Periodic task scheduling

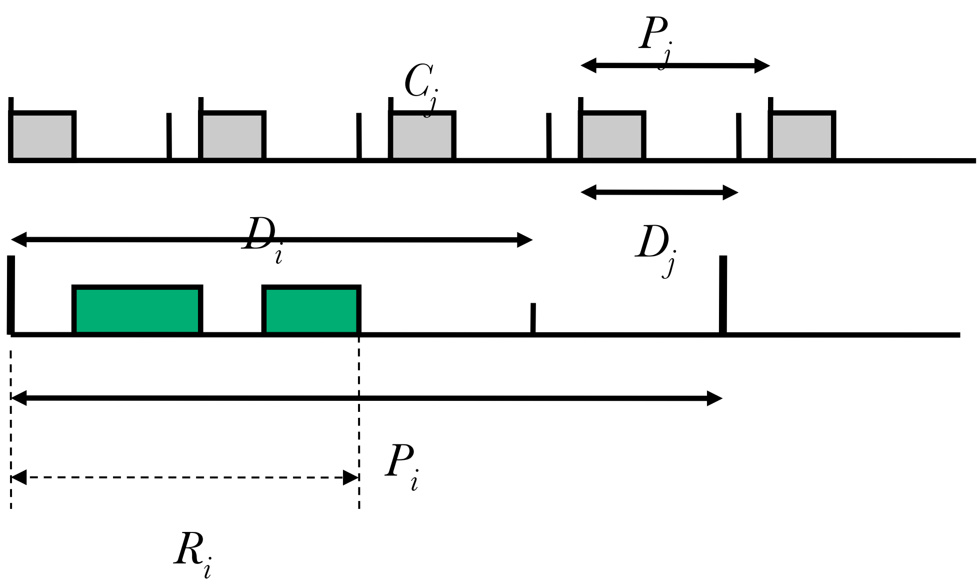
An exact test for EDF
Relative deadlines less than periods
Processor demand criterion

The earliest deadline first policy

- Optimal scheduling policy when relative deadlines are equal to task periods
- Is a dynamic priority policy
- How does it behave when relative deadlines are less than periods?
- Exact analysis for EDF

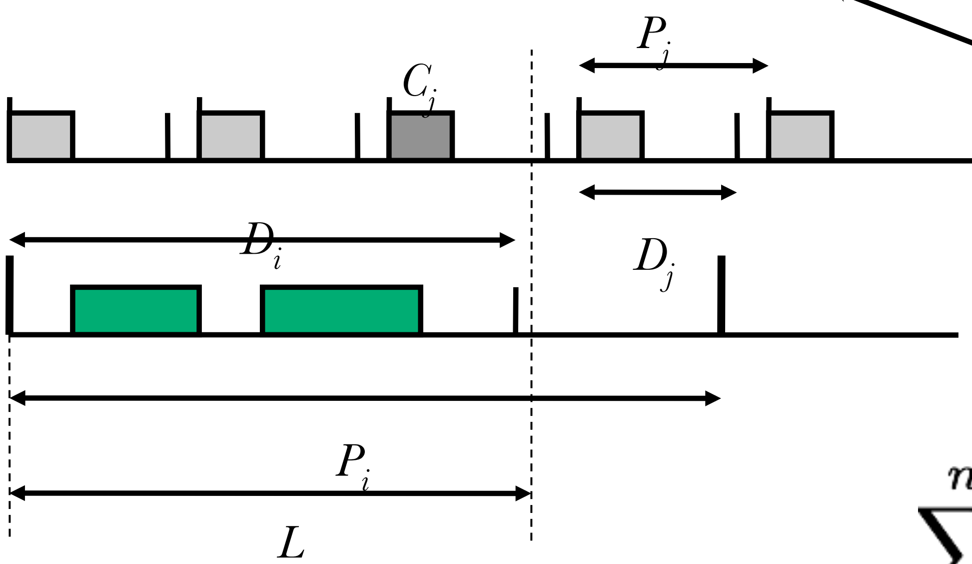
EDF and processor demand

- Interference is due to only those tasks with earlier deadlines



EDF and processor demand

- Consider demand on the processor due to tasks whose deadlines have passed
- Within any time interval, *L*, the demand must be less than *L*.



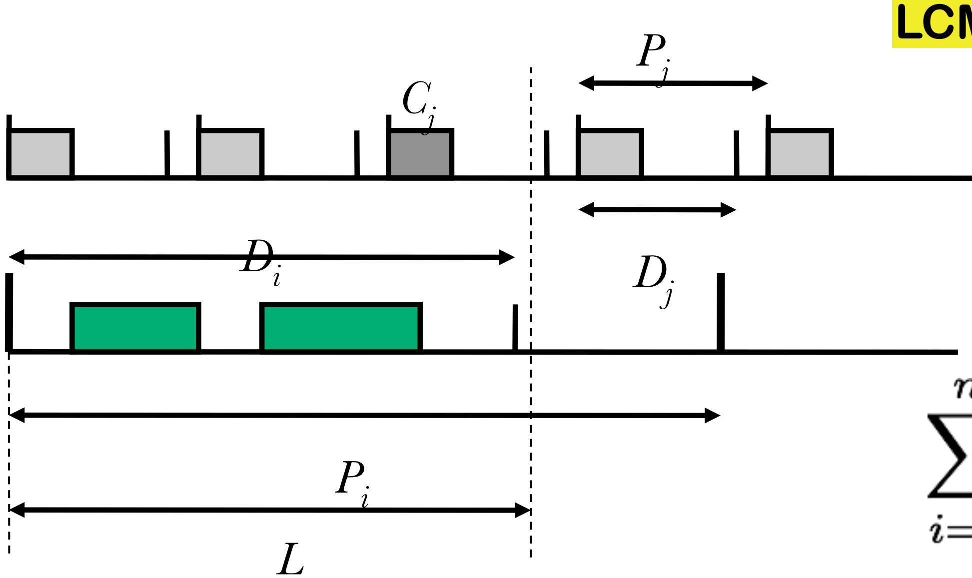
The execution time demanded by jobs with deadline less than *L* over an interval of length *L* cannot be greater than *L*

$$\sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$

Processor demand

- Checking the schedulability for at all time instants is not possible
  - Overwhelming complexity
- Observation 1: Sufficient to check up to the <u>hyperperiod</u> (schedule repeats itself)
- Observation 2: Check only at absolute deadlines

LCM of all task periods



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Processor demand

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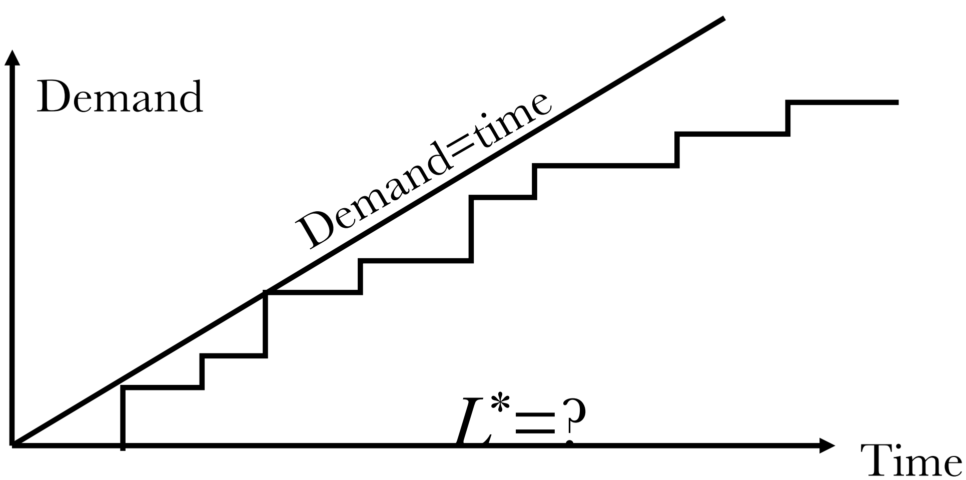
  - Overwhelming complexity
- Observation 1: Sufficient to check up to the <u>hyperperiod</u> (schedule repeats itself)
- Observation 2: Check only at absolute deadlines
- Observation 3: If $U < 1$, the demand is trivially satisfied after some time instant L^*



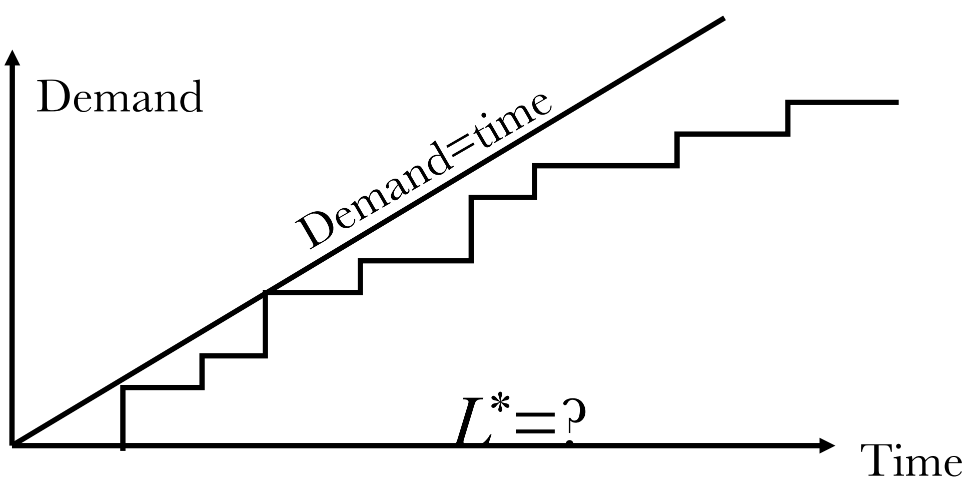
$$\sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$

Processor demand

- **Deriving *L****

$$\sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$

$$\sum_{i=1}^{n} \left\lfloor \frac{t + P_i - D_i}{P_i} \right\rfloor C_i \leq \sum_{i=1}^{n} \frac{t - D_i + P_i}{P_i} C_i = tU + \sum_{i=1}^{n} (P_i - D_i) U_i$$

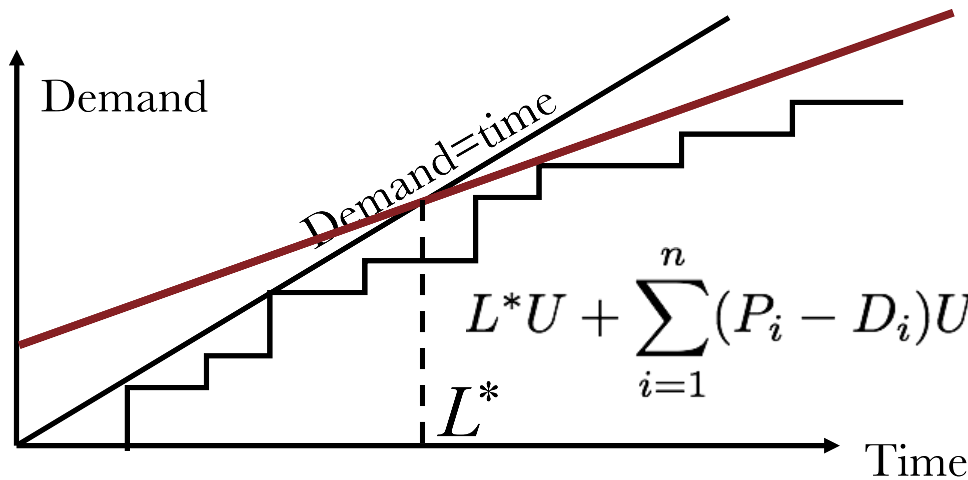


Processor demand

- **Deriving *L****

$$\sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$

$$\sum_{i=1}^{n} \left\lfloor \frac{t + P_i - D_i}{P_i} \right\rfloor C_i \leq \sum_{i=1}^{n} \frac{t - D_i + P_i}{P_i} C_i = \boxed{tU + \sum_{i=1}^{n} (P_i - D_i) U_i}$$



$$L^* U + \sum_{i=1}^{n} (P_i - D_i) U_i = L^* \Rightarrow \boxed{L^* = \frac{\sum_{i=1}^{n} (P_i - D_i) U_i}{1 - U}}$$

Processor demand criterion

- Check if $$\sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$
  - for all *L* that are absolute deadlines in the interval [0,L^*]
  - where
$$L^* = \frac{\sum_{i=1}^{n} (P_i - D_i) U_i}{1 - U}$$
- This is an exact test for EDF when relative deadlines are less than task periods

Example using processor demand

- Consider the following task set
  - T_1: (C_1=1, P_1=3, D_1=2)
  - T_2: (C_2=2, P_2=7, D_2=5.5)
  - T_3: (C_3=2, P_3=10, D_3=6)
- The task set has a hyperperiod of 210
- However, we only need to test deadlines up to $L^* = \frac{\sum_{i=1}^{n}(P_i - D_i)U_i}{1-U}$
- U = 86/105 = 0.8190; L^* = 8.63
- At $L=2$:

$$\lfloor \frac{L + P_1 - D_1}{P_1} \rfloor C_1 = \lfloor \frac{2+3-2}{3} \rfloor 1 = 1$$

Example using processor demand

- Consider the following task set
  - T_1: (C_1=1, P_1=3, D_1=2)
  - T_2: (C_2=2, P_2=7, D_2=5.5)
  - T_3: (C_3=2, P_3=10, D_3=6)
- The task set has a hyperperiod of 210
- However, we only need to test deadlines up to $L^* = \frac{\sum_{i=1}^{n}(P_i - D_i)U_i}{1-U}$
- U = 86/105 = 0.8190; L^* = 8.63
- At $L=5.5$: (also need to check at $L=5$; not shown here)

$$\lfloor \frac{L+P_1-D_1}{P_1} \rfloor C_1 + \lfloor \frac{L+P_2-D_2}{P_2} \rfloor C_2 = \lfloor \frac{5.5+3-2}{3} \rfloor 1 + \lfloor \frac{5.5+7-5.5}{7} \rfloor 2 = 2+2=4$$

Example using processor demand

- Consider the following task set
  - T_1: (C_1=1, P_1=3, D_1=2)
  - T_2: (C_2=2, P_2=7, D_2=5.5)
  - T_3: (C_3=2, P_3=10, D_3=6)
- The task set has a hyperperiod of 210
- However, we only need to test deadlines up to $L^* = \dfrac{\sum_{i=1}^{n}(P_i - D_i)U_i}{1-U}$
- U = 86/105 = 0.8190; L^* = 8.63
- At $L=6$:

$$\lfloor \frac{6+3-2}{3} \rfloor 1 + \lfloor \frac{6+7-5.5}{7} \rfloor 2 + \lfloor \frac{6+10-6}{10} \rfloor 2 = 6$$

Example using processor demand

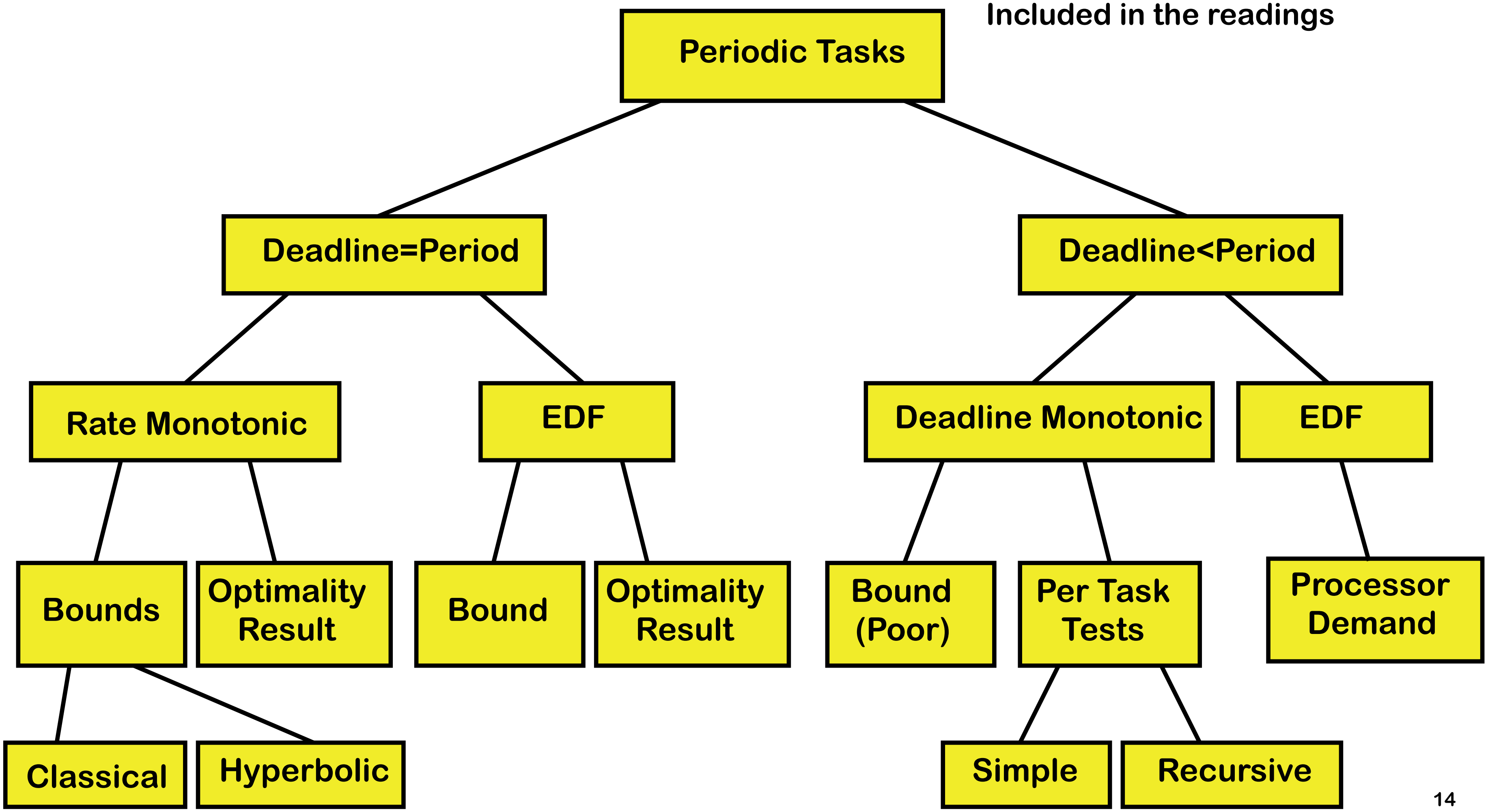
- Consider the following task set
  - T_1: (C_1=1, P_1=3, D_1=2)
  - T_2: (C_2=2, P_2=7, D_2=5.5)
  - T_3: (C_3=2, P_3=10, D_3=6)
- The task set has a hyperperiod of 210
- However, we only need to test deadlines up to $L^* = \frac{\sum_{i=1}^{n}(P_i - D_i)U_i}{1-U}$
- U = 86/105 = 0.8190; L^* = 8.63
- At $L=8$:

$$\lfloor \frac{8+3-2}{3} \rfloor 1 + \lfloor \frac{8+7-5.5}{7} \rfloor 2 + \lfloor \frac{8+10-6}{10} \rfloor 2 = 3+2+2 = 7$$

No more absolute deadlines < 8.63. We are done!

Topics covered so far

Chapter 4 of Buttazzo's text
Included in the readings



Lecture summary

- Exact test for EDF scheduling (relative deadlines < periods)
- Processor demand criterion