Quiz 2

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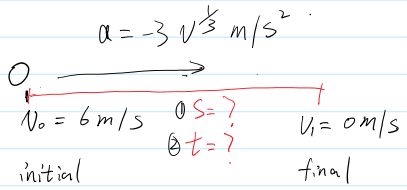
A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of a $= \left(-3v^{\frac{1}{3}}\right)\ m/s^2$ where v is in m/s. Determine how far it travels before it stops. How much time does this take?



① $s = ?$ ② $t = ?$

① $a = f(v)$ $\xrightarrow{a\,ds = v\,dv}$ $s = f(v)$ or $s(v)$ — ② $a = f(v)$ $\xrightarrow{a = \frac{dv}{dt}}$ $v = f(t)$ or $t = f(v)$

or $a(v)$

sub a:

$-3v^{\frac{1}{3}}\,ds = v\,dv$ ⇐ separable equation in 1st ODE ⇒ $-3v^{\frac{1}{3}} = \frac{dv}{dt}$

$-3\,ds = v^{\frac{2}{3}}\,dv$ — $-3\,dt = v^{-\frac{1}{3}}\,dv$

$\int_0^s -3\,ds = \int_{v_0}^{v_1} v^{\frac{2}{3}}\,dv$ ⇐ put limits ⇒ $\int_0^t -3\,dt = \int_{v_0}^{v_1} v^{-\frac{1}{3}}\,dv$

$-3s\big|_0^s = \frac{3}{5}v^{\frac{5}{3}}\big|_6^0$ — simplification — $-3t\big|_0^t = \frac{3}{2}\cdot v^{\frac{2}{3}}\big|_6^0$

$-s = \frac{1}{5}(0 - 6^{\frac{5}{3}})$ — $-3t = \frac{3}{2}(0 - 6^{\frac{2}{3}})$

$s = \frac{1}{5}\cdot 6^{\frac{5}{3}}$ — solve — $t = \frac{6}{2}$

$s = 3.96\ m/s$ — $t = 1.65\ s$

① ② $s = ?$ $\xrightarrow{v = \frac{ds}{dt}}$ $s = f(t)$ or $s(t)$

$v = f(t)$? or $v(t)$

$v = f(t) \Rightarrow \int_0^t -3\,dt = \int_{v_0}^{v} v^{-\frac{1}{3}}\,dv$, if the particle does not stop and the speed is v

$-3t\big|_0^t = \frac{3}{2}v^{\frac{2}{3}}\big|_6^v$

$-3t = \frac{3}{2}(v^{\frac{2}{3}} - 6^{\frac{2}{3}})$

$-2t + 6^{\frac{2}{3}} = v^{\frac{2}{3}}$

$v = (6^{\frac{2}{3}} - 2t)^{\frac{3}{2}}$

$v = \frac{ds}{dt} \Rightarrow (6^{\frac{2}{3}} - 2t)^{\frac{3}{2}}\,dt = ds$

$\int_0^t (6^{\frac{2}{3}} - 2t)^{\frac{3}{2}}\,dt = \int_0^s ds$

u substitution

let $u = 6^{\frac{2}{3}} - 2t$

$du/dt = -2 \Rightarrow dt = -\frac{1}{2}du$

$\int_{u_0}^{u_1} u^{\frac{3}{2}}\left(-\frac{1}{2}du\right) = \int_0^s ds$

$u_1|_{t=t_1} = 6^{\frac{2}{3}} - 2\times 1.65 = 0.0019$

$u_0|_{t=0} = 6^{\frac{2}{3}} = 3.3$

$-\frac{1}{2}\cdot\frac{2}{5}u^{\frac{5}{2}}\big|_{u_0}^{u_1} = s\big|_0^s$

$s = \frac{1}{5}(u_0^{\frac{5}{2}} - u_1^{\frac{5}{2}})$

$s = 19.782/5 = 3.96\ m$