# Notes of "Subspace"

## Jinxin Wang

## 1 Overview

- The sum of two subspaces of a vector space

  - Def: The sum of two subspaces of a vector space

    * Rmk: The sum of two subspaces is still a subspace

  - Prop: The existence of a basis of a subspace and the relation between the dimension of a space and the dimension of its subspace

  - Def: 向量空间的基与子空间相合

    * Rmk: A method to find a basis of a vector space 与其一个子空间相合

  - Thm: The existence of a basis 与两个子空间相合

    * Eg: An example that the theorem does not hold for three or more subspaces

  - Cor: The formula of the basis of the sum of two subspaces

  - TNta: k-dimensional plane, codimension (余维数) of a subspace, hyperplane (超平面), flag variety (旗簇)

- The direct sum of two subspaces of a vector space

  - Def: The linear independence and dependence of a class of subspaces

  - Prop: A necessary and sufficient condition for two subspaces to be linear independent

    * Rmk: This proposition cannot be generalized to three or more subspaces

  - Thm: A necessary and sufficient condition for a class of subspaces to be linear independent in terms of intersection

  - Thm: A necessary and sufficient condition for a class of subspaces to be linear independent in terms of basis

  - Thm: A necessary and sufficient condition for a class of subspaces to be linear independent in terms of dimension

  - Def: The (internal) direct sum of a class of subspaces of a vector space

  - Thm: The existence of the complemented subspace of a subspace

    * Rmk: Complemented subspaces of a subspace are not unique.

  - Def: The (external) direct sum of two vector spaces

  - Rmk: There is no difference in nature between internal direct sums and external direct sums

## 2 The sum of two subspaces of a vector space

## 3 The direct sum of two subspaces of a vector space