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ECE 590: Theory and Practice of Algorithms
October 9, 2024

Homework 4

Question 1.

1. Give context-free grammars for the following languages. Describe the language derived by each variable in your grammar. Is your grammer is ambigous? Why or why not?
   (a) $\{x\#y : x, y \in \{0,1\}^*, |x| \neq |y| \vee x = y^R\}$ (Recall that y^R denotes the reverese of y; for example, $0101^R = 1010$.)
   (b) $\{a^m b^n c^p : m + n = p\}$

(a)
Define a CFG for the language:

$$L = \{x\#y \mid x, y \in \{0,1\}^*,\ |x| \neq |y| \text{ or } x = y^R\}$$

where y^R denotes the reverse of y.
Grammar Definition
Variables: S — Start symbol
A — Generates strings where $|x| \neq |y|$
B — Generates strings where $x = y^R$
Productions:

$$S \to A \mid B$$
$$A \to 0A \mid 1A \mid 0A0 \mid 1A1 \mid 0B1 \mid 1B0 \mid \#A \mid \#B$$
$$B \to 0B0 \mid 1B1 \mid \epsilon$$

Language Derived: S: Derives the language L where either $|x| \neq |y|$ or $x = y^R$.
A: Derives strings where $|x| \neq |y|$.
B: Derives strings where $x = y^R$.
Ambiguity: The grammar is **unambiguous** because each string in the language can only be derived uniquely, either by satisfying $|x| \neq |y|$ (via A) or $x = y^R$ (via B).

(b) $L = \{a^m b^n c^p \mid m + n = p\}$
Grammar:

$$S \to XcS \mid \epsilon$$
$$X \to a \mid b$$

Where: X represents either an ‘a‘ or a ‘b‘, and ensures that the appropriate number of ‘c‘’s follows. S ensures that the sum of m (the number of ‘a‘’s) and n (the number of ‘b‘’s) equals p (the number of ‘c‘’s).
Ambiguity: This grammar is also **not ambiguous**, as the structure ensures a unique derivation where the number of ‘a‘’s and ‘b‘’s must match the number of ‘c‘’s.

Question 2. implementation You can find the description on Canvas under Assignments → Programming Assignments → DFA2. Submit the programming assignment to Gradescope.
Just as submitted on canvas.