Surface Wave Measurement Based on Cross-correlation

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# Introduction

Many methods have been developed over the years to measure surface wave phase velocity, which can be divided into two catalogs: measure the phase of the waveform directly ([[*Kulesh et al.*, 2005](#_ENREF_8); [*A. Levshin et al.*, 1992](#_ENREF_10); [*Yang and Forsyth*, 2006](#_ENREF_19)]) and comparing synthetic waveform with the real data ([[*Ekström et al.*, 1997](#_ENREF_3); [*Gee and Jordan*, 1992](#_ENREF_7); [*Li and Romanowicz*, 1995](#_ENREF_11); [*Tape et al.*, 2010](#_ENREF_17)]).

The methods that measure the phase directly are fast and automatic, but additional efforts need to be made to control the data quality (e.g. [[*AL Levshin and Ritzwoller*, 2001](#_ENREF_9)]) and solve the cycle skipping problem (e.g. [[*Ekström et al.*, 1997](#_ENREF_3); [*Lin and Ritzwoller*, 2011](#_ENREF_12)]).

Waveform comparison or inversion algorithms are good for dealing with higher modes interference [[*Nettles and Dziewoñski*, 2011](#_ENREF_14)], provide sensitivity for deeper structure by involving long period body modes [[*Li and Romanowicz*, 1995](#_ENREF_11)], and one more cons, but they can be computational expensive (e.g. [[*Tape et al.*, 2010](#_ENREF_17)]), not very automatic, or hard to deal with scatter problems [[*Li and Romanowicz*, 1995](#_ENREF_11)].

Multipathing and scattering of surface wave caused by the inhomogeneous structure in and out of the interested region can significantly effect phase measurement (e.g. [[*Capon*, 1970](#_ENREF_2); [*Maeda et al.*, 2011](#_ENREF_13); [*Snieder and Nolet*, 1987](#_ENREF_16)]) by introducing systematic bias in phase velocity [[*Wielandt*, 1993](#_ENREF_18)]. Many procedures have been tested to minimize this effect, which lay into two groups. Procedures in the first group use phase information only and try to trace the curved ray path from the source [[*Tape et al.*, 2010](#_ENREF_17)] if regional or global structure is concerned, or correct the incident angle of coming waves in the case of small dense array [[*Foster et al.*, 2010](#_ENREF_5); [*Pedersen et al.*, 2003](#_ENREF_15)]. However, even with the correction, good azimuthal coverage of events is needed to decrease this systemic alias [[*Bodin and Maupin*, 2008](#_ENREF_1)].

Methods in the other group try to use take amplitude measurement into consideration. Friederich and Wielandt [[1995](#_ENREF_6)] developed a method simultaneously inverse the incoming wavefield and phase velocity inside the array, and the incoming wavefield is presented by 44 parameters at each frequency. Forsyth and Li [[2005](#_ENREF_4)] simplified the incoming wavefield as the interference between two plane waves, which reduced the number of parameters to 4. The two plane wave assumption only is only valid in small area, for regional study like western US, the interested area has to be divided into several pieces and the smoothing between the pieces has to be taken good care of [[*Yang et al.*, 2011](#_ENREF_20)].

Lin and Ritzwoller [[2011](#_ENREF_12)] proposed a new method based on Helmholtz equation, which use amplitude term to build up relation between dynamic phase velocity (phase only measurement) and structure phase velocity, and succeeded applying it on USarray data to explore the upper mantle structure of western US.

Some discussion about multi-scattering problem.

# Method

Although the single station Rayleigh waveform can be quite complicated, it has great consistency amount the stations within 1-2 wavelength, as shown in Figure 1. As a result, after applied a large window to isolate the Rayleigh wave energy, we use multi-channel cross-correlation to measure the phase difference between the stations at each frequency.

Same as Generalized Seismological Data Functionals (GSDF) method [[*Gee and Jordan*, 1992](#_ENREF_7)] except we substitute the seismogram from nearby station for the synthetic waveform to measure the phase difference. Waveforms from these two stations are presented as and here.

In the first step, we applied a wide window on to form isolation filter as in GSDF method. When constructing this window, we try to include the Rayleigh wave coda as well since they also show good consistency through the array.

Now we calculate the cross-correlation between and , and get the cross-correlation function defined as:

We try to isolate the strongly correlated part by applying a narrower window around the peak of cross-correlation function. The peak of un-filtered cross-correlation relates to the group delay of central frequency, which is usually around 30s in the case of Rayleigh wave. The window function we applied here is about 140s wide and is wide enough to include the peaks of all other periods of interest.

The next step is to convolving a set of narrow-band filters with the windowed cross-correlation, which can be presented as: .

As presented by Gee and Jordan [[1992](#_ENREF_7)], the narrow-band filtered cross-correlation function can be well approximated by a five-parameter wavelet which is the product of Gaussian envelope and a cosine function:

where two important parameters are the group delay and the phase delay between these two stations, while A is a positive scale factor, is half-bandwidth and is the center frequency of the narrow-band waveform.

Lin and Ritzwoller [[2011](#_ENREF_12)] presented a new kind of tomography based on Helmholtz equation:

Data

Discussion

Conclusion

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| Figure : Waveform of three nearby stations of the event 200806171742. The distance between the stations is less than 100km. |

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