Oxford: Open-ended learning in symmetric zero-sum games

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Context

Peak deep learning

"If you have a large big dataset, and you train a very big neural network, then success is guaranteed!"
Ilya Sutskever (NIPS 2014)

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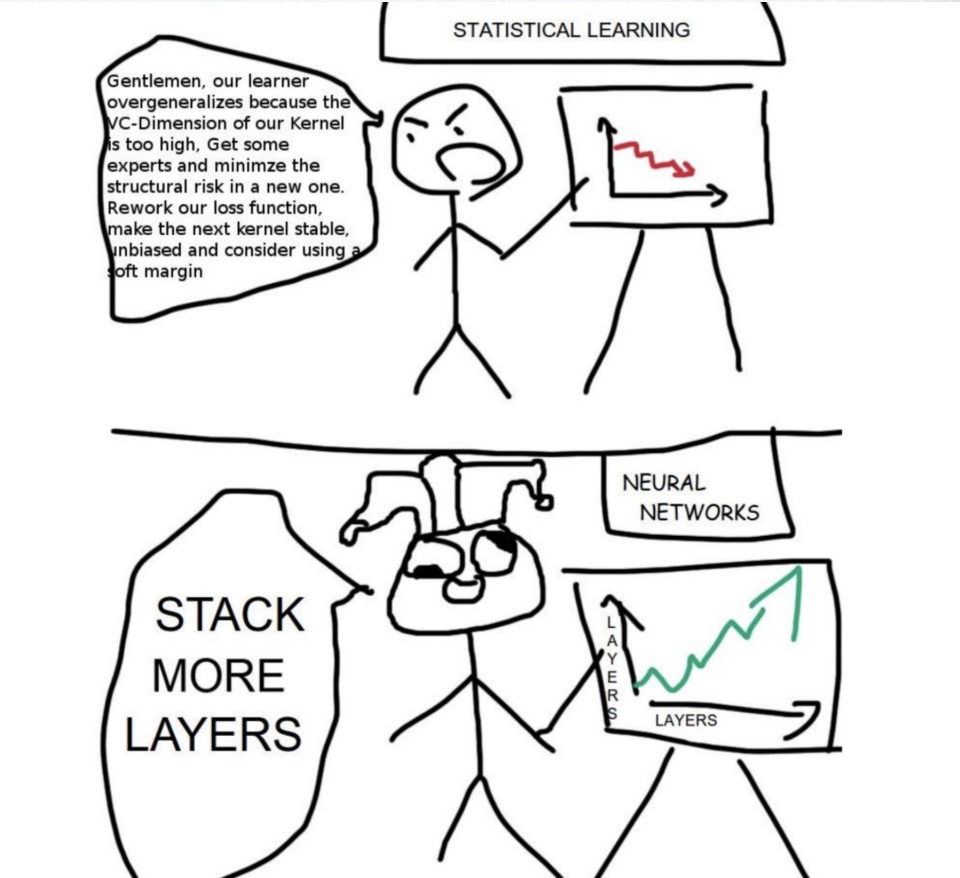

Peak deep learning

“If you have a large big dataset, and you train a very big neural network, then success is guaranteed!”
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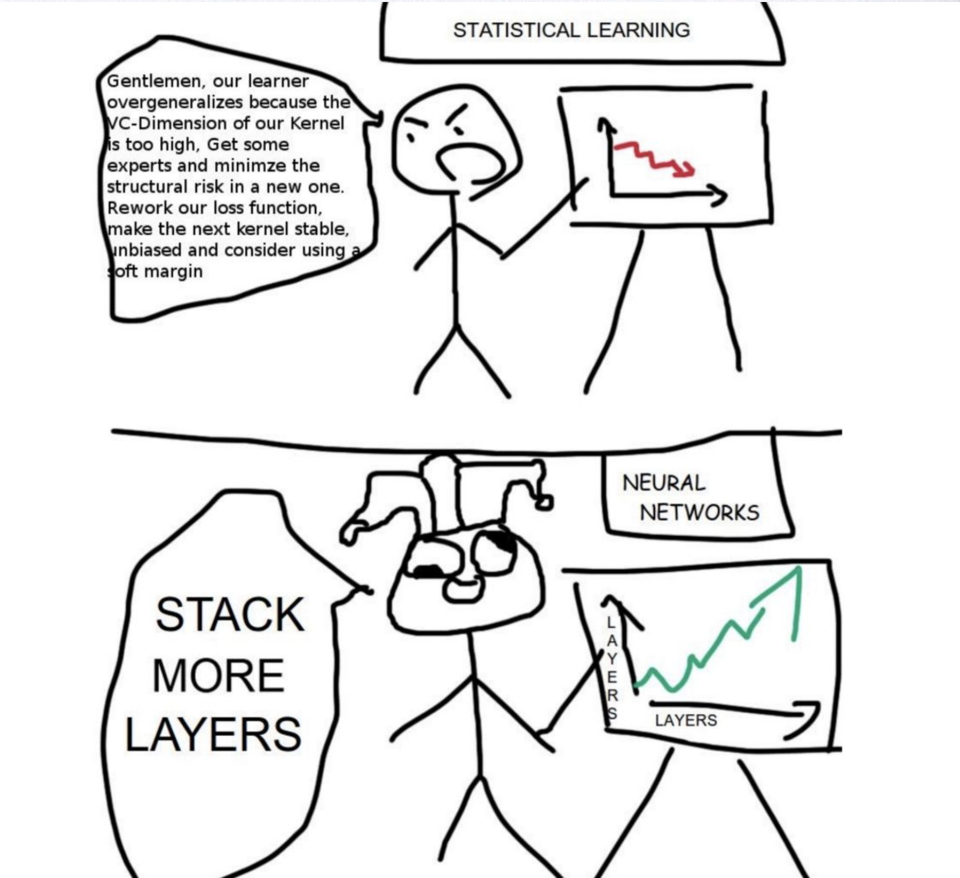
“If you formulate the right objective, and have enough capacity and compute, then success is guaranteed!”
Ilya’s law



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Long ago, not so far away (mid-1800s, Cambridge):

First tutor: **"I'm teaching the most brilliant boy in Britain"**
Second tutor: **"Well, I'm teaching the best test-taker"**

Depending on the version of the story, the first boy was either **Lord Kelvin** or **James Clerk Maxwell**. The second boy indeed *scored highest* on the Mathematical Tripos, but is otherwise long forgotten.

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Modern learning algorithms are outstanding test-takers

Intelligence is about more than taking tests
It's also about formulating useful problems

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The problem problem*

Where do ~~problems~~ objectives come from?

* Tim Lillicrap

Where do problems come from?

Answer #1:

Someone packages a dataset into a loss function

e.g. ImageNet, CIFAR, MNIST

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Answer #2:

Someone builds a task (that is, an environment sprinkled with rewards)

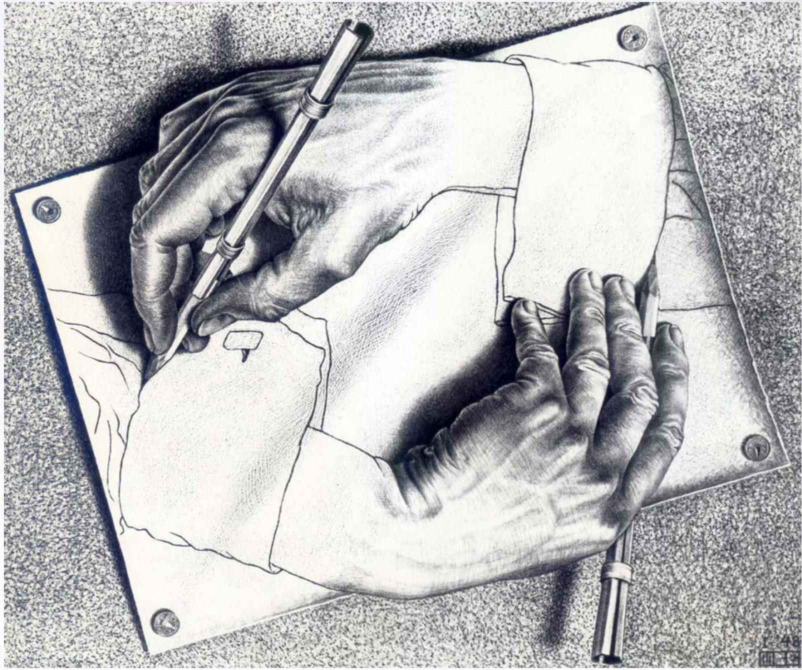
e.g. Arcade Learning Environment, DM-Lab, Open AI gym

Where do problems come from?

Answer #3:

Self-play in symmetric zero-sum games

The agent *is* the task -- create an outer loop that applies deep RL to itself



(Naive) self-play *is* an open-ended learning algorithm

It's pretty amazing

Algorithm 2 Self-play

input: agent $\mathbf{v}_1$
for $t = 1, \ldots, T$ **do**
  $\mathbf{v}_{t+1} \leftarrow \text{oracle}\left(\mathbf{v}_t, \phi_{\mathbf{v}_t}(\bullet)\right)$
end for
output: $\mathbf{v}_{T+1}$

(Naive) self-play *is* an open-ended learning algorithm

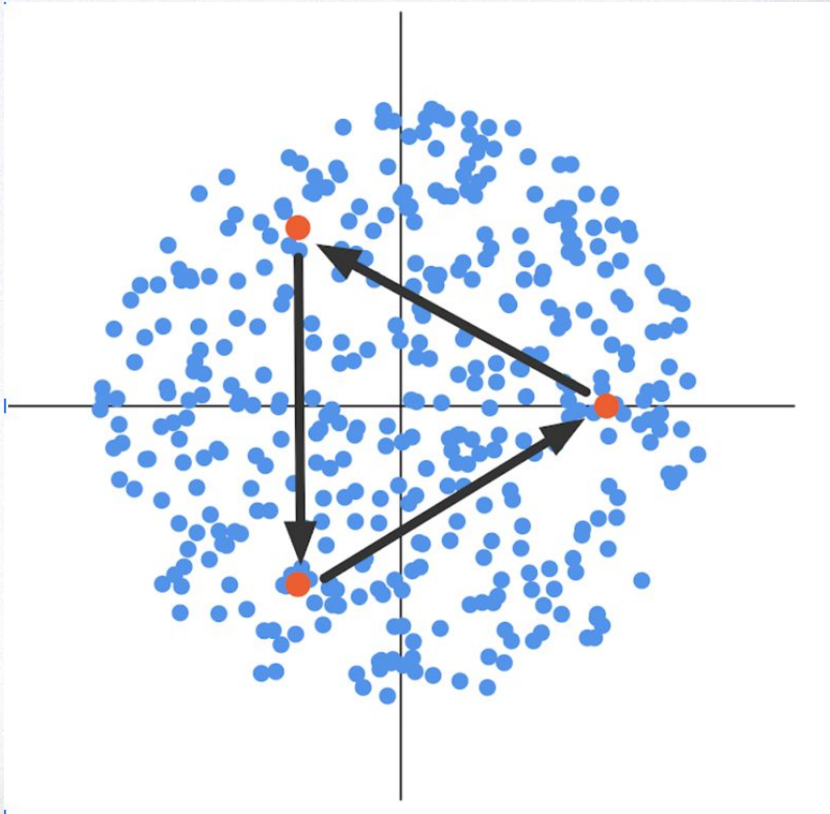
but ...

there are really simple examples

where it completely breaks down

It's **not** a general purpose learning

algorithm, *not even* for zero-sum games



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Open questions

1. **When does self-play work, and why?**
    - If self-play is the answer then what is the question?
    - "works for some games but not others" isn't good enough

2. **What is the goal in (general) zero-sum games?**
    - Yes, to win, but against whom? The opponent matters
        - "Beating a pro" isn't a formal specification
    - Is there a general algorithm?

What does success look like?

#1: Supervised learning (e.g. ImageNet)

#2: Reinforcement learning on fixed environment (e.g. DM-Lab)

#3: Self-play (e.g. on Chess, Go, Shogi, … ; but **not** on everything)

What underlies machine learning's big wins?

1. **A transitive performance measure**
    - e.g. a loss function or discounted rewards
2. **An incremental improvement operator**
    - e.g. gradient descent, RL, evolutionary algorithms

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What drives **self-play**'s success?

1. **Improvement (local):** $(A_{t+1} > A_t)$ and $(A_t > A_{t-1})$ and ...
2. **Transitivity (global):** A_{final} beats all previous agents

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Basics

On the varieties of zero-sum games

transitive: “relative skill determines who wins”

cyclic: “every strategy has a counter-strategy”

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Convex → Nonconvex → Transitive → Nontransitive

Convex objectives (ML in early 2000s): Converge to **unique global optimum**, for any initial condition and for any reasonable algorithm.

Nonconvex objectives (deep learning, ~2010...): Converge to a **local optimum** that depends on choice of initial condition and choice of algorithm.

Transitive games (e.g. self-play in Go, 2016...): Optimize and generate a sequence of objectives of the **same type** and **increasing difficulty**. Best agent at hardest objective is best overall.

Nontransitive games (e.g. AlphaStar league, 2019...): Optimum depends on who you compete with. Rather than finding a single best agent, should **invest in training a diverse "ecology" of agents.**

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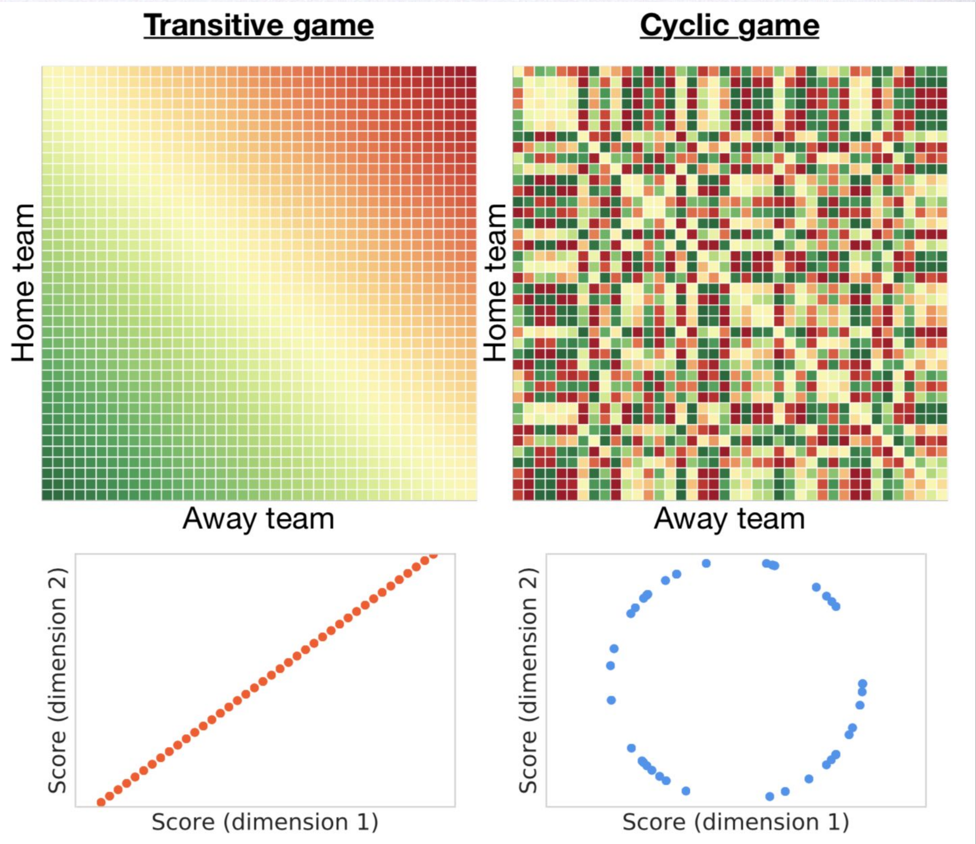
Functional-form game: a two-player symmetric zero-sum game, with differentiable parametrization:

$$\phi(\mathbf{v}, \mathbf{w})$$

Note: function approximator is “folded into” the game’s definition.

Theorem: Any symmetric zero-sum game decomposes into

[transitive] + **[cyclic]** components



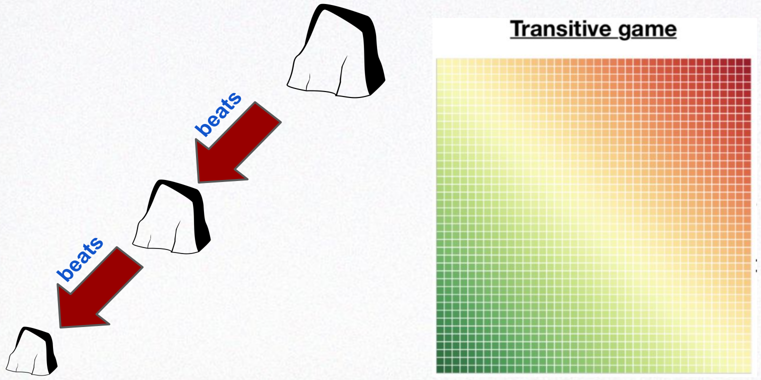
transitive: skill determines outcome

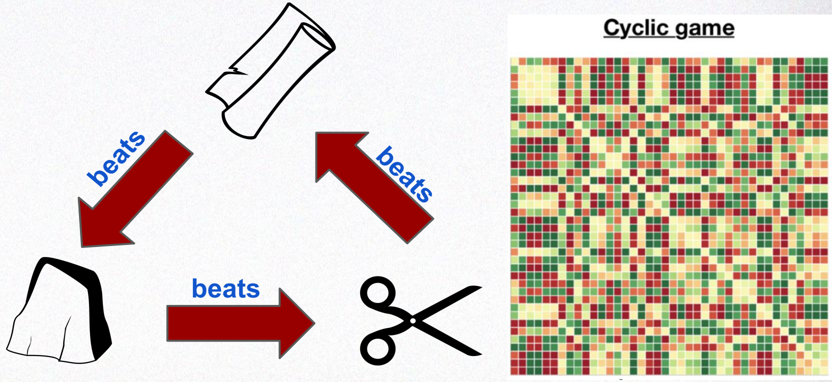
cyclic: every strategy has a counter-strategy

Transitive and cyclic games

	Good	Better	Best	Avg
Good	0	-1	-2	-1.5
Better	1	0	-1	0.0
Best	2	1	0	1.5

	Rock	Paper	Scissors	Avg
Rock	0	-1	1	0.0
Paper	1	0	-1	0.0
Scissors	-1	1	0	0.0





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Transitive games: where the opponent doesn't matter

$$\phi(\mathbf{v}, \mathbf{w}) = f(\mathbf{v}) - f(\mathbf{w})$$

- f assigns **rating** to players
- outcome is difference in ratings

→ algorithm: **optimize against a fixed opponent**

$\text{argmax}_{\mathbf{v}}\, \phi(\mathbf{v}, \mathbf{w})$ for any, fixed, $\mathbf{w}$

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Monotonic games: what self-play is "meant for"

$$\phi(\mathbf{v}, \mathbf{w}) = \sigma(\, f(\mathbf{v}) - f(\mathbf{w})\,)$$

- σ is a monotonic function (e.g. **Elo's rating model**)
- no learning signal if you train against a weak opponent (gradients vanish)
    - optimizing against a fixed opponent doesn't work

$\rightarrow$ algorithm: **self-play**

Algorithm 2 Self-play

input: agent $\mathbf{v}_1$
for $t = 1, \ldots, T$ **do**
  $\mathbf{v}_{t+1} \leftarrow \text{oracle}\left(\mathbf{v}_t, \phi_{\mathbf{v}_t}(\bullet)\right)$
end for
output: $\mathbf{v}_{T+1}$

Cyclic games: where naive self-play breaks down

A game is cyclic if $\int_W \phi(\mathbf{v}, \mathbf{w}) \cdot d\mathbf{w} = 0$

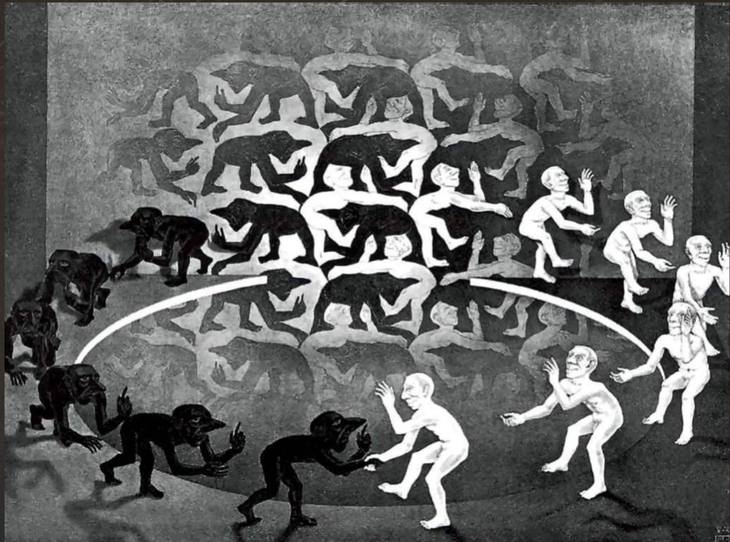
i.e **wins, against some opponents, are balanced by losses, against others.**

Implications:

- There is no best agent
- Measuring performance of individuals is nonsensical
  - which is better, paper or rock?

NEED: a **transitive objective**,

in **non-transitive games**!



How to convert **Agents** $\rightarrow$ **Objectives**

Definition: Gamescape is the convex hull of all objectives in a game

A game is a function that evaluates pairs of agents:

$$\Phi : W \times W \rightarrow R$$

$\Phi(\mathbf{v}, \mathbf{w})$ (e.g. probability that **v** beats **w**)

Fixing an opponent converts an **Agent** $\rightarrow$ **Objective function**

$$\mathbf{w} \rightarrow \Phi_{\mathbf{w}}(\ -\ ) := \Phi(\ -\ , \mathbf{w})$$

currying!

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Gamescapes

Functional Gamescape: convex hull of all objective functions $\Phi_w(\,\text{-}\,)$

- lives in function space $\mathcal{F}(W, R)$
- intractable

Empirical Gamescape: convex mixture of all rows of evaluation matrix

- proxy for functional gamescape
- tractable

Fitness landscapes are a special case that arise when the game is transitive or monotone.

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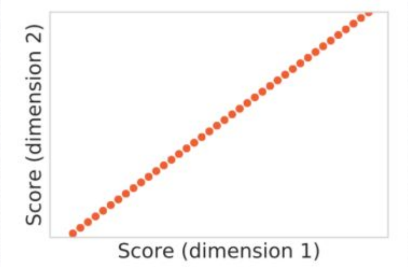
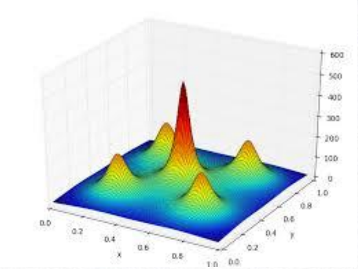
Landscape

Transitive: there's **one objective** ("improve skill") …

… so gamescape degenerates to **one dimensional fitness landscape**

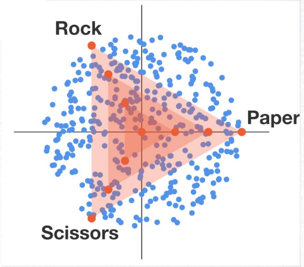




Gamescape

Nontransitive: different opponents are **different objectives** that pull in different directions

Gamescape is **multi-dimensional**



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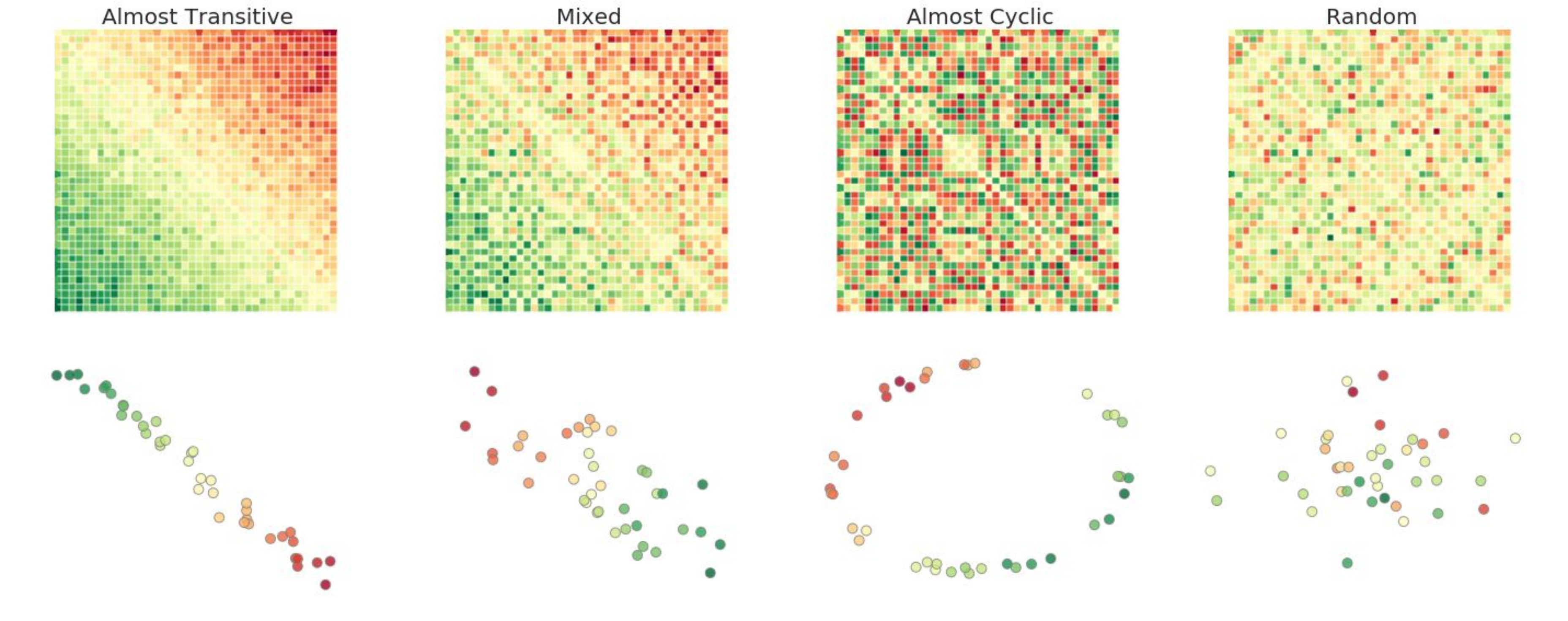
Gamescapes



mElo / Schur decomposition (roughly, in rank-2 case)

$$A_{n\times n} = W_{n\times 2} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot W^{T}_{2\times n}$$

{ agents } $\to$ { rows of **W** }

Entries of each row are the agent's **multi-dimensional Elo** scores

(let's forget about sigmoids)

Looooooong cycles

Is transitive geometry always 1-dim? **Yes**

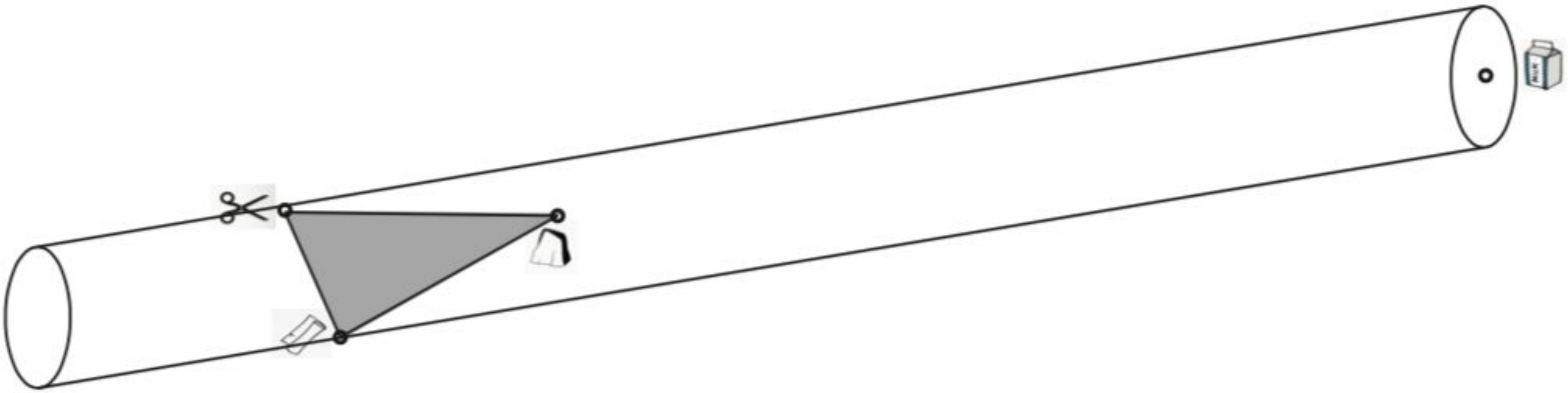
Is cyclic geometry always 2-dim? **No**

Rock → Paper → Scissors → Fire → Water → Air → Ether → Milk → … → Rock

length(cycle) = **{** 2n or 2n+1 **}** → **rank(A)** = **{** 2n-2 or 2n **}**, respectively.

What is the dimension of SC2's gamescape? No idea.

Ceci n’est pas une pipe



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So what's **the transitive objective** in nontransitive games?

Individual performance is **meaningless** in nontransitive games.

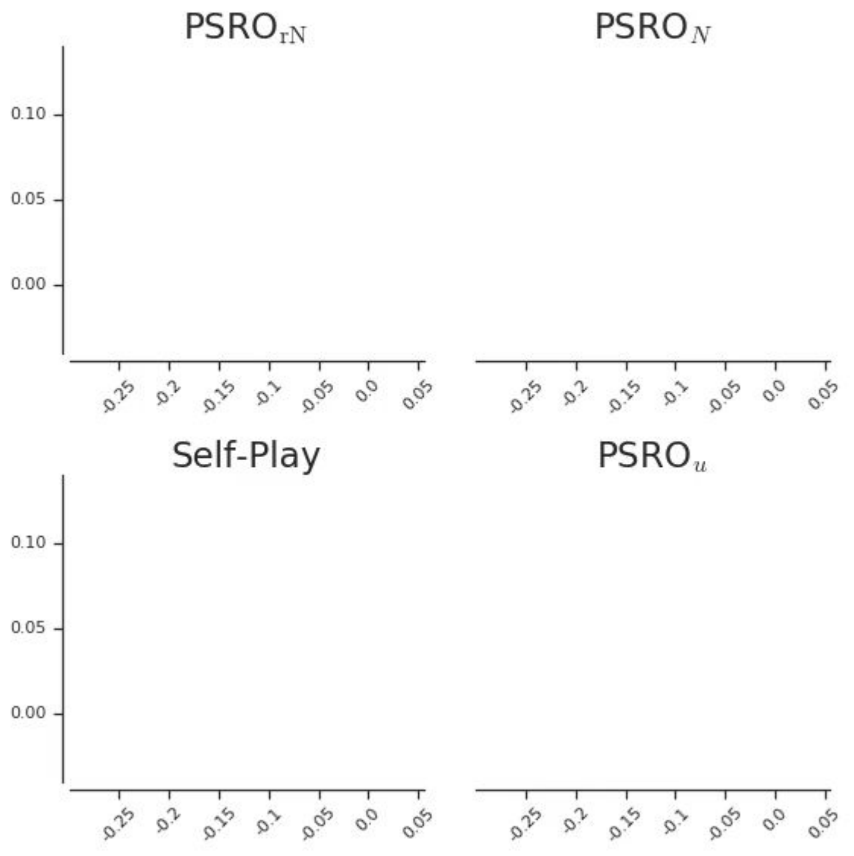
"Which is better, rock or paper?"

Idea: population-level objectives!

Grow the gamescape, to get "diverse, effective agents"

→ find strategic dimensions of game

→ and best ways of executing them



Population-level performance

Definition:

Given (m x n) evaluation matrix A for populations P and Q.

Let (p, q) be any Nash Equilibrium on zero-sum meta-game on A.

Relative performance of populations:

$$\mathbf{v(P, Q) := p^T.A.q}$$

(doesn't depend on choice of Nash).

Intuition:

- I pick my mixture of champions
- You pick yours (simultaneously)
- We play them out and
- See who wins, how much, in expectation

Population-level performance

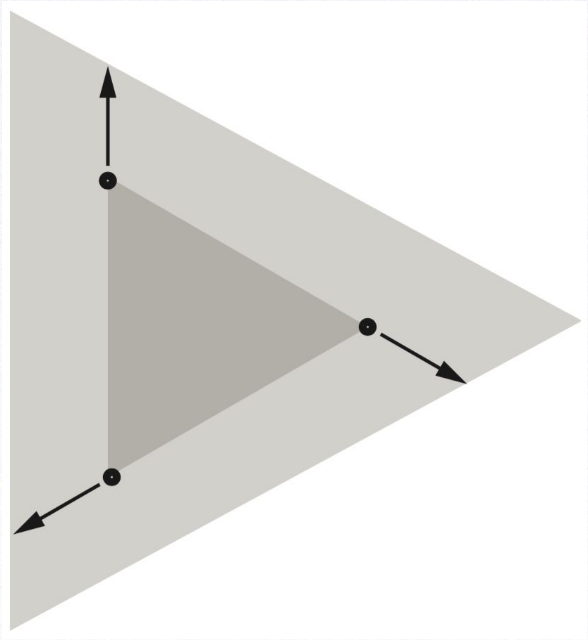
Lemma (transitivity for populations):

If agents in P are convex combinations of agents in Q then

$$v(P, Q) <= 0$$

Implication:

Algorithms that "grow polytope" are guaranteed to improve the population-level performance.



Two algorithms: one old and one new

Two ways to **track growth** ...

1. Population performance
2. Effective diversity

... and two algorithms

1. Response to Nash (double oracle)
   - train against Nash
2. Response to **rectified** Nash
   - train against agents in Nash that you beat
   - game-theoretic niches

Algorithm 3 Response to Nash (PSRO$_\mathrm{N}$)

input: population $\mathfrak{P}_1$ of agents
for $t = 1, \ldots, T$ **do**
  $\mathbf{p}_t \leftarrow$ Nash on $\mathbf{A}_{\mathfrak{P}_t}$
  $\mathbf{v}_{t+1} \leftarrow \text{oracle}\left(\mathbf{v}_t, \sum_{\mathbf{w}_i \in \mathfrak{P}_t} \mathbf{p}_t[i] \cdot \phi_{\mathbf{w}_i}(\bullet)\right)$
  $\mathfrak{P}_{t+1} \leftarrow \mathfrak{P}_t \cup \{\mathbf{v}_{t+1}\}$
end for
output: $\mathfrak{P}_{T+1}$

Algorithm 4 Response to rectified Nash (PSRO$_\mathrm{rN}$)

input: population $\mathfrak{P}_1$
for $t = 1, \ldots, T$ **do**
  $\mathbf{p}_t \leftarrow$ Nash on $\mathbf{A}_{\mathfrak{P}_t}$
  for agent $\mathbf{v}_t$ with positive mass in $\mathbf{p}_t$ **do**
    $\mathbf{v}_{t+1} \leftarrow \text{oracle}\left(\mathbf{v}_t, \sum_{\mathbf{w}_i \in \mathfrak{P}_t} \mathbf{p}_t[i] \cdot \lfloor \phi_{\mathbf{w}_i}(\bullet) \rfloor_+\right)$
  end for
  $\mathfrak{P}_{t+1} \leftarrow \mathfrak{P}_t \cup \{\mathbf{v}_{t+1} : \text{updated above}\}$
end for
output: $\mathfrak{P}_{T+1}$

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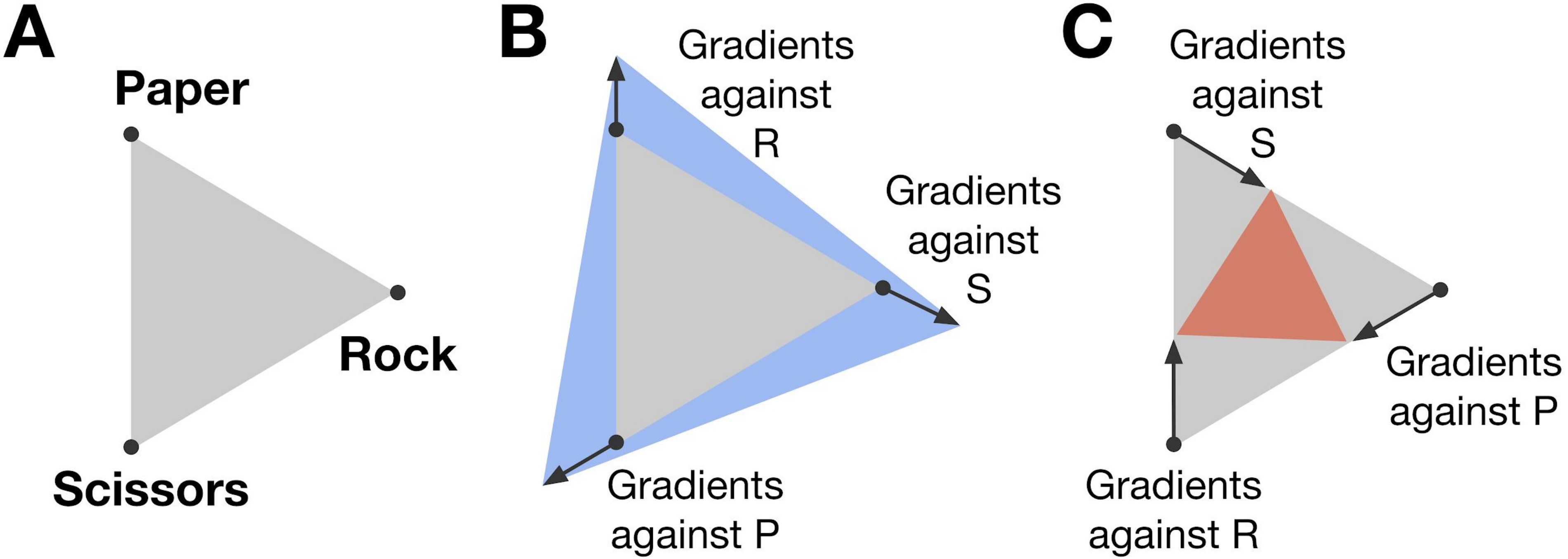
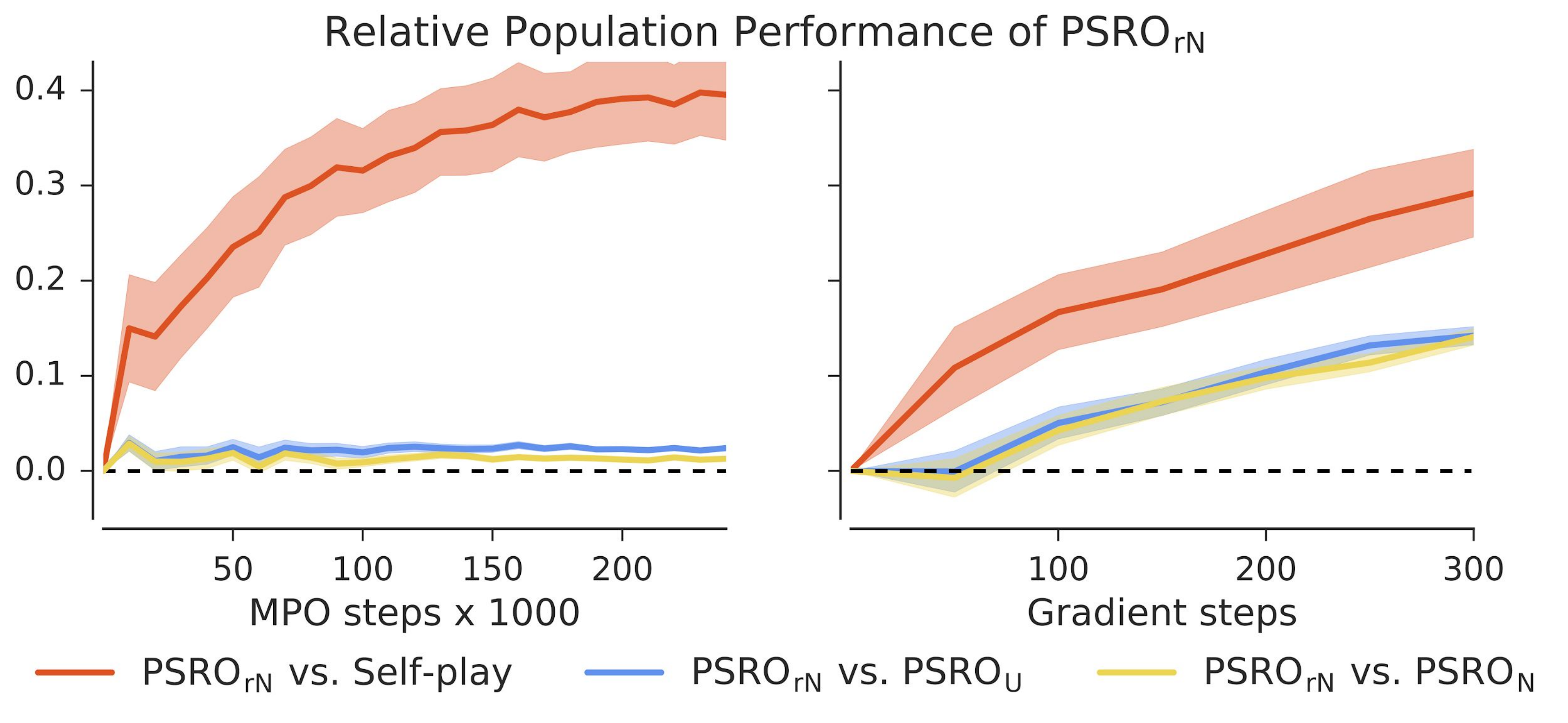
Response to Rectified Nash



Performance on Blotto and differentiable Lotto



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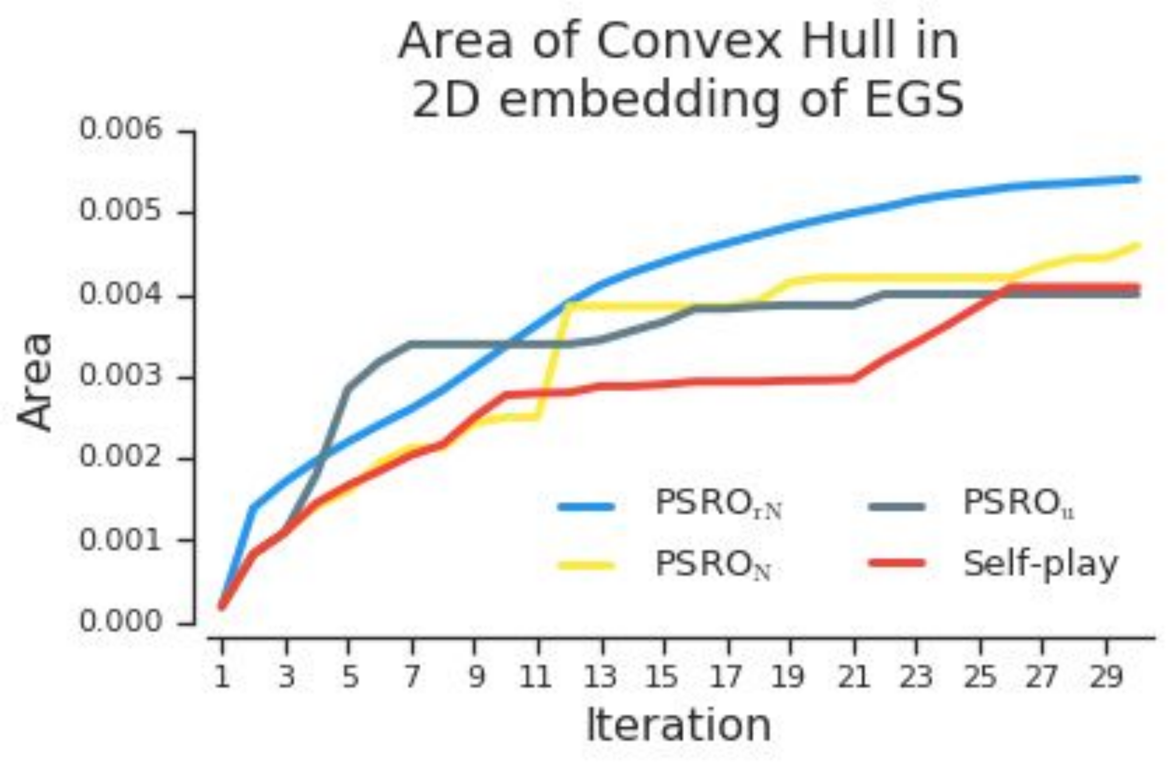
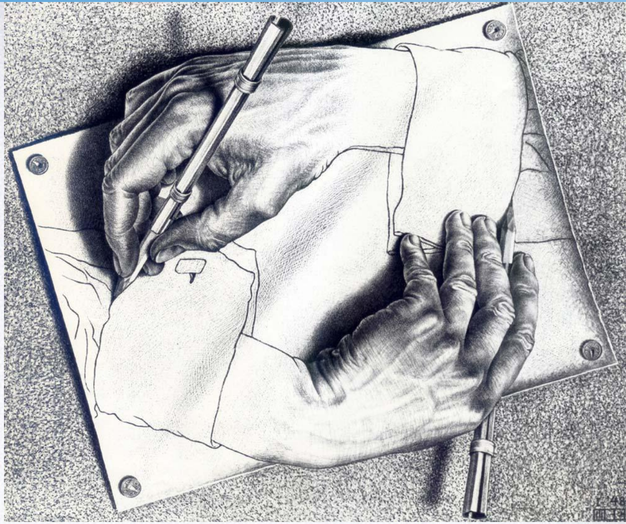
Growth of gamescapes in differentiable Lotto

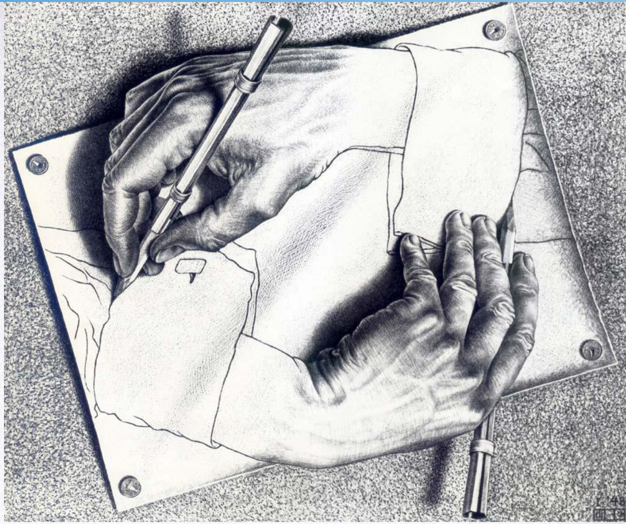


Discussion

Back to the problem problem



Back to the problem problem



1. **How can algorithms formulate problems?**
2. **Which problems are useful?**

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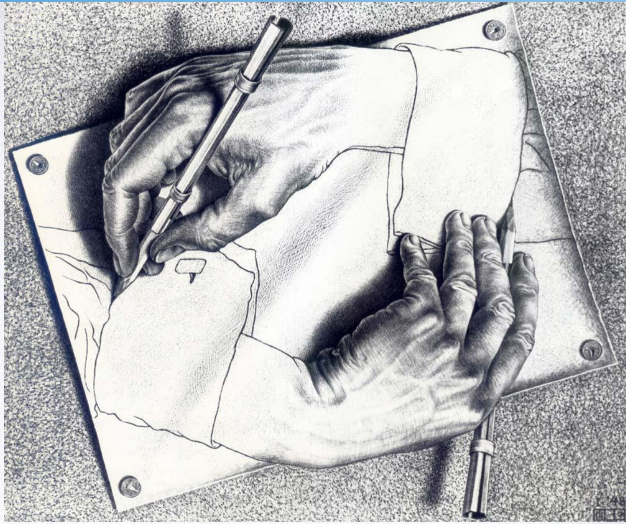
Back to the problem problem



1. **How can algorithms formulate problems?**
2. **Which problems are useful?**

Formulating problems is an **inter-agent** phenomenon

- in a zero-sum game, **"*my* behavior is *your* problem"**

Usefulness is **semantically grounded** in **winning** in ZSGs

- so **Nash formulates useful problems** via, say, response-to-Nash, response-to-rectified-Nash

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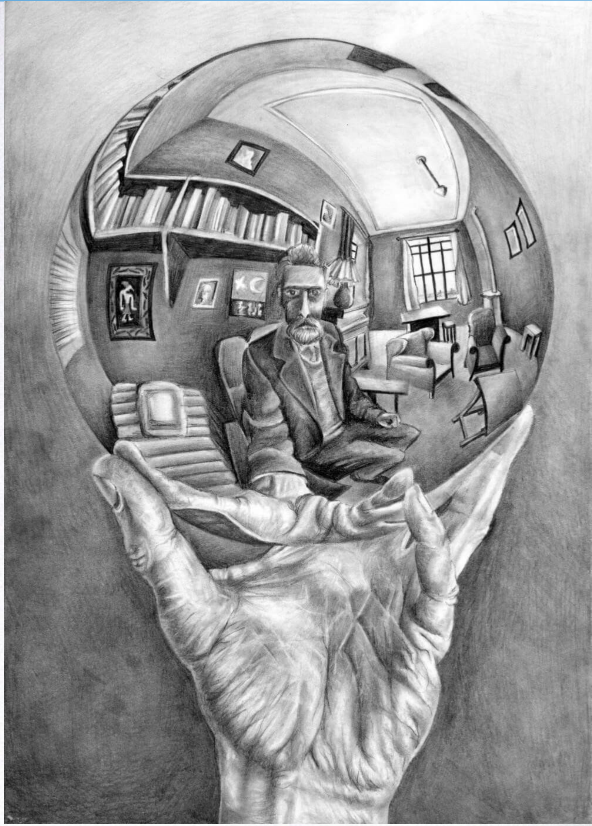
Comment on human problem posing

- The model describes posing-and-solving problems as a population-level activity

- This is how it works for humans
  - At least, population aspects are under-rated;
  - individuals get credit for work of populations

- Even humans like Einstein didn't pose **new** problems
  - "Everyone" knew Maxwell's equations aren't invariant under Galilean transforms
  - Einstein "just" took the problem seriously



Summary

Success of modern machine boils down to

- applying *many* **incremental improvements**
- to **transitive objectives**, so improvements **add up**

This talk:

- tools for **formulating objectives in a nontransitive setting**
- and **algorithms for optimizing them**

details: https://arxiv.org/abs/1901.08106