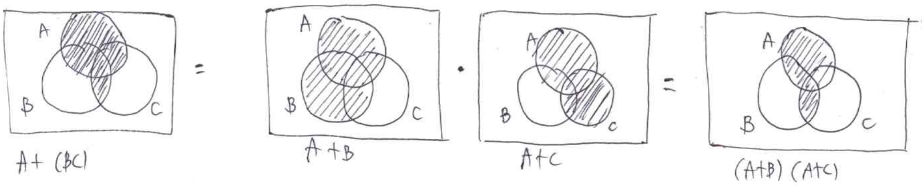
확률 및 통계 과제#1

20213064_김종민

Problem 2-1(b) : Draw a set of Venn diagrams to illustrate each of the following identites in the algebra of events.

$A+(BC) = (A+B)(A+C)$



Problem 2.2 : A sample space S is given to be $\{a_1, a_2, a_3, a_4, a_5, a_6\}$. The following events are defined on this sample space: $A_1 = \{a_1, a_2, a_4\}$, $A_2 = \{a_2, a_3, a_6\}$, and $A_3 = \{a_1, a_3, a_5\}$

(a) Find the following events:

(i) $A_1 + A_2 = \{a_1, a_2, a_3, a_4, a_6\}$

(ii) $A_1 \cdot A_2 = \{a_2\}$

(iii) $(A_1 + A_3^c) \cdot A_2 = \{a_2, a_6\}$

(b) Show the following identites:

(i) $A_1(A_2 + A_3) = A_1A_2 + A_1 \cdot A_3$

$A_1(A_2+A_3) \rightarrow A_2+A_3 = \{a_1, a_2, a_3, a_5, a_6\} \rightarrow \{a_1, a_2, a_4\} \cdot \{a_1, a_2, a_3, a_5, a_6\}$

$\therefore \{a_1, a_2\}$

$A_1A_2 + A_1A_3 \rightarrow A_1 \cdot A_2 = \{a_2\}$, $A_1 \cdot A_3 = \{a_1\} \rightarrow \{a_2\} + \{a_1\}$

$\therefore \{a_1, a_2\}$

(ii) $A_1 + A_2A_3 = (A_1 + A_2)(A_1 + A_3)$

$A_1 + A_2 \cdot A_3 \rightarrow A_2 \cdot A_3 = \{a_3\} \rightarrow \{a_1, a_2, a_4\} + \{a_3\}$

$\therefore \{a_1, a_2, a_3, a_4\}$

$(A_1+A_2)(A_1+A_3) \rightarrow (A_1+A_2) = \{a_1, a_2, a_3, a_4, a_6\}$, $(A_1+A_3) = \{a_1, a_2, a_3, a_4, a_5\}$

$\{a_1, a_2, a_3, a_4, a_6\} \cdot \{a_1, a_2, a_3, a_4, a_5\}$ $\therefore \{a_1, a_2, a_3, a_4\}$

(iii) $(A_1 + A_2)^c = A_1^c \cdot A_2^c$

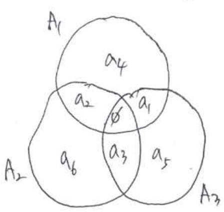
$(A_1+A_2)^c \rightarrow A_1+A_2 = \{a_1, a_2, a_3, a_4, a_6\} \rightarrow (A_1+A_2)^c = a_5$

$\therefore \{a_5\}$

$A_1^c \cdot A_2^c \rightarrow A_1^c = \{a_3, a_5, a_6\}$, $A_2^c = \{a_1, a_4, a_5\} \rightarrow \{a_3, a_5, a_6\} \cdot \{a_1, a_4, a_5\}$

$\therefore \{a_5\}$

Problem 2.5 : Consider the sample space in Prob. 2.2 in which all the outcomes are assumed equally likely. Find the following probabilities:



(a) $Pr[A_1A_2]$

$= \frac{1}{6}$

(b) $Pr[A_1+A_2]$

$= \frac{5}{6}$

(c) $Pr[(A_1 + A_3^c)A_2]$

$\hookrightarrow A_1 + A_3^c = \{a_1, a_2, a_4, a_6\}$

$(A_1+A_3^c) \cdot A_2 = \{a_1, a_2, a_4, a_6\} \cdot \{a_2, a_3, a_6\}$

$= \{a_2, a_6\}$

$\therefore \frac{2}{6} = \frac{1}{3}$

Problem 2.13

Q. In the 2003 race for Governor of California the alphabet was "reordered" so that none of the 135 candidates would feel discriminated against by the position of their name on the ballot.

(a) The week of the election it was announced that the first four letters of the reordered alphabet were R, W, Q, O. Assuming that all letters were initially equally-likely, what was the probability that this particular sequence would be chosen? (There are 26 letters in the English alphabet).

↳ 알파벳을 배열하는 전체 경우의 수는 26! 이다.
한편, R. W. Q. O는 이미 정해졌기에 나머지 22개의 알파벳을 배열하는 경우만 따지면 되는데, 이는 22!이다.

$\therefore \frac{22!}{26!} = \frac{1}{358,800} \cong 0.000002787$

(b) The letter S (for Schwarzenegger) turned up at position 10 in the new alphabet. What is the probability that S would end up in the 10th position?

↳ 위에서 보았듯이 알파벳을 배열하는 전체 경우의 수는 26! 이다.
한편, S는 10번째에 고정되어 있으므로 나머지 25개를 배열하는 경우만 따지면 되는데, 이는 25!이다.

$\therefore \frac{25!}{26!} = \frac{1}{26} \cong 0.03846$

Problem 2.14 : Consider the problem of purchasing CDs described in Example 2.2.

(a) Assuming that the probabilities of good CDs and bad CDs are equal, what is the probability that you have one or more good CDs?

↳ Sample space : GGG, GGB, GBG, BGG, GBB, BGB, BBG, BBB.
∴ 적어도 하나 이상 좋은 CD 얻을 확률은 $\frac{7}{8}$ 이다.

(b) If the probability of a good disk is 5/8, what is the probability that you have one or more good CDs?

↳ 세개 다 CD가 안 좋을 확률은 $\left(\frac{3}{8}\right) \times \left(\frac{3}{8}\right) \times \left(\frac{3}{8}\right) = \frac{27}{512}$ 이다.
전체 확률 1에서 세개 다 CD가 안 좋을 확률을 빼면 적어도 하나 CD가 좋을 확률이다.

$1 - \frac{27}{512} = \frac{485}{512}$ ∴ 0.947

Problems 2.31 : Repeat Prob. 2.30 if the probability of a memory failure is 0.02, the probability of a disk failure is 0.015, and the probability that both fail simultaneously is 0.0003.

↳ (a) 사건의 독립 여부는 확인하기 위해서는 $Pr[A] \times Pr[B] = Pr[A \cdot B]$가 성립하는지 확인하면 된다.. memory failure의 확률은 $Pr[MF] = 0.02$, disk failure의 확률은 $Pr[0.015]$ 마지막으로 동시에 failure 할 확률은 $Pr[MF \cdot HF] = 0.0003$ 라 하자.

위의 식에 대입하면 $Pr[MF] \cdot Pr[HF] = Pr[MF \cdot HF] \rightarrow 0.02 \times 0.015 = 0.0003$

식이 성립 한다. ∴ Independent 하다.

↳ (b) HF 이 발생한 후 MF이 일어날 사건인 조건부 확률을 구하는 문제이다. $Pr[MF \mid HF] = \frac{Pr[MF \cdot HF]}{Pr[HF]}$ 를 구하면 된다.

$$\frac{Pr[MF \cdot HF]}{Pr[HF]} = \frac{0.0003}{0.015} = \frac{2}{100}$$

∴ $Pr[MF \mid HF] = 0.02$ 이다.

Problems 2.32 : In the triple-core computers that Simon's Surplus has just sold back to the government, the probability of a memory failure is 0.02, while the probability of a hard disk failure is 0.015. If the probability that the memory and the hard disk failure simultaneously is 0.0003, then,

(a) Are memory failures and hard disk failure independent events?

↳ $Pr[MF] = 0.02$, $Pr[HF] = 0.015$, $Pr[MF \cdot HF] = 0.0003$

독립 사건이 성립하려면 $Pr[MF] \times Pr[HF] = Pr[MF \cdot HF]$ 식을 만족해야 한다. 값을 대입해보면.

$0.02 \times 0.015 = 0.0003$ ∴ 식을 만족하므로 independent 하다.

(b) What is the probability of a memory failure, given a hard disk failure?

↳ $$Pr[MF \mid HF] = \frac{Pr[MF \cdot HF]}{Pr[HF]} = \frac{0.0003}{0.015} = \frac{1}{50}$$

∴ 하드 디스크 손상이 있을 때 메모리 손상이 있는 조건부 확률은 0.02이다.

Problems 2.36 : Beetle Bailey has a date with Miss Buxley, but Beetle has an old jeep which will break down with probability 0.4. If his jeep breaks down he will be late with probability 0.9. If it does not break down he will be late with probability 0.2. What is the probability that Beetle will be late for his date?

↳

$Pr[\text{jeep break down}] = 0.4$ → $Pr[\text{late}_1] = 0.9$, $Pr[\text{not late}_1] = 0.1$

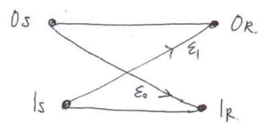
$Pr[\text{jeep not break down}] = 0.6$ → $Pr[\text{late}_2] = 0.2$, $Pr[\text{not late}_2] = 0.8$

Beetle이 date에 늦을 확률은 $Pr[\text{jeep break down}] \times Pr[\text{late}_1] + Pr[\text{jeep not break down}] \times Pr[\text{late}_2]$ 이므로

$$0.4 \times 0.9 + 0.6 \times 0.2 = 0.48$$

∴ Beetle이 데이트에 늦을 확률은 0.48이다.

Problems 2.43 : A binary communication channel is depicted below. Assume that the random experiment consists of transmitting a single binary digit and that the probability of transmitting a 0 or a 1 is the same.



(a) Draw the sample space for the experiment and label each elementary event with its probability.

↳

Sample space	Probability
(1) $0_s \to 0_R$	$\frac{1}{2}(1-\varepsilon_0)$
(2) $0_s \to 1_R$	$\frac{1}{2}\varepsilon_0$
(3) $1_s \to 0_R$	$\frac{1}{2}\varepsilon_1$
(4) $1_s \to 1_R$	$\frac{1}{2}(1-\varepsilon_1)$

(b) What is the probability of an error?

↳ $0_s \to 1_R$로 가는 경우와 $1_s \to 0_R$로 가는 경우에 에러가 발생한다.

$\therefore \frac{1}{2}\varepsilon_0 + \frac{1}{2}\varepsilon_1 = \frac{1}{2}(\varepsilon_0 + \varepsilon_1)$

(c) Given an error occured, what is the probability that a 1 was sent?

↳ (b)에서 error가 발생할 확률이 $\frac{1}{2}(\varepsilon_0 + \varepsilon_1)$임을 구했다. error가 발생했을때 1에서 보냈을 확률은 $\frac{1}{2}\varepsilon_1$이므로. $\frac{\frac{1}{2}(\varepsilon_1)}{\frac{1}{2}(\varepsilon_0+\varepsilon_1)} = \frac{\varepsilon_1}{\varepsilon_0+\varepsilon_1}$ 이다.

$\therefore$ error가 발생했을때 1에서 보냈을 확률은 $\frac{\varepsilon_1}{\varepsilon_0+\varepsilon_1}$ 이다.

(d) What is the probability a 1 was sent given that 1 was received?

↳ 1을 받는 경우의 수는 2가지인데. $0_s \to 1_R$인 경우와 $1_s \to 1_R$인 경우이다. 두 경우의 확률의 합은 $\frac{1}{2}\varepsilon_0 + \frac{1}{2}(1-\varepsilon_1)$ 이다.

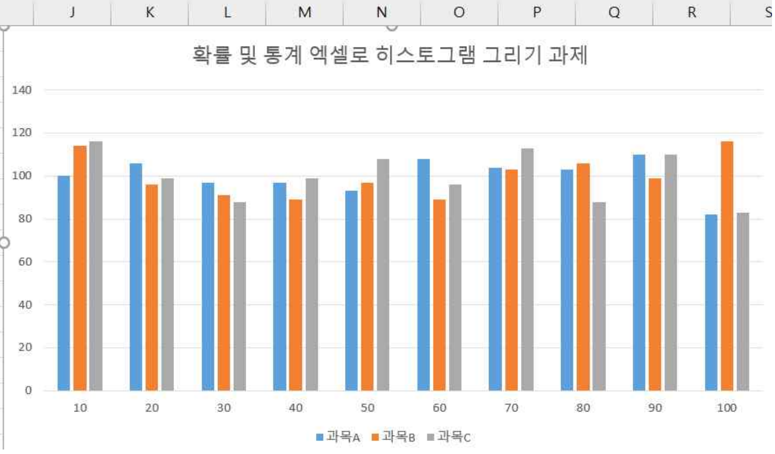
두 가지 경우에서 1_s에서 보내져서 1_R에서 받는 경우는 $1_s \to 1_R$이고 확률은 $\frac{1}{2}(1-\varepsilon_1)$ 이다.

문제에서 요구하는 확률은 $\frac{\frac{1}{2}(1-\varepsilon_1)}{\frac{1}{2}\varepsilon_0 + \frac{1}{2}(1-\varepsilon_1)} = \frac{1-\varepsilon_1}{1-\varepsilon_1+\varepsilon_0}$ 이다.

$\therefore$ 1_R에서 받을때 1_s에서 시작한 조건부 확률은 $\frac{1-\varepsilon_1}{1-\varepsilon_1+\varepsilon_0}$ 이다.

실습 1. 도수분포표와 히스토그램 (Excel)

class	과목A	과목B	과목C
10	100	114	116
20	106	96	99
30	97	91	88
40	97	89	99
50	93	97	108
60	108	89	96
70	104	103	113
80	103	106	88
90	110	99	110
100	82	116	83



	A	B	C	D	E
982	981	86	81	54	
983	982	95	30	12	
984	983	71	10	54	
985	984	68	74	24	
986	985	54	71	82	
987	986	61	32	29	
988	987	36	66	34	
989	988	90	61	86	
990	989	40	3	31	
991	990	54	5	49	
992	991	69	55	24	
993	992	53	3	10	
994	993	83	0	73	
995	994	63	4	54	
996	995	56	63	31	
997	996	63	23	56	
998	997	21	48	61	
999	998	98	89	82	
1000	999	25	83	20	
1001	1000	41	16	35	
1002	과목 평균	50.161	51.007	49.403	
1003					
1004					

실습 2. Example 2.6 (Python)

combinations_by_python.py ×

C: > Users > frive > Desktop > 학기 자료 > 숭실대 2학년 1학기 > 확률및통계 > 과제 > 1주차 과제 > combinations_by_python.py > ...

```python
# 확률 및 통계 조합 구현하기 과제
def RuleOfSum(n1, n2):
    return n1 + n2
def RuleOfProduct(n1, n2):
    return n1 * n2
def factorial(n):
    result = 1
    for i in range(1, n + 1):
        result *= i
    return result
def Combinations(n, k):
    return int(factorial(n) / factorial(k) / factorial(n - k))
def Permutations(n, k):
    return int(Combinations(n, k) * factorial(k))
def CombinationWithRepetition(n, k): # which is known as symbol "H"
    return int(Combinations(n - 1 + k, k))
def PermutationWithRepetition(n, k):
    return n ** k

# p.24 Example 2.6
sampleSpace = Combinations(25, 5)
events = Combinations(10, 2) * Combinations(15, 3) + Combinations(10, 3) * Combinations(15, 2) + \
    Combinations(10, 4) * Combinations(15, 1) + Combinations(10, 5) * Combinations(15, 0)
print(format(events / sampleSpace, ".3f"))
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
10

(base) c:\Users\frive\venv_test> c: && cd c:\Users\frive\venv_test && cmd /C "c:\Users\frive\anaconda3\pytho
2-x64\bundled\libs\debugpy\adapter/../..\debugpy\launcher 57814 -- "c:\Users\frive\Desktop\학기 자료\숭실대
0.6865612648221344

(base) c:\Users\frive\venv_test> c: && cd c:\Users\frive\venv_test && cmd /C "c:\Users\frive\anaconda3\pytho
2-x64\bundled\libs\debugpy\adapter/../..\debugpy\launcher 57868 -- "c:\Users\frive\Desktop\학기 자료\숭실대
0.687
```