Assignment: Demand Calculation

August 28, 2017

1 Assignment

The goal of the assignment is to calculate the market shares, s_j, (defined in Equation 1) from the BLP model using Julia.

1. Set-up
   (a) Go to https://github.com/jinsoo-han/CompEcon_class/tree/master/julia/blp/blp_data and download blp_example_data.csv and blp_example_grid.csv. The data consists of 50 products in 20 markets with 100 customers (i.e., simulations).
   (b) Let $u_{ij} = \beta_0 + \sum_{k=1}^{3} \beta_k x_k + \xi_j + \sum_{k=2}^{3} \theta_k x_{jk} \eta_{ik} + \epsilon_{ij} = \delta_j + \mu_{ij}$ where $\beta = (-5.0, 1.0, 1.0, 1.0)$ and $\theta = (1.4, 1.4)$.
2. Write a new Julia file and load in the data. Hint: use the package called DataFrames.
3. One way of using computers to evaluate the integral (defined in Equation 1) is to use simulations. Write a function in Julia that calculates $s_{jt}(\delta_{jt}, \theta, t|X_t, \eta) \approx \sum_i^I (\frac{exp(\delta_{jt}+\mu_{ij})}{1+\sum_{j'}^J exp(\delta_{jt}+\mu_{ij})}) w_i$ for each market t. Be sure to write the function such that shares are evaluated for each market (i.e., s_{jt} not s_j).
   (a) I is the total number of simulations and w_i is the weight associated with each simulation. These are pre-specified in blp_example_grid.csv
   (b) Step 1: construct a matrix of size $J\,x\,I$ with each element containing the value,$\sum_k \theta_k x_{jkt} \eta_{ik}$.
   (c) Step 2: construct a matrix of size $J\,x\,I$ with each element containing the value, $\frac{exp(\delta_{jt}+\mu_{ij})}{1+\sum_{j'}^J exp(\delta_{jt}+\mu_{ij})}$.
   (d) Step 3: construct a vector of size J with each element containing the value, $\sum_i^I (\frac{exp(\delta_{jt}+\mu_{ij})}{1+\sum_{j'}^J exp(\delta_{jt}+\mu_{ij})}) w_i$.
4. Write a separate Julia code that loops over each market, evaluate the function defined in 2, and store the market shares from all the markets in one large vector. Design the code as the following:
   (a) Step 1: define a J*T by 1 empty vector of S_j (i.e., $S_j = (s_{j,1}, s_{j,2}, ..., s_{j,T})$.
   (b) Step 2: write a loop over the markets. For each loop, evaluate s_{jt} defined in the previous part and update the appropriate part of S_j.
5. Optional: Parallelize the loop in 4 and measure the difference in time spent for evaluation. Hint: use the function called pmap.

2 Model

This section is not necessary to complete the assignment above. For the details of the model, look Berry, Levinsohn and Pakes (ECMA, 1995). If you want further reading, Kenneth Train's Discrete Choice Methods with Simulation is another useful reference (url: https://eml.berkeley.edu/books/choice2.html).

Consider an individual i making a discrete choice among $J+1$ alternatives/products. The utility she obtains if product j is selected is:

$$
\begin{aligned}
u_{ij} &= X_j^{'}\boldsymbol{\beta_i} + \xi_j + \epsilon_{ij} \\
&= \sum_k \beta_k x_{jk} + \sum_k \theta_k x_{jk}\eta_{ik} + \xi_j + \epsilon_{ij}
\end{aligned}
$$

where (β, θ) are model parameters to be estimated, $\eta_i \sim F_\eta$ denote the individual-specific marginal utilities for product characteristics, ξ_j are unobserved product characteristics, and ϵ_{ij} are idiosyncratic errors (i.e., i.i.d. Type-1- Extreme-Value (T1EV) random variates). With the above set-up, BLP shows that the market share of a product j is:

$$
s_j = \int (\frac{exp(\delta_j + \sum_k \theta_k x_{jk}\eta_{ik})}{1 + \sum_{j'}^{J} exp(\delta_{j' + \sum_k \theta_k x_{jk}\eta_{ik}})}) dF_\nu(\eta_i). \tag{1}
$$