

Notes of "Vector Space of Rows and Columns"

Jinxin Wang

1 Basic Definitions

Definition 1 (Row (Vector) Space and Row Vectors). *Given $n \in \mathbb{N}$,*

Properties of Vector Space

1. Associative property of addition
2. Commutative property of addition
3. Zero element
4. Inverse element of addition
5. Unit element in scalars
6. $\alpha(\beta X) = (\alpha\beta)X$
7. $(\alpha + \beta)X = \alpha X + \beta X$
8. $\alpha(X + Y) = \alpha X + \alpha Y$

There are also column vector space and column vectors.

2 Linear Combination and Linear Span

Definition 2 (Linear Combinations of Vectors).

Linear combinations have an interesting property:

Proposition 1. *A linear combination of linear combinations of vectors $X_1, X_2, \dots, X_k \in \mathbb{R}^n$ is also a linear combination of vectors $X_1, X_2, \dots, X_k \in \mathbb{R}^n$.*

证明. Hint: Express the linear combination by vectors $X_1, X_2, \dots, X_k \in \mathbb{R}^n$

□

Therefore, if we consider the set V consisting of all linear combinations of vectors $X_1, X_2, \dots, X_k \in \mathbb{R}^n$, it has the following property:

$$X, Y \in V \Rightarrow \alpha X + \beta Y \in V, \forall \alpha, \beta \in \mathbb{R}$$

Definition 3 (Linear Span (Or Linear Hull)).

Definition 4 (Linear Span of a Subset in \mathbb{R}^n). *Given a subset $S \subset \mathbb{R}^n$, the linear span of S , denoted by $\langle S \rangle$, is the set of all linear combinations of any finite numbers of vectors in S .*

Some interesting properties of linear spans:

Proposition 2. *Suppose V is a linear span in \mathbb{R}^n , then*

- $\langle V \rangle = V$.
- *If $S \subset V$, then $\langle S \rangle \subset V$.*

The second property leads to another definition of the linear span of a subset in \mathbb{R} :

Definition 5 (Linear Span (Or Linear Hull)). *The linear span of a subset $S \subset \mathbb{R}^n$ is the intersection of all linear spans in \mathbb{R}^n that contains S :*

$$\langle S \rangle = \bigcap_{S \subset V} V$$

证明. Hint:

- $\langle S \rangle \subset \bigcap_{S \subset V} V$
- $\bigcap_{S \subset V} V \subset \langle S \rangle$
- $\bigcap_{S \subset V} V$ is a linear span

□

3 Linear Dependence

Definition 6 (Linear Independent & Linear Dependent).

Remark 1. *The order of the vectors doesn't affect linear independence because the addition operation in the vector space holds the commutative property.*

4 Base and Dimension