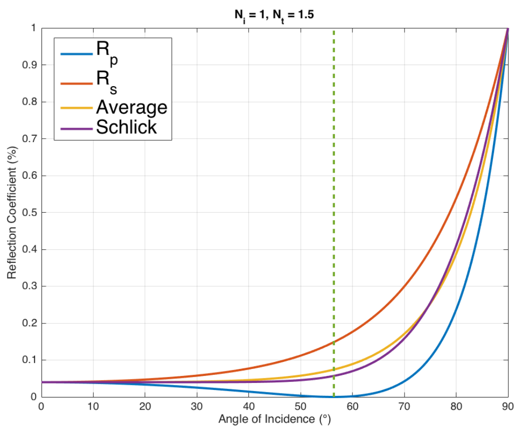
CO417 ADVANCED COMPUTER GRAPHICS
Assignment 2

Lim Yen Khye (CID:01146555) & Ju Bo (CID:01126776)

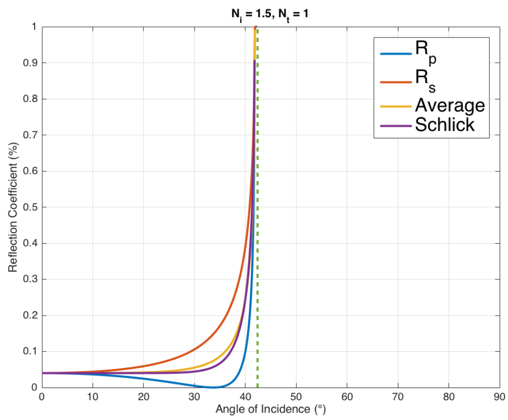
1 Generate plots of Fresnel reflectance

In the first part of this assignment, plots of Fresnel reflectance for a dielectric material are generated. The index of refractions for materials are chosen for η_i and η_t as 1.0 and 1.5 respectively. The Y-axis corresponds to the reflectance values varying with incidence angles on X-axis. The curves are also plotted for a ray exiting the material.

The average values of both parallel and perpendicular are also plotted on the curves. Next, Schlick's approximation is computed using the reflectance at normal incidence, R_0 obtained earlier.



(a) Fresnel curves with $n_i = 1$ and $n_t = 1.5$



(b) Fresnel curves with $n_i = 1.5$ and $n_t = 1$

Figure 1: Fresnel curves for different configurations of index of refraction.

From Figure 1a, the Brewster angle is approximated to be 56.4°. The reflectance at normal incidence R_0 is calculated to be 0.04 for both parallel and perpendicular components. Based on Figure 1b, total critical angle is found to be 42.4°.

Comparing the average curve of two components and the Schlick's approximation, it can be noted that the approximation is a very good estimate of the unpolarized Fresnel reflectance. It also has an added advantage where the index of refraction does not need to be estimated. This method only required the reflectance at normal incidence R_0 to be known, which is relatively simple to measure in practical.

2 Generate samples according to an Environment Map

This section highlights the details of generating samples based on the 2D cumulative density function (CDF) of the Grace Cathedral environment map. The EM is loaded in latitude-longitude format. The CDF is created based on the intensity (luminance) of each pixel. By the Jacobian of unwrapping the sphere into a latlong map, the intensity values are scaled by $sin\theta$ where θ varies from 0 to π from top to bottom of the map. This will cause the luminance of pixels at nearer to the poles to be dimmer than those at the equater.

$$Intensity = \frac{R + G + B}{3} \times sin\theta$$

To implement this, the array of intensity values act as the probability density function (PDF). This array is normalized such that the sum is equal to 1. For the CDF, the base case is initialised for the first element of CDF, $CDF[0] = PDF[0]$. Then, the subsequent CDF values are computed iteratively using the equation:

$$CDF[i] = CDF[i-1] + PDF[i]$$

Sampling row and column

The first random sample μ_1 generated from the uniform distribution between 0 and 1 is used to select the row in the latlong map. The PDF is a 512×1 array containing the sum of intensities of each row. From the CDF, the first random sample μ_1 is evaluated using the CDF and this will result as a scanline being chosen.

The second random sample μ_2 (also uniformly distributed between 0 and 1) is to find which column along the scanline to be selected. This PDF has a dimension of 1024×1 which corresponds to the intensity values of each pixel in the particular row. There are 512 of such PDFs, one for each row. By evaluating the CDF using μ_2, a random pixel in that specific row will be chosen.

By using μ_1 to select the row and μ_2 to select the column, random samples on the map can be generated. This is repeated for 64, 256 and 1024 samples on the EM. For each sample, a 5×5 neighbourhood around it will be set to green (0,1,0) for illustration. An if condition is used to check for the pixels to be coloured green is indeed within the range of the image. The results are saved with gamma correction of 2.2.

Figure 2: 64 samples (green) from the Grace Cathedral EM.

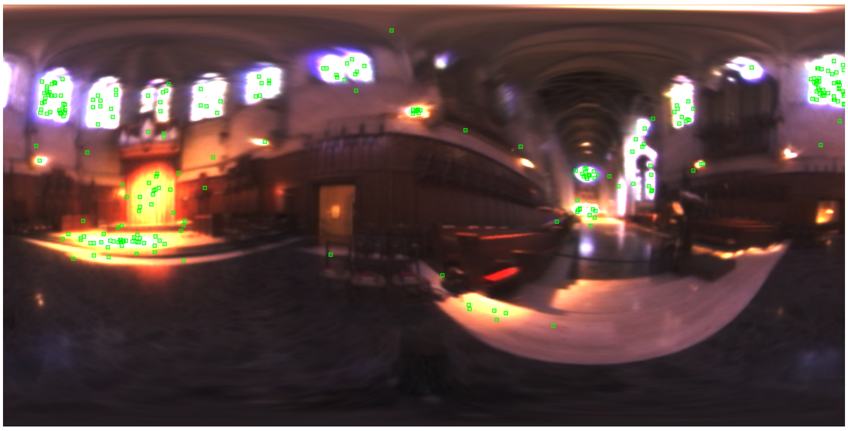


Figure 3: 256 samples (green) from the Grace Cathedral EM.

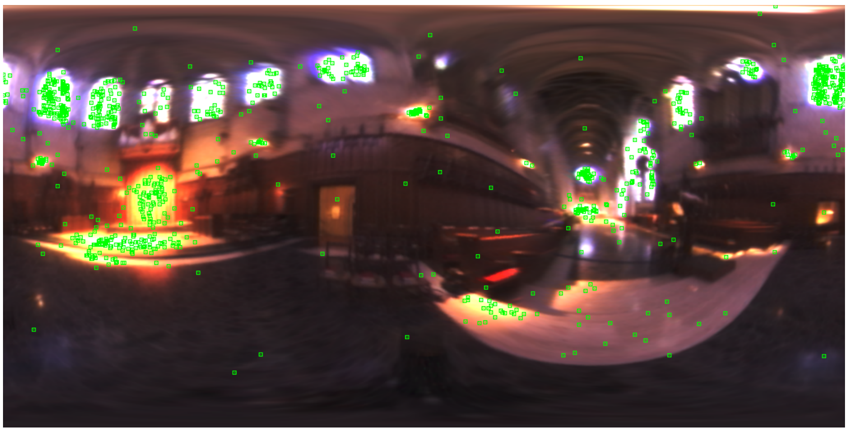


Figure 4: 1024 samples (green) from the Grace Cathedral EM.

From the output images, it can be observed that pixels with higher intensity are more likely to be sampled. Majority of the samples are along the rows where the windows are located. In the 1024 samples images, it is very clear that most of the sampled pixels in green are on the light from the windows. There are very few samples on the gap between pixels.

3 Render a sphere with sampled Environment Map

Using the function written in the previous section to generate samples from the Grace Cathedral EM, this task renders a sphere using 64, 256 and 1024 samples. The rendered image will be of 511 $\times$ 511 resolution which corresponds to a unique surface normal (similar to previous coursework). The following assumptions are made for this section:

1. Diffuse BRDF with $\rho_d = 1.0$
2. Using visibility function, $V(x, \omega_i) = 1$
3. Orthographic camera with view vector, $\omega_o = (0, 0, 1)$

The equation below is used for the Monte Carlo estimate.

$$L_{r,N}(\omega_r) = \frac{1}{N}\sum_{j=1}^{N}\frac{f_r(\omega_r, \omega_{i,j})cos\theta_{i,j}L_i(\omega_{i,j})V(\omega_{i,j})}{q(\omega_{i,j})}$$

where the proposal distribution for importance sampling, $q(\omega_{i,j}) = \frac{L_i(\omega_i)}{\int_\Omega L_i(\omega_i)d\omega_i}$

The integral is evaluated as:

$$\int_\Omega L_i(\omega_i)d\omega_i = \frac{1}{M \times N}\frac{\sum Intensity(M, N)sin\theta}{\frac{1}{2\pi}\frac{1}{\pi}}$$

As this sampling is performing using the luminance channel, the normalized RGB color of the sampled light source direction is multiplied with the BRDF. Gamma correction is then performed to visualise the image. This MC renderer uses only one set of samples from the EM to render the image. Therefore, it will not have noise/variance, but instead has bias. By increasing the number of samples from 64 to 1024, it is noted that the resulting image converge towards the actual solution.

By rotating the environment map by 90 degrees clockwise about the y-axis, the EM appears to illuminate the sphere with the centre point being the negative z-axis.

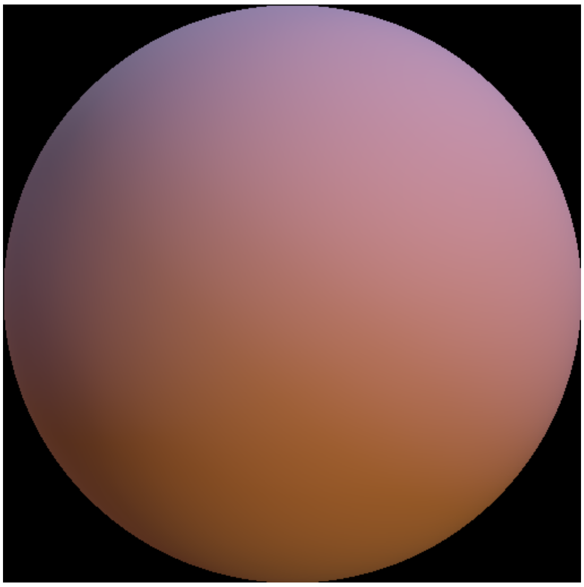
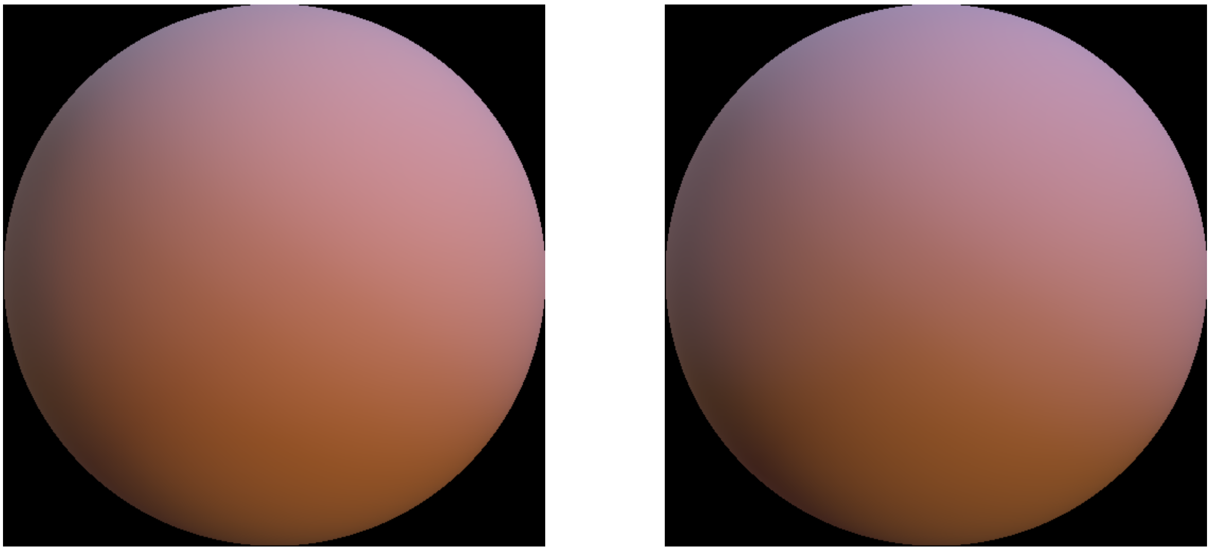


Figure 5: Diffuse sphere lit by 64 samples from Grace Cathedral EM. Photo scaled by 1.0 with 2.2 gamma value.



(a) 256 samples

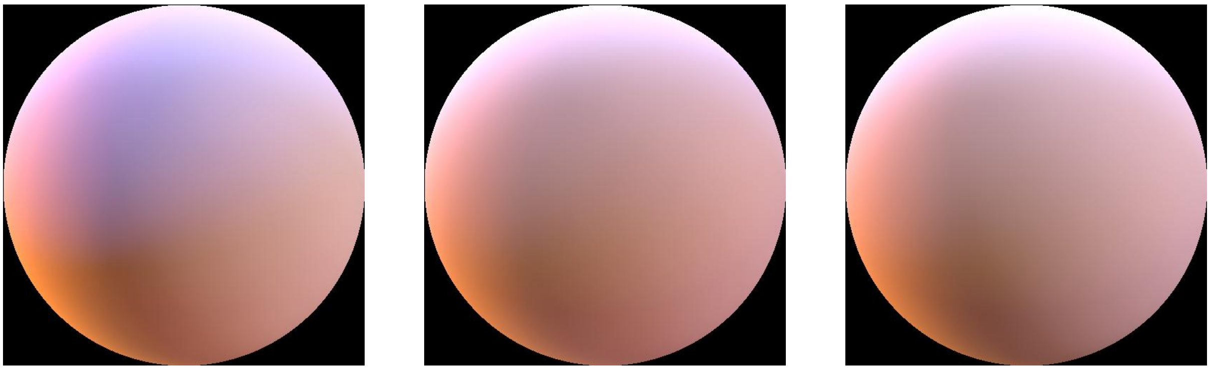
(b) 1024 samples

Figure 6: Diffuse sphere lit by the Grace Cathedral EM. Photo scaled by 1.0 with 2.2 gamma value.

4 Render a sphere lit by Grace EM using PBRT

The last task is to perform Physically Based Rendering using PBRT. The environment map used is the Grace Cathedral lighting environment in latlong format. A diffuse sphere is to be rendered for 8, 16 and 32 samples from the EM. The following are defined for this section:

1. Albedo of diffuse sphere $\rho_d = 1.0$.
2. Camera placed at [0, 0, 10], looking down on the -z-axis, with up vector +y



(a) 64 samples (b) 256 samples (c) 1024 samples

Figure 7: Diffuse sphere for EM with 90° rotation about y-axis. Photos scaled by 2 with 2.2 gamma value.

3. Orthographic camera projection

Figure 8: Rendered diffuse sphere using 8 samples in PBRT.

By increasing the number of samples, it can be noted that the Monte Carlo variance gradually decreases. As more samples are used, a better approximation is obtained and the solution will converge towards the actual result.

Discussion for diffuse sphere

Based on Section 3 and Section 4, several observations can be made. In Section 3, the same set of samples is used for calculation of all pixels. As a result, the image looks smooth without noise/variance but has some bias. However, Section 4 uses a new set of samples for each pixel. Therefore, the noise/variance is present in the images generated using PBRT. Depending on the application, it is a tradeoff between correctness and visual perception.

Figure 9: Rendered diffuse sphere using 16 samples in PBRT.

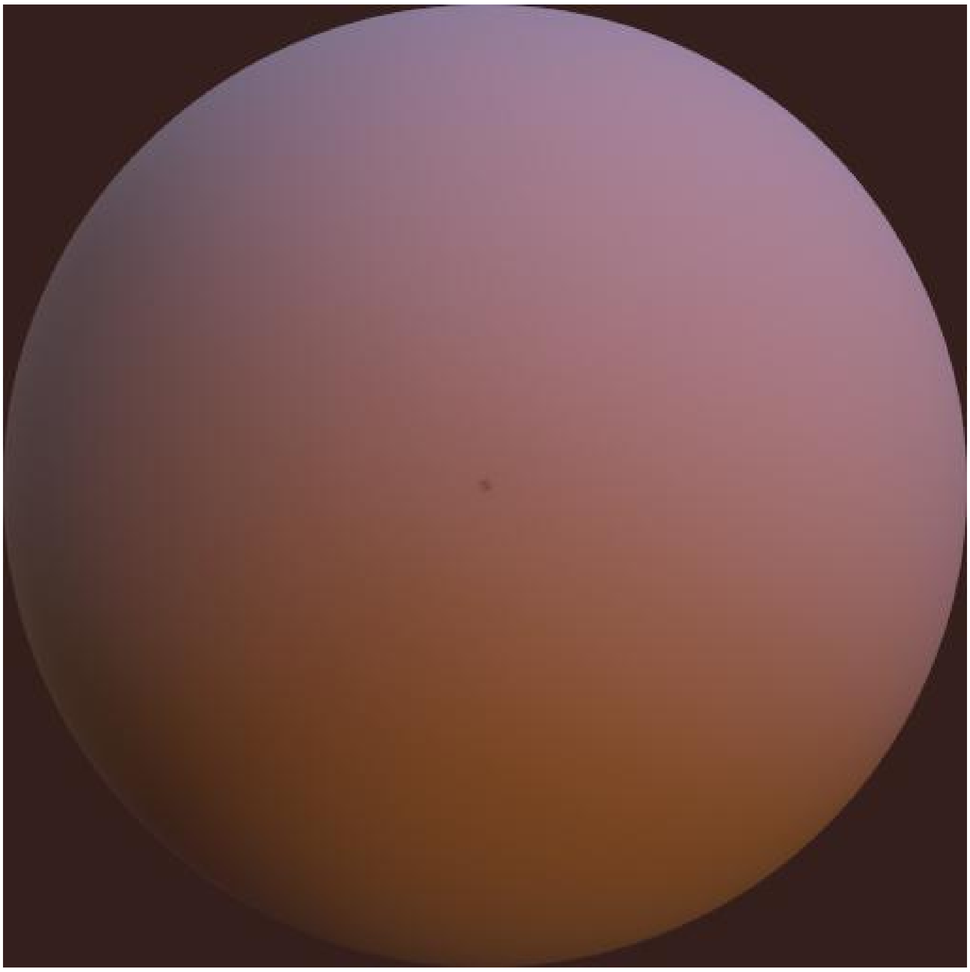


Figure 10: Rendered diffuse sphere using 32 samples in PBRT.