3.1 In a set of independent trials, a certain discrete random variable K takes on the values 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1 (a) Plot a histogram for the random variable. (b) Normalize the histogram to provide an estimate of the PMF of K.

3.2 Repeat Problem 3.1 for the discrete random variable J that takes on the values 1, 3, 0, 2, 1, 2, 0, 3, 1, 1, 3, 1, 0, 1, 2, 3, 0, 2, 1, 3, 3, 2

3.6 Sketch the PMF fK[k] for the following random variables and label the plots.

(a) Uniform. m = 0, n = 9

(b) Bernoulli. p = 1/10

(c) Binomial. n = 10, p = 1/10

(d) Geometric. p = 1/10 (plot $0 \leq k \leq 9$ only)

4.1 Rolf is an engineering student at the Technical University. Rolf plans to ask one of two women students for a Saturday night date: Claudia Sch¨onst¨uck or Ursula von Doppeldoof. Rolf is more attracted to Claudia but Ursula is more available. Further, Claudia likes the finer things in life and a night with her is more expensive. All things considered, Rolf estimates that landing a date with Claudia is a "50/50 chance" and Rolf will end up spending 200 euros. On the other hand, Ursula is about twice as likely to accept a date with Rolf as not, and Ursula is happy with beer and pretzels (40 euros). Rolf has the following procedure to determine whom he will ask first. He tosses a 1-euro coin. If it comes up "heads" he asks Claudia. If it comes up "tails," he flips it again (just to be sure). If it comes up "heads" this time he asks Claudia; otherwise he asks Ursula.

(a) What are the probabilities that Rolf will initially ask Claudia or Ursula? According to the outcome of his toss(es) of the coin, Rolf asks either Claudia or Ursula. If he asks Claudia and she turns him down, then he asks Ursula. If he asks Ursula (first) and she turns him down, he is so despondent that he does not have the courage to ask Claudia. If he does not have a date, he spends the PROBLEMS 129 night drinking beer and eating pretzels by himself, so the evening still costs 40 euros.

(b) What is the probability that Rolf has a Saturday night date with Claudia?

(c) What is the expected value of the money that Rolf spends on a Saturday night?

(d) If C is a random variable representing Rolf's expenditure on a Saturday night, sketch the PMF for C. Compute E{C} using the PMF.

4.11 Claudia is a student at the Technical University majoring in statistics. On any particular day of the week, her friend Rolf may ask her for a Saturday night date with probability 3/4. Claudia, however, is more attracted to Roberto de la Dolce, who is very handsome, drives an expensive Italian car, and really knows how to treat women! Roberto has other women he likes however, so the probability that he asks Claudia out in any particular week is only 2/3. Roberto is also very self-confident and does not plan his activities early in the week. Let D represent the event that Roberto asks Claudia for a Saturday night date. Then the day of the week on which he asks her (beginning on Monday) is a random variable K with PMF $f_{K|D}[k|D] = k/15$ for $1 \leq k \leq 5$ and 0 otherwise. Claudia is aware of this formula and the probabilities. PROBLEMS 131 Claudia needs to plan whether or not to accept if Rolf asks her for a date before Roberto; thus she decides to rate her emotional state (α) for the week on a scale of 0 to 10. A date with Roberto is actually way off the scale but she assigns it a 10. She further determines that a date with with Rolf is worth 4 points, and a Saturday night without a date is worth -5 points. She decides that if Rolf asks her out on the k th day of the week she will compute the expected value of α given that she accepts and the expected value of α given that she does not accept. Then she will make a choice according to which expected value is larger.

(a) Make a plot of the probability that Roberto does not ask Claudia for a date given that he has not asked her by the end of the l th day $1 \leq l \leq 5$.

(b) Sketch the conditional PMF for K given that Roberto asks Claudia out but has not done so by the end of the second day. Given this situation, what is the probability that Roberto asks her out (i) on the third day ($k = 3$)? (ii) on the fifth day ($k = 5$)?

(c) By the middle of the third day of the week Roberto has not asked Claudia for a date; but Rolf decides to ask her. Will she accept Rolf or not?

(d) Rolf has not studied statistics (not even probability), and thinks that his chances for a date with Claudia will be better if he asks her earlier in the week. Is he right or wrong?

(e) What is the optimal strategy for Rolf (i.e., when should he ask Claudia) in order to maximize the probability that Claudia will accept if he asks her for a Saturday night date?

4.43 The probability density function of a Laplace random variable is given by

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha|x-\mu|}, \quad -\infty < x < \infty.$$

(a) Find the mean m_X, second moment, and variance σ_X^2.

(b) Determine the following probabilities for $\alpha = 4$ and $\mu = 1$.

(i) $\Pr[|X - m_X| > 2\sigma_X]$

(ii) $\Pr[|X - m_X| < \sigma_X]$