Lecture 12: Gradient

1 Gradient

According to the chain rule, suppose that there is a function $w = w(x, y, z)$, where $x = x(t)$, $y = y(t)$, and $z = z(t)$, then

$$\begin{aligned}\frac{dw}{dt} &= w_x\frac{dx}{dt} + w_y\frac{dy}{dt} + w_z\frac{dz}{dt} \\ &=< w_x, w_y, w_z > \cdot < \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} > \\ &= \nabla w \cdot \frac{d\vec{r}}{dt} \\ \nabla w &=< w_x, w_y, w_z >\end{aligned}$$

The vector $< w_x, w_y, w_z >$ is called gradient, denoted as ∇w.

2 Directional Derivatives