Imperial College London

WEEK 5


IMPERIAL COLLEGE LONDON

DEPARTMENT OF COMPUTING


# Eigenvalues and Eigenvectors: Application to Data Problems


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Date: July 23, 2019

# 1 What are eigen-things

## 1.1 What are Eigenvalues and Eigenvectors

- linear transformations using matrices.(scalings, rotations, and shears)
  In linear algebra, an **eigenvector** or **characteristic vector** of a linear transformation is a non-zero vector that changes by only a scalar factor when that linear transformation is applied to it. More formally, if **T** is a linear transformation from a vector space **V** over a field F into itself and **v** is a vector in V that is not the zero vector, then v is an eigenvector of T if $T(\mathbf{v})$ is a scalar multiple of v. This condition can be written as the equation

$$T(\mathbf{v}) = \lambda \mathbf{v} \tag{1}$$

  where $\lambda$ is a scalar in the field F,known as the **eigenvalue**, **characteristic value**, or **characteristic root** associated with the eigenvector v

- Eigenvalues and Eigenvectors

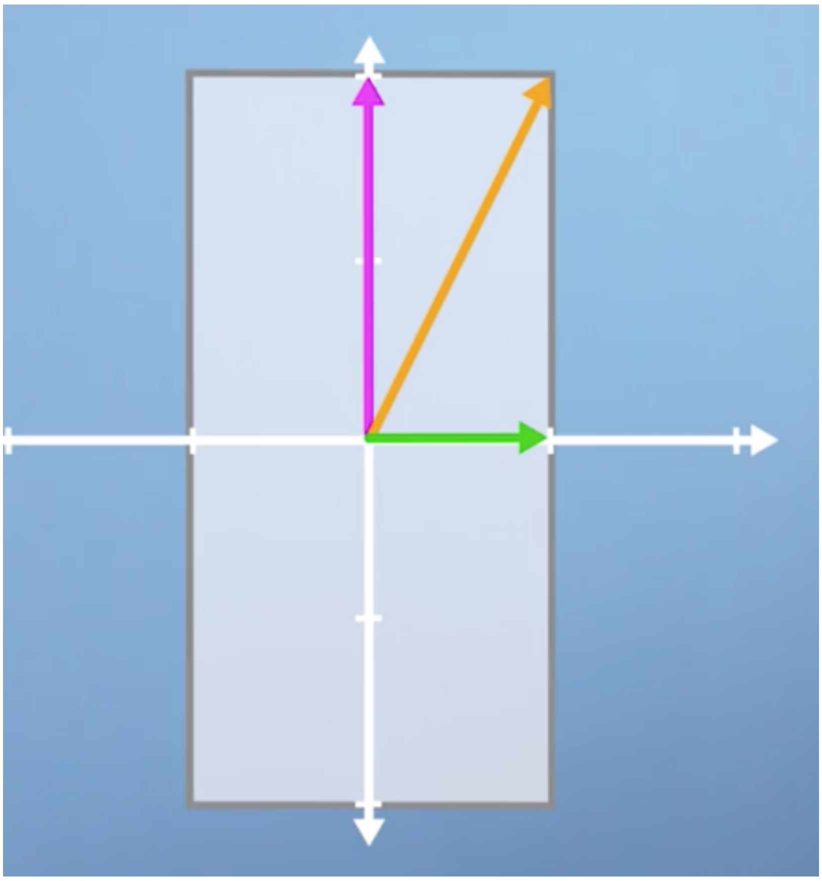

**Figure 1:** This is a vetical scale 2.

In the case above the purple and green vector only change in scale so they are Eigenvectors of this transformation.

# 2 Getting into the detail of eigenproblems

## 2.1 Special eigen-cases

recap: eigenvectors are those which lie along the same span both before and after applying a linear transform to a space. And then, eigenvalues are simply the amount that each of those vectors has been stretched in the process

- Uniform scaling: where we scale by the same amount in each direction. any vector would be an eigenvector

- Rotation: 180 degrees. all vectors for this transform are eigenvectors, and they all have eigenvalues of minus one.

- combination of a horizontal shear and a vertical scaling:

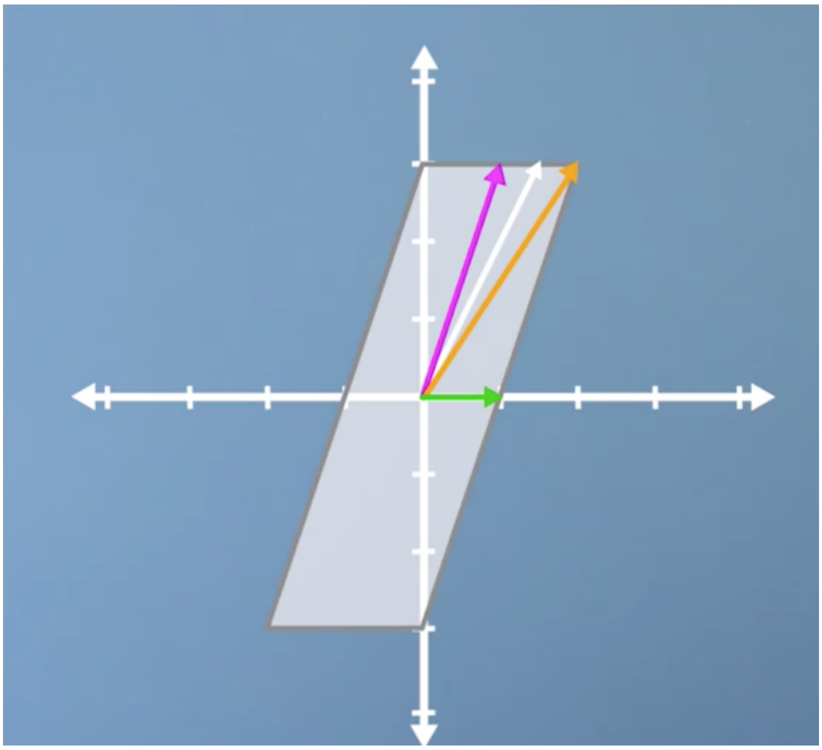

**Figure 2:** This is a combine horizontal shear and a vertical scaling

  Green and White vector are eigenvectors. (not easy to spot)

- inverse transform:

  tougher.

- 3D rotation

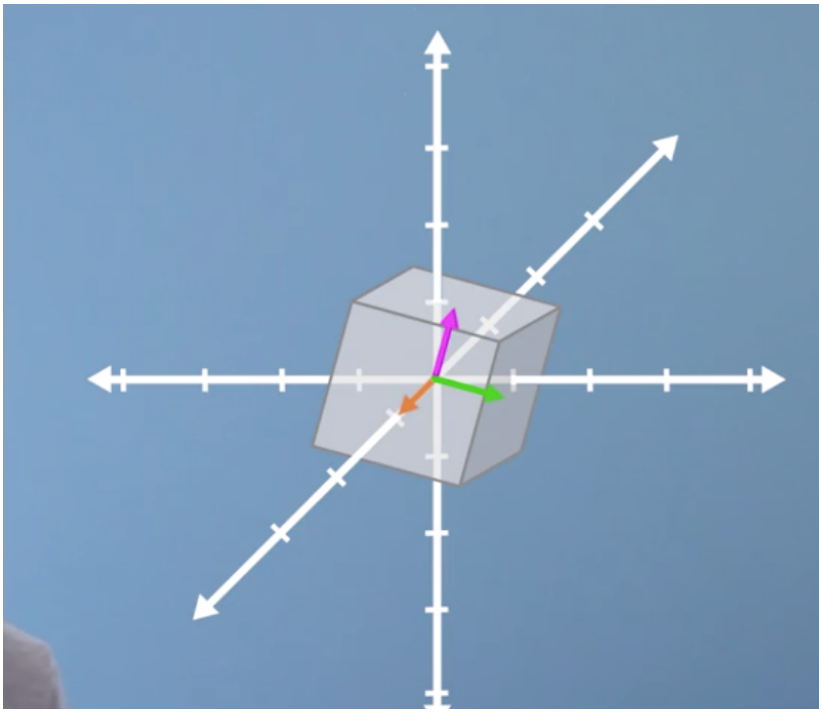

**Figure 3:** This is a 3D roation

If we find the eigenvector of a 3D rotation, it means we've also found the axis of rotation

## 2.2 Calculating eigenvectors

- Algrbra expression

$$A\mathbf{v} = \lambda\mathbf{v} \tag{2}$$

$$(A - \lambda I)\mathbf{v} = \mathbf{0} \tag{3}$$

That is, the determinant of $(A-\lambda I)\mathbf{v} = \mathbf{0}$ must equal 0. We call $p(\lambda) = \det(A-\lambda I)$ the **characteristic polynomial** of A. The eigenvalues of A are simply the roots of the characteristic polynomial of A.

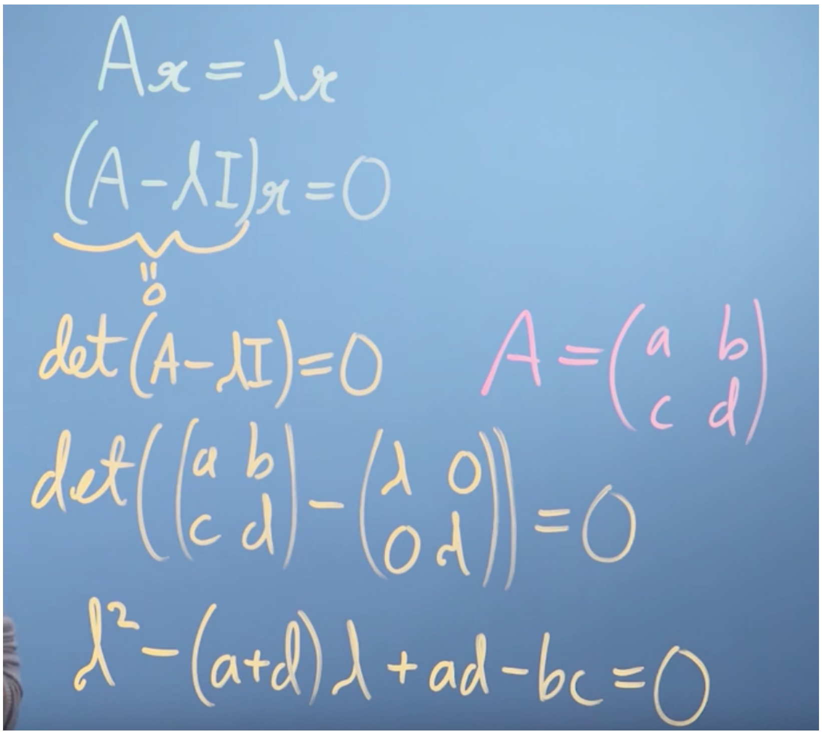


**Figure 4:** This is a the equation for 2D transfer

- vertical scaling by a factor of two,

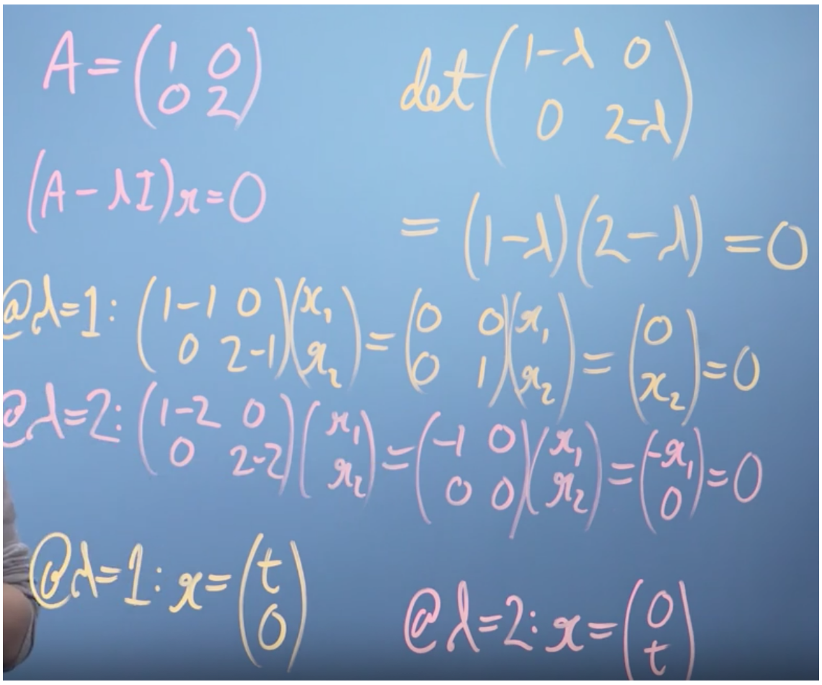


**Figure 5:** vertical scaling by a factor of two

So now we have two eigenvalues, and their two corresponding eigenvectors.

- Rotation by 90-degrees anti-clockwise

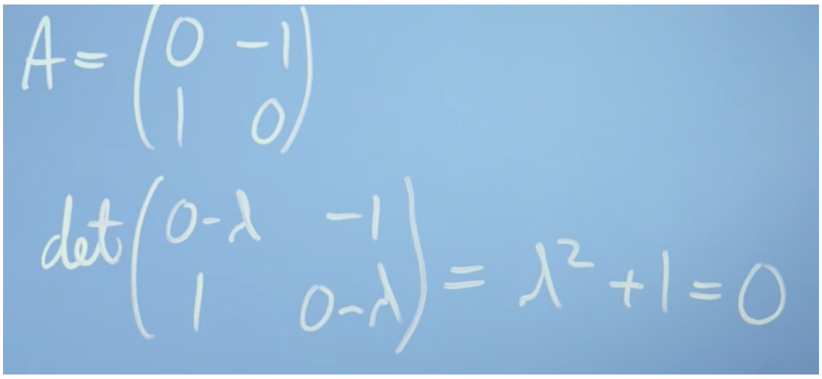


**Figure 6:** vertical scaling by a factor of two

No eigenvalue and vector.

# 3 When changing to the eigenbasis is really useful

## 3.1 Changing to the eigenbasis

- diagonalisation vector times a tranformation matrix is really costly. We want to change the T to diagonal matrix. Please look at the AV = $\lambda$ v

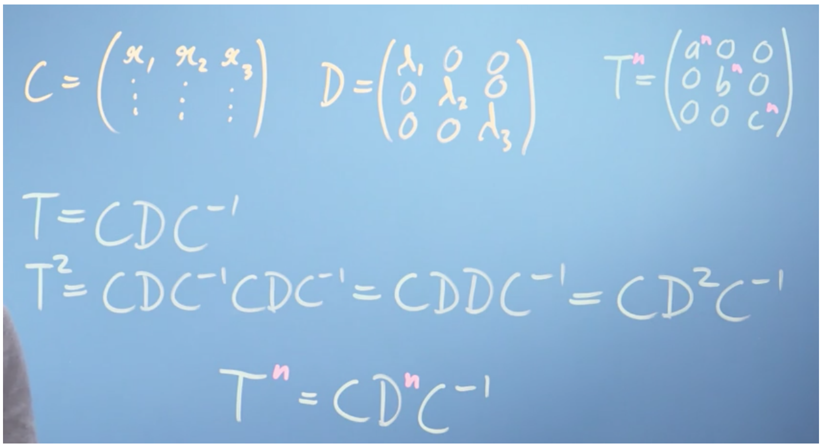


**Figure 7:** eigenanalysis

This is the eigenanalysis refer to the formulae $\boldsymbol{A}\mathbf{v} = \lambda\mathbf{v}$ A becomes T and $\lambda$ become a diagonal matrix.

## 3.2 eigenbasis example

These are some examples to show the process above;

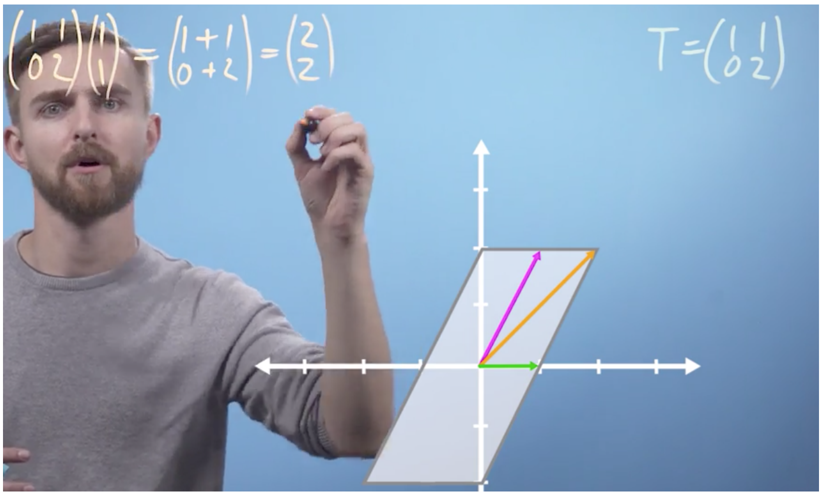


**Figure 8:** eigenanalysis example

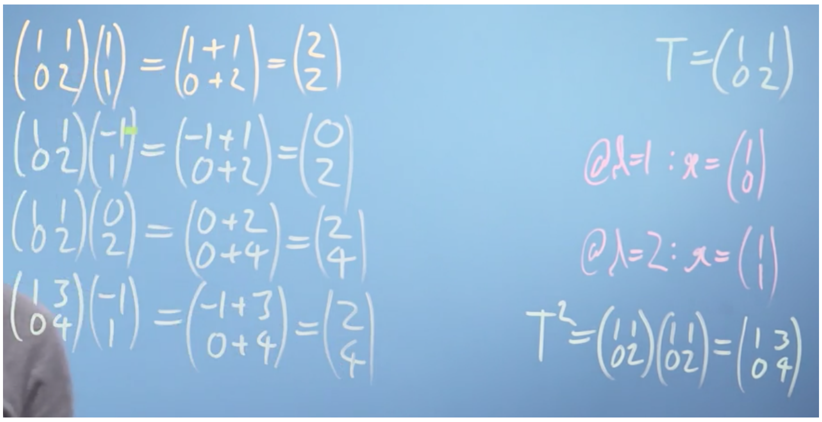


**Figure 9:** eigenanalysis example

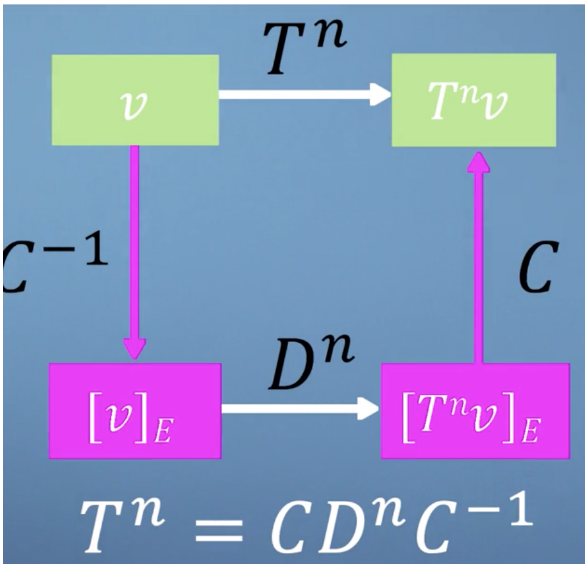


**Figure 10:** eigenanalysis example

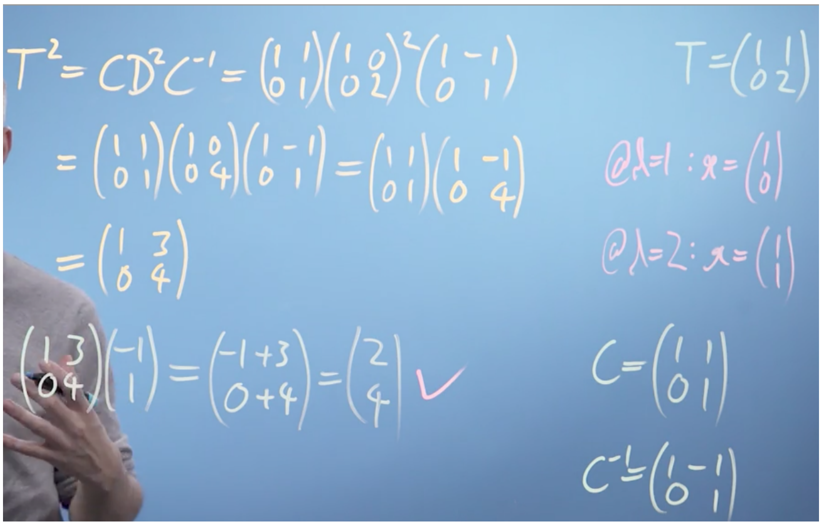


**Figure 11:** eigenanalysis example

# 4 Making the PageRank algorithm

# 5 Eigenvalues and Eigenvectors: Assessment