9.5.9 Morphological reconstruction

- Involves two images and a structuring element
- Images:

(1) Marker (contains starting points); (2) Mask (constrains transformation)

Geodesic dilation and erosion

$B \equiv$ Structuring element; $F \equiv$ Marker image; $G \equiv$ Mask image

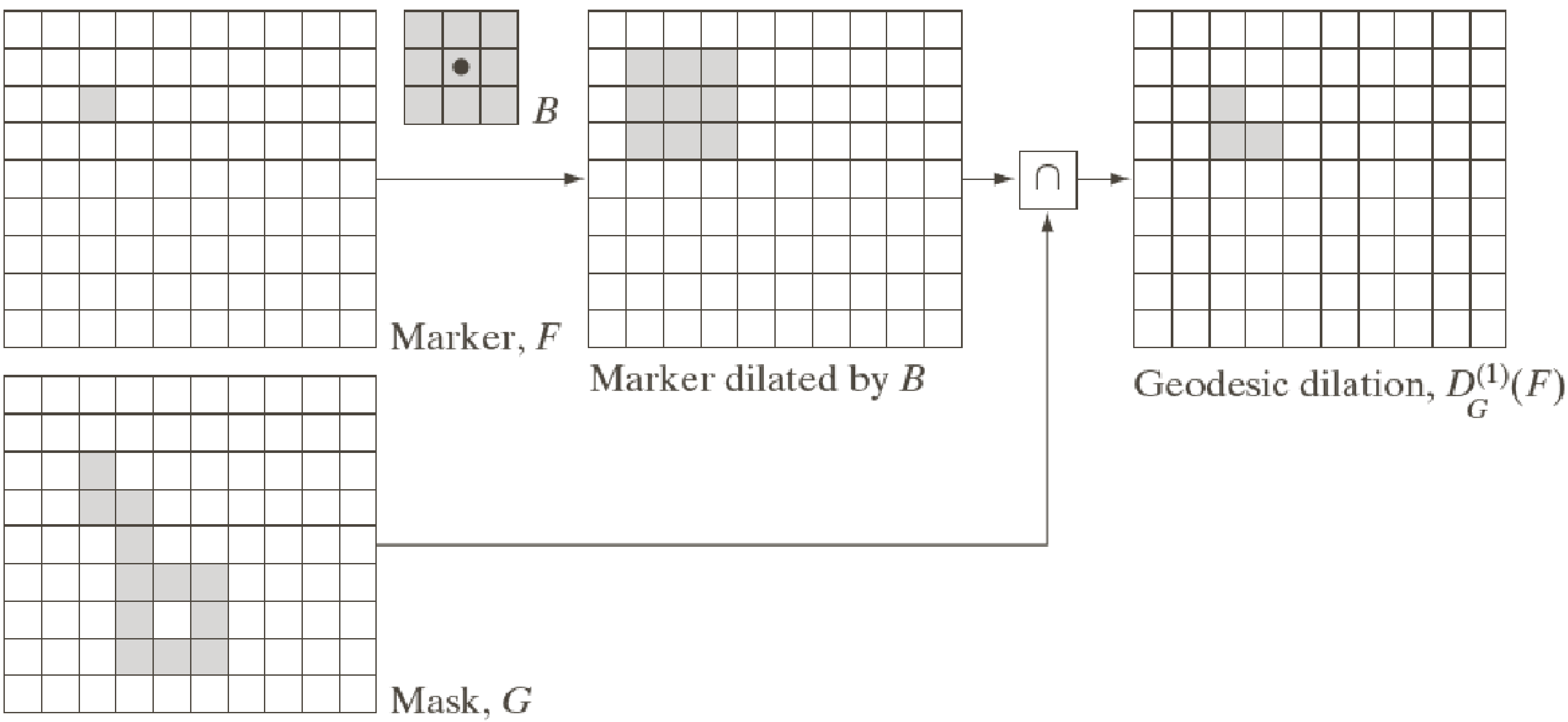
Geodesic dilation of size 1 (of F with respect to G):

$$D_G^{(1)}(F) = (F \oplus B) \bigcap G$$

Geodesic dilation of size n (of F with respect to G):

$$D_G^{(n)}(F) = D_G^{(1)}[D_G^{(n-1)}(F)], \quad D_G^{(0)}(F) = F$$

Illustration of geodesic dilation



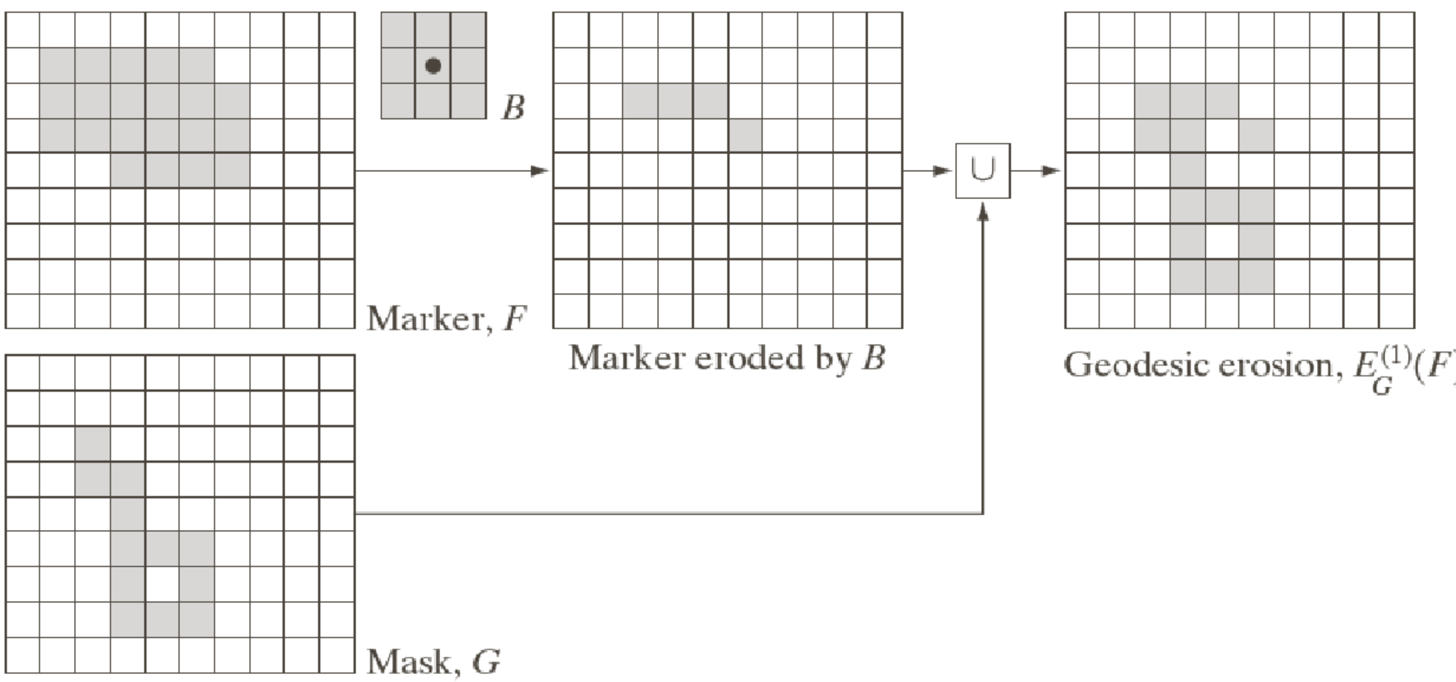
Geodesic erosion of size 1 (of F with respect to G):

$$E_G^{(1)}(F) = (F \ominus B) \bigcup G$$

Geodesic erosion of size n (of F with respect to G):

$$E_G^{(n)}(F) = E_G^{(1)}[E_G^{(n-1)}(F)], \qquad E_G^{(0)}(F) = F$$

Illustration of geodesic erosion



Morphological reconstruction by dilation

- **Geodesic dilation of F with respect to G iterated until stability is achieved**

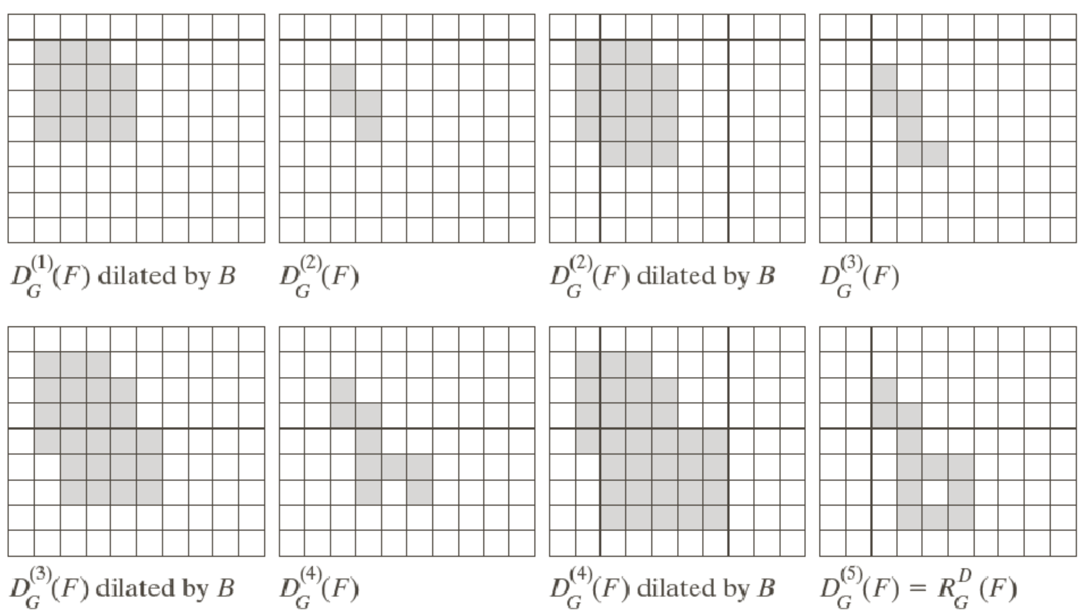
$$R_G^D(F) = D_G^{(k)}(F), \quad \textbf{with } k \textbf{ such that} \quad D_G^{(k)}(F) = D_G^{(k+1)}(F)$$

Morphological reconstruction by erosion

- **Geodesic erosion of F with respect to G iterated until stability is achieved**

$$R_G^E(F) = E_G^{(k)}(F), \quad \textbf{with } k \textbf{ such that} \quad E_G^{(k)}(F) = E_G^{(k+1)}(F)$$

Illustration of morphological reconstruction by dilation



a b c d
e f g h

FIGURE 9.28 Illustration of morphological reconstruction by dilation. F, G, B and $D_G^{(1)}(F)$ are from Fig. 9.26.

Sample applications

Opening by reconstruction

[Previously] Morphological opening $--$ Erosion removes small objects & subsequent dilation attemps to restore remaining objects

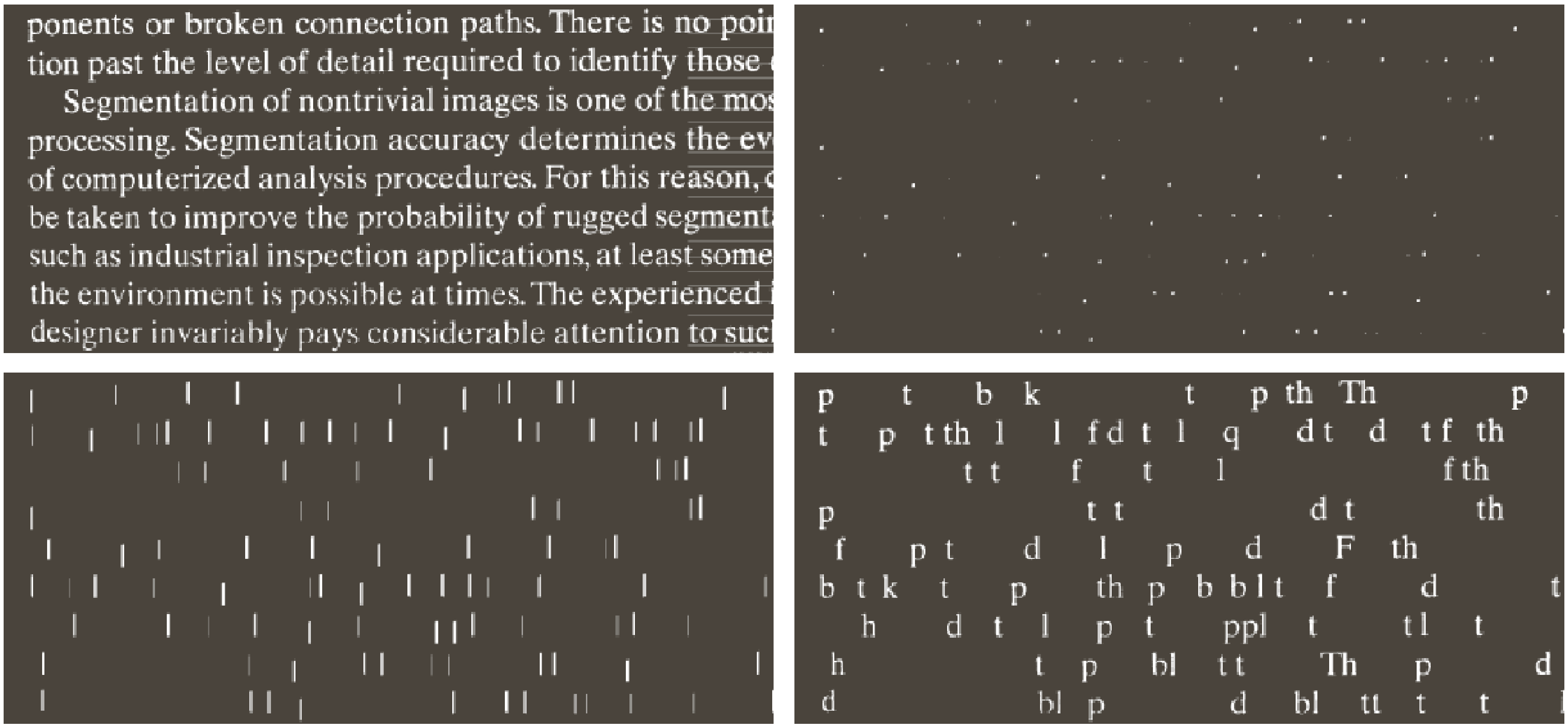
[Now] Opening by reconstruction $--$ Restores exactly the shape of objects that remain after erosion

DEF: Opening by reconstruction of size n of image F is defined as the reconstruction by dilation of F from the erosion of size n of F, that is,

$$O_R^{(n)}(F) = R_F^D[(F \ominus nB)],$$

where $(F \ominus nB)$ indicates n erosions of F by B

Example



a b
c d

FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

Filling holes

[Previously] Section 9.5.2 $--$ Algorithm for filling holes based on knowing a starting point in each hole

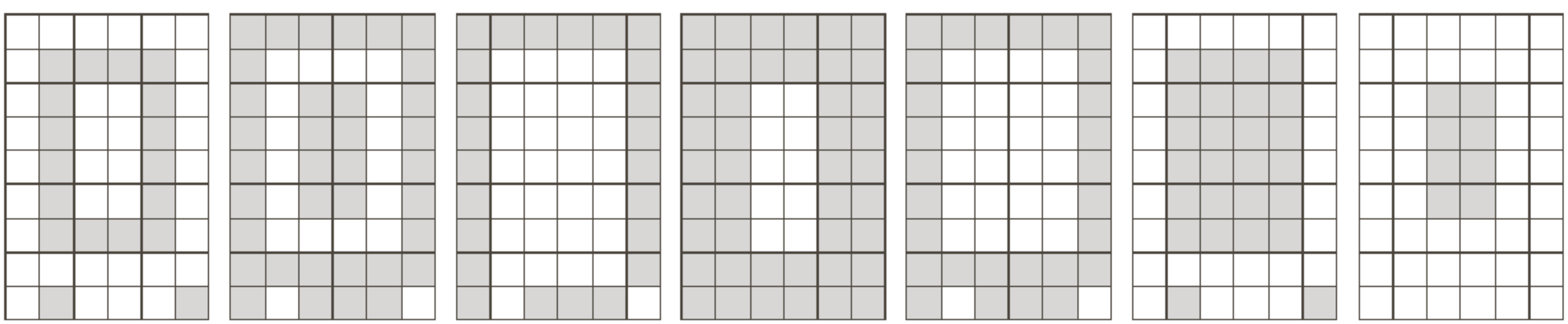
[Now] Morphological reconstruction $--$ Fully automated procedure

$I \equiv$ **Binary image;** $F \equiv$ **Marker image (**0 **everywhere, except at image border), where it is set to** $1 - I$**, that is**

$$F(x,y) = \begin{cases} 1 - I(x,y), & \textbf{if } (x,y) \textbf{ on image border of } I \\ 0, & \textbf{otherwise} \end{cases}$$

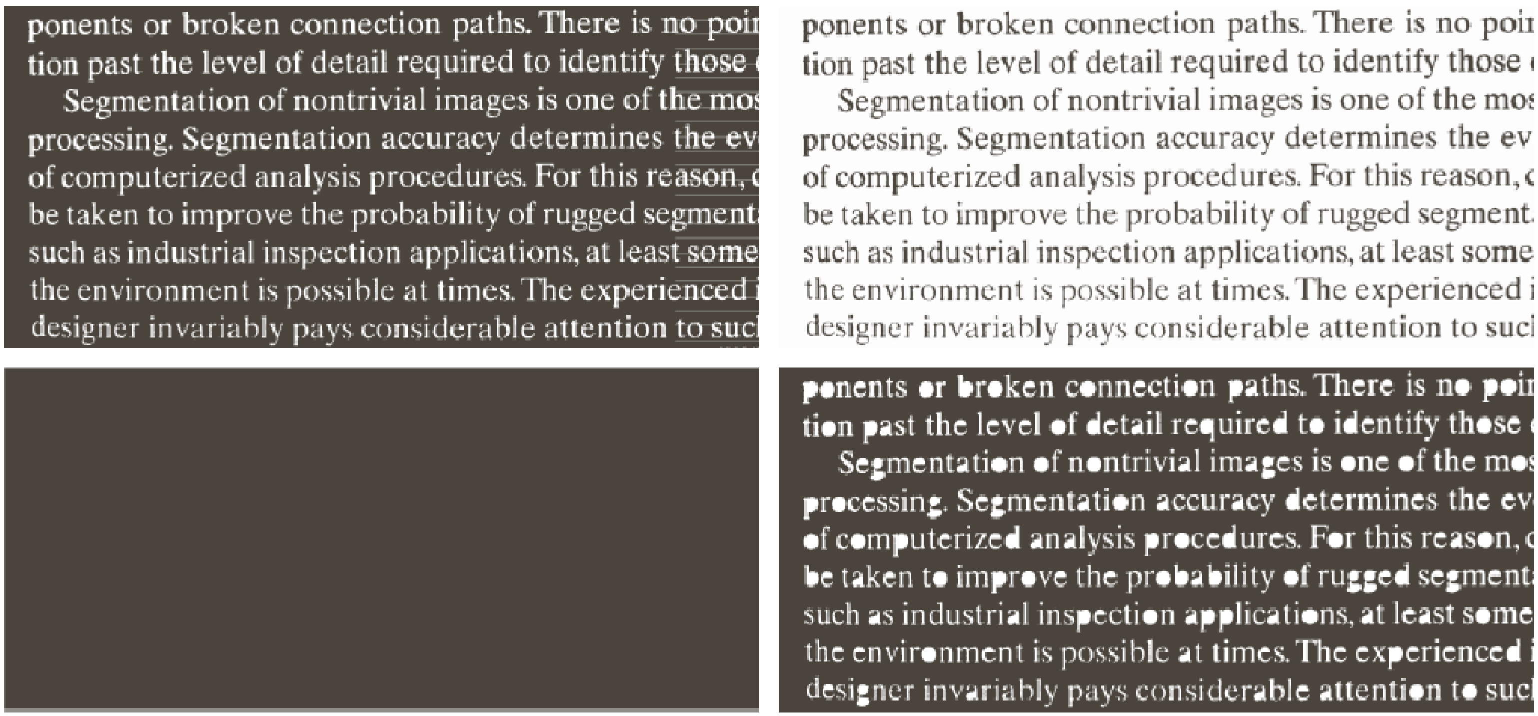
The binary image equal to I**, but with all the holes filled, is then given by**

$$H = [\ R_{I^c}^{D}(F)\ ]^c$$



I I^c F $F \oplus B$ $F \oplus B \cap I^c$ H $H \cap I^c$

Example



a b
c d

FIGURE 9.31 (a) Text image of size 918 × 2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).

Border clearing

Applications:

(1) Only complete objects remain for further processing

(2) Signal that partial objects are present in field of view

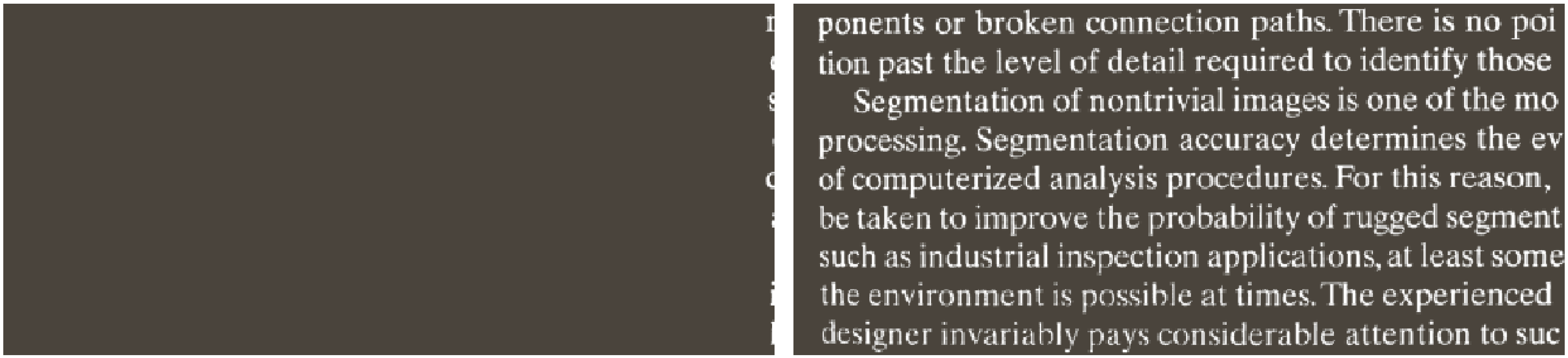
Procedure:

$$F(x,y) = \begin{cases} 1 - I(x,y), & \textbf{if } (x,y) \textbf{ on image border of } I \\ 0, & \textbf{otherwise} \end{cases}$$

$$X = I - R_I^D(F)$$

$X \equiv$ **Image with no objects touching border**

Example



a b

FIGURE 9.32
Border clearing. (a) Marker image. (b) Image with no objects touching the border. The original image is Fig. 9.29(a).