Surface Wave Measurement Based on Cross-correlation

Ge Jin and James Gaherty

# Introduction

Many methods have been developed over the years to measure surface wave phase velocity, which can be divided into two catalogs: measuring the phase of the waveform directly ([[*Kulesh et al.*, 2005](#_ENREF_9); [*A. Levshin et al.*, 1992](#_ENREF_11); [*Yang and Forsyth*, 2006](#_ENREF_23)]) and comparing synthetic waveform with the real data ([[*Ekström et al.*, 1997](#_ENREF_3); [*Gee and Jordan*, 1992](#_ENREF_7); [*Li and Romanowicz*, 1995](#_ENREF_12); [*Tape et al.*, 2010](#_ENREF_21)]).

Methods of the first catalogs are usually quite straightforward. They can be fast and automatic, but additional efforts need to be made to control the data quality (e.g. [[*AL Levshin and Ritzwoller*, 2001](#_ENREF_10)]) and solve the cycle skipping problem (e.g. [[*Ekström et al.*, 1997](#_ENREF_3); [*Lin and Ritzwoller*, 2011](#_ENREF_13)]).

Waveform comparison or inversion algorithms are good for dealing with higher modes interference [[*Nettles and Dziewoñski*, 2011](#_ENREF_17)], providing sensitivity for deeper structure by involving long period body modes [[*Li and Romanowicz*, 1995](#_ENREF_12)], controlling the data qualify automatically[[*Maggi et al.*, 2009](#_ENREF_16)], but they also can be computational expensive (e.g. [[*Tape et al.*, 2010](#_ENREF_21)]), requiring good starting model[[*Gee and Jordan*, 1992](#_ENREF_7)], or hard to deal with scatter energy [[*Li and Romanowicz*, 1995](#_ENREF_12)].

Multipathing and scattering of surface wave at inhomogeneous structures near the ray path can significantly affect the accuracy of phase measurement (e.g. [[*Capon*, 1970](#_ENREF_2); [*Maeda et al.*, 2011](#_ENREF_15); [*Snieder and Nolet*, 1987](#_ENREF_20)]). Systematic bias can be introduced when calculating the phase velocity even with a good event azimuthal coverage [[*Wielandt*, 1993](#_ENREF_22)].

Many procedures have been established to minimize this effect, which can also be categorized into two groups. Procedures in the first group use phase information only and try to either trace the curved ray path from the source [[*Tape et al.*, 2010](#_ENREF_21)] if regional or global structure is concerned, or correct the incident angle of coming waves in the case of small dense array [[*Foster et al.*, 2010](#_ENREF_5); [*Pedersen et al.*, 2003](#_ENREF_19)]. However, even with the correction, good azimuthal coverage of events is needed to reduce this systemic alias [[*Bodin and Maupin*, 2008](#_ENREF_1)].

Methods in the other group try to use take amplitude measurement to correct the interference effect. Friederich and Wielandt [[1995](#_ENREF_6)] developed a method simultaneously inverse the incoming wavefield and phase velocity inside the array, and the incoming wavefield is presented by 44 parameters at each frequency. Forsyth and Li [[2005](#_ENREF_4)] simplified the incoming wavefield as the interference between two plane waves, which reduced the number of parameters to 4. The two plane wave assumption only is only valid in small area, for regional study like western US, the interested area has to be divided into several pieces and the smoothing between the pieces has to be taken good care of [[*Yang et al.*, 2011](#_ENREF_24)].

Lin and Ritzwoller [[2011](#_ENREF_13)] proposed a new method based on Helmholtz equation, which use amplitude term to build up relation between dynamic phase velocity (phase only measurement) and structure phase velocity, and succeeded applying it on USarray data to explore the upper mantle structure of western US. Even though Helmholtz equation is only satisfied in a laterally homogeneous medium for single mode seismic surface waves, it has been proved to be able to efficiently recover the structure phase velocity from the wavefield in a slightly or smoothly inhomogeneous material [[*Wielandt*, 1993](#_ENREF_22)].

In this paper, we are presenting a new method to measure the phase and amplitude of surface wave by fitting the multi-channel cross-correlation of waveforms from the nearby stations. By applying this method, we try to make the surface wave measurement more precise and automatic. Then we update the Helmholtz tomography method so that it can directly use the phase difference measurement between the stations.

# Method

We try to follow the same work flow as the Generalized Seismological Data Functionals (GSDF) method [[*Gee and Jordan*, 1992](#_ENREF_7)] except substituting the seismogram from nearby station for the synthetic waveform to preform cross-correlation. Waveforms from these two stations are presented as and here.

### Generate Isolation Filter

The first step is to separate the signal that we are interested in time domain. In order to do that, we applied a wide window on that includes the Rayleigh wave and most of its coda. The reason that coda is included is that although the Rayleigh wave coda from individual seismic stations can be quite complicated, they show great consistency amount the stations within 1-2 wavelength, as shown in Figure 1.

Here we defined the windowed waveform from station 2 as the isolation filter, which isolates the waveform that we are interested in.

### Calculate the Cross-correlation Function

Now we do the cross-correlation between and . The cross-correlation function is defined as:

contains the delay information of all coherency signals within the window that we applied on

In order to remove the bias that window function may introduce in the measurement, we also calculate the cross-correlation between and , which is

is similar to the auto-correlation function with the group delay and phase delay close to zero. Any non-zero phase change measured in should be subtracted from the phase measurement of .

### Window the Cross-correlation Function

Although we have already isolated the fundamental mode Rayleigh wave from other phases by applying on , still contain the energy from other coherency signals like low frequency body wave or higher modes. In order to decrease the interference of these signals, we applied a much narrower hanning window around the peak of cross-correlation function. The peak of un-filtered cross-correlation relates to the group delay of central frequency (The frequency with largest amplitude), which is usually around 30mHz for Rayleigh wave. The window function we applied here has the length of 200s.

Although is long enough to include the arrivals of all other periods of interest, bias can be generated for other frequencies since the center of is not located at the group delay of these frequencies. Gee and Jordan [[1992](#_ENREF_7)] pointed out that this bias can be approximated as

where is the time location parameter usually close to 1, is the frequency we are measuring, is the center frequency, is center of the window function , and is the group delay of frequency . From this equation we can tell that this bias only be significant for those frequencies that are much lower than the center frequency.

Instead of evaluating this window bias by the equation, we applied an iterative method. For frequencies that we think this bias is important, we measure the group delay and re-center the window function for each of these frequencies. Then the difference between and can be significantly reduced and so is the bias.

### Narrow-band filter the cross-correlation function

The next step is to convolving a set of narrow-band filters with the windowed cross-correlation, which can be presented as: . The narrow-band filter we applied in this study is the zero-phase Butterworth filter with the corner frequencies 10% lower and higher than the center frequency.

### Fitting with five-parameter wavelet

As presented by Gee and Jordan [[1992](#_ENREF_7)], the narrow-band filtered cross-correlation function can be well approximated by a five-parameter wavelet which is the product of Gaussian envelope and a cosine function:

where two important parameters are the group delay and the phase delay between these two stations, while Ga is the Gaussian function, A is a positive scale factor, is half-bandwidth and is the center frequency of the narrow-band waveform.

### Amplitude measurement

In this study, amplitude measurement is performed on single station waveform. We applied same five-parameter wavelet fitting to the windowed and narrow-band filtered the auto-correlation like function , which is defined as the cross-correlation between the isolation filter and the original waveform to generate the isolation filter. The scale factor power spectrum density function at center frequency of the narrow-band filter.

### Fitting the apparent phase velocity

For each earthquake, apparent phase velocity (sometime also called dynamic phase velocity) is defined by Eikonal equation

where is the phase travel time. is the reciprocal of travel time surface gradient, which is close to the structure phase velocity, but can be distorted by the propagation pattern like multi-pathing, back-scattering and focusing of the wavefront.

In order to get the gradient , instead of fitting one smooth travel time surface and then take the gradient like described in the original Eikonal tomography paper [[*Lin et al.*, 2009](#_ENREF_14)], we inverse the slowness vector directly by applying traditional but well developed tomography technique. The phase difference between two nearby station can be described as:

where is the slowness vector and the intergrade path can be any path connecting these two stations. In this study, we adopted the great circle path between these two stations. This equation leads the method back to the classic tomography problem except for each grid, two parameters and are inversed and they can be either positive or negative depends on the direction that the wave propagates.

To stabilize the inversion, we applied a smooth kernel, which minimizes the second gradient of and . The error function we try to minimize here can be presented as:

where is the difference between observed and predicted phase delay between the station pair, and is the parameter controlling the smoothness.

### Amplitude Correction term

The bias between apparent phase velocity and structure phase velocity can be corrected by adding amplitude measurements into the inversion [[*Wielandt*, 1993](#_ENREF_22)]. Lin and Ritzwoller [[2011](#_ENREF_13)] presented a new kind of tomography based on Helmholtz equation:

where is the structure phase velocity and is the amplitude. The amplitude Laplacian term corrects the interference of non-plane wave component and recovers the real phase velocity. This method has been successfully applied on USarray data to explore the seismic structure of western US.

For the amplitude term, because the amplitude is measured by individual stations, we adopted a similar algorithm as Lin and Ritzwoller [[2011](#_ENREF_13)] by fitting a smooth amplitude surface first. However, since the correction term contain the Laplacian term of the amplitude, we put one more constrain to make sure the smoothness of the Laplacian term. The error function for the surface fitting is

where is the linear interpolation amplitude surface based on single station measurement, and control the smoothing weight for the surface and the Laplacian term of the surface.

Although we already consider the smoothness of the Laplacian term when fitting the amplitude surface, after the correction term is calculated, one further step of smoothing is still usually needed. This is because the amplitude measurement is usually less stable than the phase measurement, and to get the correction term, the second gradient of amplitude term has to be calculated using finite difference method. This will introduce some high frequency noise into the result as well.

# Data Processing

We applied our method on the data of USarray from 2006 to 2011. ?? global events over magnitude 6 and shallower than 100km are selected to inverse the dynamics and structure phase velocity maps. Software SOD [[*Owens et al.*, 2004](#_ENREF_18)] is used to download boardband seismic waveforms and remove the instrument response, and SAC [[*Goldstein et al.*, 2003](#_ENREF_8)] is used to applied the filter and cross-correlation operation.

### Auto selection of good measurement

When building this program, we try to reduce the human inter-action in the problem and hence decrease the subjectivity in the measurement as much as possible. Signal to noise ratio (SNR) and coherence of the waveforms are the two most important standards we used to exclude automatically the measurement with low quality. In this study, SNR is defined the ratio between the average amplitude of the signal inside the interested window to that outside the interested window. SNR of original seismograms and cross-correlation waveforms are calculated separately since different window functions are used.

Coherence can be calculated by comparing the amplitude of cross-correlation and two auto-correlation functions. Since we have already use five-parameter wavelet to estimate all these functions, it is convenient to use the fitting results. Coherence of a certain frequency band can be written as:

where is the amplitude of narrow-band cross-correlation wavelet while and are the amplitude of the narrow-band auto-correlation wavelet of the two stations, prospectively. In this study, we exclude all the measurement with the coherence lower than 0.5.

### Auto-Fitting the magnitude of amplitude correction term

Since the correction term has been strongly smoothed, the amplitude of this term is smaller than it should be. As a result, it cannot fully remove the bias introduced by multi-path interference and other propagation effects. However, it is well known that structure phase velocity can be well recovered by averaging the measurement of many events with good azimuthal coverage [[*Bodin and Maupin*, 2008](#_ENREF_1)]. As a result, we can rescale the amplitude correction term for each event to make the structure phase velocity map of single event close to the averaged dynamic phase velocity map of many events. Although this procedure doesn’t improve the isotropic phase velocity map a lot, it does improve the accuracy of azimuthal anisotropy measurement.

# Discussion

### Improvement compare to FTAN method

As one of the most popular methods for surface wave measurement, FTAN method has been successfully applied in many studies [[*AL Levshin and Ritzwoller*, 2001](#_ENREF_10); [*A. Levshin et al.*, 1992](#_ENREF_11); [*Yang et al.*, 2011](#_ENREF_24)]. In this method, a continuous group delay dispersion curve is first detected through different frequency bands by tracking the maximum amplitude of the envelope function, and then the phase and amplitude on this curve is measured. Finally, phase delay from the source to the station is then calculated by assuming a reference model.

The main difference between our method and FTAN method is that instead of only focusing on a small window around the peak energy, our method takes all the coherent propagating energy into the consideration. As a result, random noise can be further depressed to make the measurement more precise.

We built up a simple synthetic test to explore the robustness of these two methods. We simulate a narrow band surface wave by a Gaussian enveloped cosine function propagating with group velocity (the velocity of Gaussian envelop) 3.7km/s and phase velocity (phase velocity of the cosine function) 4.0km/s. 10% random noise is added into the waveform, and then the phase difference between 200 station pairs 50km apart are measured at 0.03Hz by both methods. The result of this synthetic test is shown in Figure 2. It shows that under the same noise level, the error produced by our method is about half of that produced by FTAN method.

Another difference is that instead of measuring the phase delay from the source to the station, we measure the phase difference between the nearby stations. And since the stations are close to each other, usually within several wavelengths even for the highest frequency, there’re no need to concern about cycle skipping problem.

### Helmholtz Tomography

One of the main purposes of this paper is to provide alternative ways to perform Helmholtz tomography developed by Lin and Ritzwoller [[2011](#_ENREF_13)]. There’re several improvements that we try to make.

First, when calculating the dynamic phase velocity map, which is the gradient of the phase arrival time, we inverse for the slowness directly instead of fitting the travel-time surface then make the gradient. Fitting the travel-time surface by minimizing the second derivative of the surface leads to minimizing the first derivative of the dynamic phase velocity, which has no control on the roughness of the phase velocity map. In our case, by separately inverse for two slowness components and , and minimizing their second derivative, we ensure the smoothness not only of the phase gradient, but also of the incident angle or propagation direction, which is an important information to explore azimuthal anisotropy.

The amplitude correction term, on the other hand, is more challenging to estimate, because it requires both amplitude distribution and the Laplacian term of it. Theoretically, at least three data points are needed to calculate the gradient of surface, while Laplacian term requires at least six data points. As a result, by nature the resolution of the amplitude correction term is only half as fine as the resolution of phase gradient. This difference can be even more significant if we compare the precision of amplitude measurement and phase measurement.

Extra efforts have to be made to stabilize the Laplacian term. Lin and Ritzwoller [[2011](#_ENREF_13)] fit the minimizing curvature amplitude surface twice iteratively while we prefer adding the fourth derivative into the smoothing kernel and applying a further smoothing operator on the Laplacian term alone. This two method is not mathematically equivalent but both targeting at producing smooth correction term. However, one of the inevitable consequences of these smoothing operations is the underestimation of the correction term. As a result, we try to reduce this bias by increasing the amplitude of this correction term by minimizing the difference between the individual event phase velocity map and the multi-event averaged dynamic phase velocity map.

# Conclusion

1. We have developed a new method to measure the phase and amplitude of surface wave more precisely and automatically.
2. Based on the phase difference measurement, we provide an alternative way to realize Helmholtz tomography, which corrects the multi-pathing effect of surface wave.
3. We applied our method on USarray data and provide Rayleigh wave phase velocity tomography of western and central US.

|  |
| --- |
| Figure : Waveform of three nearby stations of the event 200806171742. The distance between the stations is less than 100km. |

|  |
| --- |
| Macintosh HD:Users:Jingle:research:GSDF:Methodpaper:pics:xcorwaveforms:oriwaveforms.pdf  Figure : Rayleigh waveform examples of , and time domain isolation filter . Waveforms are recorded at TAarray station L21A and K22A for an earthquake near Samoa on Oct 20, 2010. |

|  |
| --- |
| Macintosh HD:Users:Jingle:research:GSDF:Methodpaper:pics:xcorwaveforms:xcorwaveforms.pdf  Figure : Waveform examples of cross-correlation function , |

|  |  |
| --- | --- |
| Macintosh HD:Users:Jingle:research:GSDF:Methodpaper:pics:GSDFvsFTAN:gsdf_errhist.eps | Macintosh HD:Users:Jingle:research:GSDF:Methodpaper:pics:GSDFvsFTAN:ftan_errhist.eps |
| Figure : Comparison between our method and FTAN method in a simple synthetic test. Left subfigure shows the error histogram of our method for 200 individual measurements with 20% noise, and right subfigure shows the FTAN measurement error on the same data. | |

# Reference:

Bodin, T., and V. Maupin (2008), Resolution potential of surface wave phase velocity measurements at small arrays, *Geophysical Journal International*, *172*(2), 698-706.

Capon, J. (1970), Analysis of Rayleigh-wave multipath propagation at LASA, *Bulletin of the Seismological Society of America*, *60*(5), 1701-1731.

Ekström, G., J. Tromp, and E. Larson (1997), Measurements and global models of surface wave propagation, *Journal of Geophysical Research*, *102*(B4), 8137-8157.

Forsyth, D. W., and A. Li (2005), Array Analysis of Two-Dimensional Variations in Surface Wave Phase Velocity and Azimuthal Anisotropy in the Presence of Multipathing Interference, *Seismic Earth: array analysis of broadband seismograms*(157), 81.

Foster, A., G. Ekstrom, and V. Hjorleifsdottir (2010), Surface wave propagation across the USArray, paper presented at AGU Fall Meeting.

Friederich, W., and E. Wielandt (1995), Interpretation of seismic surface waves in regional networks: joint estimation of wavefield geometry and local phase velocity. Method and numerical tests, *Geophysical Journal International*, *120*(3), 731-744.

Gee, L., and T. Jordan (1992), Generalized seismological data functionals, *Geophysical Journal International*, *111*(2), 363-390.

Goldstein, P., D. Dodge, M. Firpo, and L. Minner (2003), SAC2000: Signal processing and analysis tools for seismologists and engineers, *International Geophysics*, *81*, 1613-1614.

Kulesh, M., M. Diallo, and M. Holschneider (2005), Wavelet analysis of ellipticity, dispersion, and dissipation properties of Rayleigh waves, *Acoustical Physics*, *51*(4), 425-434.

Levshin, A., and M. Ritzwoller (2001), Automated detection, extraction, and measurement of regional surface waves, *Pure and Applied Geophysics*, *158*(8), 1531-1545.

Levshin, A., L. Ratnikova, and J. Berger (1992), Peculiarities of surface-wave propagation across central Eurasia, *Bulletin of the Seismological Society of America*, *82*(6), 2464-2493.

Li, X. D., and B. Romanowicz (1995), Comparison of global waveform inversions with and without considering cross-branch modal coupling, *Geophysical Journal International*, *121*(3), 695-709.

Lin, F. C., and M. H. Ritzwoller (2011), Helmholtz surface wave tomography for isotropic and azimuthally anisotropic structure, *Geophysical Journal International*.

Lin, F. C., M. H. Ritzwoller, and R. Snieder (2009), Eikonal tomography: surface wave tomography by phase front tracking across a regional broad band seismic array, *Geophysical Journal International*, *177*(3), 1091-1110.

Maeda, T., K. Obara, T. Furumura, and T. Saito (2011), Interference of long-period seismic wavefield observed by the dense Hi-net array in Japan, *Journal of Geophysical Research*, *116*(B10), B10303.

Maggi, A., C. Tape, M. Chen, D. Chao, and J. Tromp (2009), An automated time‚Äêwindow selection algorithm for seismic tomography, *Geophysical Journal International*, *178*(1), 257-281.

Nettles, M., and A. M. Dziewoñski (2011), Effect of Higher-Mode Interference on Measurements and Models of Fundamental-Mode Surface-Wave Dispersion, *Bulletin of the Seismological Society of America*, *101*(5), 2270-2280.

Owens, T. J., H. P. Crotwell, C. Groves, and P. Oliver-Paul (2004), SOD: standing order for data, *Seismological Research Letters*, *75*(4), 515-520.

Pedersen, H. A., O. Coutant, A. Deschamps, M. Soulage, and N. Cotte (2003), Measuring surface wave phase velocities beneath small broad‚Äêband arrays: tests of an improved algorithm and application to the French Alps, *Geophysical Journal International*, *154*(3), 903-912.

Snieder, R., and G. Nolet (1987), Linearized scattering of surface waves on a spherical Earth, *J. geophys*, *61*, 55-63.

Tape, C., Q. Liu, A. Maggi, and J. Tromp (2010), Seismic tomography of the southern California crust based on spectral-element and adjoint methods, *Geophysical Journal International*, *180*(1), 433-462.

Wielandt, E. (1993), Propagation and Structural Interpretation of Non-Plane Waves, *Geophysical Journal International*, *113*(1), 45-53.

Yang, Y., and D. W. Forsyth (2006), Regional tomographic inversion of the amplitude and phase of Rayleigh waves with 2-D sensitivity kernels, *Geophysical Journal International*, *166*(3), 1148-1160.

Yang, Y., W. Shen, and M. H. Ritzwoller (2011), Surface wave tomography on a large-scale seismic array combining ambient noise and teleseismic earthquake data, *Earthquake Science*, *24*(1), 55-64.