Bayesian inference

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1 Two approaches

In frequentist inference, a parameter θ is assumed as a fixed unknown quantity, however, in Bayesian inference, we assume a parameter θ to be a random variable. For example, the coefficients in linear regression model, or unknown population parameters, are random variables.

2 Bayes' rule

Let

- θ represents proportation of people who are mutants in Atlanta.
- Y is the number of mutants from a random sample in Atlanta.

Before collecting and observing the data Y, we have some beliefs (or preknowledge) about θ, $p(\theta)$; and some beliefs about Y for given each value of θ, $p(y|\theta)$. Then we construct a joint density from

- $p(\theta)$;
- $p(y|\theta)$.

After collecting and observing the data Y, we update our preknowledge (θ) and have $p(\theta|y)$, which is conditional probability. According to the Bayes' rule,

$$p(\theta|y) = \frac{p(\theta, y)}{p(y)} = \frac{p(\theta)p(y|\theta)}{p(y)} = \frac{p(\theta)p(y|\theta)}{\int_\theta p(\theta)p(y|\theta)d\theta}.$$

As the $p(y)$ does not rely on the random varialbe $\theta|y$, we can omit it and use proportion form, yielding *unnormalized posterior density* on the right side of following:

$$p(\theta|y) \propto p(\theta)p(y|\theta),$$

where $p(y|\theta)$ is taken here as a function of θ, not of y.