숭실대학교 AI 융합학부
Soongsil University School Of AI Convergence

확률 및 통계

중간고사

2024.04.26.

문제 풀이

- **예제 및 연습문제 총 15 문항**
  - 1. Example 2.6 (Combinatorics)
  - 2. Example 2.2, Problem 2.15 (Probability Model)
  - 3. Problem 2.9 (Probability Model)
  - 4. Problem 2NB3 (Bayes' Rule)
  - 5. Problem 2.41 (Bayes' Rule)
  - 6. Example 3.3 (Binomial Random Variable)
  - 7. Example 3.4 (Geometric Random Variable)
  - 8. Example 3.5 (Poisson Random Variable)
  - 9. Example 3.10 (Exponential Random Variable)
  - 10. Example 4.4 (Exponential Random Variable)
  - 11. Example 4.6, 4.7 (Moment Generating Function)
  - 12. Problem 4.50 (Laplace Random Variable + MGF)
  - 13. Example 6.1, 6.5 (Markov inequality, Gaussian Random Variable + Central Limit Theorem)
  - 14. Example 3.21 (Radar/sonar Target Detection)
  - 15. Example 2.14, 4.9 (Measuring Information and Coding)

1. Example 2.6 (Combinatorics)

DEAL Computers Incorporated manufactures some of their computers in the US and others in Lower Slobbovia. The local DEAL factory store has a stock of 3 computers that are US made and 2 that are foreign made. You order two computers from the DEAL store which are randomly selected from this stock (p.24)

1) What is the probability that only one of them are US-made?

$$\frac{\binom{3}{1}\cdot\binom{2}{1}}{\binom{5}{2}} = \frac{3\cdot 2}{10} = \mathbf{0.6}$$

1. Example 2.6 (Combinatorics)

DEAL Computers Incorporated manufactures some of their computers in the US and others in Lower Slobbovia. The local DEAL factory store has a stock of 3 computers that are US made and 2 that are foreign made. You order two computers from the DEAL store which are randomly selected from this stock (p.24)

2) What is the probability that at least one of them are US-made?

$$1 - \frac{\binom{3}{0} \cdot \binom{2}{2}}{\binom{5}{2}} = \frac{10 - 1 \cdot 1}{10} = \mathbf{0.9}$$

2. Problem 2.15 (Probability Model)

Simon has decided to improve the quality of the products he sells. Now only one out of five CDs selected from the bins is defective (i.e., the probability that a diskette is bad is 0.2). If three CDs are chosen at random (p.21),

1) What is the probability that exactly two of the three CDs are good?

BBB	BBG	BGB	$\boxed{\text{BGG}}$	GBB	$\boxed{\text{GBG}}$	$\boxed{\text{GGB}}$	GGG
A_1	A_2	A_3	$\boxed{A_4}$	A_5	$\boxed{A_6}$	$\boxed{A_7}$	A_8

$$3 \cdot 0.2^1 \cdot 0.8^2 = \mathbf{0.384}$$

2. Problem 2.15 (Probability Model)

Simon has decided to improve the quality of the products he sells. Now only one out of five CDs selected from the bins is defective (i.e., the probability that a diskette is bad is 0.2). If three CDs are chosen at random (p.21),

2) What is the probability that there are more good CDs than bad ones?

BBB	BBG	BGB	$\boxed{\text{BGG}}$	GBB	$\boxed{\text{GBG}}$	$\boxed{\text{GGB}}$	$\boxed{\text{GGG}}$
A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8

$$3 \cdot 0.2^1 \cdot 0.8^2 + 0.8^3 = \mathbf{0.896}$$

3. Problem 2.9 (Probability Model)

The following events and their probabilities are listed below.

event:	A	B	C	D
probability:	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{5}$

1) Is it possible that all of A, B, C, and D are mutually exclusive? Tell me why or why not.

No. If it were possible then

$$P[A+B+C+D] = Pr[A]+Pr[B]+Pr[C]+Pr[D] > 1$$

$\therefore$ This is not possible.

3. Problem 2.9 (Probability Model)

The following events and their probabilities are listed below.

event:	A	B	C	D
probability:	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{5}$

2) Compute Pr[A+B] assuming A and B are independent.

$$Pr[A+B] = Pr[A] + Pr[B] - Pr[A] \cdot Pr[B]$$

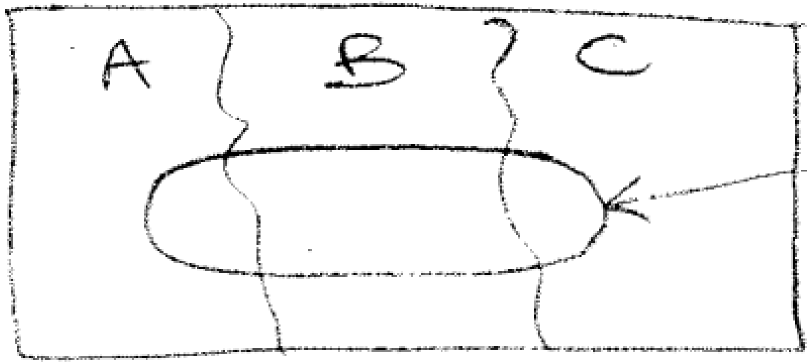
$$= \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$$

4. Problem 2NB3 (Bayes' Rule)

Components from three different manufacturers have been procured; 100 from Manufacturer A, of which 10% are defective; 300 from Manufacturer B, of which 5% are defective; and 500 from Manufacturer C, of which 20% are defective. We randomly select the shipment from "one of the manufacturers" and then randomly pick a component from it.

$$\Pr[A] = \Pr[B] = \Pr[C] = \frac{1}{3}$$

1) What is the probability that the selected component is defective?



$D \triangleq$ {Selected component is defective}

$A \triangleq$ {Component from manufacture A}

4. Problem 2NB3 (Bayes' Rule)

1) What is the probability that the selected component is defective?

By the principle of total probability, we have

$$\Pr[DA] + \Pr[DB] + \Pr[DC]$$

$$\Pr[D] = \Pr[DA] + \Pr[DB] + \Pr[DC]$$

$$= \Pr[D|A]\Pr[A] + \Pr[D|B]\Pr[B] + \Pr[D|C]\Pr[C]$$

$$= 0.10\left(\tfrac{1}{3}\right) + 0.05\left(\tfrac{1}{3}\right) + 0.20\left(\tfrac{1}{3}\right) = \frac{7}{60}$$

$$= 0.1167$$

4. Problem 2NB3 (Bayes' Rule)

2) If the selected component is found to be defective, what are the probabilities that they came from Manufacturer A, B, and C?

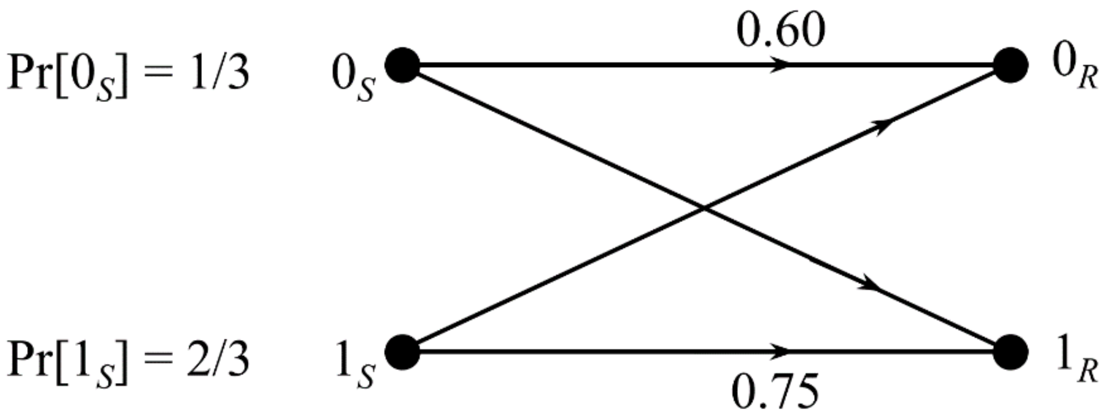
$$\text{A:}\ \Pr[\mathrm{A}|\mathrm{D}] = \frac{\Pr[\mathrm{D}\cdot\mathrm{A}]}{\Pr[\mathrm{D}]} = \frac{\Pr[\mathrm{D}|\mathrm{A}]\cdot\Pr[\mathrm{A}]}{\Pr[\mathrm{D}]} = \frac{0.1\cdot 1/3}{0.1167} \approx \mathbf{0.29}$$

$$\text{B:}\ \Pr[\mathrm{B}|\mathrm{D}] = \frac{\Pr[\mathrm{D}\cdot\mathrm{B}]}{\Pr[\mathrm{D}]} = \frac{\Pr[\mathrm{D}|\mathrm{B}]\cdot\Pr[\mathrm{B}]}{\Pr[\mathrm{D}]} = \frac{0.05\cdot 1/3}{0.1167} \approx \mathbf{0.14}$$

$$\text{C:}\ \Pr[\mathrm{C}|\mathrm{D}] = \frac{\Pr[\mathrm{D}\cdot\mathrm{C}]}{\Pr[\mathrm{D}]} = \frac{\Pr[\mathrm{D}|\mathrm{C}]\cdot\Pr[\mathrm{C}]}{\Pr[\mathrm{D}]} = \frac{0.2\cdot 1/3}{0.1167} \approx \mathbf{0.57}$$

5. Problem 2.41 (Bayes' Rule)

Consider the binary communication channel depicted below



1) Given that a 0 was sent, what is the probability that a 1 was received (= $\Pr[1_R \mid 0_S]$)?

$$\Pr[1_R \mid 0_S] + \Pr[0_R \mid 0_S] = 1$$

$$\Pr[1_R \mid 0_S] = 1 - \Pr[0_R \mid 0_S] = 1 - 0.6 = \mathbf{0.4}$$

5. Problem 2.41 (Bayes' Rule)

2) What is the probability that a 1 was received ($=\Pr[1_R]$)?

$$\Pr[1_R] = \Pr[1_R \mid 0_S]\Pr[0_S] + \Pr[1_R \mid 1_S]\Pr[1_S]$$
$$= (0.4)\frac{1}{3} + (0.75)\frac{2}{3} = \frac{19}{30} = 0.6333$$

3) What is the probability of an error ($=\Pr[error]$)?

$$\Pr[error] = \Pr[error \mid 0_S]\Pr[0_S] + \Pr[error \mid 1_S]\Pr[1_S]$$
$$= (0.4)\frac{1}{3} + (0.25)\frac{2}{3} = \frac{9}{30} = 0.3$$

4) Given that a 1 was received, what is the probability that a 0 was sent ($=\Pr[0_S \mid 1_R]$)?

$$\Pr[0_S \mid 1_R] = \frac{\Pr[1_R \mid 0_S]\ \Pr[0_S]}{\Pr[1_R]} = \frac{(0.4)\frac{1}{3}}{0.6333} = \frac{4}{19} = 0.211$$

6. Example 3.3 (Binomial Random Variable)

A modem connection has a bit error rate of p = 0.01. Given that data are sent as packets of 8 bits, what is the probability that just 1 bit is in error? Assume this is a binomial random variable (p.55)

$$\Pr[1 \text{ bit error}] = f_K[1] = \binom{8}{1} \cdot 0.01^1 \cdot 0.99^7 \approx \mathbf{0.0746}$$

7. Example 3.4 (Geometric Random Variable)

Errors in the transmission of a random stream of bytes occur with probability p = 0.01. What is the probability that the first error will occur after 10 bytes (p.56)?

$$\Pr[K>10] = \sum_{k=10}^{\infty} p(1-p)^k = (1-p)^{10} = 0.99^{10} \approx \mathbf{0.9044}$$

8. Example 3.5 (Poisson Random Variable)

Packets at a certain node on the internet arrive with a rate of 10 packets per minute. What is the probability that 1 or more packets arrive in the first 6 seconds (p.58)?

$$\alpha = 10/min \cdot \frac{6}{60} min \ = 1$$

$$f_K[0] = \frac{\alpha^k}{k!} e^{-\alpha} = \frac{1^0}{0!} e^{-1} = e^{-1}$$

$$1 - f_K[0] = 1 - e^{-1} \approx \mathbf{0.6321}$$

9. Example 3.10 (Exponential Random Variable)

In a printer queue, jobs arrive at an average rate of λ = 30 jobs per hour. What is the probability that the waiting time to the next job is between 2 and 4 minutes (p.66)?

$$\Pr\left[\frac{2}{60} < W \leq \frac{4}{60}\right] = \int_{2/60}^{4/60} 30e^{-30w}\,dw = -e^{-30w}\Big|_{2/60}^{4/60} = e^{-1} - e^{-2} = 0.233$$

10. Example 4.4 (Exponential Random Variable)

Derive the mean and variance of the exponential random variable in terms of λ. The parameter λ is the constant rate of any recurring events (p.119)

$$\boxed{f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}}$$

To help in the evaluation of moments, we can make use of the formula

$$\int_0^\infty u^n e^{-u} du = n! \qquad n = 0, 1, 2, 3, \ldots$$

10. Example 4.4 (Exponential Random Variable)

The mean is given by

$$m_X = \int_{-\infty}^{\infty} x f_X(x) dx = \int_0^{\infty} x \lambda e^{-\lambda x} dx$$

Making the change of variables $u = \lambda x$ yields

$$m_X = \frac{1}{\lambda} \int_0^{\infty} u e^{-u} du = \frac{1}{\lambda} \cdot 1! = \frac{1}{\lambda}$$

The variance is computed from

$$\begin{aligned}
\sigma_X^2 &= \int_{-\infty}^{\infty} (x - m_X)^2 f_X(x) dx = \int_0^{\infty} (x - \frac{1}{\lambda})^2 \lambda e^{-\lambda x} dx \\
&= \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx - \frac{2}{\lambda} \int_0^{\infty} x \lambda e^{-\lambda x} dx + \frac{1}{\lambda^2} \int_0^{\infty} \lambda e^{-\lambda x} dx \\
&= \frac{1}{\lambda^2} \int_0^{\infty} u^2 e^{-u} du - \frac{2}{\lambda^2} \int_0^{\infty} u e^{-u} du + \frac{1}{\lambda^2} \\
&= \frac{1}{\lambda^2} \cdot 2! - \frac{2}{\lambda^2} \cdot 1! + \frac{1}{\lambda^2} = \frac{1}{\lambda^2}
\end{aligned}$$

11. Example 4.6, 4.7 (Moment Generating Function)

What is the moment generating function (MGF) of the exponential random variable? Also, compute the mean and variance of the exponential random variable using the moment generating function (p.122)

$$M_X(s) = \mathcal{E}\{e^{sX}\} = \int_{-\infty}^{\infty} f_X(x) e^{sx} dx$$

The MGF is computed by applying (4.19)

$$\begin{aligned} M_X(s) &= \int_{-\infty}^{\infty} f_X(x) e^{sx} dx = \int_0^{\infty} \lambda e^{-\lambda x} e^{sx} dx \\ &= \lambda \int_0^{\infty} e^{-(\lambda - s)x} dx = \frac{\lambda}{-(\lambda - s)} e^{-(\lambda - s)x} \Big|_0^{\infty} = \frac{\lambda}{\lambda - s} \end{aligned}$$

11. Example 4.6, 4.7 (Moment Generating Function)

Compute the mean and variance of the exponential random variable using the moment generating function (MGF).

$$m_X = \mathcal{E}\{X\} = \left.\frac{dM_X(s)}{ds}\right|_{s=0} = \frac{d}{ds}\left(\frac{\lambda}{\lambda - s}\right) = \left.\frac{\lambda}{(\lambda - s)^2}\right|_{s=0} = \frac{1}{\lambda}$$

To compute the variance, first compute the second moment using (4.20):

$$\mathcal{E}\{X^2\} = \left.\frac{d^2 M_X(s)}{ds^2}\right|_{s=0} = \left.\frac{2\lambda}{(\lambda - s)^3}\right|_{s=0} = \frac{2}{\lambda^2}$$

Then use (4.17) to write

$$\sigma_X^2 = \mathcal{E}\{X^2\} - m_X^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

12. Problem 4.50 (Laplace Random Variable + MGF)

A random variable X is described by the Laplace density

$$f_X(x) = \frac{1}{2}e^{-|x|}$$

1) What is the mean and variance of X?

$$E[X] = \frac{1}{2}\int_{-\infty}^{\infty} x\, e^{-|x|}\, dx = 0 \qquad Var[X] = E[X^2] - \underbrace{(E[X])^2}_{0} = E[X^2]$$

$$E[X^2] = \frac{1}{2}\int_{-\infty}^{\infty} x^2 e^{-|x|}\, dx = 2\cdot\frac{1}{2}\int_{0}^{\infty} x^2 e^{-x}\, dx = 2! = 2$$

12. Problem 4.50 (Laplace Random Variable + MGF)

A random variable X is described by the Laplace density

$$f_X(x) = \frac{1}{2}e^{-|x|}$$

2) Find the MGF $M_X(s) = E[e^{sX}]$

$$M_X(s) = \int_{-\infty}^{\infty} e^{sx} \cdot \frac{1}{2} e^{-|x|} dx = \frac{1}{2}\int_{-\infty}^{0} e^{sx} e^{x} dx + \frac{1}{2}\int_{0}^{\infty} e^{sx} e^{-x} dx$$

$$= \frac{1}{2}\frac{1}{s+1} e^{(s+1)x}\Big|_{-\infty}^{0} + \frac{1}{2}\frac{1}{s-1} e^{(s-1)x}\Big|_{0}^{\infty} = \frac{1}{2}\left[\frac{1}{s+1} - \frac{1}{s-1}\right] = \frac{1}{1-s^2}$$

12. Problem 4.50 (Laplace Random Variable + MGF)

A random variable X is described by the Laplace density

$$f_X(x) = \frac{1}{2}e^{-|x|}$$

2) Find the MGF $M_X(s) = E[e^{sX}] \rightarrow E[X], E[X^2]$??

alternately:

$$\frac{dM_x'}{ds}\bigg|_{s=0} = \frac{2s}{(1-s^2)^2}\bigg|_{s=0} = 0$$

$$\frac{d^2M_x}{ds^2}\bigg|_{s=0} = \frac{(1-s^2)^2\,2 - \cancel{2s\cdot 2(1-s^2)}2s}{(1-s^2)^4}\bigg|_{s=0} = 2$$

13. Example 6.1, 6.5 (Markov inequality, Central Limit Theorem)

Consider a sequence of 100 binary values (IID Bernoulli random variables) with distribution parameter p = Pr[1] = 0.6. It is desired to determine the probability of more than 65 1s in the sequence (=Pr[Y_{100}>65])

1) Apply the Markov inequality to bound the probability of the event Y_{100}>65

The mean is computed as follows: $m_K = \mathcal{E}\{K\} = \sum_{k=0}^{1} k\, \mathrm{f}_K[k] = (1-p)\cdot 0 + p \cdot 1 = p$

The mean of the sum is $100\, m_X = 60$

$$Y_{100} = \sum_{i=1}^{100} X_i \quad \rightarrow \quad \Pr[Y_{100} > 65] \leq \frac{E[Y_{100}]}{65} = \frac{60}{65} \approx \mathbf{0.92}$$

13. Example 6.1, 6.5 (Markov inequality, Central Limit Theorem)

2) Apply the Central Limit Theorem to compute $\Pr[Y_{100} > 65]$. You can use Q(x) function.

The variance is computed by first computing the second moment:

$$\mathcal{E}\left\{K^2\right\} = \sum_{k=0}^{1} k^2 \, \mathrm{f}_K[k] = (1-p)\cdot 0^2 + p \cdot 1^2 = p$$

Then (4.17) is applied to compute the variance:

$$\sigma_K^2 = \mathcal{E}\left\{K^2\right\} - m_K^2 = p - p^2 = p(1-p)$$

$$Y_{100} = \sum_{i=1}^{100} X_i \;\rightarrow \mathrm{Var}[Y_{100}] = 100\,\sigma_X^2 = 24.$$

13. Example 6.1, 6.5 (Markov inequality, Central Limit Theorem)

2) Apply the Central Limit Theorem to compute Pr[Y_{100}>65]. You can use Q(x) function.

Since the normalized representation of Y_{100} is

$$Z_{100} = \frac{Y_{100} - 60}{\sqrt{24}},$$

we have

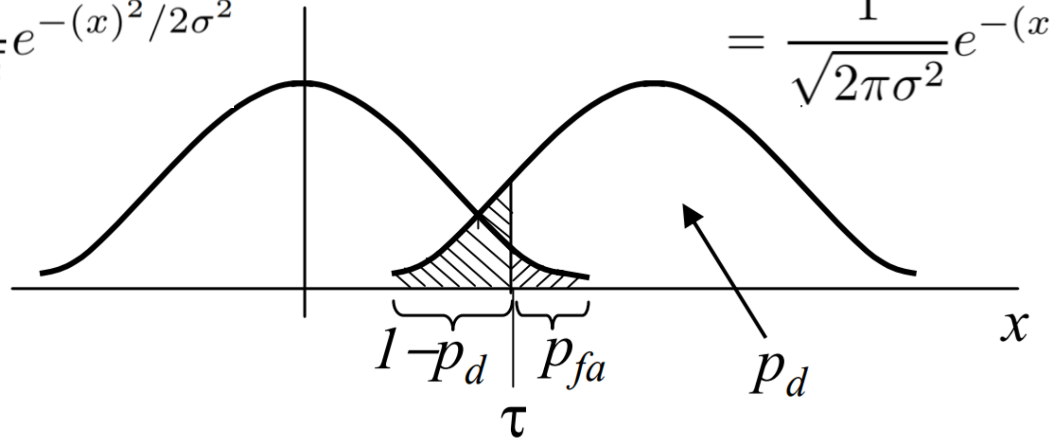
$$\begin{aligned} \Pr[Y_{100} > 65] &= \Pr\left[Z_{100} > \frac{65-60}{\sqrt{24}}\right] \\ &= \boxed{Q\left(\frac{5}{\sqrt{24}}\right) \approx Q(1.02)} \approx 0.154 \end{aligned}$$

14. Example 3.21 (Radar/sonar Target Detection)

Consider a detection problem where the signal s is equal to 2 and the noise is a zero-mean Gaussian random variable with variance $\sigma^2 = 4$. The conditional densities for the two hypotheses are given below (p.92)

$$f_{X|H_0}(x|H_0) = f_N(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x)^2/2\sigma^2}$$

$$f_{X|H_1}(x|H_1) = f_N(x-s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-s)^2/2\sigma^2}$$



1) Given the threshold τ=1, what is the probabilities of detection (p_d) and false alarm (p_{fa})?

$$p_d = Q\left(\frac{1-2}{2}\right) = 0.6915 \qquad p_{fa} = Q\left(\frac{1-0}{2}\right) = 0.3085$$

14. Example 3.21 (Radar/sonar Target Detection)

2) What is the optimal threshold of this detector when the prior probabilities are given by P_0 and P_1

Likelihood

$$f_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-n^2/2\sigma^2}$$

$$f_{X|H_1}(x|H_1) = f_N(x-s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-s)^2/2\sigma^2}$$

$$f_{X|H_0}(x|H_0) = f_N(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x)^2/2\sigma^2}$$

$$\frac{f_{X|H_1}(x|H_1)}{f_{X|H_0}(x|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \rightarrow ln\left(\frac{f_{X|H_1}(x|H_1)}{f_{X|H_0}(x|H_0)}\right) = ln\left(\frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-s)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda = ln\left(\frac{Pr(H_0)}{Pr(H_1)}\right) = ln\left(\frac{P_0}{P_1}\right)$$

$$-\frac{(x-s)^2}{2\sigma^2} + \frac{x^2}{2\sigma^2} = \frac{2sx - s^2}{2\sigma^2} \underset{H_0}{\overset{H_1}{\gtrless}} ln\left(\frac{P_0}{P_1}\right)$$

$$\boxed{x \underset{H_0}{\overset{H_1}{\gtrless}} \frac{s^2}{2s} + \frac{2\sigma^2}{2s} ln\left(\frac{P_0}{P_1}\right) = \frac{s}{2} + \frac{\sigma^2}{s} ln\left(\frac{P_0}{P_1}\right) = \tau}$$

15. Example 2.14, 4.9 (Measuring Information and Coding)

It is desired to code the message “AI CONVERGENCE” in an efficient manner using Shannon-Fano coding. The probabilities of the letters (excluding the space) are represented by their relative frequency of occurrence in the message.

1) What are the probabilities of the letters { A, C, E, G, I, N, O, R, V } to assign? (relative frequency)
2) Assign codewords for each letter according to the entropy coding.
3) What is the average length of the codewords?
4) Compute the entropy H/average information (theoretic lower bound).

15. Example 2.14, 4.9 (Measuring Information and Coding)

1) What are the probabilities of the letters { A, C, E, G, I, N, O, R, V } to assign?

"AI CONVERGENCE"

Letters	Frequency	Relative Frequency
E	3	3/13
C	2	2/13
N	2	2/13
A	1	1/13
G	1	1/13
I	1	1/13
O	1	1/13
R	1	1/13
V	1	1/13

15. Example 2.14, 4.9 (Measuring Information and Coding)

2) Assign codewords for each letter according to the entropy coding.

"AI CONVERGENCE"

Letters	Frequency	Relative Frequency	Codewords				# of Bits
E	3	3/13	0	0	End		00→2
C	2	2/13	0	1	0	End	010→3
N	2	2/13	0	1	1	End	011→3
A	1	1/13	1	0	0	0	1000→4
G	1	1/13	1	0	0	1	1001→4
I	1	1/13	1	0	1	End	101→3
O	1	1/13	1	1	0	0	1100→4
R	1	1/13	1	1	0	1	1101→4
V	1	1/13	1	1	1	End	111→3

15. Example 2.14, 4.9 (Measuring Information and Coding)

2) Assign codewords for each letter according to the entropy coding.

"AI CONVERGENCE"

(아래도 가능↓)

Letters	Frequency	Relative Frequency	Codewords				# of Bits
E	3	3/13	0	0	End		00→2
C	2	2/13	0	1	0	End	010→3
N	2	2/13	0	1	1	End	011→3
A	1	1/13	1	0	0	End	100→3
G	1	1/13	1	0	1	0	1010→4
I	1	1/13	1	0	1	1	1011→4
O	1	1/13	1	1	0	End	110→3
R	1	1/13	1	1	1	0	1110→4
V	1	1/13	1	1	1	1	1111→4

15. Example 2.14, 4.9 (Measuring Information and Coding)

3) What is the average length of the codewords?

"AI CONVERGENCE"

Letters	Frequency	Relative Frequency	# of Bits	E[X]
E	3	3/13	00→2	2 · 3/13
C	2	2/13	010→3	3 · 2/13
N	2	2/13	011→3	3 · 2/13
A	1	1/13	100→3	3 · 1/13
G	1	1/13	1010→4	4 · 1/13
I	1	1/13	1011→4	4 · 1/13
O	1	1/13	110→3	3 · 1/13
R	1	1/13	1110→4	4 · 1/13
V	1	1/13	1111→4	4 · 1/13

→ 비트 길이(X)에 대한 기댓값

$$\frac{6+6+6+11+11}{13} = \frac{40}{13} = 3.076923..\ (bits)$$

15. Example 2.14, 4.9 (Measuring Information and Coding)

4) Compute the entropy/average information H (theoretic lower bound).

"AI CONVERGENCE"

Letters	Frequency	Relative Frequency
E	3	3/13
C	2	2/13
N	2	2/13
A	1	1/13
G	1	1/13
I	1	1/13
O	1	1/13
R	1	1/13
V	1	1/13

$E[I] = E[-\log_2(Pr[X])]$ → 정보량(I)에 대한 기댓값

$-\log_2(3/13) \cdot 3/13 = 0.4892$

$-\log_2(2/13) \cdot 2/13 = 0.4154$

$-\log_2(2/13) \cdot 2/13 = 0.4154$

$-\log_2(1/13) \cdot 1/13 = 0.2846$

$-\log_2(1/13) \cdot 1/13 = 0.2846$

$-\log_2(1/13) \cdot 1/13 = 0.2846$

$-\log_2(1/13) \cdot 1/13 = 0.2846$

$-\log_2(1/13) \cdot 1/13 = 0.2846$

$-\log_2(1/13) \cdot 1/13 = 0.2846$

$$\begin{aligned} & 0.4892 \\ & + 2 \cdot 0.4154 \\ & + 6 \cdot 0.2846 \\ & = \underline{3.027} \end{aligned}$$