① Vector, modify notation

② unit vector

③ Norm & Regularization

$\therefore$ Loss Function $= \sum_i (y-\hat{y})^2 + \sum_i \beta_i^2$

④ Inverse Matrix, Transaction

square (n×n)

$(AB)^{-1} = B^{-1}A^{-1}$ $(AB)^T = B^TA^T$

$(A^{-1})^{-1} = A$ $(A^T)^T = A$

$(A+B)^{-1} \neq A^{-1}+B^{-1}$ $(A+B)^T = A^T+B^T$

⑤ Linear Combination (벡터의 가중합)

$$A = \left[\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{a_1} \underbrace{\begin{pmatrix}0\\1\end{pmatrix}}_{a_2}\right] = [a_1 \ a_2]$$

$\mathrm{Span}(A) = \mathrm{span}(w_1a_1 + w_2a_2 \mid w_1, w_2 \in R)$

$A = \begin{bmatrix} 1 & 0 & 1 & 2 & \cdots \\ 0 & 1 & 1 & 3 & \cdots \end{bmatrix} = [a_1, a_2, \cdots a_{10}]$

$\mathrm{span}(A) \in R^2$ Generating set of Vector Space

↓

minimal 〃 ⇒ Basis

〃

Linear independant G.S.O.V.S

⑥ symmetric matrix → $A = A^T \in R^{n\times n\,(\text{square})}$

$A = X\cdot X^T$ $A^T = (X\cdot X^T)^T = X\cdot X^T$

⑦ Orthogonal Matrix

$A = [a_1, a_2, a_3]$ → orthonormal

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ → $A^{-1} = A^T$

$A = \left[\begin{pmatrix}3\\6\end{pmatrix}\begin{pmatrix}6\\5\end{pmatrix}\right] = [a_1, a_2]$ → not orthonormal

⑧ Rank : linear independant 한 vector의 개수

$A = R^{m\times n} \rightarrow \mathrm{rank}(A) \leq \min(m,n)$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow \mathrm{rank}(B) = 3$ B^{-1} 존재

└ linear independant

$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \mathrm{rank}(A) = 2$

linear dependent

→ linear independent

⑨ Range Space : $A \in R^{m\times n}$ $Ax = b$, $x \in R^{n\times 1}$, $b = R^{m\times 1}$

$\mathcal{R}(A) = \{ b \in R^m \mid Ax = b \text{ for } x \in R^n \}$

$Ax = b \rightarrow \underset{\in R^{m\times m}}{A^T\cdot A}\,x = A^T\cdot b$ $\therefore x = (A^T\cdot A)^{-1}\cdot A^T\cdot b$

⑩ Null space → $N(A) = \{x \in R^n \mid Ax = 0\}$

ex) $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad R(A) = \{ b = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \}$ $\quad N(A) = \{ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mid x_1, x_2 \in R \}$

ex) $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad R(B) = \{ b = \begin{bmatrix} x_1 \\ 0 \end{bmatrix} \mid x_1 \in R \}$ $\quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$

$N(B) = \{ x = \begin{bmatrix} 0 \\ x_2 \end{bmatrix} \mid x_2 \in R \}$ $\quad Bx = 0 \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow x_1 = 0$

⑪ Norm 함수

Input	operation		output
↓	↓		↓
vector	L1 or L2	→	scalar

· 내적 (Inner Product)

· $\langle X, Y \rangle = \cancel{\|X\| \|Y\| \cos\theta}$ $\quad X, Y \in R^n \quad X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$

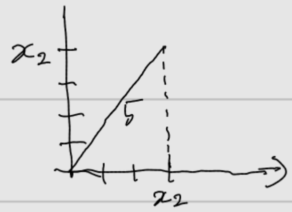
$= \sum_{i=1}^{n} x_i y_i = X^T Y$

· $\langle X, X \rangle = \sum_{i=1}^{n} x_i^2 = \|X\|_2^2 = X^T X$

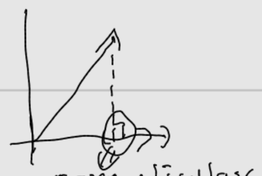
· distance

$L1 = \Sigma |x_i| \quad L2 = \sqrt{\Sigma x_i^2} \qquad d_1 = \sum_i (x_i - y_i) \quad d_2 = \sqrt{\sum_i (x_i - y_i)^2}$

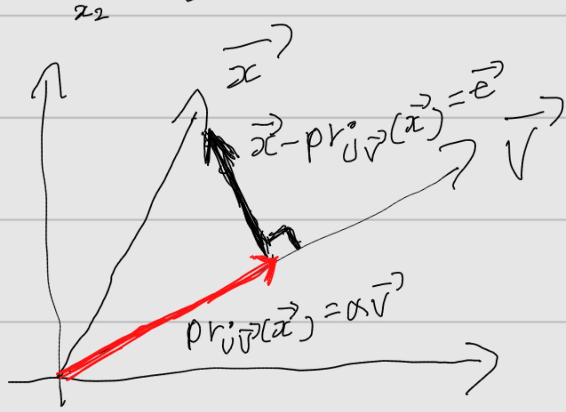
$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$



⑫ projection



perpendicular



i) $Prj_{\vec{v}}(\vec{x}) = \alpha \vec{v}$

ii) $\vec{e} = \vec{x} - Prj_{\vec{v}}(\vec{x})$

iii) $\langle v, e \rangle = v^T e = 0$

↳ $v^T(\vec{x} - Prj_{\vec{v}}(\vec{x})) = 0$

↳ $v^T \vec{x} - v^T Prj_v(x) = v^T x - v^T \alpha v$

$= v^T x - \alpha v^T v = 0$

$\therefore \quad \alpha = \frac{v^T x}{v^T v} \qquad Prj_v(x) = \alpha v = \frac{v^T x}{v^T v} v = \frac{v v^T}{v^T v} x$