# Second Coursework
# (Component Analysis and Optimisation)

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**Part I:**

You are given two facial datasets with identity information per sample (the datasets are part of YALE and PIE facial databases) You are also given two Matlab code files, one for each dataset, named `protocol.m`. The code runs a face recognition protocol and reports the results in the end (in a form of recognition error). In the provided code, no dimensionality reduction or feature extraction methodology is performed.

Your task is to investigate the role of dimensionality reduction/feature extraction for the purpose of face recognition on the two datasets. You must implement and investigate the use of the following component analysis techniques.

1. Principal Component Analysis (PCA) in Small Sampled Sized (SSS) problems.

2. Whitened PCA in SSS.

3. Linear Discriminant Analysis (LDA) in SSS.

You have to produce a report where you provide plots of the recognition error versus the number of features kept for each of the methods and each of the databases.
**Important**: Your code should contain comments about the mathematical methodology you used in your implementation.
**Notice**: For eigenanalysis you could use the `eig()` function of Matlab.

**Part II:**

Assume you are given a set of $n$ data samples $\mathbf{x}_1, \ldots, \mathbf{x}_n$. The minimum enclosing hyper-sphere of the above data samples can be found by the following constrained optimisation problem

$$\begin{array}{ll} \min_{R,\mathbf{a},\xi_i} & R^2 + C\sum_{i=1}^{n}\xi_i \\ \text{subject to} & (\mathbf{x}_i - \mathbf{a})^T(\mathbf{x}_i - \mathbf{a}) \leq R^2 + \xi_i, \forall \xi_i \geq 0. \end{array} \tag{1}$$

where $R$ is the radius of the hyper-sphere, $\mathbf{a}$ is the center of the hyper-sphere and the variable $C$ gives the trade-of between simplicity (or volume of the sphere) and the number of errors.

You should perform the following tasks

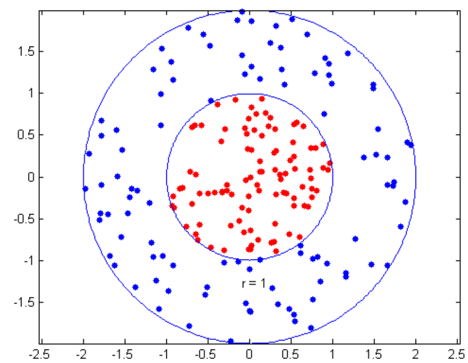


Figure 1: The two classes of simulated data (class one with red, class two with blue)

1. Formulate the Lagrangian of the above optimisation problem and derive its dual.

2. Perform the above when using arbitrary positive definite kernels (i.e., $k(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$) , i.e. find the dual of the following optimisation problem

$$\begin{array}{ll} \min_{R,\mathbf{a},\xi_i} & R^2 + C\sum_{i=1}^n \xi_i \\ \text{subject to} & (\phi(\mathbf{x}_i) - \mathbf{a})^T(\phi(\mathbf{x}_i) - \mathbf{a}) \leq R^2 + \xi_i, \forall \xi_i \geq 0. \end{array} \tag{2}$$

3. Assume you are given the following code to simulate data. Code to produce data for class one (with red in Figure 1)

```
rng(1); % For reproducibility
r = sqrt(rand(100,1)); % Radius
t = 2*pi*rand(100,1);  % Angle
data1 = [r.*cos(t), r.*sin(t)]; % Points
```

Code to produce data for class two (with blue in Figure 1)

```
r2 = sqrt(3*rand(100,1)+1); % Radius
t2 = 2*pi*rand(100,1);      % Angle
data2 = [r2.*cos(t2), r2.*sin(t2)]; % points
```

You can plot the data using the following code.

```
figure;
plot(data1(:,1),data1(:,2),'r.','MarkerSize',15)
hold on
plot(data2(:,1),data2(:,2),'b.','MarkerSize',15)
ezpolar(@(x)1);ezpolar(@(x)2);
axis equal
hold off
```

Produce data from the two classes and find their optimal enclosing hyper-sphere by solving the dual optimisation problem of (2) in Matlab (to solve the quadratic constrained optimisation problem you may use `quadprog()`).