

Lecture 3: Matrices

1 Matrices

A motivation for bringing in matrices is better expressing linear relations between variables.

Example 1. *The coordinates of a point in two different Cartesian coordinate systems have linear relations. Suppose in the first coordinate system, the point $P = (x_1, x_2, x_3)$, and in the second one, the point $P = (u_1, u_2, u_3)$. The relations between them can be described by linear equations, such as*

$$\begin{cases} u_1 = 2x_1 + x_2 + 5x_3 \\ u_2 = x_1 + 3x_2 + 2x_3 \\ u_3 = x_1 + 2x_2 + x_3 \end{cases}$$

The reason why their relation is linear is that each unit vector in the second coordinate system can be decomposed into three vectors along the directions of the unit vectors in the first coordinate system, i.e. each unit vector in the second coordinate system has a unique coordinate in the first system. Since x_1, x_2, x_3 are just scalars in three directions of the first coordinate system, the coordinates of the unit vectors of the second system can also be expressed with x_1, x_2, x_3 , a linear combination of these three scalars. Then the coordinate of the point in the second coordinate system is also a linear combination of the unit vectors in the system, so the coordinate of the point in the second coordinate system is a linear combination of the coordinate of the point in the first one.