## CHAPTER 8: Image compression (page 547)

**Definition: Reduction of amount of data used to represent an image by reducing redundant data, so that the image can be stored or transferred more efficiently**

$$\textbf{Compression:} \begin{cases} \textbf{Lossless coding:} & \textbf{Error free compression} \\ \textbf{Lossy coding:} & \textbf{Error containing compression} \end{cases}$$

**Motivation: Amount of data required to represent a $2$-hour SD TV movie...**

$$30\,\frac{\textbf{frames}}{\textbf{sec}} \times (720 \times 480)\,\frac{\textbf{pixels}}{\textbf{frame}} \times 3\,\frac{\textbf{bytes}}{\textbf{pixel}} = 31,104,000 \textbf{ bytes/sec}$$

$$31,104,000\,\frac{\textbf{bytes}}{\textbf{sec}} \times (60)^2\,\frac{\textbf{sec}}{\textbf{hour}} \times 2 \textbf{ hours} \approx 2.24 \times 10^{11} \textbf{ bytes} \approx 224 \textbf{ GB}$$

**Twenty-seven $8.5$ GB dual-layer DVDs are required to store this! To store this on a single DVD, a compression factor of $26.3$ is required. Even more compression is required for HD TV ($1920 \times 1080 \times 3$ bytes/image)**

**Other applications: Web page images, televideo conferencing, remote sensing, document and medical imaging, FAX**

## 8.1: Fundamentals

- Objective is to get rid of redundant data (data $\neq$ information)
  - Compression ratio: $C = \dfrac{b}{b'}$
  - Relative data redundancy: $R = 1 - \dfrac{1}{C}$

$$\left.\begin{array}{rl} b = & \text{amount of data in uncompressed data set} \\ b' = & \text{amount of data in compressed data set} \end{array}\right\} \approx \text{the same information}$$

If $b' = b$, then $C = 1$ and $R = 0$
If $b : b' = 10 : 1$, then $R = 0.9 \Rightarrow 90$% of data redundant

Three types of redundancy:
- (1) Coding redundancy
- (2) Spatial and temporal redundancy
- (3) Irrelevant information

### Coding redundancy

Code $\equiv$ system of symbols (letters, numbers) that represent information (set of events)

Piece of information (event) assigned sequence of code symbols $\Rightarrow$ code word

Number of symbols in a code word is its length

**Usually the $8$-bit codes that represent image intensities contain more bits than required**

## Spatial and temporal redundancy

**Pixels in images are often correlated spatially and information is unnecessarily replicated**
**In video sequences, temporally correlated pixels also duplicate information**

## Irrelevant information

**Images often contain redundant information that is ignored by the human visual system (psycho-visual redundancy)**

a b c

**FIGURE 8.1** Computer generated 256 × 256 × 8 bit images with (a) coding redundancy, (b) spatial redundancy, and (c) irrelevant information. (Each was designed to demonstrate one principal redundancy but may exhibit others as well.)

## 8.1.1 Coding redundancy

**Consider the histogram**

$$p_r(r_k) = \frac{n_k}{MN}, \quad k = 0, 1, \ldots, L-1$$

**and let**

$\ell(r_k) \equiv$ **number of bits used to represent the gray scale value** $r_k$**,**

**then**

$L_{\text{ave}} = \sum_{k=0}^{L-1} \ell(r_k)\, p(r_k) \equiv$ **average number of bits required to represent a pixel in the image**

**Therefore, for an** $M \times N$ **image, we have that**

$MNL_{\text{ave}} \equiv$ **total number a bits required to code the entire image**

**When each grey scale value is represented by an** $m$**-bit (natural) binary code, then** $L_{\text{ave}} = m$

## Example 8.1: Variable length coding (reversible)

| $r_k$ | $p_r(r_k)$ | Code 1 | $l_1(r_k)$ | Code 2 | $l_2(r_k)$ |
|---|---|---|---|---|---|
| $r_{87} = 87$ | 0.25 | 01010111 | 8 | 01 | 2 |
| $r_{128} = 128$ | 0.47 | 10000000 | 8 | 1 | 1 |
| $r_{186} = 186$ | 0.25 | 11000100 | 8 | 000 | 3 |
| $r_{255} = 255$ | 0.03 | 11111111 | 8 | 001 | 3 |
| $r_k$ for $k \neq 87, 128, 186, 255$ | 0 | — | 8 | — | 0 |

**TABLE 8.1** Example of variable-length coding.

**Natural** $8$**-bit code:** $L_{\text{ave}} = 8$ **bits**

**Variable length code:** $L_{\text{ave}} = 0.25(2) + 0.47(1) + 0.25(3) + 0.03(3) = 1.81$ **bits**

$$C = \frac{8}{1.81} \approx 4.42$$

$$R = 1 - \frac{1}{4.42} = 0.774 \quad \Rightarrow \quad 77.4\% \text{ of data is redundant}$$

**Strategy: When** $p(r_k)$ **is large,** $\ell_2(r_k)$ **should be small, and vice versa**

**Note: Best fixed-length code is** $\{00, 01, 10, 11\}$ **– the resulting compression is** $4:1$**, which is still 10% less than** $4.42:1$ **for variable-length code**

## 8.1.2 Spatial and temporal redundancy

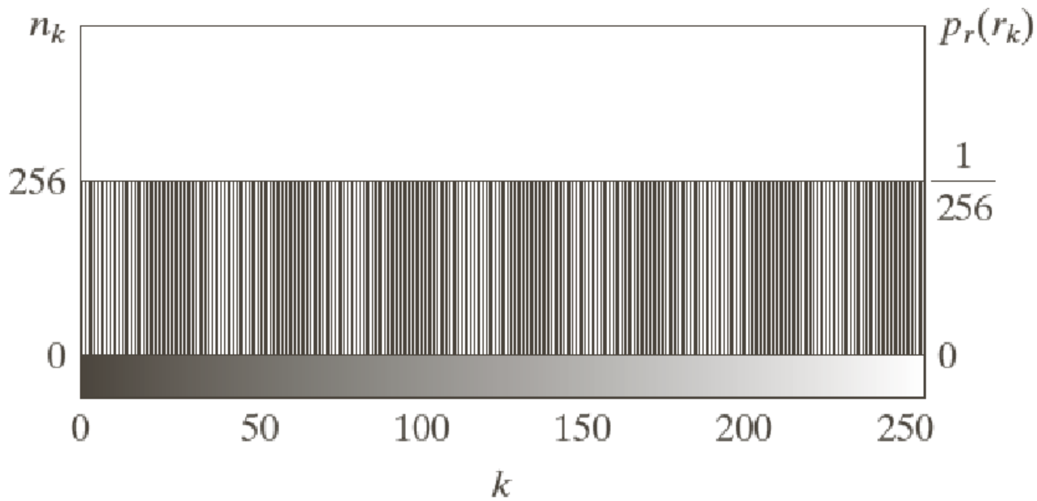



**FIGURE 8.2** The intensity histogram of the image in Fig. 8.1(b).

**(1) All intensities equally probable**

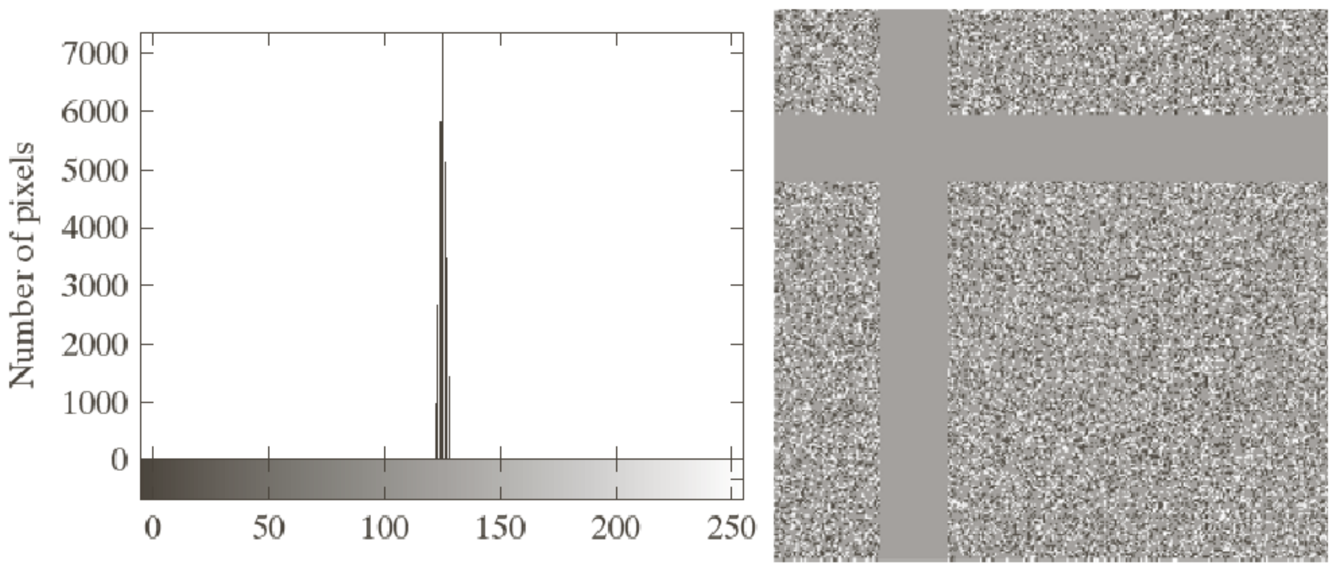
**(2) Line intensities random $\Rightarrow$ pixels independent in vertical direction**

**(3) Pixels in a specific line are maximally correlated (horizontally)**

- **Fig 8.1 (b) can not be compressed by variable-length coding alone**
- **Spatial redundancy can be eliminated by utilising run-length pairs ($128:1$) or the differences between adjacent pixels (reversible mappings)**
- **In most images, the pixels are correlated spatially ($x$ and $y$ directions), and in time (videos)**

## 8.1.3 Irrelevant information

**Fig 8.1 (c): "Appears" homogeneous $\Rightarrow$ compression of $65\,536 : 1$ may be possible!**




a b

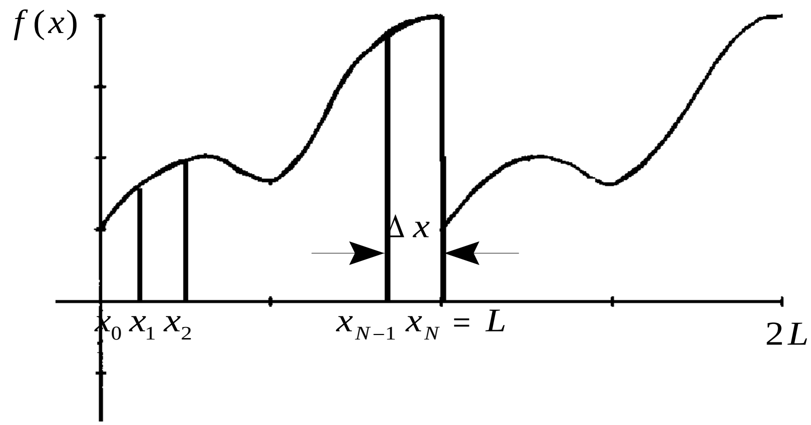
**FIGURE 8.3** (a) Histogram of the image in Fig. 8.1(c) and (b) a histogram equalized version of the image.

**Visually redundant data may be removed depending on the application**

**This removal of data is <u>irreversible</u> and referred to as <u>quantization</u>**

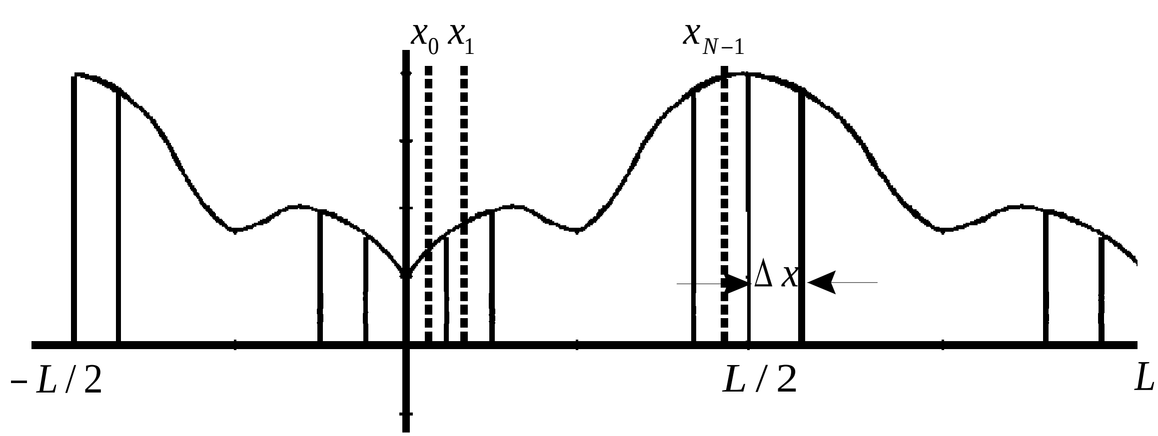
## The Discrete Cosine Transform (DCT)

**We derived the discrete Fourier transform using the example**




**In general, the periodic continuation of $f(x)$ is discontinuous, which implies that the Fourier coefficients $c_n$ decrease like $1/n$**

**We therefore rather consider the following**



**The periodic continuation of $f(x)$ is now guaranteed to be continuous, which implies that the Fourier coefficients $c_n$ will decrease at least like $1/n^2$**

$$\begin{aligned}
a_0 &= \frac{1}{L}\int_{-L/2}^{L/2} f(x)\,dx \\
&= \frac{2}{L}\int_{0}^{L/2} f(x)\,dx \\
&\approx \frac{2\Delta x}{L}\left[f(x_0) + f(x_1) + \ldots + f(x_{N-1})\right] \quad \textbf{(Rectangle rule)} \\
&= \frac{2\Delta x}{L}\sum_{j=0}^{N-1} f(x_j)
\end{aligned}$$

**But**

$$\Delta x = (L/2)/N = L/(2N) \;\Rightarrow\; (2\Delta x)/L = 1/N,$$

**therefore**

$$\hbar_0 = a_0 = \frac{1}{N}\sum_{j=0}^{N-1} f_j$$

**For** $n = 1, 2, \ldots, N-1$**, we have that**

$$\begin{aligned}
a_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(x)\cos\left\{\frac{2n\pi x}{L}\right\}\,dx \\
&= \frac{4}{L}\int_{0}^{L/2} f(x)\cos\left\{\frac{2n\pi x}{L}\right\}\,dx \\
&\approx \frac{4\,\Delta x}{L}\left\{f(x_0)\cos\left\{\frac{2n\pi x_0}{L}\right\} + \ldots + f(x_{N-1})\cos\left\{\frac{2n\pi x_{N-1}}{L}\right\}\right\} \\
&= \frac{4\,\Delta x}{L}\sum_{j=0}^{N-1} f_j\cos\left\{\frac{2n\pi x_j}{L}\right\}
\end{aligned}$$

**But** $\dfrac{2\,\Delta x}{L} = \dfrac{1}{N} \Rightarrow \dfrac{4\,\Delta x}{L} = \dfrac{2}{N}$ **and** $x_j = \left\{\dfrac{2j+1}{2}\right\}\Delta x,$ **that is**

$$\begin{aligned}
x_0 &= (1/2)\,\Delta x \\
x_1 &= (3/2)\,\Delta x \\
x_2 &= (5/2)\,\Delta x, \quad \textbf{etc.}
\end{aligned}$$

**Therefore**

$$x_j = \left\{\frac{2j+1}{2}\right\}\frac{L}{2N} \quad \Rightarrow \quad \frac{2x_j}{L} = \frac{(2j+1)}{2N}$$

**and**

$$\hbar_n = a_n = \frac{2}{N}\sum_{j=0}^{N-1} f_j \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\},\ n = 1, 2, \ldots, N-1$$

**To summarize...**

$$\hbar_n = \alpha_n^2 \sum_{j=0}^{N-1} f_j \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\}$$

**where**

$$\alpha_n^2 = \begin{cases} \dfrac{1}{N}, & n = 0 \\ \\ \dfrac{2}{N}, & n = 1, 2, \ldots, N-1 \end{cases}$$

**Multiply on both sides with** $\cos\left\{\dfrac{(2k+1)\,n\pi}{2N}\right\}$ **and** $\sum\limits_{n=0}^{N-1}$**:**

$$\sum_{n=0}^{N-1} \hbar_n \;\cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\}$$

$$= \sum_{n=0}^{N-1} \alpha_n^2 \sum_{j=0}^{N-1} f_j \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\} \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\}$$

$$= \sum_{j=0}^{N-1} f_j \sum_{n=0}^{N-1} \alpha_n^2 \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\} \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\}$$

**It can be shown that**

$$\sum_{n=0}^{N-1} \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right] \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\} = \begin{cases} \frac{N+1}{2}, & j = k \\ \frac{1}{2}, & j \neq k \end{cases}$$

**thus**

$$\sum_{n=0}^{N-1} \alpha_n^2 \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\} \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\} = \begin{cases} \left\{\frac{2}{N}\right\}\left\{\frac{N+1}{2}\right\} - \frac{1}{N} = 1, & j = k \\ \\ \left\{\frac{2}{N}\right\}\left\{\frac{1}{2}\right\} - \frac{1}{N} = 0, & j \neq k \end{cases}$$

**This implies that**

$$\sum_{n=0}^{N-1} \hbar_n \;\cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\} = f_k \;\times\; 1$$

**and**

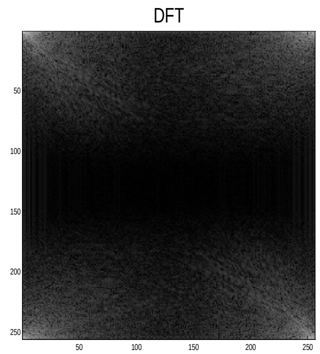
$$f_j = \sum_{n=0}^{N-1} \hbar_n \;\cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\}$$

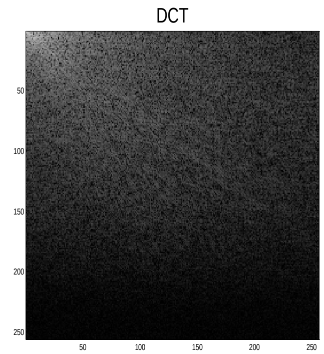
**Now let** $C_n = \dfrac{1}{\alpha_n}\hbar_n$**, then**

$$\begin{aligned} C_n &= \textbf{DCT}\{\, f_j \,\} &= \alpha_n \sum_{j=0}^{N-1} f_j \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\} \\ f_j &= \textbf{DICT}\{\, C_n \,\} &= \sum_{n=0}^{N-1} \alpha_n\, C_n \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\} \end{aligned}$$

**where**

$$\alpha_n = \begin{cases} \sqrt{\dfrac{1}{N}}, & n = 0 \\[2em] \sqrt{\dfrac{2}{N}}, & n = 1, 2, \ldots, N-1 \end{cases}$$




## 8.1.4 Measuring image information (READ)

## 8.1.5 Fidelity criteria

**Objective fidelity criteria**

### Root-mean-square error

$$e_{\mathrm{rms}} = \left[ \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[ \hat{f}(x,y) - f(x,y) \right]^2 \right]^{1/2}$$

### Mean-square signal-to-noise ratio

$$\mathrm{SNR}_{\mathrm{ms}} = \frac{\displaystyle\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\displaystyle\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[ \hat{f}(x,y) - f(x,y) \right]^2}$$

## Subjective fidelity criteria

- **Often more appropriate!!!**

| Value | Rating | Description |
|---|---|---|
| 1 | Excellent | An image of extremely high quality, as good as you could desire. |
| 2 | Fine | An image of high quality, providing enjoyable viewing. Interference is not objectionable. |
| 3 | Passable | An image of acceptable quality. Interference is not objectionable. |
| 4 | Marginal | An image of poor quality; you wish you could improve it. Interference is somewhat objectionable. |
| 5 | Inferior | A very poor image, but you could watch it. Objectionable interference is definitely present. |
| 6 | Unusable | An image so bad that you could not watch it. |

**TABLE 8.2** Rating scale of the Television Allocations Study Organization. (Frendendall and Behrend.)