

# Notes of "Vector Space of Rows and Columns"

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## 1 Basic Definitions

**Definition 1** (Row (Vector) Space and Row Vectors). *Given  $n \in \mathbb{N}$ ,*

Properties of Vector Space

1. Associative property of addition
2. Commutative property of addition
3. Zero element
4. Inverse element of addition
5. Unit element in scalars
6.  $\alpha(\beta X) = (\alpha\beta)X$
7.  $(\alpha + \beta)X = \alpha X + \beta X$
8.  $\alpha(X + Y) = \alpha X + \alpha Y$

There are also column vector space and column vectors.

## 2 Linear Combination and Linear Span

**Definition 2** (Linear Combinations of Vectors).

Linear combinations have an interesting property:

**Proposition 1.** *A linear combination of linear combinations of vectors  $X_1, X_2, \dots, X_k \in \mathbb{R}^n$  is also a linear combination of vectors  $X_1, X_2, \dots, X_k \in \mathbb{R}^n$ .*

证明. Hint: Express the linear combination by vectors  $X_1, X_2, \dots, X_k \in \mathbb{R}^n$

□

Therefore, if we consider the set  $V$  consisting of all linear combinations of vectors  $X_1, X_2, \dots, X_k \in \mathbb{R}^n$ , it has the following property:

$$X, Y \in V \Rightarrow \alpha X + \beta Y \in V, \forall \alpha, \beta \in \mathbb{R}$$

**Definition 3** (Linear Span (Or Linear Hull)).

**Definition 4** (Linear Span of a Subset in  $\mathbb{R}^n$ ). *Given a subset  $S \subset \mathbb{R}^n$ , the linear span of  $S$ , denoted by  $\langle S \rangle$ , is the set of all linear combinations of any finite numbers of vectors in  $S$ .*

Some interesting properties of linear spans:

**Proposition 2.** *Suppose  $V$  is a linear span in  $\mathbb{R}^n$ , then*

- $\langle V \rangle = V$ .
- *If  $S \subset V$ , then  $\langle S \rangle \subset V$ .*

The second property leads to another definition of the linear span of a subset in  $\mathbb{R}$ :

**Definition 5** (Linear Span (Or Linear Hull)). *The linear span of a subset  $S \subset \mathbb{R}^n$  is the intersection of all linear spans in  $\mathbb{R}^n$  that contains  $S$ :*

$$\langle S \rangle = \bigcap_{S \subset V} V$$

证明. Hint:

- $\langle S \rangle \subset \bigcap_{S \subset V} V$
- $\bigcap_{S \subset V} V \subset \langle S \rangle$
- $\bigcap_{S \subset V} V$  is a linear span

□

### 3 Linear Dependence

**Definition 6** (Linear Independent & Linear Dependent).

**Remark 1.** *The order of the vectors doesn't affect linear independence because the addition operation in the vector space holds the commutative property.*

### 4 Base and Dimension