Due:Oct 3

20 points

1. Give context-free grammars for the following languages. Describe the language derived by each variable in your grammar. Is your grammer is ambigous? Why or why not?

   (a) $\{x\#y : x, y \in \{0,1\}^*, |x| \neq |y| \vee x = y^R\}$ (Recall that y^R denotes the reverese of y; for example, $0101^R = 1010$.)

   Solution:

   $L = L_1 \cup L_2$
   $L_1 = \{x\#y : x, y \in \{0,1\}^*, |x| \neq |y|\}$
   $L_2 = \{x\#y : x, y \in \{0,1\}^*, x = y^R\}$

   $G_1 = (V_1, \Sigma, R_1, S)$
   $V_1 = \{S_1, L, R, A, B\}$
   $\Sigma = \{0,1\}$
   $S = S_1$
   R_1 :

$S_1 \to BL\|RB$	S_1 derives $x\#y$ where $\|x\| \geq \|y\| \vee \|x\| \leq \|y\|$
$L \to BL\|A$	L derives $x\#y$ where $\|x\| \geq \|y\|$
$R \to RB\|A$	R derives $x\#y$ where $\|x\| \leq \|y\|$
$A \to BAB\|\#$	A derives $x\#y$ where $\|x\| = \|y\|$
$B \to 0\|1$	B derives 0 or 1

   $G_2 = (V_2, \Sigma, R_2, S)$
   $V_2 = \{S_2\}$
   $\Sigma = \{0,1\}$
   $S = S_2$
   R_2 :

$S_2 \to 0S_20\|1S_21\|\#$	S_2 derives $x\#y$ where $x = y^R$

   $G = (V, \Sigma, R, S)$
   $V = \{S_1, S_2, L, R, A, B\}$
   $\Sigma = \{0,1\}$

$S = S$

R :

$S \to S_1 \mid S_2$	
$S_1 \to BL \mid RB$	S_1 derives $x\#y$ where $\lvert x\rvert \geq \lvert y\rvert \vee \lvert x\rvert \leq \lvert y\rvert$
$L \to BL \mid A$	L derives $x\#y$ where $\lvert x\rvert \geq \lvert y\rvert$
$R \to RB \mid A$	R derives $x\#y$ where $\lvert x\rvert \leq \lvert y\rvert$
$A \to BAB \mid \#$	A derives $x\#y$ where $\lvert x\rvert = \lvert y\rvert$
$B \to 0 \mid 1$	B derives 0 or 1
$S_2 \to 0S_20 \mid 1S_21 \mid \#$	S_2 derives $x\#y$ where $x = y^R$

Then,
$L(G_1) = L_1$ and $L(G_2) = L_2$ therefore $L(G) = L(G_1) \cup L(G_2) = L$

(b) $\{a^m b^n c^p : m + n = p\}$

Solution:

$G = (V, \Sigma, R, S)$
$V = \{S, T\}$
$\Sigma = \{a, b, c\}$
$S = S$
R :

$S \to aSc \mid T$	S derives $a^m b^n c^p : m + n = p$
$T \to bTc \mid \epsilon$	T derives $b^n c^n : n \geq 0$