CMSC 330: Organization of Programming Languages

OCaml 3
Higher Order Functions,
Closures, Currying

This Lecture

- Higher order functions
  - Map, fold
- Static & dynamic scoping
- Environments & closures
- Currying

Higher-Order Functions

- In OCaml you can pass functions as arguments, and return functions as results

```
let plus_three x = x + 3
let twice (f, z) = f (f z)
twice (plus_three, 5)  = 11
  // twice : ('a->'a) * 'a -> 'a

let plus_four x = x + 4
let pick_fn n =
  if n > 0 then plus_three else plus_four
(pick_fn 5)  0  = 3
  // pick_fn : int -> (int->int)
```


The map Function

- Let's write the map function (just like Ruby's collect)
  - Takes a function and a list, applies the function to each element of the list, and returns a list of the results

```
let rec map (f, l) = match l with
    [] -> []
  | (h::t) -> (f h)::(map (f, t))
```

```
let add_one x = x + 1
let negate x = -x
map (add_one, [1; 2; 3]) = [2; 3; 4]
map (negate, [9; -5; 0]) = [-9; 5; 0]
```


The map Function (cont.)

- What is the type of the map function?

```
let rec map (f, l) = match l with
    [] -> []
  | (h::t) -> (f h)::(map (f, t))
```

$\underbrace{\texttt{('a -> 'b)}}_{\texttt{f}} \texttt{ * } \underbrace{\texttt{'a list}}_{\texttt{l}} \texttt{ -> 'b list}$

Anonymous Functions

- Passing functions around is very common
  - So often we don't want to bother to give them names
- Use fun to make a function with no name

Parameter Body

```
fun x -> x + 3
```

```
twice ((fun x -> x + 3), 5)     = 11
map ((fun x -> x+1), [1; 2; 3])= [2; 3; 4]
```

Pattern Matching with fun

- match can be used within fun

```
map ((fun l -> match l with (h::_) -> h),
     [ [1; 2; 3]; [4; 5; 6; 7]; [8; 9] ])
     = [1; 4; 8]
```

- But use named functions for complicated matches
- May use standard pattern matching abbreviations

```
map ((fun (x, y) -> x+y), [(1,2); (3,4)])
     = [3; 7]
```


All Functions Are Anonymous

- Functions are first-class, so you can bind them to other names as you like

```
let f x = x + 3
let g = f
g 5  = 8
```

- In fact, let for functions is just shorthand

```
let f x = body
```

↓ stands for

```
let f = fun x -> body
```


Examples – Anonymous Functions

- `let next x = x + 1`
  - Short for `let next = fun x -> x + 1`
- `let plus (x, y) = x + y`
  - Short for `let plus = fun (x, y) -> x + y`
  - Which is short for

```
let plus = fun z ->
        (match z with (x, y) -> x + y)
```


Examples – Anonymous Functions

```
let rec fact n =
     if n = 0 then 1 else n * fact (n-1)
```

- Short for `let rec fact = fun n ->`

```
(if n = 0 then 1 else n * fact (n-1))
```


The fold Function

- Common pattern
  - Iterate through list and apply function to each element, keeping track of partial results computed so far

```
let rec fold (f, a, l) = match l with
    [] -> a
  | (h::t) -> fold (f, f (a, h), t)
```

  - a = "accumulator"
  - Usually called fold left to remind us that f takes the accumulator as its first argument
- What's the type of fold?

```
= ('a * 'b -> 'a) * 'a * 'b list -> 'a
```


Example

```
let rec fold (f, a, l) = match l with
    [] -> a
  | (h::t) -> fold (f, f (a, h), t)
```

```
let add (a, x) = a + x
fold (add, 0, [1; 2; 3; 4]) →
fold (add, 1, [2; 3; 4]) →
fold (add, 3, [3; 4]) →
fold (add, 6, [4]) →
fold (add, 10, []) →
10
```

We just built the `sum` function!

Another Example

```
let rec fold (f, a, l) = match l with
    [] -> a
  | (h::t) -> fold (f, f (a, h), t)
```

```
let next (a, _) = a + 1
fold (next, 0, [2; 3; 4; 5]) →
fold (next, 1, [3; 4; 5]) →
fold (next, 2, [4; 5]) →
fold (next, 3, [5]) →
fold (next, 4, []) →
4
```

We just built the `length` function!

Using fold to Build rev

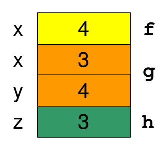
```
let rec fold (f, a, l) = match l with
    [] -> a
  | (h::t) -> fold (f, f (a, h), t)
```

- Can you build the reverse function with fold?

```
let prepend (a, x) = x::a
fold (prepend, [], [1; 2; 3; 4]) →
fold (prepend, [1], [2; 3; 4]) →
fold (prepend, [2; 1], [3; 4]) →
fold (prepend, [3; 2; 1], [4]) →
fold (prepend, [4; 3; 2; 1], []) →
[4; 3; 2; 1]
```


The Call Stack in C/Java/etc.

```
void f(void) {
  int x;
  x = g(3);
}
int g(int x) {
  int y;
  y = h(x);
  return y;
}
int h (int z) {
  return z + 1;
}
int main(){
  f();
  return 0;
}
```



Nested Functions

- In OCaml, you can define functions anywhere
  - Even inside of other functions

```
let sum l =
  fold ((fun (a, x) -> a + x), 0, l)
```

```
let pick_one n =
  if n > 0 then (fun x -> x + 1)
  else (fun x -> x - 1)
(pick_one -5) 6   (* returns 5 *)
```


Nested Functions (cont.)

- You can also use let to define functions inside of other functions

```
let sum l =
  let add (a, x) = a + x in
  fold (add, 0, l)
```

```
let pick_one n =
  let add_one x = x + 1 in
  let sub_one x = x - 1 in
  if n > 0 then add_one else sub_one
```


How About This?

```
let addN (n, l) =
  let add x = n + x in
  map (add, l)
```

Accessing variable from outer scope

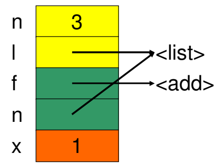
- (Equivalent to...)

```
let addN (n, l) =
  map ((fun x -> n + x), l)
```

Consider the Call Stack Again

```
let map (f, n) = match n with
  [] -> []
| (h::t) -> (f h)::(map (f, t))
let addN (n, l) =
  let add x = n + x in
  map (add, l)

addN (3, [1; 2; 3])
```



- Uh oh...how does add know the value of n?
  - Dynamic scoping: it reads it off the stack
    - The language could do this, but can be confusing (see above)
  - OCaml uses static scoping like C, C++, Java, and Ruby

Static Scoping

- In static or lexical scoping, (nonlocal) names refer to their nearest binding in the program text
  - Going from inner to outer scope
  - In our example, add refers to addN's n
  - C example:

Refers to the **x** at file scope – that's the nearest **x** going from inner scope to outer scope in the source code

```
int x;
void f() { x = 3; }
void g() { char *x = "hello"; f(); }
```


Returned Functions

- As we saw, in OCaml a function can return another function as a result
  - So consider the following example

```
let addN n = (fun x -> x + n)
(addN 3) 4   (* returns 7 *)
```

  - When the anonymous function is called, n isn't even on the stack any more!
    - We need some way to keep n around after addN returns

Environments and Closures

- An environment is a mapping from variable names to values
  - Just like a stack frame

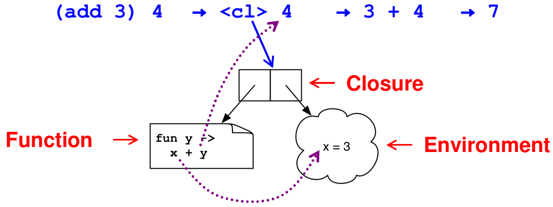
- A closure is a pair (f, e) consisting of function code f and an environment e

- When you invoke a closure, f is evaluated using e to look up variable bindings

Example – Closure 1

```
let add x = (fun y -> x + y)
```

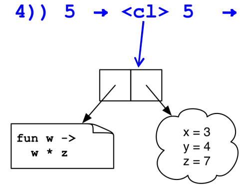
(add 3) 4 → <cl> 4 → 3 + 4 → 7



Example – Closure 2

```
let mult_sum (x, y) =
  let z = x + y in
    fun w -> w * z
```

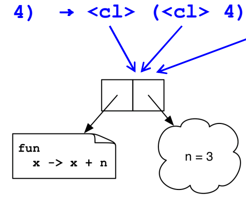
(mult_sum (3, 4)) 5 → <cl> 5 → 5 * 7 → 35



Example – Closure 3

```
let twice (n, y) =
  let f x = x + n in
    f (f y)
```

```
twice (3, 4)  → <cl> (<cl> 4) → <cl> 7 → 10
```

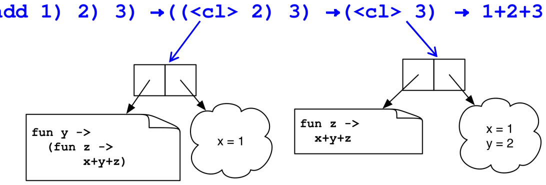


Example – Closure 4

```
let add x = (fun y -> (fun z -> x + y + z))
```

add( ) took 3 arguments?

```
(((add 1) 2) 3) →((<cl> 2) 3) →(<cl> 3) → 1+2+3
```



Currying

- We just saw another way for a function to take multiple arguments
  - The function consumes one argument at a time, creating closures until all the arguments are available
- This is called currying the function
  - Named after the logician Haskell B. Curry
  - But Schönfinkel and Frege discovered it
    - So it should probably be called Schönfinkelizing or Fregging

Curried Functions in OCaml

- OCaml has a really simple syntax for currying

```
let add x y = x + y
```

  - This is identical to all of the following

```
let add = (fun x -> (fun y -> x + y))
let add = (fun x y -> x + y)
let add x = (fun y -> x+y)
```


Curried Functions in OCaml (cont.)

- What is the type of add?

```
let add x y = x + y
```

- Answer
  - `add` has type `int -> (int -> int)`
  - `add 3` has type `int -> int`
    - `add 3` is a function that adds 3 to its argument
  - `(add 3) 4 = 7`
- This works for any number of arguments

Curried Functions in OCaml (cont.)

- Currying is so common, OCaml uses the following conventions
  - `->` associates to the right
    - `int -> int -> int` is the same as `int -> (int -> int)`
  - Function application `( )` associates to the left
    - `add 3 4` is the same as `(add 3) 4`

Another Example of Currying

- A curried add function with three arguments

```
let add_th x y z = x + y + z
```

  is the same as

```
let add_th x = (fun y -> (fun z -> x+y+z))
```

- Then...
  - `add_th` has type `int -> (int -> (int -> int))`
  - `add_th 4` has type `int -> (int -> int)`
  - `add_th 4 5` has type `int -> int`
  - `add_th 4 5 6` is `15`

Recall Functions map & fold

- Map

```
let rec map (f, l) = match l with
    [] -> []
  | (h::t) -> (f h)::(map (f, t))
```

  - Type = `('a -> 'b) * 'a list -> 'b list`

- Fold

```
let rec fold (f, a, l) = match l with
    [] -> a
  | (h::t) -> fold (f, f (a, h), t)
```

  - Type = `('a * 'b -> 'a) * 'a * 'b list -> 'a`

Currying and the map Function

- New Map

```
let rec map f l = match l with
    [] -> []
  | (h::t) -> (f h)::(map f t)
```

- Examples

```
let negate x = -x
map negate [1; 2; 3] (* [-1; -2; -3 ] *)
let negate_list = map negate
negate_list [-1; -2; -3] (* [1; 2; 3 ] *)
let sum_pair_l = map (fun (a, b) -> a + b)
sum_pair_l[(1, 2); (3, 4)] (* [3; 7] *)
```

- What is the type of this form of map?
  `('a -> 'b) -> 'a list -> 'b list`

Currying and the fold Function

- New Fold

```
let rec fold f a l = match l with
    [] -> a
  | (h::t) -> fold f (f a h) t
```

- Examples

```
let add x y = x + y
fold add 0 [1; 2; 3]    (* 6 *)
let sum = fold add 0
sum [1; 2; 3]           (* 6 *)
let next n _ = n + 1
let len = fold next 0   (* len not polymorphic! *)
len [4; 5; 6; 7; 8]     (* 5 *)
```

- What is the type of this form of fold?
  `('a -> 'b -> 'a) -> 'a -> 'b list -> 'a`

Another Convention

- Since functions are curried, function can often be used instead of match
  - function declares anonymous function w/ one argument
  - Instead of

```
let rec sum l = match l with
    [] -> 0
  | (h::t) -> h + (sum t)
```

  - It could be written

```
let rec sum = function
    [] -> 0
  | (h::t) -> h + (sum t)
```


Another Convention (cont.)

- Instead of

```
let rec map f l = match l with
    [] -> []
  | (h::t) -> (f h)::(map f t)
```

- It could be written

```
let rec map f = function
    [] -> []
  | (h::t) -> (f h)::(map f t)
```

Currying is Standard in OCaml

- Pretty much all functions are curried
  - Like the standard library map, fold, etc.
  - See /usr/local/ocaml/lib/ocaml on linuxlab
    - In particular, look at the file list.ml for standard list functions
    - Access these functions using `List.<fn name>`
    - E.g., `List.hd`, `List.length`, `List.map`

- OCaml plays a lot of tricks to avoid creating closures and to avoid allocating on the heap
  - It's unnecessary much of the time, since functions are usually called with all arguments