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ECE 590: Theory and Practice of Algorithms
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Homework 2

Question 1. Describing DFAs

For the following deterministic finite automaton M1:

Determine what language M1 recognizes.

The string can start with both 1 and 0, when it meets the first 0 , it converts qo > q1. Then if it is 1, it will come back to q0 unless the two 0's are continuous. Then it will converts to q2 (accept state) no matter how many 0's it has (like000000 at the end), but when it gives 1 it will turn back to q0.

∴L(M1) = { w: w end with "00" } ;

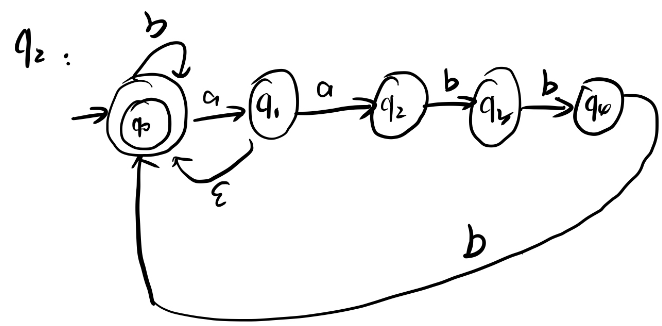
Question 2. Draw an NFA whose alphabet is a, b and which accepts the language of strings such that any two consecutive as are followed by exactly 3 consecutive bs. Note that this does not require that 2 consecutive as ever appear in the input, just that anytime you see aa, you must then see exactly 3 consecutive bs. The following are examples of strings in the language of this NFA: (epsilon)

a

babaabbba

aabbbaabbbabababbbbb

sol:



Question 3. Describing NFAs Give the formal definition (Q, Σ, , q0, F) of the following NFA,and show that it accepts the string "aab" according to the formal definition of accepting.

Q = {q0,q1,q2}

q0 = {q0}

F = {q1}

Σ = {a,b}

{q0,q1}=a ; {q1,q0}=ε; {q0,q2}=a; {q2,q1}=b;

To prove that it accepts "aab",

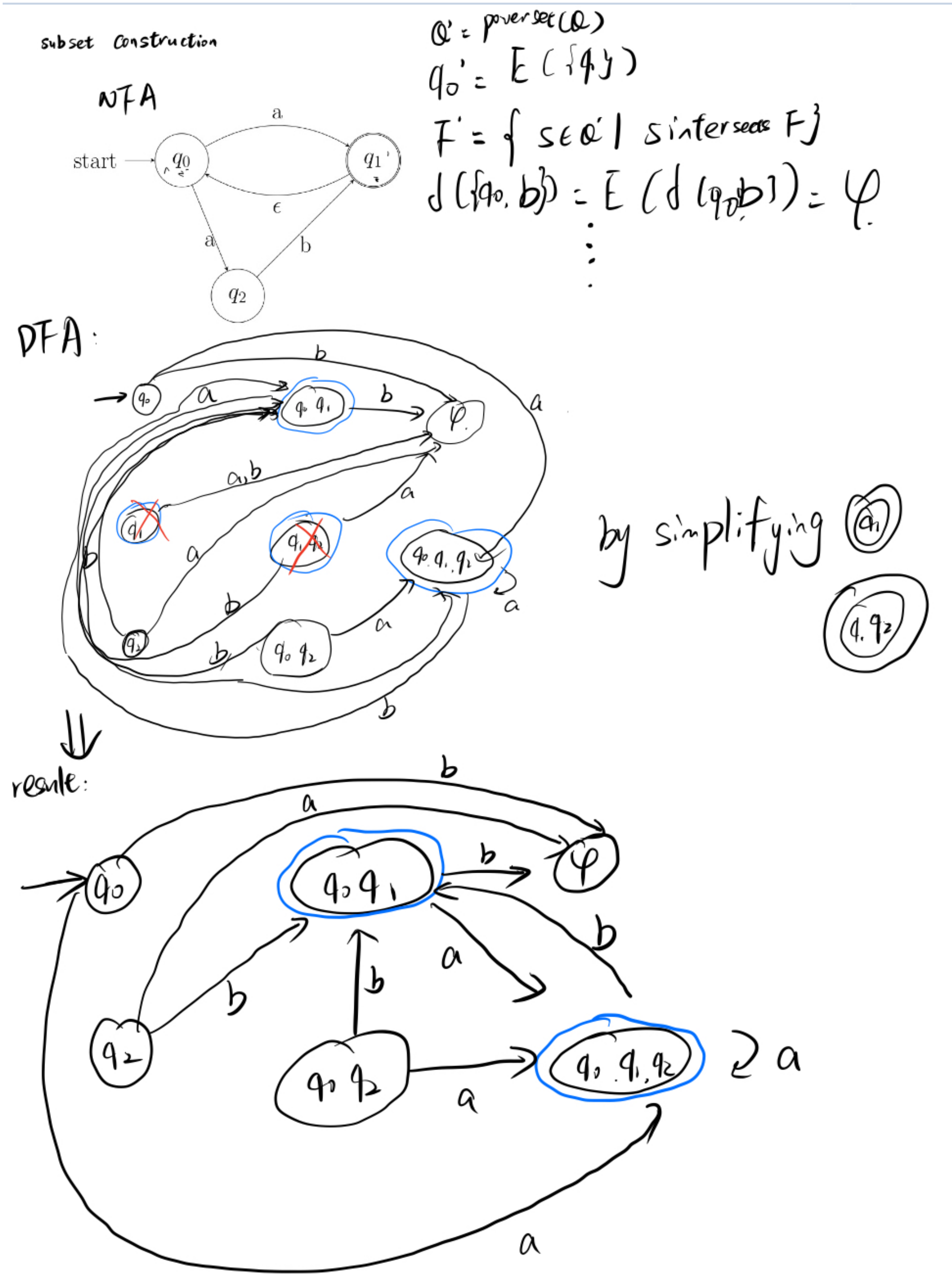
(1) w1 = a , w2 = ε, w3 = a ,w4 = b

(2)r1 = q0, r2 = q1, r3 = q0. r4 = q2, r5 = q1

(3) {q0,q1}=a; {q1,q0}=ε; {q0,q2}=a ; {q2,q1}=b

∴It accepts the string "aab"

Question 4. Subset Construction Turn the above NFA into DFA using the subset construction. It should be clear from your drawing of a DFA how the subset construction was applied.



Question 5. Intersection Prove that the set of regular languages is closed under the intersection operation, i.e., prove that if languages A and B are regular, then A B is also regular. You may assume that A and B share the same alphabet.

To prove that the set of regular languages is closed under intersection, we show that if A and B are regular, then $A \cap B$ is also regular.

Since A and B are regular, there exist DFAs $M_A = (Q_A, \Sigma, \delta_A, q_A^0, F_A)$ and $M_B = (Q_B, \Sigma, \delta_B, q_B^0, F_B)$ that recognize A and B, respectively.

We construct a DFA $M_\cap$ for $A \cap B$ using the product construction:

States: $Q_\cap = Q_A \times Q_B$

Alphabet: Σ (shared by both A and B)
Transition function:

$$\delta_\cap((q_A, q_B), a) = (\delta_A(q_A, a), \delta_B(q_B, a)) \quad \text{for } a \in \Sigma$$

Initial state: $q_\cap^0 = (q_A^0, q_B^0)$
Accepting states: $F_\cap = F_A \times F_B = \{(q_A, q_B) \mid q_A \in F_A \text{ and } q_B \in F_B\}$
The DFA $M_\cap$ accepts a string if and only if both M_A and M_B accept it. Therefore, $A \cap B$ is regular.

$$\boxed{A \cap B \text{ is regular.}}$$