Due:Sept 26

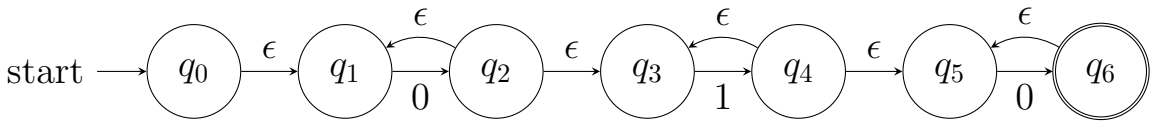
40 points

1. Give regular expressions that describe each of the following languages

   (a) $L_1 = \{$w: w is a numerical constant that may include a fractional part and/or a positive or negative sign$\}$ over the alphabet $\Sigma = \{+, -, ., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
   The following are examples of strings in this language: 110, 3.76, +10, +1., -.05, -10.
   Solution:
   $(+ \cup - \cup \epsilon)(D^+ \cup D^+.D^* \cup D^*.D^+)$

   (b) $L_2 = \{$w: every even position of w is an a$\}$ over the alphabet $\Sigma = \{a, b\}$.
   The following are examples of strings in this language: b, ba, baaa, babab.
   Solution:
   $((b + a)a)^*(\epsilon + (b + a))$

2. Convert the following regular expression $0^*1^*0^+$ to an equivalent NFA using the conversion process seen in class.



3. For each of the following languages, determine if the language is regular or not. If the language *is* regular, demonstrate its regularity by either writing a regular expression which accepts the language, or drawing an NFA which accepts the language. If the language *is not* regular, **prove** that is not regular.

   (a) $L_1 = \{0^i 1^j : i, j \geq 0 \text{ and } 5i < j\}$

   (b) The language of strings over the alphabet $\{0, 1\}$ of the form $0^i 1^j$ where $(i \bmod 2) + 1 = j \bmod 3$. Examples of strings in this language include:1, 011111, 0000011. **Solution**:
   $(i \bmod 2) + 1 = j \bmod 3$

Case 1: i is odd
$i = 2 * k + 1$, where $k \in N$
$(i \bmod 2) + 1 = 2$
$j \bmod 3 = (i \bmod 2) + 1 = 2$
$j = 3 * k + 2$, where $k \in N$
Case 2: i is even
$i = 2 * k$, where $k \in N$
$i \bmod 2 + 1 = 1$
$j \bmod 3 = i \bmod 2 + 1 = 1$, meaning $j = 3 * k + 1$ where $k \in N$
Therefore we can write a regular expression descibing this language meaning this language is Regular.
$0(00)^*(111)^*11|(00)^*(111)^*1$

(c) The language of strings over the alphabet $\{0\}$ of the form 0^i where $\exists k.i = k^2$ (*i.e.*, sequences of 0s where the number of 0s is a perfect square). Examples of strings in this language include:ϵ, 0000, 0000000000000000.