Part I: True/ False

Question 1 of 20 10 Points

Suppose G is a connected, weighted, undirected graph with n vertices and m edges and the weight of each edge is 1. Assume G has at least 10 vertices. Suppose u and v are vertices in G. The fastest algorithm for computing the length of a shortest path from u to v is:

A. The BFS-based algorithm for computing shortest path lengths

B. Run DFS starting at u and count the number of vertices visited until v is reached

C. Dijkstra's algorithm using a priority queue.

D. The trivial algorithm works: Just output the number 1. (Runs in O(1) time.)

Reset Selection

Question 2 of 20 10 Points

Suppose a disconnected graph G has exactly two connected components, H and K. Let n be the number of vertices in G and m the number of edges in G. Which of the following is true?

A. G has at least $n - 1$ edges.

B. G has at most $\frac{(n-1)(n-2)}{2}$ edges

C. G has at least n^2 edges

D. If each of the components contains a cycle, G has at least $n + 1$ edges

Reset Selection

Question 3 of 20 10 Points

For any natural number $n > 1$, every graph having n vertices and $n + 1$ edges is connected.

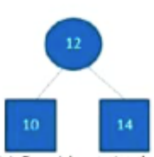
True

False

Reset Selection

Question 4 of 20 10 Points

A red-black tree T is given in the attached image. (Black nodes are represented by circles and red nodes by squares.) Using the red-black insertion procedure (discussed in class), to insert the value 15 in the tree, what is the first step that must be done? (It is expected that there will be other steps that follow this first step -- you only need to decide which step must come first.)



A. Do a right rotation that lifts 10 and lowers 12

B. Do a color flip that would change the color of all nodes to red

C. Create a right child of the node 14, color it red, and place the value 15 in the node

D. Do a color flip that would change the color of just the bottom two nodes, from black to red

Question 5 of 20 | 10 Points

Let k be a positive integer. Every heap having exactly k nonempty levels has at least 2^{k-1} nodes.

- ○ True
- ● False

Reset Selection

Question 6 of 20 | 10 Points

There is a graph G that has the following two properties:

(1) Every vertex of G has degree 2.

(2) G is not a simple cycle.

- ○ True
- ● False

Reset Selection

Question 7 of 20 | 10 Points

There exists a simple undirected graph having 10 vertices and 45 edges.

- ○ True
- ● False

Reset Selection

Question 8 of 20	10 Points

Assume that P ≠ NP. Let LIST be the following decision problem:

LIST: Given a list L with n elements and an object z, is z an element of L?

Which of the following is true?

- ○ A. HamiltonianCycle is polynomial reducible to TSP but TSP is not polynomial reducible to HamiltonianCycle
- ○ B. LIST is polynomial reducible to VertexCover but VertexCover is not polynomial reducible to LIST.
- ○ C. VertexCover is polynomial reducible to HamiltonianCycle but HamiltonianCycle is not polynomial reducible to VertexCover
- ○ D. All of the above
- ● E. None of the above

Question 9 of 20	10 Points

Suppose a heap H has 57 nodes. Which of the following statements is true?

- ○ A. The height of H must be exactly 5
- ● B. The height of H must be exactly 6
- ○ C. The height of H must be exactly 7
- ○ D. The height of H could be either 5, 6, or 7, depending on how H is structured
- ○ E. We do not have enough information to determine the height of H

Reset Selection

Question 10 of 20	10 Points

If BFS is run on any connected acyclic graph having n vertices, the running time will be O(n) in the worst case.

- ● True
- ○ False

Reset Selection

Question 11 of 20 | 10 Points

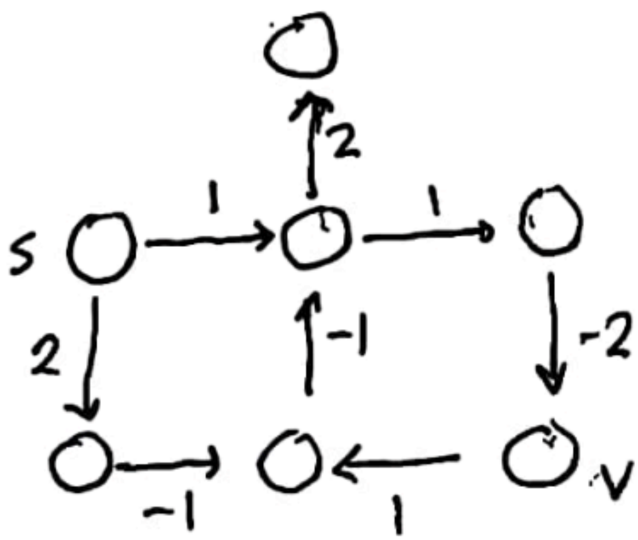
There is an O(1) algorithm that can detect whether any connected graph having n vertices contains a cycle. More precisely, there is an algorithm that accepts as inputs simple connected undirected graphs and that outputs "true" if the graph contains a cycle, "false" otherwise. (This algorithm's behavior is not guaranteed if disconnected graphs are given as input -- you do not need to consider this possibility.)

- (●) True
- ( ) False

Reset Selection

Question 12 of 20 | 10 Points

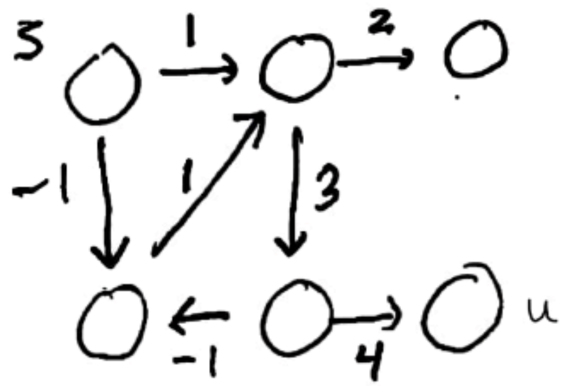
Which of the algorithms below will correctly compute the integer that specifies the length of a shortest path from s to v (see the attached image)? Select the best answer.



- ( ) A. Slow Dijkstra for directed graphs
- ( ) B. Bellman-Ford
- ( ) C. The dynamic programming algorithm for computing shortest paths
- ( ) D. Both A and B, but not C
- ( ) E. Both B and C, but not A
- ( ) F. A, B, and C
- (●) G. None of the above

Question 13 of 20	10 Points

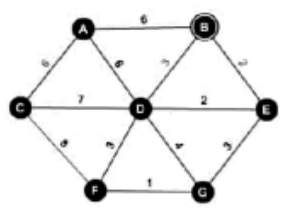
Which of the algorithms below will correctly compute the integer that specifies the length of a shortest path from s to u (see the attached image)? Select the best answer.



- ○ A. Slow Dijkstra for directed graphs
- ○ B. Bellman-Ford
- ○ C. The dynamic programming algorithm for computing shortest paths
- ○ D. Both A and B, but not C
- ○ E. Both B and C, but not A
- ● F. A, B, and C

Question 14 of 20	10 Points

A contractor needs to find a way to build a railway network that allows commuters to commute between cities – a typical example of such a network is shown in the image (in which the cost of laying track from one city to an adjacent city is indicated by edge weights), but the way tracks have been laid out in this example will be very costly. What graph algorithm would be most useful in determining the least costly way of laying out track in such a way that it will be possible for a commuter to travel from any city in the network to any other city in the network?



- ○ A. Dijkstra's algorithm for undirected graphs.
- ○ B. Dijkstra's algorithm for directed graphs.
- ○ C. A suitable subclass of the BFS class.
- ● D. Kruskal's algorithm

Question 15 of 20 | 10 Points

It is possible to find a graph G having all three of the following characteristics:
(1) G has more than 40 vertices
(2) G has a vertex cover of size 10
(3) When the VertexCoverApprox algorithm is run on the graph, it outputs a vertex cover of size 25

- (●) True
- ( ) False

Reset Selection

Question 16 of 20 | 10 Points

Suppose Kruskal's algorithm is applied to an undirected weighted graph G, building a set T of edges to be returned as output at the end. At each step of the algorithm, for each vertex v, it is always true that the edges from T whose endpoints lie in the cluster C(v) form a minimum spanning tree for the subgraph G[C(v)] induced by C(v).

- (●) True
- ( ) False

Reset Selection

Question 17 of 20 | 10 Points

Even though the AKS primality algorithm does not execute as efficiently as the Solovay-Strassen probabilistic algorithm, AKS is the algorithm that is used in the industry as part of mission-critical financial applications that depend on correct detection of primes. The reason for this preference is that the output of Solovay-Strassen (and other similar probabilistic algorithms to detect primes) has a slight chance of being incorrect (whereas AKS always has correct outputs), and even a small chance of error is not acceptable in mission-critical applications.

- (●) True
- ( ) False

Reset Selection

Question 18 of 20 | 10 Points

You are designing a software application that, during processing, will need to determine whether a large undirected graph (which is part of the input for the application) has a Hamiltonian cycle. It is known in advance that each vertex of graphs that are used as input has degree at least n/2, where n is the number of vertices of the graph. Although this is supposed to be true, the algorithm that will look for a Hamiltonian cycle will first need to verify that each vertex of the graph does in fact have degree at least n/2 (if not, an exception will be thrown). What is the running time of the most efficient possible algorithm to determine whether the input graph for this application has a Hamiltonian cycle? Be sure to factor in the running time for checking the degree of each vertex.

- ( ) A. $O(1)$
- ( ) B. $O(n)$
- ( ) C. $O(n^2)$
- (●) D. $O(2^n)$

Reset Selection

Question 19 of 20	10 Points

Consider the following variation of the SubsetSum decision problem:

> **SubsetSumLite**: Given a set S = $\{s_0, s_1, \ldots, s_{n-1}\}$ of positive integers. Is there a subset of S whose sum is <= 10?

There is an algorithm to solve SubsetSumLite that runs in O(n) time.

- (●) True
- ( ) False

Reset Selection

Question 20 of 20	10 Points

If someone one day discovers an $O(n^2)$ algorithm that solves the VertexCover decision problem, it will follow that P = NP.

- (●) True
- ( ) False

Reset Selection

Questions

1) illustrate kruskal algorithm

2) Show that problem R is NP complete, with assuming Hamiltonian cycle is NP complete problem

3) H is an heap, R is a red black tree, node counts of H and R is equal, draw H and R provides that height of R => h(R) >= 1.5 * h(H) <=height of H
True false and multiple choise questions were about NP and graphs
Array based heapsort illustration

4) G(V,E) is a graph. u, w, v are some nodes in G. Write the algorithm, can we reach from all u,w,v to another ones in u,w,v or we cannot reach.
Solution: both has just 1 node

5) Draw a the graph that has the features:

At least one edge is negative,
Dijkstra cannot be applicable,
Dynamic shortest path algorithms CANNOT be applicable too

There is a similar question in the past papers, it says dynamic shortest path algorithms CAN be applicable, difference is in this question he expects to draw CANNOT applicable shortest path algorithms