Imperial College London

WEEK 4

IMPERIAL COLLEGE LONDON

DEPARTMENT OF COMPUTING

Matrices make linear mappings

Author:
Jinwei Zhang (CID: 01540854)

Date: July 22, 2019

1 Einstein summation convention and the symmetry of the dot product

- Einstein's Summation Convention

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mp} \end{pmatrix} \tag{2}$$

the matrix product $\mathbf{C} = \mathbf{AB}$ (denoted without multiplication signs or dots) is defined to be the $n \times p$ matrix $c_{ij} = a_{i1}b_{1j} + \cdots + a_{im}b_{mj} = \sum_{k=1}^{m} a_{ik}b_{kj}$.

$\mathbf{C} = \mathbf{AB}$
$C_{ik} = a_{ij}b_{jk}$

- projection is symmetric and the dot product is symmetric and why projection is the dot product

2 Matrices changing basis

- Transforms Bear's vectors into my world

- Translate my world into Bear's world(usually)

The matrix inverse of a square matrix m may be taken in the Wolfram Language using the function `Inverse`[*m*].

For a 2×2 matrix

$$A \equiv \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

the matrix inverse is

$$\begin{aligned} A^{-1} &= \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{a\,d - b\,c} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}. \end{aligned}$$

For a 3×3 matrix

$$A \equiv \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

the matrix inverse is

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}.$$

Figure 1: This is matrix inverse.

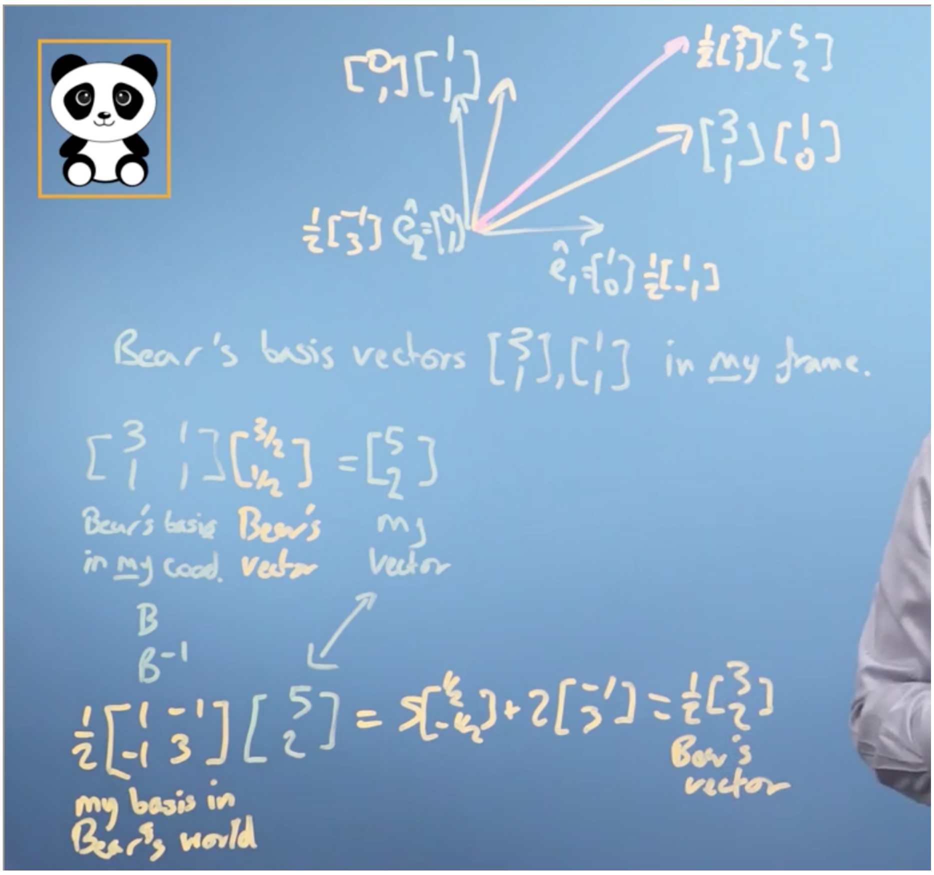

Figure 2: This is matrix basis convert. (yellow bear)

- we could do this just by using projections, if the new basis vectors were orthogonal (need normalise)

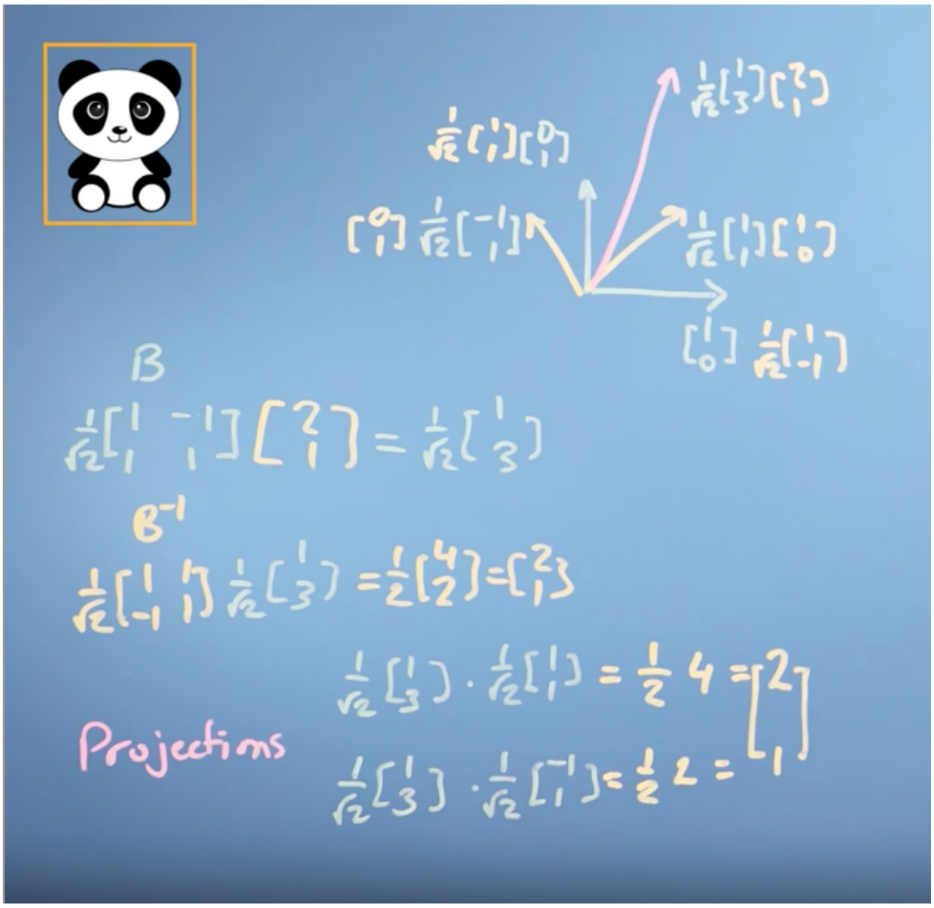

Figure 3: This is matrix basis convert. (yellow bear)

3 Doing a transformation in a changed basis

4 Orthogonal matrices

5 The GramSchmidt process

6 Example: Reflecting in a plane