확률 및 통계

Quiz 1,

Assignment 1

2024.04.05.

퀴즈 답안

- **퀴즈 #1 (4/5)**

1. A test in which the outcome is uncertain. → random experiment (확률 실험)

2. A complete collection of outcomes. All of the possible outcomes of an experiment.
   → sample space (표본공간)

3. A subset of the sample space of a random experiment. A single outcome or a collection of outcomes from a sample space. → event (사건)

4. A real number on a scale of 0 to 1 that represent the likelihood or chance of a certain event occurring. → probability (확률)

5. The ratio of the number of outcomes in which a specified event occurs to the total number of trials. The occurrence of an event in a large number of repetitions of the experiment. → relative frequency (상대빈도)

6. A branch of mathematics that deals with counting and enumerating the number of ways to arrange or select objects from a set. It involves the study of permutations, combinations, and other related concepts. → combinatorics (조합론)

퀴즈 답안

- **퀴즈 #1 (4/5)**

7. A branch of mathematics that deal with the collection, organization, analysis, interpretation, and presentation of data. → statistics (통계)

8. A branch of statistics that describes or summarizes features from a collection of information. Without assumptions about the underlying probability distribution, it is concerned with summarizing and presenting data in a meaningful and informative way.
→ descriptive statistics (기술통계)

9. A function of sample values. Any quantity computed from values in a sample which is considered for a statistical purpose. → statistic (통계량)

10. A measure of central tendency. The sum of a collection of values divided by the number of the values in the collection. → arithmetic mean (산술평균)

11. A measure of dispersion. The average of the squared differences of each data point from the mean → variance (분산)

퀴즈 답안

- **퀴즈 #1 (4/5)**

  12. A statement or principle that is accepted to be true, but need not be so. Fundamental assumption without proof. → axiom (공리)

  13. Events that cannot occur simultaneously. Only one event can occur at a time, i.e., the occurrence of one event precludes the occurrence of other events.
  → Mutually exclusive (상호배타)

  14. Events that together cover all possible outcomes of a particular experiment. One of the events must always occur. → collectively exhaustive (전체포괄)

  15. Two experiments or events that have nothing to do with each other. The occurrence of the first event does not affect the probability of the occurrence of the second event.
  → statistically independent (통계적독립)

  16. The probability of an event occurring given that another event has already occurred. When two events are independent, conditioning one upon the other has no effect.
  → conditional probability (조건부확률)

퀴즈 답안

- **퀴즈 #1 (4/5)**

17. It describes the probability of an event, based on prior knowledge of conditions that might be related to the event, i.e., one conditional probability to be computed from the other. This rule can be used to compute the probability of an event based on new evidence or information. → Bayes' rule (베이즈규칙)

18. A mapping from the sample space to the real line. A variable whose numerical value is determined by the outcome of a random experiment. → random variable (확률변수)

19. A function that describes the probabilities of values of a random variable.
→ probability distribution (확률분포)

20. A random variable all of whose values are equally likely.
→ uniform random variable (균일 확률변수)

21. A discrete random variable that takes on only one of two discrete values. e.g., 0 or 1.
→ Bernoulli random variable (베르누이 확률변수)

퀴즈 답안

- **퀴즈 #1 (4/5)**

22. A discrete random variable that represents the number of successes in a fixed number of independent and identical Bernoulli trials.
→ binomial random variable (이항 확률변수)

23. A discrete random variable that represents the number of rare events that occur at a constant rate in a fixed interval of time or unit space.
→ Poisson random variable (푸아송 확률변수)

24. A discrete random variable that represents the number of independent and identical Bernoulli trials required to obtain the first successes.
→ geometric random variable (기하 확률변수)

25. A discrete random variable that represents the number of independent and identical Bernoulli trials required to obtain a certain number of successes.
→ Pascal random variable (파스칼 확률변수)

퀴즈 답안

- **퀴즈 #1 (4/5)**

26. A continuous random variable that represents the time between two successive events that occur independently. It is also closely related to the Poisson random variable.

→ exponential random variable (지수 확률변수)

27. A continuous random variable that represents the waiting time for a certain number of/multiple events that occur independently. → Erlang random variable (얼랑 확률변수)

28. A continuous random variable that has a symmetric/double exponential probability distribution with respect to its mean and a scale parameter. Its probability distribution is expressed in terms of the absolute difference from the mean.

→ Laplace random variable (라플라스 확률변수)

29. A continuous random variable that has a bell-shaped probability distribution with a single peak at its mean value and a dispersion parameter. Its probability distribution is expressed in terms of the squared difference from the mean.

→ Gaussian/Normal random variable (가우시안 확률변수)

30. A sum of the squares of a certain number of independent standard normal random variables. Its probability distribution can be interpreted as a special case of the gamma distribution. → chi-square random variable (카이제곱 확률변수)

과제 범위

- **Assignment #1 (~4/5)**
  - 연습 1. Problems 2.1 (b)
  - 연습 2. Problems 2.2
  - 연습 3. Problems 2.5
  - 연습 4. Problems 2.13
  - 연습 5. Problems 2.14
  - 연습 6. Problems 2.31
  - 연습 7. Problems 2.32
  - 연습 8. Problems 2.36
  - 연습 9. Problems 2.43
  - 실습 1. 도수분포표와 히스토그램 (Excel)
  - 실습 2. Example 2.6 (Python)

연습 1. Problems 2.1 (b)

2.1 Draw a set of Venn diagrams to illustrate each of the following identities in the algebra of events.

(a) $A \cdot (B + C) = AB + AC$

(b) $A + (BC) = (A + B)(A + C)$

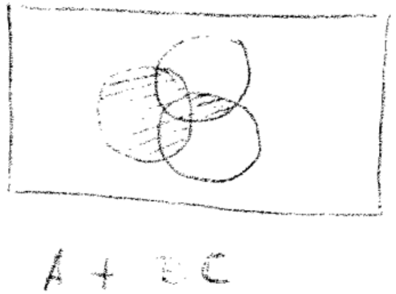
(c) $(AB)^c = A^c + B^c$

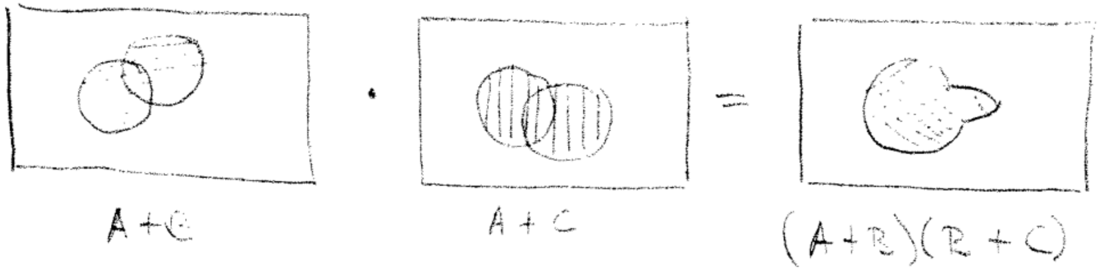
(d) $(A + B)^c = A^c B^c$

(e) $A + A^c B = A + B$

(f) $AB + B = B$

(g) $A + AB + B = A + B$





연습 1. Problems 2.1

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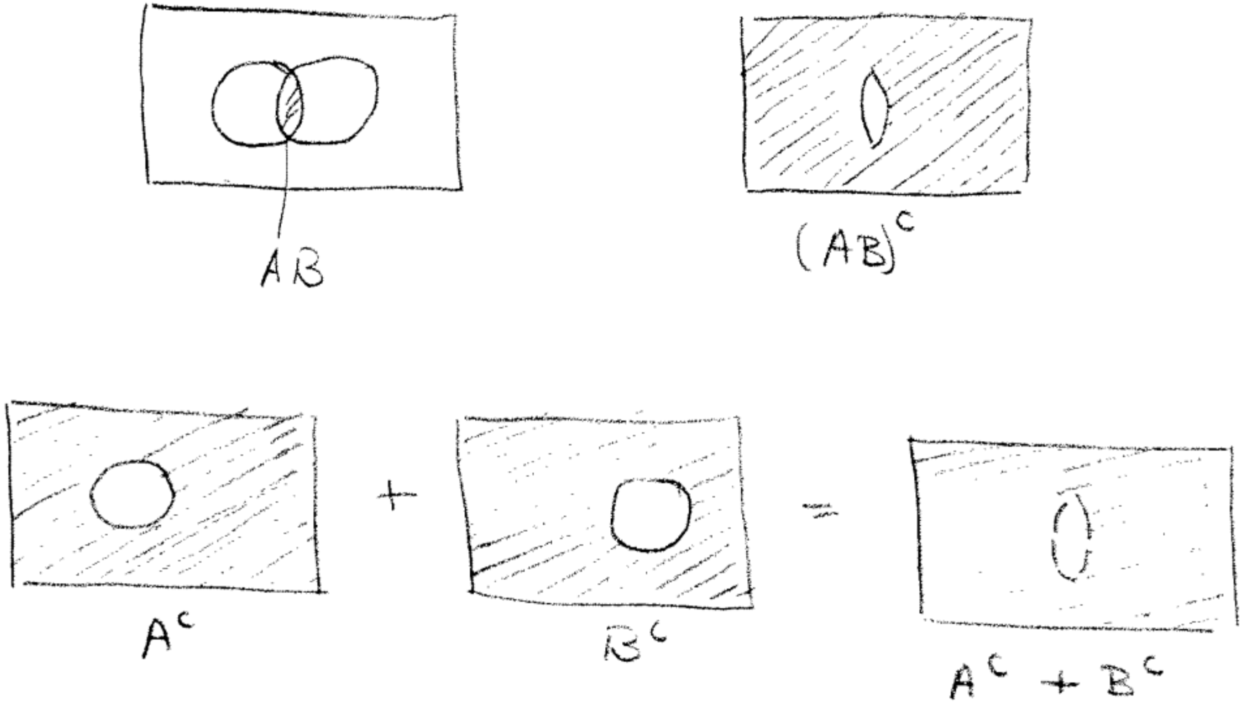

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[✓ (d)] $(A + B)^c = A^c B^c$

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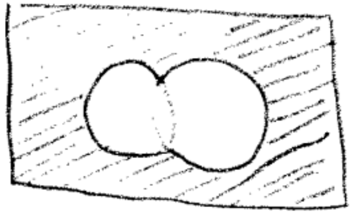
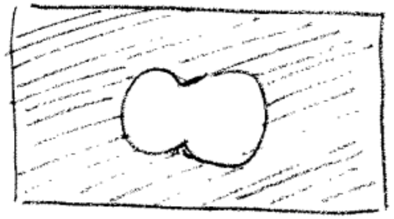
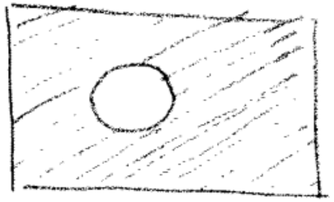


$(A+B)^c$

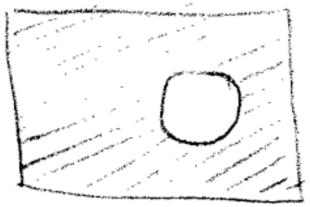
see above for diagrams of A^c and B^c ⇒



$A^c \cdot B^c$

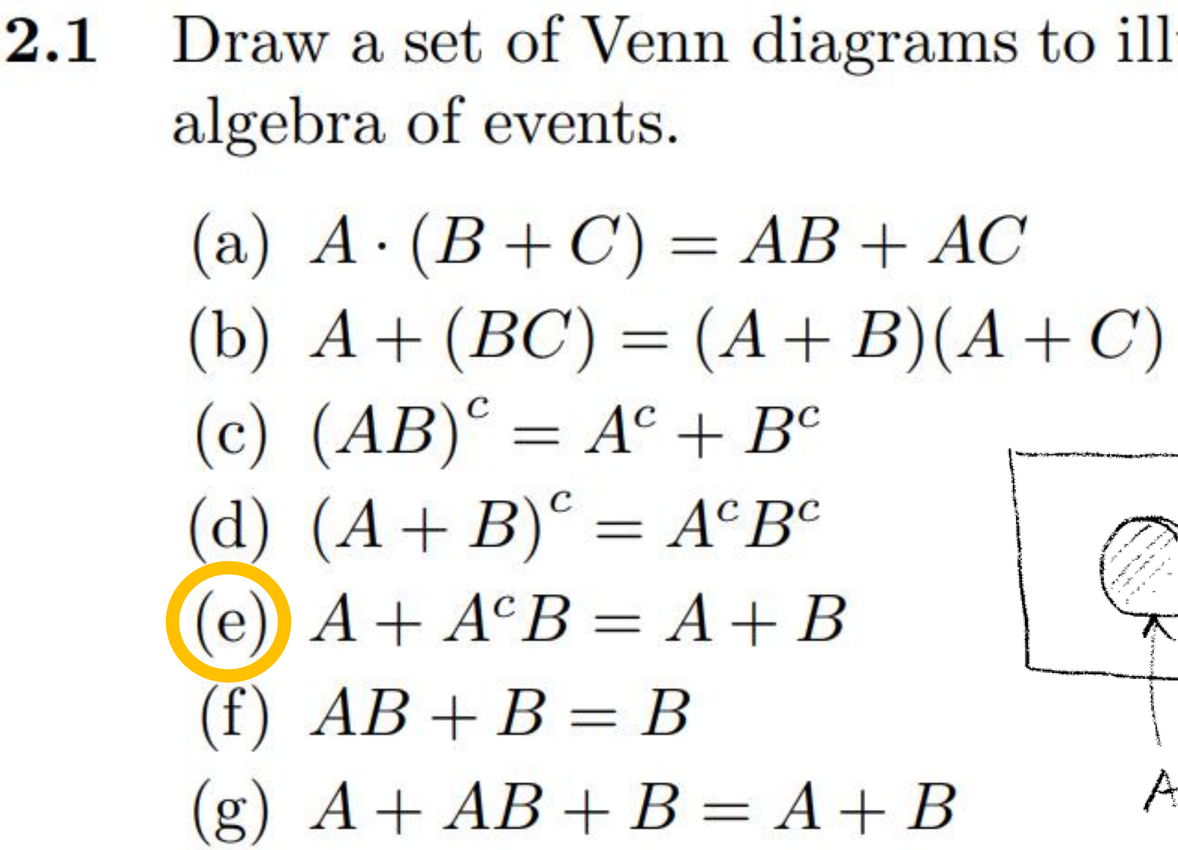


A^c



B^c

연습 1. Problems 2.1



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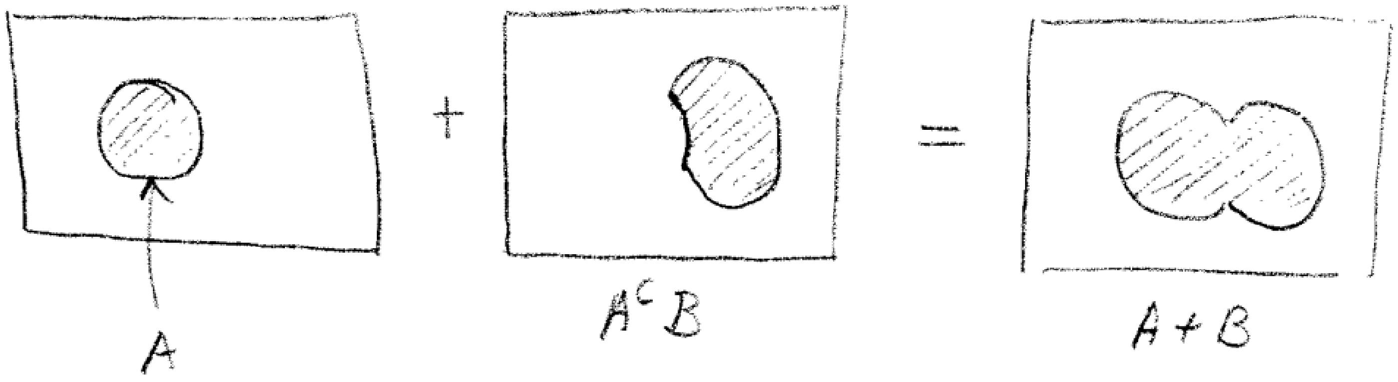

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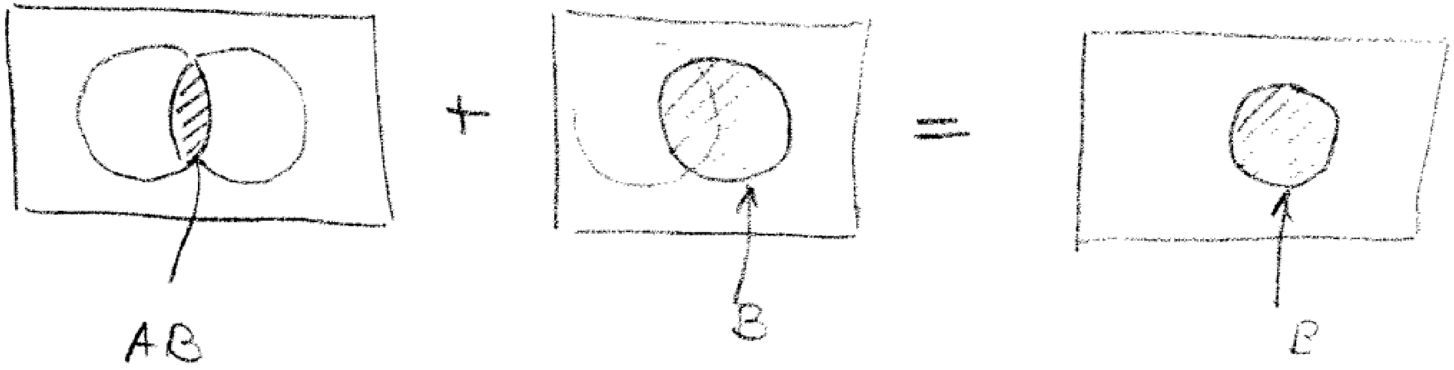

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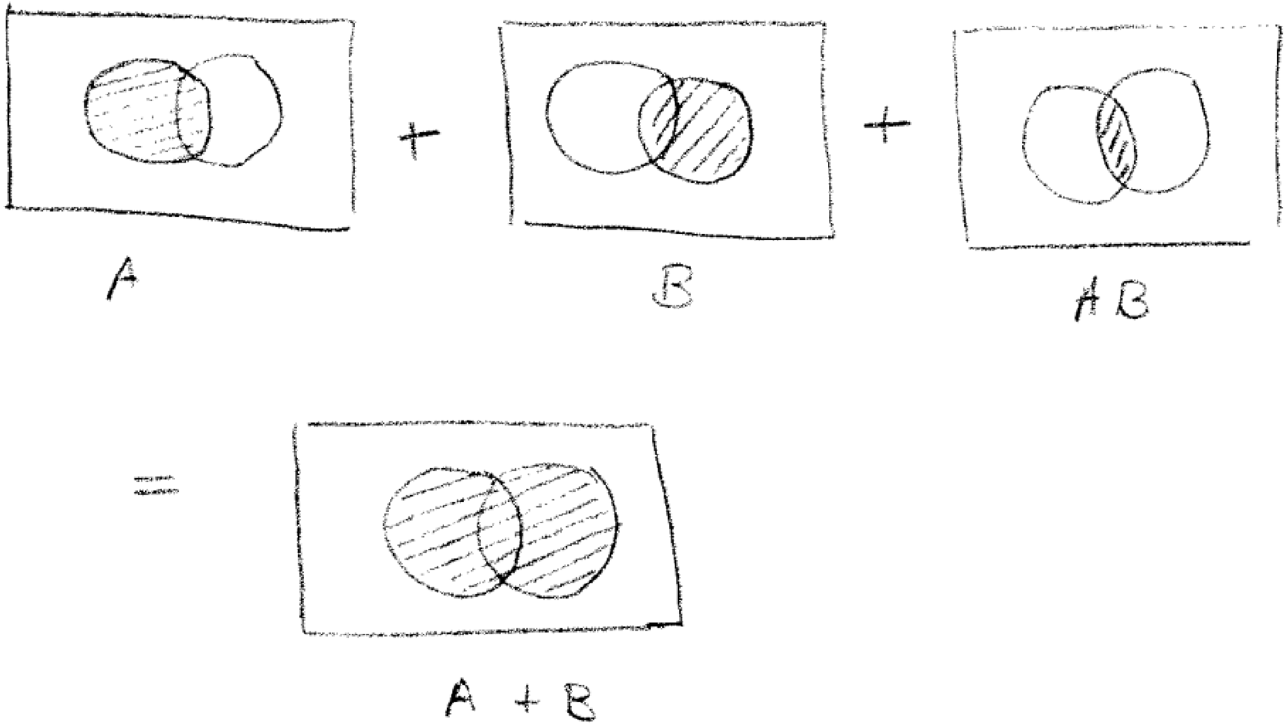


연습 2. Problems 2.2

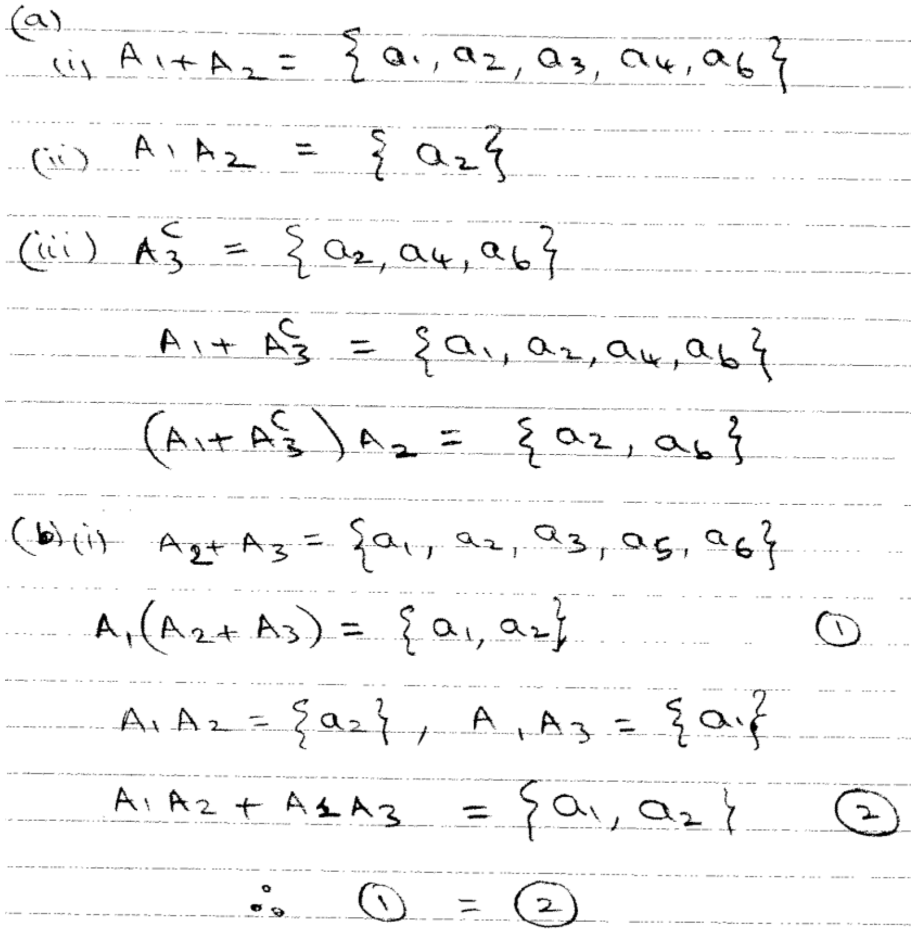
2.2 A sample space S is given to be $\{a_1, a_2, a_3, a_4, a_5, a_6\}$. The following events are defined on this sample space: $A_1 = \{a_1, a_2, a_4\}$, $A_2 = \{a_2, a_3, a_6\}$, and $A_3 = \{a_1, a_3, a_5\}$.

(a) Find the following events:

- (i) $A_1 + A_2$
- (ii) $A_1 A_2$,
- (iii) $(A_1 + A_3^c)A_2$

(b) Show the following identities:

- (i) $A_1(A_2 + A_3) = A_1A_2 + A_1A_3$
- (ii) $A_1 + A_2A_3 = (A_1 + A_2)(A_1 + A_3)$
- (iii) $(A_1 + A_2)^c = A_1^c A_2^c$



(a)

(i) $A_1 + A_2 = \{a_1, a_2, a_3, a_4, a_6\}$

(ii) $A_1 A_2 = \{a_2\}$

(iii) $A_3^c = \{a_2, a_4, a_6\}$

$A_1 + A_3^c = \{a_1, a_2, a_4, a_6\}$

$(A_1 + A_3^c)A_2 = \{a_2, a_6\}$

(b)(i) $A_2 + A_3 = \{a_1, a_2, a_3, a_5, a_6\}$

$A_1(A_2 + A_3) = \{a_1, a_2\}$ ①

$A_1A_2 = \{a_2\}$, $A_1A_3 = \{a_1\}$

$A_1A_2 + A_1A_3 = \{a_1, a_2\}$ ②

∴ ① = ②

연습 2. Problems 2.2

2.2 A sample space S is given to be $\{a_1, a_2, a_3, a_4, a_5, a_6\}$. The following events are defined on this sample space: $A_1 = \{a_1, a_2, a_4\}$, $A_2 = \{a_2, a_3, a_6\}$, and $A_3 = \{a_1, a_3, a_5\}$.

(a) Find the following events:

- (i) $A_1 + A_2$
- (ii) $A_1 A_2$,
- (iii) $(A_1 + A_3^c)A_2$

(b) Show the following identities:

- (i) $A_1(A_2 + A_3) = A_1 A_2 + A_1 A_3$
- (ii) $A_1 + A_2 A_3 = (A_1 + A_2)(A_1 + A_3)$
- (iii) $(A_1 + A_2)^c = A_1^c A_2^c$

$A_1 + A_2 A_3 = (A_1 + A_2)(A_1 + A_3)$

$A_2 A_3 = \{a_3\}$

$A_1 + A_2 A_3 = \{a_1, a_2, a_3, a_4\}$ ①

$A_1 + A_2 = \{a_1, a_2, a_3, a_4, a_6\}$

$A_1 + A_3 = \{a_1, a_2, a_3, a_4, a_5\}$

$(A_1 + A_2)(A_1 + A_3) = \{a_1, a_2, a_3, a_4\}$ ②

$\therefore$ ① = ②

연습 2. Problems 2.2

2.2 A sample space S is given to be $\{a_1, a_2, a_3, a_4, a_5, a_6\}$. The following events are defined on this sample space: $A_1 = \{a_1, a_2, a_4\}$, $A_2 = \{a_2, a_3, a_6\}$, and $A_3 = \{a_1, a_3, a_5\}$.

(a) Find the following events:

(i) $A_1 + A_2$
(ii) $A_1 A_2$,
(iii) $(A_1 + A_3^c)A_2$

(b) Show the following identities:

(i) $A_1(A_2 + A_3) = A_1 A_2 + A_1 A_3$
(ii) $A_1 + A_2 A_3 = (A_1 + A_2)(A_1 + A_3)$
(iii) $(A_1 + A_2)^c = A_1^c A_2^c$

$A_1 + A_2 = \{a_1, a_2, a_3, a_4, a_6\}$

$(A_1 + A_2)^c = \{a_5\}$ ①

$A_1^c = \{a_3, a_5, a_6\}$, $A_2^c = \{a_1, a_4, a_5\}$

$A_1^c A_2^c = \{a_5\}$ ②

∴ ① = ②

연습 3. Problems 2.5

2.5 Consider the sample space in Prob. 2.2 in which all the outcomes are assumed equally likely. Find the following probabilities:

(a) $\Pr[A_1 A_2]$

(b) $\Pr[A_1 + A_2]$

(c) $\Pr[(A_1 + A_3^c)A_2]$

(i) ~~(ii)~~ $\Pr[A_1 A_2] = \frac{1}{6}$

(ii) ~~(i)~~ $\Pr[A_1 + A_2] = \Pr[A_1] + \Pr[A_2] - \Pr[A_1 A_2]$

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6}$

(iii) ~~(iii)~~ $\Pr[(A_1 + A_3^c)A_2] = \Pr[\{a_2, a_6\}]$

$= \frac{2}{6}$

연습 4. Problems 2.13

2.13 In the 2003 race for Governor of California the alphabet was "reordered" so that none of the 135 candidates would feel discriminated against by the position of their name on the ballot.

(a) The week of the election it was announced that the first four letters of the reordered alphabet were R, W, Q, O. Assuming that all letters were initially equally-likely, what was the probability that this *particular* sequence would be chosen? (There are 26 letters in the English alphabet.)

(b) The letter S (for Schwarzenegger) turned up at position 10 in the new alphabet. What is the probability that S would end up in the 10th position?

(a)

$N = 26 \cdot 25 \cdot 24 \cdot 23$

= ~~388,800~~ 358,800

$\text{Prob} = \dfrac{1}{358{,}800} = 2.787 \times 10^{-6}$

(b)

$\Pr[S \text{ in } 10^{th} \text{ position}] = \dfrac{1}{26}$

$= 0.038$

연습 5. Problems 2.14

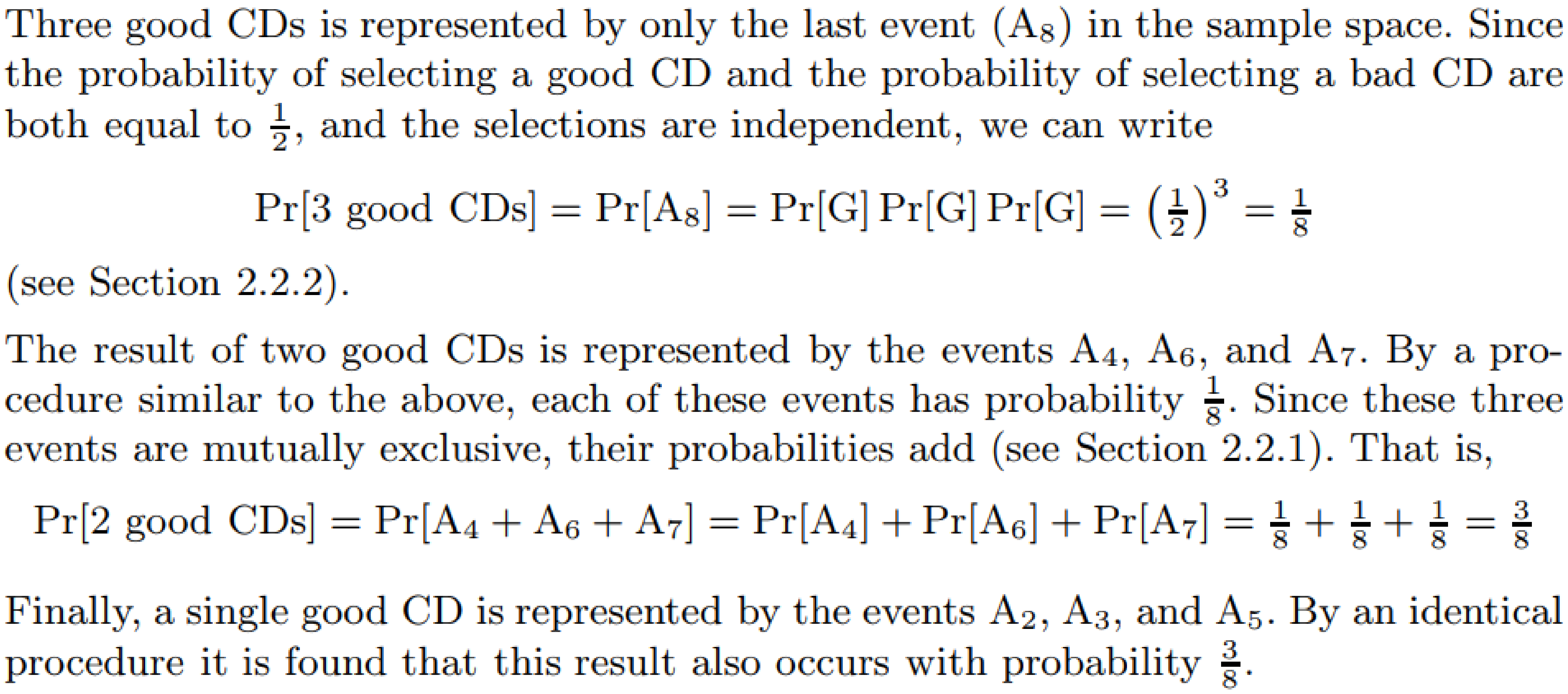
Example 2.2: CDs selected from the bins at Simon's Surplus are as likely to be good as to be bad. If three CDs are selected independently and at random, what is the probability of getting exactly *three* good CDs? Exactly *two* good CDs? How about *one* good CD?

Evidently buying a CD at Simon's is like tossing a coin. The sample space is represented by the listing of outcomes shown below: where G represents a good CD and B

BBB	BBG	BGB	BGG	GBB	GBG	GGB	GGG
A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8

represents a bad one. Each outcome is labeled as an event A_i; note that the events A_i are mutually exclusive and collectively exhaustive.

연습 5. Problems 2.14



Three good CDs is represented by only the last event (A_8) in the sample space. Since the probability of selecting a good CD and the probability of selecting a bad CD are both equal to $\frac{1}{2}$, and the selections are independent, we can write

$$\Pr[3 \text{ good CDs}] = \Pr[A_8] = \Pr[G]\Pr[G]\Pr[G] = \left(\tfrac{1}{2}\right)^3 = \tfrac{1}{8}$$

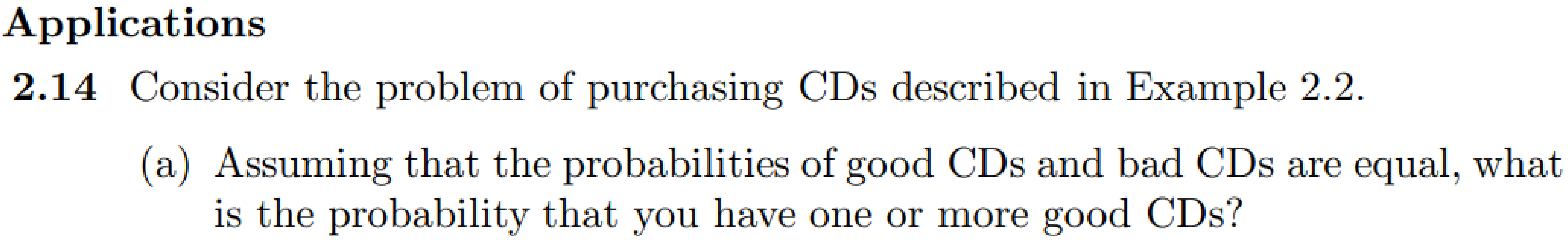
(see Section 2.2.2).

The result of two good CDs is represented by the events A_4, A_6, and A_7. By a procedure similar to the above, each of these events has probability $\frac{1}{8}$. Since these three events are mutually exclusive, their probabilities add (see Section 2.2.1). That is,

$$\Pr[2 \text{ good CDs}] = \Pr[A_4 + A_6 + A_7] = \Pr[A_4] + \Pr[A_6] + \Pr[A_7] = \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} = \tfrac{3}{8}$$

Finally, a single good CD is represented by the events A_2, A_3, and A_5. By an identical procedure it is found that this result also occurs with probability $\frac{3}{8}$.

연습 5. Problems 2.14



Applications

2.14 Consider the problem of purchasing CDs described in Example 2.2.

(a) Assuming that the probabilities of good CDs and bad CDs are equal, what is the probability that you have one or more good CDs?

$$\Pr[\text{one or more good}] = 1 - \Pr[\text{all bad}]$$

$$= 1 - \Pr[A_1] = 1 - \frac{1}{8} = \frac{7}{8}$$

연습 5. Problems 2.14

(b) If the probability of a good disk is 5/8, what is the probability that you have one or more good CDs?

In this case it is easiest to use the alternate approach above;

$$\Pr[\text{one or more good}] = 1 - \Pr[A_1]$$

However, in this case the probability of a bad disk is $\frac{3}{8}$, so

$$\Pr[A_1] = \left(\frac{3}{8}\right)^3 = \frac{27}{512}$$

$$\Pr[\text{one or more good}] = 1 - \frac{27}{512} = \frac{485}{512}$$

$$\simeq 0.947$$

연습 6. Problems 2.31

2.30 In a certain computer, the probability of a memory failure is 0.01, while the probability of a hard disk failure is 0.02. If the probability that the memory and the hard disk fail simultaneously is 0.0014, then

(a) Are memory failures and hard disk failures independent events?

(b) What is the probability of a memory failure, given a hard disk failure?

(a)

$$\Pr[M] \cdot \Pr[D] = (0.01)(0.02) = 0.0002$$

$$\Pr[MD] = 0.0014$$

M = memory failure

D = hard disk failure

$\therefore$ Not independent

(b)

$$\Pr[M|D] = \frac{\Pr[MD]}{\Pr[D]} = \frac{0.0014}{0.02} = .07$$

연습 6. Problems 2.31

2.31 Repeat Prob. 2.30 if the probability of a memory failure is 0.02, the probability of a disk failure is 0.015, and the probability that both fail simultaneously is 0.0003.

$$\Pr[M]\cdot\Pr[D] = (0.02)(0.015) = 0.0003$$

$\Pr[MD] = 0.0003$ ∴ <u>independent</u>

$$\Pr[M|D] = \frac{\Pr[MD]}{\Pr[D]} = \frac{0.0003}{0.015} = 0.02$$

$\Pr[M|D] = \Pr[M]$ since M & D

are independent

연습 7. Problems 2.32

2.32 In the triple-core computers that Simon's Surplus has just sold back to the government, the probability of a memory failure is 0.02, while the probability of a hard disk failure is 0.015. If the probability that the memory and the hard disk fail simultaneously is 0.0003, then

(a) Are memory failures and hard disk failures independent events?

(b) What is the probability of a memory failure, given a hard disk failure?

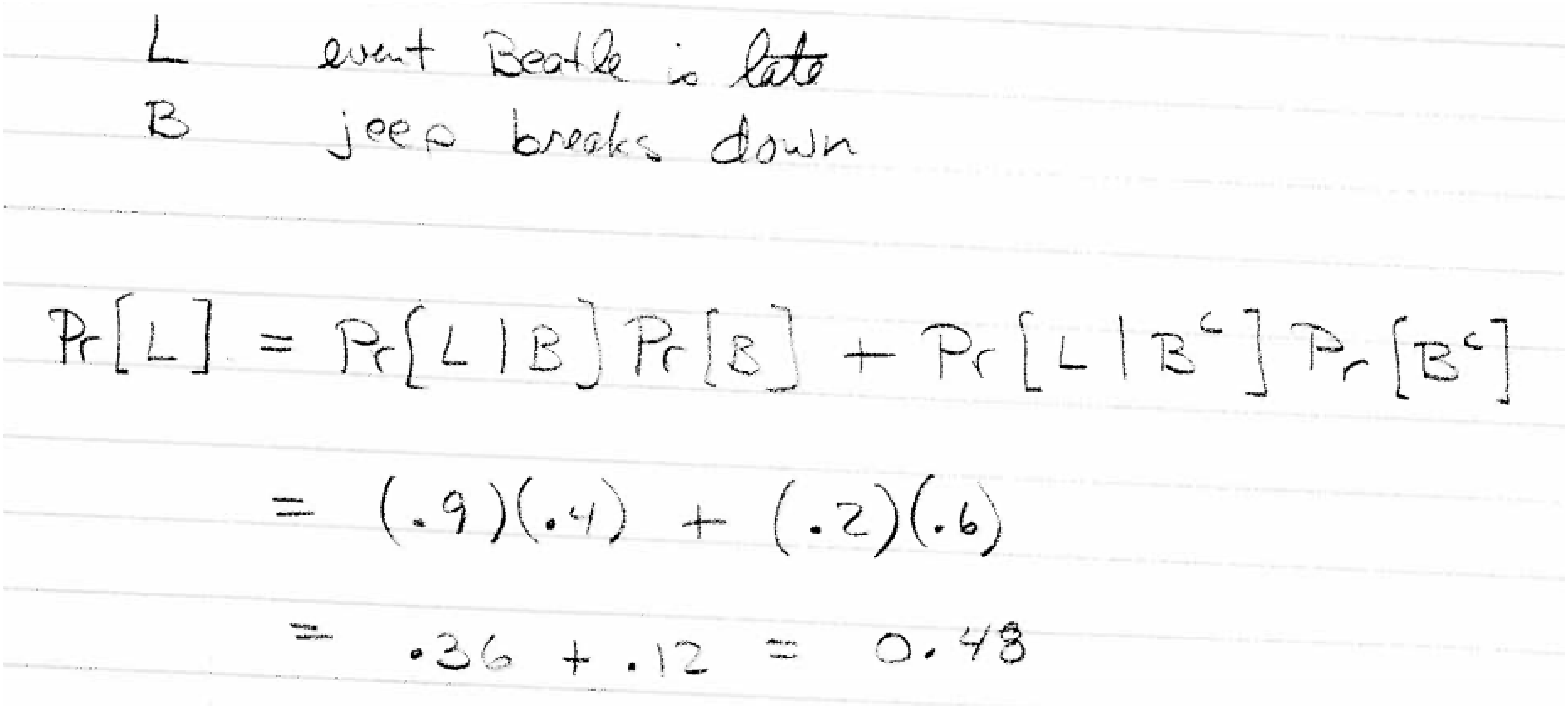
$\Pr[M]\cdot\Pr[D] = (0.02)(0.015) = 0.0003$

$\Pr[MD] = 0.0003$ ∴ independent

$\Pr[M|D] = \Pr[M]$ since M & D are independent

연습 8. Problems 2.36

2.36 Beetle Bailey has a date with Miss Buxley, but Beetle has an old jeep which will break down with probability 0.4. If his jeep breaks down he will be late with probability 0.9. If it does not break down he will be late with probability 0.2. What is the probability that Beetle will be late for his date?



L event Beetle is late

B jeep breaks down

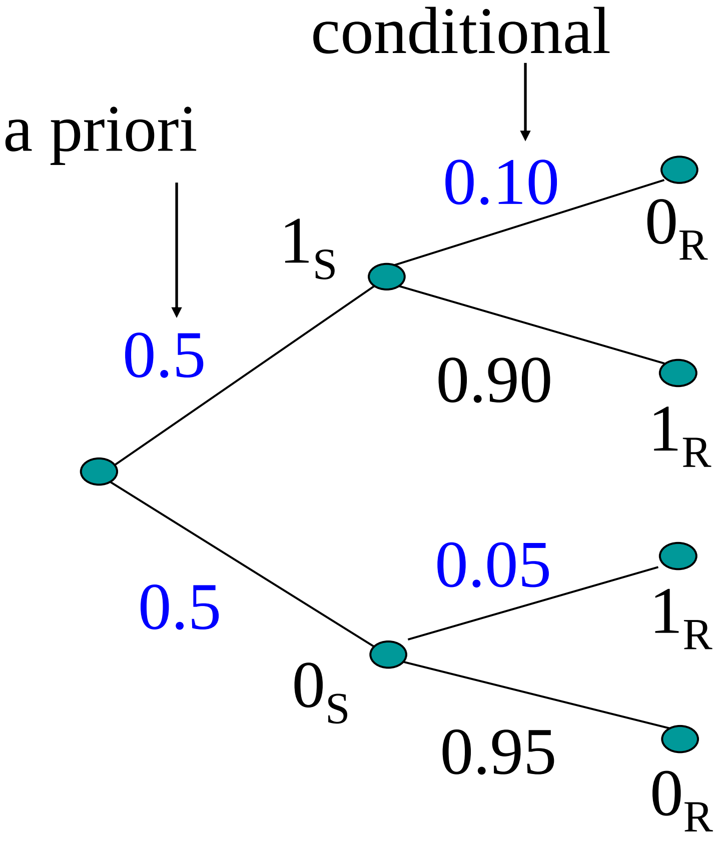
$$\Pr[L] = \Pr[L \mid B]\Pr[B] + \Pr[L \mid B^c]\Pr[B^c]$$

$$= (.9)(.4) + (.2)(.6)$$

$$= .36 + .12 = 0.48$$

연습 9. Problems 2.43

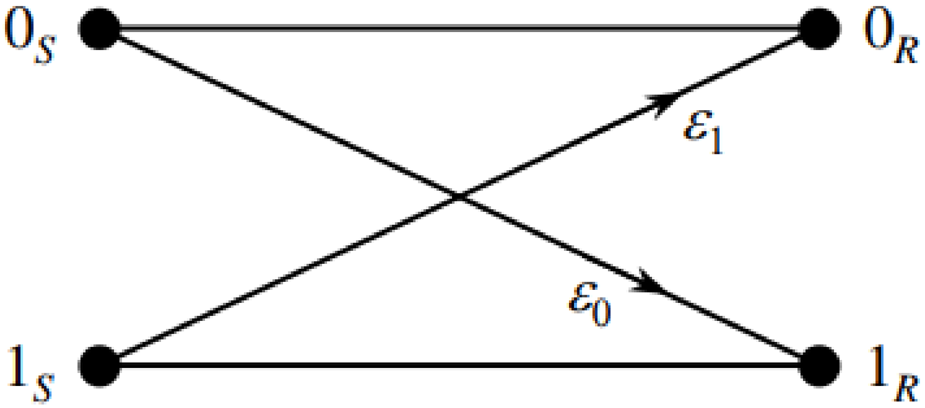
2.43 A binary communication channel is depicted below. Assume that the random experiment consists of transmitting a single binary digit and that the probability of transmitting a 0 or a 1 is the same.



	Probability
1_S0_R (error)	**0.050**
1_S1_R	0.450
0_S1_R (error)	**0.025**
0_S0_R	0.475

연습 9. Problems 2.43

2.43 A binary communication channel is depicted below. Assume that the random experiment consists of transmitting a single binary digit and that the probability of transmitting a 0 or a 1 is the same.



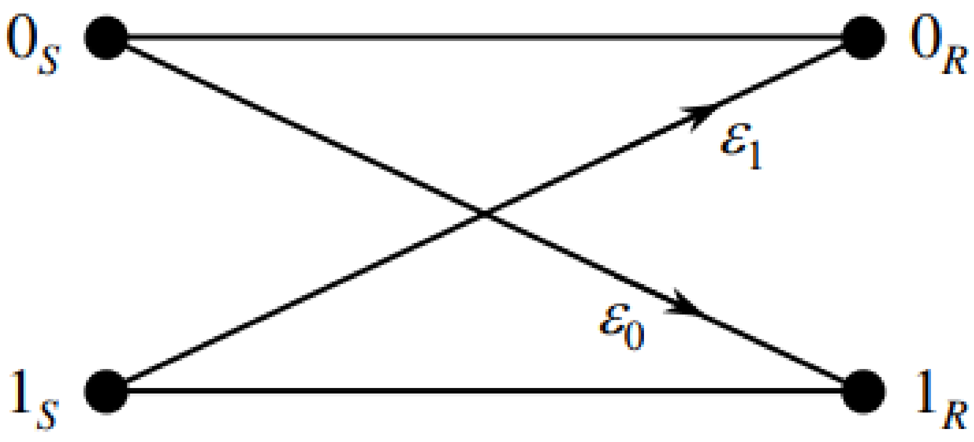
(a) Draw the sample space for the experiment and label each elementary event with its probability.

(b) What is the probability of an error?

(c) Given that an error occurred, what is the probability that a 1 was sent?

(d) What is the probability a 1 was sent given that a 1 was received?

연습 9. Problems 2.43



(a)

S	R	Prob
0	0	$\frac{1}{2}(1-\varepsilon_0)$
0	1	$\frac{1}{2}\varepsilon_0$
1	0	$\frac{1}{2}\varepsilon_1$
1	1	$\frac{1}{2}(1-\varepsilon_1)$

(b) "Error" = "0 1" ∪ "1 0"

$$Pr[\text{Error}] = \frac{1}{2}\varepsilon_0 + \frac{1}{2}\varepsilon_1 = \frac{1}{2}(\varepsilon_0 + \varepsilon_1)$$

(c)

$$Pr[1 \text{ sent} | \text{Error}] = \frac{Pr[1 \text{ sent} \cap \text{Error}]}{Pr[\text{Error}]}$$

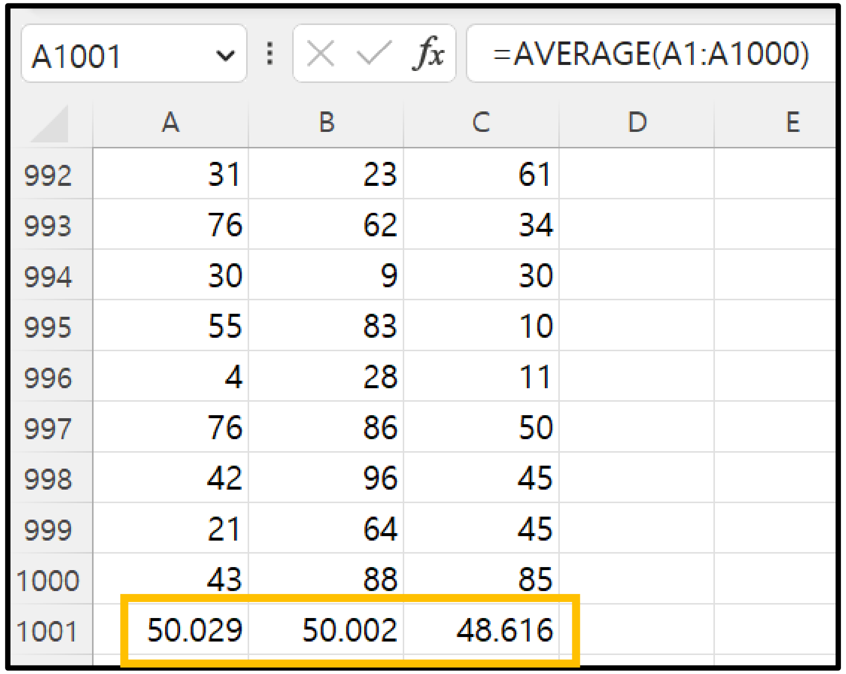
$$= \frac{\frac{1}{2}\varepsilon_1}{\frac{1}{2}(\varepsilon_0+\varepsilon_1)} = \frac{\varepsilon_1}{\varepsilon_0+\varepsilon_1}$$

(d)

$$Pr[1 \text{ sent} / 1 \text{ rcvd}] = \frac{Pr[1 \text{ rcvd} | 1 \text{ sent}] \cdot Pr[1 \text{ sent}]}{Pr[1 \text{ rcvd}]}$$

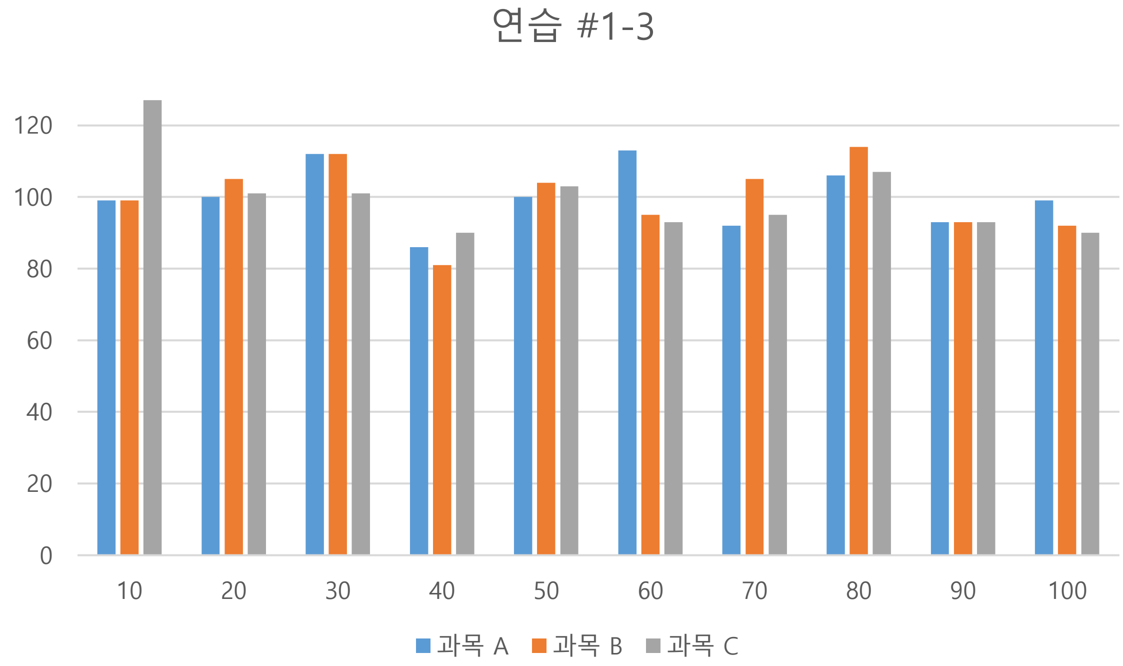
$$= \frac{\frac{1}{2}(1-\varepsilon_1)}{\frac{1}{2}\varepsilon_0 + \frac{1}{2}(1-\varepsilon_1)} = \frac{1-\varepsilon_1}{1-\varepsilon_1+\varepsilon_0}$$

실습 1. 도수분포표와 히스토그램 (Excel)

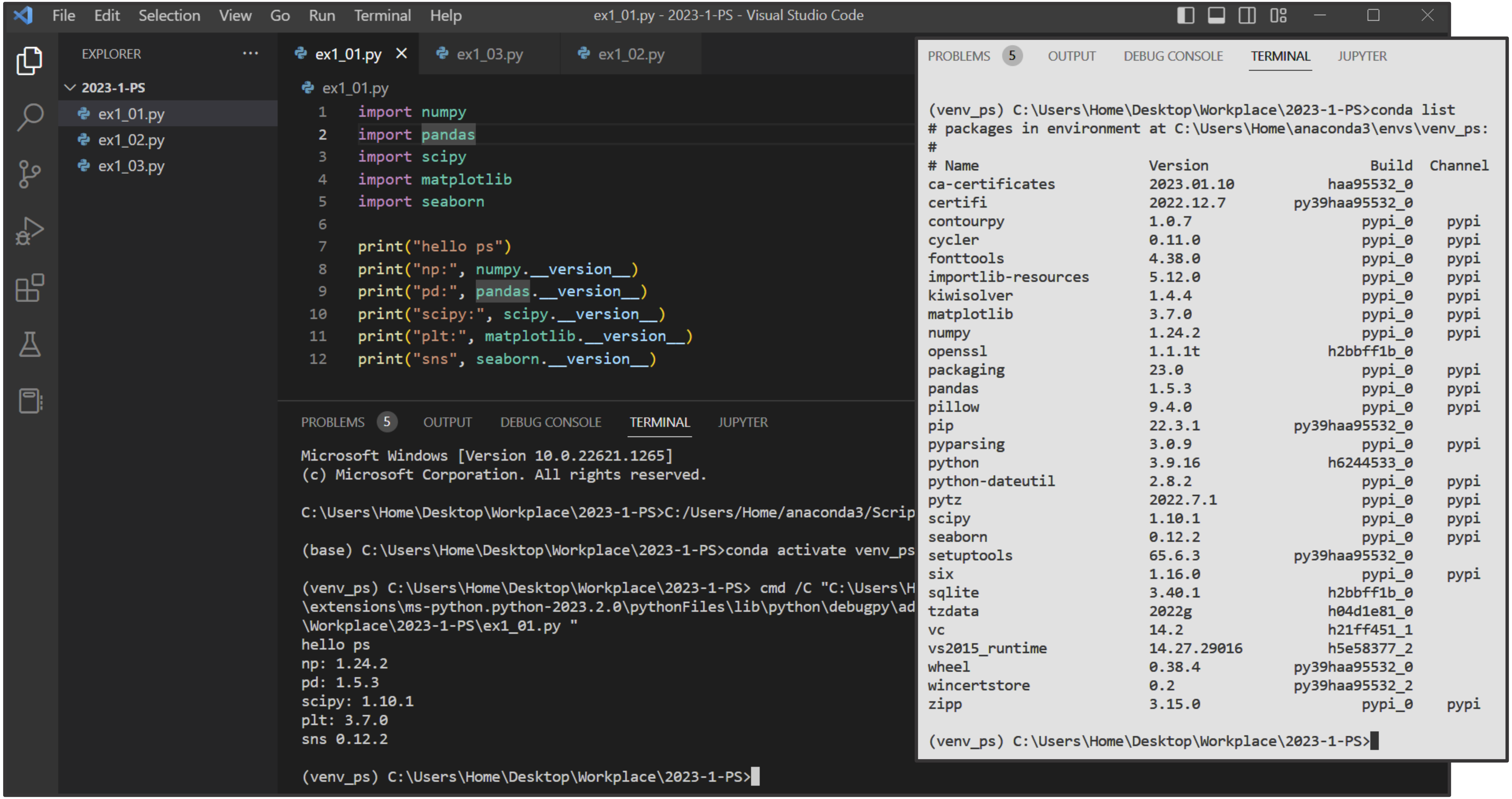


A1001 =AVERAGE(A1:A1000)

	A	B	C	D	E
992	31	23	61		
993	76	62	34		
994	30	9	30		
995	55	83	10		
996	4	28	11		
997	76	86	50		
998	42	96	45		
999	21	64	45		
1000	43	88	85		
1001	50.029	50.002	48.616		



실습 2. Example 2.6 (Python)



```python
import numpy
import pandas
import scipy
import matplotlib
import seaborn

print("hello ps")
print("np:", numpy.__version__)
print("pd:", pandas.__version__)
print("scipy:", scipy.__version__)
print("plt:", matplotlib.__version__)
print("sns", seaborn.__version__)
```

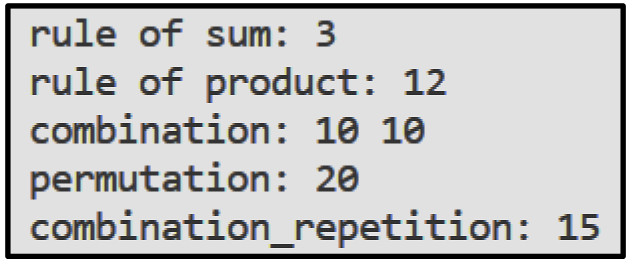
```
Microsoft Windows [Version 10.0.22621.1265]
(c) Microsoft Corporation. All rights reserved.

C:\Users\Home\Desktop\Workplace\2023-1-PS>C:/Users/Home/anaconda3/Scrip

(base) C:\Users\Home\Desktop\Workplace\2023-1-PS>conda activate venv_ps

(venv_ps) C:\Users\Home\Desktop\Workplace\2023-1-PS> cmd /C "C:\Users\H
\extensions\ms-python.python-2023.2.0\pythonFiles\lib\python\debugpy\ad
\Workplace\2023-1-PS\ex1_01.py "
hello ps
np: 1.24.2
pd: 1.5.3
scipy: 1.10.1
plt: 3.7.0
sns 0.12.2

(venv_ps) C:\Users\Home\Desktop\Workplace\2023-1-PS>
```

```
(venv_ps) C:\Users\Home\Desktop\Workplace\2023-1-PS>conda list
# packages in environment at C:\Users\Home\anaconda3\envs\venv_ps:
#
# Name                    Version                   Build  Channel
ca-certificates           2023.01.10           haa95532_0
certifi                   2022.12.7        py39haa95532_0
contourpy                 1.0.7                    pypi_0    pypi
cycler                    0.11.0                   pypi_0    pypi
fonttools                 4.38.0                   pypi_0    pypi
importlib-resources       5.12.0                   pypi_0    pypi
kiwisolver                1.4.4                    pypi_0    pypi
matplotlib                3.7.0                    pypi_0    pypi
numpy                     1.24.2                   pypi_0    pypi
openssl                   1.1.1t               h2bbff1b_0
packaging                 23.0                     pypi_0    pypi
pandas                    1.5.3                    pypi_0    pypi
pillow                    9.4.0                    pypi_0    pypi
pip                       22.3.1           py39haa95532_0
pyparsing                 3.0.9                    pypi_0    pypi
python                    3.9.16               h6244533_0
python-dateutil           2.8.2                    pypi_0    pypi
pytz                      2022.7.1                 pypi_0    pypi
scipy                     1.10.1                   pypi_0    pypi
seaborn                   0.12.2                   pypi_0    pypi
setuptools                65.6.3           py39haa95532_0
six                       1.16.0                   pypi_0    pypi
sqlite                    3.40.1               h2bbff1b_0
tzdata                    2022g                h04d1e81_0
vc                        14.2                 h21ff451_1
vs2015_runtime            14.27.29016          h5e58377_2
wheel                     0.38.4           py39haa95532_0
wincertstore              0.2              py39haa95532_2
zipp                      3.15.0                   pypi_0    pypi

(venv_ps) C:\Users\Home\Desktop\Workplace\2023-1-PS>
```

실습 2. Example 2.6 (Python)

ex1_02.py > ...

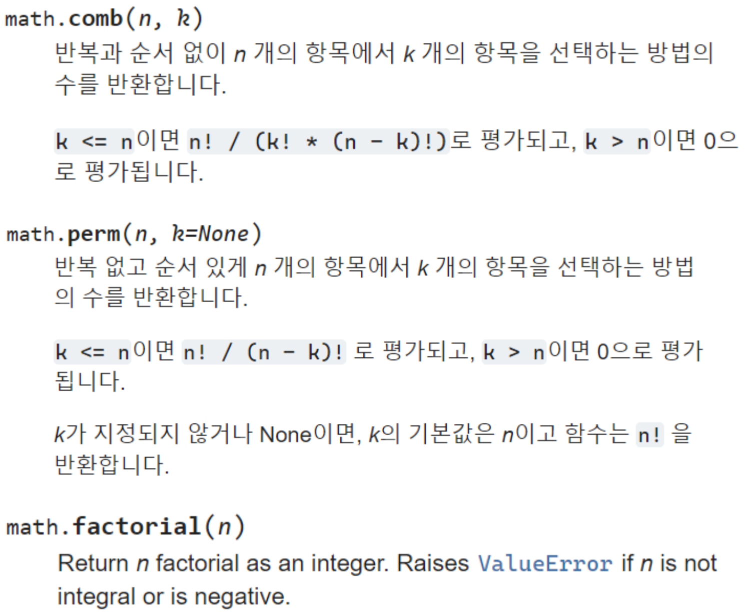
```python
import math
from scipy.stats import hypergeom

# Basic Combinatorics
def fun_factorial(n):
    if n==0: return 1
    else: return n*fun_factorial(n-1)

def fun_sum(N1, N2): return N1+N2
def fun_product(N1, N2): return N1*N2
def fun_combination(N, k):
    return int(fun_factorial(N) /
                (fun_factorial(N-k) * fun_factorial(k)))
def fun_permutation(N, k): return fun_combination(N,k)*fun_factorial(k)
def fun_combination_repetition(N, k): return fun_combination(N+k-1,k)

if __name__ == '__main__':
    # Functional verification
    print("rule of sum:", fun_sum(1,2))
    print("rule of product:", fun_product(3,4))
    print("combination:", fun_combination(5,2), math.comb(5,2))
    print("permutation:", fun_permutation(5,2))
    print("combination_repetition:", fun_combination_repetition(5,2))
```



```
rule of sum: 3
rule of product: 12
combination: 10 10
permutation: 20
combination_repetition: 15
```

이항계수 연산 관련:

https://docs.python.org/ko/3/library/math.html



`math.comb(n, k)`

반복과 순서 없이 *n* 개의 항목에서 *k* 개의 항목을 선택하는 방법의 수를 반환합니다.

`k <= n`이면 `n! / (k! * (n - k)!)`로 평가되고, `k > n`이면 0으로 평가됩니다.

`math.perm(n, k=None)`

반복 없고 순서 있게 *n* 개의 항목에서 *k* 개의 항목을 선택하는 방법의 수를 반환합니다.

`k <= n`이면 `n! / (n - k)!` 로 평가되고, `k > n`이면 0으로 평가됩니다.

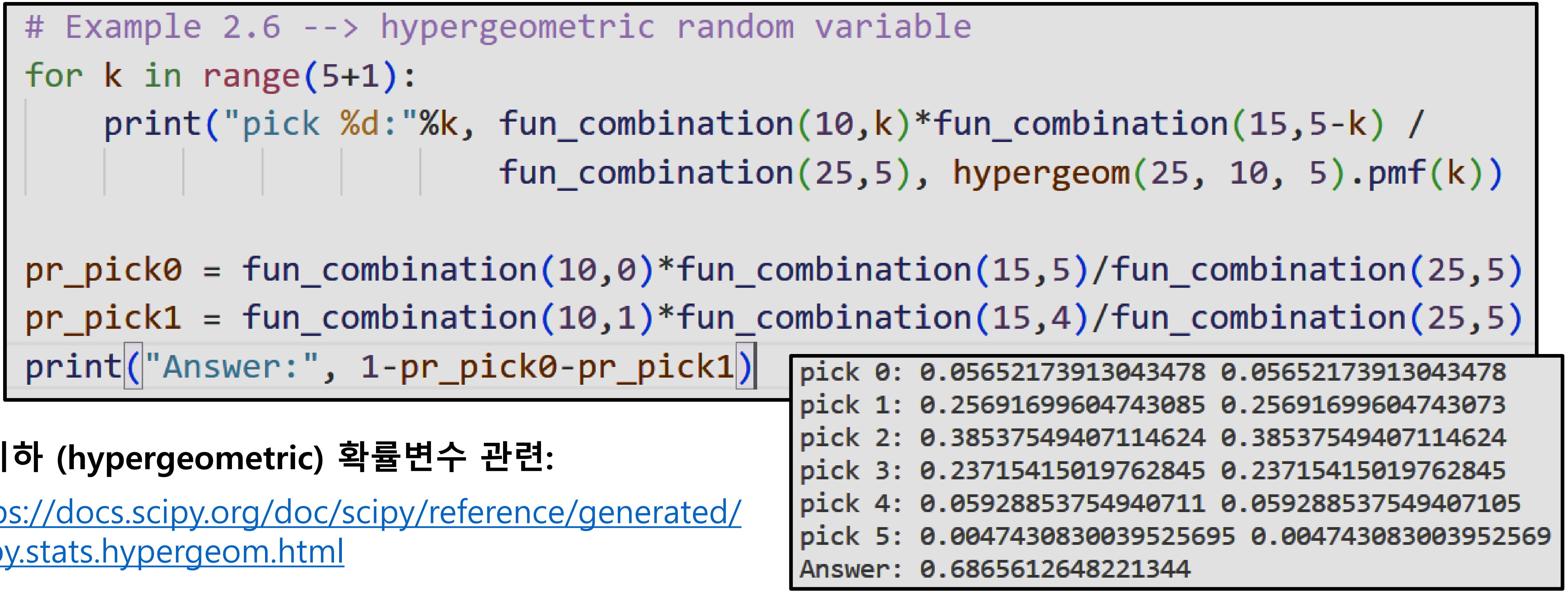
*k*가 지정되지 않거나 None이면, *k*의 기본값은 *n*이고 함수는 `n!` 을 반환합니다.

`math.factorial(n)`

Return *n* factorial as an integer. Raises `ValueError` if *n* is not integral or is negative.

실습 2. Example 2.6 (Python)

Example 2.6: DEAL Computers Incorporated manufactures some of their computers in the US and others in Lower Slobbovia. The local DEAL factory store has a stock of 10 computers that are US made and 15 that are foreign made. You order five computers from the DEAL store which are randomly selected from this stock. What is the probability that two or more of them are US-made?



```
# Example 2.6 --> hypergeometric random variable
for k in range(5+1):
    print("pick %d:"%k, fun_combination(10,k)*fun_combination(15,5-k) /
                        fun_combination(25,5), hypergeom(25, 10, 5).pmf(k))

pr_pick0 = fun_combination(10,0)*fun_combination(15,5)/fun_combination(25,5)
pr_pick1 = fun_combination(10,1)*fun_combination(15,4)/fun_combination(25,5)
print("Answer:", 1-pr_pick0-pr_pick1)
```

```
pick 0: 0.05652173913043478 0.05652173913043478
pick 1: 0.25691699604743085 0.25691699604743073
pick 2: 0.38537549407114624 0.38537549407114624
pick 3: 0.23715415019762845 0.23715415019762845
pick 4: 0.05928853754940711 0.059288537549407105
pick 5: 0.0047430830039525695 0.004743083003952569
Answer: 0.6865612648221344
```

초기하 (hypergeometric) 확률변수 관련:

https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.hypergeom.html