[과제 #4]

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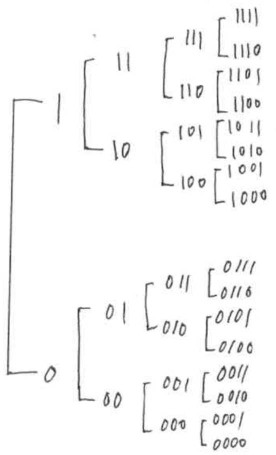
Assignment #4

problems 2.35 A random hexadecimal character in the form of four binary digits is read from a storage device.

(a) Draw the tree diagram and the sample space for this experiment.

↳



sample space =

{0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111

, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111}

(b) Given that the first bit is zero, what is the probability of more zeros than ones?

↳ 첫번째 비트가 0인 경우 = {0111, 0110, 0101, 0100, 0011, 0010, 0001, 0000} ... 8개

위의 집합에서 1보다 0이 더 많은 경우 = {0100, 0010, 0001, 0000} ... 4개

$\therefore$ probability is $\frac{4}{8} = \frac{1}{2}$

(c) Given that the first two bits are 10, what is the probability of more zeros than ones?

↳ 시작 비트가 10인 경우 = {1011, 1010, 1001, 1000} ... 4개

위의 집합에서 1보다 0이 더 많은 경우 = {1000} ... 1개

$\therefore$ probability is $\frac{1}{4}$

(d) Given that there are more zeros than ones, what is the probability that the first bit is a zero?

↳ 1보다 0이 더 많은 경우 = {1000, 0100, 0010, 0001, 0000} ... 5개

· 위의 집합에서 시작 비트가 0인 경우 = {0100, 0010, 0001, 0000} ... 4개

$\therefore$ probability is $\frac{4}{5}$

problems 7.10 For each of the following random variables:

(i) geometric, $p = 1/8$ (ii) uniform, $m=0$, $n=7$ (iii) Poisson, $\alpha = 2$

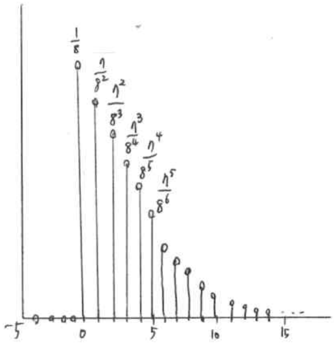
(a) sketch the PMF

↳ (i) geometric PMF

$f_K[k] = p(1-p)^k \quad 0 \le k < \infty$

$p = \frac{1}{8}$

$f_K(k) = \frac{1}{8}\left(\frac{7}{8}\right)^k$

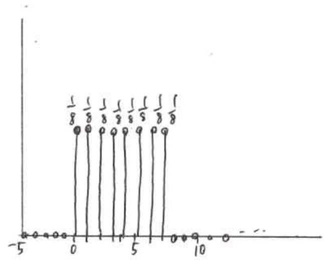


(ii) uniform PMF

$$f_K[k] = \begin{cases} \frac{1}{n-m+1} & m \le k \le n \\ 0 & \text{otherwise} \end{cases}$$

$m=0$, $n=7$

$$f_K[k] = \begin{cases} \frac{1}{7-0+1} = \frac{1}{8} & 0 \le k \le 7 \\ 0 & \text{otherwise} \end{cases}$$

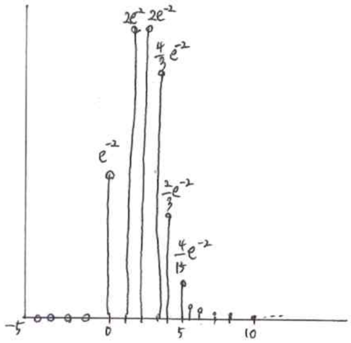


(iii) Poisson PMF

$f_K(k) = \frac{\alpha^k}{k!} e^{-\alpha} \quad 0 \le k < \infty$

$\alpha = 2$

$f_K(k) = \frac{2^k}{k!} e^{-2}$



(b) Compute the probability that the random variable is greater than 5.

↳ 확률변수가 5보다 큰 확률은 전체 확률 1에서 $f_K(0)+f_K(1)+f_K(2)+f_K(3)+f_K(4)+f_K(5)$를 빼주면 된다. 즉, 여사건을 이용한다.

(i) geometric (type 0의 경우)

probability $= 1-(f_K(0)+f_K(1)+f_K(2)+f_K(3)+f_K(4)+f_K(5)) = 1-\left(\frac{1}{8}+\frac{1}{8}\left(\frac{7}{8}\right)+\frac{1}{8}\left(\frac{7}{8}\right)^2+\frac{1}{8}\left(\frac{7}{8}\right)^3+\frac{1}{8}\left(\frac{7}{8}\right)^4+\frac{1}{8}\left(\frac{7}{8}\right)^5\right)$

∴ geometric random variable이 5보다 큰 확률은 0.4488

(ii) uniform

probability $= 1-(f_K(0)+f_K(1)+f_K(2)+f_K(3)+f_K(4)+f_K(5)) = 1-\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right) = 1-\frac{6}{8}$

∴ uniform random variable이 5보다 큰 확률은 0.25

(iii) Poisson

probability $= 1-(f_K(0)+f_K(1)+f_K(2)+f_K(3)+f_K(4)+f_K(5)) = 1-\left(e^{-2}+2e^{-2}+2e^{-2}+\frac{4}{3}e^{-2}+\frac{2}{3}e^{-2}+\frac{4}{15}e^{-2}\right) = 1-\frac{109}{15}e^{-2}$

∴ poisson random variable이 5보다 큰 확률은 0.0166

(c) Find the value of the parameters such that $Pr[K>5] \le 0.5$

↳ (i) type 0 geometric random variable

$Pr(K>5) = 1-Pr[K\le 5] = 1-(f_K(0)+f_K(1)+f_K(2)+f_K(3)+f_K(4)+f_K(5))$

$= 1-(p+p(1-p)+p(1-p)^2+p(1-p)^3+p(1-p)^4+p(1-p)^5)$

↓

$Pr(K>5) = (1-p)^6$

$(1-p)^6 \le 0.5$를 만족하는 p를 찾으면 된다.

$(1-p)^6 = 0.5$라 할때, 이를 만족하는 p는 0.1091이다.

$\therefore p = 0.1091$

(ii) uniform

$Pr(K>5) = 1 - (f_K(0) + f_K(1) + f_K(2) + f_K(3) + f_K(4) + f_K(5))$

uniform random variable은 parameter가 m, n 두 개 입니다.

처음 문제에서 주어진 바와 같이 $m=0$이라고 하겠습니다.

$Pr(K>5) = 1 - \left(\frac{1}{n+1} + \frac{1}{n+1} + \frac{1}{n+1} + \frac{1}{n+1} + \frac{1}{n+1} + \frac{1}{n+1}\right) = 1 - \frac{6}{n+1} = \frac{1}{2}$

$\frac{(n+1)-6}{n+1} = \frac{1}{2}$, $\frac{n-5}{n+1} = \frac{1}{2}$. $n = 11$

$\therefore m = 0, \ n = 11$

(iii) poisson

poisson의 PMF는 앞서 구한바와 같이 $f_K(k) = \frac{\alpha^k}{k!} e^{-\alpha}$ 입니다.

$Pr(K>5) = 1 - (f_K(0) + f_K(1) + f_K(2) + f_K(3) + f_K(4) + f_K(5))$

$$1 - \left(\frac{\alpha^0}{0!}e^{-\alpha} + \frac{\alpha^1}{1!}e^{-\alpha} + \frac{\alpha^2}{2!}e^{-\alpha} + \frac{\alpha^3}{3!}e^{-\alpha} + \frac{\alpha^4}{4!}e^{-\alpha} + \frac{\alpha^5}{5!}e^{-\alpha}\right) \le \frac{1}{2}$$

$e^{-\alpha}\left(1 + \alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{6} + \frac{\alpha^4}{24} + \frac{\alpha^5}{120}\right) = \frac{1}{2}$을 만족하는

α를 구하면 $\alpha = 5.67$

$\therefore \alpha = 5.67$

해를 구하는 과정의 추가적인 설명이 필요한 것 같아 위의 geometric random variable과 poisson random variable의 parameter를 구하는 파이썬 코드를 추가로 첨부합니다.

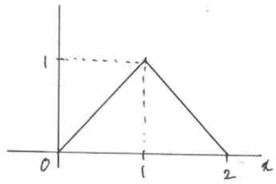
In [4]:
```python
#geometric random variable의 해를 구하기
import numpy as np
from scipy.optimize import fsolve
# 방정식을 정의합니다
def equation(p):
    return (1 - p)**6 - 1/2
# 초기 추정치를 설정합니다
initial_guess = 0.5
# fsolve를 사용하여 방정식을 풉니다
solution = fsolve(equation, initial_guess)
# 결과를 출력합니다
p_value = solution[0]
print(f"방정식을 만족하는 p 값은: {p_value}")
```

```
방정식을 만족하는 p 값은: 0.10910128185966073
```

In [2]:
```python
#poisson random variable의 해를 구하기
import numpy as np
from scipy.optimize import fsolve
# 방정식 정의
def equation(x):
    return np.exp(-x) * (1 + x + x**2/2 + x**3/6 + x**4/24 + x**5/120) - 0.5
# 초기 추정값 설정
initial_guess = 0.5
# fsolve를 사용하여 방정식 풀이
solution = fsolve(equation, initial_guess)
print(f"미지수 x의 값은: {solution[0]}")
```

```
미지수 x의 값은: 5.670161188712067
```

problems 4.41 The probability density function of a random variable X is shown below:



(a) write a set of algebraic expressions that describe the density function.

↳
$$f_X(x) = \begin{cases} x & (0 \le x < 1) \\ 2-x & (1 \le x < 2) \\ 0 & \text{otherwise} \end{cases}$$

(b) Compute the mean and variance of this random variable.

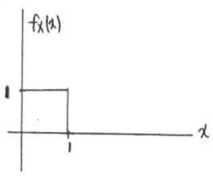
↳ $E(x) = \int_0^2 x \cdot f_X(x)\,dx = \int_0^1 x\cdot x\,dx + \int_1^2 x(2-x)\,dx = \frac{1}{3} + \frac{2}{3} = 1$

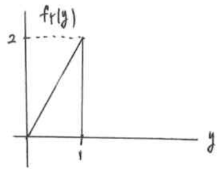
$E(x^2) = \int_0^2 x^2 \cdot f_X(x)\,dx = \int_0^1 x^2 \cdot x\,dx + \int_1^2 x^2(2-x)\,dx = \frac{1}{4} + \frac{11}{12} = \frac{7}{6}$

$V(x) = E(x^2) - (E(x))^2 = \frac{7}{6} - 1 = \frac{1}{6}$

$\therefore E(x) = 1, \quad V(x) = \frac{1}{6}$

problems 5.27 Random variables X and Y are independent and described by the density function shown below:





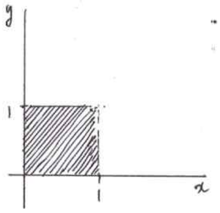
(a) Give an algebraic expression for the joint density $f_{XY}(x,y)$ (including limits) and sketch the region where it is nonzero below.

↳ $f_X(x) = \begin{cases} 1 & (0 \le x \le 1) \\ 0 & \text{otherwise} \end{cases}$ $\quad f_Y(y) = \begin{cases} 2y & (0 \le y \le 1) \\ 0 & \text{otherwise} \end{cases}$

확률변수 X와 Y가 independent 하므로 $f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$가 성립한다.

$f_{X,Y}(x,y) = 1 \cdot 2y = 2y$

$$f_{X,Y}(x,y) = \begin{cases} 2y & (0 \le x \le 1) \text{ and } (0 \le y \le 1) \\ 0 & \text{otherwise} \end{cases}$$



nonzero region은 (0,0), (0,1), (1,0), (1,1)을 꼭짓점으로 하는 정사각형

(b) What is $Pr[X \le Y]$?

↳ $0 \le X \le Y \le 1$

$$\int_0^1 \int_0^y f_{X,Y}(x,y)\,dx\,dy = \int_0^1 \int_0^y 2y\,dx\,dy = 2\int_0^1 y \int_0^y 1 \cdot dx \cdot dy = 2\int_0^1 y^2\,dy = \frac{2}{3}$$

$\therefore Pr[X \le Y] = \frac{2}{3}$

(c) What is the variance of Y. ?

↳ $E(Y) = \int_0^1 y \cdot f_Y(y)\,dy = \int_0^1 2y^2\,dy = \frac{2}{3}$

$E(Y^2) = \int_0^1 y^2 \cdot f_Y(y)\,dy = \int_0^1 2y^3\,dy = \frac{1}{2}$

$V(Y) = E(Y^2) - (E(Y))^2 = \frac{1}{2} - (\frac{2}{3})^2 = \frac{1}{18}$ $\therefore V(Y) = \frac{1}{18}$

(d) What is the correlation coefficient ρ_{XY}?

↳ correlation coefficient $\rho_{XY} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$ 로 정의됩니다.

$COV(X,Y) = E(XY) - m_X m_Y$

X와 Y가 independent 하기 때문에 $COV(X,Y) = E(X)E(Y) - m_X m_Y$ 가 성립하고 이는 0입니다.

즉, $\rho_{XY} = \frac{0}{\sigma_X \sigma_Y} = 0$ $\therefore$ correlation coefficient $\rho_{XY} = 0$.

problems 6.11 In a communication receiver, IID signal samples (random variables) X_i collected at a 25-element antenna are linearly combined with equal weights, i.e., $Y = \sum_{i=1}^{25} X_i$. The signal sample X_i collected at the ith antenna element is uniformly distributed over the interval [-1, 1]. Consider that the central Limit Theorem applies. Calculate the probability $Pr[Y \ge 0]$.

↳ 균등 확률변수는 구간 (a, b)에서 평균이 $\frac{a+b}{2}$, 분산이 $\frac{(b-a)^2}{12}$ 입니다.

각각의 X_i는 [-1, 1] 구간에서 균등하게 분포하므로 X_i의 평균 $\mu_{X_i} = \frac{a+b}{2} = \frac{-1+1}{2} = 0$ 이고

X_i의 분산 $\sigma_{X_i}^2 = \frac{(b-a)^2}{12} = \frac{(1-(-1))^2}{12} = \frac{1}{3}$입니다.

$Y = \sum_{i=1}^{25} X_i$ 이고 중심극한 정리에 의해, 많은 수의 IID 확률변수들의 합이 $n\mu$와 $n\sigma^2$을 갖는 정규분포로 근사할 수 있습니다.

$\mu_Y = 25\mu$ (μ는 X_i들의 평균) $= 25 \times 0 = 0$.

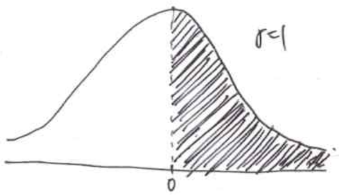
$\sigma_Y^2 = 25\sigma^2$ (σ^2은 X_i들의 분산) $= 25 \times \frac{1}{3} = \frac{25}{3}$

Y는 $N(0, \frac{25}{3})$인 정규분포를 따릅니다.

이를 표준 정규분포로 정규화 해주면

$$Z = \frac{Y-\mu_Y}{\sigma_Y} = \frac{Y-0}{\frac{5}{\sqrt{3}}} = \frac{Y\sqrt{3}}{5}$$

$$Pr[Y \geq 0] = Pr\left[\frac{Y\sqrt{3}}{5} \geq 0\right] = Pr[Z \geq 0]$$



표준 정규 분포 Z는 $N(0,1)$을 따르고 PDF는 오른쪽과 같습니다.

구하고자 하는 확률은 빗금친 부분이고 이는 전체 확률의 절반, $\frac{1}{2}$입니다.

$$\therefore Pr[Y \geq 0] = \frac{1}{2}$$

problems 7.1 The joint PDF of two random variables X_1 and X_2 is given by

$$f_{X_1X_2}(x_1, x_2) = \begin{cases} 1/12 & -1 \leq x_1 \leq 1,\ -3 \leq x_2 \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the first moment of the random variables in vector notation.

↳ $E(X_1) = \int_{-1}^{1}\int_{-3}^{3} x_1 \cdot f_{X_1X_2}(x_1,x_2)\, dx_2\, dx_1 = \int_{-1}^{1}\int_{-3}^{3} \frac{x_1}{12}\, dx_2\, dx_1 = \int_{-1}^{1} \frac{x_1}{2}\, dx_1 = 0.$

$E(X_2) = \int_{-1}^{1}\int_{-3}^{3} x_2 \cdot f_{X_1X_2}(x_1,x_2)\, dx_2\, dx_1 = \int_{-1}^{1}\int_{-3}^{3} \frac{x_2}{12}\, dx_2\, dx_1 = \int_{-1}^{1} 0\, dx_1 = 0.$

$$\therefore \mathbf{m_X} = \mathbf{E}(\mathbf{X}) = \begin{pmatrix} E(X_1) \\ E(X_2) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(b) Determine the correlation matrix.

↳ correlation matrix 는 $\mathbf{R_X} = E\{\mathbf{X}\mathbf{X}^T\} = \begin{bmatrix} E(X_1^2) & E(X_1X_2) & \cdots & E(X_1X_k) \\ E(X_2X_1) & E(X_2^2) & \cdots & E(X_2X_k) \\ \vdots & & & \vdots \\ E(X_kX_1) & E(X_kX_2) & \cdots & E(X_k^2) \end{bmatrix}$ 입니다.

$E(X_1^2) = \int_{-1}^{1}\int_{-3}^{3} x_1^2 \cdot f_{X_1X_2}(x_1,x_2)\, dx_2\, dx_1 = \frac{1}{12}\int_{-1}^{1} x_1^2 \int_{-3}^{3} 1\, dx_2\, dx_1 = \frac{1}{2}\int_{-1}^{1} x_1^2\, dx_1 = \frac{1}{3}$

$E(X_2^2) = \int_{-1}^{1}\int_{-3}^{3} x_2^2\, f_{X_1X_2}(x_1,x_2)\, dx_2\, dx_1 = \frac{1}{12}\int_{-1}^{1}\int_{-3}^{3} x_2^2\, dx_2\, dx_1 = \frac{3}{2}\int_{-1}^{1} 1\, dx_1 = 3$

$E(X_1X_2) = \int_{-1}^{1}\int_{-3}^{3} x_1x_2 \cdot f_{X_1X_2}(x_1,x_2)\, dx_2\, dx_1 = \int_{-1}^{1} x_1 \int_{-3}^{3} \frac{x_2}{12}\, dx_2\, dx_1 = \frac{1}{12}\int_{-1}^{1} x_1 \cdot 0\, dx_1 = 0.$

$$\therefore \mathbf{R_X} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 3 \end{bmatrix}$$

(c) Determine the covariance matrix.

↳ $C_x = R_x - m_x m_x^T$

$$R_x = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 3 \end{bmatrix} \qquad m_x m_x^T = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_x = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 3 \end{bmatrix} \qquad \therefore C_x = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 3 \end{bmatrix}$$

problems 7.4 Using (7.19) through (7.21), show that the estimate for the covariance matrix can be put in the form (7.22)

↳ (7.19) $M_n = \frac{1}{n} \sum_{i=1}^{n} x_i$ (7.20) $R_n = \frac{1}{n} \sum_{i=1}^{n} x_i x_i^T$ (7.21) $C_n = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - M_n)(x_i - M_n)^T$

(7.22) $C_n = \frac{n}{n-1} (R_n - M_n M_n^T)$

지금부터 (7-19), (7-20), (7-21)의 식을 이용하여 (7-22)의 식을 유도해보겠습니다.

우선 (7-21) 식을 정리하겠습니다. $C_n = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - M_n)(x_i - M_n)^T = \frac{1}{n-1} \sum_{i=1}^{n} (x_i x_i^T - x_i M_n^T - M_n x_i^T + M_n M_n^T)$

$$= \frac{1}{n-1} \left(\sum_{i=1}^{n} x_i x_i^T - \sum_{i=1}^{n} x_i M_n^T - M_n \sum_{i=1}^{n} x_i^T + \sum_{i=1}^{n} M_n M_n^T \right)$$

$\sum_{i=1}^{n} x_i x_i^T$는 (7-20)을 이용하여 $n \cdot R_n$으로 표현할수 있고 $\sum_{i=1}^{n} x_i M_n^T$는 (7-19)를 이용하여 $\left(\sum_{i=1}^{n} x_i \right) M_n^T = n \cdot \left(\frac{1}{n} \sum_{i=1}^{n} x_i \right) M_n^T$ $= n M_n M_n^T$, $M_n \sum_{i=1}^{n} x_i^T$도 마찬가지로 $M_n \sum_{i=1}^{n} x_i^T = M_n \cdot n \left(\frac{1}{n} \sum_{i=1}^{n} x_i^T \right) = n \cdot M_n M_n^T$, 마지막으로 $\sum_{i=1}^{n} M_n M_n^T = n M_n M_n^T$로 나타낼수 있습니다. 이는 위의 식에 적용하면 아래와 같습니다.

$$\frac{1}{n-1} \left(\sum_{i=1}^{n} x_i x_i^T - \sum_{i=1}^{n} x_i M_n^T - M_n \sum_{i=1}^{n} x_i^T + \sum_{i=1}^{n} M_n M_n^T \right) = \frac{1}{n-1} \left(n \cdot R_n - n M_n M_n^T - n M_n M_n^T + n M_n M_n^T \right) = \frac{1}{n-1} \left(n R_n - n M_n M_n^T \right)$$

$$= \frac{n}{n-1} \left(R_n - M_n M_n^T \right) \quad (7.22)$$

∴ (7-19), (7-20), (7-21)을 통해 (7-22)는 유도했습니다.

$\boxed{\text{problems 1.10}}$ A pair of random variables X_1 and X_2 is defined by $X_1 = 3U - 4V$, $X_2 = 2U + V$

where U and V are independent random variables with mean 0 and variance 1.

(a) Find a matrix A such that $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = A \begin{bmatrix} U \\ V \end{bmatrix}$

↳ $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 3U - 4V \\ 2U + V \end{bmatrix} = A \begin{bmatrix} U \\ V \end{bmatrix}$, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$라 하자.

$$\begin{bmatrix} 3U - 4V \\ 2U + V \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} a_{11}U + a_{12}V \\ a_{21}U + a_{22}V \end{bmatrix}$$

$a_{11} = 3$, $a_{12} = -4$
$a_{21} = 2$, $a_{22} = 1$

$$\therefore A = \begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix}$$

(b) What are R_X and C_X of the random vector X?

↳ $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$, $K = \begin{bmatrix} U \\ V \end{bmatrix}$, $X = AK$

$E(U) = 0$ $\quad$ $E(V) = 0$ $\quad$ $E(X_1) = E(3U - 4V) = 3E(U) - 4E(V) = 0$.

$V(U) = 1$ $\quad$ $V(V) = 1$ $\quad$ $E(X_2) = E(2U + V) = 2E(U) + E(V) = 0$.

$V(U) = E(U^2) - (E(U))^2$ $\quad$ $V(V) = E(V^2) - (E(V))^2$

$E(U^2) = 1$ $\quad$ $E(V^2) = 1$

$$R_X = E(XX^T) = E((AK)(AK)^T) = E(AK \cdot K^T A^T) = AE(KK^T)A^T = A \cdot R_K A^T$$

$$R_K = \begin{bmatrix} E(U^2) & E(UV) \\ E(VU) & E(V^2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_X = A R_K A^T = \begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 25 & 2 \\ 2 & 5 \end{bmatrix}$$

$m_X = \begin{bmatrix} E(X_1) \\ E(X_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $m_X^T = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$$C_X = R_X - m_X m_X^T = \begin{bmatrix} 25 & 2 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 25 & 2 \\ 2 & 5 \end{bmatrix}$$

$$\therefore R_X = \begin{bmatrix} 25 & 2 \\ 2 & 5 \end{bmatrix}, \quad C_X = \begin{bmatrix} 25 & 2 \\ 2 & 5 \end{bmatrix}$$

(c) What are the means and variance of X_1 and X_2 ?

↳ $E(X_1) = E(3U-4V) = 3E(U) - 4E(V) = 0.$

$V(X_1) = V(3U-4V) = 3^2 V(U) + 4^2 V(V) = 25.$

$E(X_2) = E(2U+V) = 2E(U) + E(V) = 0.$

$V(X_2) = V(2U+V) = 2^2 V(U) + 1^2 \cdot V(V) = 5.$

$\therefore E(X_1)=0, \quad E(X_2)=0, \quad V(X_1)=25, \quad V(X_2)=5.$

(d) What is the correlation $E(X_1 X_2)$?

↳ $E(X_1 X_2) = E\left((3U-4V)(2U+V)\right) = E\left(6U^2 - 5UV - 4V^2\right) = 6E(U^2) - 5E(UV) - 4E(V^2)$

(b)에서 이미 $E(U^2)=1$, $E(V^2)=1$을 구했고 U, V는 독립이기 때문에 $E(UV) = E(U) \cdot E(V)$입니다.

$6E(U^2) - 5E(UV) - 4E(V^2) = 6 \cdot 1 - 5 \cdot E(U) \cdot E(V) - 4E(V^2) = 6 - 0 - 4 = 2$

$\therefore E(X_1 X_2) = 2.$