linear subspace

Subspace of R^n $V \leftarrow$ subset of R^n $\left\{\begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix} x_i \in R \quad 1 \le i \le n \right\}$

V subspace of R^n

V contain 0 vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\underbrace{\vec{x} \text{ in } V \quad \vec{x} \text{ 스칼라 곱은 in } V}_{c\vec{x}}$ → subspace

$\vec{a}$ in V, $\vec{b}$ in V

$\vec{a}$ and $\vec{b}$ in V

$\underline{\vec{a} + \vec{b} \text{ in } V}$

$V = \{0\} = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$ V is a subset of R^3?

$c\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ 이 집합은 곱셈에 닫혀있다.

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ 덧셈에 닫혀있다.

$S = \left\{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in R^2 \mid x_1 \ge 0 \right\}$

1, 4사분면

is S subspace of R^2?

contains $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c \\ b+d \end{bmatrix}$ closed under Addition.

$-1\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -a \\ -b \end{bmatrix}$ not closed under scalar multiplication.

곱셈이 닫혀있지 않음

$U = \text{span}(v_1, v_2, v_3)$ ·· valued subspace of $\mathbb{R}^2$

$0\vec{v_1} + 0\vec{v_2} + 0\vec{v_3} = 0$

$\vec{x} = c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3}$

$a\vec{x} = ac_1\vec{v_1} + ac_2\vec{v_2} + \overrightarrow{ac_3 v_3}$ closed multiplication

$= c_4\vec{v_1} + c_5\vec{v_2} + c_6\vec{v_3}$ closed Addition.

$\vec{y} = d_1\vec{v_1} + d_2\vec{v_2} + d_3\vec{v_3}$ $\vec{x} + \vec{y} = (c_1+d_1)\vec{v_1} + (c_2+d_2)\vec{v_2} + (c_3+d_3)\vec{v_3}$

V_1, V_2, V_3의 선형결합

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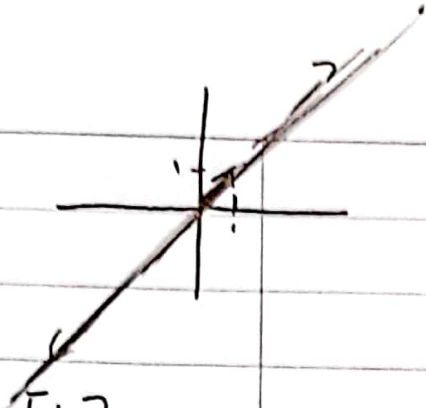
$U = \text{span}(\begin{bmatrix}1\\1\end{bmatrix})$



$\vec{a}$ $\vec{b}$

$c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}1\\1\end{bmatrix} = (c_1+c_2)\begin{bmatrix}1\\1\end{bmatrix}$

$= c_3\begin{bmatrix}1\\1\end{bmatrix}$

부분공간의 기저. $\underset{\text{subspace}}{V} = \text{Span}(\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n)$

$\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n)$ 선형 독립

$c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n = 0$

$c_1 + c_2 + \cdots + c_n = 0$

$S = \{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\}$

S is $\underset{\text{기저}}{\text{basis}}$ for V

$T = \{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n, \vec{v}_k\}$ v_1+v_2

선형독립X 선형종속O

span (T) = V

여전히 부분공간이다.

T is not basis for V

Basis is "minimum set of vectors that spans the subspace"
기저

$S = \{\begin{bmatrix}2\\3\end{bmatrix}, \begin{bmatrix}7\\0\end{bmatrix}\}$ $\text{span}(S) = R^2$

선형독립인가?

$c_1\begin{bmatrix}2\\3\end{bmatrix} + c_2\begin{bmatrix}7\\0\end{bmatrix} = \begin{bmatrix}x_1\\x_2\end{bmatrix}$ $x_1, x_2 \in R$

$c_1\begin{bmatrix}2\\3\end{bmatrix} + c_2\begin{bmatrix}7\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$2c_1 + 7c_2 = x_1$ $c_2 = (x_1 - \frac{2}{3}x^2)/7$

$3c_1 + 0 = x_2$ $c_1 = \frac{x_2}{3}$

C1과 C2가 0일때이므로 선형독립이다.

S is basis of R^2

→ 표준 기저 집합 이라고 함

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$T = \{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \}$ $\quad span(T) = \mathbb{R}^2$

T is also basis for R^2

$x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ 선형독립.

$\{V_1, V_2, \cdots V_n\}$ = Basis of for U ← subspace

$\vec{a} \in U$

$$\begin{aligned} \vec{a} &= c_1\vec{v_1} + c_2\vec{v_2} + \cdots + c_n\vec{v_n} \quad \text{선형결합.} \\ - \quad \vec{a} &= d_1\vec{v_1} + d_2\vec{v_2} + \cdots + d_n\vec{v_n} \\ \hline \vec{0} &= \underbrace{(c_1-d_1)}_{0}\vec{v_1} + \underbrace{(c_2-d_2)}_{0}\vec{v_2} + \cdots + \underbrace{(c_n-d_n)}_{0}\vec{v_n} \end{aligned}$$

여기서 위에서는 $c_1 = d_1 \quad c_2 = d_2 \quad c_n = d_n$ 선형독립.

기저 : 어떤 벡터 공간의 기저는 그 벡터 공간을 선형생성하는 선형독립된 벡터들이다.

벡터 공간의 임의의 벡터에게 선형결합으로서 유일한 표현을 부여하는 벡터

일반적으로 기저는 다음 두 조건을 만족하는 V의 부분집합 $B \subseteq V$이다.

선형독립 : 임의의 $c_1, c_2, \cdots, c_n \in K$, $b_1, b_2, \cdots, b_n \in B$에 대하여

만약 $c_1b_1 + c_2b_2 + \cdots + c_nb_n = 0$이면

$c_1 = \cdots = c_n = 0$이다.

선형생성. 임의의 벡터 $v \in V$는 어떤 $c_1, \cdots, c_n \in K$ 및

$b_1, \ldots b_n \in B$를 써서 $v = c_1b_1 + \cdots + c_nb_n$와 같이 표현된다.