Notes of "Fundamental Theorem of Affine Transformations"

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1 Overview

2 The Vector Transformation Decided by an Affine Transformation

3 Fundamental Theorem of Affine Transformations

Theorem 1 (Fundamental Theorem of Affine Transformations)**.** *Let π be a plane.*

T1 *Suppose $f : \pi \to \pi$ is an affine transformation, $I = [O; \vec{e}_1, \vec{e}_2]$ is an affine coordinate system on π, then $I' = [f(O); f(\vec{e}_1), f(\vec{e}_2)]$ is also an affine coordinate system on π, and for all $P \in \pi$, the coordinates of P in I are the same as the one of $f(P)$ in I'.*

T2 *Let $I = [O; \vec{e}_1, \vec{e}_2]$ and $I' = [O'; \vec{e}'_1, \vec{e}'_2]$ be two affine coordinate systems on π. There exists an mapping $f : \pi \to \pi$ as following: for all $P \in \pi$ with the coordinates (x, y) in I, let $f(P)$ be the point with the same coordinates (x, y) in I', and $f : \pi \to \pi$ is an affine transformation.*

证明. (TODO) $\square$

Remark 1. *A conclusion from the fundamental theorem of affine transformations is that for any two affine coordinate systems I and I' on a plane π, there exists a unique affine transformation $f : \pi \to \pi$ such that $f(I) = I'$.*

T2 *proves the existence of $f : \pi \to \pi$.*

T1 *proves the uniqueness of $f : \pi \to \pi$. If there are two affine transformations $f_1 : \pi \to \pi$ and $f_2 : \pi \to \pi$ with $f_1(I) = I'$ and $f_2(I) = I'$, then for all $P \in \pi$, $f_1(P)$ and $f_2(P)$ have the same coordinates in I', which is the coordinates of P in I, and thus $f_1(P) = f_2(P)$. Hence $f_1 = f_2$.*