

Notes of "Polynomials"

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1 一元多项式

Definition 1 (数域 P 上的一元多项式).

Definition 2 (一元多项式相等及零多项式).

Definition 3 (数域 P 上的一元多项式环).

2 整除的概念

Proposition 1 (带余除法的成立).

$$f(x) = q(x)g(x) + r(x) \tag{1}$$

$q(x)$ and $r(x)$ are uniquely determined. $r(x) = 0$ or $\deg(r(x)) < \deg(g(x))$.

Definition 4 (整除).

Properties of Divisors:

- 对偶等价于相差常数倍
- 传递性
- 整除组合

3 最大公因式

Definition 5 (Common Divisor and Greatest Common Divisor).

Lemma 1 (Common Divisors in Euclidean Division). $f(x)$ and $g(x)$, $g(x)$ and $r(x)$ have the same common divisors.

Theorem 1 (Theorem of Polynomial Greatest Common Divisor).

Example 1 (Euclid's Algorithm).