

2023-04-05 week 5

연습문제 3.3 Part 3

4.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

$a^{(11)}$ 값을 이항 $\boxed{a = -2}$

6.

$$(1) \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \times 2 + R_2 \rightarrow R_2} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array} \right] \xrightarrow{R_2 \times -1 + R_1 \rightarrow R_1} \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

$$(2) \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \times -2 + R_2 \rightarrow R_2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -3 & -2 & 1 \end{array} \right] \xrightarrow{R_2 \times -\frac{1}{3} \rightarrow R_2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \end{array} \right] \xrightarrow{R_2 \times -2 + R_1 \rightarrow R_1} \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \end{array} \right]$$

$$-\frac{1}{3}A^{-1} = A^{-1} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

가역적 판별식은 $\det(A) \neq 0$ 인지를 판별하는 것으로 계산해보면,

$$A = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 0 & 2 \\ -4 & -9 & 7 \end{bmatrix} \quad \det(A) = -3 \cdot \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} + (-5) \cdot \begin{bmatrix} 1 & 0 \\ -4 & -9 \end{bmatrix}$$

$$\det(A) = 0 \quad \text{이므로} \quad \text{비가역적 행렬이다.}$$

10.

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 2 & 3 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_1 \times -1 + R_2 \rightarrow R_2 \\ R_1 \times -2 + R_3 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 2 & -1 & 1 & 0 \\ 0 & 2 & 5 & -2 & 0 & 1 \end{array} \right]$$

$$R_2 \times -1 + R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 2 & -1 & 1 & 0 \\ 0 & 0 & 3 & -1 & -1 & 1 \end{array} \right]$$

$$R_2 \times \frac{1}{2} \rightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 3 & -1 & -1 & 1 \end{array} \right]$$

$$R_3 \times \frac{1}{3} + R_1 \rightarrow R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 3 & -1 & -1 & 1 \end{array} \right]$$

$$R_3 \times -\frac{1}{3} + R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{6} & \frac{5}{6} & -\frac{1}{3} \\ 0 & 0 & 3 & -1 & -1 & 1 \end{array} \right]$$

$$\frac{1}{3} R_3 \times \frac{1}{3} \rightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{6} & \frac{5}{6} & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array} \right]$$

$$A^{-1} = \left[\begin{array}{ccc} \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{5}{6} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array} \right]$$

12.

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$$

$$M_{11} : -16$$

$$M_{12} : -2$$

$$M_{13} : 4$$

$$M_{21} : 11$$

$$M_{22} : 14$$

$$M_{23} : -5$$

$$M_{31} : -10$$

$$M_{32} : 4$$

$$M_{33} : -8$$

$$10 - \frac{14}{4} = 56$$

$$(-4) \begin{bmatrix} 2 & -4 \\ 1 & 5 \end{bmatrix} = -56$$

$$2 \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} = -2$$

$$A^{-1} = \begin{bmatrix} -16 & 2 & 4 \\ -11 & 14 & 5 \\ -10 & -4 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} -16 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

$$-46 \begin{bmatrix} -16 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

$$\det(A) = -46$$

0.14.1 3.4 part 3

1.1

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

$$\det(A) = -3$$

$$\boxed{[6, -2]}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}^{-1}$$

$$\frac{1}{\det(A)} = -\frac{1}{3}$$

$$\begin{bmatrix} -18 & 6 \end{bmatrix} \times -\frac{1}{3} = \begin{bmatrix} +6 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

$$-\frac{14}{3} + \frac{14}{3} = \begin{bmatrix} \frac{10}{3} & \frac{22}{3} \end{bmatrix}$$

$$\frac{8}{3} + \frac{14}{3}$$

$$\begin{bmatrix} 2 & 2 & -1 \\ 1 & 1 & -1 \\ 3 & 2 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\det(A) = 2 \cdot \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix} - (2) \cdot \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} - 1 \cdot \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\frac{1}{\det(A)} = -1$$

$$-1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 4 & -1 \\ 0 & 3 & 1 \\ -1 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2, 0, 1 \\ -2, 0, -1 \end{bmatrix}$$

$$\boxed{[-2, 0, -1]}$$

$$M_{11} = -1 \quad M_{12} = 0 \quad M_{13} = -1$$

$$M_{21} = -4 \quad M_{22} = -3 \quad M_{23} = -2$$

$$M_{31} = -1 \quad M_{32} = -1 \quad M_{33} = 0$$

$$\begin{bmatrix} -1 & 0 & -1 \\ 4 & -3 & 2 \\ -1 & 1 & 0 \end{bmatrix}^{-1}$$

$$\Rightarrow \begin{bmatrix} -1 & 4 & -1 \\ 0 & 3 & 1 \\ -1 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} 18 \\ 24 \\ 4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\det(A) = 2 \cdot (-16) - 4 \cdot (-26) + 6 \cdot 11 = -32 - 16 + 66 = 18$$

$$x_1 = \frac{18 \cdot 4 - 4 \cdot 24 + 6 \cdot 18}{18} = 4$$

$$\begin{aligned} m_{11} &= -16 & m_{12} &= -12 & m_{13} &= 4 \\ m_{21} &= -14 & m_{22} &= -60 & m_{23} &= 2 \\ m_{31} &= -6 & m_{32} &= -36 & m_{33} &= -6 \end{aligned}$$

$$x_1 = 4$$

$$18 \cdot 4 - 4 \cdot 24 + 6 \cdot 18 = 72 - 96 + 108 = 84$$

$$24 - 10 = 14$$

$$\begin{array}{r} 4 \\ 14 \\ \hline 18 \\ 24 \\ \hline 42 \\ 144 \\ \hline 186 \end{array}$$

$$18 \cdot (-16) - 4 \cdot (-26) + 6 \cdot 11 = -288 + 104 + 66 = -118$$

$$18 \cdot \begin{bmatrix} 5 & 1 \\ 1 & -2 \end{bmatrix} - 4 \cdot \begin{bmatrix} 24 & 6 \\ 4 & -2 \end{bmatrix} + 6 \cdot \begin{bmatrix} 24 & 5 \\ 4 & 6 \end{bmatrix}$$

$$18 \cdot (-16) + (-12) \cdot 4 + 24 \cdot 4 = -288 - 48 + 96 = -240$$

$$24 -$$

$$108 - 144 = -36$$

$$\begin{cases} x_1 = 4 \\ x_2 = -2 \\ x_3 = 3 \end{cases}$$

$$\begin{aligned} 4x_2 + 6x_3 &= 10 & x_3 &= \\ 18 - 6x_3 - 5x_2 &= 8 \end{aligned}$$

$$7x_2 = 2$$

$$x_2 = -2$$

$$\begin{bmatrix} 1 & -3 & -2 \\ 2 & -4 & -3 \\ -3 & 6 & 8 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ -5 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\det(A) = 1 \cdot (-14)$$

$$\begin{cases} x=1 \\ y=3 \\ z=2 \end{cases}$$

$$\det(A) = 7$$

$$\begin{aligned} -2x + 7 - 38 &= -30(-3) + 21 \\ -32 + 14 &= -4 \\ x &= 1 \\ y &= -3 \end{aligned}$$

$$\begin{bmatrix} 6 & -3 & -2 \\ 8 & -4 & -3 \\ -5 & 6 & 8 \end{bmatrix}$$

$$-3y - 2z = 5$$

$$-4y - 3z = 6$$

$$\begin{cases} 2y + 2z = -4 \\ -3y - 2z = 5 \end{cases}$$

$$-3y - 2z = 5$$

$$-4y - 3z = 6$$

$$y = 1$$

$$y + z = -2$$

$$-3y - 2z = 5$$

$$6 \cdot \begin{bmatrix} -4 & -3 \\ 6 & 8 \end{bmatrix} + (-3) \cdot \begin{bmatrix} 8 & -3 \\ -5 & 8 \end{bmatrix} + (-2) \cdot \begin{bmatrix} 6 & -4 \\ -5 & 6 \end{bmatrix}$$

$$-32 + 16$$

$$64 - 15$$

$$48 - 20$$

$$6(-14)$$

$$-(-3 \cdot 49)$$

$$\begin{array}{r} 1 \\ 28 \\ \times 2 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 2 \\ 14 \\ \times 6 \\ \hline 84 \end{array}$$

$$\begin{array}{r} 2 \\ 49 \\ \times 3 \\ \hline 147 \end{array}$$

$$-56$$

$$147$$

$$-56$$

$$-84$$

$$7$$

$$140$$