Notes of "Differential Calculus Used to Study Functions"

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1 L'Hopital's Rule

Proposition 1. *Suppose the function* $f : (a, b) \to \mathbb{R}$ *and* $g : (a, b) \to \mathbb{R}$ *are differentiable on the open interval* $(a, b)(-\infty \leq a < b \leq +\infty)$ *with* $g'(x) \neq 0$ *on* (a, b) *and*

$$\frac{f'(x)}{g'(x)} = A \text{ as } x \to a + 0 \ (-\infty \leq A \leq +\infty)$$

Then

$$\frac{f(x)}{g(x)} = A \text{ as } x \to a + 0$$

in each of the following two cases:

1^0 $(f(x) \to 0) \wedge (g(x) \to 0)$ *as* $x \to a + 0$

2^0 $g(x) \to \infty$ *as* $x \to a + 0$

A similar assertion holds as $x \to b - 0$

证明. □