

$$\hat{y}_i = ax_i + b$$

(a : coef, b : intercept , N = data[↑])

$$\begin{aligned} \text{RSS} &= \sum_i^N (y_i - \hat{y}_i)^2 \\ &= \sum_i^N (y_i - ax_i - b)^2 \end{aligned}$$

x	...	y	\hat{y}
x_1	...	y_1	\hat{y}_1
x_2	...	y_2	\hat{y}_2
:		:	:
x_i	...	y_i	\hat{y}_i
:	...	:	:
x_N	...	y_N	\hat{y}_N

$$RSS = \sum_i^N (y_i - ax_i - b)^2$$

$$\frac{\partial RSS}{\partial a} = 0 \quad and \quad \frac{\partial RSS}{\partial b} = 0$$

위의 두 식을 만족 시키는 a,b 를 구하면 RSS가 최소!

미분의 주요 공식 복습!

1. $F(x) = f_1(x) + f_2(x) + \dots + f_n(x) = \sum f_i(x)$

$$\frac{dF(x)}{dx} = \frac{df_1(x)}{dx} + \frac{df_2(x)}{dx} + \dots + \frac{df_n(x)}{dx} = \sum \left[\frac{df_i(x)}{dx} \right]$$

2. $\frac{d(c*f(x))}{dx} = c * \frac{df(x)}{dx}$ (c는 상수)

3. $\frac{dx^n}{dx} = nx^{n-1}$,

4. 합성함수의 미분 $h=g(x)$, $f(g(x)) = f(h)$

$$\frac{df(g(x))}{dx} = \frac{df(h)}{dh} * \frac{dg(x)}{dx}$$

• 미분의 예제

- $f(x) = x^2$
- $g(x) = 3x+2$
- $f(g(x)) = (3x+2)^2 = 9x^2 + 12x + 4$
- $\frac{df(g(x))}{dx} = ?$

• 풀이

$$\begin{aligned} \frac{df(g(x))}{dx} &= \frac{d(9x^2+12x+4)}{dx} \\ &= 2*9x + 12 + 0 = 18x+12 \end{aligned}$$

• 합성함수의 미분을 이용한 풀이

- $h = g(x) = 3x+2$
- $\frac{df(h)}{dh} = \frac{d(h^2)}{dh} = 2*h = 2(3x+2)$
- $\frac{dg(x)}{dx} = \frac{d(3x+2)}{dx} = \frac{d(3x)}{dx} + \frac{d(2)}{dx} = 3+0 = 3$
- $\frac{df(g(x))}{dx} = \frac{df(h)}{dh} * \frac{dg(x)}{dx} = 2(3x+2)*3 = 18x+12$

$$\frac{\partial RSS}{\partial a} = \frac{\partial \sum_i^N (y_i - ax_i - b)^2}{\partial a} = 0$$

이제 $h = y_i - ax_i - b$ 이라고 하면 $\frac{\partial h}{\partial a} = -x_i$

$$\begin{aligned} \frac{\partial RSS}{\partial a} &= \sum_i^N \left(\frac{\partial h^2}{\partial h} * \frac{\partial h}{\partial a} \right) = \sum_i^N (2h * -x_i) \\ &= -2 \sum_i^N [(y_i - ax_i - b) x_i] \\ &= -2 \sum_i^N [x_i y_i - a(x_i^2) - bx_i] \\ &= -2 [\sum_i^N (x_i y_i) - a \sum_i^N (x_i^2) - b \sum_i^N x_i] = 0 \end{aligned}$$

$$\sum_i^N (x_i y_i) = a \sum_i^N (x_i^2) + b \sum_i^N x_i$$

$$\frac{\partial RSS}{\partial b} = \frac{\partial \sum_i^N (y_i - ax_i - b)^2}{\partial b} = 0$$

이제 $h = y_i - ax_i - b$ 이라고 하면 $\frac{\partial h}{\partial b} = -1$

$$\begin{aligned} \frac{\partial RSS}{\partial a} &= \sum_i^N \left(\frac{\partial h^2}{\partial h} * \frac{\partial h}{\partial b} \right) = \sum_i^N (2h * -1) \\ &= -2 \sum_i^N (y_i - ax_i - b) \\ &= -2 \left[\sum_i^N y_i - a \sum_i^N x_i - N * b \right] = 0 \end{aligned}$$

$$\sum_i^N y_i = a \sum_i^N x_i + N * b$$

양변을 N으로 나누면

$$\bar{y} = a \bar{x} + b \quad \left(\bar{x} = \frac{\sum_i^N x_i}{N}, \bar{y} = \frac{\sum_i^N y_i}{N} \right)$$

$$\text{정규방정식} \left\{ \begin{array}{l} \sum_i^N (x_i y_i) = \mathbf{a} \sum_i^N (x_i^2) + \mathbf{b} \sum_i^N x_i \\ \bar{y} = \mathbf{a} \bar{x} + \mathbf{b} \quad (\bar{x} = \frac{\sum_i^N x_i}{N}, \bar{y} = \frac{\sum_i^N y_i}{N}) \end{array} \right.$$

두 식중 아래식의 양변에 $\sum_i^N x_i$ 를 곱하여 두 식의 \mathbf{b} 에 대한 계수를 같게 만들면 ,

$$\begin{aligned} \sum_i^N (x_i y_i) &= \mathbf{a} \sum_i^N (x_i^2) + \mathbf{b} \sum_i^N x_i \\ \bar{y} \sum_i^N x_i &= \mathbf{a} \bar{x} \sum_i^N x_i + \mathbf{b} \sum_i^N x_i \end{aligned}$$

위 식에서 아래 식을 빼고, \mathbf{a} 에 대하여 정리하면,

$$\mathbf{a} = \frac{\sum_i^N (x_i y_i) - \bar{y} \sum_i^N x_i}{\sum_i^N (x_i^2) - \bar{x} \sum_i^N x_i}$$

$$\mathbf{a} = \frac{\sum_i^N (x_i y_i) - \bar{y} \sum_i^N x_i}{\sum_i^N (x_i^2) - \bar{x} \sum_i^N x_i}, \quad \mathbf{b} = \bar{y} - a\bar{x} \quad (\bar{x} = \frac{\sum_i^N x_i}{N}, \bar{y} = \frac{\sum_i^N y_i}{N})$$

마지막으로 a 의 분모와 분자를 다른 형태로 정리해보자!

$$\begin{aligned} \sum_i^N (x_i - \bar{x})(y_i - \bar{y}) &= \sum_i^N (x_i y_i - \bar{x} y_i - \bar{y} x_i + \bar{x} \bar{y}) \\ &= \sum_i^N (x_i y_i) - \bar{x} \sum_i^N y_i - \bar{y} \sum_i^N x_i + \bar{x} \bar{y} \\ &= \sum_i^N (x_i y_i) - \bar{x} \sum_i^N y_i - \bar{y} \sum_i^N x_i + N * \bar{x} \bar{y} \\ &= \sum_i^N (x_i y_i) - \bar{y} \sum_i^N x_i - \overline{N * \bar{x} \bar{y}} + N * \bar{x} \bar{y} \\ &= \sum_i^N (x_i y_i) - \bar{y} \sum_i^N x_i = \text{분자} \end{aligned}$$

$$\begin{aligned} \mathbf{a} &= \frac{\sum_i^N (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^N (x_i - \bar{x})^2} \\ &= \frac{S_{xy}}{S_{xx}} \end{aligned}$$

$$\begin{aligned} \sum_i^N (x_i - \bar{x})^2 &= \sum_i^N (x_i^2 - 2x_i \bar{x} + \bar{x} * \bar{x}) \\ &= \sum_i^N (x_i^2) - 2\bar{x} \sum_i^N x_i + N * \bar{x} * \bar{x} \\ &= \sum_i^N (x_i^2) - 2\bar{x} \sum_i^N x_i + \bar{x} * \sum_i^N x_i \\ &= \sum_i^N (x_i^2) - \bar{x} \sum_i^N x_i = \text{분모} \end{aligned}$$

$$\mathbf{a} = \frac{S_{xy}}{S_{xx}} \quad \bar{y} = a\bar{x} + b$$