Cell Transmission Model and its Applications

INTRODUCTION

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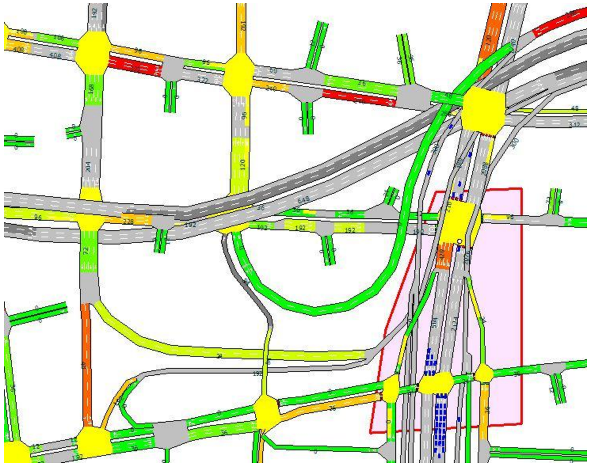
TRAFFIC SIMULATION APPROACHES

- **Microscopic simulation (vehicle-based)**
  - Simulates detailed behavior of every individual vehicle
  - Based on vehicle behaviors such as car-following and lane-change theories
- **Macroscopic simulation (flow-based)**
  - Considers vehicle platoons together instead of individual vehicles
  - Based on the relationship between flow, speed and density
- **Mesoscopic simulation**
  - Combines properties of both microscopic and macroscopic simulation models



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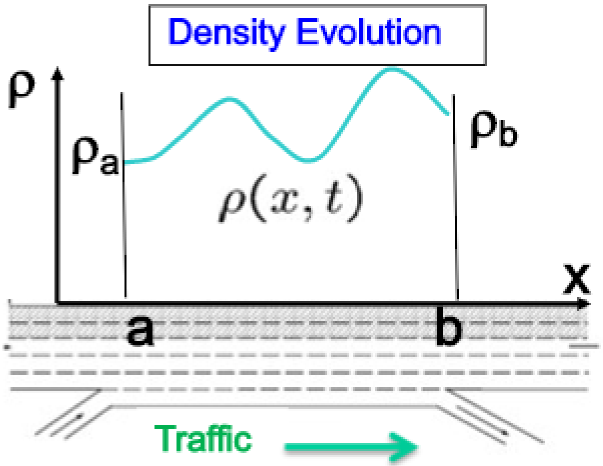
- **Cell transmission model (CTM)** is a macroscopic simulation approach first proposed by Dr. Carlos Daganzo in his paper: The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B: Methodological* 28.4 (1994): 269-287.
- CTM is a discretized framework for solving the kinematic wave theory, the Lighthill-Whitham-Richards (LWR) Model (Lighthill and Whitham 1955, Richards 1956), to model the traffic flow characteristics.

➢ LWR partial differential equation

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$



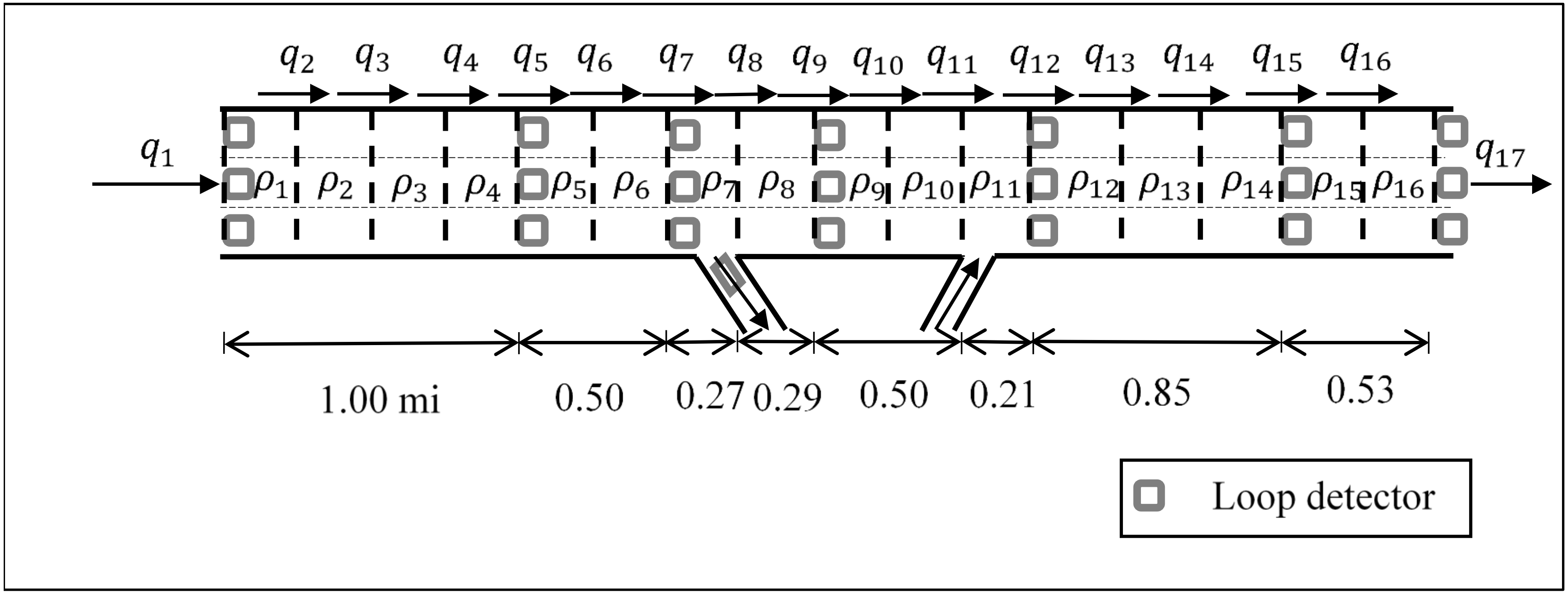
WHY CTM?

- It is **trustworthy** as it’s based on sound traffic theory (LWR model);
- It is **parsimonious** as it only needs a few parameters which can be estimated both online and off-line;
- It requires quite **low computation effort** to predict the traffic states in real-time;
- It is compatible with data from modern advanced transportation information systems (ATIS), e.g., inductive loop detectors (ILD);
- It can be easily used for simulating and evaluating various traffic control strategies which are usually optimized with respect to macroscopic traffic measures, e.g., traffic volume, travel time (derived from traffic speed).

Basic Concepts

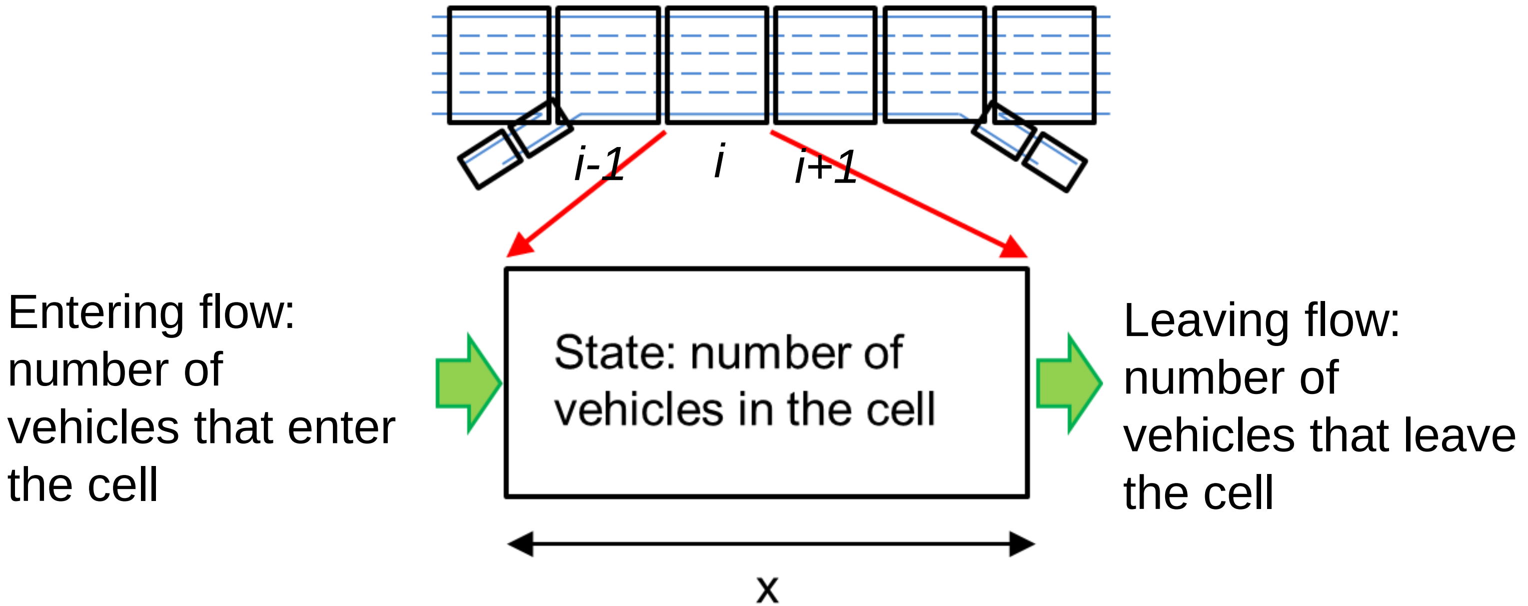
Configuration

- A segment is divided into homogeneous cells, and any cell length cannot be shorter than the distance travelled by the free flow traffic in a time step: $\min(L_c) \geq v_f \cdot T$



- v_f =65 mi/h, $T = 5\ s = \frac{5}{3600} h$, $v_f \cdot T$=0.090 mi
- $\min(L_c) = 0.21\ mi > 0.090\ mi$

TRAFFIC EVOLUTION IN CTM



- Equation of the vehicle count of Cell *i* at time step *t+1*:

$$n_i(t+1) = n_i(t) + y_i(t) - y_{i+1}(t) = n_i(t) + R_i(t) \cdot T - S_i(t) \cdot T$$

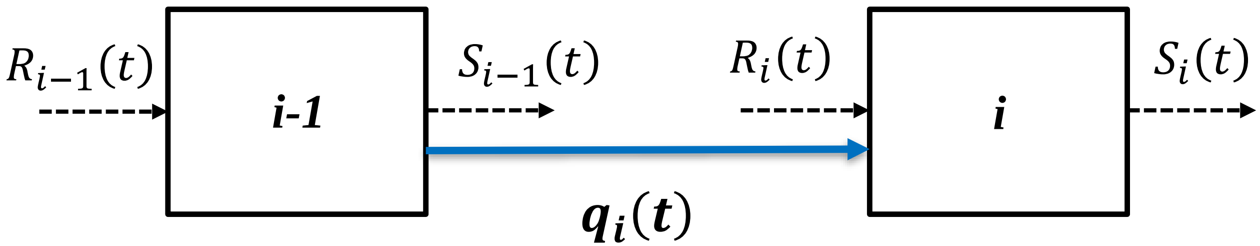
$y_i(t)$: the number of vehicles entering Cell *i* in next time step, *t*

- Equation of traffic density of Cell *i* at time step *t+1*:

$$\rho_i(\mathrm{t}+1) = \rho_i(t) + \frac{T}{l_i}(q_i(t) - q_{i+1}(t)), \quad q_i(t)\text{: the flow rate}$$

Sending and Receiving Functions

- Sending function ($S_i(t)$): the maximum flow rate that can be supplied by Cell *i* in next time step, *t*
- Receiving function ($R_i(t)$): the maximum flow rate that can be received by Cell *i* in next time step, *t*



$q_i(t)$=min{$S_{i-1}(t)$, $R_i(t)$} for $1<i<N$

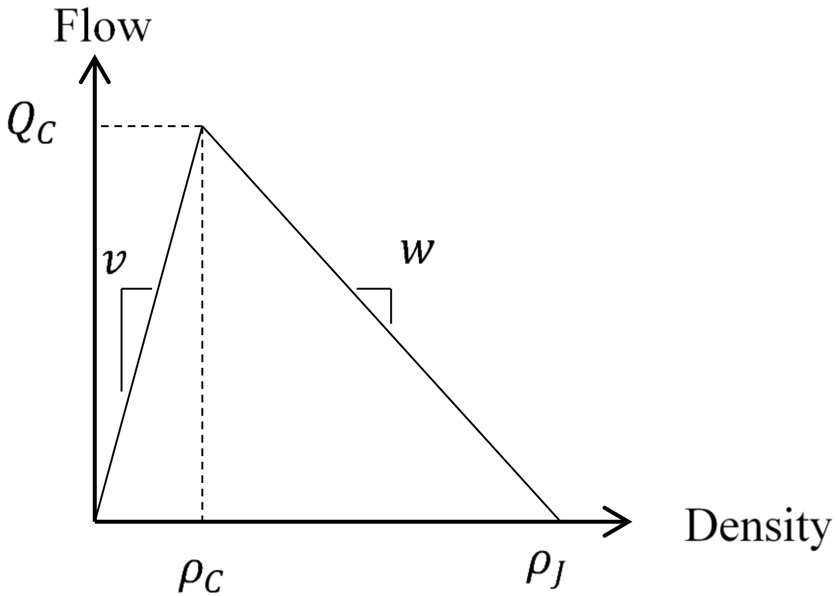
$q_1(t)$=min{$q_U(t)$, $R_i(t)$} for the first cell

$q_N(t)$=min{$S_{i-1}(t)$,$q_D(t)$} for the last cell

U: upstream
D: downstream

Fundamental Diagram (FD)

- A FD governs the relationship between the flow and density.
- A triangular FD is often used.



Q_C: capacity flow

ρ_C: critical density

ρ_J: jam density

v: free-flow speed

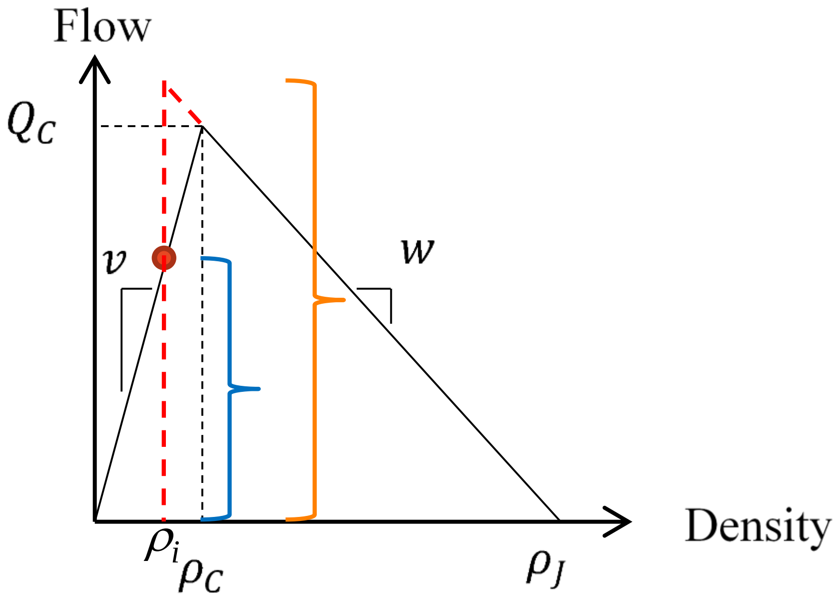
w: shockwave speed

$$S_i(t) = \min\{v_i \rho_i(t), Q_{C,i}\}$$

$$R_i(t) = \min\{Q_{C,i}, w_i\left(\rho_{J,i} - \rho_i(t)\right)\}$$

$$q_i(t)=\min\{S_{i-1}(t),\ R_i(t)\}=\min\{v_{i-1}\rho_{i-1}(t), Q_{C,i-1},\ Q_{C,i}, w_i\left(\rho_{J,i} - \rho_i(t)\right)\}$$

FUNDAMENTAL DIAGRAM (FD)



$$S_i(t) = \min\{v_i\rho_i(t), Q_{C,i}\}$$

$$R_i(t) = \min\{Q_{C,i}, w_i\left(\rho_{J,i} - \rho_i(t)\right)\}$$

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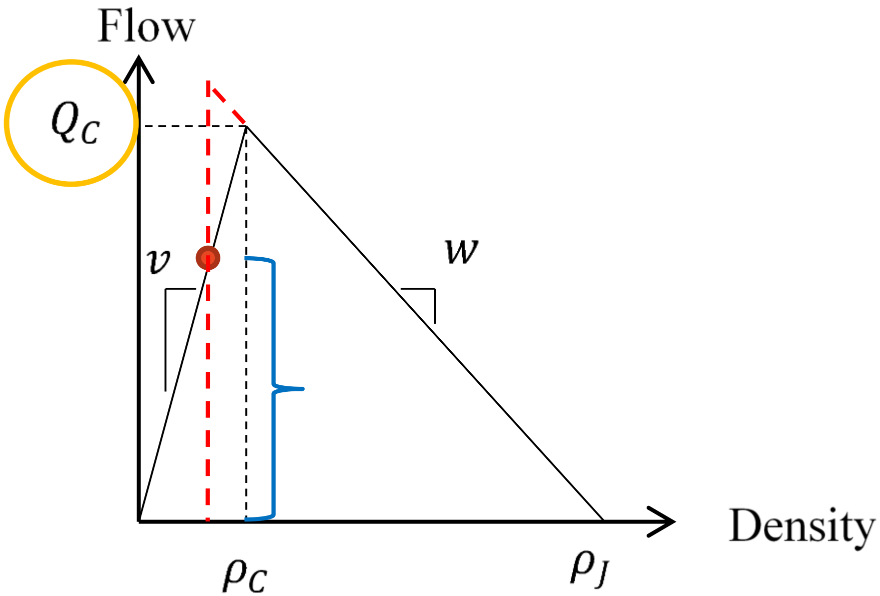
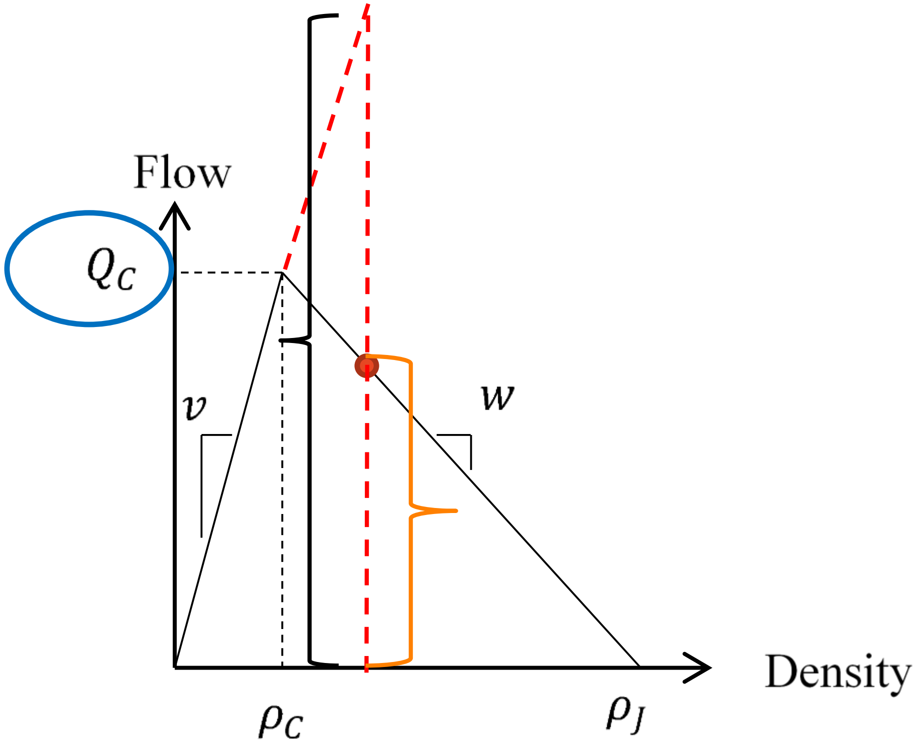


$$S_i(t) = \min\{v_i \rho_i(t), Q_{C,i}\}$$

$$R_i(t) = \min\{Q_{C,i}, w_i\left(\rho_{J,i} - \rho_i(t)\right)\}$$

- If $\rho_i(t) < \rho_C$ (free-flow), $S_i(t) = v_i \rho_i(t)$, $R_i(t) = Q_{C,i}$

FUNDAMENTAL DIAGRAM (FD)



$$S_i(t) = \min\{v_i\rho_i(t), Q_{C,i}\}$$

$$R_i(t) = \min\{Q_{C,i}, w_i\left(\rho_{J,i} - \rho_i(t)\right)\}$$

- If $\rho_i(t) > \rho_C$ (congested), $S_i(t) = Q_{C,i}$, $R_i(t) = w_i\left(\rho_{J,i} - \rho_i(t)\right)$

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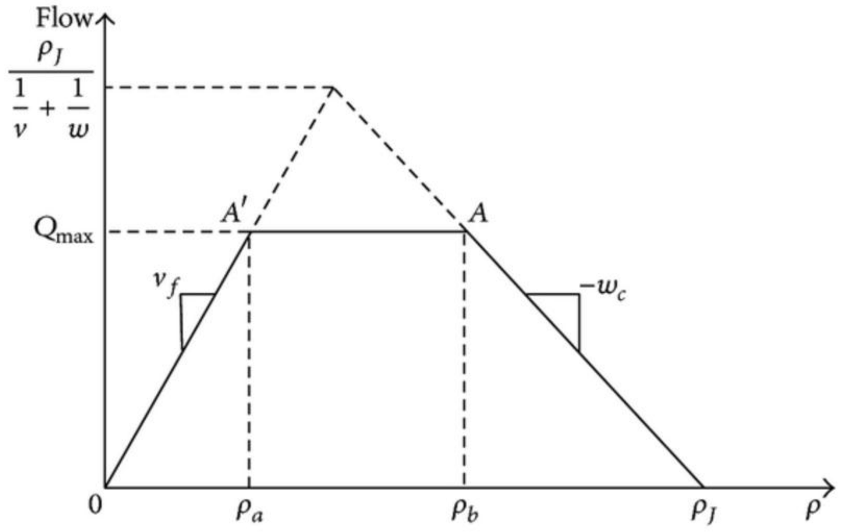
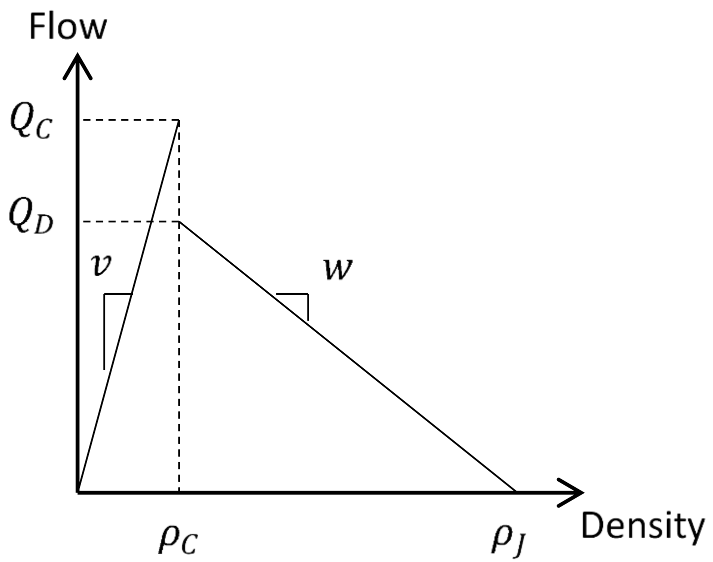
FUNDAMENTAL DIAGRAM (FD)

- Other FD types:



Trapezoid



Inverse-Lamda

Modelling Traffic Phenomena in CTM

SHOCKWAVE

- Illustration of shockwave when there is state transition ($\rho_C = 5$)

Clock Tick #	Cell #, j: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
2	4	4	4	4	4	4	4	4	4	7	1	4	4	4	4
3	4	4	4	4	4	4	4	4	4	10	1	1	4	4	4
4	4	4	4	4	4	4	4	4	4	13	1	1	1	4	4
5	4	4	4	4	4	4	4	4	6	14	1	1	1	1	4
6	4	4	4	4	4	4	4	4	9	14	1	1	1	1	1
7	4	4	4	4	4	4	4	4	12	14	1	1	1	1	1
8	4	4	4	4	4	4	4	5	14	14	1	1	1	1	1
9	4	4	4	4	4	4	4	8	14	14	1	1	1	1	1
10	4	4	4	4	4	4	4	11	14	14	1	1	1	1	1
11	4	4	4	4	4	4	4	14	14	14	1	1	1	1	1
12	4	4	4	4	4	4	7	14	14	14	1	1	1	1	1
13	4	4	4	4	4	4	10	14	14	14	1	1	1	1	1
14	4	4	4	4	4	4	13	14	14	14	1	1	1	1	1
15	4	4	4	4	4	6	14	14	14	14	1	1	1	1	1
16	4	4	4	4	4	9	14	14	14	14	1	1	1	1	1
17	4	4	4	4	4	12	14	14	14	14	1	1	1	1	1
18	4	4	4	4	5	14	14	14	14	14	1	1	1	1	1
19	4	4	4	4	8	14	14	14	14	14	1	1	1	1	1
20	4	4	4	4	11	14	14	14	14	14	1	1	1	1	1
21	4	4	4	4	14	14	14	14	14	14	1	1	1	1	1
22	4	4	4	7	14	14	14	14	14	10	5	1	1	1	1
23	4	4	4	10	14	14	14	14	10	10	5	5	1	1	1
24	4	4	4	13	14	14	14	10	10	10	5	5	5	1	1
25	4	4	6	14	14	14	10	10	10	10	5	5	5	5	1
26	4	4	9	14	14	10	10	10	10	10	5	5	5	5	5
27	4	4	12	14	10	10	10	10	10	10	5	5	5	5	5
28	4	5	14	10	10	10	10	10	10	10	5	5	5	5	5
29	4	8	10	10	10	10	10	10	10	10	5	5	5	5	5
30	4	7	10	10	10	10	10	10	10	10	5	5	5	5	5
31	4	6	10	10	10	10	10	10	10	10	5	5	5	5	5
32	4	5	10	10	10	10	10	10	10	10	5	5	5	5	5
33	4	4	10	10	10	10	10	10	10	10	5	5	5	5	5
34	4	4	9	10	10	10	10	10	10	10	5	5	5	5	5
35	4	4	8	10	10	10	10	10	10	10	5	5	5	5	5
36	4	4	7	10	10	10	10	10	10	10	5	5	5	5	5
37	4	4	6	10	10	10	10	10	10	10	5	5	5	5	5
38	4	4	5	10	10	10	10	10	10	10	5	5	5	5	5
39	4	4	4	10	10	10	10	10	10	10	5	5	5	5	5
40	4	4	4	9	10	10	10	10	10	10	5	5	5	5	5
41	4	4	4	8	10	10	10	10	10	10	5	5	5	5	5
42	4	4	4	7	10	10	10	10	10	10	5	5	5	5	5
43	4	4	4	6	10	10	10	10	10	10	5	5	5	5	5
44	4	4	4	5	10	10	10	10	10	10	5	5	5	5	5
45	4	4	4	4	10	10	10	10	10	10	5	5	5	5	5

Model Oscillation

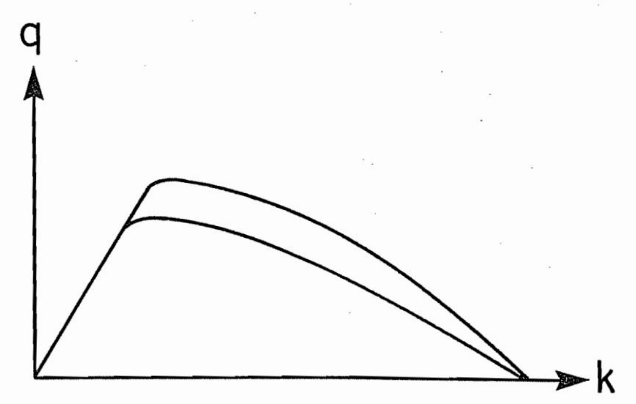
- Oscillation/ Stop-and-go traffic:

Traffic oscillations, also known as the "stop-and-go" traffic, refer to the phenomenon that congested traffic tends to oscillate between slow-moving and fast-moving states rather than maintain a steady state.

Newell (1963) proposed a possible explanation: "an instability would arise if drivers catching up with denser/slower traffic ahead were to delay braking, perhaps in the hope that traffic would clear up before they had to slow down. This behavior would result in average spacings shorter than usual when traffic was decelerating and would cause an instability"

- Strategy:

Vehicles are regulated by the top curve if the density in the "look-ahead" section (e.g., the current cell) is greater than the traffic density in the vehicles' immediate neighborhood (e.g., the preceding cell); otherwise, vehicles are regulated by the lower curve.



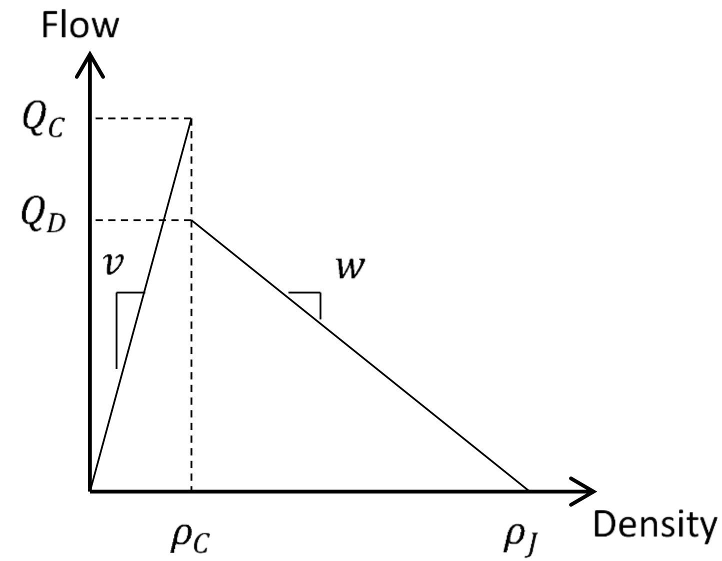
Capacity Drop

- Capacity drop:

Studies of freeway bottlenecks have shown that discharge flow decreases once queues form just upstream (Hall and Agyemang-Duah, 1991; Banks, 1991; Cassidy and Bertini, 1999).

- Strategy:

An inverse-lamda FD is applied to model the capacity drop: once the density at a bottleneck exceeds the critical density, i.e., the congestion forms, the capacity drops from Q_C to Q_D.



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CTM Application

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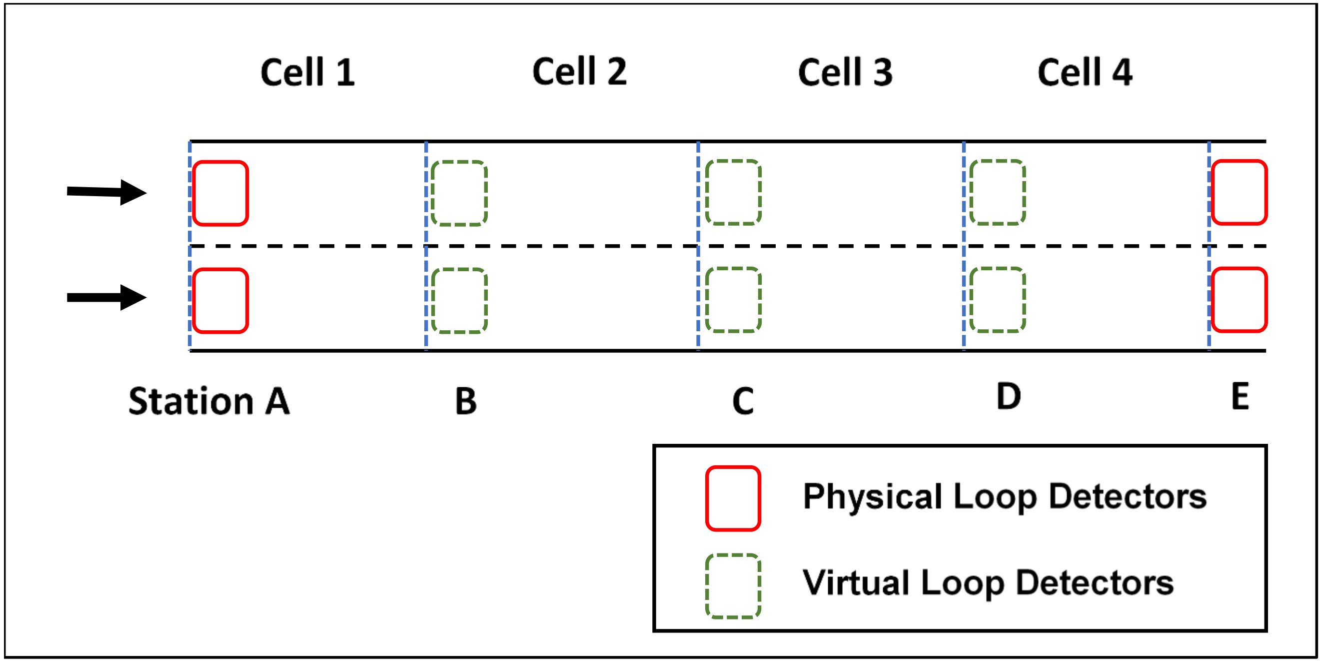
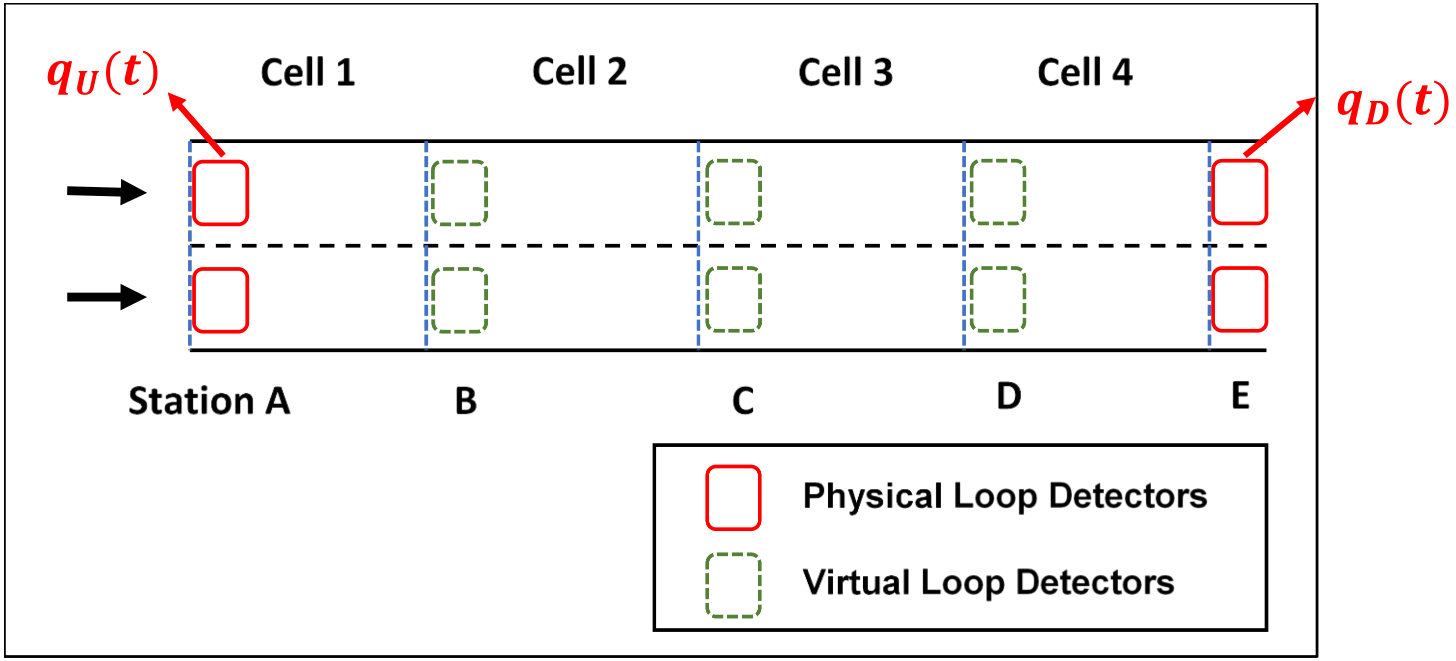
Traffic State Estimation

- CTM can take aggregated data such as flow and density as inputs, and it thus operates sufficiently with aggregated data measured from ILD stations and can be applied to simulate traffic conditions at unmeasured collections.



- To run the CTM, we need to know the initial densities of all cells, $q_U(t)$ and $q_D(t)$, and FD parameters.

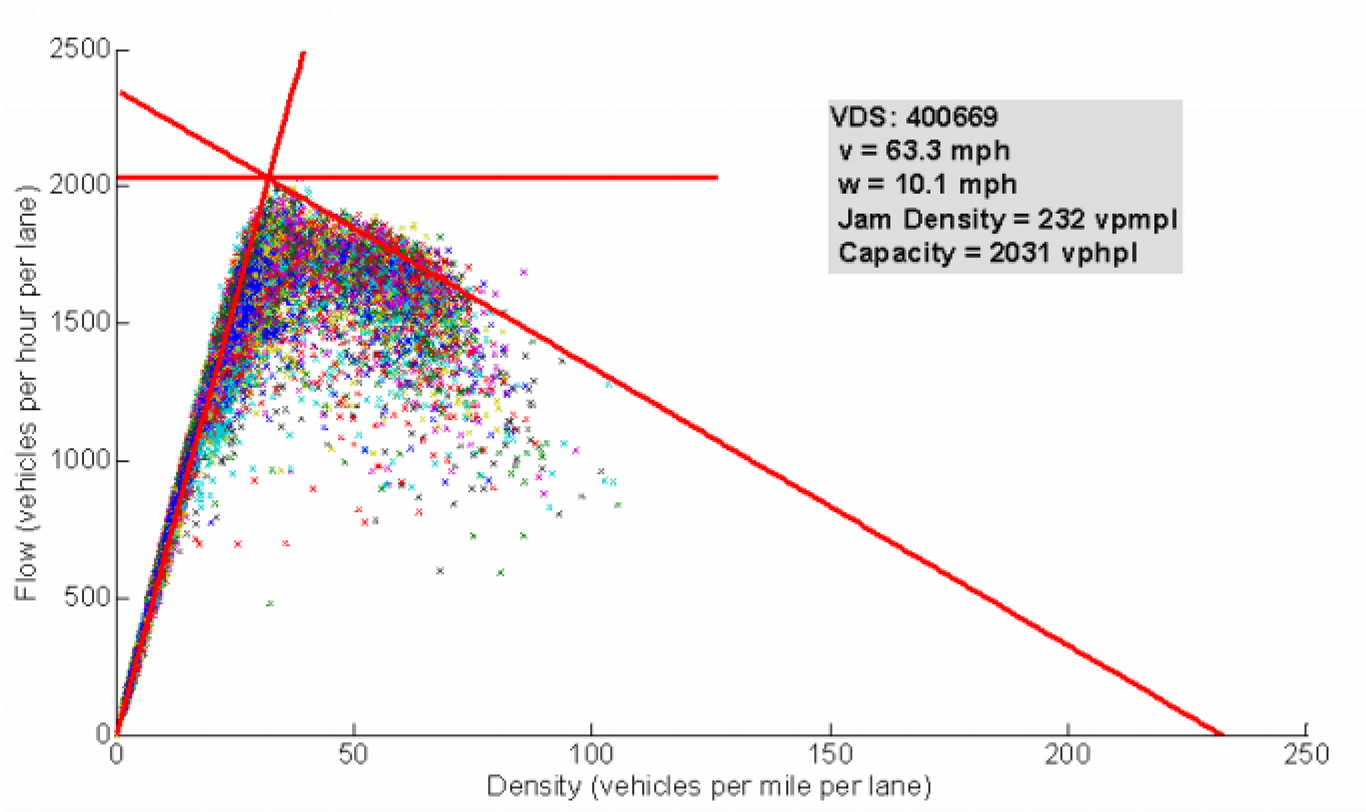


➢ Initial densities of all cells can be estimated using ILD data:

$$\rho_1(0) = \rho_2(0) = \rho_U(0)$$

$$\rho_3(0) = \rho_4(0) = \rho_D(0)$$

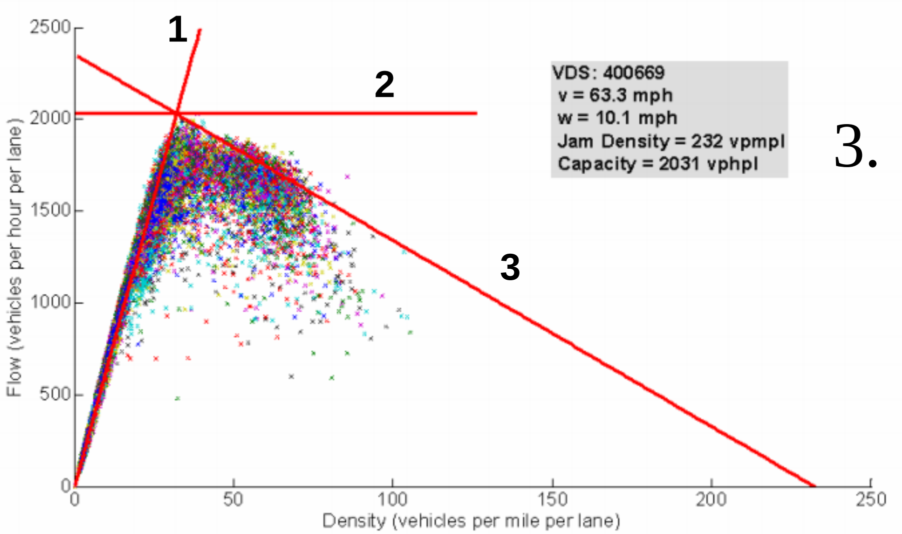
- FD parameters can be estimated from ILD data (FD calibration)
  - **Method 1:** Fit the FD using discrete flow-density data points (it requires a large amount of data)



A Calibrated FD

Credit: Dervisoglu et al. 2009

Source: *Dervisoglu G. Gomes G. Kwan J. Muralidharan A. Horowitz R. Horowitz R. 2009. Automatic calibration of the fundamental diagram and empirical observations on capacity. Transportation Research Board 88th Annual Meeting.*



1. Estimate the free-flow speed, v, using **least-squared method** with flow-density pairs in the free-flow conditions (e.g., *v*>Speed limit -10 mph);
2. Find the maximum measured flow rate, q_{max}, as the capacity, Q_C. Critical density is determined by $\rho_C = \frac{Q_C}{v}$;
3. Estimate the shockwave speed, w, and the jam density, ρ_J, using the **approximate quantile regression** with flow-density pairs exceeding the critical density.

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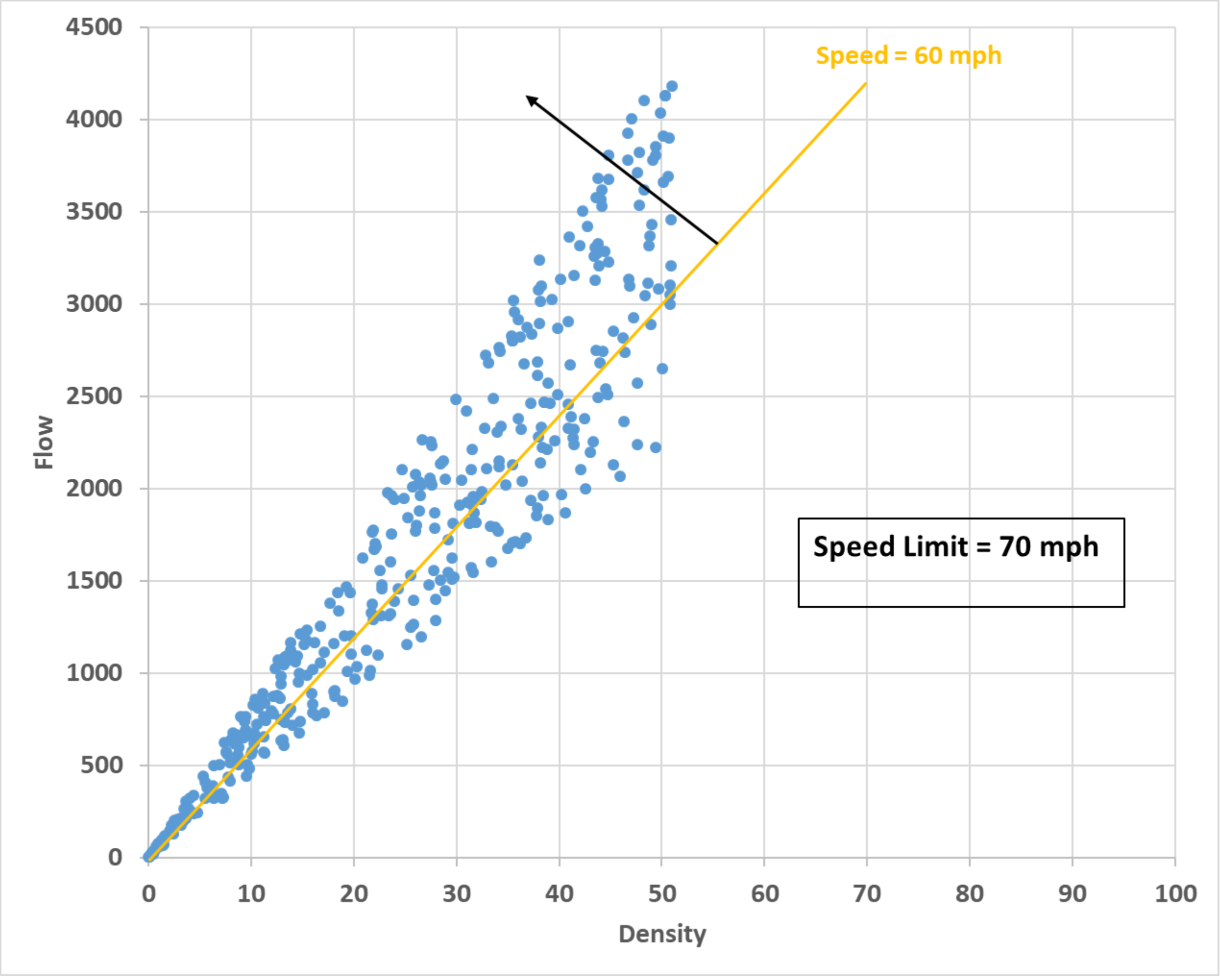

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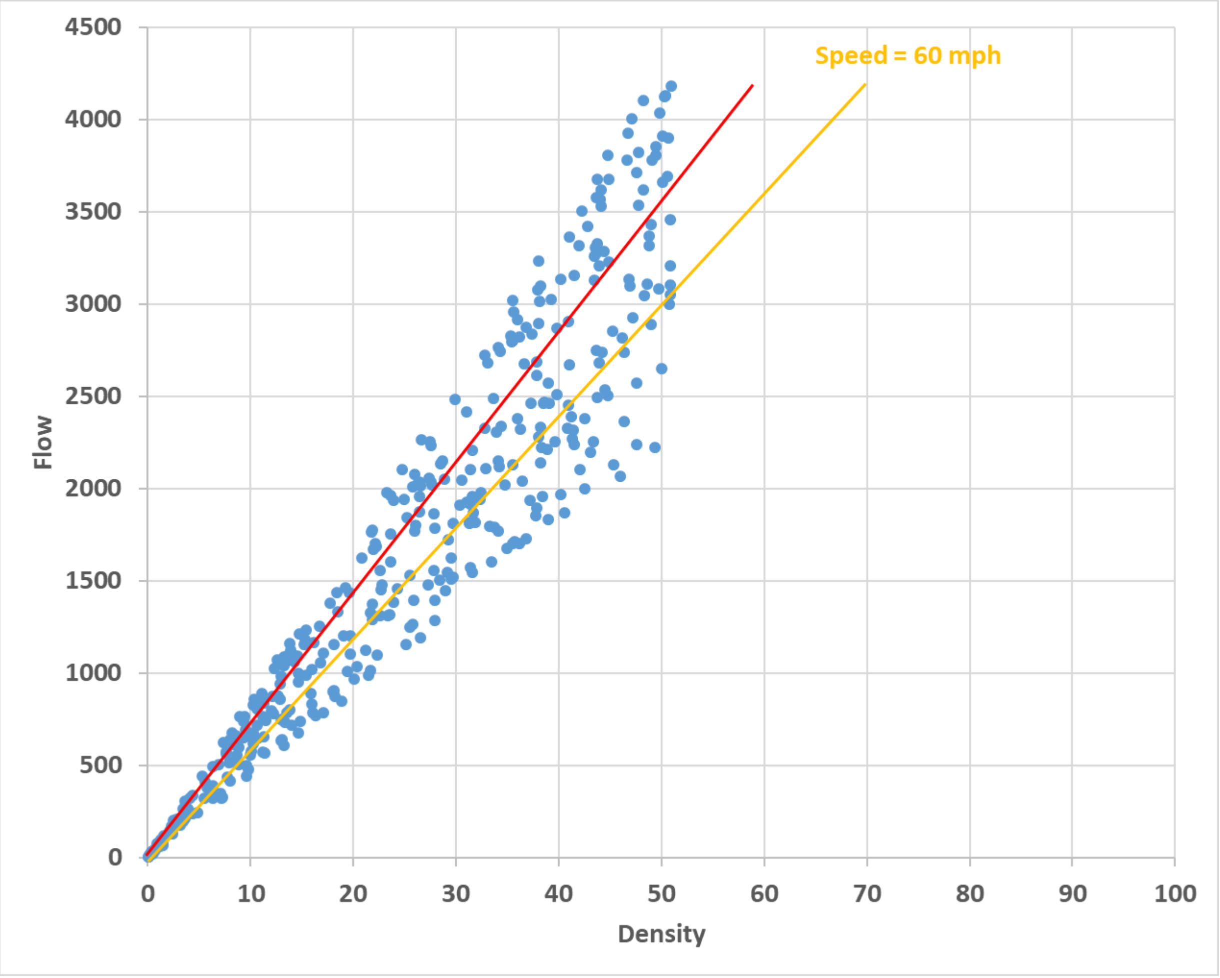

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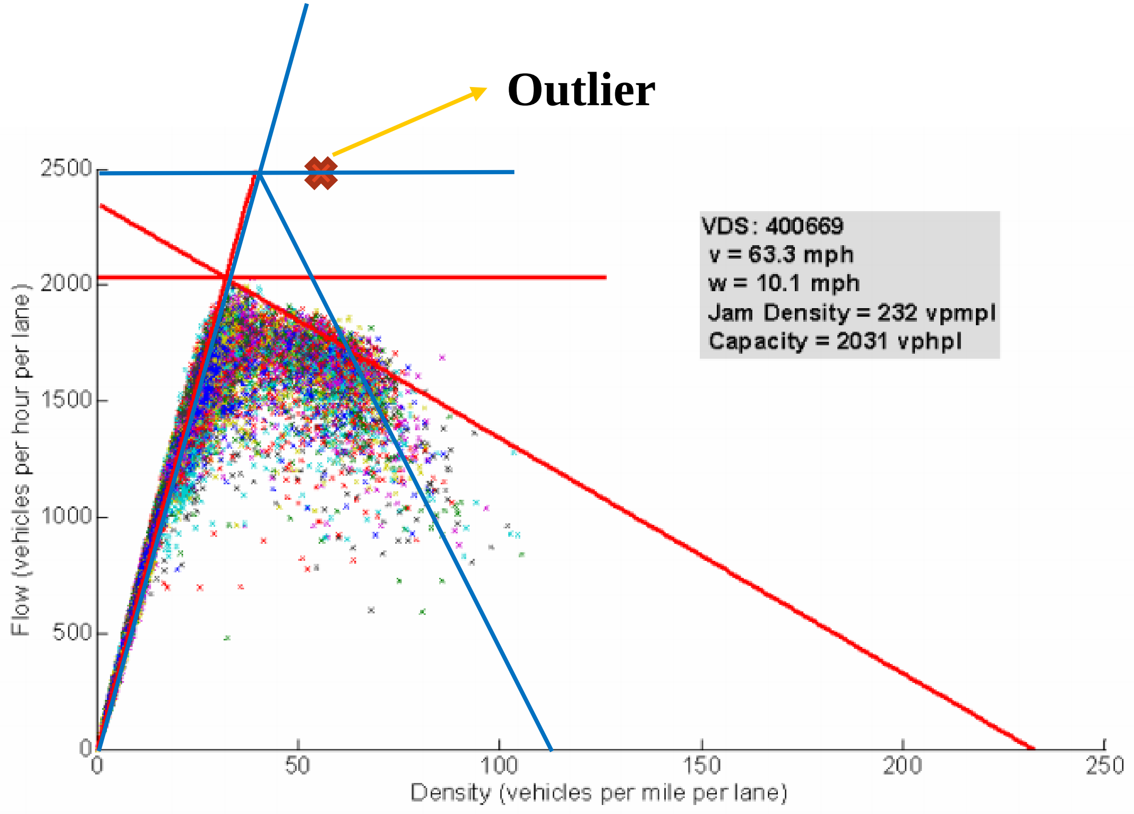
Outlier Correction:

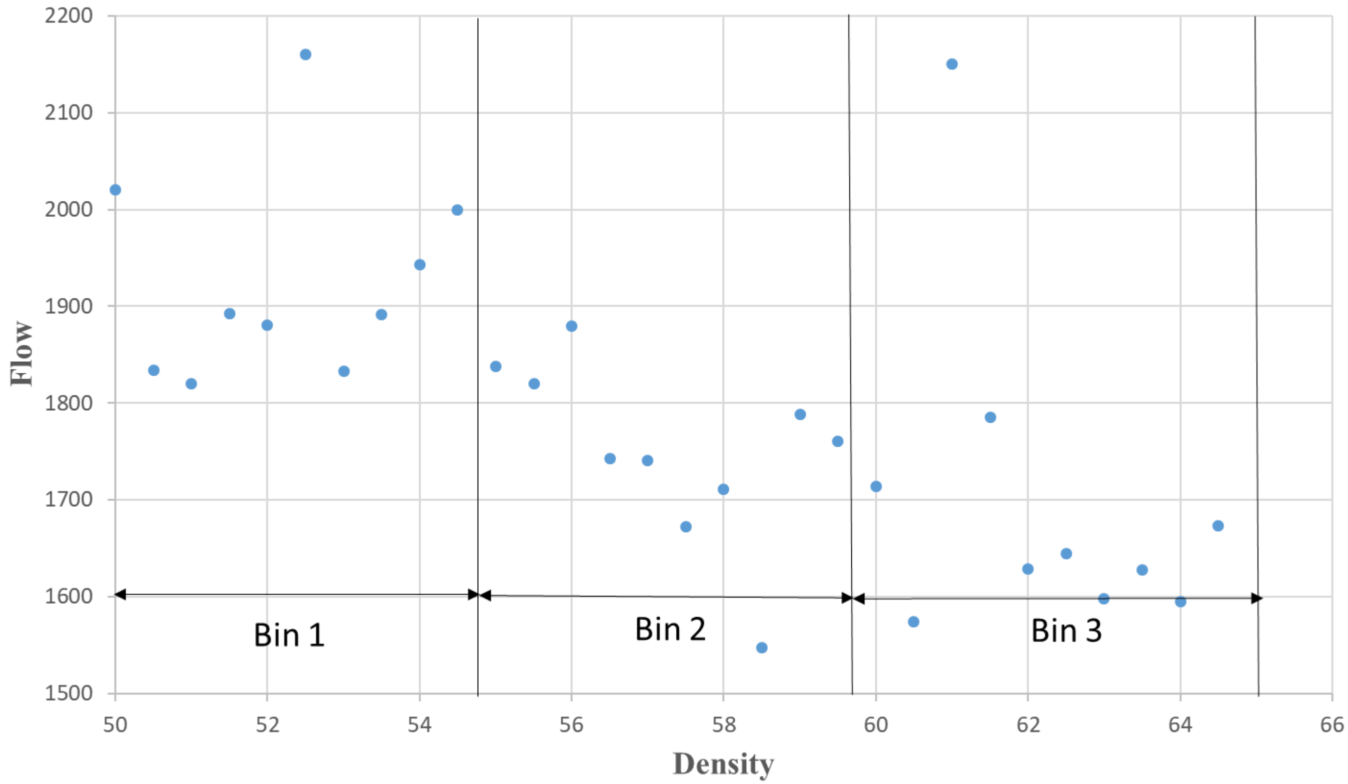
$$\boldsymbol{Q_C} = \begin{cases} \mathbf{min}(\boldsymbol{q_{max}}, \mathbf{2400})\,, & \boldsymbol{if\ FFS \geq 70m\,i/h} \\ \mathbf{min}(\boldsymbol{q_{max}}, \mathbf{2400 - 10 \times (70 - FFS)}), & \boldsymbol{if\ FFS < 70m\,i/h} \end{cases}$$



3. Estimate the shockwave speed, w, and the jam density, ρ_J, using the **approximate quantile regression** with flow-density pairs exceeding the critical density.

➢ Approximate quantile regression

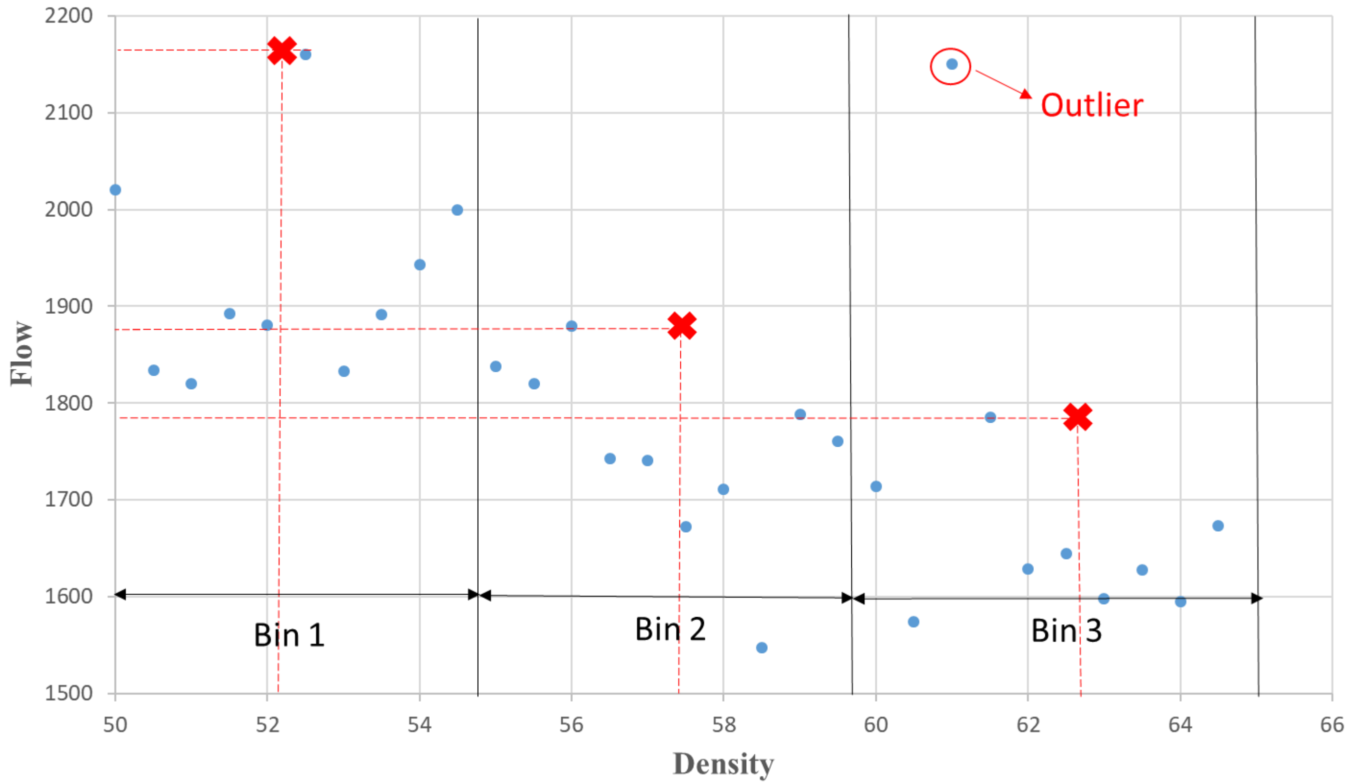
Step 1: Divide data points into non-overlapping bins with N (e.g.,10) points in each bin



➢ Approximate quantile regression

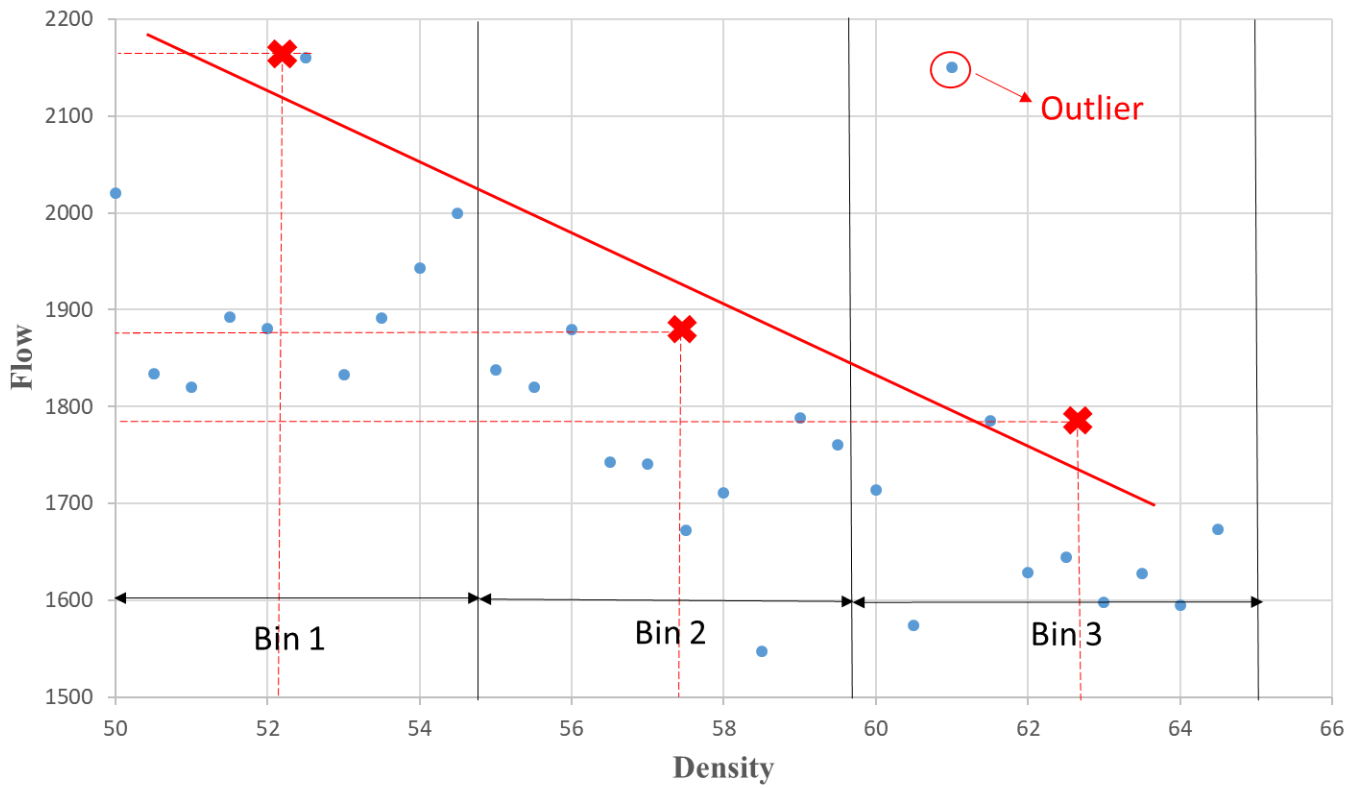
Step 2: Determine the density and flow of each bin

$$BinDensity = mean(\rho_1, \rho_2, \dots, \rho_{10})$$
$$Bin = \{f_1, f_2, \dots, f_{10}\}$$
$$BinFlow = \max_{f_i}(f_i | f_i \in Bin, f_i < Q3 + 1.5Q1)$$



- Approximate quantile regression

Step 3: Estimate the shockwave speed and jam density using a least-squared method with bin flow-density pairs



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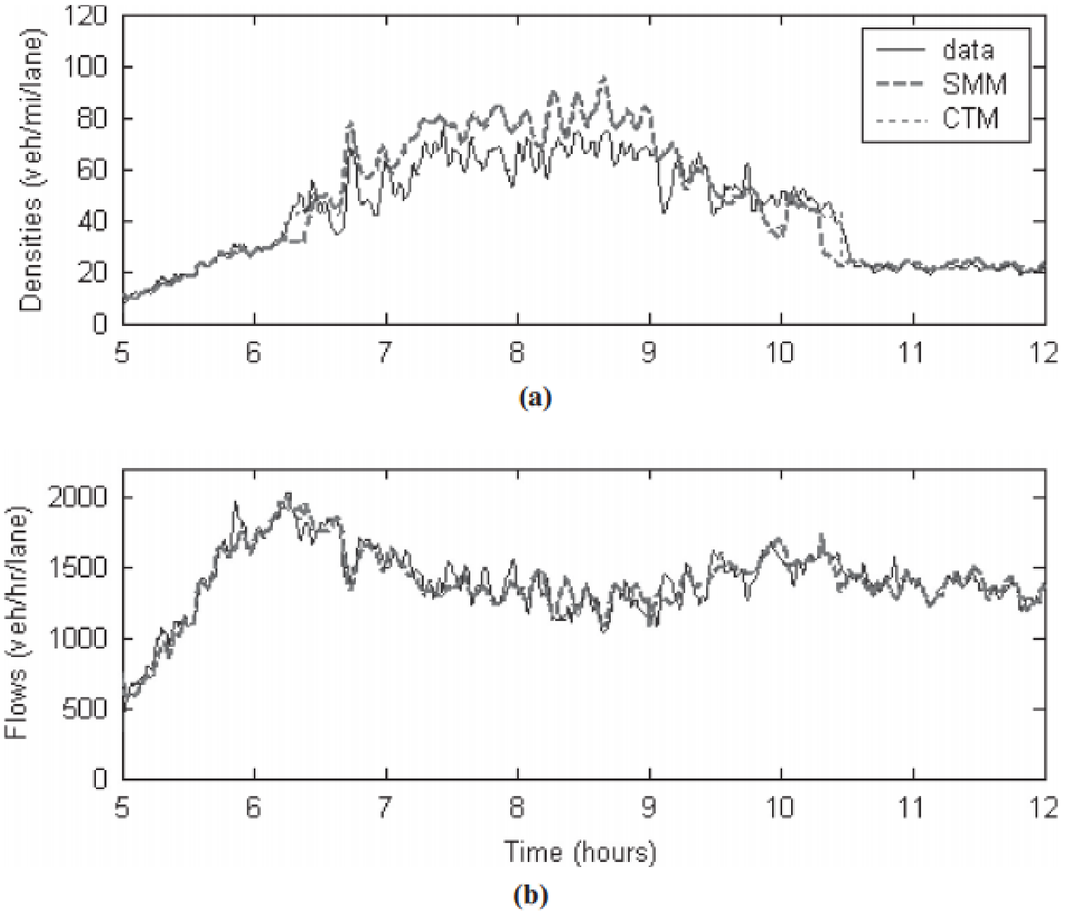
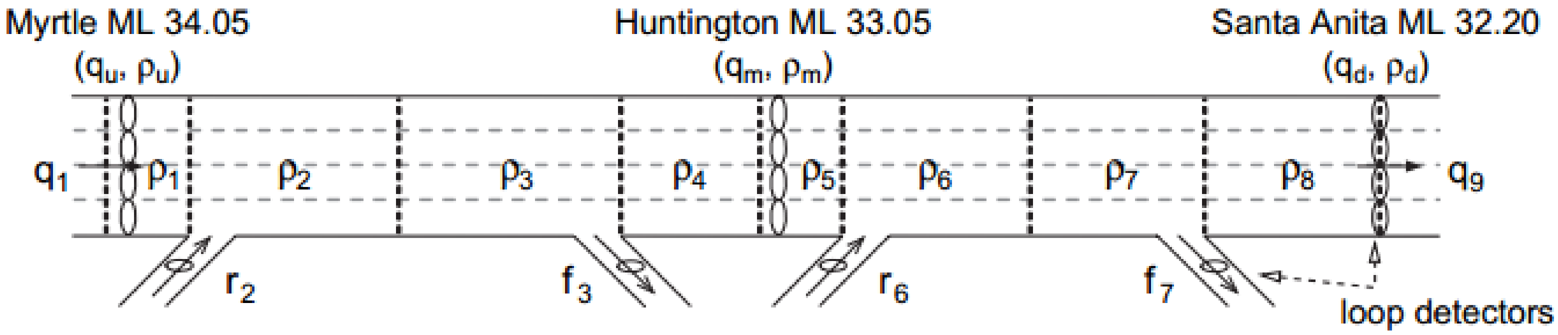
- FD calibration
  - **Method 2:** Optimize FD parameters to minimize the error between the observed and estimated data (it usually uses data of one day



Comparison between Observed and Estimated Traffic Data
Credit: Muñoz et al. 2006

❖ Method 2

- Use data from three ILD stations
  1. Data from the first and the last stations provides the initial densities of all cells, $q_U(t)$ and $q_D(t)$
  2. Data from the middle station is used as benchmark, i.e., used to optimize the error



A Layout of Three ILD Stations
Credit: Muñoz et al. 2006

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❖ Method 2

Source: Zhong, R., Chen, C., Chow, A. H., Pan, T., Yuan, F., & He, Z. (2016). Automatic calibration of fundamental diagram for first-order macroscopic freeway traffic models. *Journal of Advanced Transportation*, *50*(3), 363-385.

$$\text{Problem NLP}: \quad x^* = \arg\min_x g = \arg\min_x (\hat{\rho} - \rho)^T (\hat{\rho} - \rho)$$

Equality constraints:

$$\rho_i(k+1) = \rho_i(k) + \frac{\Delta T}{l_i}\left(q_i(k) - q_{i+1}(k) + r_i(k) - f_i(k)\right),$$

$$q_1(k) = min\{q_u(k), \widetilde{R}_1(k)\}, \quad \forall k,$$

$$q_i(k) = min\left\{\widetilde{S}_{i-1}(k), \widetilde{R}_i(k)\right\}, i = 2, \cdots, n, \forall k,$$

$$q_{n+1}(k) = min\left\{\widetilde{S}_n(k), \quad q_d(k)\right\}, \forall k,$$

$$\widetilde{S}_i(k) = min\{v_{f,i}\rho_i(k) - f_i(k), Q_{M,i} - f_i(k)\},$$

$$\widetilde{R}_i(k) = min\{Q_{M,i} - r_i(k), w_{c,i}(\rho_{J,i} - \rho_i(k)) - r_i(k)\}$$

Definition constraints:

$$v_{f,i}\Delta T \le l_i, \forall i,$$

$$Q_{M,i} = v_{f,i}\rho_{c,i}, \forall i,$$

$$w_{c,i} = \frac{Q_{M,i}}{\rho_{J,i} - \rho_{c,i}}, \forall i,$$

$$w_{c,i} < v_{f,i}, \forall i,$$

$$v_{f,i}, w_{c,i}, Q_{M,i}, \rho_{c,i}, \rho_{J,i} > 0, \forall i$$