Notes of "Affine Characteristics of Conic Sections"

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Affine characteristics of conic sections refer to their geometric characteristics that are not affected by metric, such as:

- Intersection between a lines and a curve
- Center of an ellipse or hyperbola
- Asymptotes of a hyperbola
- Open direction of a parabola

Our task in this section is to study the affine characteristics of conic sections using equations and their coefficients, to find the correspondance between an affine characteristic and an algebraic characteristic. As a result, we'll give algebraic definitions of these affine characteristics. The common ground on which we try to link an affine characteristic and an algebraic characteristic is the intersection between a line and a conic section.

Notice that the algebraic definitions of these affine characteristics are independent from the types of conic sections. In other words, for any type of conic sections we can determine these affine characteristics based on the algebraic definitions. However, the geometric interpretation of the affine characteristics must be discussed with the type of the conic section involved.

1 Intersection between a Line and a Quadratic Curve

Let $F(x,y) = (x,y,1)A[x,y,1]$ in which

$$A = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{12} & a_{22} & b_2 \\ b_1 & b_2 & c \end{pmatrix}$$

$$\begin{aligned} F_1(x,y) &= a_{11}x + a_{12}y + b_1 \\ F_2(x,y) &= a_{12}x + a_{22}y + b_2 \\ F_3(x,y) &= b_1x + b_2y + c \end{aligned}$$

$$\begin{aligned} F(x,y) &= (x,y,1)A[x,y,1] \\ &= (x,y,1)[F_1(x,y), F_2(x,y), F_3(x,y)] \\ &= xF_1(x,y) + yF_2(x,y) + F_3(x,y) \end{aligned}$$

2 Center

Definition 1.

3 Asymptotic Direction

4 Diameter and Conjugate

5 Line Tangent to Conic Sections

6 Open Direction of a Parabola