

Due: Nov 14

45 points

For this homework you are doing a True or False question on **complexity**, a proof on **complexity**, a question on recurrences equation and two questions on **complexity** which are a mix of pencil and paper (showing the  $O^1$  of the runtime of your algorithm), implementation, graphing, and comparing the runtime you came up with to the experimental results;

This means that for each of the last two problems below you should:

- Devise an algorithm for that problem and write pseudocode.
- Implement (in Python) the algorithm for that problem—this part will get submitted separately as the implementation portion. Note that for each Python program you write, if we run it from the command line, *e.g.* :

```
python3 rmdup.py
```

It should measure the runtimes (as described below) and print out the data in a comma separated format, with the size first and the values for each type of data as separate columns, such as (note these numbers are just made up):

```
4096,12345,12456,13987
8192,24690,24900,30012
16384,50073,52045,61875
etc..
```

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<sup>1</sup>Your bounds should be tight, so really  $\Theta$

If we load your file into another with `import` (*i.e.*, to test its behavior and grade correctness) your performance experiments should **NOT** run. Please use

```
if __name__ == "__main__":
```

to make your performance experiments run only if we run your python file directly.

- Analyze the runtime of your algorithm as we did in class: for each line, state how many times that line happens, and how long it takes. Then give an overall runtime. Please be sure to include the pseudocode and code in the writeup so that the TAs can grade this while looking only at the pencil and paper part (your pencil and paper submission should be “self contained”).
- Measure the runtime for various data sizes. Include in your writeup both a graph, and a table of the raw data (with units!). Note that for each problem below we will say a bit more about the range of input sizes you should measure. We will also give you some different kinds of data that you should measure on for each problem. These different types should all be put into one graph (separate lines) and one table (a column for each). Please also include a brief description of any experimental methodology of note—*e.g.*, if you did multiple runs, if so how many and how you combined the results (average, median, etc). If you present error bars, please describe what they mean, etc.
- Analyze your results. If theory (2 above) and practice (3 above) diverge, please note such, and give any thoughts you have on why this divergence may happen. If different categories of inputs

exhibit different behaviors, give your thoughts as to why those difference happen.

1. Indicate which of the following are True (provide a sentence explaining why your choices are correct)[8 points]:

(a)  $2n^3 - 8n^2 + 32n + 9 \in O(n^3)$

(b)  $n^p \in O(e^n)$ , where  $p \in \mathbb{R}$  and  $p \geq 0$

(c)  $e^n \in O(n^p)$ , where  $p \in \mathbb{R}$  and  $p \geq 0$

(d)  $\sqrt{n} \in O(1)$  (side note:  $O(1)$  is called ‘constant time’)

2. Show that for any real constants  $x$  and  $y$ , where  $y > 0$ ,  
 $(n + x)^y = \theta(n^y)$  [5 points].

3. Give asymptotic upper and lower bounds for  $T(n)$  in each of the following recurrences. Assume that  $T(n)$  is constant for sufficiently small  $n$ . Make your bounds as tight as possible, and justify your answers[8 points].

(a)  $T(n) = 16T(\frac{n}{4}) + n^2$ .

(b)  $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$ .

(c)  $T(n) = T(n - 2) + n^2$ .

(d)  $T(n) = 3T(\frac{n}{3}) + n/\lg n$ .

4. The first problem you should implement is in `matrixmult.py`. Here, you are going to write matrix multiplication. Note that there are many fancy matrix multiplication algorithms with improvements to the runtime over the “standard” approach (that you would do by hand). You are welcome to do a fancy algorithm if you want to learn such—however, please note that what

you write should reflect your understanding of that algorithm and not copying code. If you do learn a fancy algorithm, please cite any relevant sources.

Also please note that the standard algorithm you would do by hand/what you would have learned in school is fine, and you will not receive more or fewer points for it than a fancy algorithm.

For this problem, you should consider data sizes with  $N$  ranging from 4 to 512 (as before, doubling at each step: 4, 8, 16...). You will also consider three categories of the “shapes” of the matrix:

**Many rows by few columns**  $(N*4) \times N * N \times (N/4)$

**Square**  $N \times N * N \times N$

**Few rows by many columns**  $(N/4) \times N * N \times (N*4)$

Note that for all three you can consider  $N$  to be the  $x$ -value in your plots.

5. For this problem, you are going to work in `eqsubstr.py`, and write The `matching_length_substrs` function. This function takes a string (`s`) and two characters (`c1` and `c2`). The goal of this function is to find consecutive sequences of `c1` and consecutive sequences of `c2` that are the same length. It should return a *set* of three-tuples where each element of the tuple is (`c1`-index, `c2`-index, length). For example, an element in the answer with value (5, 12, 3) means “there are 3 `c1`s at index 5 and also 3 `c2`s at index 12”. In this problem, we only consider the total sequence length (if you see 4 “a”s in a row, you only call it a sequence of length 4, not also a sequence of length 3), and we only consider lengths  $> 0$ .

Your function should return a set containing *all* pairings of equal length sequences. If there are 2 places that 4 of `c1` appear and

3 places that 4 of  $c_2$  appear, then your answer should have all 6 pairings of those. Likewise, if  $c_1$  has a length 5 sequence but  $c_2$  does not have any length 5 sequences, then no tuples with length 5 will appear in the output.

There may be characters other than  $c_1$  and  $c_2$  in the input string. For this problem, you should consider three categories. However, two of them you need to describe in more detail—namely, give a regular expression which describes those cases.

**Best Case (all  $c_1$  and  $c_2$ ):** What is the best case for the runtime of your algorithm on a string that only has  $c_1$  and  $c_2$  in it (no other characters)? Describe the *entire* category with a **regular expression**. Note that we restrict it to no other characters as an input of  $d^*$  when  $c_1 = a$  and  $c_2 = b$  is likely to be trivial.

**Worst Case:** What is the worst case? Again, please write a **regular expression** that describes all strings that give this worst case behavior.

**Random Input:** Use the `rndstr` function we have provided. This function takes an number `n` and produces a string of length `n` which is approximately  $3/7$  “a”s,  $3/7$  “b”s and  $1/7$  a capital letter. You should use this with  $c_1=a$  and  $c_2=b$ .

For this problem, you should consider inputs sizes of 512 to 16K (16,384) in steps of 2x. If your algorithm has significantly worse runtimes than ours, you can stop at whatever input size takes about 1 minute for the worst case, but as before, start correspondingly earlier.

Hint: you can make a list of functions in Python, so I wrote

`best(n)` and `worst(n)` and then made the list `strgens=[best,worst,rndstr]` which could iterate over like any other list:

```
strgens=[best,worst,rndstr]
for sg in strgens:
    #....
    s=sg(size)
    #....
```

To submit:

**Typed PDF** Q1–5 Big-O analysis, recurrence equations, pseudocode, graphs, tables, analysis of results.

**Implementation** Your modified `eqsubstr.py`, and `matrixmult.py` files.