5.1 A model of the image degradation/restoration process (page 334)

Spatial domain

$$g(x,y) = h(x,y) \bigstar f(x,y) + \eta(x,y)$$

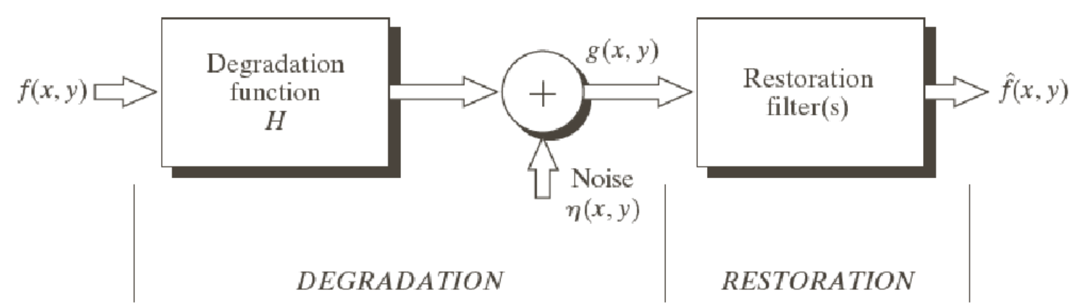
Frequency domain

$$G(u,v) = H(u,v)\ F(u,v) + N(u,v)$$

$H(u,v)$: **Degradation function**
$\eta(x,y)$: **Additive noise term**

The objective is to find an estimate $\hat{f}(x,y)$ of the original image $f(x,y)$. The more we know about H and η, the closer $\hat{f}(x,y)$ will be to $f(x,y)$.

FIGURE 5.1 A model of the image degradation/restoration process.



5.2 Noise models

(page 335)

Sources of noise: image acquisition and/or transmission

5.2.1 Spatial and Frequency Properties of Noise

- What is the spatial characteristics of the noise?
- Is the noise correlated with the image?
- What is the frequency content of the noise?
  When the Fourier spectrum is constant: white noise

Assumptions: Noise independent of spatial coordinates
Noise uncorrelated with respect to image

5.2.2 Noise PDFs

Statistical behavior of grey-level values of noise component

Gaussian noise

- Popular model: sometimes used when marginally applicable

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

Rayleigh noise

- **Useful for approximating skewed histograms**

$$p(z) = \begin{cases} (2/b)\,(z-a)\,e^{-(z-a)^2/b}, & z \geq a \\ 0, & z < a \end{cases}$$

$$\overline{z} = a + \sqrt{\pi b/4}; \quad \sigma^2 = \frac{b(4-\pi)}{4}$$

Erlang/Gamma noise

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!}\,e^{-az}, & z \geq 0 \\ 0, & z < 0 \end{cases}; \quad a > 0; \quad b \textbf{ positive integer}$$

$$\overline{z} = \frac{b}{a}; \quad \sigma^2 = \frac{b}{a^2}$$

Exponential noise

- **Special case of Erlang with** $b = 1$

$$p(z) = \begin{cases} a\,e^{-az}, & z \geq 0 \\ 0, & z < 0 \end{cases}; \quad a > 0$$

$$\overline{z} = \frac{1}{a}; \quad \sigma^2 = \frac{1}{a^2}$$

Uniform noise

$$p(z) = \begin{cases} \dfrac{1}{b-a}, & a \leq z \leq b \\ 0, & \textbf{otherwise} \end{cases}$$

$$\overline{z} = \frac{a+b}{2}; \quad \sigma^2 = \frac{(b-a)^2}{12}$$

Impulse (salt-and-pepper) noise

- **Noise impulses can be positive or negative**
- **Usually large with respect to image signal**

$\Rightarrow$ digitized as black or white

$$p(z) = \begin{cases} P_a, & z = a \\ P_b, & z = b \\ 0, & \textbf{otherwise} \end{cases}$$

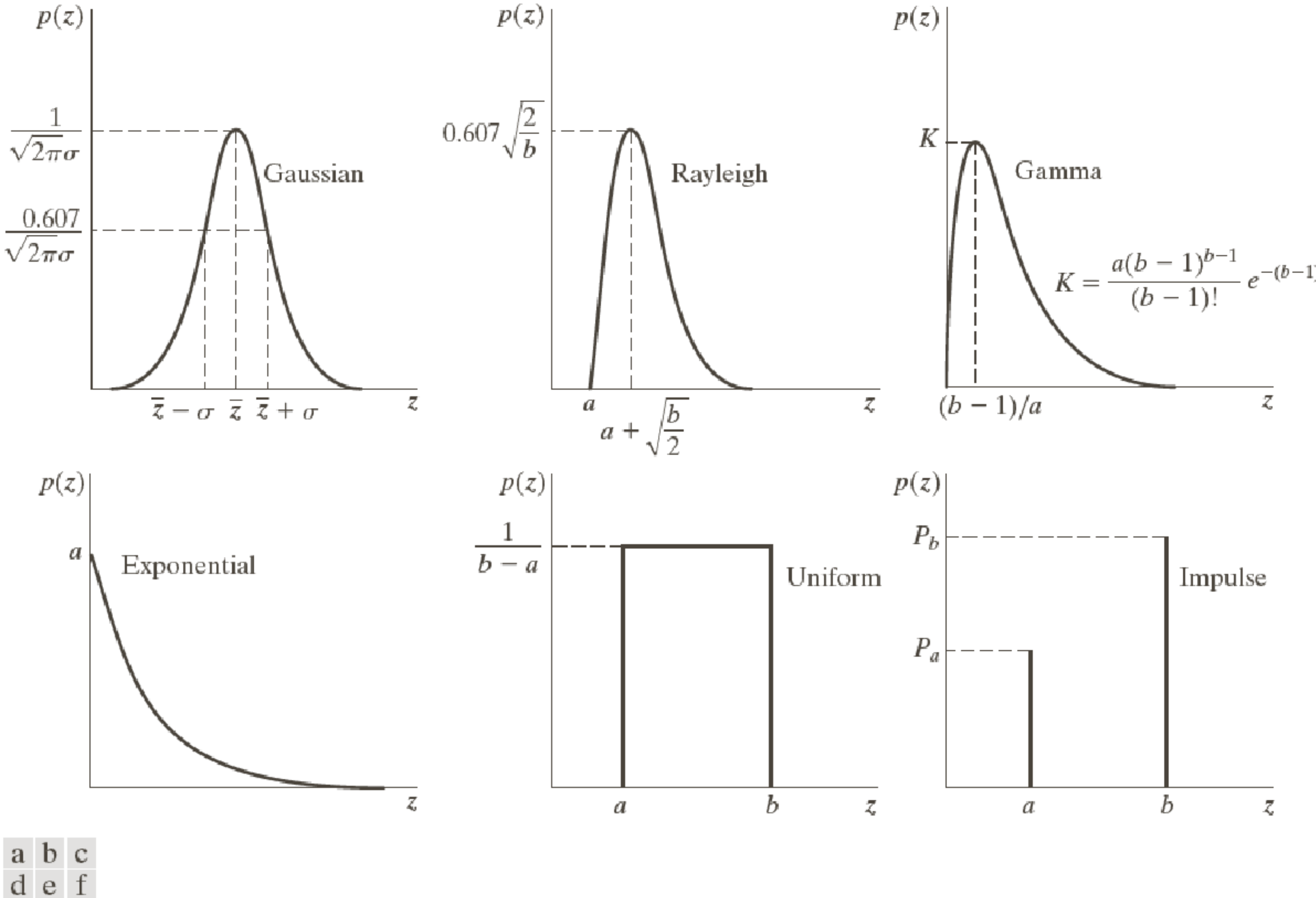


FIGURE 5.2 Some important probability density functions.

These PDFs usually model specific noise corruption situations (p 339)

Example 5.1
Noisy images and their histograms

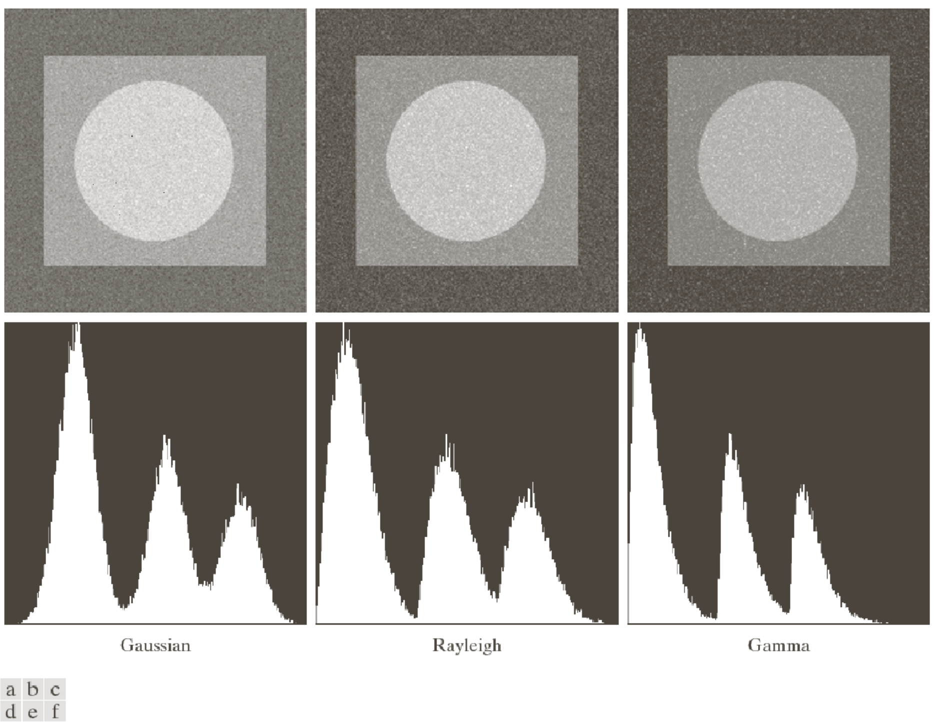


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

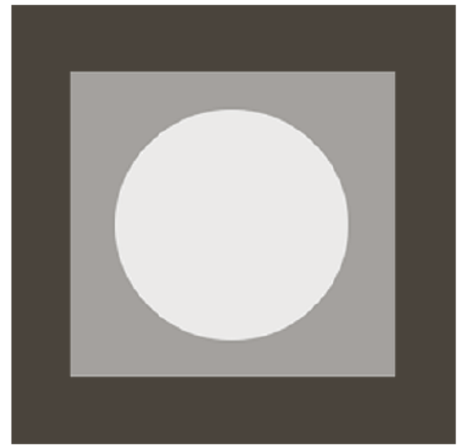


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Gassian, Rayleigh, and gamma noise added

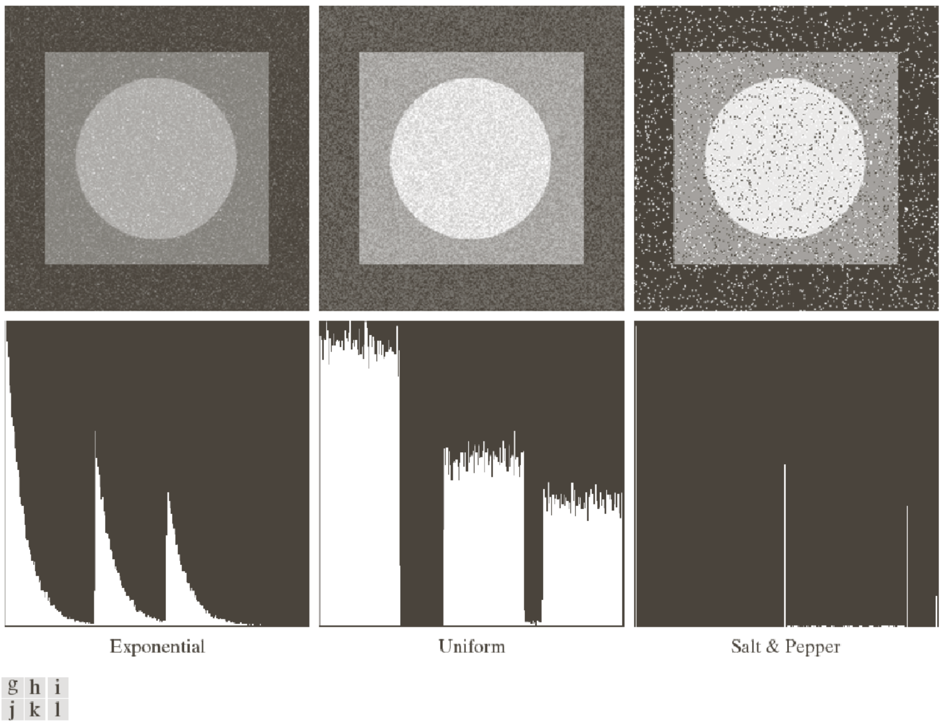


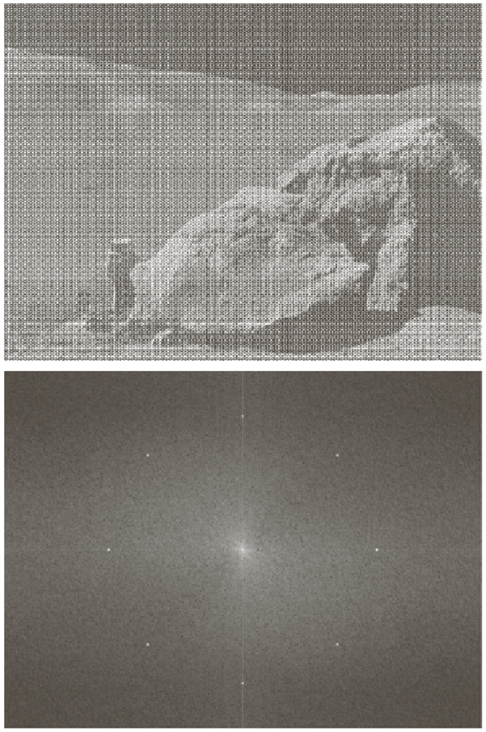
FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Exponential, uniform, and salt-and-pepper noise added

- **Close correspondence between histograms and PDFs**
- **Corrupted images similar - histograms significantly different**

5.2.3 Periodic noise

- **This is the only space dependent noise we shall consider**
- **Can be reduced by frequency domain filtering**



a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

5.2.4 Estimation of noise parameters

Periodic noise

- Inspect Fourier spectrum: spikes
- Automated analysis possible when spikes are pronounced

PDFs

- Imaging system available: capture uniform environment
- Imaging system not available: consider small patches of constant gray scale

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

$$\overline{z} = \sum_{i=0}^{L-1} z_i \, p_S(z_i)$$

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \overline{z})^2 \, p_S(z_i)$$

- **Shape of histogram identifies closest PDF match**

5.3 Restoration in the presence of noise only - spatial filtering

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

- **Noise term unknown**
- **When only additive noise present: usually spatial filtering**

5.3.1 Mean Filters

Arithmetic mean filter: $$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- **Noise reduced as result of blurring**

Geometric mean filter: $$\hat{f}(x,y) = \left\{ \prod_{(s,t)\in S_{xy}} g(s,t) \right\}^{\frac{1}{mn}}$$

- **Loose less detail than arithmetic mean filter**

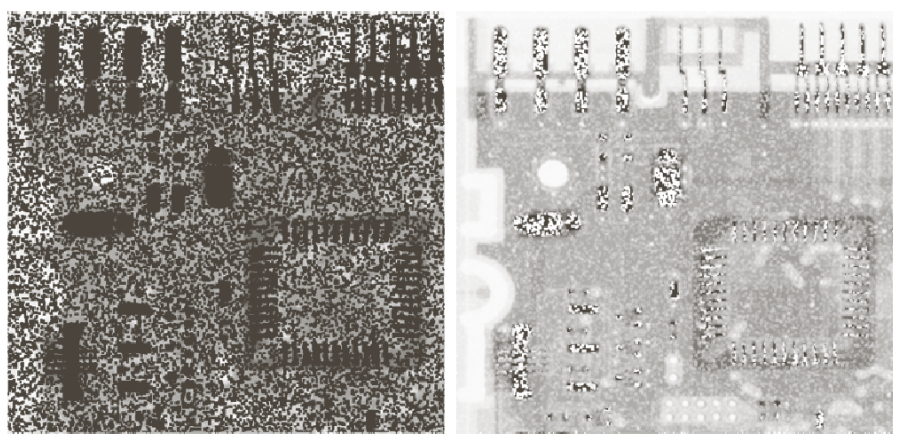
Harmonic mean filter: $$\hat{f}(x,y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- Works well for salt - fails for pepper
- Works well for other types of noise

Contraharmonic mean filter

$$\hat{f}(x,y) = \frac{\displaystyle\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\displaystyle\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

- Q: Order of filter
- Well-suited for salt-and-pepper noise
- Q positive: eliminates pepper noise
- Q negative: eliminates salt noise



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Example 5.2

Illustration of mean filters

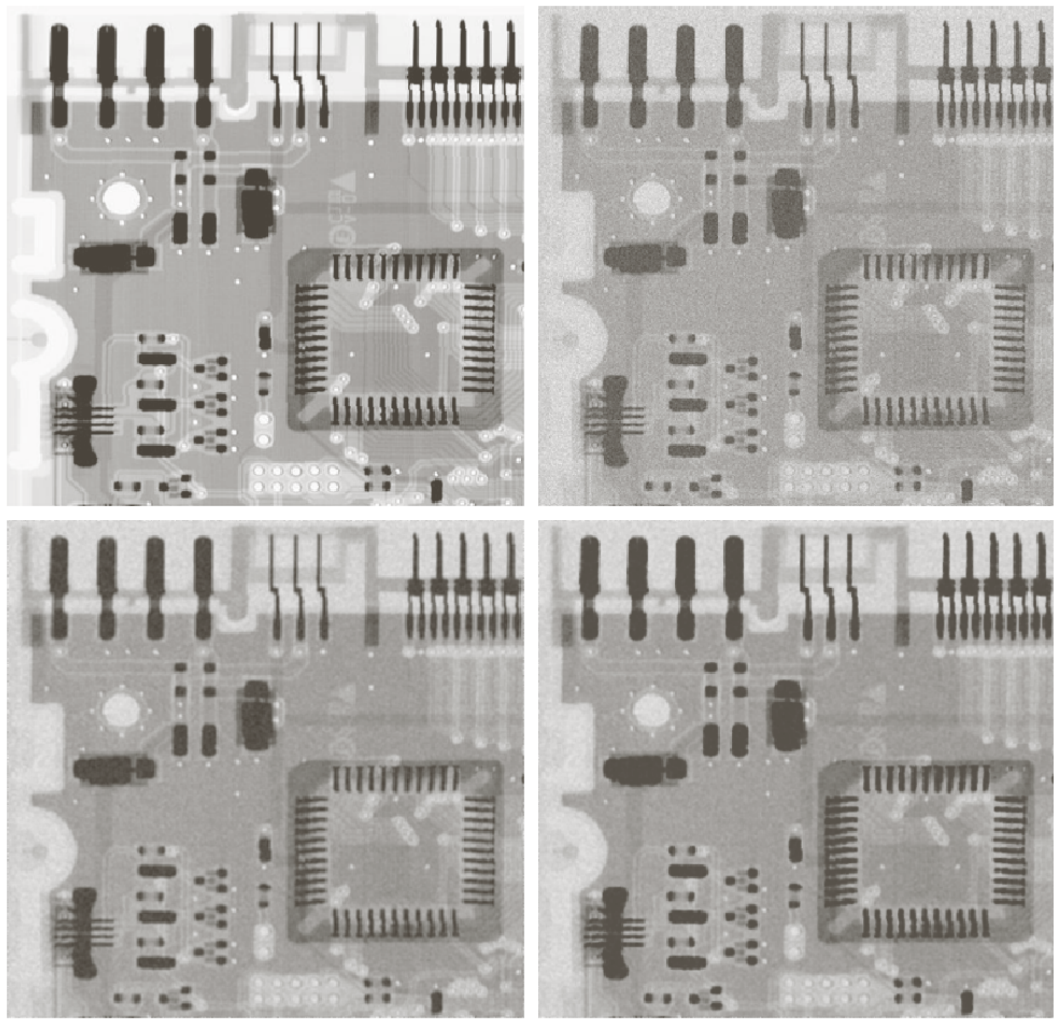
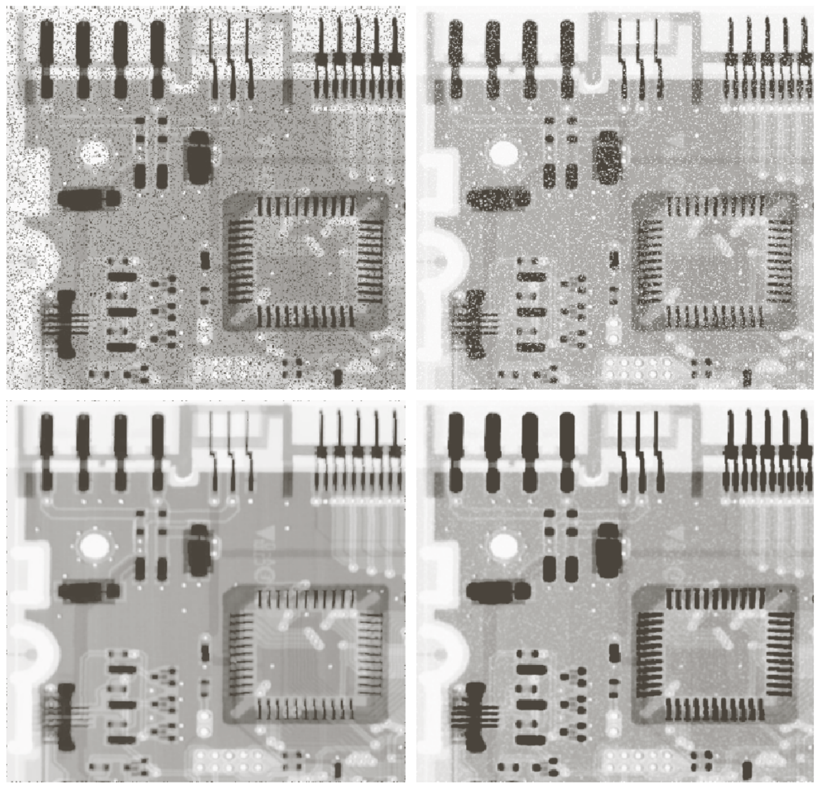


FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Example 5.2

Illustration of mean filters



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Summary

- **Arithmetic and geometric mean filters: Gaussian, uniform noise**
- **Contraharmonic mean filter: impulse noise, but have to choose proper sign**

5.3.2 Order-statistics filters

Median filter

$$\hat{f}(x,y) = \text{median}_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Max and min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left\{ \max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\} \right\}$$

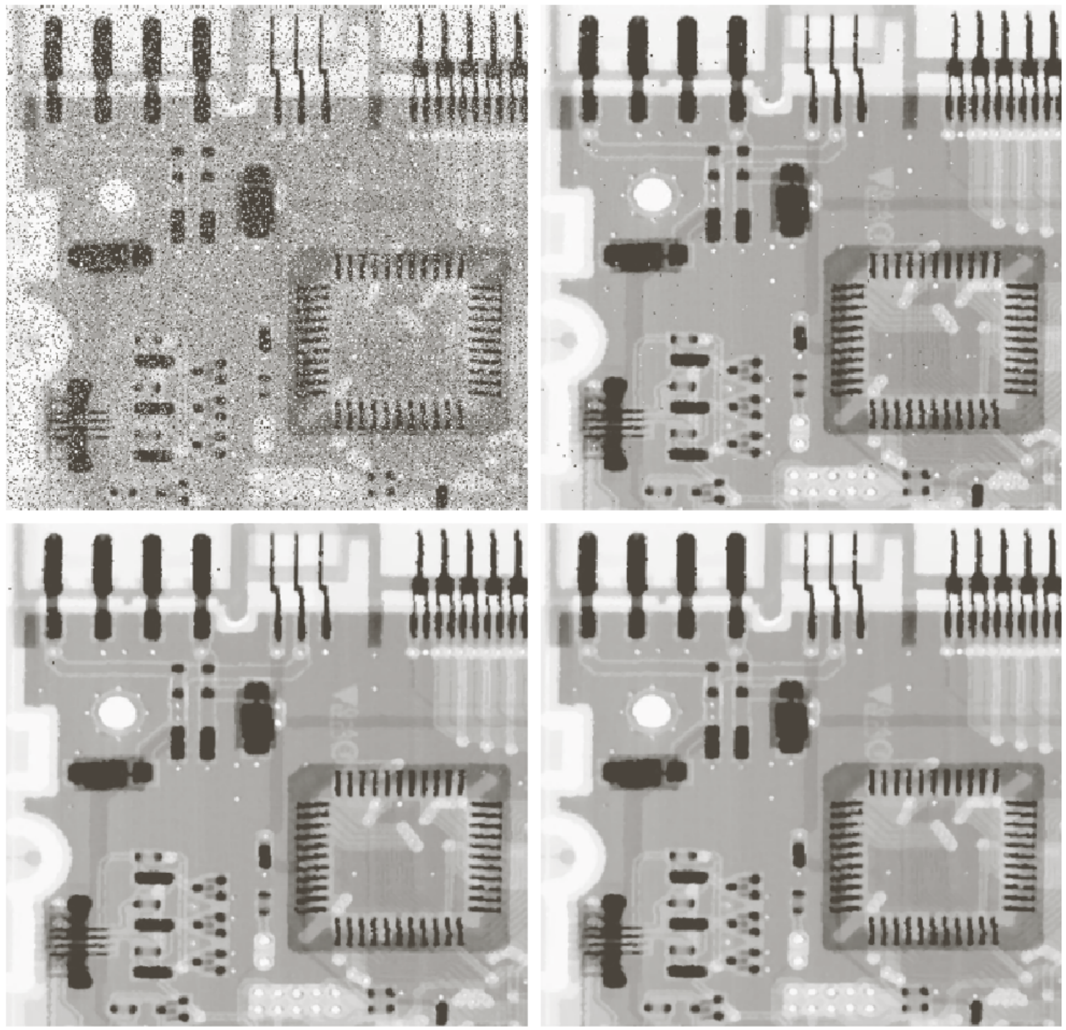
Alpha-trimmed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

- $d = 0$: **arithmetic mean filter**
- $d = (mn-1)/2$: **median filter**
- **suitable for combination of different types of noise**

Example 5.3

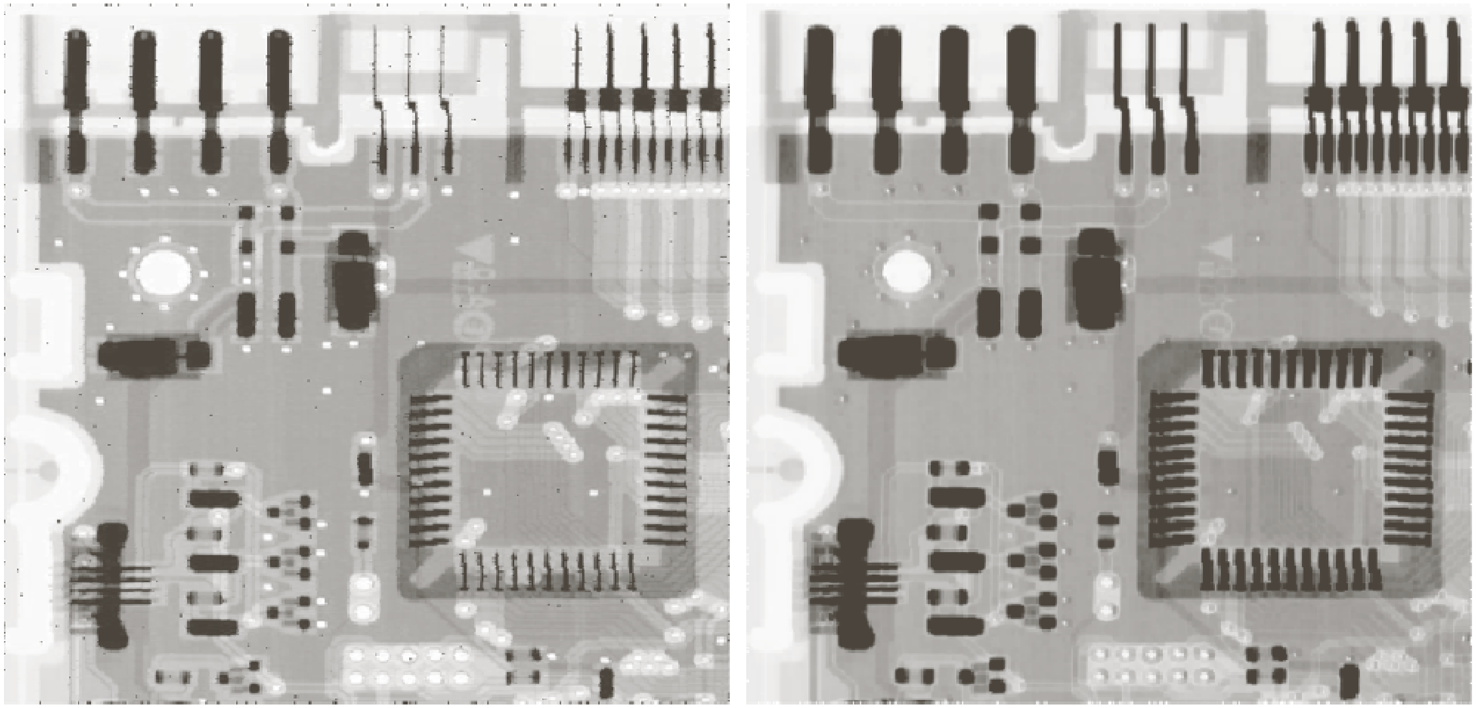
Illustration of order-statistic filters



a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Example 5.3
Illustration of order-statistic filters



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.

Example 5.3

Illustration of order-statistic filters

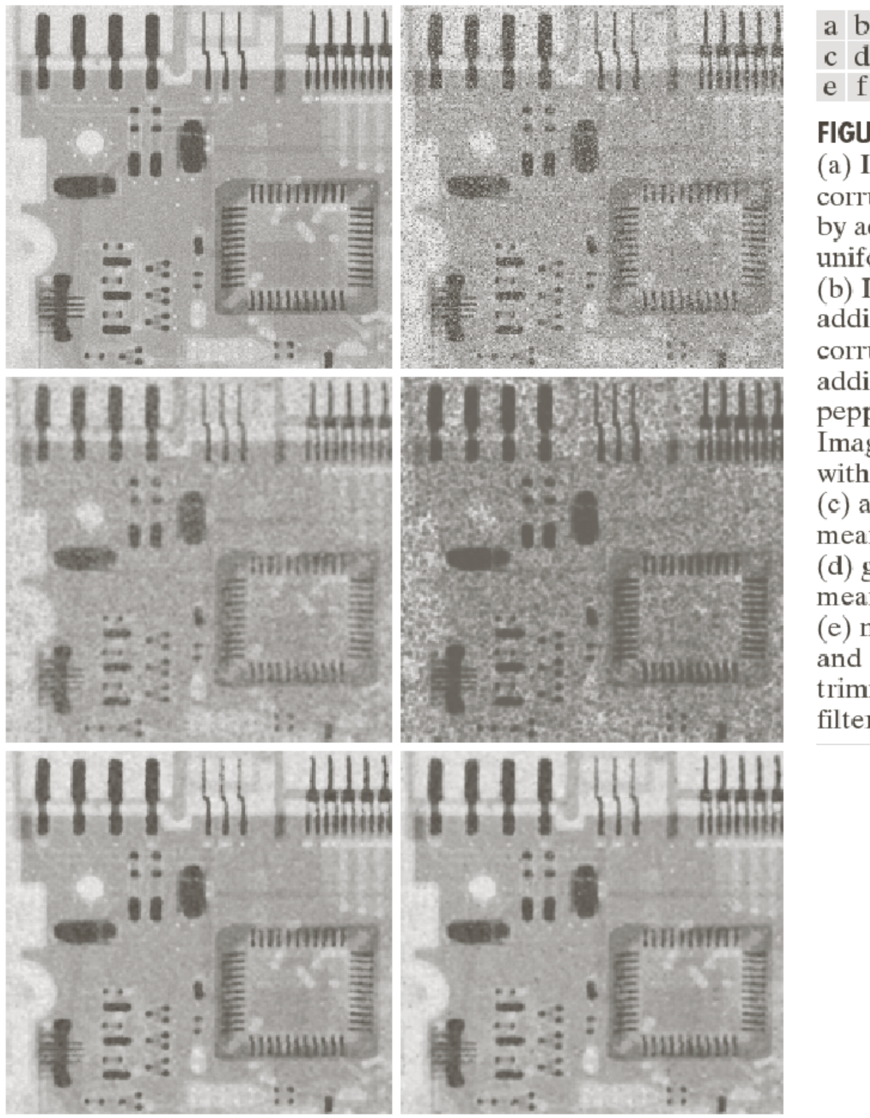


FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

5.3.3 Adaptive Filters

- **Behavior changes according to statistical characteristics of sub-image in filter region**

Adaptive, local noise reduction filter

$$g(x,y) = f(x,y) + \eta(x,y)$$

Parameters in local region S_{xy}:

(a) $g(x,y)$, **that is g at (x,y)**

(b) σ_η^2, **that is the global variance of $\eta(x,y)$**

(c) m_L, **that is the local mean of S_{xy}**

(d) σ_L^2, **that is the local variance of S_{xy}**

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[\, g(x,y) - m_L \,\right]$$

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[\, g(x,y) - m_L \,\right]$$

(1) If $\sigma_\eta^2 = 0$ then $\hat{f}(x,y) = g(x,y)$: zero noise

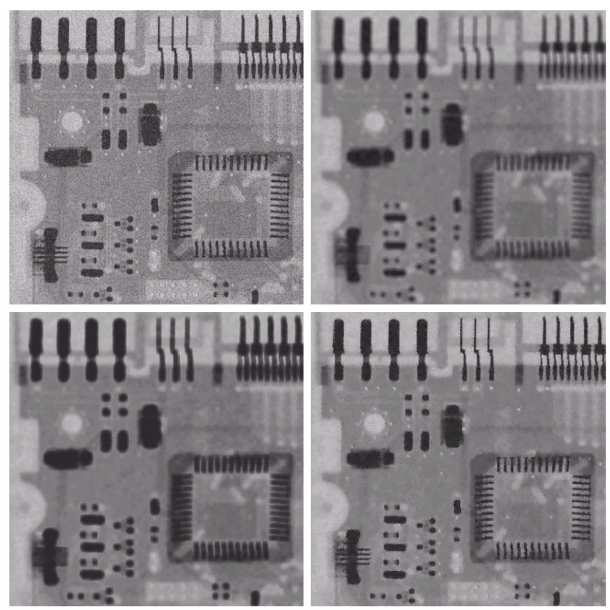
(2) If $\sigma_L^2 \gg \sigma_\eta^2$ then $\hat{f}(x,y) \approx g(x,y)$: edges preserved

(3) If $\sigma_L^2 \approx \sigma_\eta^2$ then $\hat{f}(x,y) = m_L$: arithmetic mean

- **We only need to know/estimate σ_η^2**
- **We assume that $\sigma_L^2 \geq \sigma_\eta^2$**
- **When $\sigma_L^2 < \sigma_\eta^2$, then let $\sigma_L^2 = \sigma_\eta^2$: makes filter non-linear**

a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



Adaptive median filter

- **Advantages**

(1) **The standard median filter does not perform well when P_a or P_b is greater than 0.2, while the adaptive median filter can better handle these P's**

(2) **The adaptive median filter preserves detail and smooth non-impulsive noise, while the standard median filter does not**

- **Adaptive median filter changes size of S_{xy} during operation**

- **Notation**

$$
\begin{aligned}
z_{\min} &= \text{minimum gray level value in } S_{xy} \\
z_{\max} &= \text{maximum gray level value in } S_{xy} \\
z_{\text{med}} &= \text{median of gray levels in } S_{xy} \\
z_{xy} &= \text{gray level at coordinates } (x, y) \\
S_{\max} &= \text{maximum allowed size of } S_{xy}
\end{aligned}
$$

- **Algorithm**

Level A: $A1 = z_{\text{med}} - z_{\text{min}}$
$A2 = z_{\text{med}} - z_{\text{max}}$
if $A1 > 0$ **AND** $A2 < 0$**, go to level B**
else increase the window size
if window size $\leq S_{\text{max}}$**, repeat level A**
else output z_{xy}

Level B: $B1 = z_{xy} - z_{\text{min}}$
$B2 = z_{xy} - z_{\text{max}}$
if $B1 > 0$ **AND** $B2 < 0$**, output** z_{xy}
else output z_{med}

- **Purpose**

(1) **Remove impulse noise**

(2) **Smoothing of other noise**

(3) **Reduce distortion, like excessive thinning or thickening of object boundaries**

- **Explanation**

Level A: **IF** $z_{\min} < z_{\text{med}} < z_{\max}$, **then**

- z_{med} **is not an impulse**

(1) go to level B to test if z_{xy} **is an impulse ...**

ELSE

- z_{med} **is an impulse**

(1) the size of the window is increased and

(2) level A is repeated until ...

(a) z_{med} **is not an impulse and go to level B or**

(b) $S_{\max}$ **reached: output is** z_{xy} **(∗∗)**

Level B: **IF** $z_{\min} < z_{xy} < z_{\max}$, **then**

- z_{xy} **is not an impulse**

(1) output is z_{xy} **(distortion reduced)**

ELSE

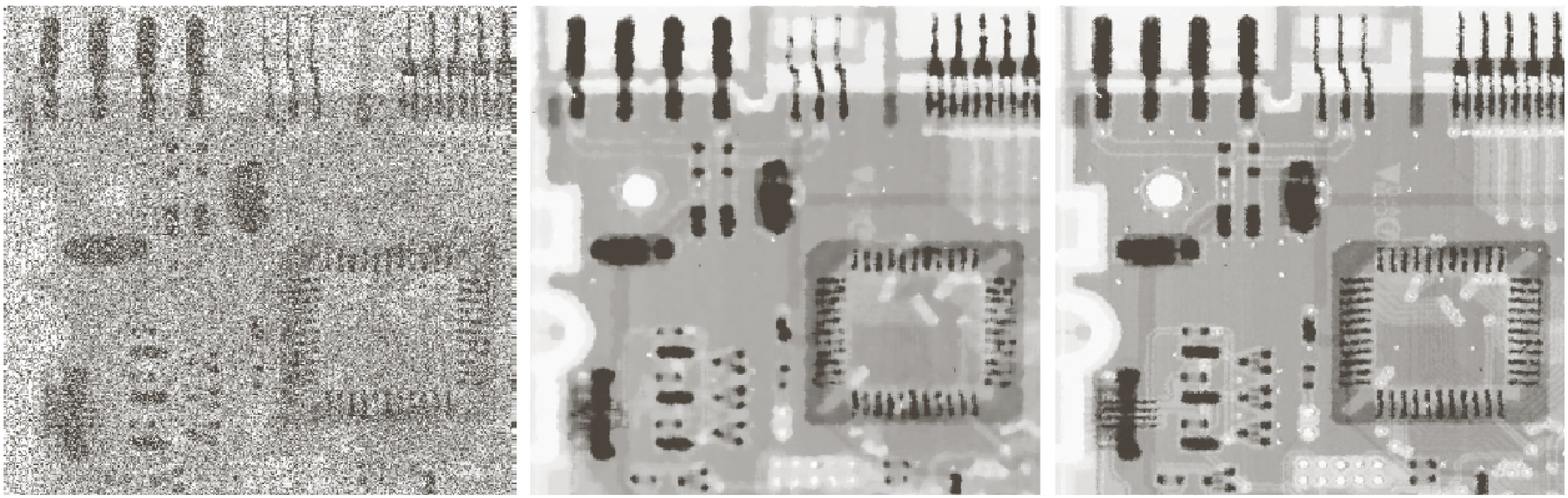
- **either** $z_{xy} = z_{\min}$ **or** $z_{xy} = z_{\max}$

(2) output is z_{med} **(standard median filter)**

  - z_{med} **is not an impulse (from level A)**

(∗∗) No guarantee that this is not an impulse. The smaller P_a **and/or** P_b **are, or the larger** $S_{\max}$ **is, the less likely a premature exit will be**

Example 5.5
Illustration of adaptive median filtering



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.