Lecture 8: Partial Derivatives

1 Multivariable Functions

Multivariable functions are functions whose values depend on more than one independent variables.
Examples of Multivariable functions:

- $f(x, y) = x^2 + y^2$
- $f(x, y) = \frac{1}{x+y}$, $x + y \neq 0$
- $f(x, y) = \frac{1}{\sqrt{y}}$, $y > 0$
- The temperature of a certain point on earth at a given time is a multivariable function of latitude, longitude, and height.

For simplicity, in this course we will mainly study multivariable functions with two or three variables, but the concepts apply to any multivariable functions with any number of variables.

2 Visualization of Multivariable Functions with Two Variables

2.1 Function Graph

For a single-variable function f, we plot the function graph as $(x, f(x))$, which is very straightforward.
For a multivariable function with two variables f, we can similarly plot the function graph as $(x, y, f(x, y))$, which is a surface in 3D space.

Example 1. *Plot the graph of the function* $f(x, y) = -y$.
The equation of the function graph is $z = -y$*, which is a plane.*

Example 2. *Plot the graph of the function* $f(x, y) = 1 - x^2 - y^2$.
The equation of the function graph is $z = 1 - x^2 - y^2$*, whose shape is not so easy to be identified.*
We can first take a look at the intersection between the function graph and the $x - z$ *plane, which means let* $y = 0$*. Hence, the equation of the intersection is* $z = 1 - x^2$*, which is a parabola.*

We can then take a look at the intersection between the function graph and the $y - z$ plane, which means let $x = 0$. Hence, the equation of the intersection is $z = 1 - y^2$, which is a parabola.
Finally, we can take a look at the intersection between the function graph and the $x - y$ plane, which means let $z = 0$. Hence the equation of the intersection is $x^2 + y^2 = 1$, which is a unit circle.
Based on the above information, we can guess the shape of the function graph is as follows:

From the above examples, we can see that the graph of multivariable functions with two variables are not only hard to plot, but also often hard to read.

2.2 Contour Plot

Contour Plot is another way to visualize multivariable functions with two variables. It basically shows all the points where $f(x, y) =$ some fixed constants, which are chosen at regular intervals. Examples include contour map, isotherm map, etc.
From geometric point of view, contour plot is equivalent to using horizontal planes, $z =$ some fixed constants, to slice the function graph and combining the intersection points.

3 Partial Derivatives