Notes of "Linear Map between Vector Spaces"

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1 Overview

- Definition and related concepts

    - Def: A linear map from a space on K to the other on K

    - Examples of linear maps

        * Eg: A linear form of a vector space
        * Eg: Rotation on $\mathbb{R}^2$
        * Eg: The projection from $\mathbb{R}^3$ to its subspace $\{(a, b, 0) \mid a, b \in \mathbb{R}\}$
        * Eg: $C^k(\mathbb{R}) \to \mathbb{R}_{k+1}[x]$
        * Eg: $\mathcal{D} : K[x] \to K[x], f(x) \mapsto f'(x)$

    - Def: The kernel and image of a linear map

        * Rmk: The kernel and image of a linear map are subspaces of the domain and codomain of the map respectively

    - Thm: The relationship of the dimensions of the kernel, the image and the domain of a linear map

    - Cor: The dimension of the image of a linear map is no greater than the dimension of the domain of the map

- The matrix and the rank of a linear map

    - Rmk: Generalize some properties of cases of linear maps

    - Prop: 一个线性映射与它在一组基上的像一一对应

    - Prop: 一个线性映射关于 domain 和 codomain 的各一组基由一个矩阵唯一表示

    - Thm: 线性映射与矩阵的对应关系

- The algebraic structure of the set of linear maps from a space to the other

- The correspondence between linear maps and matrices in terms of coordinate transformation

2   Definition and related concepts

3   The matrix and the rank of a linear map

4   The algebraic structure of the set of linear maps from a space to the other

5   The correspondence between linear maps and matrices in terms of coordinate transformation