



## CHAPTER 10: IMAGE SEGMENTATION

Image Analysis: { Chapter 10: Segmentation  
Chapter 11: Representation and description  
Chapter 12: Object recognition

Segmentation: Subdivision of an image into its constituent parts

The level of segmentation depends on the application

### 10.1 Fundamentals

Segmentation based on: { (1) Discontinuities { (1) Isolated points  
(2) Lines  
(3) Edges  
(2) Similarity { (1) Thresholding  
(2) Region growing  
(3) Region splitting/merging

(1) Edge-based segmentation

(2) Region-based segmentation



Let  $R$  represent the entire image region

The segmentation process partitions  $R$  into  $n$  subregions,  $R_1, R_2, \dots, R_n$ , such that...

(a)  $\bigcup_{i=1}^n R_i = R$

(b)  $R_i$  is a connected set,  $i = 1, 2, \dots, n$

(c)  $R_i \cap R_j = \emptyset$  for all  $i$  and  $j, i \neq j$

(d)  $Q(R_i) = \text{TRUE}$  for  $i = 1, 2, \dots, n$

(e)  $Q(R_i \cup R_j) = \text{FALSE}$  for any adjacent regions  $R_i$  and  $R_j$

Here  $Q(R_k)$  is logical predicate defined over all points in  $R_k$

(a) Every pixel must be in a region

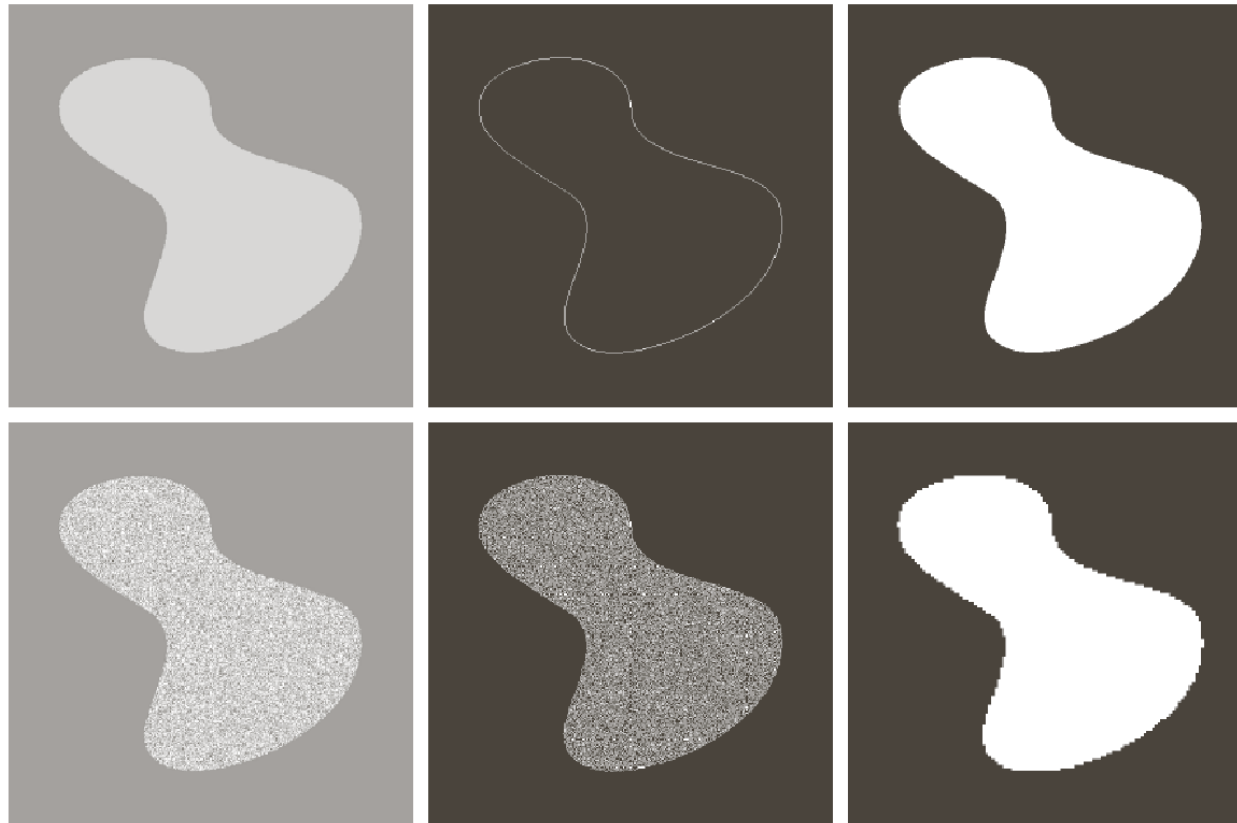
(b) All the points in a region must be “connected”

(c) Regions must be disjoint

(d) For example  $Q(R_i) = \text{TRUE}$  if all the pixels in  $R_i$  have the same gray level

(e) Regions  $R_i$  and  $R_j$  are different in some sense

## Example



**FIGURE 10.1** (a) Image containing a region of constant intensity. (b) Image showing the boundary of the inner region, obtained from intensity discontinuities. (c) Result of segmenting the image into two regions. (d) Image containing a textured region. (e) Result of edge computations. Note the large number of small edges that are connected to the original boundary, making it difficult to find a unique boundary using only edge information. (f) Result of segmentation based on region properties.



## 10.2 Point, line and edge detection

### 10.2.1 Background

$$\frac{\partial f}{\partial x} = f'(x) = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial f'(x)}{\partial x}$$

$$= f'(x+1) - f'(x)$$

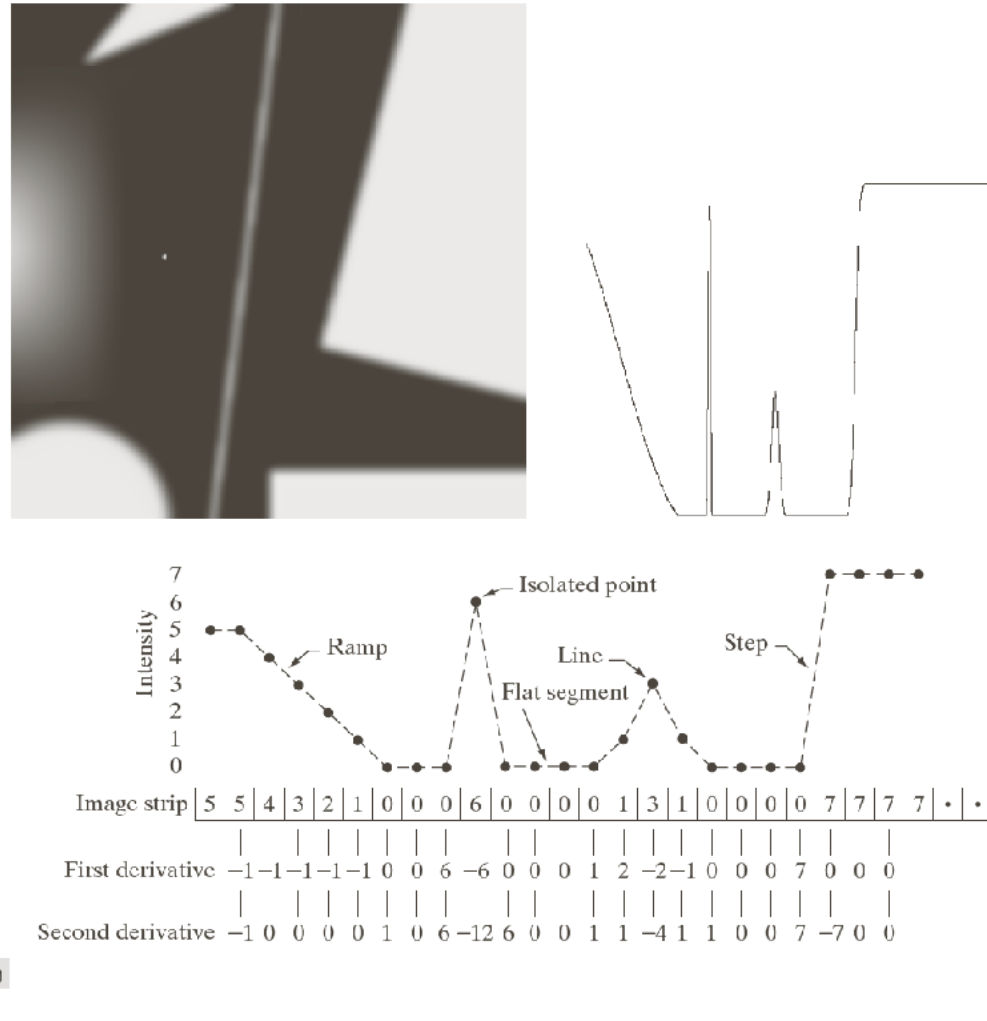
$$= f(x+2) - f(x+1) - f(x+1) + f(x)$$

$$= f(x+2) - 2f(x+1) + f(x)$$

**The above represents a truncated Taylor expansion about the point  $x+1$**

**Expansion about point  $x$ :**

$$\frac{\partial^2 f}{\partial x^2} = f''(x) = f(x+1) + f(x-1) - 2f(x)$$



**FIGURE 10.2** (a) Image. (b) Horizontal intensity profile through the center of the image, including the isolated noise point. (c) Simplified profile (the points are joined by dashes for clarity). The image strip corresponds to the intensity profile, and the numbers in the boxes are the intensity values of the dots shown in the profile. The derivatives were obtained using Eqs. (10.2-1) and (10.2-2).



**General linear filter: (Response)**

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \dots + w_{mn} z_{mn} \\ &= \sum_{k=1}^{mn} w_k z_k \\ &= \mathbf{w}^T \mathbf{z} \end{aligned}$$

**3x3 example: (Response)**

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \dots + w_9 z_9 \\ &= \sum_{k=1}^9 w_k z_k \\ &= \mathbf{w}^T \mathbf{z} \end{aligned}$$

$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$



## 10.2.2 Detection of isolated points

### Laplacian derivative operator

#### Continuous form:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

#### Discrete form: $x$ -direction

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

#### Discrete form: $y$ -direction

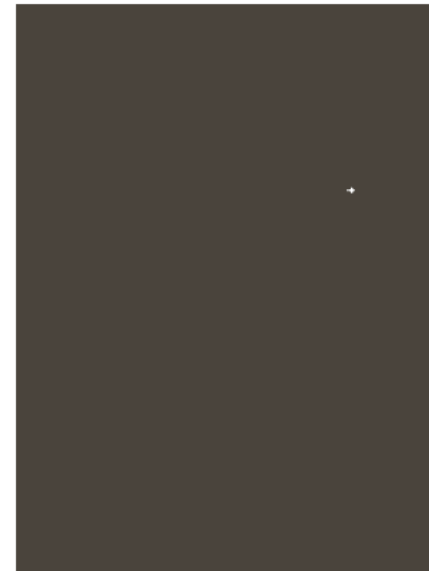
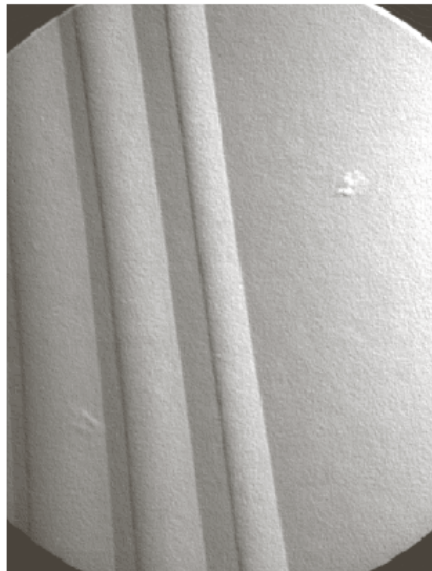
$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

#### Discrete form: 2-D Laplacian - sum of the two components

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

$$g(x, y) = \begin{cases} 1, & \text{if } |R(x, y)| \geq T \\ 0, & \text{otherwise} \end{cases}$$

1	1	1
1	-8	1
1	1	1



a  
b c d

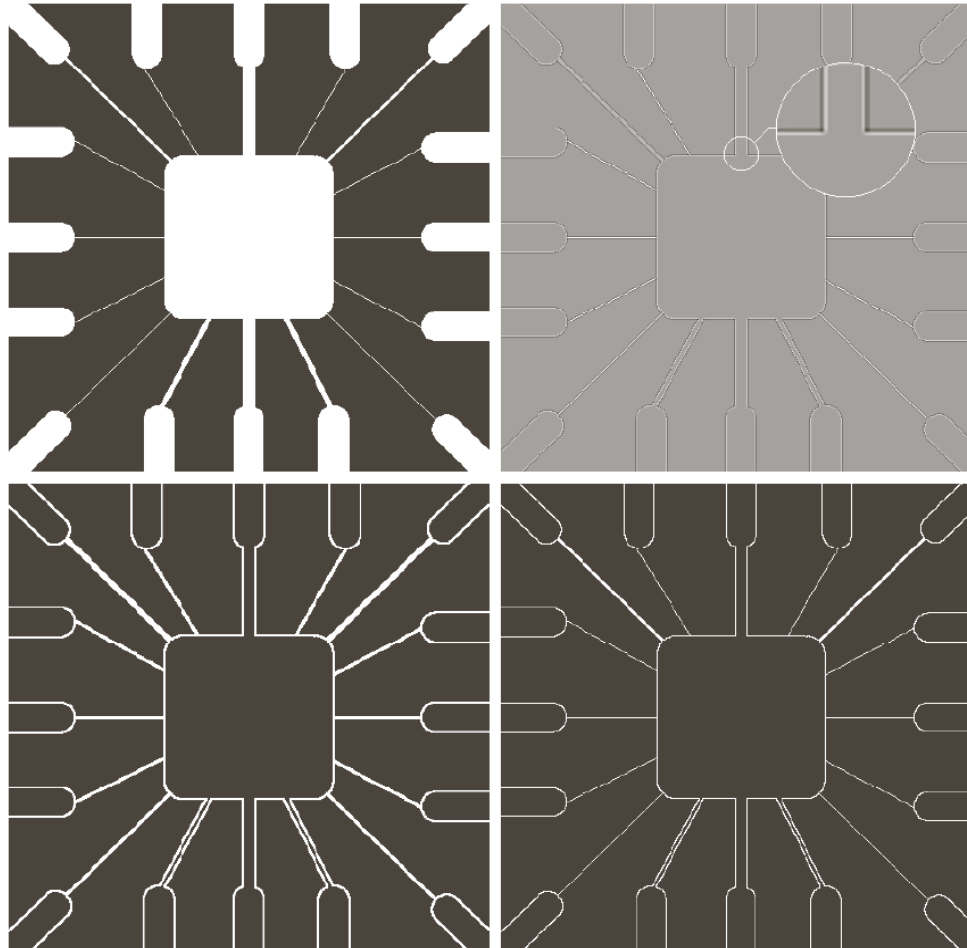
**FIGURE 10.4**

(a) Point detection (Laplacian) mask. (b) X-ray image of turbine blade with a porosity. The porosity contains a single black pixel. (c) Result of convolving the mask with the image. (d) Result of using Eq. (10.2-8) showing a single point (the point was enlarged to make it easier to see). (Original image courtesy of X-TEK Systems, Ltd.)



### 10.2.3 Line detection

Laplacian is isotropic:  $R$  independent of direction



a	b
c	d

**FIGURE 10.5**

(a) Original image.

(b) Laplacian image; the magnified section shows the positive/negative double-line effect characteristic of the Laplacian.

(c) Absolute value of the Laplacian.

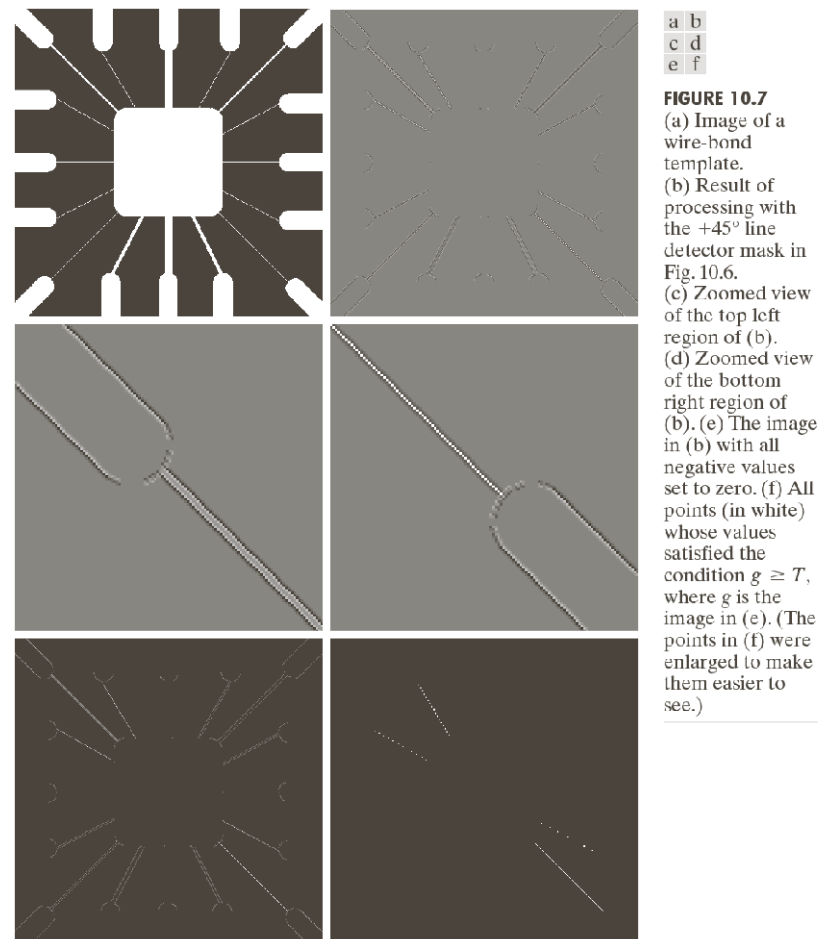
(d) Positive values of the Laplacian.

**Detection of lines in a specified direction:** If  $|R_i| \geq |R_j| \forall j \neq i$  the point in question probably lies on a line that is detected by mask  $i$



-1	-1	-1	2	-1	-1	-1	2	-1	-1	-1	2
2	2	2	-1	2	-1	-1	2	-1	-1	2	-1
-1	-1	-1	-1	-1	2	-1	2	-1	2	-1	-1
Horizontal			+45°			Vertical			-45°		

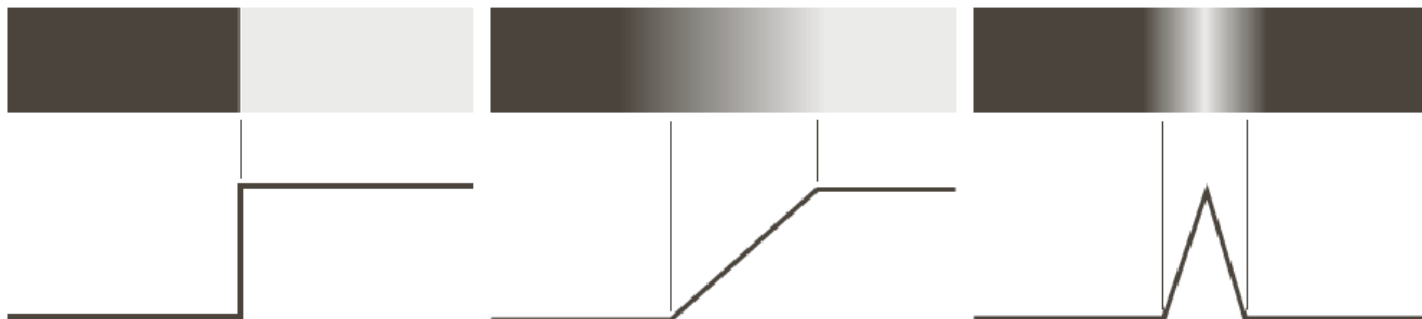
**FIGURE 10.6** Line detection masks. Angles are with respect to the axis system in Fig. 2.18(b).



## 10.2.4 Edge models

Classified according to their intensity profiles:

- (1) Step model (used to derive Canny algorithm [Section 10.2.6])
- (2) Ramp model (slope inversely proportional to degree of blurring)
- (3) Roof edge (models line through a region)

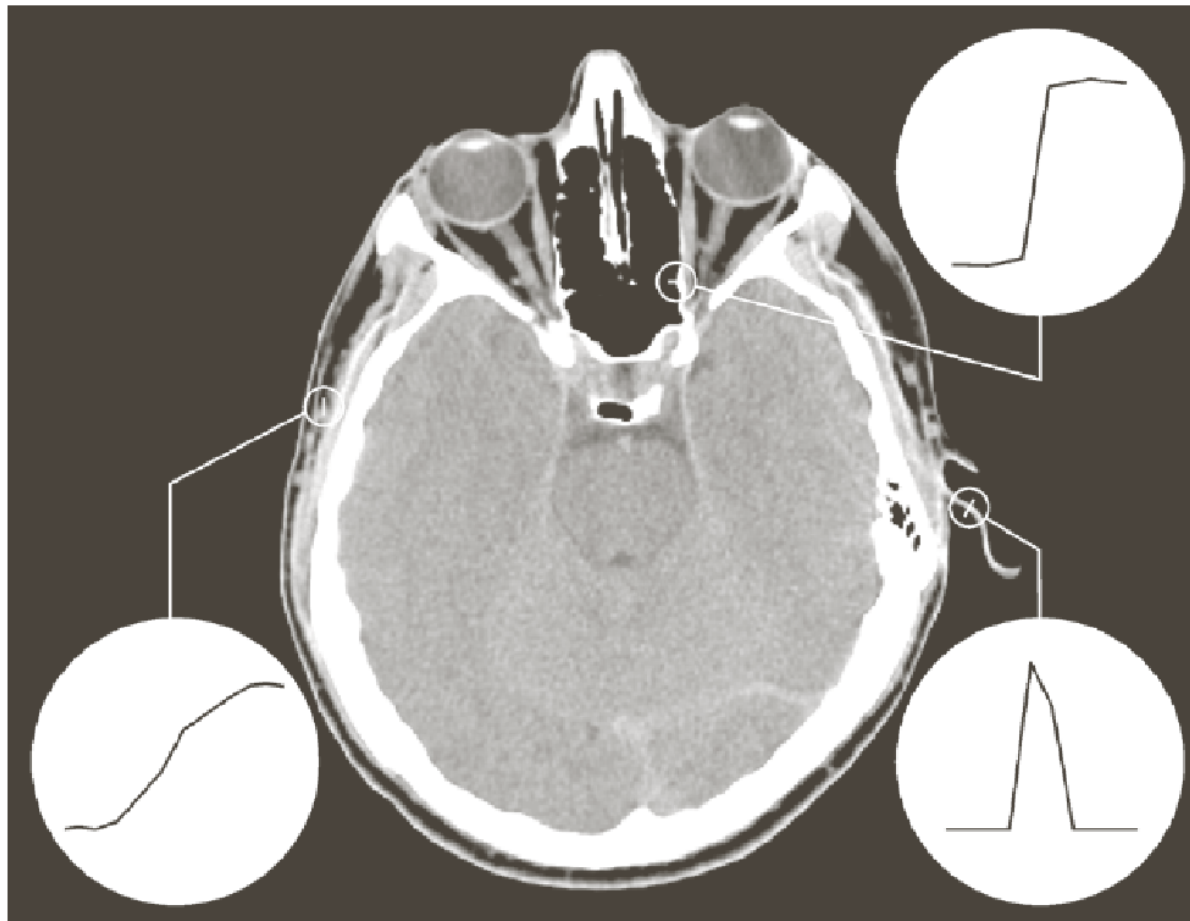


a b c

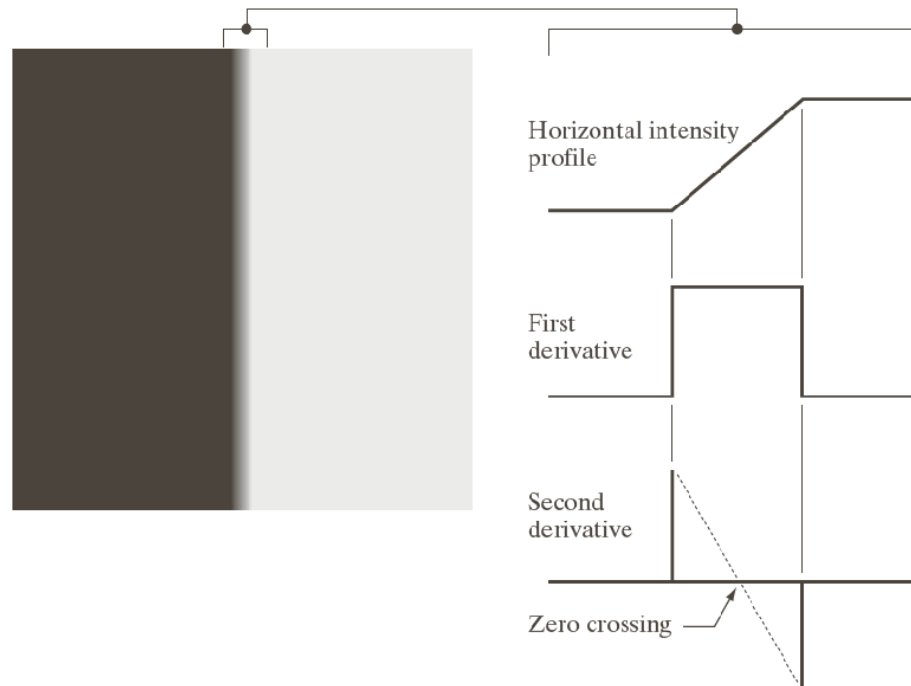
**FIGURE 10.8**

From left to right, models (ideal representations) of a step, a ramp, and a roof edge, and their corresponding intensity profiles.

**Example:** Image that contains all three types of edges



**FIGURE 10.9** A  $1508 \times 1970$  image showing (zoomed) actual ramp (bottom, left), step



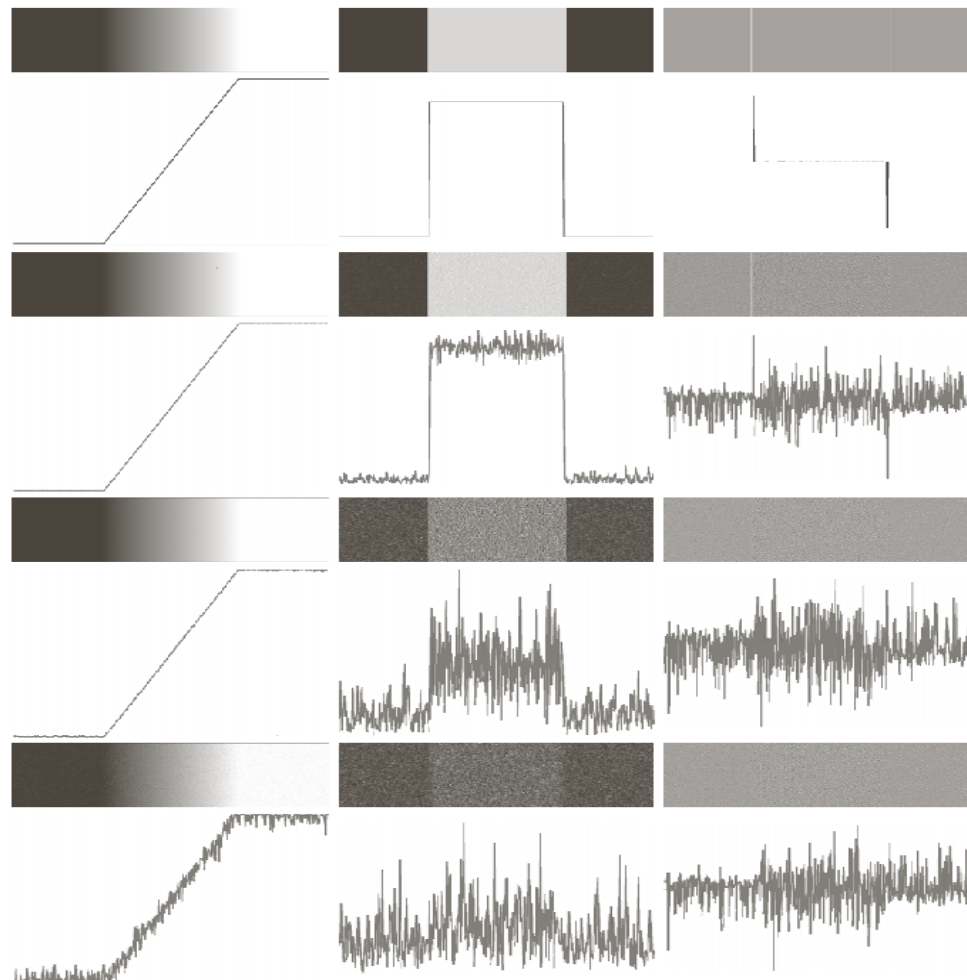
a b

**FIGURE 10.10**  
 (a) Two regions of constant intensity separated by an ideal vertical ramp edge.  
 (b) Detail near the edge, showing a horizontal intensity profile, together with its first and second derivatives.

## Observations:

- (1) Magnitude of first derivative detect presence of edge
- (2) Sign of second derivative determines whether edge pixel is on dark or light side
- (3) Second derivative produces two values for every edge (undesirable)
- (4) The zero crossings of (3) can be used to locate centers of thick edges

## Effect of noise...



**FIGURE 10.11** First column: Images and intensity profiles of a ramp edge corrupted by random Gaussian noise of zero mean and standard deviations of 0.0, 0.1, 1.0, and 10.0 intensity levels, respectively. Second column: First-derivative images and intensity profiles. Third column: Second-derivative images and intensity profiles.



**Conclusion: Three fundamental steps in edge detection**

**(1) Image smoothing for noise reduction**

**(2) Detection of edge points**

- Find potential candidates to become edge points

**(3) Edge localization**

- Select true edge points from candidates