$$\hat{y}_i = \mathrm{a}x_i + \mathrm{b}$$

(a : coef, b : intercept , N = data수)

$$\mathrm{RSS} = \sum_i^N (y_i - \hat{y}_i)^2$$
$$= \sum_i^N (y_i - \mathrm{a}x_i - \mathrm{b})^2$$

x	…	y	$\hat{y}$
x_1	…	y_1	$\hat{y}_1$
x_2	…	y_2	$\hat{y}_2$
:		:	:
$\boldsymbol{x_i}$	…	$\boldsymbol{y_i}$	$\boldsymbol{\hat{y}_i}$
:	…	:	:
x_N	…	y_N	$\hat{y}_N$

$$\mathrm{RSS} = \sum_i^N (y_i - \mathrm{a}x_i - \mathrm{b})^2$$

$$\frac{\partial RSS}{\partial a} = 0 \quad and \quad \frac{\partial RSS}{\partial b} = 0$$

위의 두 식을 만족 시키는 a,b 를 구하면 RSS가 최소!

미분의 주요 공식 복습!

1. F(x) = f_1 (x) + f_2(x) + … + f_n(x) = $\sum f_i(x)$

$$\frac{dF(x)}{dx} = \frac{df1(x)}{dx} + \frac{df2(x)}{dx} + \ldots + \frac{dfn(x)}{dx} = \sum\left[\frac{dfi(x)}{dx}\right]$$

2. $\frac{d\, c*f(x)}{dx}$ = c*$\frac{df(x)}{dx}$ (c는 상수)

3. $\frac{dx^n}{dx}$ = nx^{n-1} ,

4. 합성함수의 미분 h=g(x), f(g(x)) = f(h)

$$\frac{df(g(x))}{dx} = \frac{df(h)}{dh} * \frac{dg(x)}{dx}$$

- 미분의 예제
  - f(x) = x^2
  - g(x) = 3x+2
  - f(g(x)) = $(3x+2)^2$ = $9x^2 + 12x^1 + 4$
  - $\frac{\boldsymbol{df(g(x))}}{\boldsymbol{dx}}$ = ?
- 풀이
  - $\frac{\boldsymbol{df(g(x))}}{\boldsymbol{dx}}$ = $\frac{\boldsymbol{d(9x^2+12x^1+4)}}{\boldsymbol{dx}}$
    = 2*9x + 12 +0 = 18x+12
- 합성함수의 미분을 이용한 풀이
  - h = g(x) = 3x+2
  - $\frac{\boldsymbol{df(h)}}{\boldsymbol{dh}}$ = $\frac{\boldsymbol{d(h^2)}}{\boldsymbol{dh}}$ = 2*h = 2(3x+2)
  - $\frac{\boldsymbol{dg(x)}}{\boldsymbol{dx}}$ = $\frac{\boldsymbol{d(3x+2)}}{\boldsymbol{dx}}$ = $\frac{\boldsymbol{d(3x)}}{\boldsymbol{dx}}$ + $\frac{\boldsymbol{d(2)}}{\boldsymbol{dx}}$ = 3+0 = 0
  - $\frac{\boldsymbol{df(g(x))}}{\boldsymbol{dx}}$ = $\frac{\boldsymbol{df(h)}}{\boldsymbol{dh}} * \frac{\boldsymbol{dg(x)}}{\boldsymbol{dx}}$ = 2(3x+2)*3
    =18x+12

$$\frac{\partial RSS}{\partial a} = \frac{\partial \sum_i^N (y_i - abx_i - b)^2}{\partial a} = 0$$

이제 h = $y_i - ax_i - b$이라고 하면 $\frac{\partial h}{\partial a}$ = -x_i

$$\frac{\partial RSS}{\partial a} = \sum_i^N \left(\frac{\partial h^2}{\partial h} * \frac{\partial h}{\partial a}\right) = \sum_i^N (2h * -x_i)$$

$$= -2 \sum_i^N [(y_i - ax_i - b)\, x_i]$$

$$= -2 \sum_i^N [x_i\, y_i - a({x_i}^2) - bx_i]$$

$$= -2 \left[\sum_i^N (x_i y_i) - a \sum_i^N ({x_i}^2) - b \sum_i^N x_i\right] = 0$$

$$\boldsymbol{\sum_i^N (x_i y_i) =} a \boldsymbol{\sum_i^N ({x_i}^2)} + b \boldsymbol{\sum_i^N x_i}$$

$$\frac{\partial RSS}{\partial b} = \frac{\partial \sum_i^N (y_i - ax_i - b)^2}{\partial b} = 0$$

이제 h = $y_i - ax_i - b$이라고 하면 $\frac{\partial h}{\partial b}$ = -1

$$\frac{\partial RSS}{\partial a} = \sum_i^N \left(\frac{\partial h^2}{\partial h} * \frac{\partial h}{\partial b}\right) = \sum_i^N (2h \ * -1)$$

$$= -2 \sum_i^N (y_i - ax_i - b)$$

$$= -2 \left[\sum_i^N y_i - a \sum_i^N x_i - \mathrm{N} * b \right] = 0$$

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$$\sum_i^N y_i = a \sum_i^N x_i + \mathrm{N} * b$$

양변을 N으로 나누면

$$\boldsymbol{\bar{y} = a\,\bar{x} + b} \quad \left(\bar{x} = \frac{\sum_i^N x_i}{N},\ \bar{y} = \frac{\sum_i^N y_i}{N}\right)$$

정규방정식
$$\begin{cases} \sum_i^N (x_i y_i) = a\sum_i^N (x_i^2) + b\sum_i^N x_i \\ \bar{y} = a\bar{x} + b \quad (\bar{x} = \frac{\sum_i^N x_i}{N}, \ \bar{y} = \frac{\sum_i^N y_i}{N}) \end{cases}$$

두 식중 아래식의 양변에 $\sum_i^N x_i$ 를 곱하여 두 식의 $\mathbf{b}$ 에 대한 계수를 같게 만들면 ,

$$\sum_i^N (x_i y_i) = a\sum_i^N (x_i^2) + b\sum_i^N x_i$$
$$\bar{y}\sum_i^N x_i = a\bar{x}\sum_i^N x_i + b\sum_i^N x_i$$

위 식에서 아래 식을 빼고, $\mathbf{a}$ 에 대하여 정리하면,

$$\mathbf{a} = \frac{\sum_i^N (x_i y_i) - \bar{y}\sum_i^N x_i}{\sum_i^N (x_i^2) - \bar{x}\sum_i^N x_i}$$

$$\mathbf{a} = \frac{\sum_i^N (x_i y_i) - \bar{y}\sum_i^N x_i}{\sum_i^N (x_i{}^2) - \bar{x}\sum_i^N x_i} \quad , \; b = \bar{y} - a\bar{x} \;\; \left(\bar{x} = \frac{\sum_i^N x_i}{N} \, , \; \bar{y} = \frac{\sum_i^N y_i}{N}\right)$$

마지막으로 a 의 분모와 분자를 다른 형태로 정리해보자!

$$\begin{aligned}
\sum_i^N (x_i - \bar{x})(y_i - \bar{y}) &= \sum_i^N (x_i y_i - \bar{x} y_i - \bar{y} x_i + \bar{x}\,\bar{y}) \\
&= \sum_i^N (x_i y_i) - \bar{x} y_i - \bar{y} x_i + \bar{x}\,\bar{y}) \\
&= \sum_i^N (x_i y_i) - \bar{x}\sum_i^N y_i - \bar{y}\sum_i^N x_i + N * \bar{x}\,\bar{y} \\
&= \sum_i^N (x_i y_i) - \bar{y}\sum_i^N x_i - N * \bar{x}\,\bar{y} + N * \bar{x}\,\bar{y} \\
&= \sum_i^N (x_i y_i) - \bar{y}\sum_i^N x_i = \text{분자}
\end{aligned}$$

$$\begin{aligned}
\sum_i^N (x_i - \bar{x})^2 &= \sum_i^N (x_i{}^2 - 2x_i\bar{x} + \bar{x} * \bar{x}) \\
&= \sum_i^N (x_i{}^2) - 2\bar{x}\sum_i^N x_i + \mathrm{N} * \bar{x} * \bar{x} \\
&= \sum_i^N (x_i{}^2) - 2\bar{x}\sum_i^N x_i + \bar{x} * \sum_i^N x_i \\
&= \sum_i^N (x_i{}^2) - \bar{x}\sum_i^N x_i = \text{분모}
\end{aligned}$$

$$\mathbf{a} = \frac{\sum_i^N (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^N (x_i - \bar{x})^2} = \frac{s_{xy}}{s_{xx}}$$

$$\mathbf{a} = \frac{S_{xy}}{S_{xx}} \qquad \bar{y} = a\bar{x} + b$$