Notes of "Change of Basis and Quadric Surfaces"

Jinxin Wang

1 The General Theory of Change of Basis

1.1 Transition Matrix and Change of Basis Formula of Points and Vectors

Remark 1. 由于坐标变换前后的方程描述的是同一个几何对象，因此许多几何性质在坐标变换前后保持不变，比如直线在变换后还是直线。但某些性质也会改变，如由于坐标轴单位的不同，不同方向上的比例关系发生了改变。

1.2 Change of Basis Formula of a Graph

Example 1 (Change of Basis of a Plane)**.**

Example 2 (Change of Basis of a Line)**.** *Suppose the transition matrix from I to I' is* $C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$*. The standard equation of a line in I is $\frac{x-x_0}{u_x} = \frac{y-y_0}{u_y} = \frac{z-z_0}{u_z}$. To find the equation of the line in I', we have two methods:*

- *To utilize the method of change of basis of a plane, we transform the standard equation of the line to the general form:*

$$\begin{cases} \frac{x-x_0}{u_x} = \frac{y-y_0}{u_y} \\ \frac{y-y_0}{u_y} = \frac{z-z_0}{u_z} \end{cases}$$

1.3 Properties of Transition Matrix

1.4 Transition Matrix of Change of Basis Between Cartesian Systems

2 Types of Quadric Curve

3 Determine the Type and Invariants of a Quadric Curve