NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS AND APPLIED PROBABILITY
ST5210: Multivariate Data Analysis
SEMESTER I, AY 2020/2021

Typing Mathematics in LumiNUS

For the final examinations, you will be expected to type in short responses (at most 4 to 5 sentences) which will (sometimes) include mathematical expressions.

Unfortunately, LumiNUS does not support the typesetting of mathematical expressions in the Quiz function. Thus you will need to input mathematical expressions in plain text.

As this might be new to some of you, this document details how some simple mathematical expressions, relevant to this course, can be typed in plain text.

Please take some time to go through and familiarize yourself with these before examination day.

The following are some codes that are acceptable in the exam. Of course, plain English or Latex are also acceptable.

Math	Code
$\underline{x}$	\ul x (note: \ul means \underline)
$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$	\ul x = c(x_1, x_2, \cdots, x_n)^T
$\underline{x}^T = (x_1 \quad x_2 \quad \cdots \quad x_n)$	\ul x^T = c(x_1, x_2, \cdots, x_n)
$\begin{pmatrix} x_{11} & \cdots & x_{13} \\ \vdots & \ddots & \vdots \\ x_{n,1} & \cdots & x_{n,3} \end{pmatrix}$	rbind(c(x_11, \cdots, x_13), c(\vdots,\ddots,\vdots), c(x_{n,1}, \cdots,x_{n,3}))
$n\left(\underline{\bar{X}} - \underline{\mu}\right)^T S^{-1}\left(\underline{\bar{X}} - \underline{\mu}\right) \sim T^2(p, n-1)$	n (\ul \bar{X} -\ul \mu)^T S^{-1} (\ul \bar{X} -\ul \mu) ~ T^2 (p, n-1)
$Y_{11} = \underline{\hat{u}}_1^T \underline{X}_1$	Y_11 =\ul \hat(u)_1^T \ul X_1

Table 1: Greek Letters – You will only use a small subset of these; the others are listed for completeness.

α	\alpha	θ	\theta	o	o	υ	\upsilon
β	\beta	ϑ	\vartheta	π	\pi	ϕ	\phi
γ	\gamma	ι	\iota	ϖ	\varpi	φ	\varphi
δ	\delta	κ	\kappa	ρ	\rho	χ	\chi
ϵ	\epsilon	λ	\lambda	ϱ	\varrho	ψ	\psi
ε	\varepsilon	μ	\mu	σ	\sigma	ω	\omega
ζ	\zeta	ν	\nu	ς	\varsigma		
η	\eta	ξ	\xi	τ	\tau		
Γ	\Gamma	Λ	\Lambda	Σ	\Sigma	Ψ	\Psi
Δ	\Delta	Ξ	\Xi	Υ	\Upsilon	Ω	\Omega
Θ	\Theta	Π	\Pi	Φ	\Phi		

Table 2: Common Operators.

min	\min	max	\max	$\int$	\int
$\sum$	\sum	$\prod$	\prod		
$\cup$	\cup	$\cap$	\cap		

Table 3: Common expressions used in multivariate statistics

Math	Code
$S = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$	S = rbind(c(2,3), c(3,4)) or S = cbind(c(2,3), c(3,4))
$\Sigma = V^{1/2} R V^{1/2}$	\Sigma = V^{1/2} R V^{1/2}

$$f_{\underline{X}}(\underline{x}) = \frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}\left(\underline{x}-\underline{\mu}\right)^T \Sigma^{-1}\left(\underline{x}-\underline{\mu}\right)\right]$$

F_{\ul X} (\ul x) = 1/{{2\pi}^{p/2} |\Sigma |^{1/2} } exp (-1/2(\ul x- \ul \mu)^T \Sigma^{-1} (\ul x - \ul\mu))

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_2\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 & 1.6 \\ 1.6 & 4 \end{pmatrix}\right)$$

c(X_1, X_2)^T ~ N_2 (c(2,3)^T, rbind(c(1,1.6),c(1.6,4)))

$$\left(\underline{X}_2 | \underline{X}_1 = \underline{x}_1\right) \sim N_{p-r}\left(\underline{\mu}_2 + \Sigma_{21}\Sigma_{11}^{-1}\left(\underline{x}_1 - \underline{\mu}_1\right), \Sigma_{22.1}\right)$$

(\ul X_2 | \ul X_1 = x_1) ~ N_{p-r} (\ul \mu_2 +\Sigma_21 \Sigma_11^{-1}(\ul x - \ul \mu_1), \Sigma_{22.1})

$$\left(\underline{\bar{X}} - \underline{\mu}_0\right)^T S^{-1}\left(\underline{\bar{X}} - \underline{\mu}_0\right) > \frac{(n-1)p}{n(n-p)} F_\alpha(p, n-p)$$

(\ul \bar{X} - \ul \mu_0)^T S^{-1} (\ul \bar{X} - \ul \mu_0) > {(n-1)p}/{n-p} F_{\alpha} (p, n-p)

The following is part of Question 1 in Tutorial 2. This is to illustrate how one may answer this type of "essay type" questions in the final exam.

Question

Suppose $\underline{Y} \sim N_3(\underline{\mu}, \Sigma)$, where

$$\underline{Y} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}, \underline{\mu} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 6 & 1 & -2 \\ 1 & 13 & 3 \\ -2 & 3 & 4 \end{pmatrix}$$

(a) Find a vector $\underline{a}$ such that $\underline{a}^T\underline{Y} = 2Y_1 - Y_2 + 3Y_3$. Hence, find the distribution of $Z = 2Y_1 - Y_2 + 3Y_3$.

(b) Find a matrix A such that $A\underline{Y} = \begin{pmatrix} Y_1 + Y_2 + Y_3 \\ Y_1 - Y_2 + 2Y_3 \end{pmatrix}$. Hence, find the joint distribution of $\underline{W} = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$, where $W_1 = Y_1 + Y_2 + Y_3$ and $W_2 = Y_1 - Y_2 + 2Y_3$.

You should include the following in your answer to part (a).

(1) The vector $\underline{a}$. i.e. $\underline{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

(2) Some key steps in getting the answer. Since $Z = \underline{a}^T \underline{Y}$, and $\underline{Y} \sim N(\underline{\mu}, \Sigma)$ therefore $Z \sim N\left(\underline{a}^T \underline{\mu}, \underline{a}^T \Sigma \underline{a}\right)$.

(3) The distribution of Z. i.e. $Z \sim N(17, 27)$.

This is how you may type out your answer in *plain text* in LumiNUS.

(a)
\ul a = c(2, -1, 3).
Since Z =\ul a^T * \ul Y, where \ul Y ~ N(\ul \mu, \Sigma), therefore Z ~ N(\ul a^T * \ul\mu, \ul a^T * \Sigma * \ul a).
Hence, Z ~ N(17, 27).
(b)
A = rbind(c(1,1,1), c(1,-1,2)).
Since \ul W = A * \ul Y, where \ul Y ~ N(\ul \mu, \Sigma), therefore W ~ N_2 (A* \ul\mu, A * \Sigma * A^T).
Hence, \ul W ~ N_2 (c(8, 10), rbind(c(27,-2), c(-2,13))).

Prepared by Y M Chan
6/11/2020