Decision Theory

X = datum

$f(x)$ = decision

Y = correct label

Risk $R(f) = E[L(f(x), Y)] = \sum_Y \sum_X L(f(x), y) P(X=x, Y=y)$

False alarm: $P(f(x)=CPU \mid Y=pow) = 0$

Miss detection: $P(f(x)=pow \mid Y=cpu) = 1$

Fairness

$P(Y=1 \mid A=1) = \frac{2}{3}$

Y	$P(\hat{Y}=0 \mid Y, A=1)$	$P(\hat{Y}=1 \mid Y, A=1)$
0	0.8	0.2
1	0.4	0.6

$$P(Y=1 \mid \hat{Y}=1, A=1) = \frac{P(Y=1, \hat{Y}=1 \mid A=1)}{P(\hat{Y}=1 \mid A=1)}$$

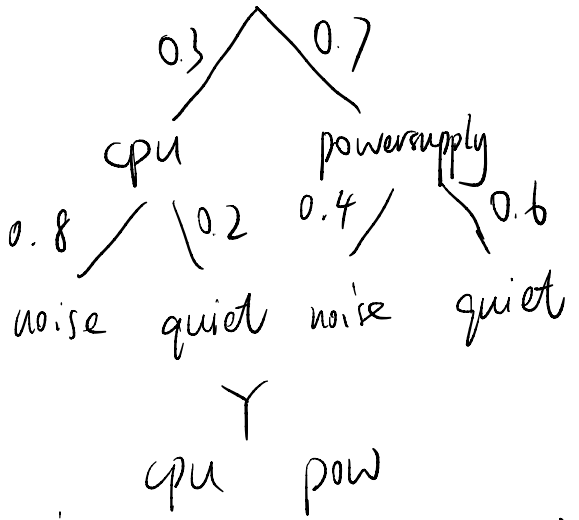
$$= \frac{P(Y=1 \mid A=1) P(\hat{Y}=1 \mid Y=1, A=1)}{P(Y=1 \mid A=1) P(\hat{Y}=1 \mid Y=1, A=1) + P(Y=0 \mid A=1) P(\hat{Y}=1 \mid Y=0, A=1)}$$

Demo Parity $P(\hat{Y} \mid A=1) = P(\hat{Y} \mid A=0)$

Equal Odds $P(\hat{Y} \mid Y=1, A=1) = P(\hat{Y} \mid Y=1, A=0)$

Maximum a posteriori

$f(x) = \text{argmax}_Y P(Y=f(x) \mid X=x)$



Y

cpu pow

X noise $(0.3)(0.8)$ $(0.7)(0.4)$

quiet $(0.3)(0.2)$ $(0.7)(0.6)$

$f(x) = \text{argmax}_Y P(Y \mid x) = \text{argmax}_Y P(Y, x)$

$= \begin{cases} cpu, & \text{never} \\ pow, & \text{if } x = \text{noisy, quiet.} \end{cases}$

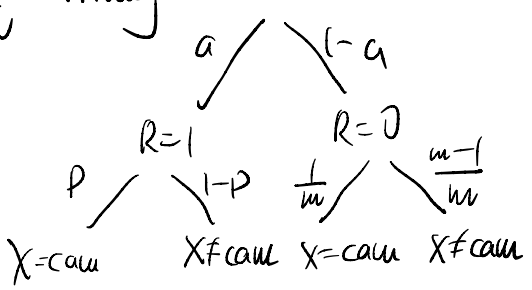
Bayes' rule: $P(Y=y \mid X=x) = \frac{P(Y=y) P(X=x \mid Y=y)}{P(X=x)}$

$f(x) = \text{argmax}_y P(Y=y) P(X=x \mid Y=y)$

Bayes Error Rate: $\sum P(X=x) \min_y P(Y \neq y \mid X=x)$

$= P(f(x) \neq Y) = P(y = CPU) = 0.3.$

Privacy



$P(X=cam) = b$

$= P(X=cam, R=1) + P(X=cam, R=0)$

Optimization

random restart

for each restart P

for each iteration q

for each coordinate m

for each value n

exhaustive search $O(n^m)$.