CHAPTER 9: Morphological image processing

- **Language of mathematical morphology: set theory**
- **Sets** $\equiv$ **objects in an image**
- **Binary images: sets** $\in Z^2$
- **Gray-scale images: sets** $\in Z^3$

9.1 Preliminaries

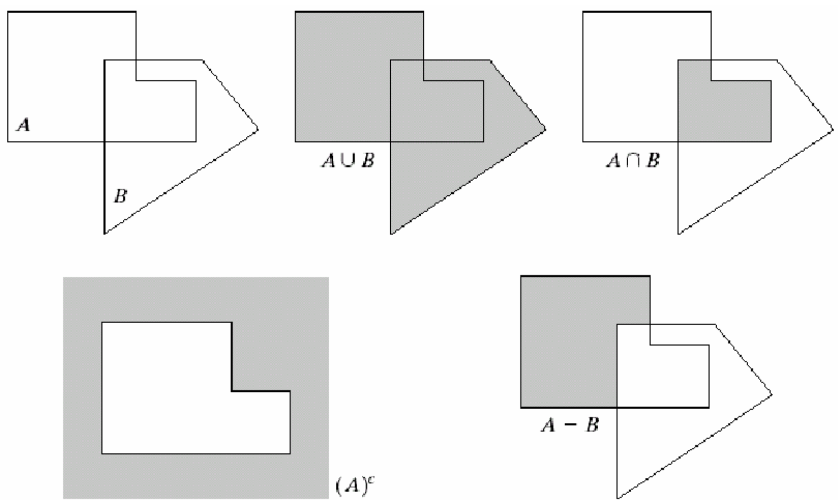
- **Let** A **be a set in** Z^2**. If** $a = (a_1, a_2)$ **is an element of** A**, then we write** $a \in A$
- **Subset, union, intersection:**

$$A \subseteq B,\ C = A \bigcup B,\ D = A \bigcap B$$

- **Disjoint or mutually exclusive:** $A \bigcap B = \emptyset$
- **Complement:** $A^c = \{w | w \notin A\}$
- **Difference:** $A - B = \{w | w \in A, w \notin B\} = A \bigcap B^c$

- $\boxed{\textbf{Reflection:}}$ $\hat{B} = \{w | w = -b, \text{ for } b \in B\}$
- $\boxed{\textbf{Translation}}$ **of set** A **by point** $z = (z_1, z_2)$**:** $(A)_z = \{c | c = a + z, \text{ for } a \in A\}$

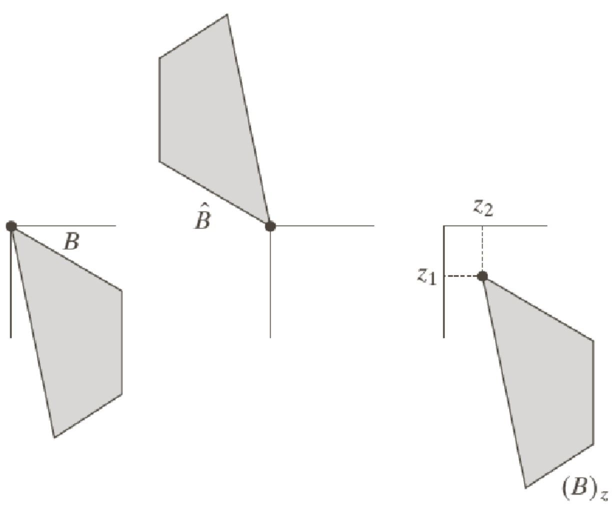
(Ed 2)



a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

(Ed 3)



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

- **Reflection and translation are employed to formulate operations based on structuring elements (SEs): small sets (subimages) used to probe an image for properties of interest**

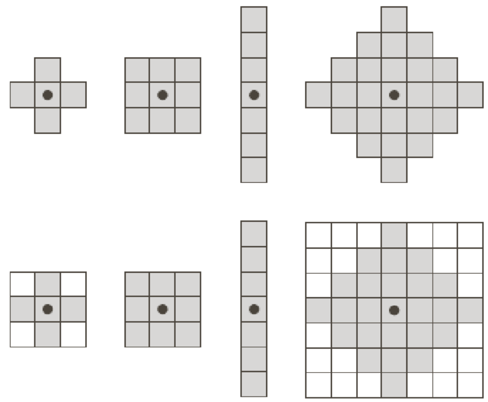


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

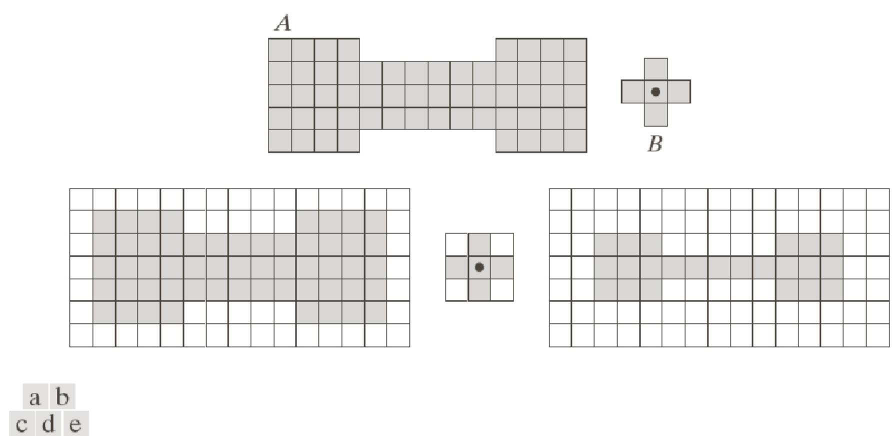


FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

9.2 Erosion and dilation

These operations are fundamental to morphological processing

9.2.1 Erosion

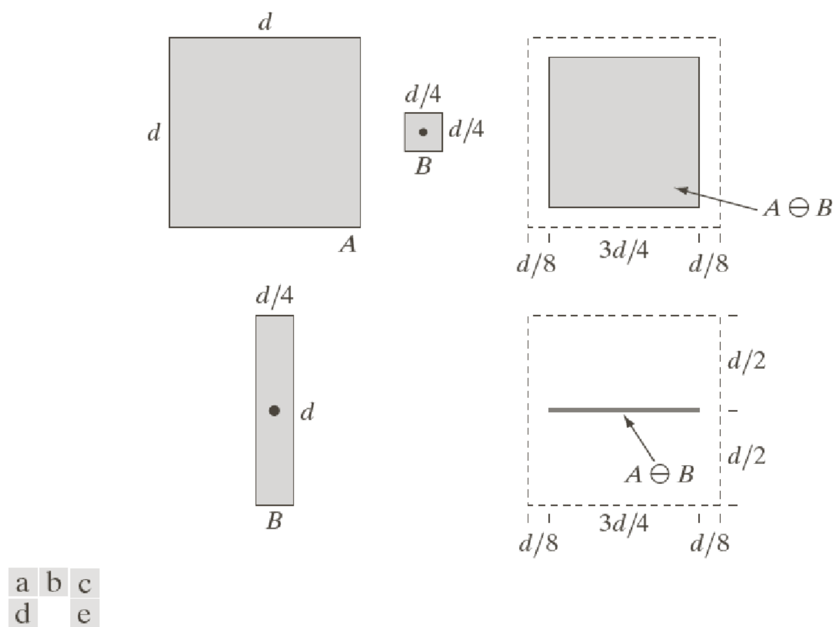
With A and B sets in Z^2, the erosion of A by B, is defined as

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

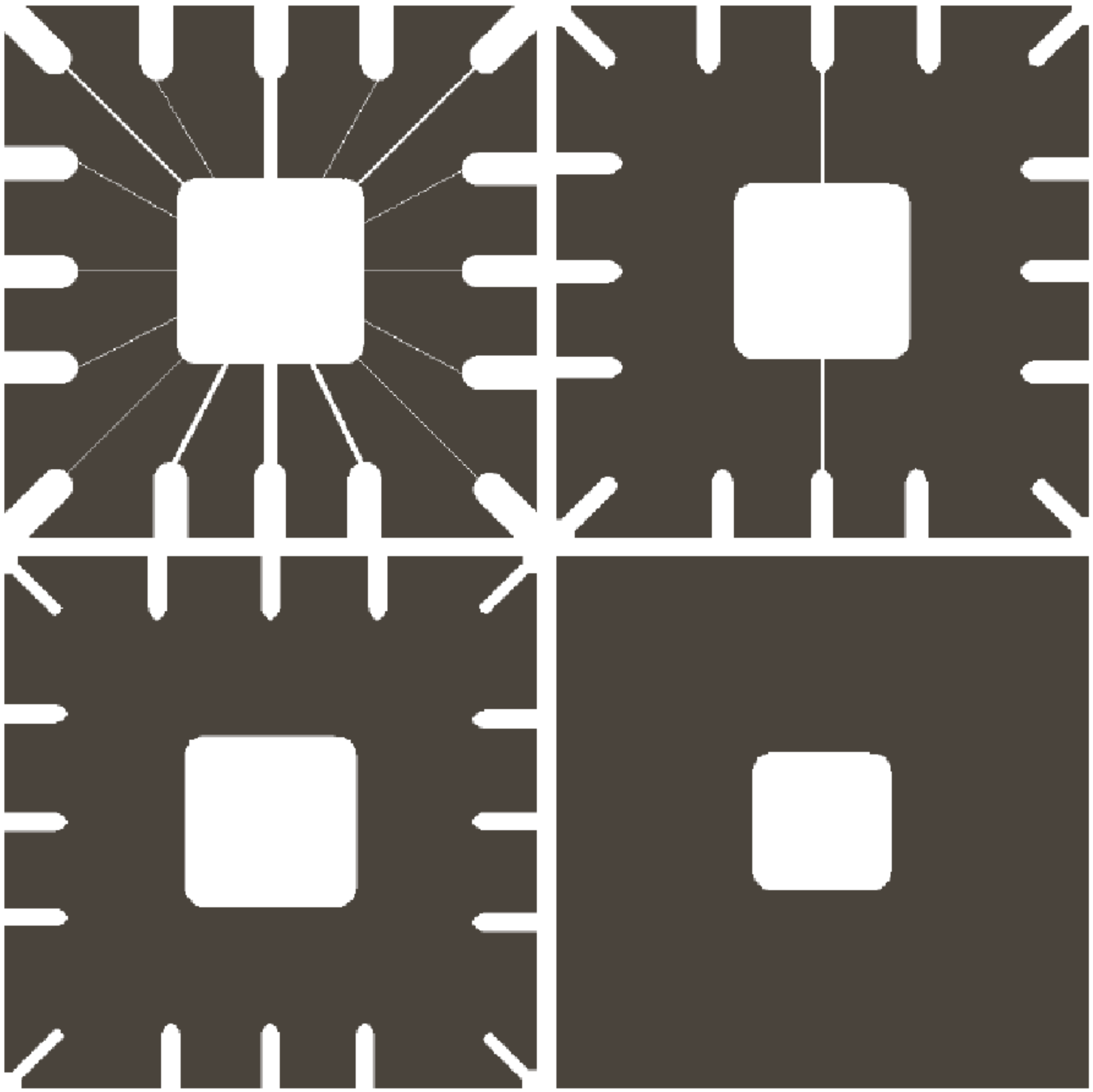
or alternatively

$$A \ominus B = \left\{ z | (B)_z \bigcap A^c = \emptyset \right\}$$

- B **is the SE**
- **Convolution process**



Example 9.1



a	b
c	d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

9.2.2 Dilation

With A and B sets in Z^2, the dilation of A by B, is defined as

$$A \oplus B = \left\{ z | (\hat{B})_z \bigcap A \neq \emptyset \right\}$$

or alternatively

$$A \oplus B = \left\{ z | \left[(\hat{B})_z \bigcap A \right] \subseteq A \right\}$$

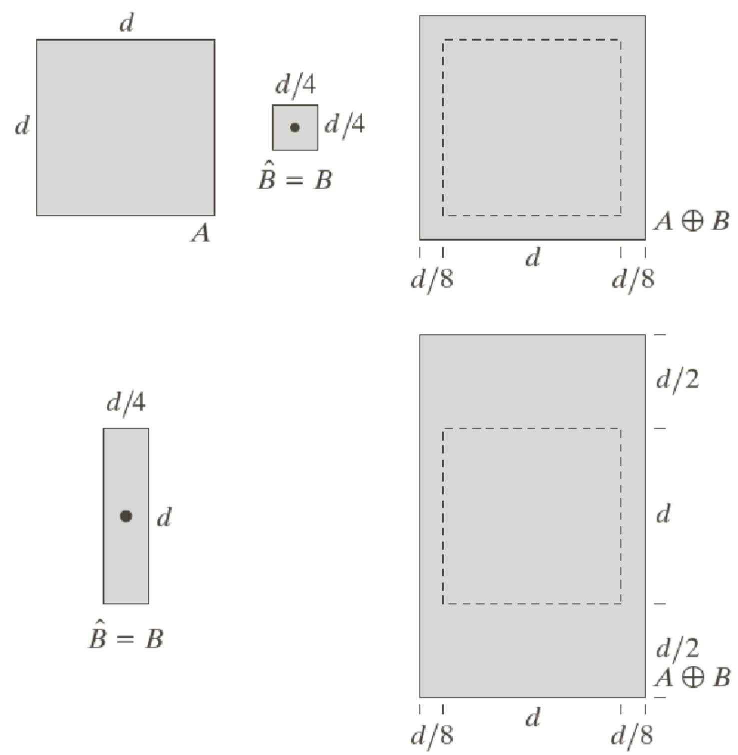
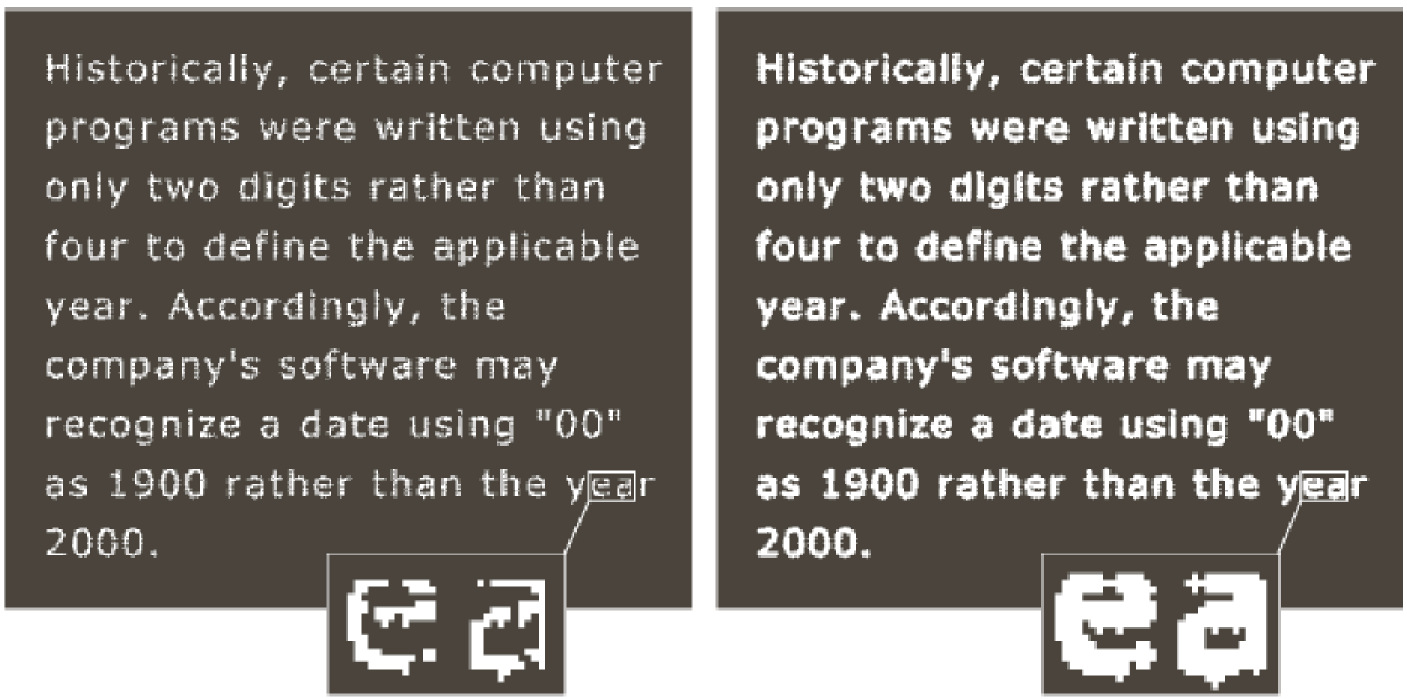


FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Example 9.2



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

9.2.3 Duality

Dilation and erosion are duals of each other with respect to set complementation and reflection, that is

$$\boxed{(A \ominus B)^c = A^c \oplus \hat{B}} \qquad (*)$$

and

$$\boxed{(A \oplus B)^c = A^c \ominus \hat{B}}$$

Proof of $(*)$:

$$\begin{aligned}(A \ominus B)^c &= \{z|(B)_z \subseteq A\}^c \\ &= \{z|(B)_z \cap A^c = \emptyset\}^c \\ &= \{z|(B)_z \cap A^c \neq \emptyset\} \\ &= A^c \oplus \hat{B}\end{aligned}$$

9.3 Opening and closing

USES

Opening: Smoothes the contour of an object
Breaks narrow isthmuses ("bridges")
Eliminates thin protrusions

Closing: Smoothes sections of contours
Fuses narrow breaks and long thin gulfs
Eliminates small holes in contours
Fills gaps in contours

Definitions

The opening of set A by structuring element B:

$$A \circ B = (A \ominus B) \oplus B$$

The closing of set A by structuring element B:

$$A \bullet B = (A \oplus B) \ominus B$$

Illustration of opening...

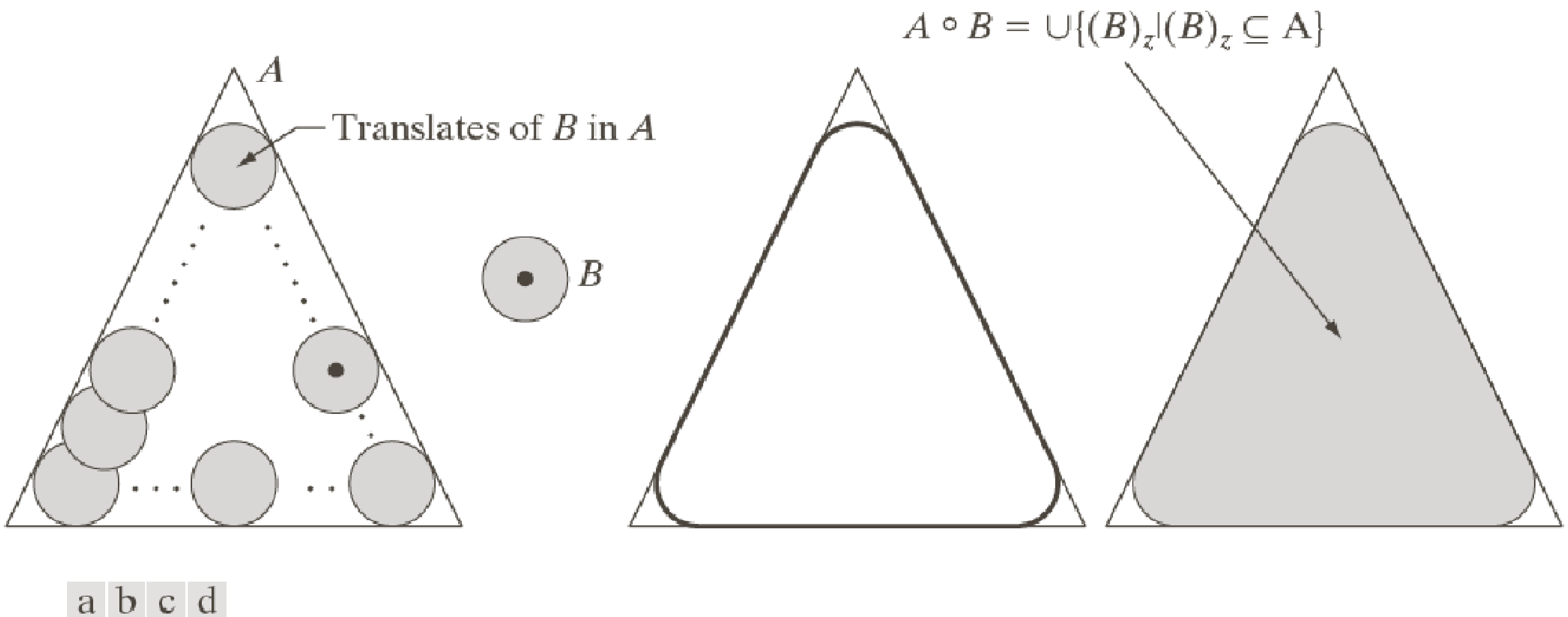


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Alternative definition for opening:

$$A \circ B = \bigcup \{(B)_z | (B)_z \subseteq A\}$$

Illustration of closing...

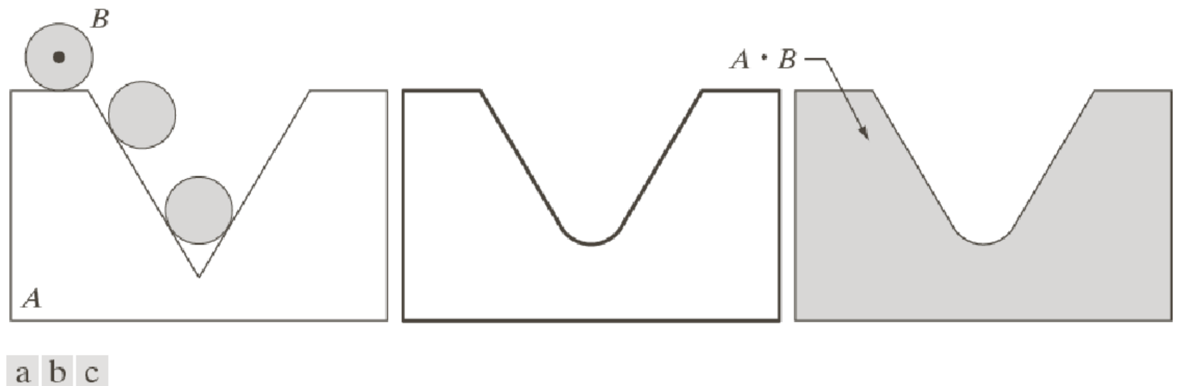


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Alternative definition for closing:

A point w is an element of $A \bullet B$ if and only if $(B)_z \cap A \neq \emptyset$ for any translate of $(B)_z$ that contains w

Opening and closing are also duals of each other with respect to set complementation and reflection, that is

$$(A \bullet B)^c = A^c \circ \hat{B}$$

(Verify)

The opening operation satisfies the following properties:

(i) $A \circ B \subseteq A$ **(ii)** **If** $C \subseteq D$**, then** $C \circ B \subseteq D \circ B$ **(iii)** $(A \circ B) \circ B = A \circ B$

The closing operation satisfies the following properties:

(i) $A \subseteq A \bullet B$ **(ii)** **If** $C \subseteq D$**, then** $C \bullet B \subseteq D \bullet B$ **(iii)** $(A \bullet B) \bullet B = A \bullet B$

Example 9.3

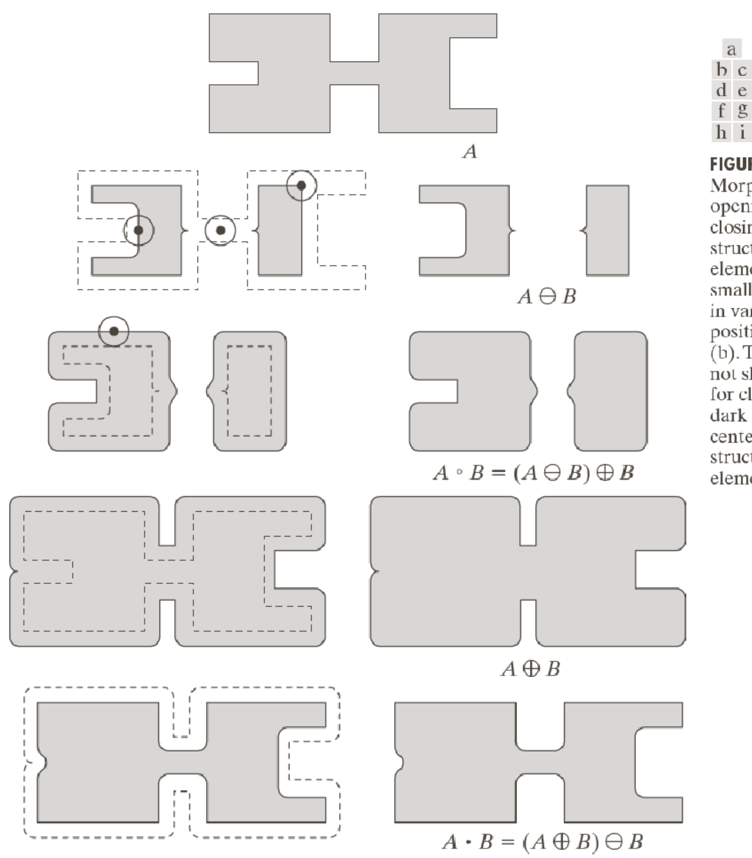
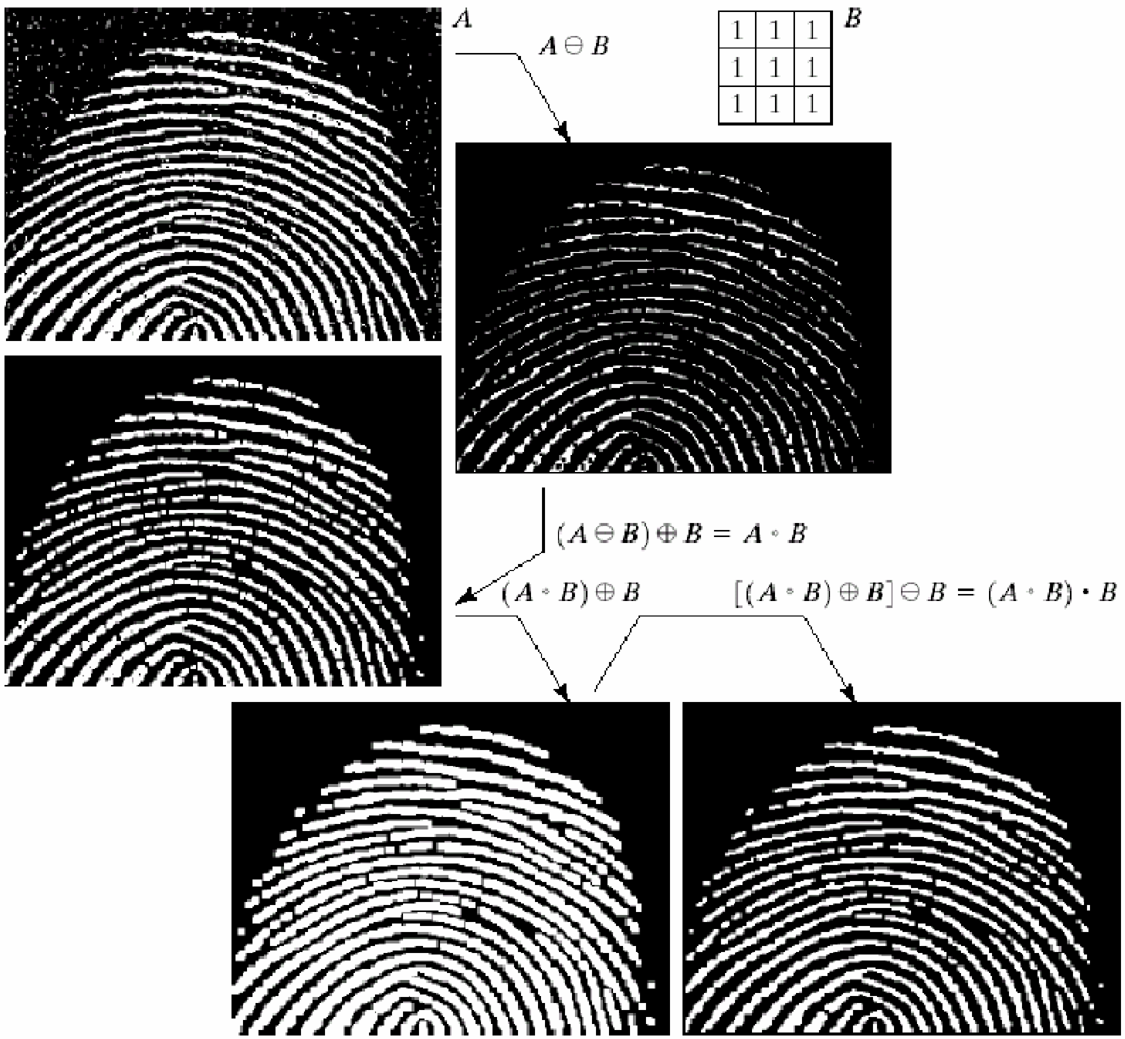


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Example 9.4:



a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)