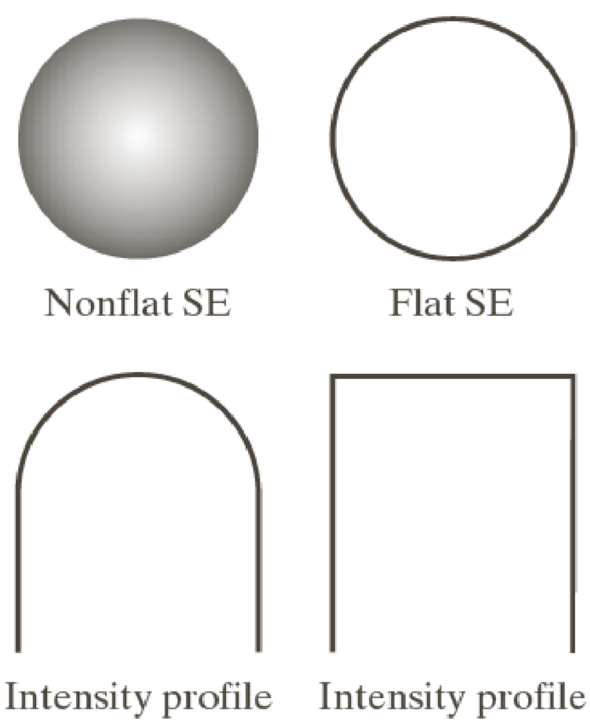
9.6 Gray-scale morphology

$f(x, y)$: **Input image**
$b(x, y)$: **Structuring element (SE) image**

Two types of SEs: non-flat and flat

Grey-scale SEs are used infrequently in practice



a b
c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

9.6.1 Erosion and dilation

The erosion of f by a flat SE b at (x, y) is defined as the minimum value of the image in the region coincident with b when the origin of b is at (x, y):

$$[f \ominus b](x, y) = \min_{(s,t)\in b} \{f(x+s, y+t)\}$$

The dilation of f by a flat SE b at (x, y) is defined as the maximum value of the image in the region coincident with $\hat{b}$ when the origin of $\hat{b}$ is at (x, y):

$$[f \oplus b](x, y) = \max_{(s,t)\in b} \{f(x-s, y-t)\}$$

Note: $\hat{b} = b(-x, -y)$

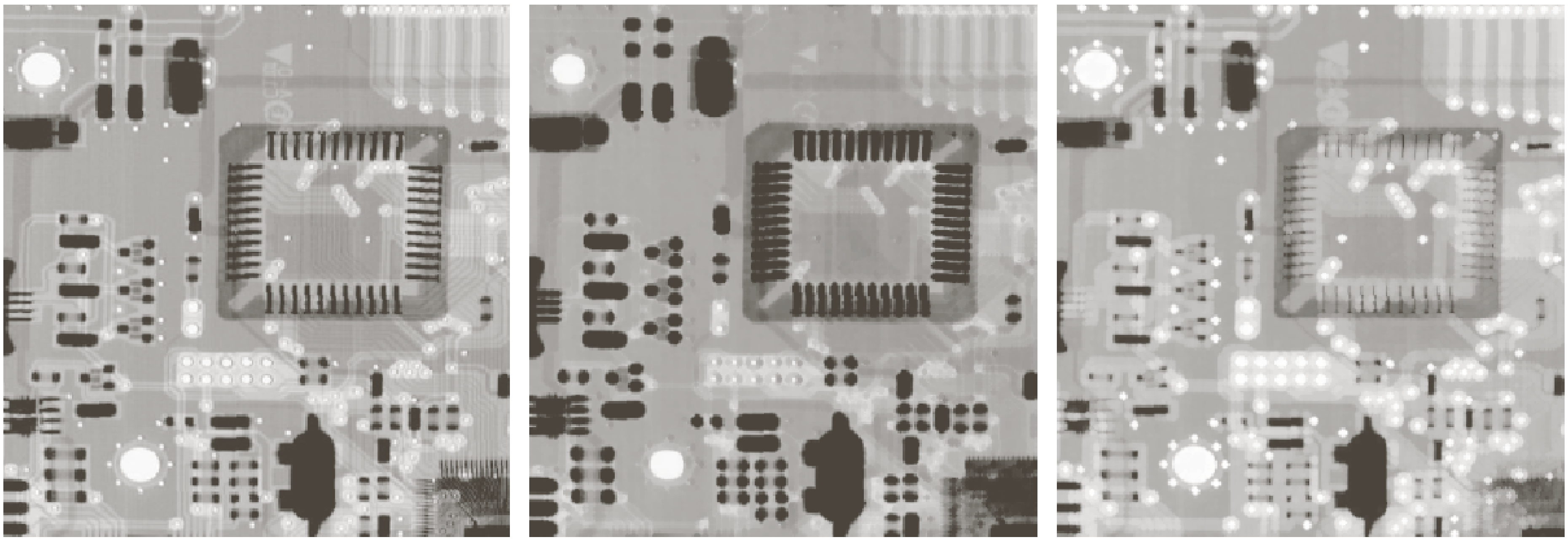
The erosion of f by a nonflat SE b_N at (x, y):

$$[f \ominus b_N](x, y) = \min_{(s,t)\in b_N} \{f(x+s, y+t) - b_N(s, t)\}$$

The dilation of f by a nonflat SE b_N at (x, y):

$$[f \oplus b](x, y) = \max_{(s,t)\in b_N} \{f(x-s, y-t) + b_N(s, t)\}$$

Example



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

9.6.2 Opening and closing

The opening of f by SE b: $\boxed{f \circ b = (f \ominus b) \oplus b}$

The closing of f by SE b: $\boxed{f \bullet b = (f \oplus b) \ominus b}$

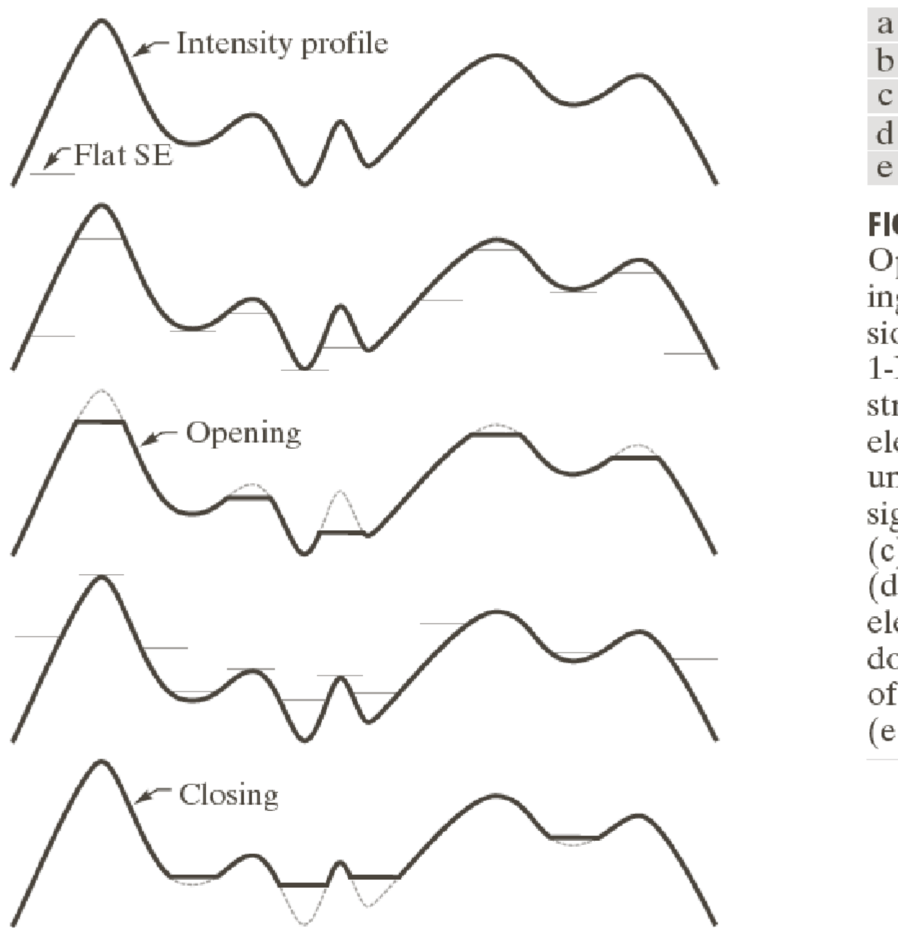


FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

Example

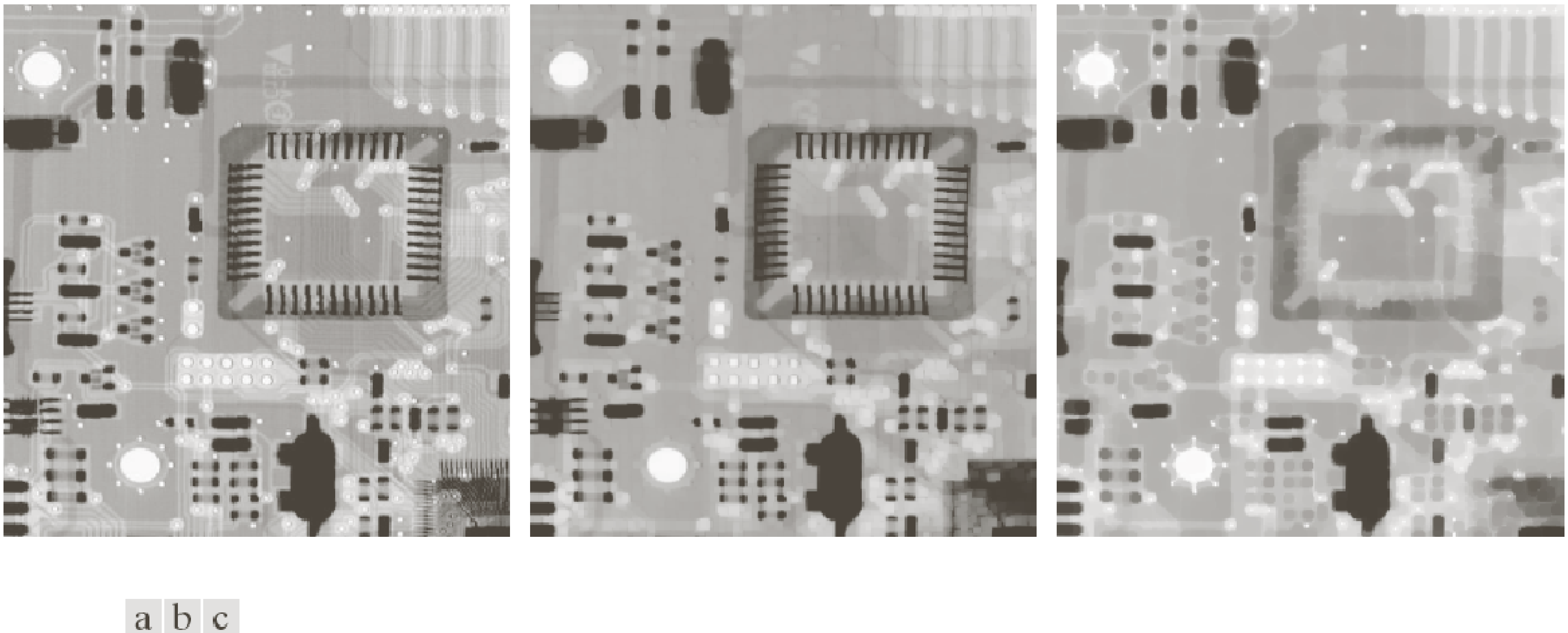
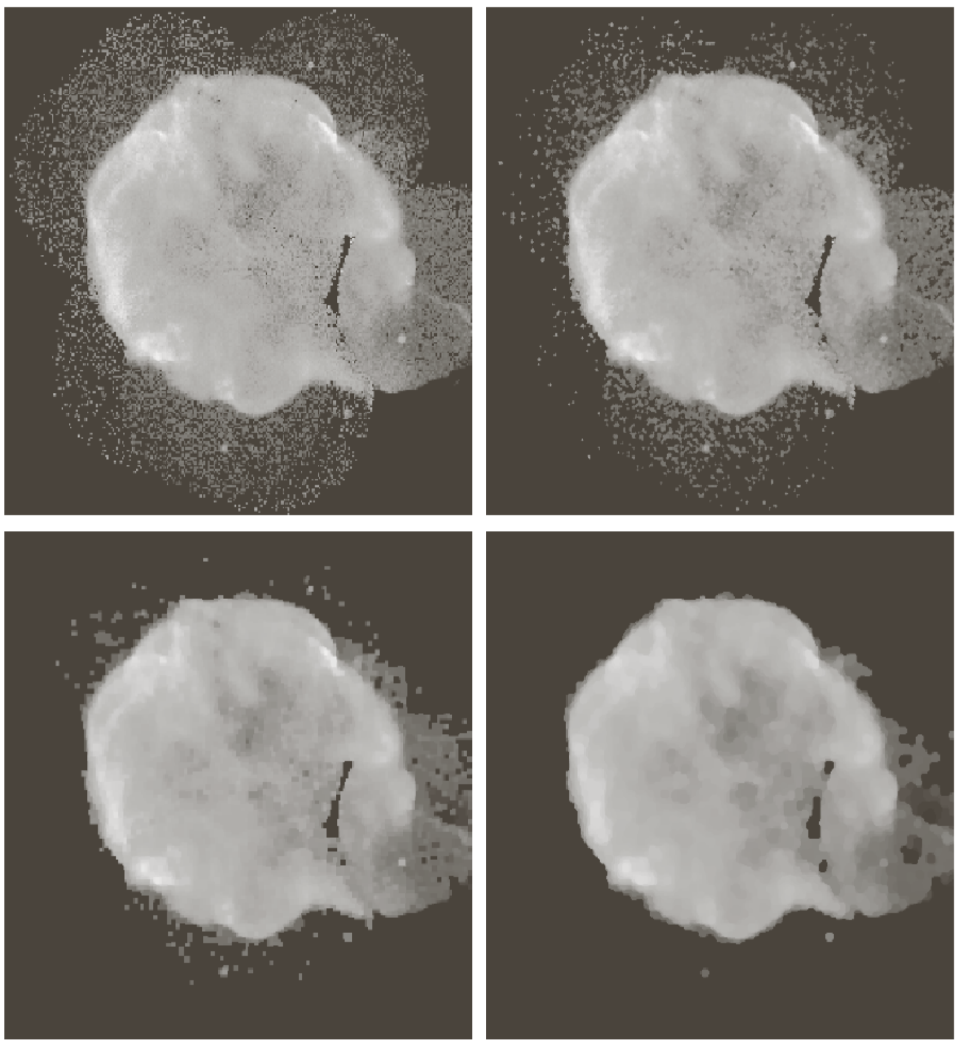


FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

9.6.3 Some basic gray-scale morphological algorithms

Morphological smoothing

Example



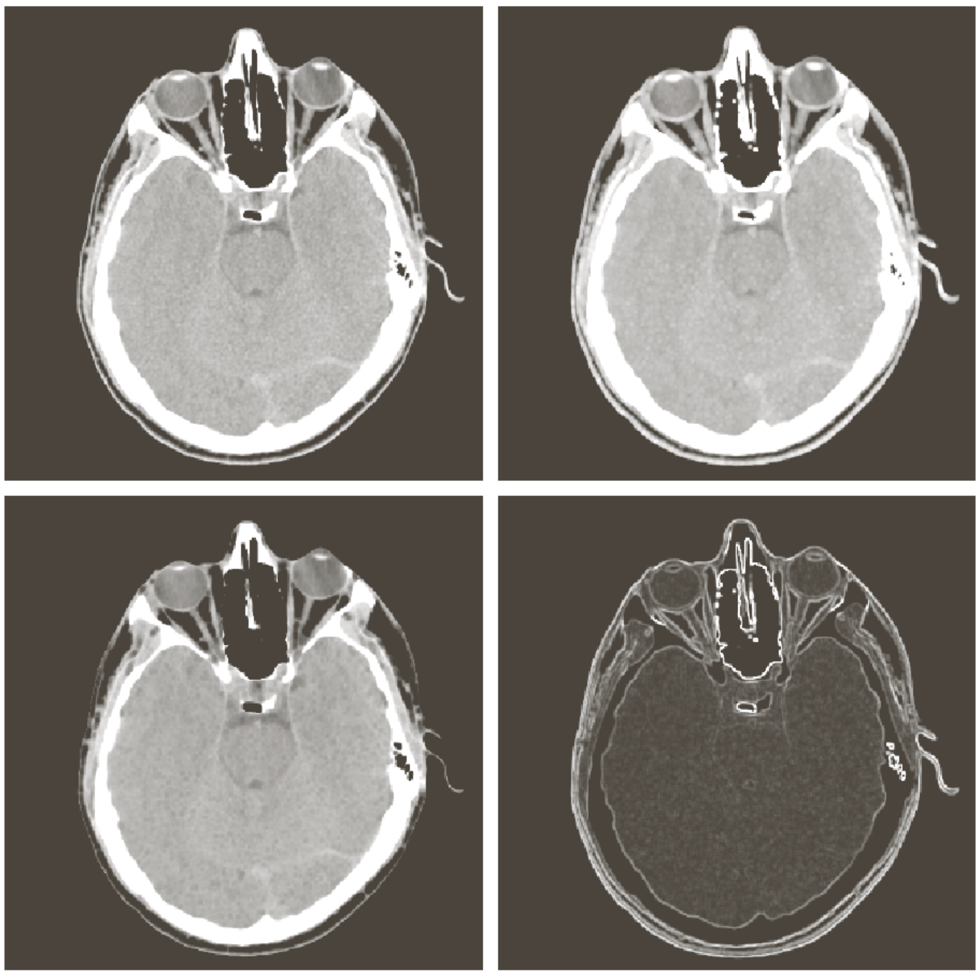
a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Morphological gradient

$$g = (f \oplus b) - (f \ominus b)$$

Example



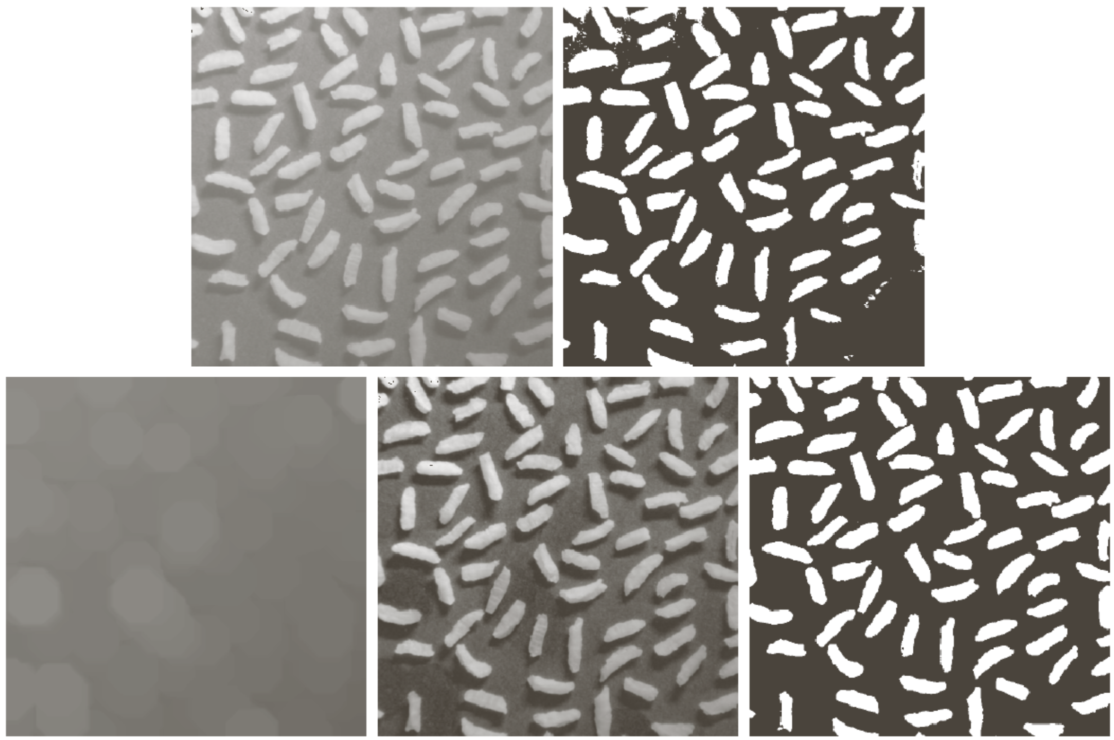
a b
c d

FIGURE 9.39 (a) 512×512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and bottom-hat transformations

$$\boxed{T_{\text{hat}} = f - (f \circ b)} \qquad \boxed{B_{\text{hat}} = (f \bullet b) - f}$$

Example



a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Granulometry

Granulometry: Field that deals with determining the size distribution of particles in an image

Algorithm:

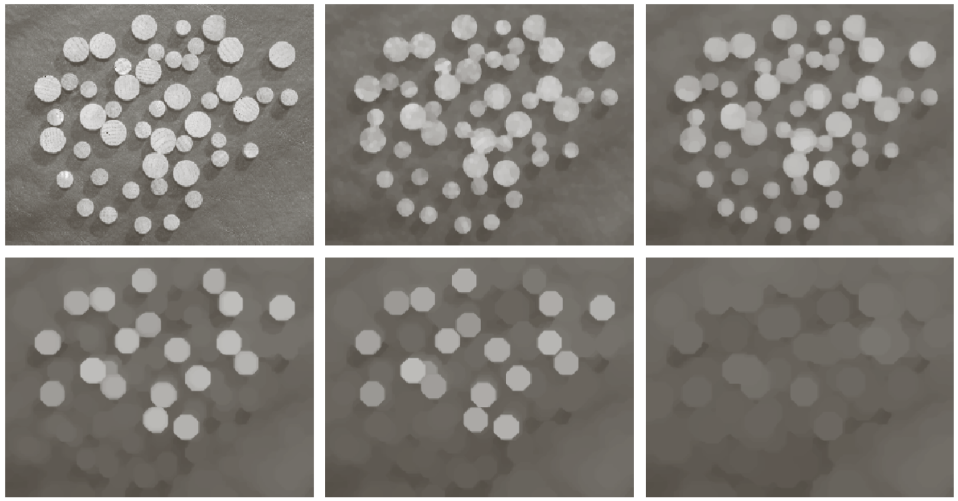
(1) Open with structuring elements of increasing size

(2) For each opening, calculate the sum (surface area) of pixel values

(3) Construct array of differences between successive surface areas

(4) Prominent peaks in difference array indicate predominant particle sizes

Example



a b c
d e f

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

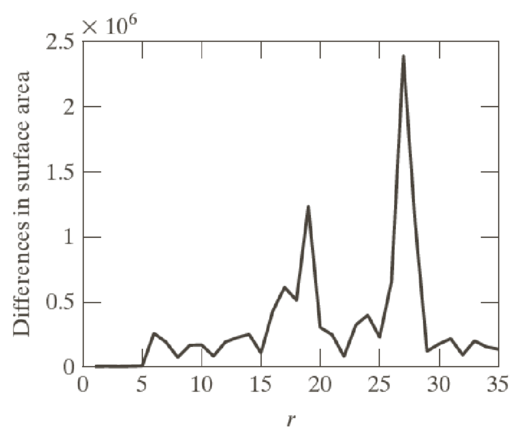


FIGURE 9.42 Differences in surface area as a function of SE disk radius, r. The two peaks are indicative of two dominant particle sizes in the image.

Textural segmentation

Input: Two texture regions

Output: Boundary between the two regions

Algorithm:

(1) Close input image using successively larger SEs. When size(SE) $\approx$ size(small blobs), blobs are removed

(2) Single opening with SE that is large in relation to separation between large blobs $\leadsto$ light patches between blobs removed $\leadsto$ light region on left, dark region on right

(3) Morphological gradient operation $\leadsto$ boundary

Example

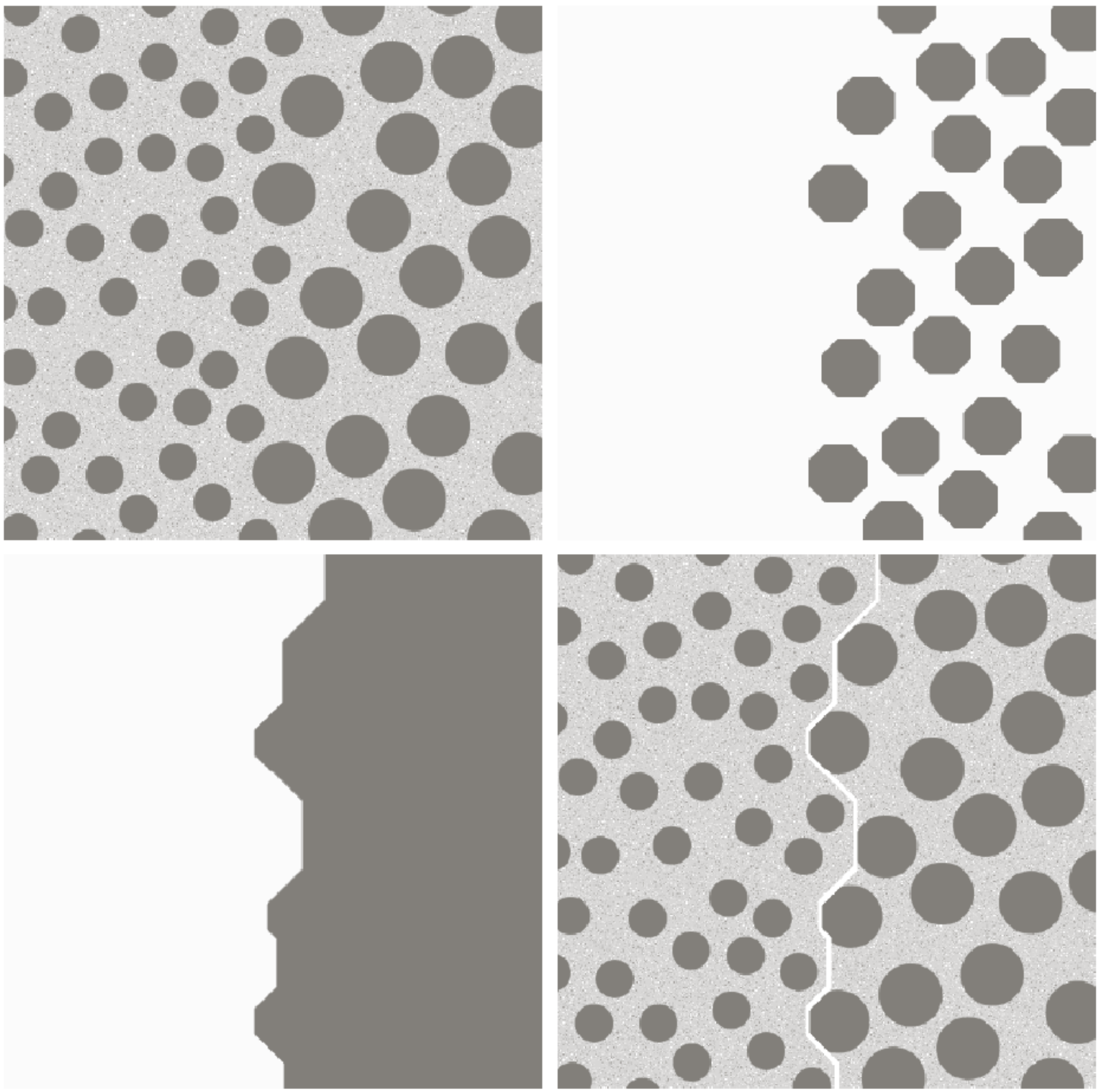
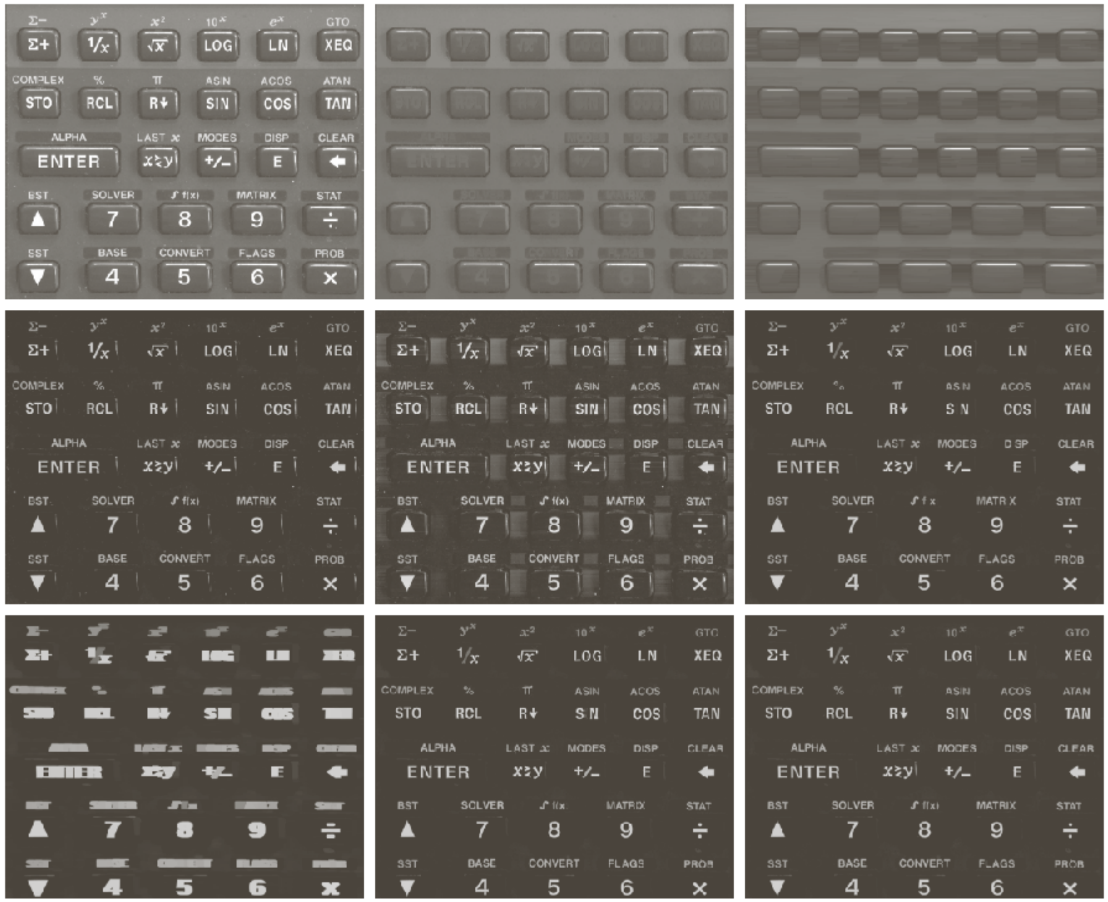


FIGURE 9.43 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

9.6.4 Gray-scale morphological reconstruction



a b c
d e f
g h i

FIGURE 9.44 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a) using a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same line. (d) Top-hat by reconstruction. (e) Top-hat. (f) Opening by reconstruction of (d) using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

Homework

due: next week

a5-mophorlogicalfilter