선형대수학 과제 - 14

3. $L: R^2 \Rightarrow R^3$ $\quad L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$

$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

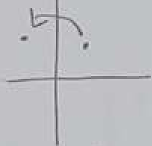
$L[u+v] = \begin{bmatrix} u_1+u_2 \\ v_1+v_2 \\ 0 \end{bmatrix}$ $\quad L(u)+L(v) = \begin{bmatrix} u_1 \\ v_1 \\ 0 \end{bmatrix} + \begin{bmatrix} u_2 \\ v_2 \\ 0 \end{bmatrix} = \begin{bmatrix} u_1+u_2 \\ v_1+v_2 \\ 0 \end{bmatrix}$

$L[\alpha u] = \begin{bmatrix} \alpha u_1 \\ \alpha u_2 \\ 0 \end{bmatrix}$ $\quad \alpha L(u) = \alpha \begin{bmatrix} u_1 \\ u_2 \\ 0 \end{bmatrix}$ 에서 $\left( \begin{array}{l} L(u+v) = L(u) + L(v) \\ L(\alpha u) = \alpha L(u) \end{array} \right.$

성립하므로 선형변환 된다.

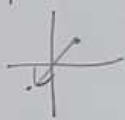
4. (1)

$L\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} -a_1 \\ a_2 \end{bmatrix}$



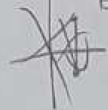
y축대칭

$\begin{bmatrix} -a_1 \\ -a_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$



원점대칭

$\begin{bmatrix} -a_2 \\ a \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$



반시계 ~~방향~~ 90° 회전

5. $(a, b, c) \mid (b, c, a)$ $\quad \begin{bmatrix} a \\ b \\ c \end{bmatrix} \boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} b \\ c \\ a \end{bmatrix}$

6.

$L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$, $L\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{Bmatrix} 3 \\ -5 \end{Bmatrix}$, $L\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 2 \\ -4 \end{bmatrix} - 2\begin{bmatrix} 3 \\ -5 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \end{bmatrix}$

9.

(1) $L: R \Rightarrow R$ $\quad \sin 1 + \sin 1 \neq \sin 2$ 이므로 성립하지 않음.

$L(x) = \sin x$

(2) $L: R^3 \Rightarrow R^2$ $\quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ $\quad L(u) + L(v) = \begin{bmatrix} u_1^2 \\ 0 \end{bmatrix} + \begin{bmatrix} v_1^2 \\ 0 \end{bmatrix}$ 이므로 성립하지 않음.

$L(x, y, z) = (x^2, 0)$ $\quad v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

10.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$$

$Ax = b$

$x = A^{-1}b$

$x_1 - 2x_3 = -1$

$-2x_1 + x_2 + 6x_3 = 7$

$3x_1 - 2x_2 - 5x_3 = -3$

$x_1 - 2x_3 = -1$

$x_1 - x_2 + x_3 = 4$

$x_2 - 5x_3 = -5$

$3x_1 + x_2 = 2.$

$$\mathrm{adj}(A) = \begin{bmatrix} 7 & 4 & 2 \\ 8 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

$\det(A) = 7 + 0 - 2 \times 1 = 5$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 7 & 4 & 2 \\ 8 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$x = \frac{1}{5}\begin{bmatrix} 7 & 4 & 2 \\ 8 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\det(A) \neq 0$ 이기 때문에 유일한 해이다.

11.

$$u = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

(1) $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = 1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$

(2) $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = 2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - 1\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}$

(3) $T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = 3\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ -3 \end{bmatrix}$ 이다.