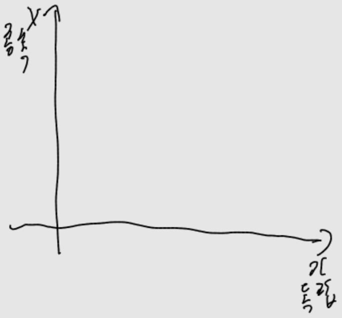
- 다중 선형회귀



$x \in R$ (single) : 단순선형회귀 → $y = wx + b$

$x \in R^n$ (multiple) : 다중선형회귀 → $y = w_1 x \cdots$

$x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{im} \end{bmatrix}$ (w_1, …, w_m) → $y_i = w_1 x_{i1} + w_2 x_{i2} + \cdots + w_m x_{im} + b$ ⇒ $x_i \in R^m$

$y \quad \hat{y}$ ↓ $\in R^n$

$\begin{pmatrix} 1 & x_1 \in R^m & y_1 & \hat{y} \\ 2 & & & \\ \vdots & & & \\ n & x_n \in R^m & & \end{pmatrix}$

⟨single L-R⟩ - $y = \alpha + \beta x$

i) 편미분 : $\begin{cases} \alpha = \bar{y} - \beta \bar{x} \\ \beta = \dfrac{\sum x(y-\bar{y})}{\sum x(x-\bar{x})} \end{cases}$

ii) 대수적 $\theta = \begin{bmatrix} w \\ b \end{bmatrix}$ $\hat{Y} = X\theta \simeq Y$ $\theta = (X^T X)^{-1} X^T Y$

ㄴ 예제는 강의자료 참고

완벽하게 맞아떨어지지 않음

Y = 정답 = Lable $\hat{Y}$ = 측정값

i) $\hat{Y} = X\theta$ ii) $E = (Y - \hat{Y})^2$

$(A+B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

$(ABC)^T = C^T B^T A^T$

$A \in R^1 \Rightarrow A = A^T$

- 벡터 $A^2 = A^T A$

ex) $A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ $L_1 = |\sum x_i| = \|X\|_1$, $L_2 = \sqrt{\sum x_i^2} = \|X\|_2$ $\|A\|_2^2 = a_1^2 + a_2^2 + a_3^2 = (a_1\ a_2\ a_3)\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$

- $E = (Y - \hat{Y})^2 = (Y - \hat{Y})^T (Y - \hat{Y}) = (Y - X\theta)^T (Y - X\theta)$

$= (Y^T - (X\theta)^T)(Y - X\theta)$

$= (Y^T - \theta^T X^T)(Y - X\theta)$

$= Y^T Y - Y^T X\theta - \theta^T X^T Y - \theta^T X^T X\theta$

$= Y^T Y - Y^T X\theta - (Y^T X\theta)^T - \theta^T X^T X\theta$

$Y \in R$
X
θ

$x = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix} \in R^{3\times2}$ $\theta = \begin{bmatrix} w \\ b \end{bmatrix} \in R^2$ $\hat{Y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} \in R^3$

$Y^T X\theta$ → $(1\times3)(3\times2)(2\times1)$
$(1\times2)(2\times1) = 1$: 스칼라

i) $E = Y^TY - Y^TX\theta - (Y^TX\theta)^T - \theta^TX^TX\theta = Y^TY - 2Y^TX\theta - \theta^TX^TX\theta$

ii) $\frac{dE}{d\theta} = -2X^TY + 2X^TX\theta = 0 \quad \theta = (X^TX)^{-1}X^TY$ (가정X)

① $Y = AX \Rightarrow \frac{dY}{dX} = A \quad A = R^{m\times n}$

② $K = Y^TAX \Rightarrow \frac{dK}{dX} = \frac{d(Y^TAX)}{dx} = Y^TA$

$\hookrightarrow \frac{d(Y^TAX)}{dY} = (AX)^T = X^TA^T$

를 Y^T라고 가정

③ X^TAX — A 는 정방행렬이어야 함

$\Rightarrow \underbrace{\frac{d(X^TAX)}{dX}}_{X^TA^T} + \underbrace{\frac{d(X^TAX)}{dX}}_{X^TA} = X^T(A+A^T)$

iii) $\frac{dE}{d\theta} = \frac{d}{d\theta}\left[\cancel{Y^TY} - 2Y^TX\theta - \theta^TX^TX\theta\right]$

① $\frac{d}{d\theta}\left[Y^TX\theta\right] = Y^TX$

② $\frac{d}{d\theta}\left[\theta^TX^TX\theta\right] \to \frac{d}{d\theta}\left[\theta^T\underbrace{(X^TX)}_{A}\theta\right] = \theta^T\left(X^TX + (X^TX)^T\right) = \theta^T(X^TX + \underbrace{XX^T}_{X^TX})$

$= 2\theta^TX^TX$

$\therefore \frac{dE}{d\theta} = \cancel{2}Y^TX - \cancel{2}\theta^TX^TX = 0$

$(Y^TX)^T \to X^TY = (\theta^TX^TX)^T \to X^TX\theta \to \theta = (X^TX)^{-1}X^TY$

• $\hat{y} = w_1x_{i1} + w_2x_{i2} \cdots w_mx_{im} + b$

$x_i = \begin{bmatrix} x_{i1} \\ \vdots \\ x_{im} \end{bmatrix} \in R^m \quad \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix} \in R^n \quad \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} \in R^n \quad \theta = \begin{bmatrix} w_1 \\ \vdots \\ w_m \\ b \end{bmatrix} \in R^{m+1} \quad X = \overbrace{\begin{bmatrix} X_{11} & \cdots & X_{1m} & 1 \\ \vdots & & \vdots & \vdots \\ X_{n1} & \cdots & X_{nm} & 1 \end{bmatrix}}^{m+1} n \in R^{n\times(m+1)}$

$\hat{Y} = X \cdot \theta$

$\in R^{n\times 1} \quad \in R^{n\times(m+1)} \quad \in R^{(m+1)\times 1}$

$(n\times 1) \quad (n\times(m+1)) \quad ((m+1)\times n)$