Notes of "Definition and Properties of the Limit of a Sequence"

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1 Overview

2 Definition of the Limit of a Sequence

Definition 1 (The Limit of a Sequence)**.** *(ϵ-N)*

(neighborhood-N)

Remark 1. *The two definitions of the limit of a sequence is equivalent. The equivalence relation between them relies on the definition of a neighborhood.*

Definition 2 (Divergence)**.**

3 Properties of the Limit of a Sequence

3.1 General Properties

- 有限点无关性：A finite number of terms of a sequence doesn't affect the convergence of the sequence. (Proof: definition)
- 唯一性：The limit of a convergent sequence is unique. (Proof: contradiction + definition)
- 有界性：A convergent sequence is bounded. (Proof: definition)

3.2 Properties Involving Arithmetic Operations

Theorem 1 (极限的四则运算)**.**

3.3 Properties Involving Inequalities

Theorem 2 (保序性)**.**

Theorem 3 (夹逼性)**.**

4 Infinity

4.1 Definition of Infinity

Definition 3 (Infinity, Positive Infinity, Negative Infinity)**.**

Corollary 1 (Relation between Infinity and Infinitesimal)**.**

Definition 4 (Not an Infinity)**.**

4.2 Operations Involving Infinity