Notes of "Binary Operation"

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1 Overview

- Operations, semigroups and monoids
  - Def: An n-ary operation on a set
  - Examples of n-ary operations on a set
    - Examples of unary operations:
    - Examples of binary operations:
  - Def: A semigroup
  - Def: An identity element and a monoid
    - Rmk: The uniqueness of the identity element in a monoid
  - Examples of semigroups and monoids
    - Eg: The set of all transformations on a set
  - Def: A subsemigroup and A submonoid
    - Rmk: The equality between the identity element of a monoid and the one of its submonoid
  - Examples of subsemigroups and submonoids
    - Eg: The set of all even integers with the addition in $(\mathbb{Z}, +)$
    - Eg: The set of all inversible $M_n(\mathbb{R})$ with the multiplication of matrices in $(M_n(\mathbb{R}), \cdot)$
- Properties of associative operations
  - Prop: The result of an associative binary operation of multiple operands is independent from the order of carrying out the operations
  - Rmk: Examples of binary operations on a set that is not associative
- Exponentions and multiples
  - Def: Exponentions with non-negative integer exponents
  - Rmk: Properties of exponentions with non-negative integer exponents
  - Def: Multiples with non-negative integer multipliers
  - Rmk: Properties of multiples with non-negative integer multipliers
- Inversible elements

  - Def: An inversible element and its inverse element
    * Rmk: The prerequisite of discussing the inversibility of an element is having the identity element
    * Rmk: The uniqueness of the inverse element of an inversible element in a monoid
    * Rmk: The inversibility of the operation result of two inversible elements
  - Def: Negative exponentions and negative multiples
  - Prop: Properties of exponentions with integer exponents

2 Operations, semigroups and monoids

Remark 1 (The equality between the identity element of a monoid and the one of its submonoid)**.** *The identity element of a monoid doesn't necessarily equal to the one in a submonoid, if the submonoid doesn't contain the identity of the monoid.*

One example is that the monoid is $(M_2(\mathbb{R}), \cdot)$*, whose identity is* $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$*, and the submonoid is the set of real-valued matrices of order 2 with the form like* $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$*, whose identity is* $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

3 Properties of associative operations

4 Exponentions and multiples

5 Inversible elements