



## 9.5.9 Morphological reconstruction

- Involves two images and a structuring element
- Images:
  - (1) Marker (contains starting points); (2) Mask (constrains transformation)

### Geodesic dilation and erosion

$B \equiv$  Structuring element;  $F \equiv$  Marker image;  $G \equiv$  Mask image

**Geodesic dilation of size 1** (of  $F$  with respect to  $G$ ):

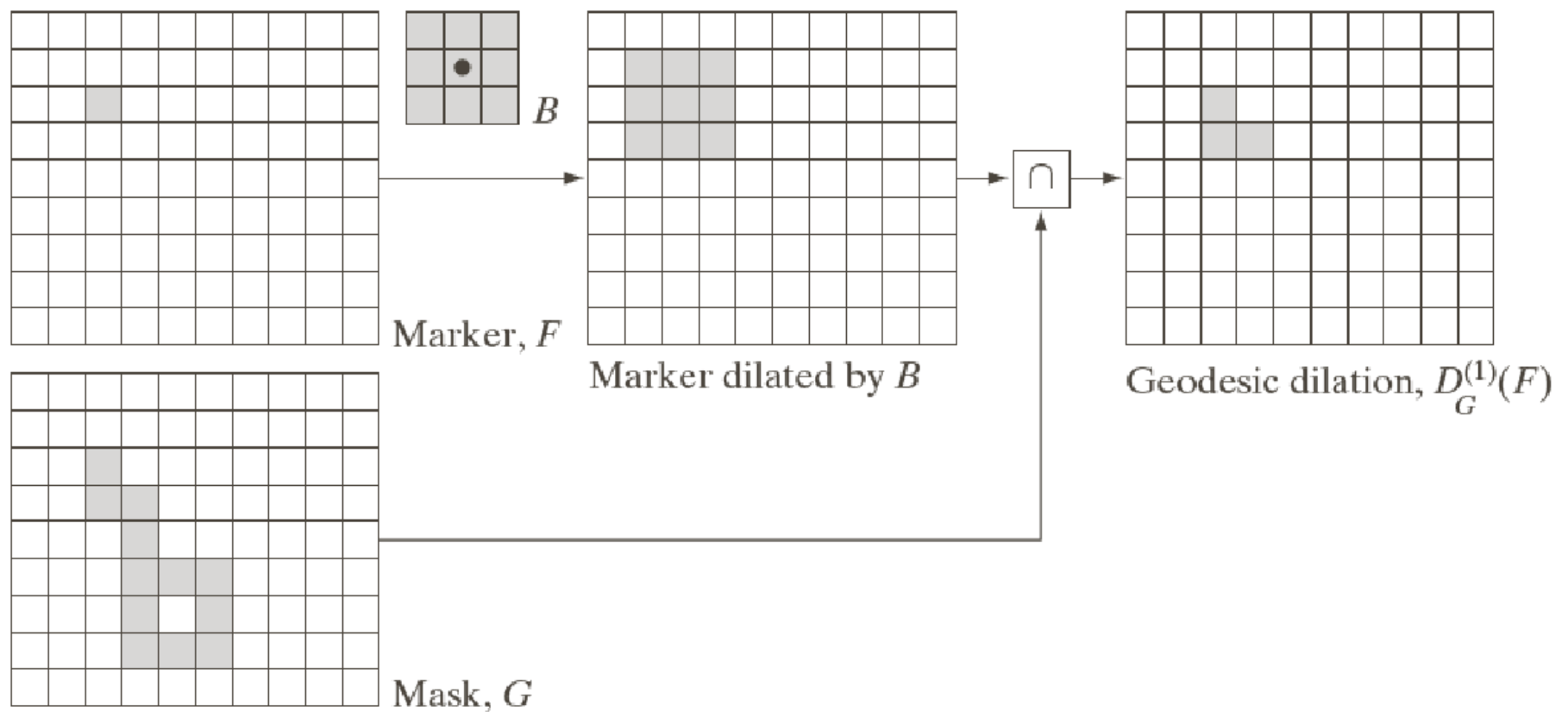
$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

**Geodesic dilation of size  $n$**  (of  $F$  with respect to  $G$ ):

$$D_G^{(n)}(F) = D_G^{(1)}[D_G^{(n-1)}(F)], \quad D_G^{(0)}(F) = F$$



## Illustration of geodesic dilation





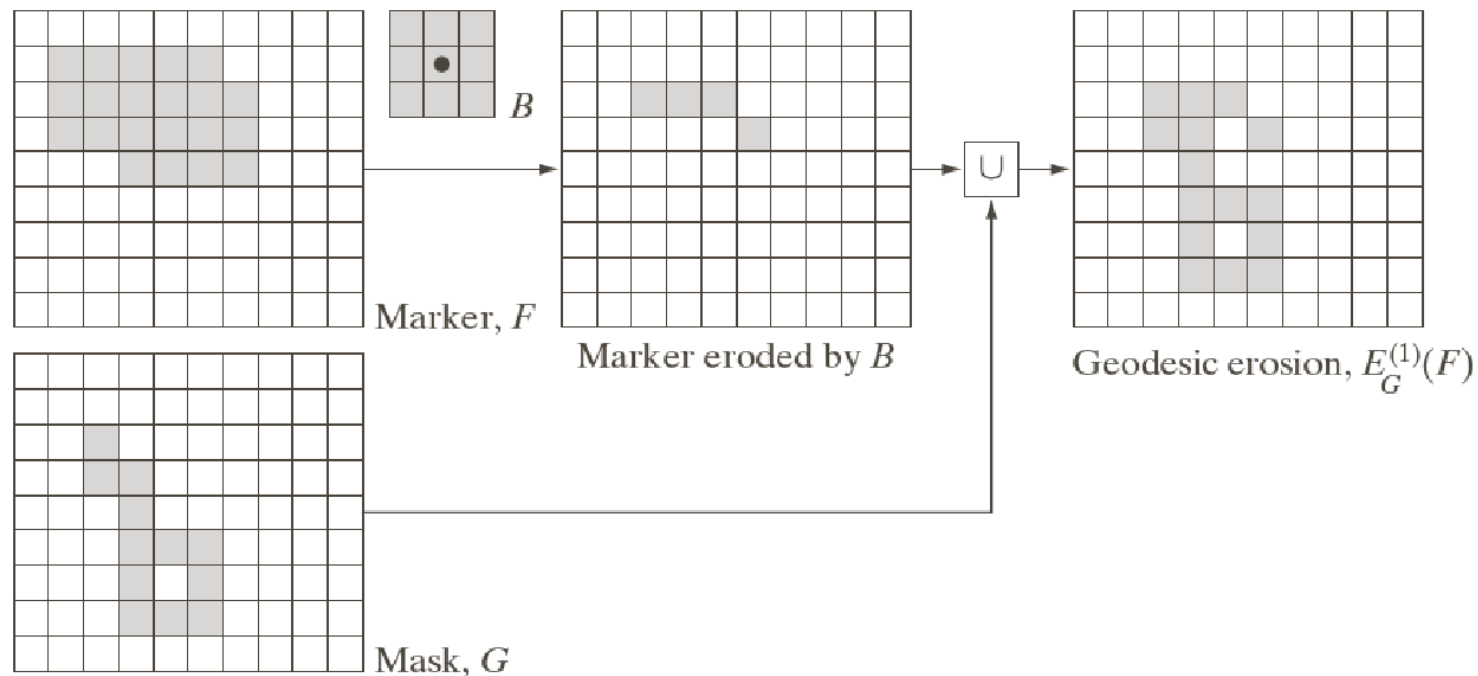
**Geodesic erosion of size 1 (of  $F$  with respect to  $G$ ):**

$$E_G^{(1)}(F) = (F \ominus B) \bigcup G$$

**Geodesic erosion of size  $n$  (of  $F$  with respect to  $G$ ):**

$$E_G^{(n)}(F) = E_G^{(1)}[E_G^{(n-1)}(F)], \quad E_G^{(0)}(F) = F$$

### Illustration of geodesic erosion



## Morphological reconstruction by dilation

- Geodesic dilation of  $F$  with respect to  $G$  iterated until stability is achieved

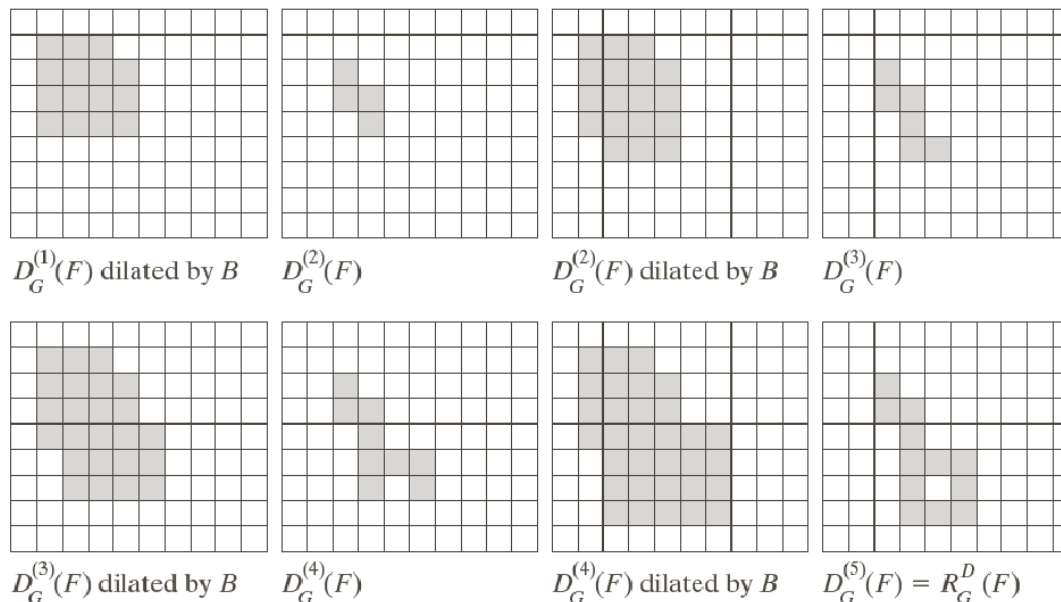
$$R_G^D(F) = D_G^{(k)}(F), \quad \text{with } k \text{ such that } D_G^{(k)}(F) = D_G^{(k+1)}(F)$$

## Morphological reconstruction by erosion

- Geodesic erosion of  $F$  with respect to  $G$  iterated until stability is achieved

$$R_G^E(F) = E_G^{(k)}(F), \quad \text{with } k \text{ such that } E_G^{(k)}(F) = E_G^{(k+1)}(F)$$

## Illustration of morphological reconstruction by dilation



a	b	c	d
e	f	g	h

**FIGURE 9.28**  
Illustration of morphological reconstruction by dilation.  $F$ ,  $G$ ,  $B$  and  $D_G^{(1)}(F)$  are from Fig. 9.26.



## Sample applications

### Opening by reconstruction

**[Previously]** Morphological opening — Erosion removes small objects & subsequent dilation attempts to restore remaining objects

**[Now]** Opening by reconstruction — Restores exactly the shape of objects that remain after erosion

**DEF:** Opening by reconstruction of size  $n$  of image  $F$  is defined as the reconstruction by dilation of  $F$  from the erosion of size  $n$  of  $F$ , that is,

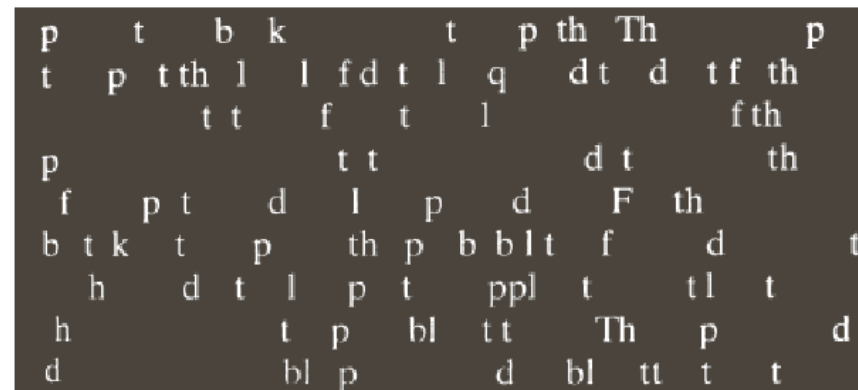
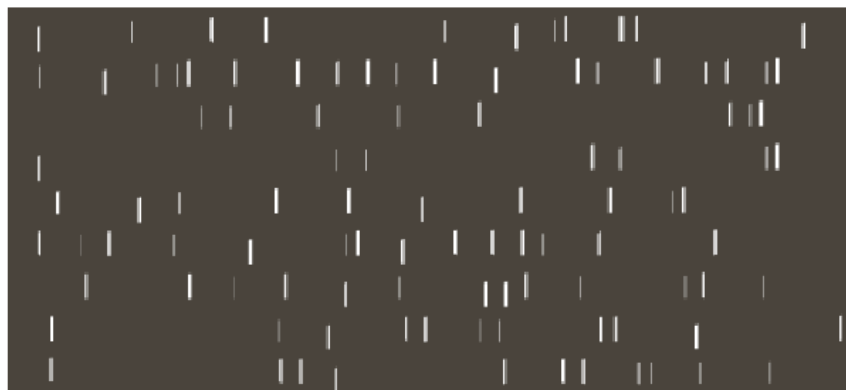
$$O_R^{(n)}(F) = R_F^D[(F \ominus nB)],$$

where  $(F \ominus nB)$  indicates  $n$  erosions of  $F$  by  $B$

## Example

ponents or broken connection paths. There is no position past the level of detail required to identify those

Segmentation of nontrivial images is one of the most difficult tasks in image processing. Segmentation accuracy determines the effectiveness of computerized analysis procedures. For this reason, considerable effort must be taken to improve the probability of rugged segmentation. This is especially true in such applications as industrial inspection applications, at least some of which are possible at times. The experienced image processing designer invariably pays considerable attention to such



a	b
c	d

**FIGURE 9.29** (a) Text image of size  $918 \times 2018$  pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size  $51 \times 1$  pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.



## Filling holes

**[Previously]** Section 9.5.2 — Algorithm for filling holes based on knowing a starting point in each hole

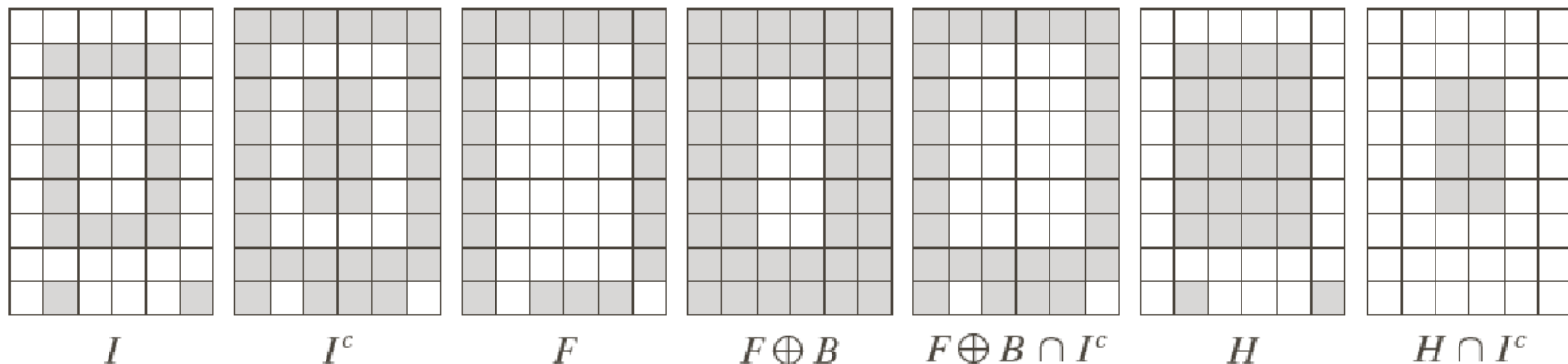
**[Now]** Morphological reconstruction — Fully automated procedure

$I \equiv$  Binary image;  $F \equiv$  Marker image (0 everywhere, except at image border), where it is set to  $1 - I$ , that is

$$F(x, y) = \begin{cases} 1 - I(x, y), & \text{if } (x, y) \text{ on image border of } I \\ 0, & \text{otherwise} \end{cases}$$

The binary image equal to  $I$ , but with all the holes filled, is then given by

$$H = [R_{I^c}^D(F)]^c$$





## Example

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**FIGURE 9.31**  
(a) Text image of size  $918 \times 2018$  pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).





## Border clearing

### Applications:

- (1) Only complete objects remain for further processing
- (2) Signal that partial objects are present in field of view

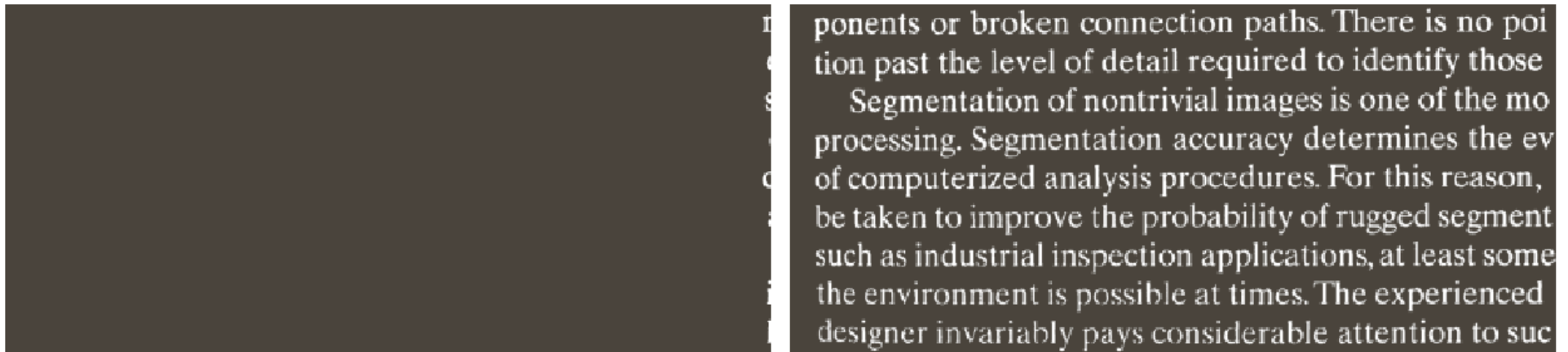
### Procedure:

$$F(x, y) = \begin{cases} 1 - I(x, y), & \text{if } (x, y) \text{ on image border of } I \\ 0, & \text{otherwise} \end{cases}$$

$$X = I - R_I^D(F)$$

$X \equiv$  Image with no objects touching border

## Example



a b

### FIGURE 9.32

Border clearing.  
(a) Marker image.  
(b) Image with no  
objects touching  
the border. The  
original image is  
Fig. 9.29(a).