Homework 2

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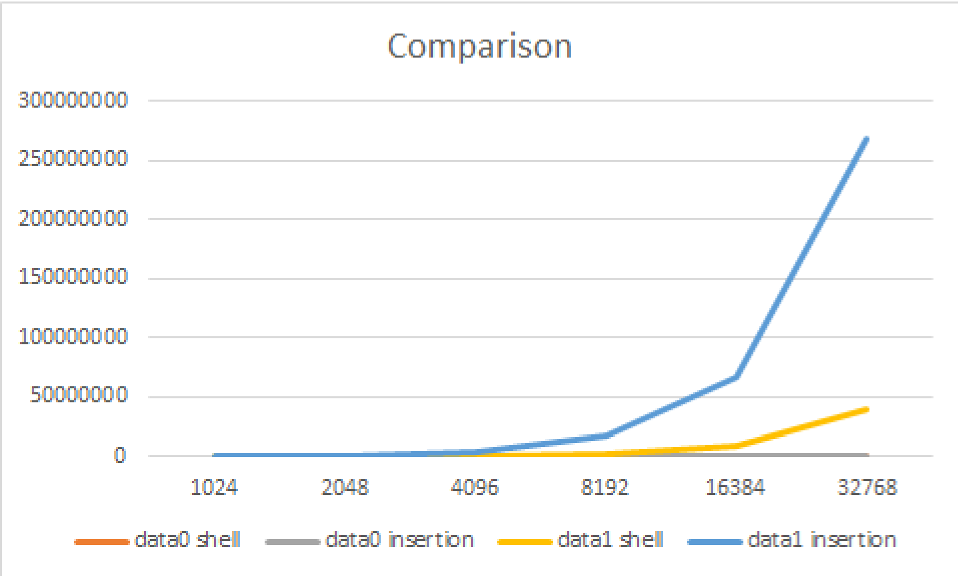
Q1

Result

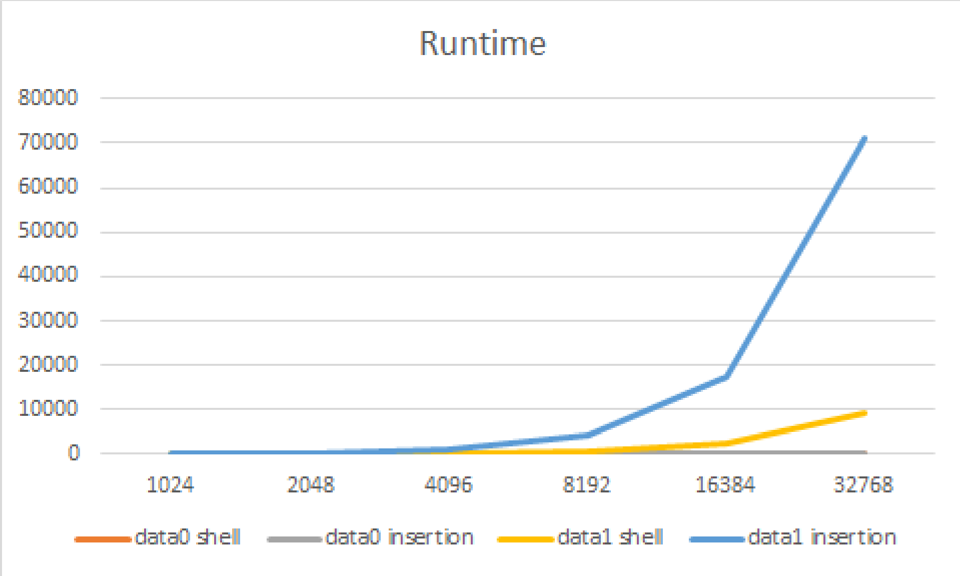
Comparisons:

Compare	Size	1024	2048	4096	8192	16384	32768
data0	shell	3061	6133	12277	24565	49141	98293
data0	insertion	1023	2047	4095	8191	16383	32767
data1	shell	46728	169042	660619	2576270	9950992	39442456
data1	insertion	265553	1029278	4187890	16936946	66657561	267966668



Runtime (ms) :

Time	Size	1024	2048	4096	8192	16384	32768
data0	shell	0.274	0.558	1.096	2.329	4.468	9.129
data0	insertion	0.094	0.196	0.414	0.808	1.525	3.276
data1	shell	10.429	39.563	154.881	609.344	2300.576	9007.49
data1	insertion	63.018	243.17	989.307	3888.903	15290.861	61916.212



Analysis

- Insertion sort performs better than shell sort when the sequence is in order. The reason is that insertion sort just goes over the sequence for only one time, while the shell sort needs to repeat comparison on the same element when the step **H** is changed.
- In average case, shell sort outplays insertion sort. The reason is shell sort can quickly reduce the numbers of inversions by doing **H-sort**, so there are relatively few inversions when **H** is 1, compared to much more inversions for insertion sort.

Q2

Implementation

I read the idea from https://en.wikipedia.org/wiki/Kendall_tau_distance.

First, I create a "revert mapping function" based on ranking a.

```
map[element]=index for index,element in enumerate(a)
```

Then, I perform this map on another ranking b, to get the "reverted index" of ranking b.

```
bIndex[index]=map[b[index]] for index in b
```

This **bIndex** is the rearranged index using the map on ranking a, which means we just need to calculate the inversions number of bIndex, and that will be the relative distance from b to a, also known as Kendall Tau distance.

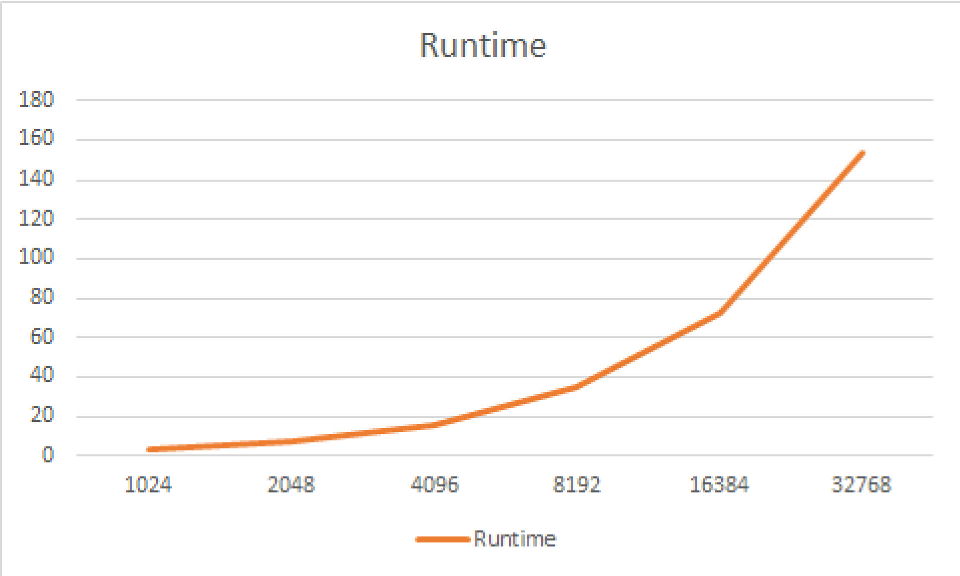
In order to count the inversion numbers in less than $O(N^2)$, the most frequently used method is merge sort. This is because we need a **stable** sorting algorithm(in case missing counting the inversion when doing long distance swap), and the insertion sort is $O(N^2)$, so merge sort is the only choice.

When doing **merge()** operation in merge sort, if a right element is smaller, count **mid-i+1** :

```
# merge()
for k in range(lo, hi+1):
    if i > mid:
        a[k: hi + 1] = aux[j: hi + 1]
        break
    elif j > hi:
        a[k:hi+1] = aux[i: mid+1]
        break
    elif aux[j] < aux[i]:               # right is smaller
        a[k], j = aux[j], j + 1
        num += mid-i+1                  #count elements from i to mid
    else:
        a[k], i = aux[i], i+1
```

Result

Size	1024	2048	4096	8192	16384	32768
Inversions	264541	1027236	4183804	16928767	66641183	267933908
Runtime/ms	3.461	7.731	16.009	35.064	72.925	153.886



Analysis

As the result shows this implementation has $O(NlogN)$ time complexity.

Q3

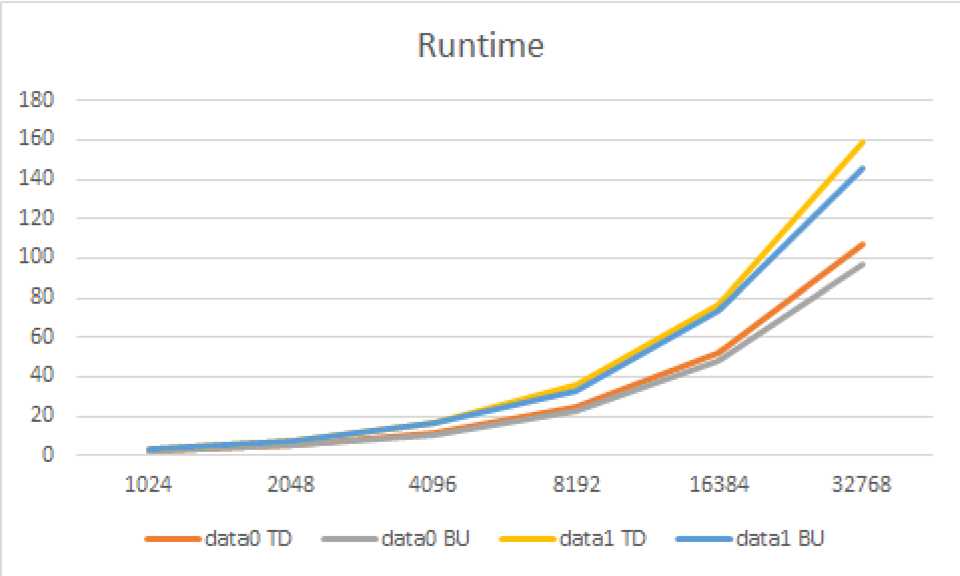
Result

Comparison:

Size	1024	2048	4096	8192	16384	32768
data0	5120	11264	24576	53248	114688	245760
data1	8954	19934	43944	96074	208695	450132

Runtime (ms):

Time	Size	1024	2048	4096	8192	16384	32768
data0	TD	2.72	5.73	11.59	24.69	51.62	107.15
data0	BU	2.47	5.33	10.73	22.73	47.54	97.02
data1	TD	3.56	7.88	16.68	36.26	76.51	158.88
data1	BU	3.39	7.29	16.49	33.18	73.36	146.07



Analysis

- The grow rate for comparisons is roughly $O(N)$, this property makes merge sort fast and stable.
- The time complexity is $O(NlogN)$.
- Bottom-up is slightly faster than Top-down, that is because TD version spent some time on recursion.

Q4

Analysis

I choose insertion sort.

Given that this dataset is already in order, the insertion sort would be the fastest one because it only has $N-1$ times data comparison, which means $2(N-1)$ data access and $N-1$ comparison, that is $O(3(N-1))$.

For other sorting algorithms:

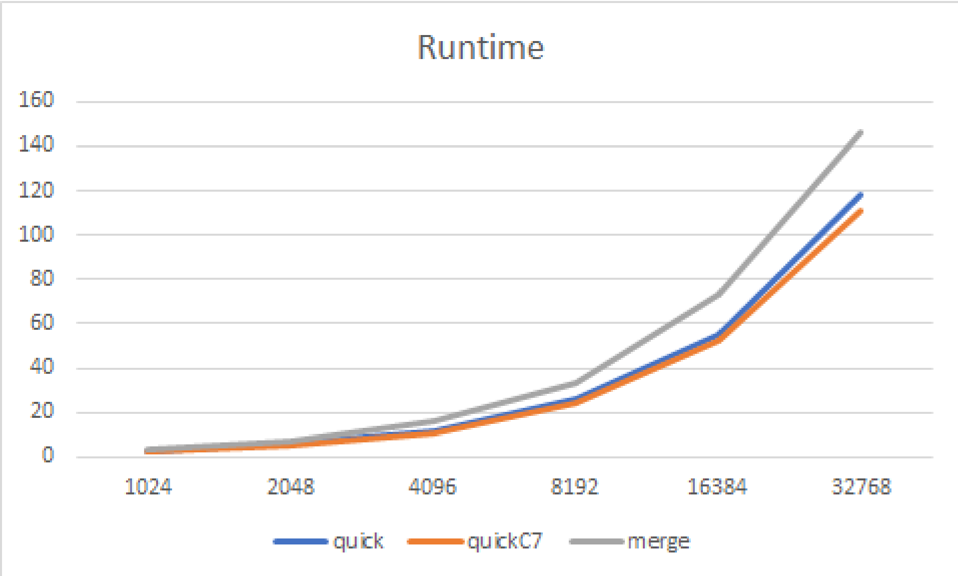
- Selection sort is $O(N^2)$
- Shell sort has more data comparison according to **Q1**
- Merge sort would spend extra time on reading or writing **aux**
- Quick sort would shuffle the original dataset making it slower
- ...

Q5

Result

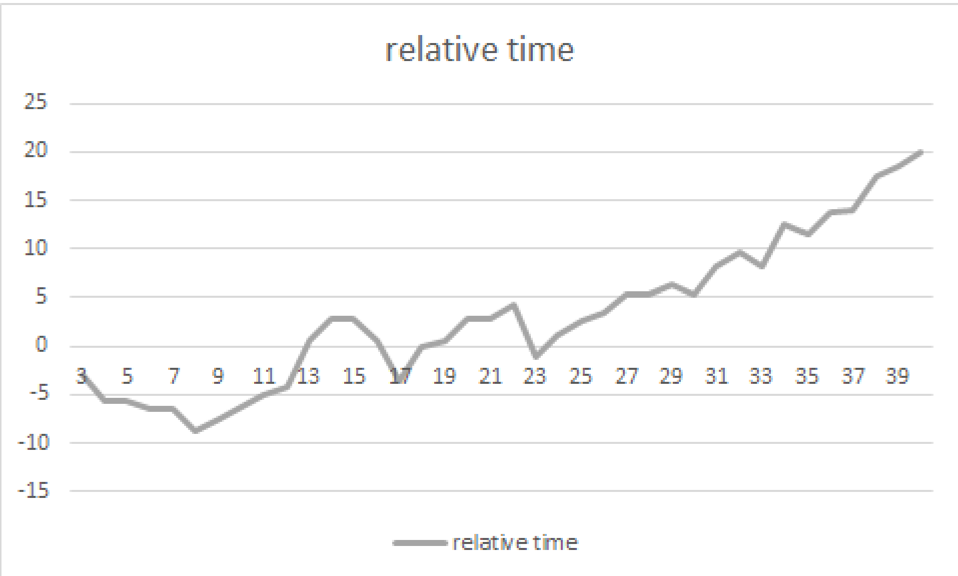
Runtime (ms) :

Time	Size	1024	2048	4096	8192	16384	32768
data0	quickSort	1.77	3.74	7.19	13.81	28.59	59.08
data0	qSortCF7	1.21	2.67	4.71	10.12	21.5	46.21
data1	quickSort	2.57	5.83	11.84	25.71	55.11	118.16
data1	qSortCF7	2.32	5.06	10.8	23.94	52.16	110.77



Cut-off experiment:

I have recorded the runtime on **data1.32768** for cut-off from 3 to 40 and do $T_c - 118.16$.



Analysis

- Performance: quick sort with cut-off-7 is the fastest, quick sort is at the median, and merge sort is the slowest.
- There is little different between quick sort with cut-off-7 and quick sort.

- The switching point is 4 and 12. For cut-off in $[4, 12]$, quick sort with cut-off is better, for the others, quick sort is better.

Q6

1. Merge Sort(bottom-up)
2. Quick Sort
3. Knuth Shuffle
4. Merge Sort(top-down)
5. Insertion Sort
6. Heap Sort
7. Selection Sort
8. Quick Sort