

Notes of "Linear Function"

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1 Overview

- Def: A linear form (or linear functional or covector) of a vector space
- Examples of linear forms of vector spaces
 - Eg: $f(x_1, x_2, \dots, x_n) = \sum_{i=1}^n a_i x_i$ is a linear map from K^n to K
 - Eg: $\int_a^b f(x)dx$ is a linear map from the space of all continuous functions on $[a, b]$ to \mathbb{R}
 - Eg: $Tr : M_n(\mathbb{R}) \rightarrow \mathbb{R}, M \mapsto Tr(M)$ is a linear map from $M_n(\mathbb{R})$ to \mathbb{R}
- Rmk: A necessary and sufficient condition to determine a linear form of a space
- Rmk: The set of linear forms from a vector space V to its field K is a subspace of K^V
- Def: The dual space of a vector space
- Rmk: A basis of the dual space of a finite-dimensional space
- Thm: The dimension of the dual space of a finite-dimensional space and the dual basis of a basis of the space
- Examples of dual space and dual basis
 - Eg: