

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
CS440/ECE448 Artificial Intelligence
Exam 2
Spring 2022

April 4, 2022

Your Name: _____

Your NetID: _____

Instructions

- Please write your NetID on the top of every page.
- This is a **CLOSED BOOK** exam. You will be permitted to bring one 8.5x11 page of handwritten notes (front & back).
- Calculators are not permitted. You need not simplify explicit numerical expressions.

Possibly Useful Formulas**Search:**

$$\text{Admissible Heuristic: } h(n) \leq d(n)$$

$$\text{Consistent Heuristic: } h(m) - h(n) \leq d(m) - d(n) \text{ if } d(m) - d(n) \geq 0$$

Belief Propagation:

$$P(A, B, C) = P(A)P(B|A)P(C|A, B)$$

$$P(A, C) = \sum_b P(A, B = b, C)$$

$$P(A|C) = \frac{P(A, C)}{P(C)}$$

Expectation Maximization:

$$P(B = b|A = a) \leftarrow \frac{E[\# \text{ times } A = a, B = b]}{E[\# \text{ times } A = a]}$$

$$E[\# \text{ times } A = a, B = b] = \sum_t P(A_t = a, B_t = b | \text{observations on day } t)$$

HMM:

$$P(Y_1, X_1, \dots, Y_T, X_T) = \prod_{t=1}^T P(Y_t | Y_{t-1}) P(X_t | Y_t)$$

Viterbi Algorithm:

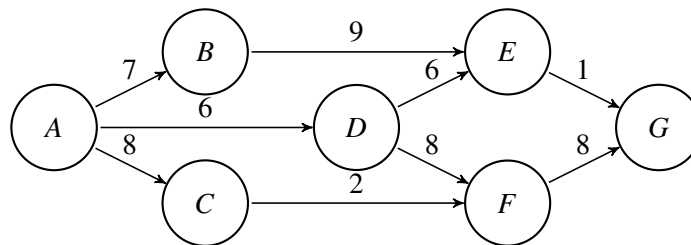
$$e_{i,j,t} = a_{j,i} b_{j,k}$$

$$v_{i,1} = \pi_i b_{i,x_1}$$

$$v_{j,t} = \max_i v_{i,t-1} e_{i,j,t}$$

Question 1 (7 points)

Consider the following search graph. The starting state is A, the goal state is G, and the cost of each possible action is shown on the corresponding edge:



Suppose that the states shown have the following heuristics:

	A	B	C	D	E	F	G
$h(n)$:	8	8	4	4	1	4	0

A* search (with repetitions avoided using an explored set) is applied to this graph to find the shortest path.

- What states are expanded, and
- what is the shortest path?

A B D C E F G

A D E G

Question 2 (7 points)

A typical freight management problem seeks to deliver several large objects from point A to point B using a truck that can carry up to M kilograms. First, you weigh each of the objects, so that you know its mass. Then you use the following search problem to devise an optimal plan:

- State: S = a list of the objects that have not yet been delivered.
- Action: Load a set of objects onto the truck, take them from A to B , then return the truck from B to A .
- Cost: Each trip from A to B has a cost of 1, regardless of the weight of the objects on the truck. Your goal is simply to minimize the number of trips.

Define a nonzero heuristic for this problem, and prove that your heuristic is admissible.

Question 3 (7 points)

The figure below shows a map of the 7 provinces of Costa Rica, donated to Wikipedia by Pixeltoo.



Your goal is to color each of these 7 provinces Red, Green, or Blue, in such a way that no two neighboring provinces have the same color.

Answer the following three questions using the LRV, MCV, and LCV heuristics.

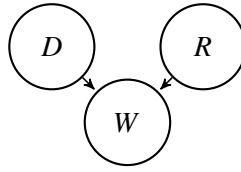
- (a) Suppose that none of the regions have been colored yet. Which region should be colored first, and why?

- (b) Suppose that region 1 has been colored Red, and no other regions have been colored. Now you want to find a color for region 2. What color should region 2 be colored, and why?

- (c) Suppose that region 1 has been colored Red, and region 2 has been colored Green. What region should be colored next, and why?

Question 4 (7 points)

Consider the following Bayesian network:



Suppose that the model parameters are as follows:

$$P(D = d) = \frac{1}{2} \text{ for } d \in \{1, 2\}$$

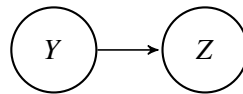
$$P(R = r) = \frac{1}{2} \text{ for } r \in \{1, 2\}$$

$$P(W = T | D = d, R = r) = \begin{cases} \frac{2}{3} & d \geq r \\ \frac{1}{3} & d < r \end{cases}$$

What is $P(D = 2 | W = T)$?

Question 5 (7 points)

Consider the following Bayesian network, showing the relationship between two binary variables Y and Z :



Suppose that you've been given the following initial estimates of the model parameters, where a , b , and c are some arbitrary constants:

$$P(Y = T) = a, \quad P(Z = T|Y = F) = b, \quad P(Z = T|Y = T) = c$$

You are now trying to re-estimate the values of these model parameters. You have observed the values of the variables on five consecutive days, but on the fifth day, the value of Y was unobserved (labeled with a “?” in the table below:

	Day 1	Day 2	Day 3	Day 4	Day 5
Value of Y :	T	F	T	F	?
Value of Z :	F	F	F	T	T

For this table, in terms of the current model parameters a , b , and c , what is the expected number of days on which $Y = T$?

Question 6 (7 points)

Your apartment is haunted by a ghost. Like most ghosts, your ghost tends to sleep for several days at a time. Let $Y_t = T$ if the ghost is awake on day t . You can't see the ghost, but if the ghost is awake, your cat tends to hide under the bed; let $X_t = T$ if your cat is hiding under the bed on day t . Suppose that these probabilities are given by the following distribution, where a , b , c and d are arbitrary constants:

Y_{t-1}	$P(Y_t = T Y_{t-1})$
F	a
T	b

Y_t	$P(X_t = T Y_t)$
F	c
T	d

Suppose you know that the ghost was asleep on day 0 ($Y_0 = F$). You don't know whether or not it was awake on day 1, but you know that your cat hid under the bed ($X_1 = T$). In terms of a , b , c and/or d , find $P(Y_1 = T | Y_0 = F, X_1 = T)$.

Question 7 (7 points)

Consider an HMM with state variables Y_1, \dots, Y_T and observations X_1, \dots, X_T . Suppose that the model has the following parameters, where a, b, c and d are some arbitrary constants:

Y_{t-1}	$P(Y_t = 2 Y_{t-1})$
1	a
2	b

Y_t	$P(X_t = 2 Y_t)$
1	c
2	d

For a particular observation sequence $X_1 = x_1, \dots, X_T = x_T$, define the Viterbi vertex probability to be

$$v_{j,t} = \max_{y_1, \dots, y_{t-1}} P(Y_1 = y_1, X_1 = x_1, \dots, Y_{t-1} = y_{t-1}, X_{t-1} = x_{t-1}, Y_t = j, X_t = x_t)$$

Suppose that $v_{j,t}$ has been calculated, and has been found to have the following values, where e and f are some arbitrary constants:

$$v_{1,t} = e$$

$$v_{2,t} = f$$

Furthermore, suppose that $x_{t+1} = 2$. In terms of the constants a, b, c, d, e , and/or f , what are $v_{1,t+1}$ and $v_{2,t+1}$?