Notes of "Subspace"

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1 Overview

- The sum of two subspaces of a vector space
  - Def: The sum of two subspaces of a vector space
    - Rmk: The sum of two subspaces is still a subspace
  - Prop: The existence of a basis of a subspace and the relation between the dimension of a space and the dimension of its subspace
  - Def: 向量空间的基与子空间相合
    - Rmk: A method to find a basis of a vector space 与其一个子空间相合
  - Thm: The existence of a basis 与两个子空间相合
    - Eg: An example that the theorem does not hold for three or more subspaces
  - Cor: The formula of the basis of the sum of two subspaces
  - TNta: k-dimensional plane, codimension (余维数) of a subspace, hyperplane (超平面), flag variety (旗簇)
- The direct sum of two subspaces of a vector space
  - Def: The linear independence and dependence of a class of subspaces
  - Prop: A necessary and sufficient condition for two subspaces to be linear independent
    - Rmk: This proposition cannot be generalized to three or more subspaces
  - Thm: A necessary and sufficient condition for a class of subspaces to be linear independent in terms of intersection
  - Thm: A necessary and sufficient condition for a class of subspaces to be linear independent in terms of basis
  - Thm: A necessary and sufficient condition for a class of subspaces to be linear independent in terms of dimension
  - Def: The (internal) direct sum of a class of subspaces of a vector space
  - Prop: Two necessary and sufficient conditions of the sum of a class of subspaces to be a direct sum
  - Thm: The existence of the complemented subspace of a subspace

    * Rmk: Complemented subspaces of a subspace are not unique.
  - Def: The (external) direct sum of two vector spaces
  - Rmk: There is no difference in nature between internal direct sums and external direct sums

2 The sum of two subspaces of a vector space

3 The direct sum of two subspaces of a vector space

Proposition 1 (Two necessary and sufficient conditions of the sum of a class of subspaces to be a direct sum)**.** *Let V be a vector space and $W_1, W_2, \ldots, W_k$ be subspaces of V. The following statements are equivalent:*

P1 $W_1 + W_2 + \cdots + W_k = W_1 \oplus W_2 \oplus \cdots \oplus W_k$

P2 $W_1 + W_2 + \cdots + W_k$ 中的任意向量分解为求和的各子空间的向量之和的方式唯一。

P3 V 中零元分解为求和的各子空间的向量之和的方式唯一，即全为零向量。