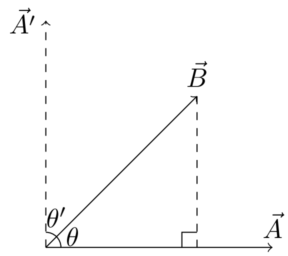
Lecture 2: Determinants, Cross Product

1 Determinants

With two vectors, how do we calculate the area of the triangle enclosed by these two vectors?



Apparently

$$Area = \frac{1}{2}|\vec{A}||\vec{B}|\sin\theta$$

But it is hard to calculate $\sin\theta$, whereas it is easy to calculate $\cos\theta$ with dot products. Therefore, we can rotate $\vec{A}'$ by $90°$ to get $\vec{A}'$ and hence

$$\sin\theta = \cos(90° - \theta) = \cos(\theta')$$

$$\begin{aligned} Area &= \frac{1}{2}|\vec{A}||\vec{B}|\sin\theta \\ &= \frac{1}{2}|\vec{A}'||\vec{B}|\cos\theta' \\ &= \frac{1}{2}\vec{A}' \cdot \vec{B} \end{aligned}$$

Suppose that $\vec{A} =< a_1, a_2 >$, and $\vec{B} =< b_1, b_2 >$, then $\vec{A}' =< -a_2, a_1 >$. Therefore,

$$\begin{aligned} \vec{A}' \cdot \vec{B} &=< -a_2, a_1 > \cdot < b_1, b_2 > \\ &= a_1 b_2 - a_2 b_1 \\ &= det(\vec{A}, \vec{B}) \\ &= \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \end{aligned}$$

Notice that it is also possible that $\vec{A}' = < a_2, -a_1 >$, depending on the relative positions of $\vec{A}$ and $\vec{B}$, so the result could be the negative value of the area.