Shuyi Jin sj445
ECE 590: Theory and Practice of Algorithms
September 26, 2024

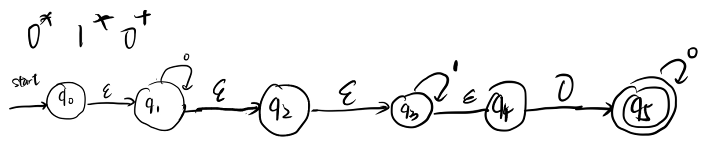
Homework 3

Question 1. 1. Give regular expressions that describe each of the following languages (a) L1 = w: w is a numerical constant that may include a fractional part and/or a positive or negative sign over the alphabet Σ = +, −, ., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The following are examples of strings in this language: 110, 3.76, +10, +1., -.05, -10. (b) L2 = w: every even position of w is an a over the alphabet Σ = a, b. The following are examples of strings in this language: b, ba, baaa, babab.

(a)RE: L1 $= (+ \cup - \cup \varepsilon)\,(0-9)^*\,(\varepsilon \cup (0-9)^*)$

(b)RE: L2 $= \Sigma(a\Sigma)^* \cup (\Sigma a)^*$

Question 2. Convert the following regular expression 0∗1∗ 0+ to an equivalent NFA using the conversion process seen in class.



Question 3. For each of the following languages, determine if the language is regular or not. If the language is regular, demonstrate its regularity by either writing a regular expression which accepts the language, or drawing an NFA which accepts the language. If the language is not regular, prove that is not regular.

(a)

The language L1 is not regular.

Proof (using the Pumping Lemma): The pumping lemma states that for a regular language L, there exists a pumping length p such that any string $w \in L$ with $|w| \geq p$ can be split into three parts $w = xyz$, satisfying:

1. $|xy| \leq p$,
2. $|y| > 0$,
3. $xy^k z \in L$ for all $k \geq 0$.

Consider the string $w = 0^p 1^{5p+1} \in L_1$, where $i = p$ and $j = 5p + 1$. Clearly, $5i = 5p < 5p + 1 = j$, so $w \in L_1$.

According to the pumping lemma, w can be split into three parts $w = xyz$, where $|xy| \leq p$. Since $|xy| \leq p$, the substring y consists only of '0's. Therefore, $y = 0^k$ for some $k > 0$.

Now, pump y with $k = 2$, yielding the new string $w' = xyyz = 0^{p+k} 1^{5p+1}$. This new string has more '0's but the same number of '1's, and it no longer satisfies the condition $5i < j$. Specifically, the new number of '0's is $i' = p + k$, and the number of '1's is still $j = 5p + 1$, so:

$$5i' = 5(p + k) > 5p + 1 = j$$

Thus, $w' \notin L_1$, contradicting the pumping lemma.

So L1 is not regular language.

(b) Language:

$$L_2 = \{0^i 1^j \mid (i \mod 2) + 1 = j \mod 3\}$$

No, the language L2 is not regular.

Proof : Applying the Pumping Lemma to L_2:

Consider the string $w = 0^{2p}1^{3p+1} \in L_2$. For this string: $i = 2p$ and $i \mod 2 = 0$, $j = 3p + 1$ and $j \mod 3 = 1$, so the condition $(i \mod 2) + 1 = j \mod 3$ is satisfied, as $0 + 1 = 1$.

Now, according to the pumping lemma, we can split $w = xyz$ where $|xy| \leq p$. This means that the part y contains only '0's because the first p characters of w are all '0's.

Let $y = 0^k$, where $k > 0$.We can choose k = 2 here. After pumping y (i.e., repeating y), the new string $w' = xyyz = 0^{2p+k}1^{3p+1}$ will have more '0's, but the same number of '1's.

Now, for the new string w': The new number of '0's is $i' = 2p + k$, so $i' \mod 2 \neq 0$, The number of '1's remains $j = 3p + 1$, so $j \mod 3 = 1$.

Thus, the condition $(i' \mod 2) + 1 = j \mod 3$ is no longer satisfied because $(i' \mod 2) + 1 = 1 + 1 = 2$, while $j \mod 3 = 1$.

This means that $w' \notin L_2$, which contradicts the pumping lemma.

Thus L2 is not regular language

(c) Language:

$$L_3 = \{0^i \mid \exists k \in \mathbb{N}, i = k^2\}$$

This language consists of strings of '0's where the number of '0's is a perfect square.

No, the language is not regular.

Proof:

Let $w = 0^{p^2} \in L_3$, where $i = p^2$ for some pumping length p. By the pumping lemma, w can be split into three parts $w = xyz$, where:

$|xy| \leq p$,

$|y| > 0$,

$xy^kz \in L_3$ for all $k \geq 0$.

Since $|xy| \leq p$, the substring y consists only of '0's, and $y = 0^k$ for some $k > 0$.

Now, consider pumping y with $k = 2$. The new string becomes $w' = 0^{p^2+k}$. The length of w' is no longer a perfect square because $p^2 + k$ is not generally a perfect square for non-zero k.

Thus, $w' \notin L_3$, contradicting the pumping lemma.

Since L_3 fails the pumping lemma test, it is not a regular language.

Question 4. implementation You can find the description and the sample input files on Canvas under Assignments → Programming Assignments → DFA1. Submit the programming assignment to Gradescope.

Just as submitted on canvas.