Notes of "Types of Quadric Curves"

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The problem we are going to deal with in this section is to classify quadric curves into different types based on their graphs. We already studied three types of quadric curves with standard equations before: ellipse, hyperbola, and parabola. Are there any other types of quadric curves with different graphs?

From the previous studying experience of the three types of quadric curves, it is clear that a standard equation, or a simple equation, reduces the difficulty of graphing an equation and thus classifying it into different types of quadric curves. Now with the tool of coordinate transformation, given a quadric equation in a coordinate system, we can always transform it into another coordinate system to make the equation simpler. This is our main method in this section.

Since the standard equations of the three quadric curves are in a right-handed Cartesian coordinate system, we are going to transform a general quadric equation into a right-handed Cartesian coordinate system. If the equation is in an affine coordinate system, the information of the system (such as the vectors in the basis) is needed to transform it into another coordinate system. To make our problem simpler, in this section we only deal with general quadric equations in a right-handed Cartesian coordinate system.

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2b_1x + 2b_2y + c = 0$$

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