

# Schedulability with resource sharing

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Priority inheritance protocol

Priority ceiling protocol

Stack resource policy

# Lecture overview

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- We have discussed the occurrence of **unbounded priority inversion**
- We know about **blocking** and **blocking times**
- Now: Evaluating schedulability in combination with protocols for avoiding unbounded priority inversion
- **Priority ceiling protocol** to prevent deadlocks
- **Stack-based resource policy**
  - Improves on other policies
  - Extends to EDF

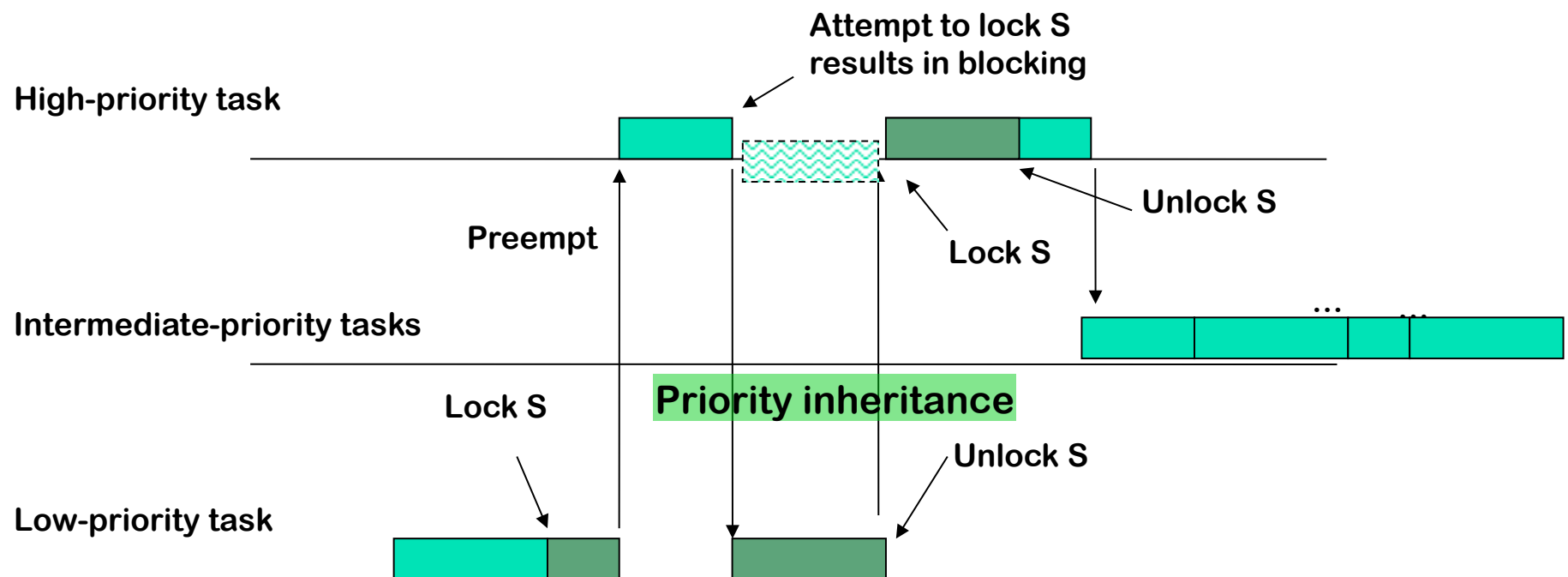
# Blocking

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- Tasks have synchronization constraints
  - Use semaphores to protect critical sections
- Blocking can cause a *higher priority task to wait for a lower priority task to unlock a resource*
  - We always assumed that higher priority tasks can preempt lower priority tasks
  - To make rules consistent, we discussed the priority inheritance approach

# The priority inheritance protocol

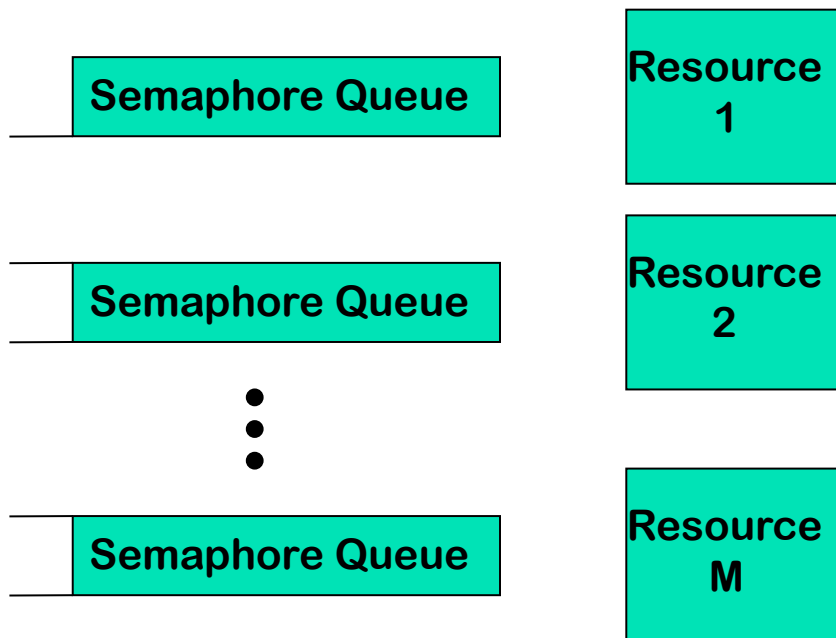
- Allow a task to inherit the priority of the highest priority task that it is blocking



# Maximum blocking time

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- If all critical sections are of equal length,  $B$ 
  - Blocking time =  $B \times \min(N, M)$
  - Why?
  - And what if the critical sections are of differing lengths?



If I am a task, priority inversion occurs when

- (a) Lower priority task holds a resource I need (direct blocking)
- (b) Lower priority task inherits a higher priority than me because it holds a resource the higher-priority task needs (push-through blocking)

# Maximum blocking time

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- If all critical sections are of equal length,  $B$ 
  - Blocking time =  $B \times \min(N, M)$
  - Why?
- And what if the critical sections are of differing lengths?
  - Find the maximum length critical section for each resource
  - Add the top  $\min(N, M)$  sections in size
  - The total priority inversion time experienced by Task  $T_i$  is denoted  $B_i$
- **Remember: when computing the blocking time, you need only consider tasks with lower priority.**
  - **And a task may be blocked at most once by a lower priority task.**

# Schedulability tests

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- For the fixed-priority scheduling case
  - We can use the Liu & Layland bound with some modifications
- For task  $T_k$ : we need to consider the blocking by lower priority tasks

$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \leq k(2^{1/k} - 1)$$

Each instance of a task may experience blocking (worst case); equivalent to increasing the execution time of the task by the blocking time.

For task  $T_k$ , we need to consider:

- a) preemption by higher priority tasks
- b) blocking from lower priority tasks
- c) bound for  $T_k$  involves only  $k$  tasks

Why do we test each task separately? Why can we not have one utilization bound test like we did earlier?

# Example: blocking and schedulability

- Consider the following set of tasks, which share resources  $R_1$ ,  $R_2$  and  $R_3$ 
  - Relative deadline are equal to periods; tasks scheduled using RM policy
  - $T_1$ :  $P_1=20$ ,  $e_1=3$ , uses  $R_1$  and  $R_2$  separately for 1 time unit each
  - $T_2$ :  $P_2=30$ ,  $e_2=6$ , uses  $R_2$  and  $R_3$  simultaneously for 2 time units
  - $T_3$ :  $P_3=50$ ,  $e_2=10$ , uses  $R_1$  and  $R_3$  separately for 3 and 4 time units respectively
  - $T_4$ :  $P_4=80$ ,  $e_2=8$ , uses  $R_2$  for 5 time units

Is there a difference?

We will see that there is no difference in this example. In other cases, maybe.

Without resource constraints

$$U = \frac{3}{20} + \frac{6}{30} + \frac{10}{50} + \frac{8}{80} = 0.65 < 0.69$$

The task set satisfies the Liu and Layland bound; easily schedulable by RM



# Example: blocking and schedulability

- Consider the following set of tasks, which uses resources  $R_1$ ,  $R_2$  and  $R_3$ 
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  - $T_4$ :  $P_4=80$ ,  $e_2=8$ , uses  $R_2$  for 5 time units

## With resource constraints

$T_1$  can potentially be blocked by  $T_2$ ,  $T_3$  and  $T_4$

It can be blocked by  $T_2$  on resource  $R_2$  for upto 2 time units

It can be blocked by  $T_3$  on resource  $R_1$  for upto 4 time units

It can be blocked by  $T_4$  on resource  $R_2$  for upto 5 time units

The worst-case wait for  $R_1$  is 3 units (only  $T_3$  can block  $T_1$ )

The worst-case wait for  $R_2$  is 5 units ( $T_2$  can block  $T_1$  for 2 units or  $T_4$  can block  $T_1$  for 5 units)

Maximum wait for resources is  $B_1 = 3+5 = 8$

$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \leq k(2^{1/k} - 1)$$

$$\frac{8}{20} + \frac{3}{20} < 1$$

$T_1$  is schedulable

# Example: blocking and schedulability

- Consider the following set of tasks, which uses resources  $R_1$ ,  $R_2$  and  $R_3$ 
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$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \leq k(2^{1/k} - 1)$$

## With resource constraints

$T_2$  can be blocked by  $T_3$  and  $T_4$

$T_3$  can block  $T_2$  in two ways:

directly on  $R_3$  (upto 4 units)

by obtaining priority of  $T_1$  when using  $R_1$  (upto 3 units) (push-through)

$T_4$  can block  $T_2$  in two ways:

directly when using  $R_2$  (upto 5 units)

by obtaining priority of  $T_1$  when using  $R_2$  (upto 5 units) (push-through)

The worst-case blocking by  $T_3$  is 4 time units

The worst-case blocking by  $T_4$  is 5 time units

Maximum wait for resources is  $B_2 = 5+4 = 9$

**A low priority task can block a high priority task at most once. With priority inheritance, it will get a higher priority and continue till it releases the lock. Therefore, it can block a high priority task at most once.**

$$\frac{9}{30} + \left( \frac{3}{20} + \frac{6}{30} \right) = 0.65 < 0.82$$

**$T_2$  is schedulable**

# Example: blocking and schedulability

- Consider the following set of tasks, which uses resources  $R_1$ ,  $R_2$  and  $R_3$ 
  - Relative deadline are equal to periods; tasks scheduled using RM policy
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**With resource constraints**

$T_3$  can be blocked by  $T_4$

*even when it shares no resource with  $T_4$  (lower priority task)*

Notice that  $T_4$  may execute with priority of  $T_1$  (priority inheritance)

$T_4$  may execute with the priority of  $T_1$  for at most 5 time units

**Classic case of push-through blocking**

Maximum blocking due to  $T_4$  is 5 time units;  $B_3 = 5$

$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \leq k(2^{1/k} - 1)$$

$$\frac{5}{50} + \left( \frac{3}{20} + \frac{6}{30} + \frac{10}{50} \right) = 0.65$$

**$T_3$  is schedulable**

# Example: blocking and schedulability

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- Consider the following set of tasks, which uses resources  $R_1$ ,  $R_2$  and  $R_3$ 
  - Relative deadline are equal to periods; tasks scheduled using RM policy
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$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \leq k(2^{1/k} - 1)$$

**With resource constraints**

$T_4$  can never be blocked

because it is the lowest priority task

Maximum wait for resources is  $B_4 = 0$

$$\left( \frac{3}{20} + \frac{6}{30} + \frac{10}{50} + \frac{8}{80} \right) = 0.65$$

**$T_4$  is schedulable**

# General approach to computing blocking times

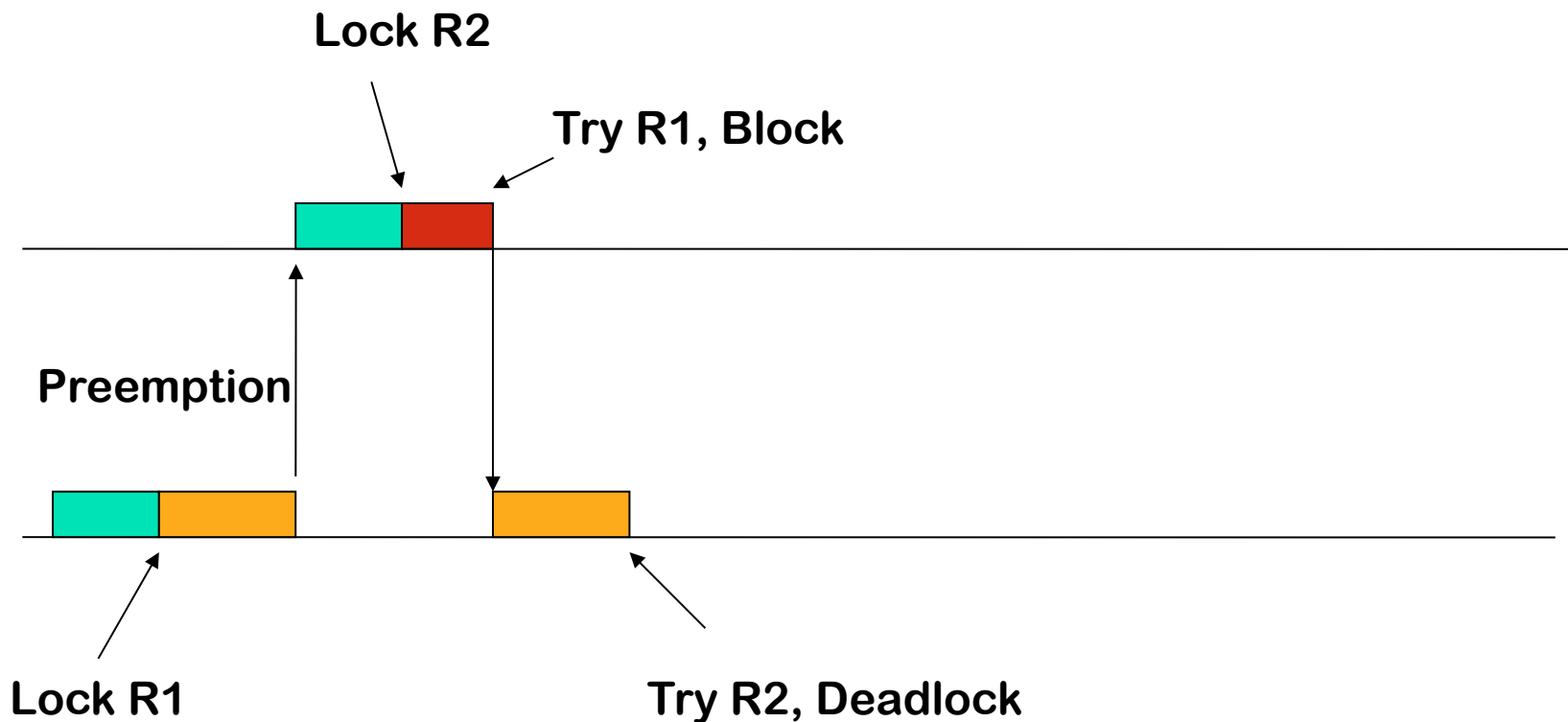
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- For a high-priority task
  - Examine all tasks with lower priority
    - Determine the worst-case blocking that it may offer (consider the highest priority that it can inherit)
  - Examine all semaphores/resources
    - Determine the worst-case blocking due to that resource
    - Consider lower-priority tasks that may inherit a higher priority when they hold the semaphore

# Does priority inheritance solve all problems?

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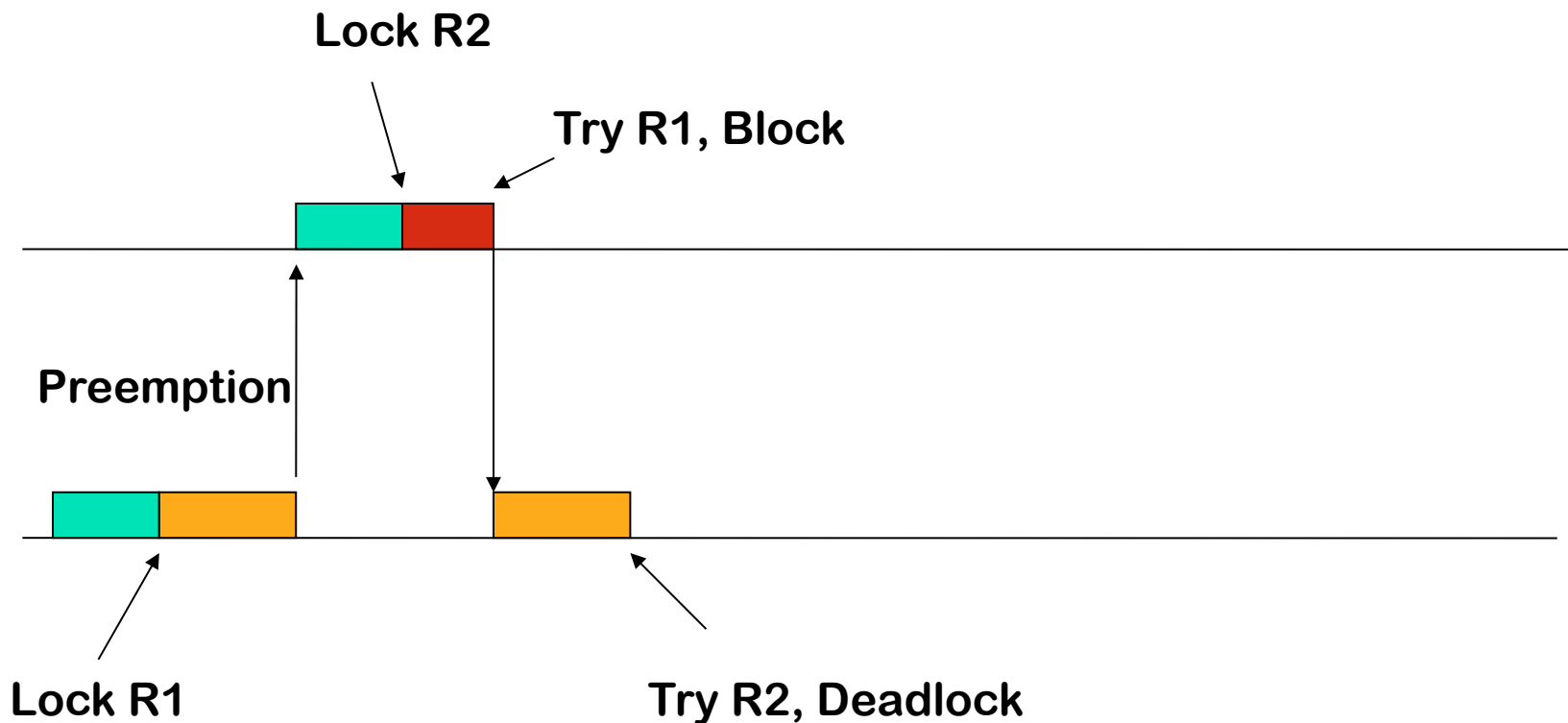
- Actually, not all problems
- We can still have a deadlock if resources are locked in opposing orders
- As we saw two lectures back



# Deadlocks

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- Can attribute it to sloppy programming
- But can we solve the problem in a different way
- Avoid deadlocks by designing a suitable protocol



# Priority ceiling protocol

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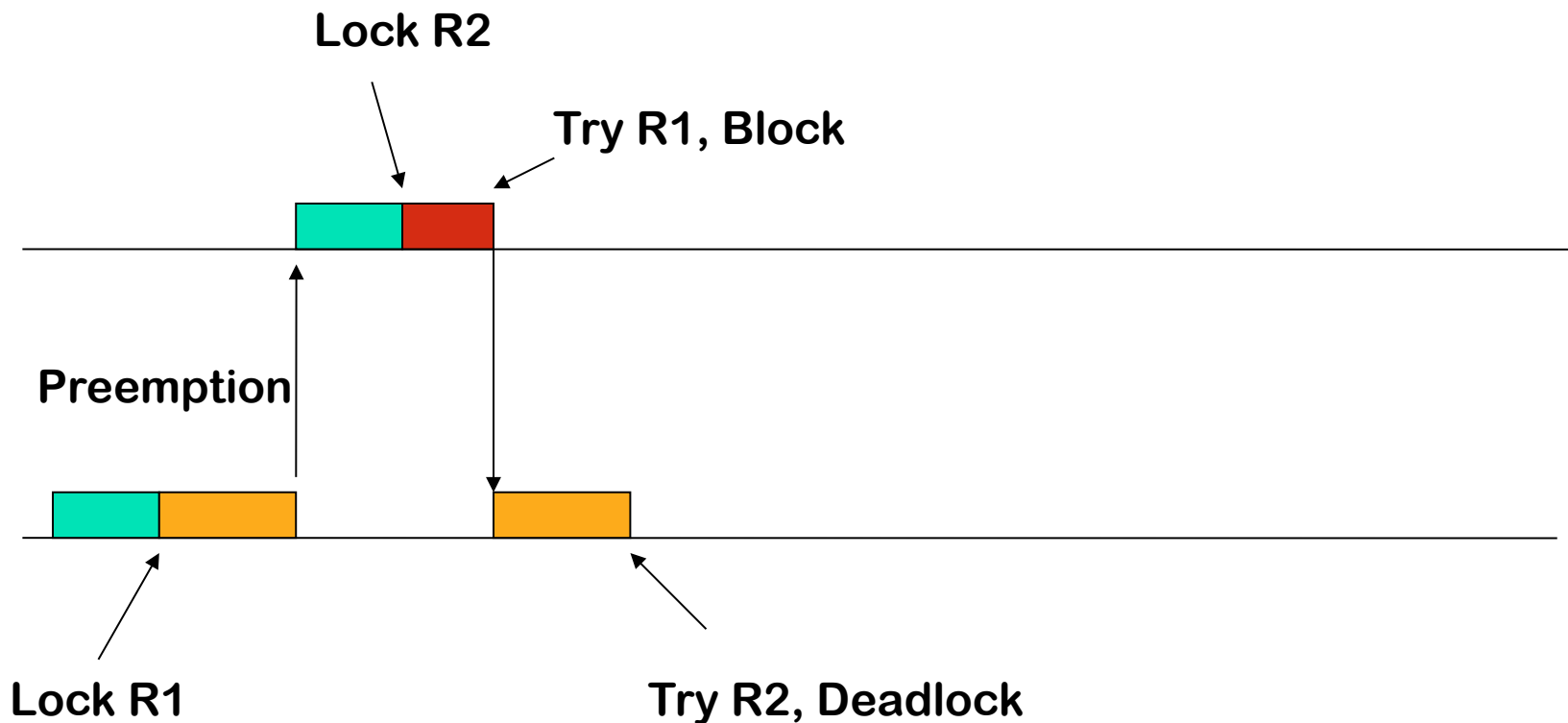
- **Definition:** the **priority ceiling** of a semaphore is the highest priority among all tasks that can lock the semaphore
- A task that requests lock  $R_k$  is denied if its priority is not higher than the highest priority ceiling of all currently locked semaphores (let us say this belongs to semaphore  $R_h$ )
  - The task is said to be blocked by the task holding semaphore  $R_h$
- A task inherits the priority of the top higher-priority task it is blocking



# Deadlocks?

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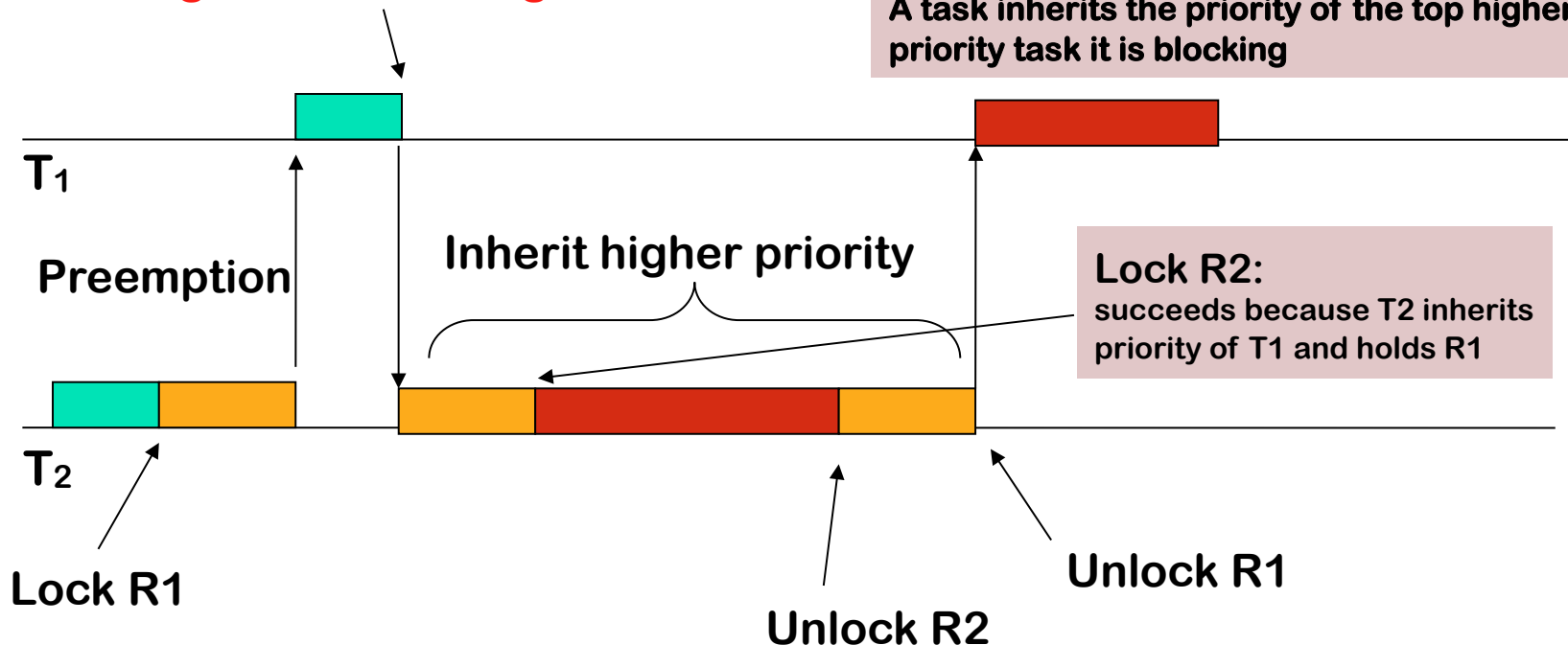
- A deadlock can occur if two tasks locked semaphores in opposite order. Can it occur with the priority ceiling protocol?



# Priority ceilings

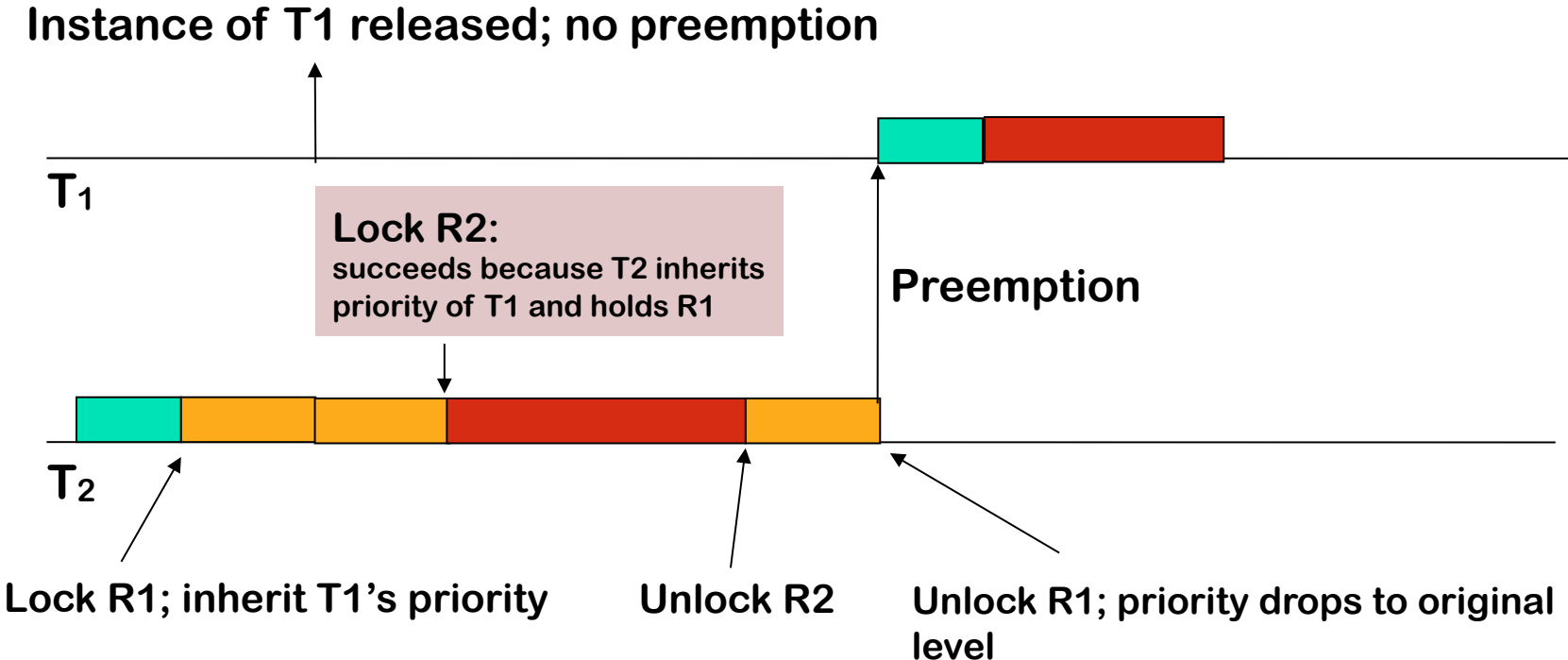
- $T_1$  and  $T_2$  use  $R_1$  and  $R_2$ : the priority ceiling of a resource is the priority of the highest priority task that uses it, therefore the priority ceilings of  $R_1$  and  $R_2$  are the same: the priority of  $T_1$

Lock R2: **Denied because its priority is not higher than ceiling of R1**



# Immediate inheritance

- Priority ceiling protocol with slight difference: when a semaphore is locked, the locking task raises its priority to the ceiling priority of the semaphore (**immediate inheritance**). When the semaphore is unlocked the task's priority is restored.



# Schedulability test for priority ceiling protocol

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- The same as the priority inheritance protocol: worst-case blocking time does not change

$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \leq k(2^{1/k} - 1)$$

For task  $\tau_k$

# Stack-based resource policy

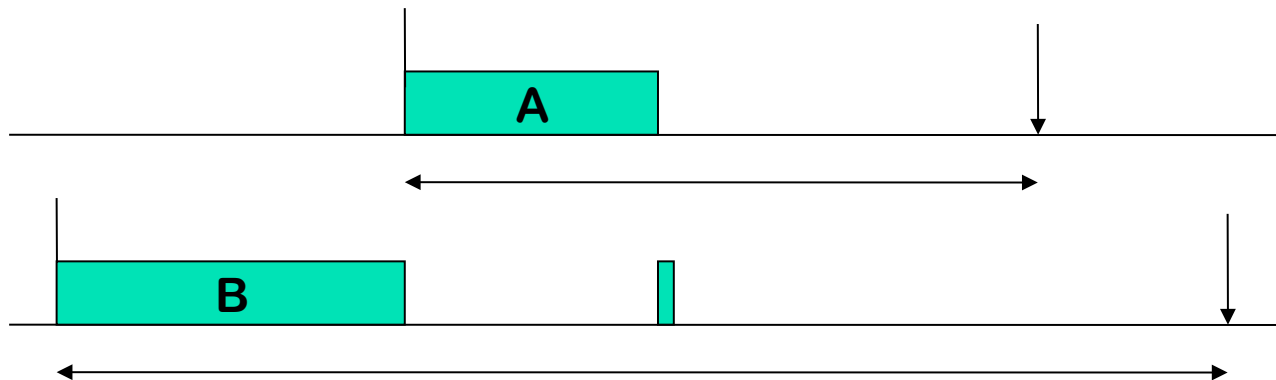
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- Priority inheritance protocol and priority ceiling protocol are easy to analyze in a fixed-priority setting
- What about dynamic priority scheduling?
- **Stack-based resource policy [SRP]**
  - **Preemption level**: Any fixed value that satisfies the statement “if A arrives after B and  $\text{priority}(A) > \text{priority}(B)$ , then  $\text{PreemptionLevel}(A) > \text{PreemptionLevel}(B)$ .”
  - **Resource ceiling** for resource  $R$ : Highest preemption level of all tasks that may access the resource  $R$
  - **System ceiling**: Highest resource ceiling among all currently locked resources
  - A task can preempt another task if
    - it has the highest priority and
    - its preemption level is higher than the system ceiling

# Stack-based resource policy with EDF

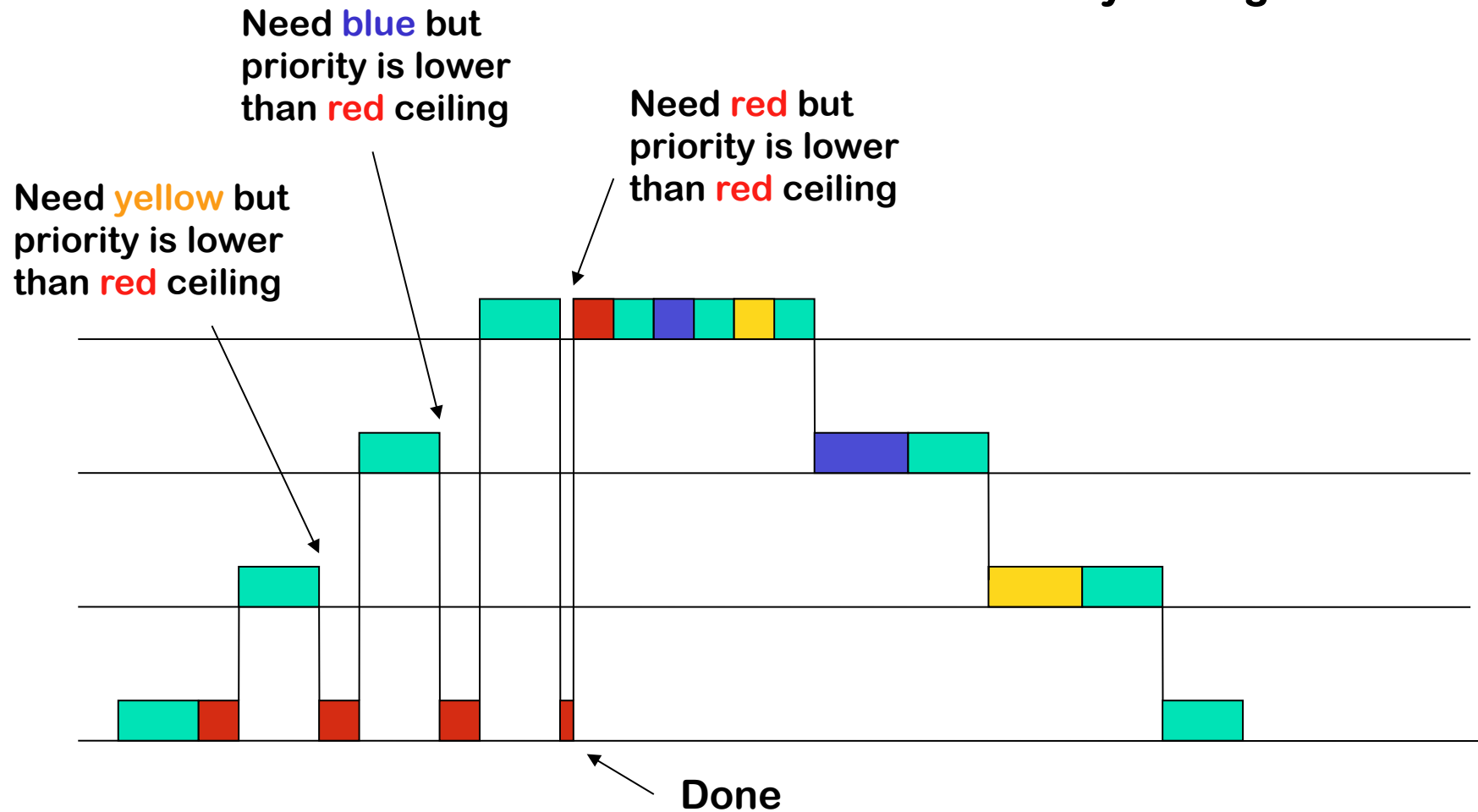
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- Priority is inversely proportional to the absolute deadline
- Preemption level is inversely proportional to the relative deadline
- Observe that:
  - If A arrives after B and  $\text{Priority}(A) > \text{Priority}(B)$  then  $\text{PreemptionLevel}(A) > \text{PreemptionLevel}(B)$



# Priority ceiling vs. stack-based resource policy

## Priority Ceiling Protocol

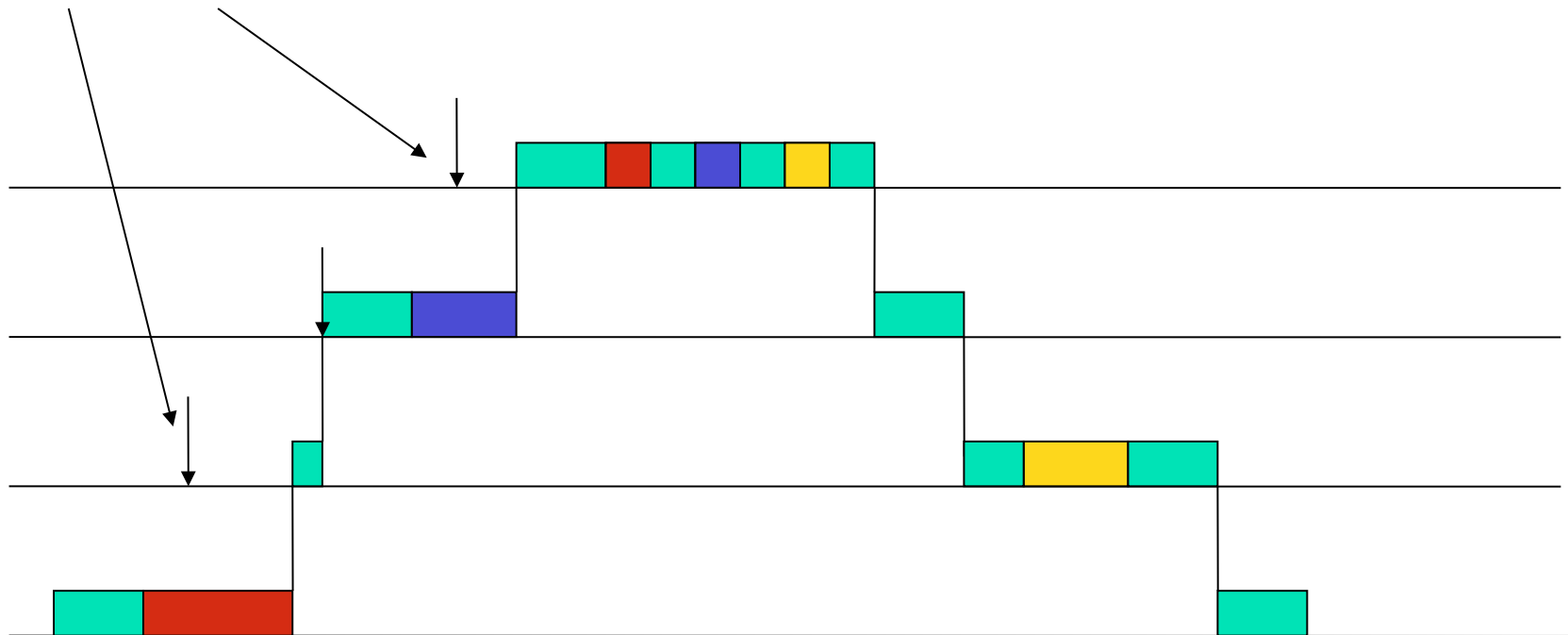


# Priority ceiling vs. stack-based resource policy

## Stack-based Resource Policy

Can't preempt.  
Preemption level is not  
higher than ceiling.

Notice that SRP is similar to immediate inheritance in PCP.  
However, with no static priority levels, it needs a preemption level.

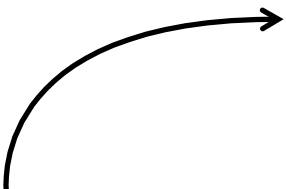




# Analysis with EDF and SRP

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- As simple as other protocols


$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_k}{P_k} \leq 1$$

For task  $T_k$

Maximum blocking due to task with lower preemption level; in the case of EDF: with period  $P_j$  such that  $P_k < P_j$ .

Tasks are sorted such that the task with shortest period is  $T_1$  and so on.

# Highlights

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- Schedulability analysis needs to account for blocking due to low priority tasks
- **Priority inheritance protocol (PIP)** may not prevent deadlocks
- **Deadlocks** can be prevented with the **priority ceiling protocol (PCP)**
- To deal with dynamic priority policies (such as EDF), we need a different policy: the **stack-based resource policy (SRP)**
- SRP (and the immediate inheritance version of the PCP) have efficient implementations
  - Reduce the number of context switches
  - SRP also prevents deadlocks (note the similarities between PCP and SRP)