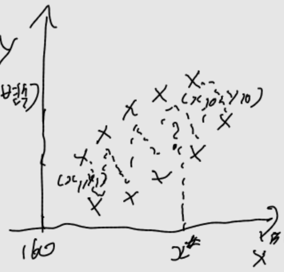
ML / DL { 분류 / 회귀 (종속변수)



$n = 10$

(x_1, y_1)

⋮

(x_{10}, y_{10})

x → 아파트 { 평수 ⋮ (독립변수) 가격

점 2개 — 직선 가능 (표현)

점 3개 이상 직선 불가 (복잡)

→ 직선이 제일 표현함

→ 직선으로 (선형) 돌아가고 싶음 (회귀) ⇒ 선형 회귀 (Linear Regression)

– 선형회귀 종류

- 독립변수 1개 → 단순 선형회귀
- 독립변수 1개 초과 → 다중 선형회귀 ⇒ $x \in R \to x \in R^m$

– 직선 표현 : 기울기, y절편

– $\hat{y} = \alpha + \beta x$, $\alpha, \beta \in R$

(x_1, y_1) → $\hat{y}_1 = \alpha + \beta x_1$

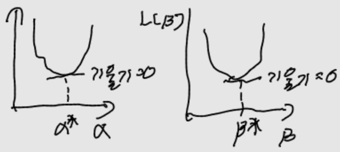
⋮

(x_{10}, y_{10}) → $\hat{y}_{10} = \alpha + \beta x_{10}$

⇓ training data

$$L(\text{Loss}) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum (y - \hat{y})^2 = \sum (y - \alpha - \beta x)^2$$

→ loss function, cost 〃, target 〃



$$\frac{dL}{d\alpha}, \frac{dL}{d\beta} \to \alpha^*, \beta^*$$

i) $L = \sum (y - \alpha - \beta x)^2$

$$\frac{dL}{d\alpha} \Rightarrow \nabla L(\alpha) = \nabla_\alpha L$$

$$= \cancel{-2} \sum (y - \alpha - \beta x) = 0$$

$$\sum y - \sum \alpha - \sum \beta x = 0$$

$$n\alpha = \sum y - \sum \beta x \quad \alpha = \frac{\sum y - \beta \sum x}{n} = \bar{y} - \beta \bar{x}$$

$\bar{y}, \bar{x}$ = x, y의 평균

$$\frac{dL}{d\beta} = \cancel{-2} \sum (y - \alpha - \beta x) x = 0 \qquad \alpha = \bar{y} - \beta \bar{x}$$

$$= \sum (y - (\bar{y} - \beta \bar{x}) - \beta x) x = 0$$

$$= \sum x ((y - \bar{y}) - \beta (x - \bar{x})) = 0$$

$$= \beta = \frac{\sum x (y - \bar{y})}{\sum_{i=1}^{n} x_i (x_i - \bar{x}_i)} = 0$$

$$\hat{y} = \alpha + \beta x \begin{cases} \alpha = \bar{y} - \beta \bar{x} \\ \beta = \dfrac{\sum x (y - \bar{y})}{\sum x (x - \bar{x})} \end{cases}$$

자료 ⇒ $\beta = \dfrac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

i) 자료(분자) → $\sum (x - \bar{x})(y - \bar{y})$

$$= \sum (x(y - \bar{y}) - \bar{x}(y - \bar{y}))$$

$$= \sum x (y - \bar{y}) - \sum \bar{x} (y - \bar{y})$$

상수 0

→ $y = 4, 6, 8$ $\bar{y} = 6$ $\sum (y - \bar{y}) = 0$

$$= \sum x (y - \bar{y})$$

ii) 자료(분모) → $\sum (x - \bar{x})^2$

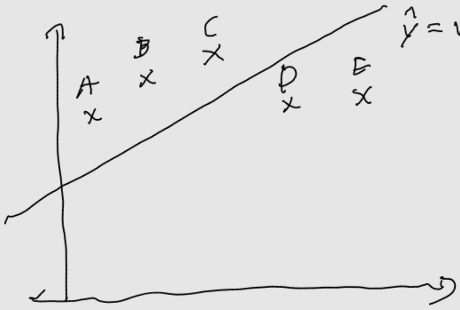
$$= \sum (x - \bar{x})(x - \bar{x})$$

$$= \sum (x(x - \bar{x}) - \bar{x}(x - \bar{x}))$$

0

$$= \sum x (x - \bar{x})$$

- $y = \alpha x + \beta x_2 + c x_3 = D$ → 일일이 편미분하기 귀찮음



$\hat{y} = wx + b$

$A(x_1, y_1) \Rightarrow \hat{y} = wx_1 + b$

$\vdots$

$E(x_5, y_5) \Rightarrow \hat{y} = wx_5 + b$

$$\hat{Y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_5 \end{bmatrix} \in \mathbb{R}^5, \quad X = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ x_4 & 1 \\ x_5 & 1 \end{bmatrix} \in \mathbb{R}^{5 \times 2} \quad \theta = \begin{bmatrix} w \\ b \end{bmatrix} \in \mathbb{R}^2$$

$\hat{Y} = X\theta \in \mathbb{R}^5$

$(5\times1) \quad (5\times2)(2\times1)$

$X\theta = Y$ 가 되면 best

↳ $\theta = X^{-1} Y$? (X)

$X^T X \theta = X^T Y$

$(2\times5)(5\times2)(2\times1) \quad (2\times5)(5\times1)$

$(2\times2) \qquad (2\times1)$

(2×1)

$\therefore \theta = (X^T X)^{-1} X^T Y \in \mathbb{R}^2$