

# Notes of "Permutations"

Jinxin Wang

## 1 The Concept of Permutations

1. definition of permutations
2. notation of permutations
3. permutation group
- 3.1 the set of permutations of  $n$  elements
- 3.2 the operations on permutations
- 3.3 check it is a group
- 3.4 the cardinal of the permutation group

## 2 The Cyclic Structure of Permutations

1. Motivation: with operation, we try to turn a permutation into the result of some smaller permutations.
2. the concept of cycles and disjoint cycles
3. the definition of the power of a permutation
4. the order of a permutation
5. the equivalence relation determined by a permutation, and the orbits of a permutation
6. any permutation can be expressed as the product of disjoint cycles, and this expression is unique up to the order of the cycles.
7. the concept of a transposition
8. any permutation can be expressed as the product of transpositions.

## 3 The Parity of Permutations

1. the first definition of the parity of a permutation based on the number of transpositions.
2. proposition: the parity of a permutation is unique and independent from the expression of transpositions.
3. the concept of odd permutations and even permutations.
4. derive the parity of a permutation from its expression of the product of cycles.

## 4 Permutation Group's Action on Functions

1. the definition of a permutation's action on a function.
2. the definition of a skew-symmetric function
3. the proof of the uniqueness and independence of the parity of a permutation by the action of a permutation and the skew-symmetric function.