Notes of "Linear Relation between Vectors"

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1 Overview

- Linear combination and linear span
  - Def: A linear combination of a set of vectors
  - Def: The linear span of a set of vectors
    - Rmk: The linear span of a set of vectors is a subspace of the space to which the set of vectors belongs
    - Rmk: The linear span of a set of vectors is the minimal subspace containing the set of vectors
- Linear dependence and independence
  - Def: A set of vectors is linear dependent (independent)
  - Examples of linear dependent and independent vectors
    - Eg:
- Properties of linear independent and dependent vectors
  - Thm: A few sufficient or necessary and sufficient conditions for a set of vectors to be linear dependent
  - Thm: The linear dependence of m linear combinations out of n vectors
  - Thm: Some conclusions about the relation between linear combinations and linear dependence

2 Linear combination and linear span

Definition 1 (The linear span of a set of vectors)**.** *The linear span of a subset M of a vector space V is defined as*

$$\langle M\rangle = \left\{a_1x_1 + a_2x_2 + \cdots + a_kx_k \mid a_1, a_2, \ldots, a_k \in K, x_1, x_2, \ldots, x_k \in M, k \in \mathbb{N}^+\right\} \tag{1}$$

3 Linear dependence and independence

4 Properties of linear independent and dependent vectors

Theorem 1 (A few sufficient or necessary and sufficient conditions for a set of vectors to be linear dependent)**.** *We have the following conditions for a set of vectors to be linear dependent:*

T1 *A set of vectors containing the zero vector are linear dependent.*

T2 *A set of at least 2 vectors are linear dependent if and only if one of the vectors is a linear combination of the rest.*

T3 *If a subset is linear dependent, then the whole set is linear dependent.*

Theorem 2 (The linear dependence of m linear combinations out of n vectors)**.** *Let* $u_1, u_2, \ldots, u_m$ *are linear combinations of* $v_1, v_2, \ldots, v_n$*:*

T1 *If* $m > n$*, then* $u_1, u_2, \ldots, u_m$ *are linear dependent.*

T2 *If* $u_1, u_2, \ldots, u_m$ *are linear independent, then* $m \leq n$*.*

Theorem 3 (Some conclusions about the relation between linear combinations and linear dependence)**.** *We have the following conclusions about linear combinations of a set of vectors:*

T1 *If* $v_1, v_2, \ldots, v_n$ *are linear independent,* $v_1, v_2, \ldots, v_n, v$ *are linear dependent, then* v *is a linear combination of* $v_1, v_2, \ldots, v_n$*.*

T2 *If* $v_1, v_2, \ldots, v_n$ *are linear independent,* v *is not a linear combination of* $v_1, v_2, \ldots, v_n$*, then* $v_1, v_2, \ldots, v_n, v$ *are linear independent.*

T3 *If* u *is a linear combination of* $v_1, v_2, \ldots, v_n$*, and every* v_i *is a linear combination of* $w_1, w_2, \ldots, w_m$*, then* u *is a linear combination of* $w_1, w_2, \ldots, w_m$*.*