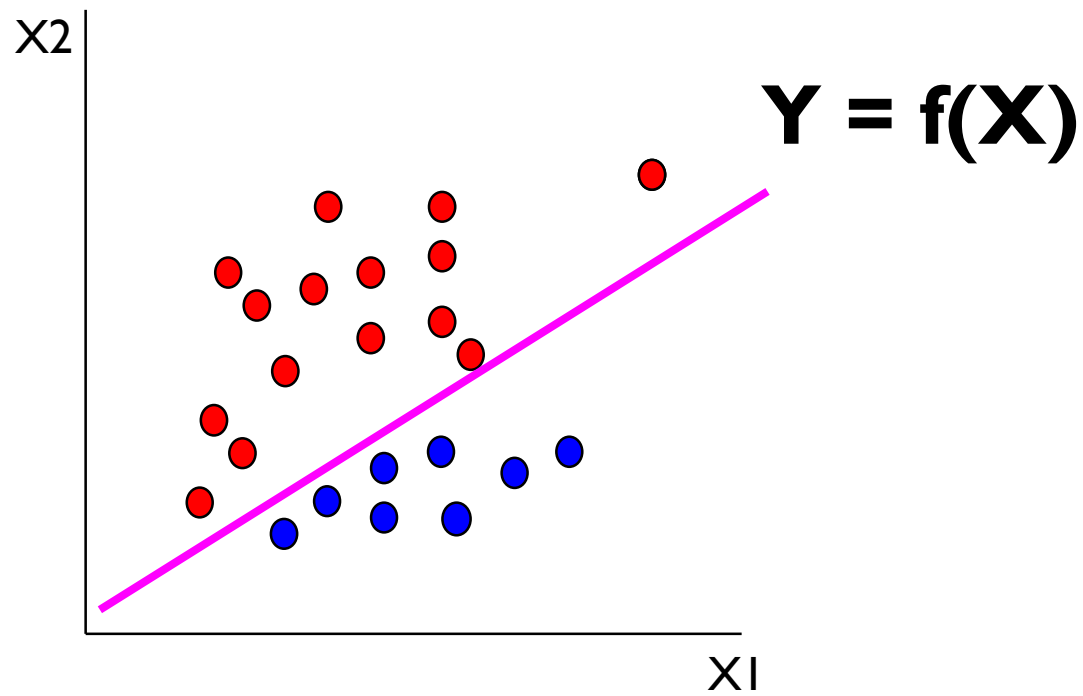
수치예측 / 범주예측 (분류)



범주예측 모델링 개요

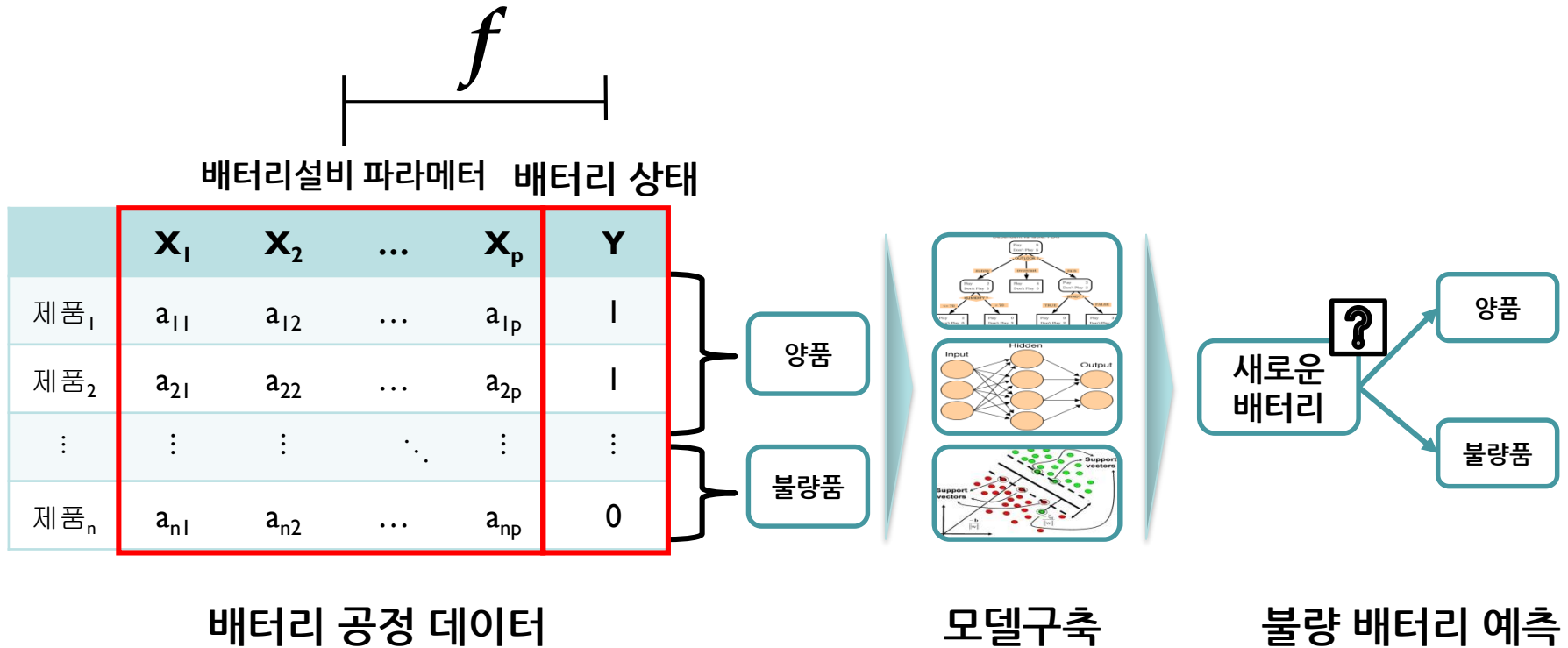
● 불량범주

● 양품범주



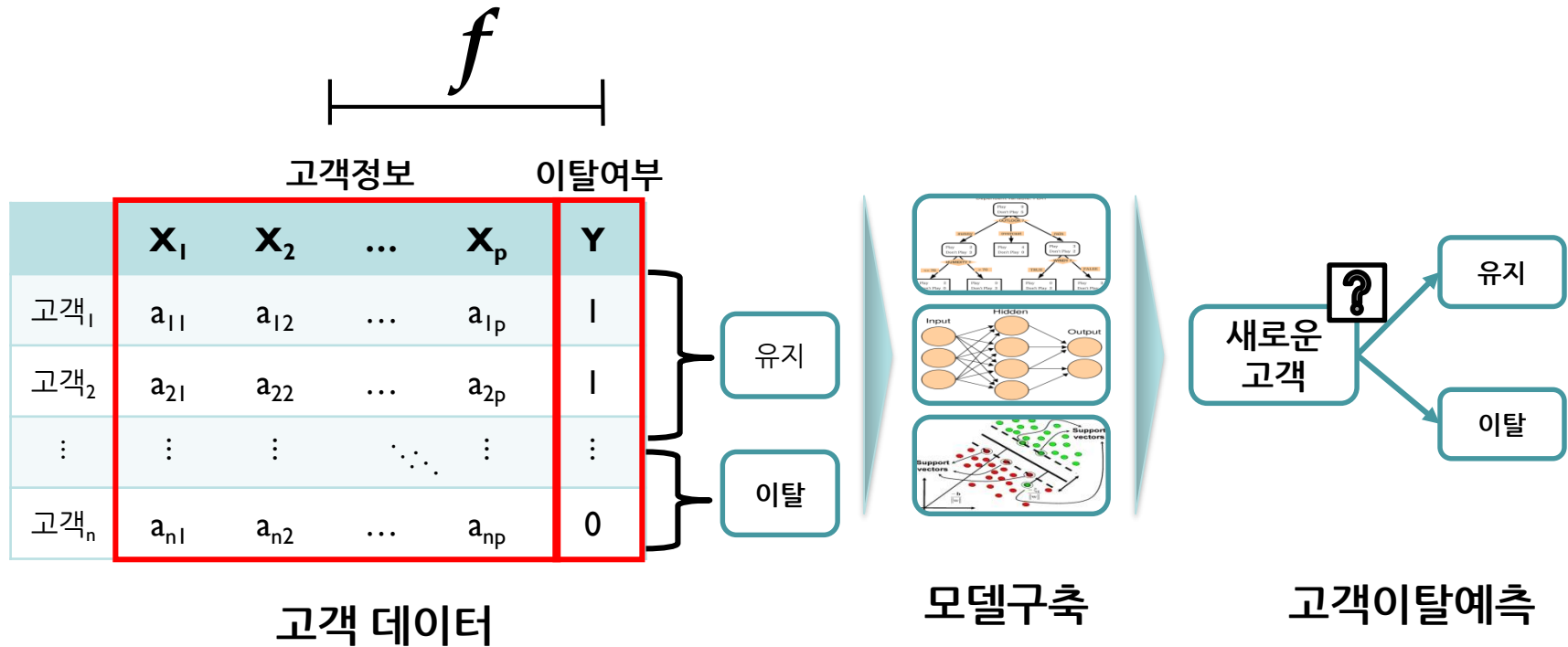
범주예측 예제 - 불량 예측

- 배터리 공정에서 설비 파라미터 측정값들을 이용하여,
배터리가 양품인지 불량품인지 여부를 예측



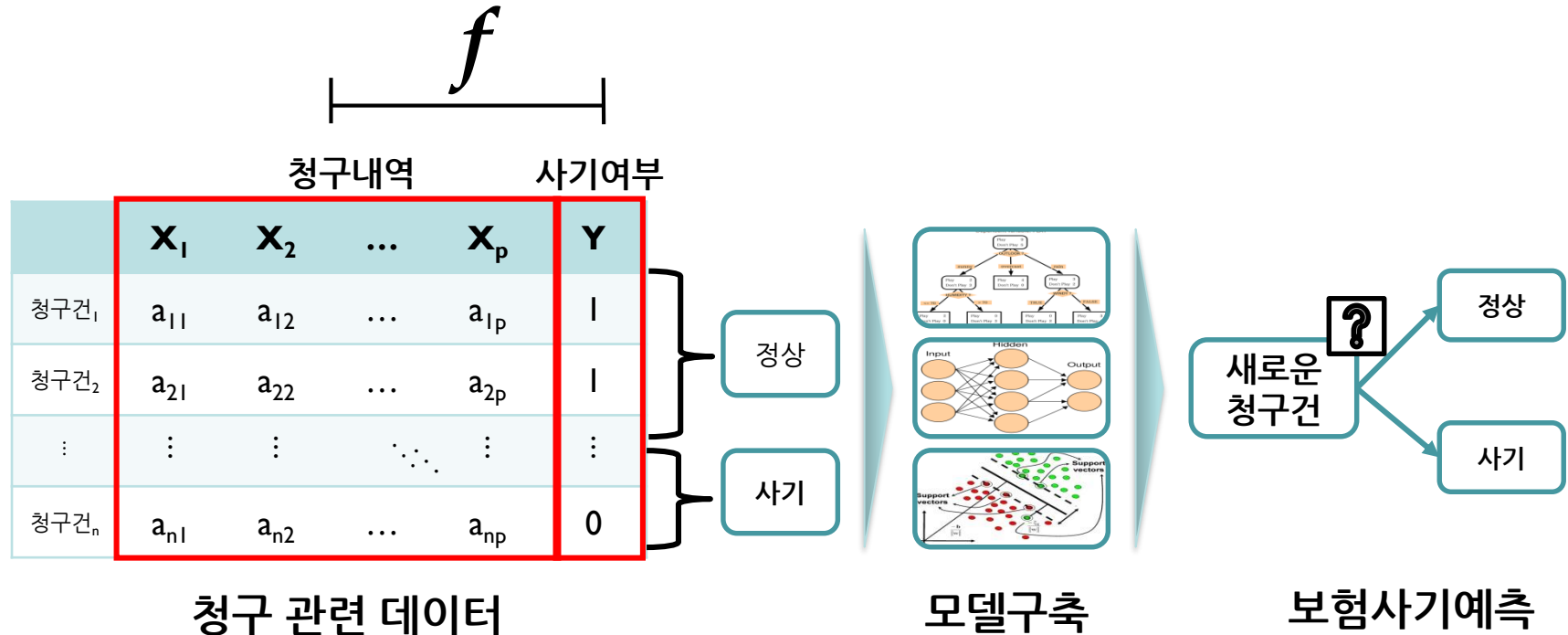
범주예측 예제 - 고객 이탈 예측

- 고객의 정보(성별, 연령, 직업, 연봉 등)를 이용하여,
고객 이탈 여부를 예측



범주예측 예제 - 보험 사기 여부 예측

- 각 청구 건에 대한 내역 분석을 통해 청구 건에 대한 사기 여부 예측



데이터

Y (결과): 종속변수, 반응변수, 출력변수

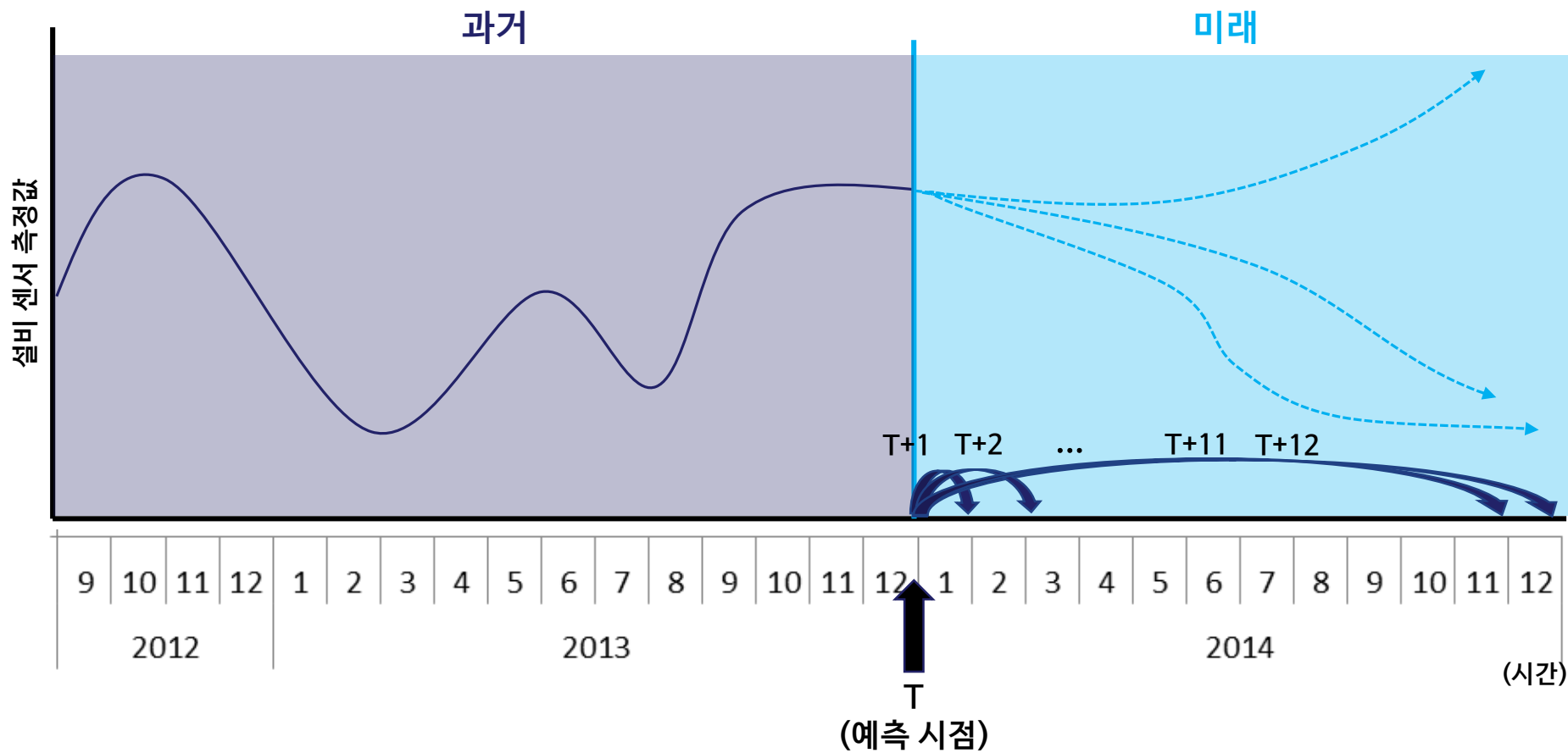
X (원인): 독립변수, 예측변수, 입력변수

<div>변수</div> <div>관측치</div>	X_1	...	X_i	...	X_p	Y
N_1	x_{11}	...	x_{1i}	...	x_{1p}	20.5
N_2	x_{21}	...	x_{2i}	...	x_{2p}	22.2
...
N_{n-1}	x_{n-11}	...	x_{n-1i}	...	x_{n-1p}	72.3
N_n	x_{n1}	...	x_{ni}	...	x_{np}	82.8

단변량 시계열 예측

X 변수						Y 변수
변수 관측치	X_1	...	X_i	...	X_p	Y
N_1	x_{11}	...	x_{1i}	...	x_{1p}	20.5
N_2	x_{21}	...	x_{2i}	...	x_{2p}	22.2
...
N_{n-1}	x_{n-11}	...	x_{n-1i}	...	x_{n-1p}	72.3
N_n	x_{n1}	...	x_{ni}	...	x_{np}	82.8

단변량 시계열 예측



많은 현상을 X와 Y로 설명할 수 있어...



어떤 고객들이 이탈할까?



고장을 미리 예측 할 수 있을까?



최적의 투자전략은 무엇인가?



식품 판매량 (수요) 예측?



보험 과다 청구 여부?



출시 예정 상품이 시장에서 어떤 반응을 보일까?

X와 Y의 관계를 찾는 것!

우리의 주 관심은 Y (예측하려는 대상)

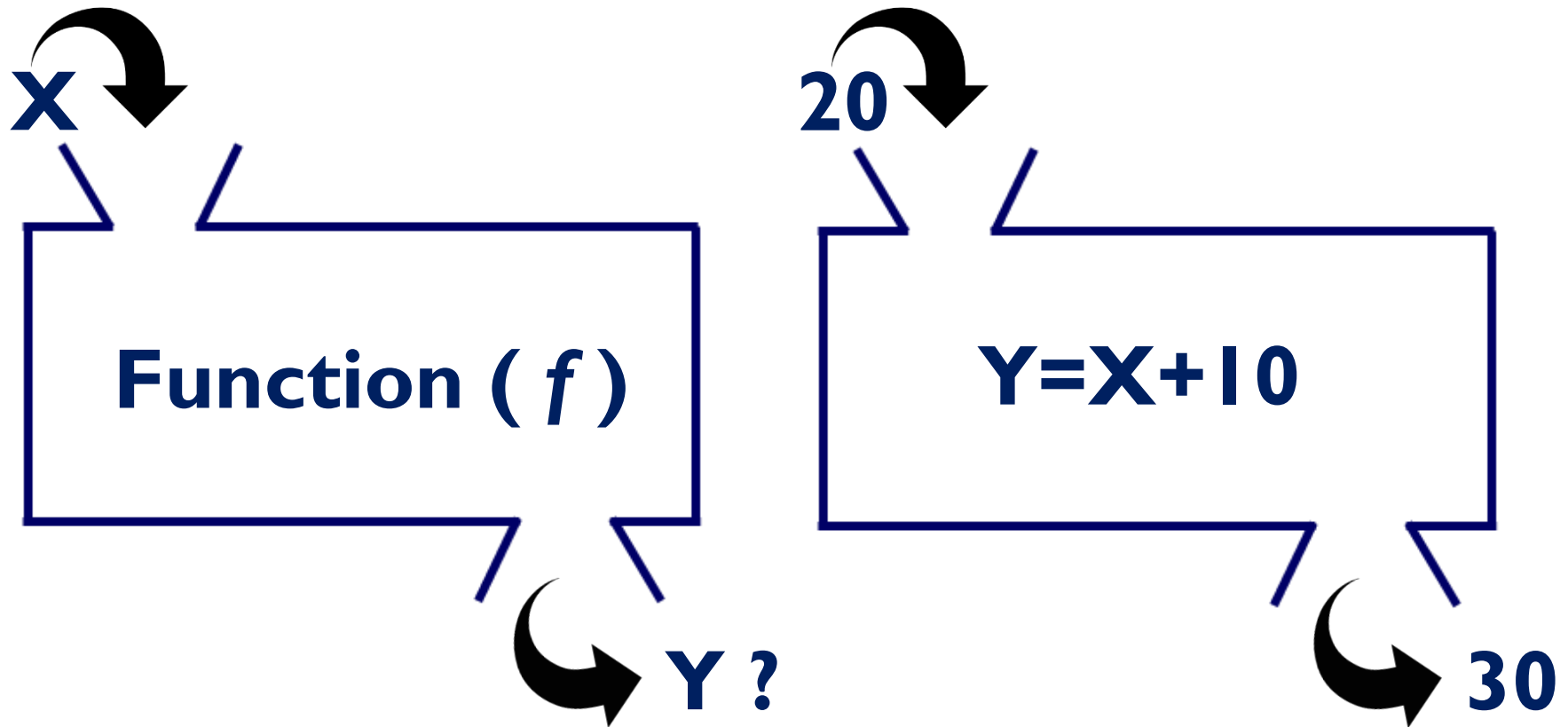
Y를 설명하는 X변수는 보통 여러 개

여러 개의 X와 Y의 관계를 찾는 것!

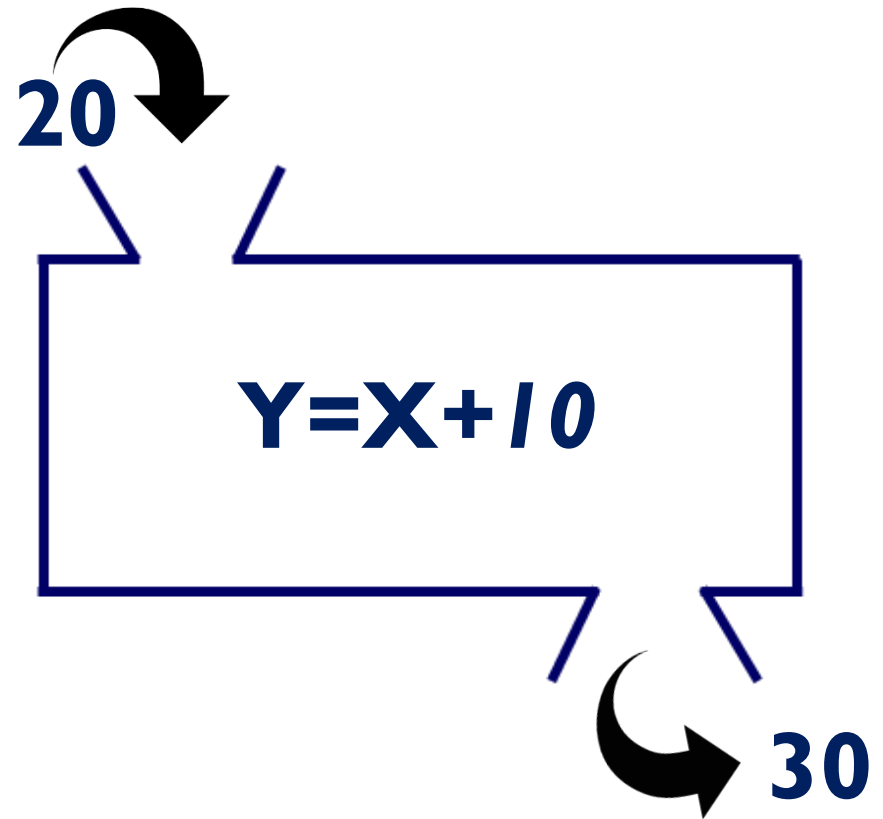
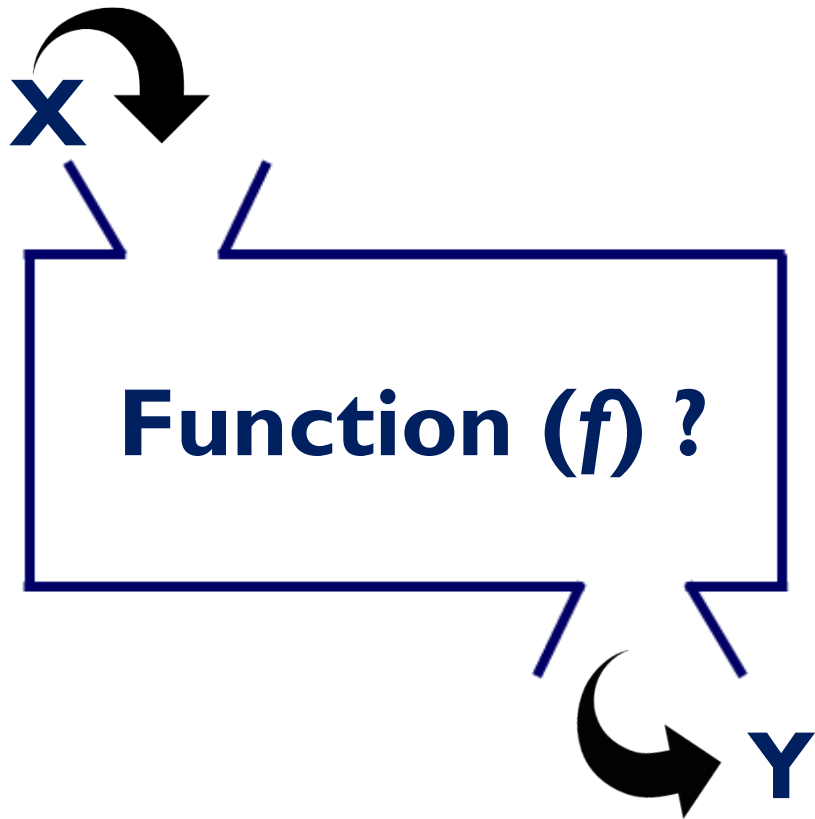
X변수들을 조합(결합)하여 Y를 표현

수학적으로는, $Y = f(X_1, X_2, \dots, X_p)$

X와 Y의 관계 찾기



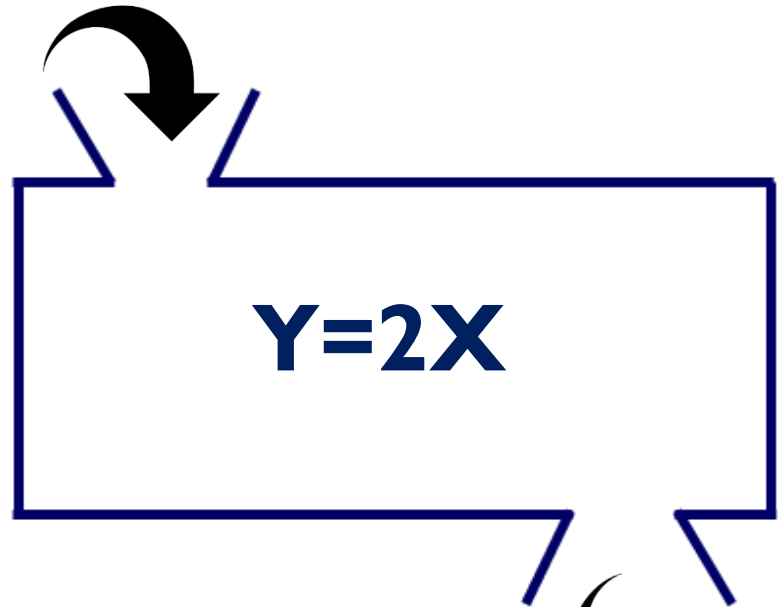
X와 Y의 관계 찾기



X와 Y의 관계 찾기

X	Y
0	0
1	2
2	4
3	6

0, 1, 2, 3

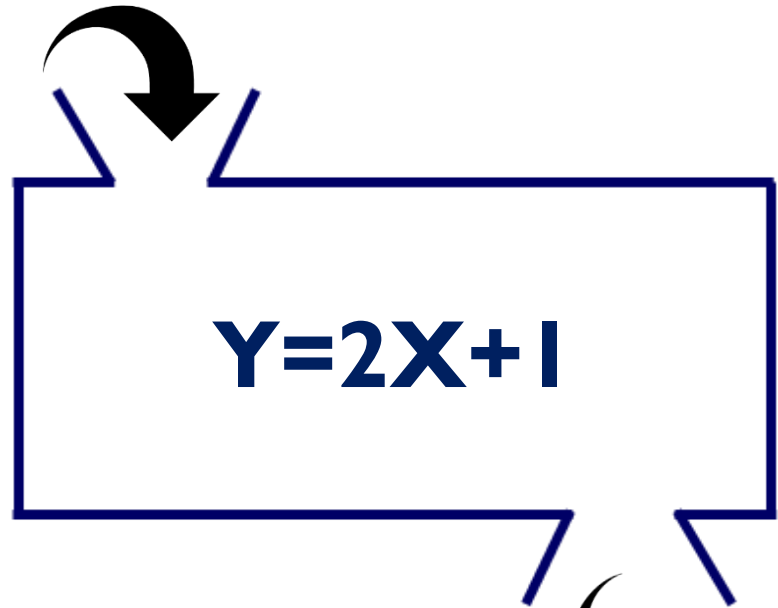


0, 2, 4, 6

X와 Y의 관계 찾기

X	Y
0	1
1	3
2	5
3	7

0, 1, 2, 3



1, 3, 5, 7

X와 Y의 관계 찾기

X	Y
0	2
1	2.5
2	3
3	3.5

0, 1, 2, 3

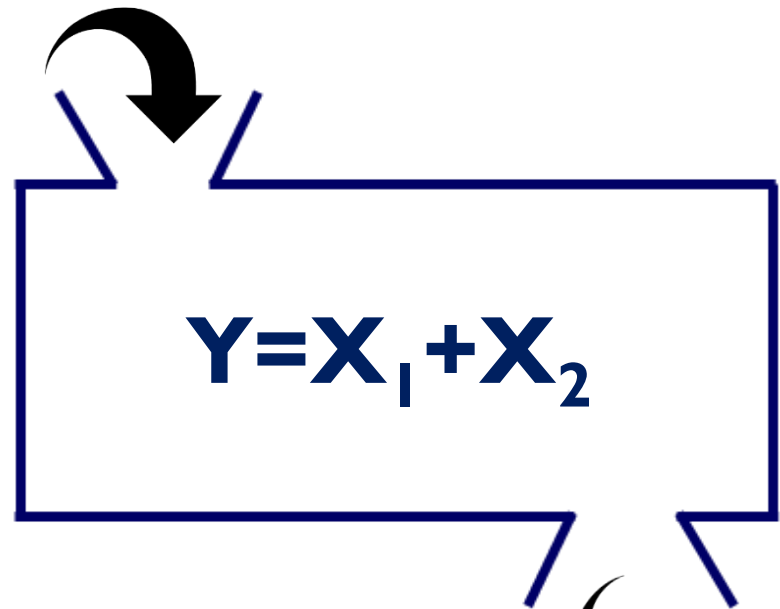
$$Y=0.5X+2$$

2, 2.5, 3, 3.5

X와 Y의 관계 찾기

X_1	X_2	Y
0	2	2
1	3	4
2	4	6
3	5	8

$(0, 2), (1, 3), (2, 4), (3, 5)$

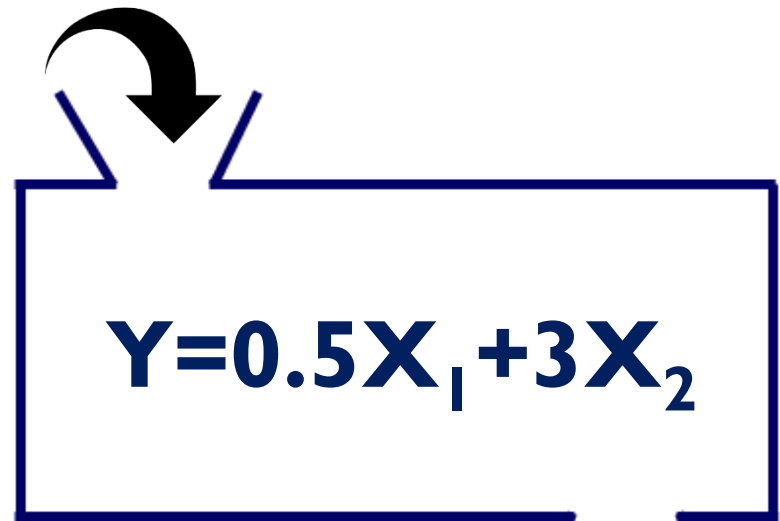


$2, 4, 6, 8$

X와 Y의 관계 찾기

X_1	X_2	Y
0	2	6
1	3	9.5
2	4	13
3	5	16.5

$(0, 2), (1, 3), (2, 4), (3, 5)$


$$Y = 0.5X_1 + 3X_2$$

$6, 9.5, 13, 16.5$

X와 Y의 관계 찾기

X_1	X_2	Y
0	2	6
1	3	9
2	4	11.5
3	5	14.5

$(0, 2), (1, 3), (2, 4), (3, 5)$

$$Y = ?X_1 + ?X_2$$

$6, 9, 11.5, 14.5$

X와 Y의 관계 찾기

모델	X1	X2	X3	Y
	주행거리	마력	용량	가격
TOYOTA Corolla 2.0 D4D HATCHB TERRA 2/3-Doors	46,986	90	2,000	13,500
TOYOTA Corolla 1800 T SPORT VVT I 2/3-Doors	19,700	192	1,800	21,500
TOYOTA Corolla 1.9 D HATCHB TERRA 2/3-Doors	71,138	69	1,900	12,950
TOYOTA Corolla 1.8 VVTL-i T-Sport 3-Drs 2/3-Doors	31,461	192	1,800	20,950
TOYOTA Corolla 1.8 16V VVTLI 3DR T SPORT BNS 2/3-Doors	43,610	192	1,800	19,950
TOYOTA Corolla 1.6 VVTI Linea Terra Comfort 2/3-Doors	21,716	110	1,600	17,950
TOYOTA Corolla 1.6 16v LSOL 2/3-Doors	25,563	110	1,600	16,750
TOYOTA Corolla 1.6 16V VVT I 3DR TERRA 2/3-Doors	64,359	110	1,600	16,950
TOYOTA Corolla 1.6 16V VVT I 3DR SOL AUT4 2/3-Doors	43,905	110	1,600	16,950
TOYOTA Corolla 1.6 16V VVT I 3DR SOL 2/3-Doors	56,349	110	1,600	15,950
TOYOTA Corolla 1.4 VVTI Linea Terra 2/3-Doors	9,750	97	1,400	12,950
TOYOTA Corolla 1.4 16V VVT I 3DR 2/3-Doors	27,500	97	1,400	14,750
TOYOTA Corolla 1.4 16V VVT I 3DR 2/3-Doors	49,059	97	1,400	13,950
TOYOTA Corolla 1.4 16V VVT I 3DR 2/3-Doors	44,068	97	1,400	16,750
TOYOTA Corolla 1.4 16V VVT I 3DR 2/3-Doors	46,961	97	1,400	13,950
TOYOTA Corolla 2.0 D4D 90 5DR TERRA COMFORT 4/5-Doors	110,404	90	2,000	16,950
TOYOTA Corolla 2.0 D4D 90 5DR TERRA COMFORT 4/5-Doors	100,250	90	2,000	16,950
TOYOTA Corolla 2.0 D4D 90 5DR SOL 4/5-Doors	84,000	90	2,000	19,000
TOYOTA Corolla 2.0 D4D 90 5DR TERRA 4/5-Doors	79,375	90	2,000	17,950
TOYOTA Corolla 1.4 16V VVT I 5DR TERRA COMFORT 4/5-Doors	75,048	97	1,400	15,800

$$Y = ?X_1 + ?X_2 + ?X_3 + \epsilon$$

X로 설명되는 부분

그렇지 않은 부분

$$Y = ?X_1 + ?X_2 + \varepsilon$$

$$Y = w_1X_1 + w_2X_2 + \varepsilon$$

$$w_1? \quad w_2?$$

Given X_1, X_2, Y (데이터)

$$Y = w_1 x_1 + w_2 x_2 + \varepsilon$$

파라미터 (母數)(媒介變數)

데이터가 주어졌을 때 모델의 파라미터 찾기!

$$Y = w_1 X_1 + w_2 X_2 + \varepsilon$$
$$= f(X) + \varepsilon$$

$$\varepsilon = Y - f(X) \Rightarrow \text{오차}$$

 Loss function
(손실함수)

$$Y - f(X) = 0, \varepsilon = 0$$

$$\varepsilon = Y - f(X) \quad \text{Loss function} \\ \text{(손실함수)}$$

$$f(X) = w_1 X_1 + w_2 X_2 + \varepsilon$$

$$\varepsilon = Y - (w_1 X_1 + w_2 X_2)$$

$$\varepsilon_i = Y_i - (w_1 X_{1i} + w_2 X_{2i}), i=1,2,\dots,n$$

$$\varepsilon_i = Y_i - (\mathbf{w}_1 X_{1i} + \mathbf{w}_2 X_{2i}), i=1, 2, \dots, n$$

$$\sum_{i=1}^n \{Y_i - (\mathbf{w}_1 X_{1i} + \mathbf{w}_2 X_{2i})\} = 0$$

$$\sum_{i=1}^n \{Y_i - (\mathbf{w}_1 X_{1i} + \mathbf{w}_2 X_{2i})\}^2$$



Cost function
(비용함수)

$$\sum_{i=1}^n \{Y_i - (\mathbf{w}_1 X_{1i} + \mathbf{w}_2 X_{2i})\}^2 \quad \text{Cost function} \\ \text{(비용함수)}$$

비용함수를 최소로 하는 \mathbf{w}_1 와 \mathbf{w}_2 를 찾자!

$$\min_{\mathbf{w}_1, \mathbf{w}_2} \sum_{i=1}^n \{Y_i - (\mathbf{w}_1 X_{1i} + \mathbf{w}_2 X_{2i})\}^2$$

파라미터 추정

$$\min_{w_1, w_2} \sum_{i=1}^n \{Y_i - (w_1 X_{1i} + w_2 X_{2i})\}^2$$

답: \hat{w}_1, \hat{w}_2

$$\hat{f}(X) = \hat{w}_1 X_{1i} + \hat{w}_2 X_{2i}$$

비용함수

- **Regression (Y가 연속형)**

Mean squared error (MSE)

$$C(Y, f(X)) = \sum_{i=1}^n \{Y_i - f(X_i)\}^2$$

- **Classification (Y가 범주형)**

Cross Entropy

$$C(Y, f(X)) = \sum_{i=1}^N \{-Y_i \cdot \log(f(X_i)) - (1 - Y_i) \cdot \log(1 - f(X_i))\}$$

모델 결정

$$C(Y, \boxed{f(X)})$$

$$f(X) = w_0 + w_1 X_1 + w_2 X_2$$

선형회귀 모델

$$f(X) = \frac{1}{1 + e^{-(w_0 + w_1 X_1 + w_2 X_2)}}$$

로지스틱회귀 모델

$$f(X) = \sum_{m=1}^n k(m) I\{(x_1, x_2) \in R_m\}$$

의사결정나무 모델

$$f(X) = \frac{1}{1 + \exp \left(- \left(w_0 + w_1 \left(\frac{1}{1 + e^{-(w_{01} + w_{11} X_1 + w_{21} X_2)}} \right) \right) + w_2 \left(\frac{1}{1 + e^{-(w_{02} + w_{12} X_1 + w_{22} X_2)}} \right) \right)}$$

뉴럴네트워크 모델

모델 결정 → 파라미터 추정

$$\min_W \sum_{i=1}^n \{Y_i - f(X_i)\}^2$$

$f(X_i) = w_0 + w_1 X_{1i} + w_2 X_{2i}$ 선형회귀 모델

$$\min_{w_0, w_1, w_2} \sum_{i=1}^n \{Y_i - (w_0 + w_1 X_{1i} + w_2 X_{2i})\}^2$$

Least square estimation algorithm
(최소제곱법)

$$\hat{f}(X) = \hat{w}_0 + \hat{w}_1 X_1 + \hat{w}_2 X_2$$

모델 결정 → 파라미터 추정

$$\min_W \sum_{i=1}^N \{-Y_i \cdot \log(f(X_i)) - (1 - Y_i) \cdot \log(1 - f(X_i))\}$$

$$f(X_i) = \frac{1}{1 + e^{-(w_0 + w_1 X_{1i} + w_2 X_{2i})}} \quad \text{로지스틱회귀 모델}$$

$$\min_{w_0, w_1, w_2} \sum_{i=1}^N \left\{ -Y_i \log \left(\frac{1}{1 + e^{-(w_0 + w_1 X_{1i} + w_2 X_{2i})}} \right) - (1 - Y_i) \log \left(1 - \frac{1}{1 + e^{-(w_0 + w_1 X_{1i} + w_2 X_{2i})}} \right) \right\}$$

Conjugate gradient algorithm

$$\hat{f}(X) = \frac{1}{1 + e^{-(\hat{w}_0 + \hat{w}_1 X_1 + \hat{w}_2 X_2)}}$$

모델 결정 → 파라미터 추정

$$\min_W \sum_{i=1}^N \{-Y_i \cdot \log(f(X_i)) - (1 - Y_i) \cdot \log(1 - f(X_i))\}$$

뉴럴네트워크 모델

$$f(X_i) = \frac{1}{1 + \exp \left(- \left(w_0 + w_1 \left(\frac{1}{1 + e^{-(w_{01} + w_{11}X_{1i} + w_{21}X_{2i})}} \right) \right) + w_2 \left(\frac{1}{1 + e^{-(w_{02} + w_{12}X_{1i} + w_{22}X_{2i})}} \right) \right)}$$

$$\min_{w_0, \dots, w_{22}} \sum_{i=1}^N \left\{ -Y_i \cdot \log \left(\frac{1}{1 + \exp \left(- \left(w_0 + w_1 \left(\frac{1}{1 + e^{-(w_{01} + w_{11}X_{1i} + w_{21}X_{2i})}} \right) \right) + w_2 \left(\frac{1}{1 + e^{-(w_{02} + w_{12}X_{1i} + w_{22}X_{2i})}} \right) \right)} \right) - (1 - Y_i) \cdot \log \left(1 - \frac{1}{1 + \exp \left(- \left(w_0 + w_1 \left(\frac{1}{1 + e^{-(w_{01} + w_{11}X_{1i} + w_{21}X_{2i})}} \right) \right) + w_2 \left(\frac{1}{1 + e^{-(w_{02} + w_{12}X_{1i} + w_{22}X_{2i})}} \right) \right)} \right) \right\}$$



Backpropagation algorithm
(오차역전파)

$$\hat{f}(X) = \frac{1}{1 + \exp \left(- \left(\hat{w}_0 + \hat{w}_1 \left(\frac{1}{1 + e^{-(\hat{w}_{01} + \hat{w}_{11}X_1 + \hat{w}_{21}X_2)}} \right) \right) + \hat{w}_2 \left(\frac{1}{1 + e^{-(\hat{w}_{02} + \hat{w}_{12}X_1 + \hat{w}_{22}X_2)}} \right) \right)}$$

모델 결정 → 파라미터 추정

$$f(X) = w_0 + w_1X_1 + w_2X_2 \quad \hat{f}(X) = \hat{w}_0 + \hat{w}_1X_1 + \hat{w}_2X_2$$

다중선형회귀 **모델** Least square estimation **algorithm**

$$f(X) = \frac{1}{1 + e^{-(w_0 + w_1X_1 + w_2X_2)}} \quad \hat{f}(X) = \frac{1}{1 + e^{-(\hat{w}_0 + \hat{w}_1X_1 + \hat{w}_2X_2)}}$$

로지스틱회귀 **모델** Conjugate gradient **algorithm**

$$f(X) = \frac{1}{1 + \exp\left(-\left(w_0 + w_1\left(\frac{1}{1 + e^{-(w_{01} + w_{11}X_1 + w_{21}X_2)}}\right) + w_2\left(\frac{1}{1 + e^{-(w_{02} + w_{12}X_1 + w_{22}X_2)}}\right)\right)\right)}$$

뉴럴네트워크 **모델** Backpropagation **algorithm**

$$\hat{f}(X) = \frac{1}{1 + \exp\left(-\left(\hat{w}_0 + \hat{w}_1\left(\frac{1}{1 + e^{-(\hat{w}_{01} + \hat{w}_{11}X_1 + \hat{w}_{21}X_2)}}\right) + \hat{w}_2\left(\frac{1}{1 + e^{-(\hat{w}_{02} + \hat{w}_{12}X_1 + \hat{w}_{22}X_2)}}\right)\right)\right)}$$

EOD