Solution to Q2 in W1D1 and W1D2

Let F(n) denote the nth Fibonacci number. Prove $F(n) > (4/3)^n$ for $n > 4$.

Proof.

Induction Basis:

Let $n = 5$. Then $F(5) = 5 > 1024/243 = (4/3)^5$.

Let $n = 6$. Then $F(6) = 8 > 4096/729 = (4/3)^6$

Induction Hypothesis:

Assume the result is true for all n in the in the interval [5, k]
In particular, the result is true for $n = k$ and $n = k - 1$.

$F(k) > (4/3)^k$ and $F(k - 1) > (4/3)^{(k-1)}$.

Induction Step: We need to prove the result for n = k+1.

Now $F(k + 1) = F(k) + F(k -1) > (4/3)^k + (4/3)^{(k-1)} = (4/3)^{(k-1)}[(4/3) + 1]$
$= (4/3)^{(k-1)}[(7/3)] = (4/3)^{(k-1)}[(21/9)] > (4/3)^{(k-1)}[(16/9)] =(4/3)^{(k-1)}[(4/3)^2] = (4/3)^{(k+1)}$

1, 10	$\Theta(1)$
log(log n)	$\Theta(\log(\log n))$
log n, ln n	$\Theta(\log n)$
$n^{1/k}$(k>3)	$\Theta(n^{1/k})$ (k>3)
$n^{1/3}$	$\Theta(n^{1/3})$
$n^{1/3}$ log n	$\Theta(n^{1/3} \log n)$
$n^{1/2}$	$\Theta(n^{1/2})$
$n^{1/2}$ log n	$\Theta(n^{1/2} \log n)$
log n^n, n log n	$\Theta(n \log n)$
n^2	$\Theta(n^2)$
n^3	$\Theta(n^3)$
n^k (k>3)	$\Theta(n^k)$ (k>3)
2^n	$\Theta(2^n)$
3^n	$\Theta(3^n)$
n!	$\Theta(n!)$
n^n	$\Theta(n^n)$