

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
CS440/ECE448 Artificial Intelligence

Exam 1
Spring 2022

February 21, 2022

Your Name: _____

Your NetID: _____

Instructions

- Please write your name on the top of every page.
- This will be a **CLOSED BOOK, CLOSED NOTES** exam. You are permitted to bring and use only one 8.5x11 page of hand-written notes, front and back.
- No electronic devices (phones, tablets, calculators, computers etc.) are allowed.
- No calculators are permitted. You need not simplify explicit numerical expressions.

Possibly Useful Formulas

$$\textbf{Probability: } P(B = 1|A = 1) = \frac{P(A = 1, B = 1)}{P(A = 1)}$$

$$\textbf{Naïve Bayes: } P(X = x|Y = y) \approx \prod_{i=1}^n P(W = w_i|Y = y)$$

$$\textbf{Laplace Smoothing: } P(w) = \frac{\text{Count}(w) + k}{\sum_w \text{Count}(w) + k(1 + \sum_w 1)}$$

$$\textbf{Perceptron: } \vec{w}_y = \vec{w}_y + \eta \vec{x}, \quad \vec{w}_{f(\vec{x})} = \vec{w}_{f(\vec{x})} - \eta \vec{x}$$

$$\textbf{Linear Regression w/SGD: } \vec{w} \leftarrow \vec{w} - \frac{\eta}{2} \nabla_{\vec{w}} \epsilon_i^2 = \vec{w} - \eta \epsilon_i \vec{x}_i$$

$$\textbf{Logistic Regression: } \nabla_{\vec{w}_c} \mathcal{L}_i = \nabla_{\vec{w}_c} \left(-\ln \frac{e^{\vec{w}_c^T \vec{x}_i}}{\sum_k e^{\vec{w}_k^T \vec{x}_i}} \right) = \left(\frac{e^{\vec{w}_c^T \vec{x}_i}}{\sum_k e^{\vec{w}_k^T \vec{x}_i}} - y_{i,c} \right) \vec{x}_i$$

$$\textbf{Neural Net: } \xi_j^{(l)} = b_j^{(l)} + \sum_k w_{j,k}^{(l)} h_k^{(l-1)}, \quad h_j^{(l)} = g^{(l)} \left(\xi_j^{(l)} \right)$$

$$\textbf{Back-Propagation: } \frac{\partial \mathcal{L}}{\partial h_k^{(l-1)}} = \sum_j \frac{\partial \mathcal{L}}{\partial h_j^{(l)}} \frac{\partial h_j^{(l)}}{\partial h_k^{(l-1)}}$$

$$\textbf{Pinhole Camera: } \frac{x'}{f} = -\frac{x}{z}, \quad \frac{y'}{f} = -\frac{y}{z}$$

Question 1 (7 points)

Consider two binary random variables, X and Y . Suppose that

$$P(X = 1) = a$$

$$P(Y = 1) = b$$

$$P(X = 1, Y = 0) = c$$

In terms of a , b , and/or c , what is $P(Y = 1|X = 1)$?

Question 2 (7 points)

You've been asked to create a naïve Bayes model of the candy produced by the Santa Claus Candy Company. As your training dataset, you've been given a box containing 80 pieces of candy, of which 8 are strawberry, 48 are raspberry, and 24 are blueberry. In terms of the Laplace smoothing parameter k , estimate the following probabilities:

$P(\text{flavor}=\text{strawberry}|\text{Santa Claus Candy Company})=$

$P(\text{flavor}=\text{raspberry}|\text{Santa Claus Candy Company})=$

$P(\text{flavor}=\text{blueberry}|\text{Santa Claus Candy Company})=$

$P(\text{flavor}=\text{other}|\text{Santa Claus Candy Company})=$

Question 3 (7 points)

Describe, in one sentence each, the purpose of (1) a training set, (2) a development test set, (3) an evaluation test set.

**Question 4** (7 points)

You're trying to create a multi-class perceptron that will classify animals as being either fish, birds, or reptiles. Your feature vector is $\vec{x} = [x_1, x_2, x_3, 1]^T$, where

x_1 = fraction of time the animal spends under water

x_2 = fraction of time the animal spends on land

x_3 = fraction of time the animal spends flying

- Based on your extensive prior knowledge of zoology, you initialize your perceptron with the following weight vectors: $\vec{w}_{\text{fish}} = [1, 0, 0, 0]^T$, $\vec{w}_{\text{reptile}} = [0, 1, 0, 0]^T$, and $\vec{w}_{\text{bird}} = [0, 0, 1, 0]^T$.
- Your first training token is a crocodile, for which $y = \text{reptile}$, and $\vec{x} = [0.7, 0.3, 0, 1]^T$.

After training with this training token, what are the numerical values of \vec{w}_{fish} , \vec{w}_{reptile} , and \vec{w}_{bird} ? Assume a learning rate of $\eta = 1$.

Question 5 (7 points)

In stochastic gradient descent, we train using one training token at a time. Suppose

$$\mathcal{L} = (\vec{w}^T \vec{x} - y)^2$$

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ b \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

In terms of \vec{x} , \vec{w} , w_1 , w_2 , b , x_1 , x_2 , and/or y , what is $\frac{d\mathcal{L}}{dw_2}$?

Question 6 (7 points)

Suppose that

$$\begin{aligned}f &= w_{1,1}^{(2)}h_1 + w_{1,2}^{(2)}h_2 + b^{(2)} \\h_1 &= \text{ReLU}\left(w_{1,1}^{(1)}x_1 + w_{1,2}^{(1)}x_2 + b_1^{(1)}\right) \\h_2 &= \text{ReLU}\left(w_{2,1}^{(1)}x_1 + w_{2,2}^{(1)}x_2 + b_2^{(1)}\right)\end{aligned}$$

Assume, for a particular training token, that $h_1 > 0$ and $h_2 > 0$. For that particular training token, what is $\frac{\partial f}{\partial w_{1,1}^{(1)}}$? Express your answer in terms of x_j , h_j , $w_{j,k}^{(l)}$, and/or $b_k^{(l)}$ for any values of j , k , and/or l that may be useful to you.

Question 7 (7 points)

In the real world, the (x, y, z) coordinates of Joe's face and Mike's face are $(14, 3, 7)$ and $(14, 3, 17)$, respectively, where z is distance from the camera. In the image plane (x', y') , which person (Joe or Mike) is closer to the center of the image (the point $(x', y') = (0, 0)$), and why?